解上三角

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$\cdots \qquad \cdots$$

$$a_{n-1,n-1}x_{n-1} + a_{n-1,n}x_n = b_{n-1}$$

$$a_{nn}x_n = b_n$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ & a_{22} & \cdots & a_{2n} \\ & & \ddots & \vdots \\ & & & a_{nn} \end{bmatrix}$$

要求：对角线上元素不为0. $a_{ii} \neq 0 \quad (i = 1, 2, \cdots, n)$

解法：

① $x_n = \dfrac{b_n}{a_{nn}}$

② $x_k = \dfrac{b_k - a_{k,k+1}x_{k+1} - \cdots - a_{kn}x_n}{a_{kk}} \quad (k = n-1, n-2, \cdots, 1)$

算法：

Input : A , b

Output : x

① $x_n = \dfrac{b_n}{a_{nn}}$

② for $k = [n-1, n-2, \cdots, 1]$

$t = a_{k,k+1:n} \cdot x_{k+1:n}$ → $[a_{k,k+1} \; a_{k,k+2} \; \cdots \; a_{k,n}] \begin{bmatrix} x_{k+1} \\ x_{k+2} \\ \vdots \\ x_n \end{bmatrix}$

$x_k = \dfrac{b_k - t}{a_{kk}}$

end

get : $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

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