Morning

Properties of CFL

1. Pumping lemma for CFL

2. Closure properties

Pumping Lemma for CFL

Let L be a CFL . Then there exists some positive integer n such that any $w \in L$ with $|w| \geq n$ can be de-composed as

$$w = uvxyz$$

with

$$|vxy| \leq n$$

and

$$|vy| \geq 1$$

such that

$$uv^ixy^iz \in L$$

for all i=0,1,2,......

Proof

L is a CFL $\Rightarrow$ There is a CFG $G=(V,T,R,S)$ generating L.

V is finite $\Rightarrow$ $m=|V|$

$|\alpha|$ is finite for all $A\rightarrow\alpha$ $\Rightarrow$ $k=\max\{|\alpha|$ for all $A\rightarrow\alpha\}$

Let $n=k^m$

For any $w\in L$ with $|w|\geq n$, there must be some variable A that appears at least two times in the parse tree.

That is : $S \overset{*}{\Rightarrow} uAz \overset{*}{\Rightarrow} uvAyz \overset{*}{\Rightarrow} w$

$$S \stackrel{*}{\Rightarrow} w_1 S_1 v_1 \qquad |w_1 S_1 v_1| \leq k$$

$$\stackrel{*}{\Rightarrow} w_1 w_2 S_2 v_2 v_1 \qquad |w_2 S_2 v_2| \leq k$$

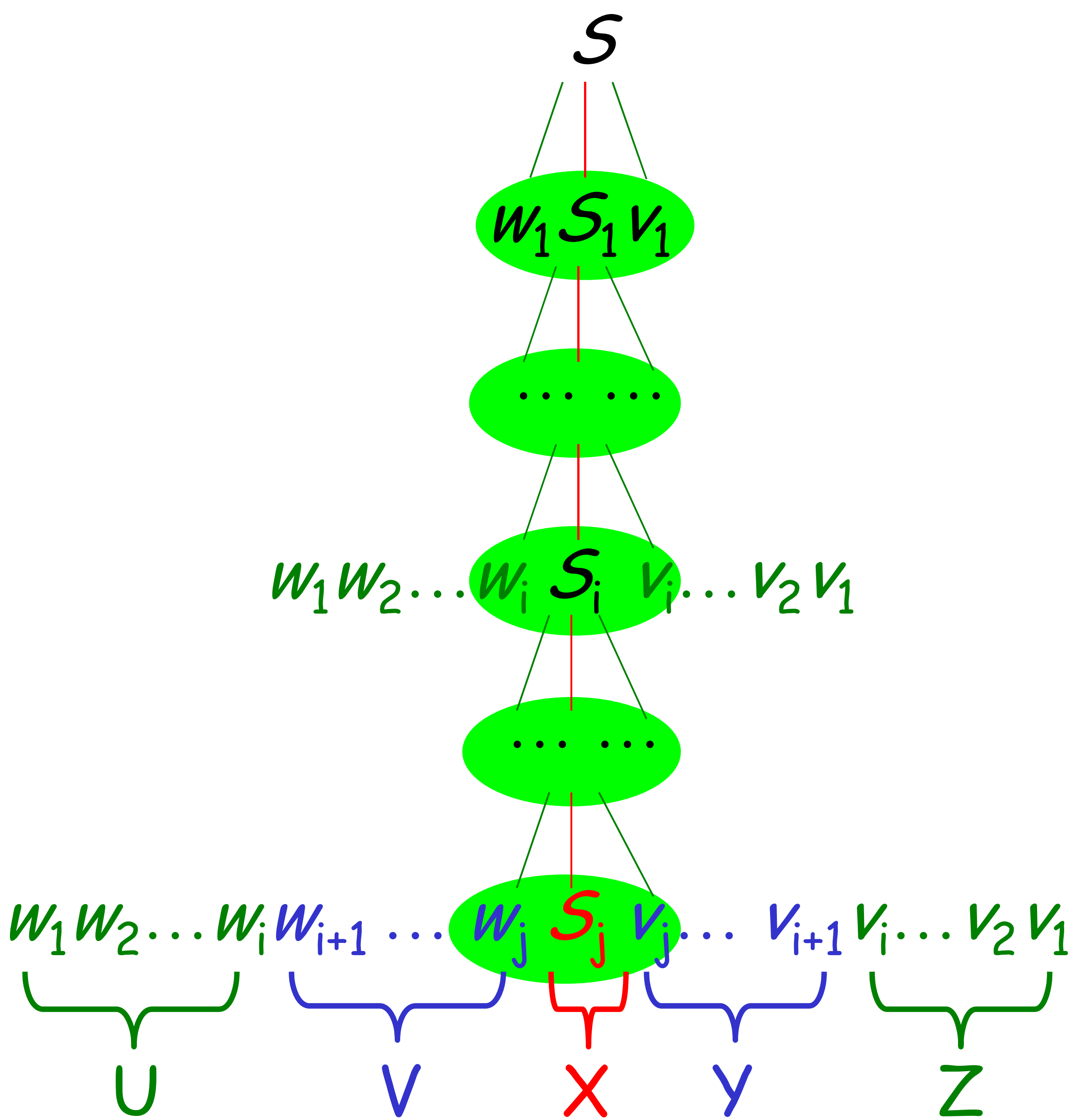
$$\stackrel{*}{\Rightarrow} w_1 w_2 w_3 S_3 v_3 v_2 v_1 \qquad |w_3 S_3 v_3| \leq k$$

$$\stackrel{*}{\Rightarrow} \ldots \ \ldots$$

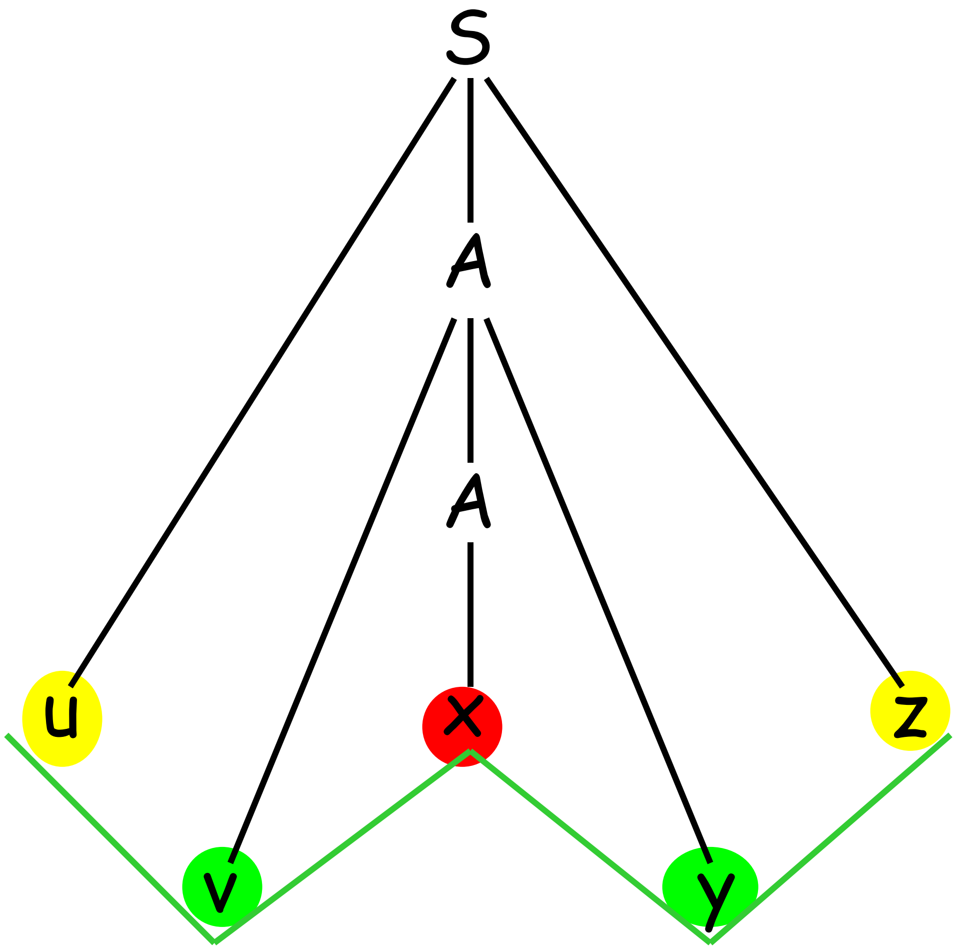
$$\stackrel{*}{\Rightarrow} w_1 w_2 \ldots w_m S_m v_m \ldots v_2 v_1 \qquad |w_m S_m v_m| \leq k$$

where

$$w_1, w_2, \ldots, w_m,\ v_1, v_2, \ldots, v_m \in T^*,\ S_1, S_2, \ldots, S_m \in V$$



S
w1 S1 v1
... ...
w1 w2 ... wi Si vi ... v2 v1
... ...
w1 w2 ... wi wi+1 ... wj Sj vj ... vi+1 vi ... v2 v1
U
V
X
Y
Z



$$S \stackrel{*}{\Rightarrow} uAz \stackrel{*}{\Rightarrow} uvAyz \stackrel{*}{\Rightarrow} w$$

Example 10.1 Show that the language

$$L = \{ ww \mid w \in \{0,1\}^* \}$$

is not context-free.

Example 10.2 Show that the language

$$L = \{ 0^n 1^m \mid n = m^2 \}$$

is not context-free.

Closure Properties of CFL

- Union

If L_1 and L_2 are CFL , then so is $L_1 \cup L_2$.

Proof

Let $G(L_1)=(V_1,T_1,R_1,S_1)$, $G(L_2)=(V_2,T_2,R_2,S_2)$

Then $G(L_1 \cup L_2)=(V_1 \cup V_2, T_1 \cup T_2, R, S)$

$R = \{S \to S_1 \mid S_2\} \cup R_1 \cup R_2$

Closure Properties of CFL

- Concatenation

  If L_1 and L_2 are CFL , then so is $L_1 L_2$.

Proof

Let $G(L_1)=(V_1,T_1,R_1,S_1)$, $G(L_2)=(V_2,T_2,R_2,S_2)$

Then $G(L_1\ L_2\ )=(V_1\cup V_2,T_1\cup T_2,\ R,S)$

$R= \{S\rightarrow S_1\ S_2\} \cup R_1\cup R_2$

Closure Properties of CFL

- Star

  If L is a CFL , then so is L^*.

Proof

Let $G(L)=(V,T,R,S)$

Then $G(L^*)=(V,T,\{S \to SS|\varepsilon\} \cup R,S)$

Closure Properties of CFL

- Reversal

  If L is a CFL , then so is L^R .

Proof

Let $G(L)=(V,T,R,S)$

Then $G(L^R)=(V,T, \{A\to\alpha^R \mid A\to\alpha\in R\},S)$

Closure Properties of CFL

- Intersection

CFL is not closed under intersection.

Proof

$$L_1 = \{ a^n b^n c^m \mid n \geq 0, m \geq 0 \}$$

$$L_2 = \{ a^n b^m c^m \mid n \geq 0, m \geq 0 \}$$

$$L_1 \cap L_2 = \{ a^n b^n c^n \mid n \geq 0 \}$$

- Intersection

If L_1 is a CFL and L_2 is a RL , then $L_1 \cap L_2$ is CFL.

Proof

$$P(L_1) = (Q_1, \Sigma_1, \Gamma, \delta_1, q_1, z_0, F_1)$$

$$A(L_2) = (Q_2, \Sigma_2, \delta_2, q_2, F_2)$$

$$P(L_1 \cap L_2) = (Q_1 \times Q_2, \Sigma_1 \times \Sigma_2, \Gamma, \delta, (q_1, q_2), z_0, F_1 \times F_2)$$

$$\delta((q,p),a,X)=((r,s), \alpha)$$

where $\delta_1(q,a,X)=(r,\alpha)$, $\delta_2(p,a)=s$

Example 10.3 Show that the language

$$L = \{ 0^n 1^n \mid n \geq 0,\ n \neq 100 \}$$

is context-free.

Example 10.4 Show that the language

$$L = \{ w \mid w \in \{a,b,c\}^*,\ n_a(w) = n_b(w) = n_c(w) \}$$

is not context-free.

Good good study

day day up !