Formal Definition of ε-NFA

An NFA with ε transition is a five-tuple ,

such as $M = (Q, \Sigma, \delta, q_0, F)$

Where Q is a finite set of *states* ,

Σ is a finite set of *input symbols s* ,

q_0 is *start state* ,

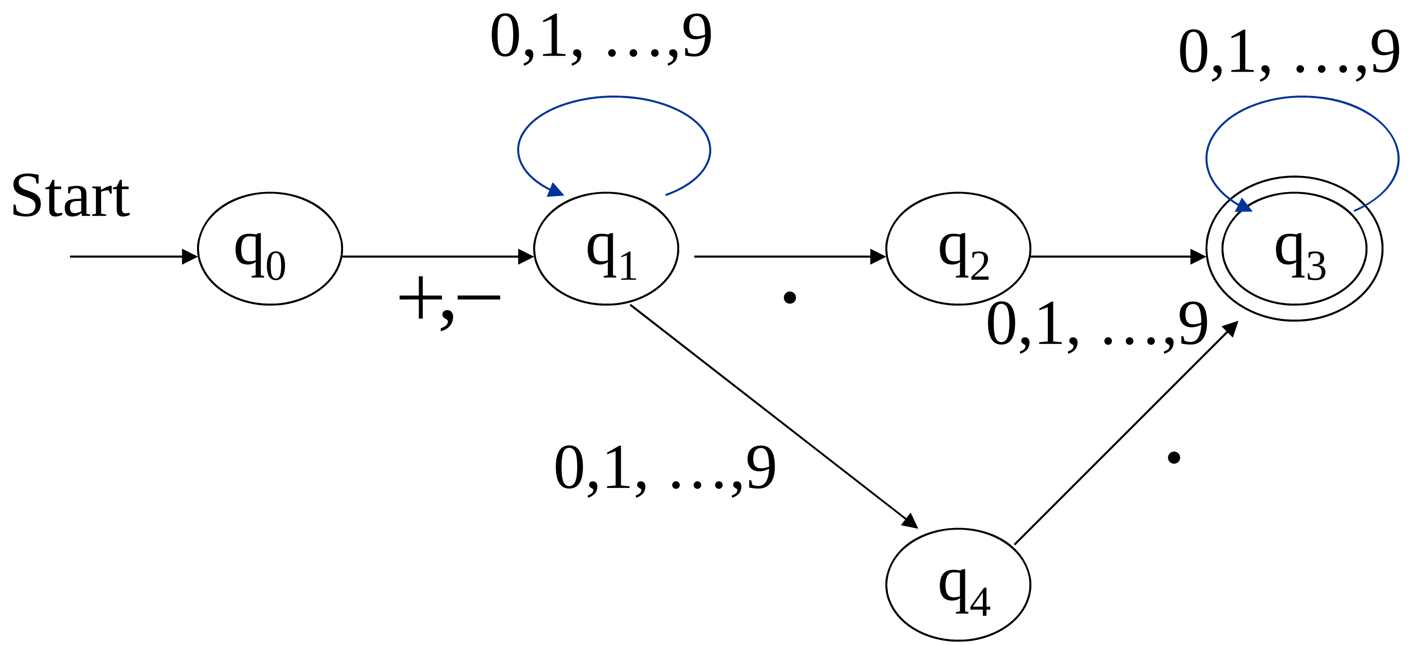
F is *a set of final state* ,

δ is *transition function* , which is a mapping

from $Q \times (\Sigma \cup \{\varepsilon\})$ to 2^Q .

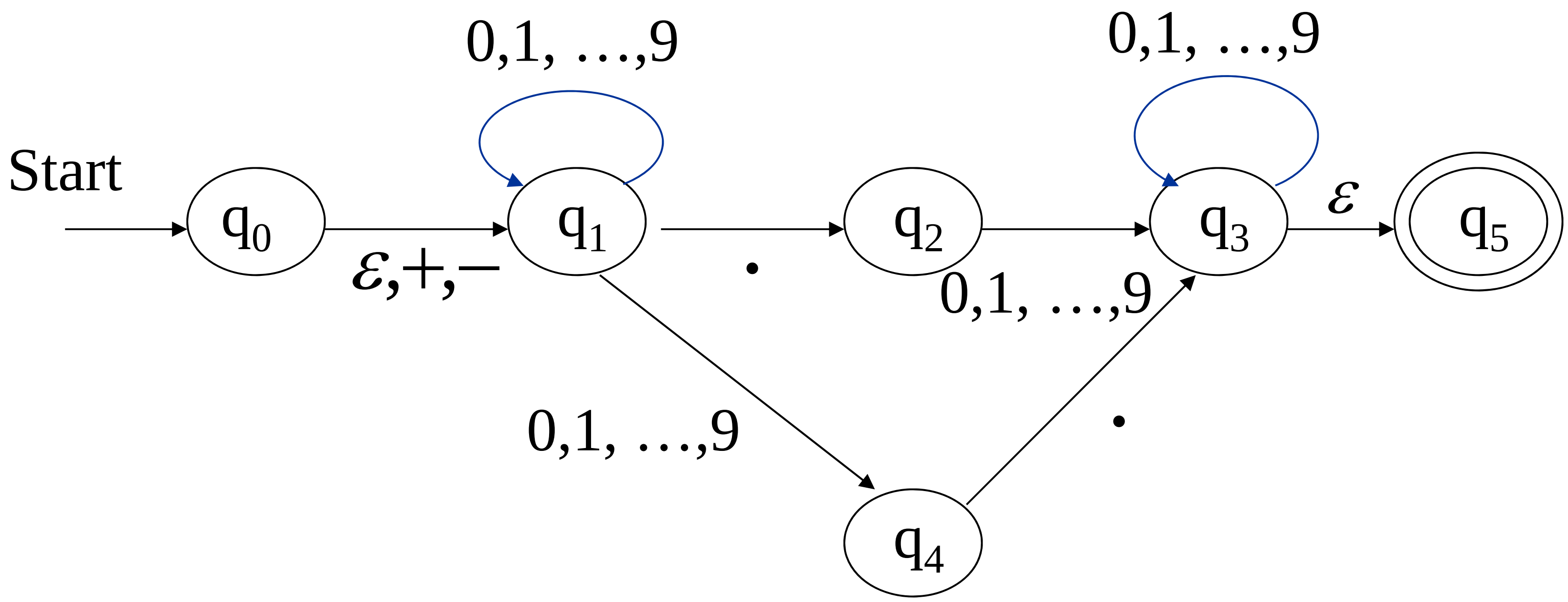
Example4. 1

Describe the language accepted by this NFA :



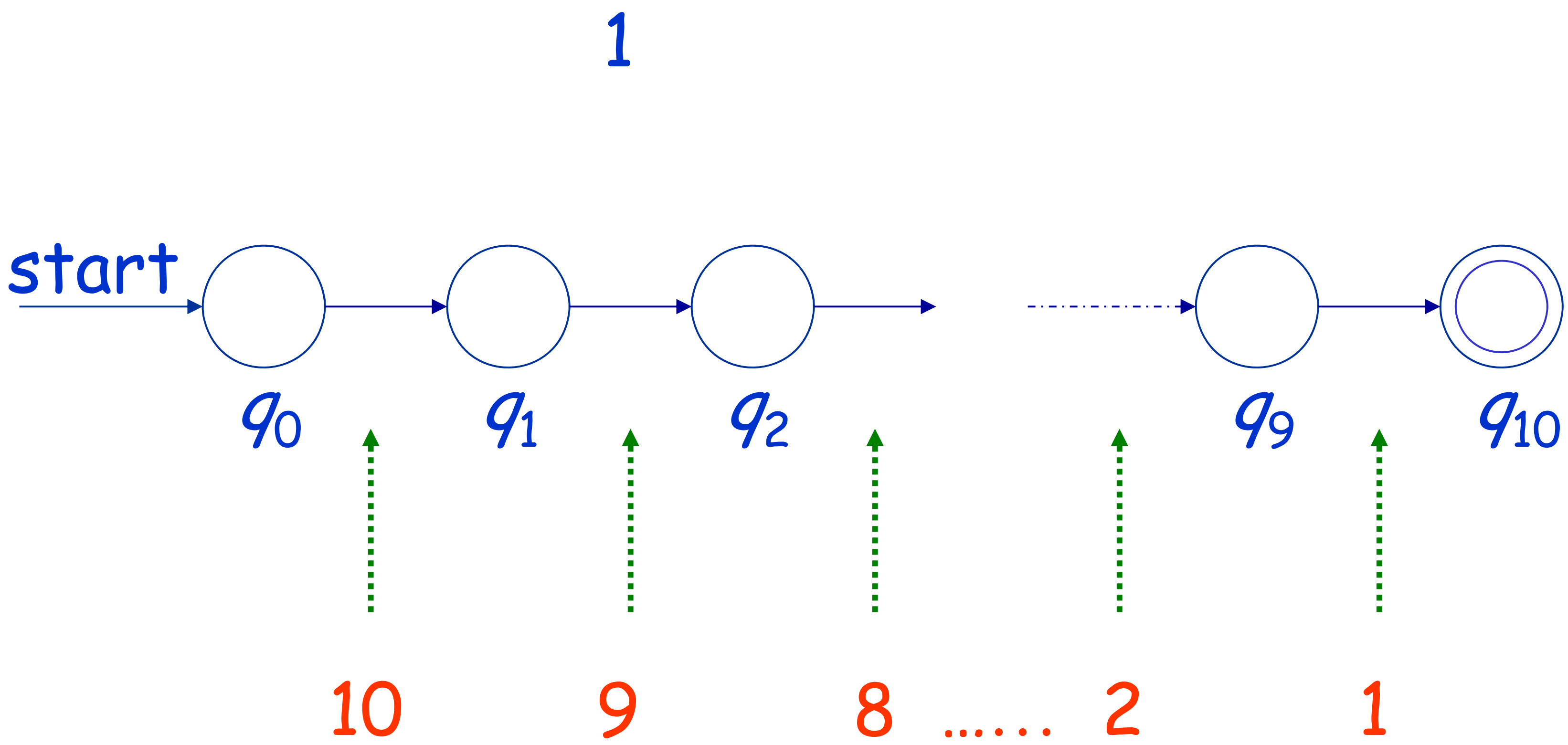
What about the NFA just accept decimal numbers ?

An ε- NFA for decimal numbers

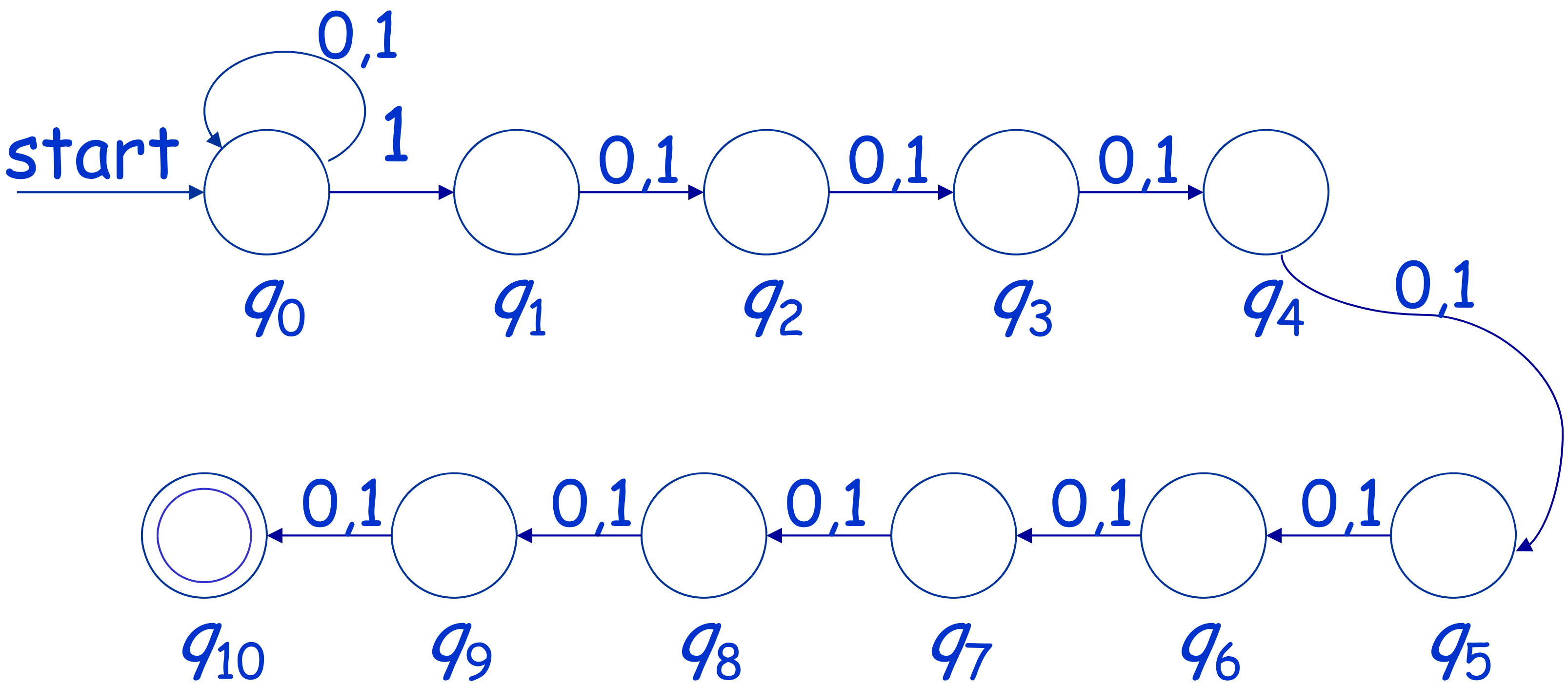


Example 4.2 Design an ε-NFA for following language

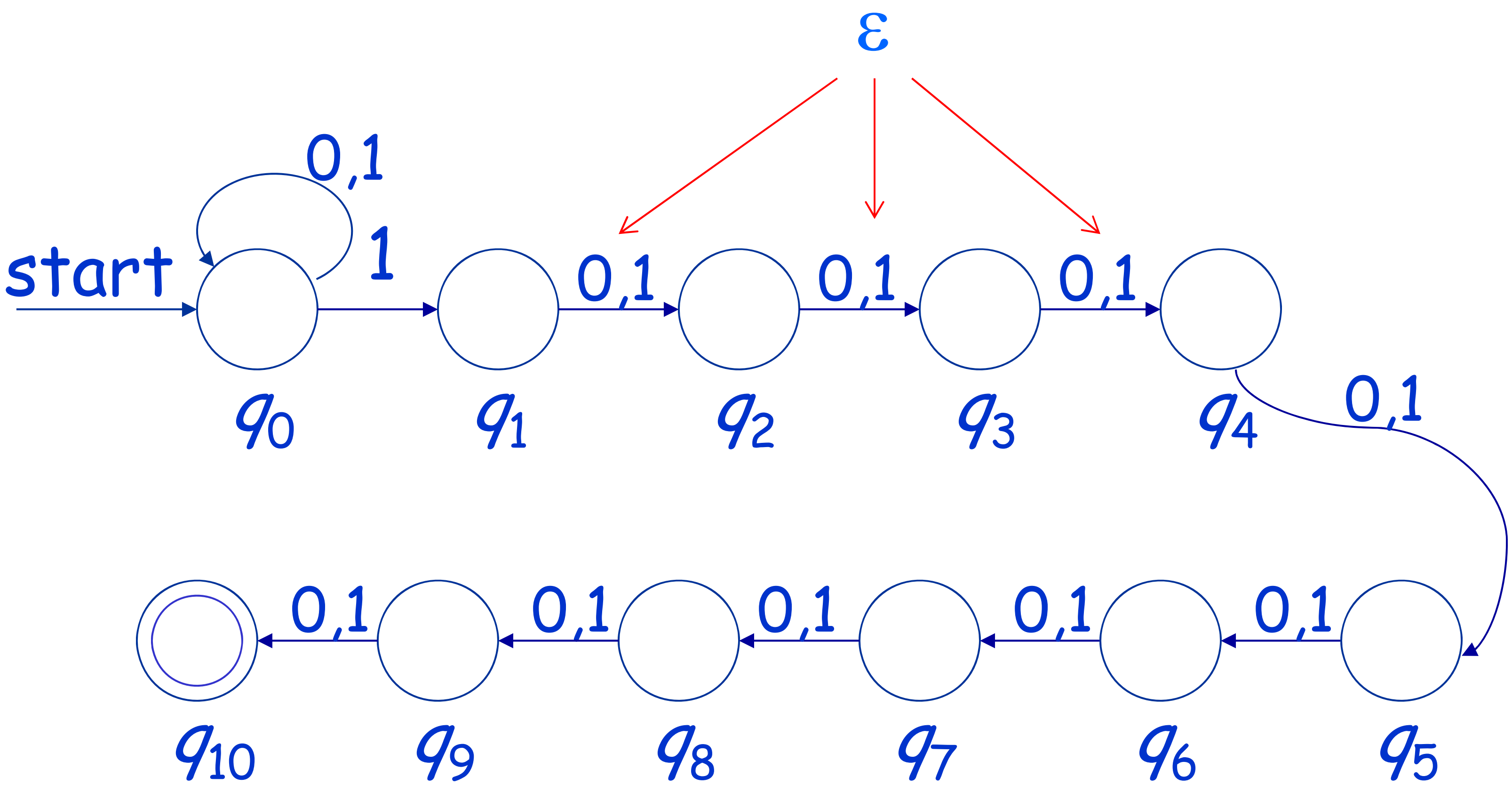
The set of strings of 0's and 1's such that at least one of the last ten positions is a 1 .



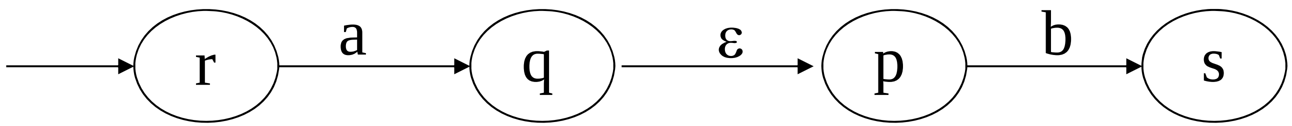
How about this NFA



How about this ε - NFA



ε- transition

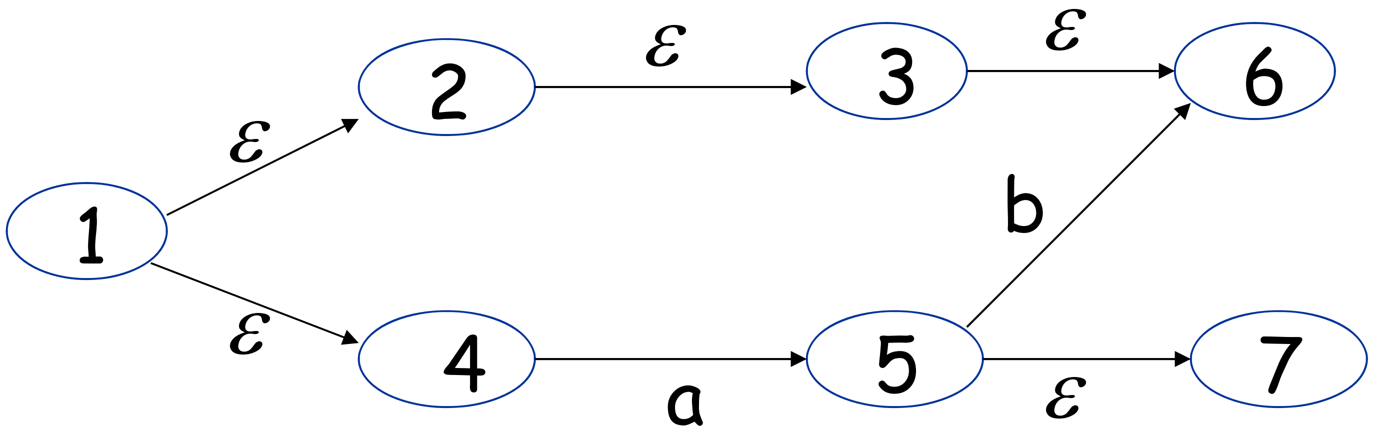


$\delta(r, a) = ?$ $\delta(q, b) = ?$

ε- closure

BASIS : State q is in ECLOSE(q)

INDUCTION : If state p is in ECLOSE(q),and there is a transition from state p to state r labeled ε ,then r is in ECLOSE(q).



Extending transition to strings

BASIS : $\hat{\delta}(q,\varepsilon) = ECLOSE(q).$

INDUCTION :

Surpose $w = xa\,,\ \hat{\delta}(q,x) = \{p_1, p_2, \cdots, p_k\}$

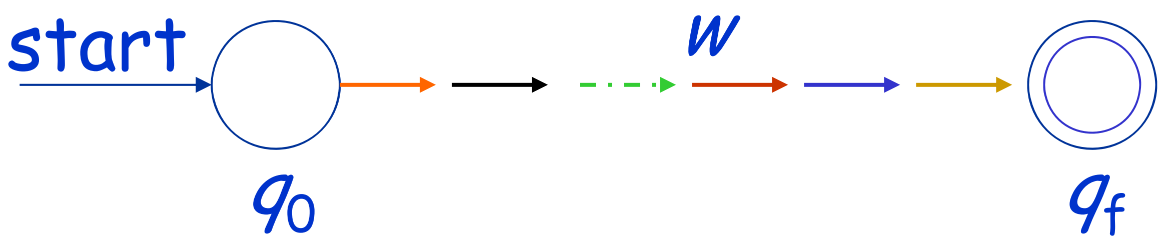
Let $\bigcup_{i=1}^{k} \delta(p_i, a) = \{r_1, r_2, \cdots, r_m\}$

Then $\hat{\delta}(q,w) = \bigcup_{i=1}^{m} Eclose(r_i)$

The language of ε-NFA

Definition *The language* of an ε-NFA *A* is denoted *L*(*A*), and defined by

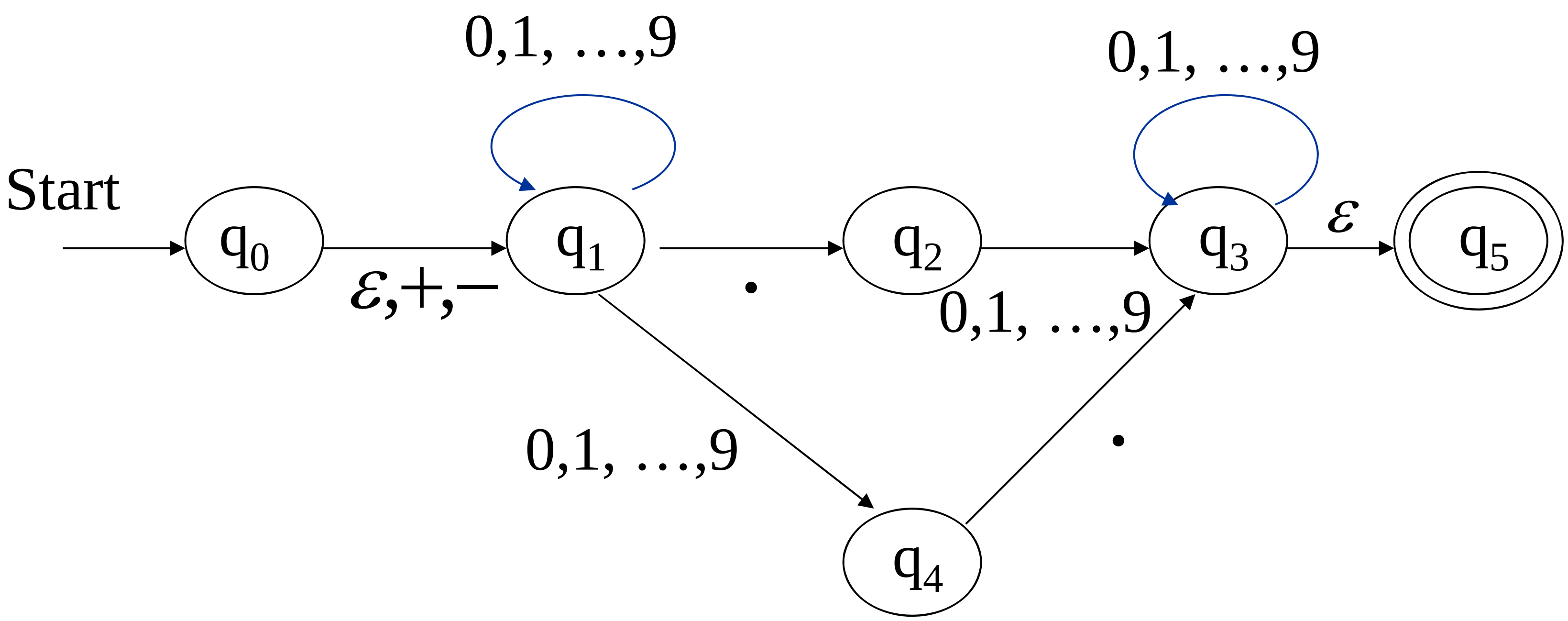
$$L(A) = \{w \mid \hat{\delta}(q_0, w) \cap F \neq \phi\}$$



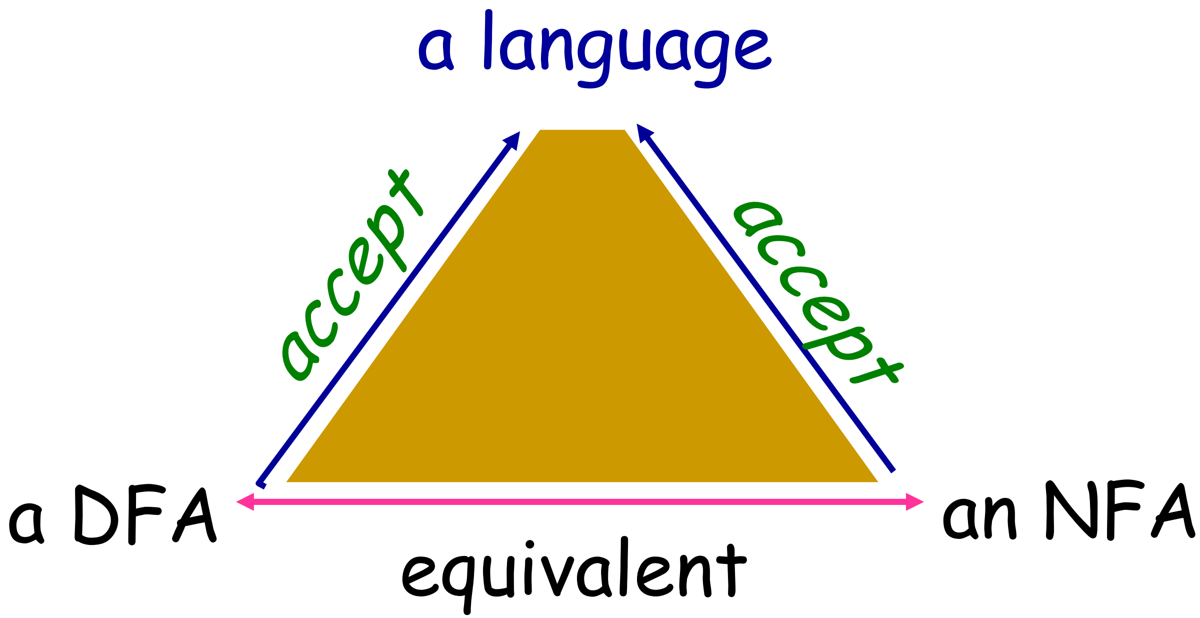
There is at least a path, labeled with w, from start state to final state.

Example 4.3

Compute : $\hat{\delta}(q_0, 5.6)$



Equivalence of DFA and NFA



If a DFA and an NFA accepts the same language , then we say that they are equivalent.

Equivalence : NFA $\Rightarrow$ DFA

Given an NFA : $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$

Construct a DFA : $A = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$

Such that :

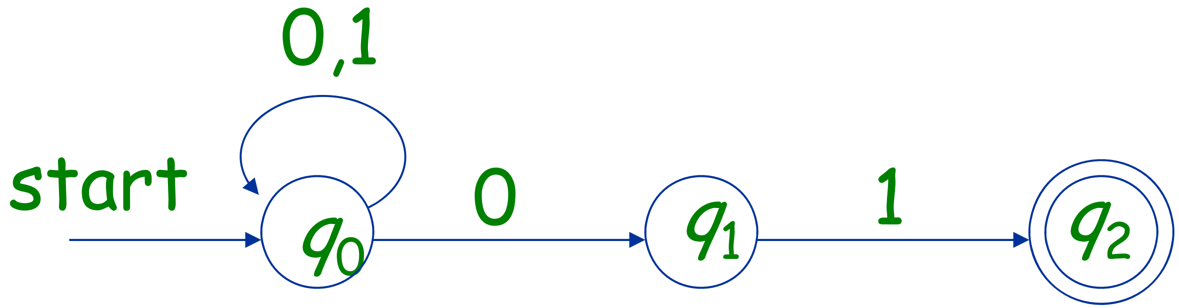
$$Q_D = 2^{Q_N}$$

$$\delta_D(S, a) = \bigcup_{p \ in \ S} \delta_N(p, a)$$

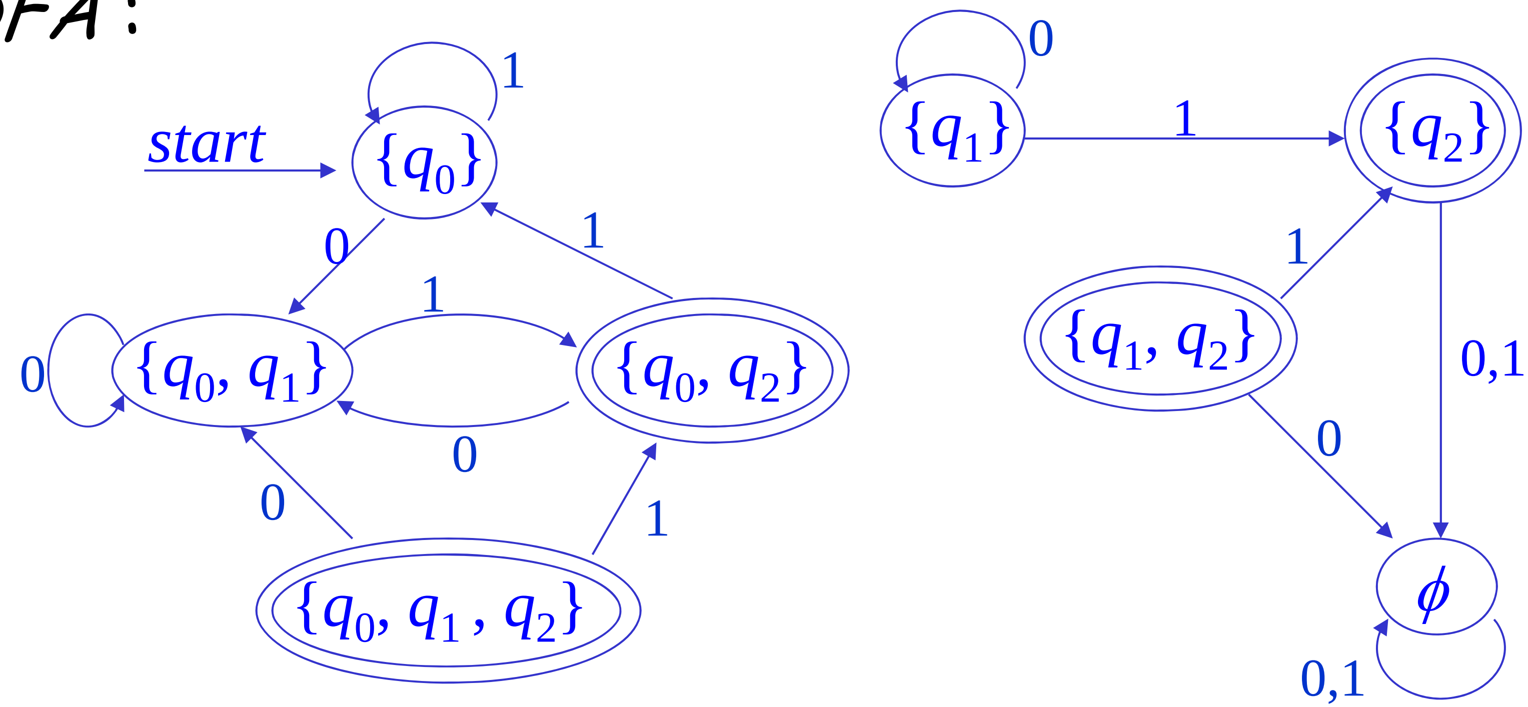
$$F_D = \{ S \mid S \subseteq Q_N \ and \ S \cap F_N \neq \phi \}$$

Example 4.4

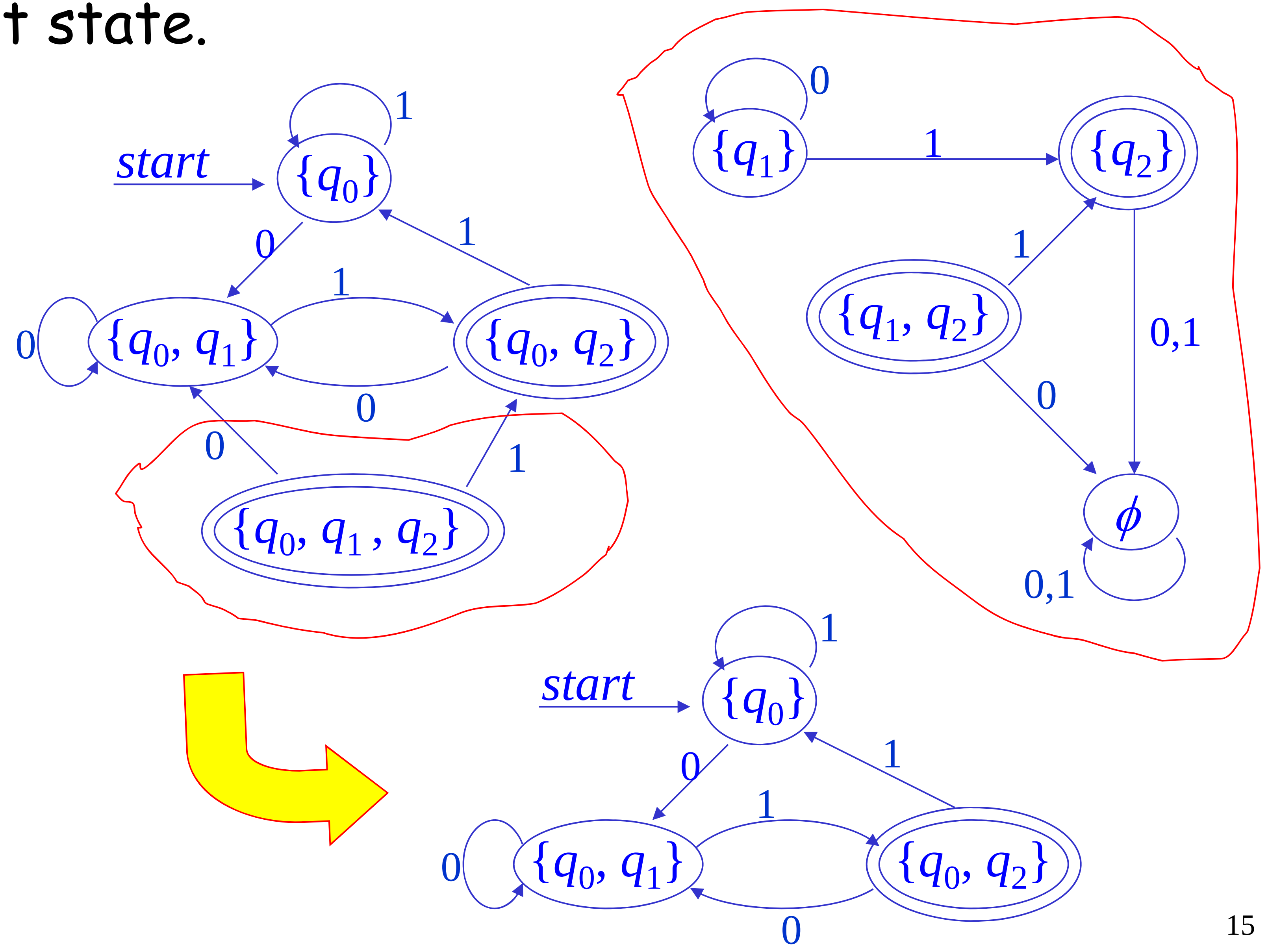
L_{x01}={x01 | x is any strings of 0's and 1's}

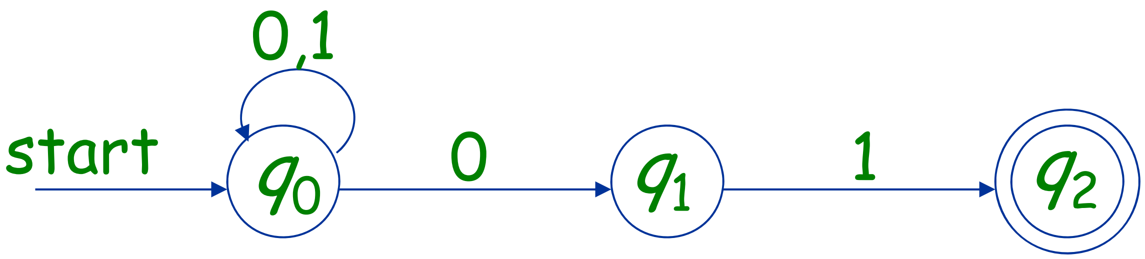


DFA :

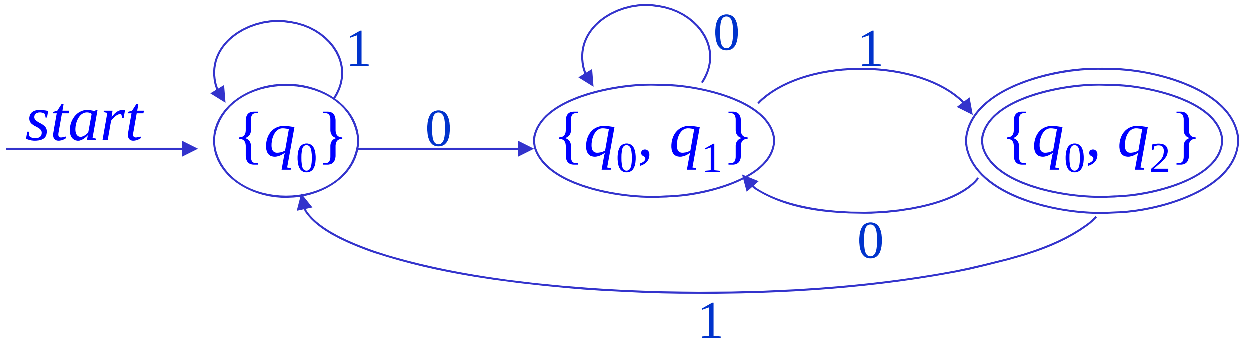


Eliminate the states which can't be reached from start state.



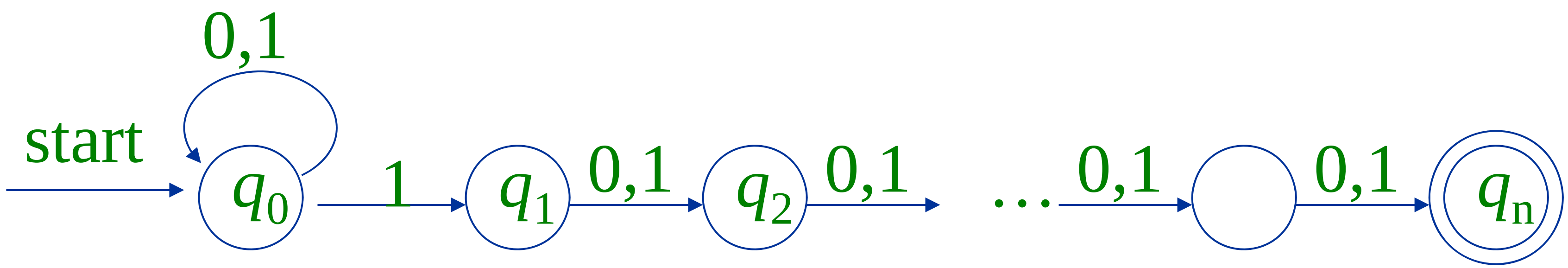


"Lazy evaluation" :



Bad case

$L = \{w \mid w$ consists of 0's and 1's , and the tenth symbol from the right end is 1 $\}$



Equivalence : DFA $\Rightarrow$ NFA

Given a DFA : $A = (\mathrm{Q}_{\mathrm{D}}, \Sigma, \delta_D, q_0, F_D)$

Construct an NFA : $N = (\mathrm{Q}_{\mathrm{N}}, \Sigma, \delta_N, q_0, F_N)$

Such that :

$$Q_{\mathrm{N}} = Q_D$$

$$\delta_N(q, a) = \{\delta_D(q, a)\}$$

$$F_N = F_D$$

Good good study

day day up !