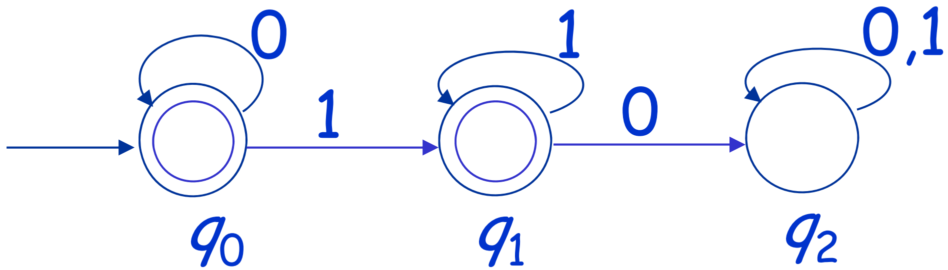
Pushdown Automata(PDA)

1. Definition
2. Construction
3. Configuration
4. Two types of accepting language
5. Deterministic PDA

The limit of FA in recognizing languages

$L=\{ 0^n 1^n \mid n \geq 0 \}$ $M=\{0^n 1^m \mid n \geq 0, m \geq 0 \}$



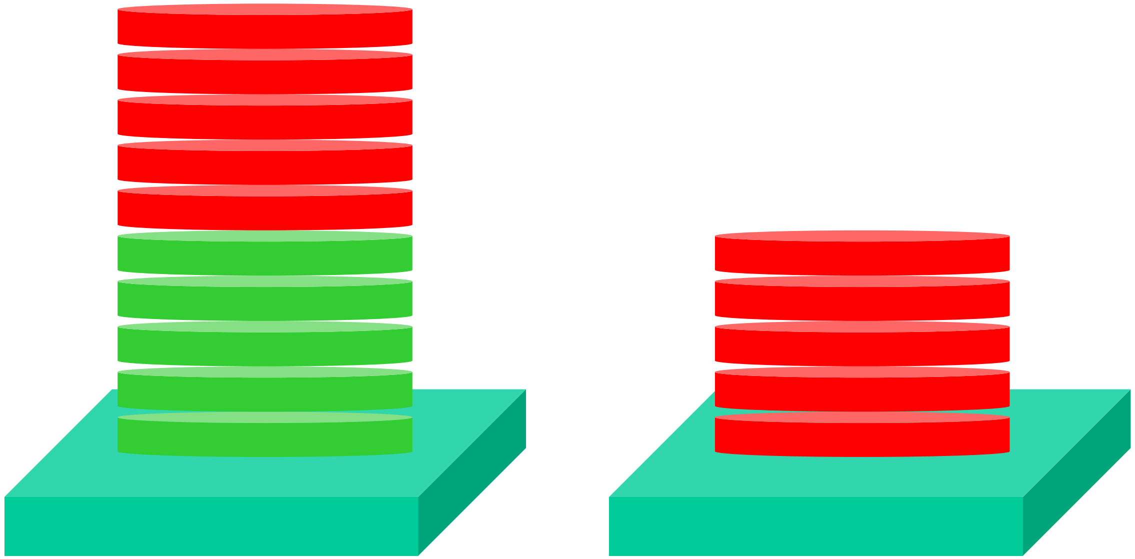
Why is there no any FA to recognize L ?

$L=\{$ 01, 0011, 000111, 00001111, 0000011111 , …… $\}$

---- Remember the same number of 0's and 1's

A practice problem

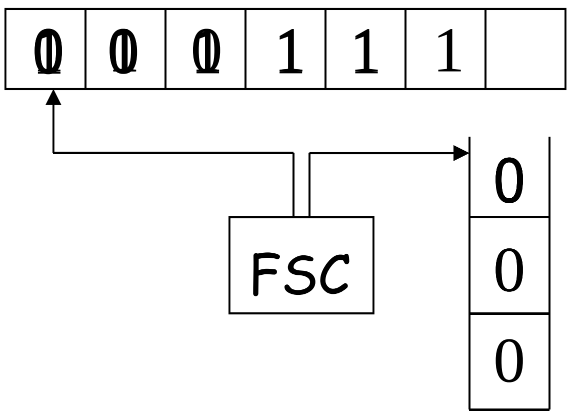
Is same the number of red and green discs ?



- Take red off , and put it on right table, one by one
- Take green off with red corresponding to it, one by one

Modification of FA

$L = \{\, 0^n 1^n \mid n \geq 1 \,\}$



read	:	1	1	1
pop	:	0	0	0

- read one 0, push one 0
- read one 1, pop one 0

$$(q, a, X) \Rightarrow (p, \alpha)$$

Formal Definition

PDA is a seven-tuple $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

- Q is finite set of states
- Σ is finite set of input symbols
- Γ is finite set of stack symbols
- δ is transition function : $Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \Rightarrow Q \times \Gamma^*$

$$\delta(q, a, X) = \{(p, \alpha) \mid p \in Q, \alpha \in \Gamma^*\}$$

- q_0 is start state
- z_0 is initial stack symbol
- F is finite set of accepting state

Example 8.1 PDA for $L = \{0^n 1^n \mid n \geq 1\}$

$$P(L) = (\{q, p, r\}, \{0,1\}, \{0, z\}, \delta, q, z, \{r\})$$

δ is defined as follows :

$\delta(q, 0, z) = (q, 0z)$

$\delta(q, 0, 0) = (q, 00)$

$\delta(q, 1, 0) = (p, \varepsilon)$

$\delta(p, 1, 0) = (p, \varepsilon)$

$\delta(p, \varepsilon, z) = (r, z)$

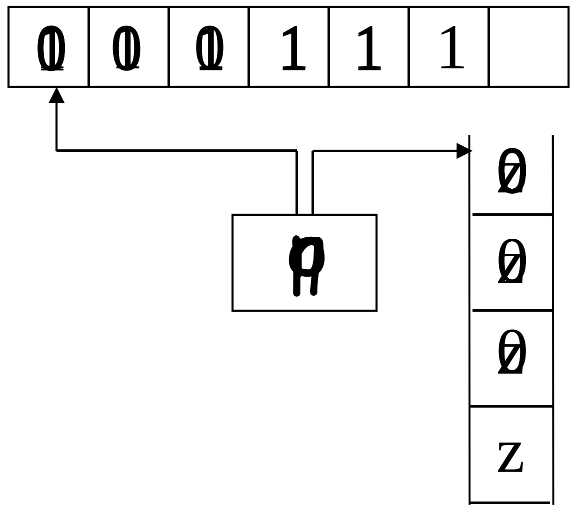
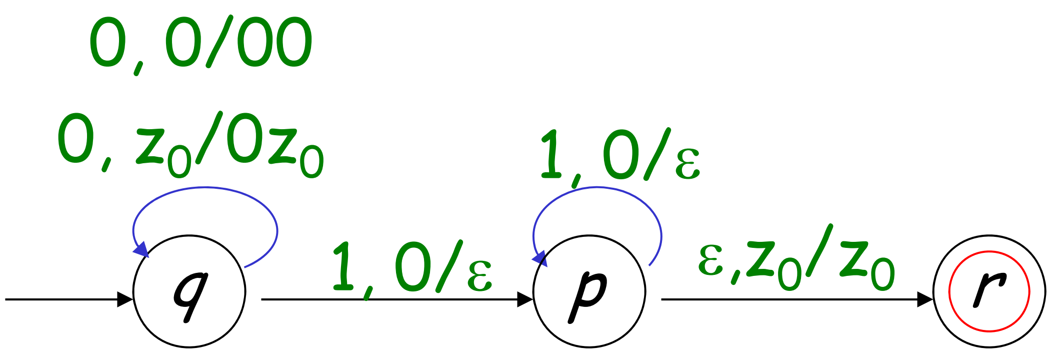


Diagram notation

- adding stack symbol to arc
- diagram of PDA for $L=\{ 0^n1^n \mid n \geq 1 \}$



$$\delta(q, 0, z_0) = (q, 0z_0)$$
$$\delta(q, 0, 0) = (q, 00)$$
$$\delta(q, 1, 0) = (p, \varepsilon)$$
$$\delta(p, 1, 0) = (p, \varepsilon)$$
$$\delta(p, \varepsilon, z_0) = (r, z_0)$$

- What is the diagram for PDA of $L=\{0^n1^n \mid n \geq 0\}$?

Example 8.2 Construct PDA for $L=\{ ww^R \mid w \in \{0,1\}^* \}$

- let ww^R=11011011(w=1101)

- step 1. Push w into stack one by one

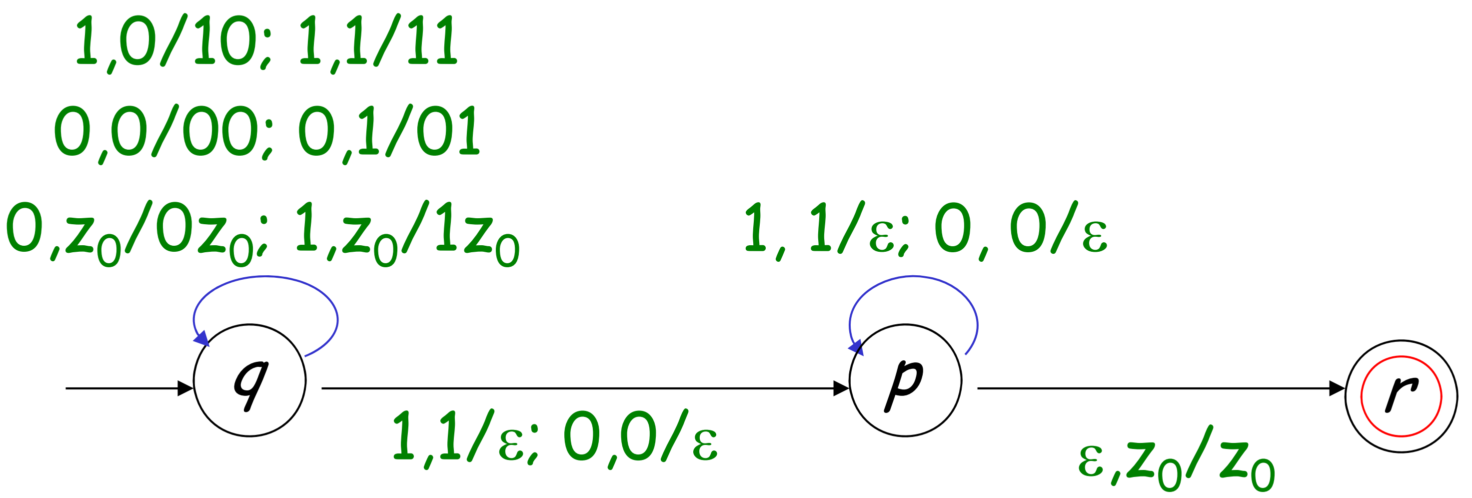
$$\delta(q, 0, z_0) = (q, 0z_0),\ \delta(q, 1, z_0) = (q, 1z_0)$$
$$\delta(q, 0, 0) = (q, 00),\quad \delta(q, 1,1) = (q, 11)$$
$$\delta(q, 0, 1) = (q, 01),\quad \delta(q, 1,0) = (q, 10)$$

- step 2. Pop w^R out of stack one by one

$$\delta(q, 1, 1) = (p, \varepsilon),\quad \delta(q, 0, 0) = (p, \varepsilon)$$
$$\delta(p, 1, 1) = (p, \varepsilon),\quad \delta(p, 0, 0) = (p, \varepsilon)$$

- finally $\delta(p,\varepsilon,z_0) = (r, z_0)$

- diagram of PDA for $L=\{ ww^R \mid w \in \{0,1\}^* \}$



Is it right enough ?

Configuration

configuration--> (q, w, α)

q : state in which the PDA is

w : left symbols that PDA is going to read

α : string within stack

In example 8.1 Let w=0011

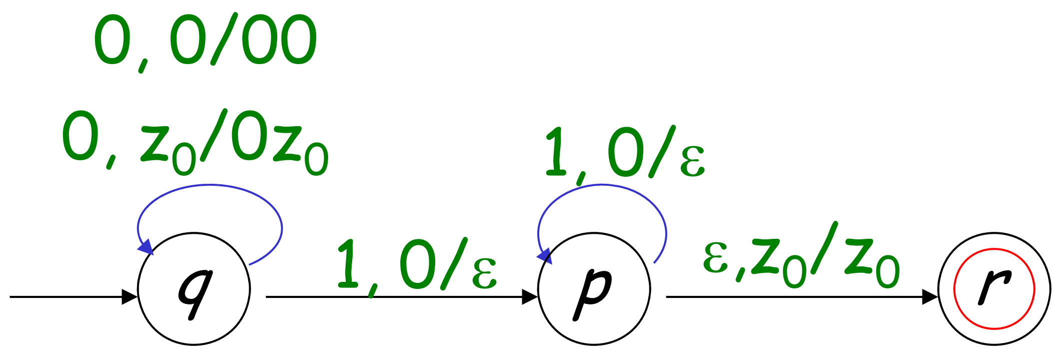
Initial configuration : $(q, 0011, z)$

Inner configuration : $(q, 011, 0z)$, $(q, 11, 00z)$

Final configuration : (r, ε, z)

Instantaneous Description(ID)

- diagram of PDA for $L=\{0^n1^n \mid n\geq 1\}$



Let w=0011,

$(q,0011,z_0)\vdash(q,011,0z_0)\vdash(q,11,00z_0)\vdash(p, 1,0z_0)$

$\vdash(p, \varepsilon , z_0)\vdash(r, \varepsilon , z_0)$

Configuration

- the derivation of ID --> $(q, aw, X\beta) \vdash (p, w, \alpha\beta)$
- the sequence of ID of w = 0011 for $P(0^n1^n)$

$(q,0011,z_0)\vdash(q,011,0z_0)\vdash(q,11,00z_0)\vdash(p, 1,0z_0)$

$\vdash(p, \varepsilon , z_0)\vdash(r, \varepsilon , z_0)$

Compact : $(q,0011,z_0) \overset{*}{\vdash} (r, \varepsilon , z_0)$

Language of PDA

- Acceptance by final state

$$L(P) = \{w \mid (q_0, w, z_0) \vdash^* (q, \varepsilon, \alpha),\ q \in F\}$$

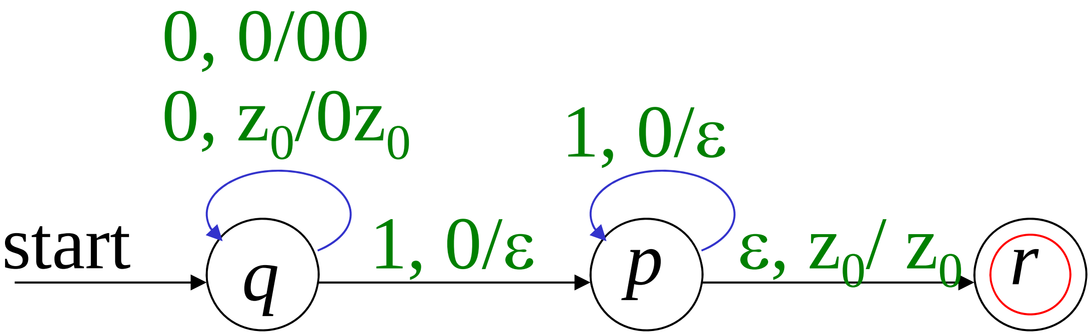
- Acceptance by empty stack

$$N(P) = \{w \mid (q_0, w, z_0) \vdash^* (q, \varepsilon, \varepsilon)\}$$

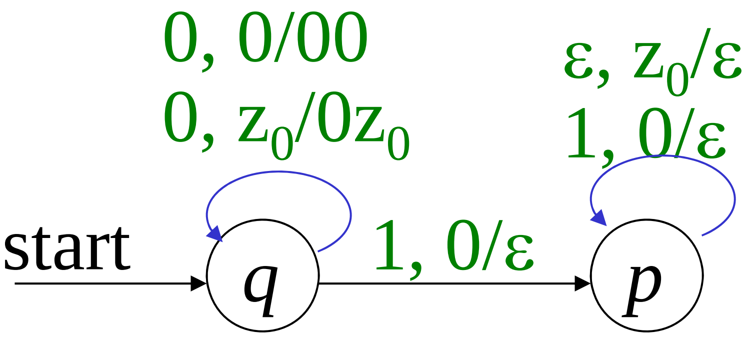
- Equivalence of two acceptance

$$L(P) \Leftrightarrow N(P)$$

Equivalence of two acceptance



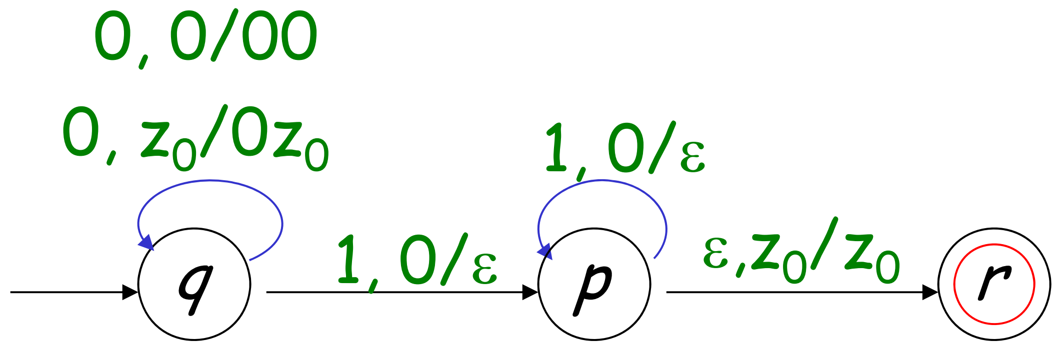
Accept by final state



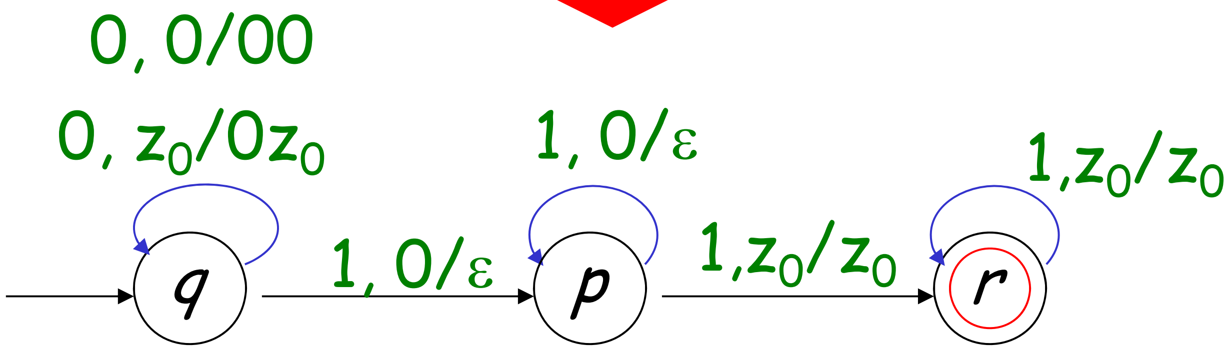
Accept by empty stack

Example 8.3 PDA for $L = \{ 0^n 1^m \mid n < m \}$

$w = 0^n 1^m = 0^n 1^n 1^{m-n}$, $m - n > 0$

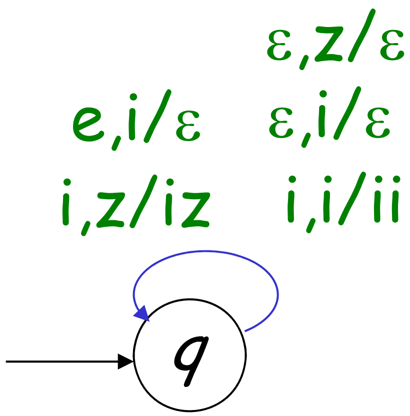


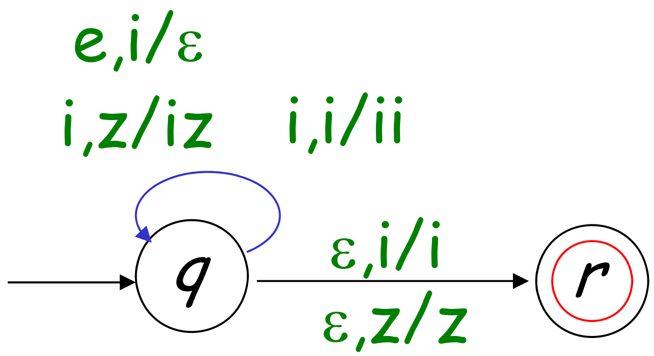
m=n



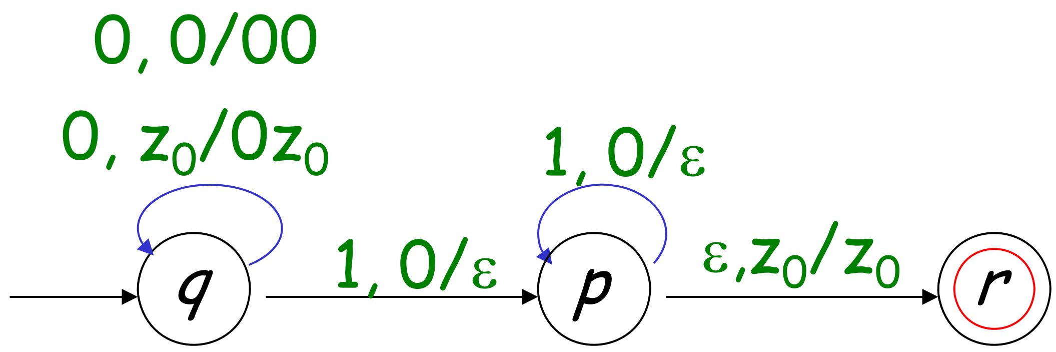
m>n

Example 8.4 Design a PDA that processes sequences of if's and else's in C language.





Deterministic Push-down Automata

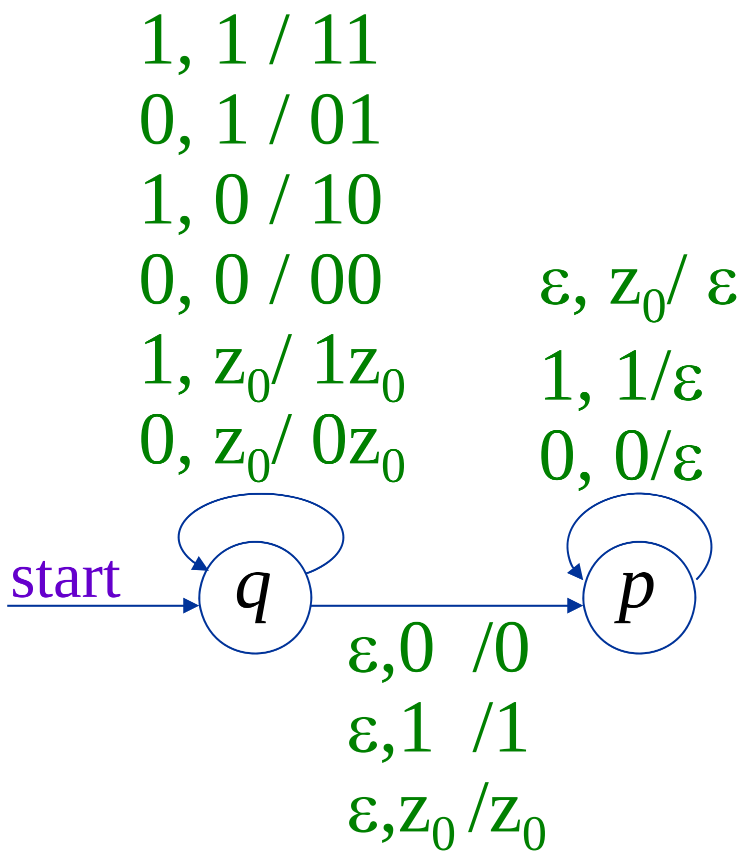


Definition of DPDA

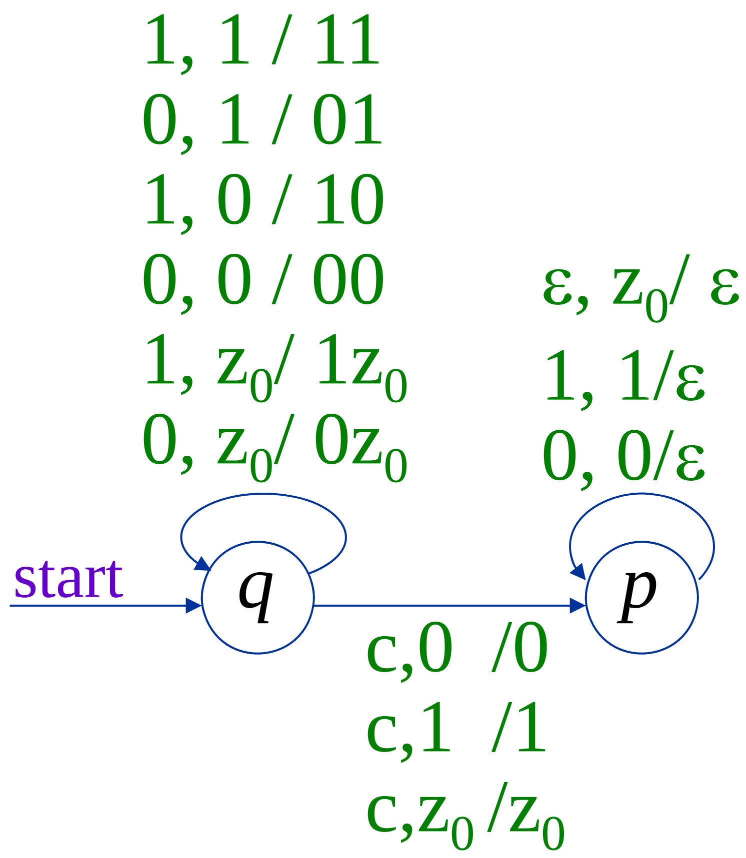
A PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ is said to be deterministic , when

- $\delta(q, a, X)$ has at most one member for any q in Q, a in Σ or a=ε , and X in Γ
- If $\delta(q, a, X)$ is nonempty for some a in Σ , then $\delta(q, \varepsilon, X)$ must be empty .

PDA & DPDA

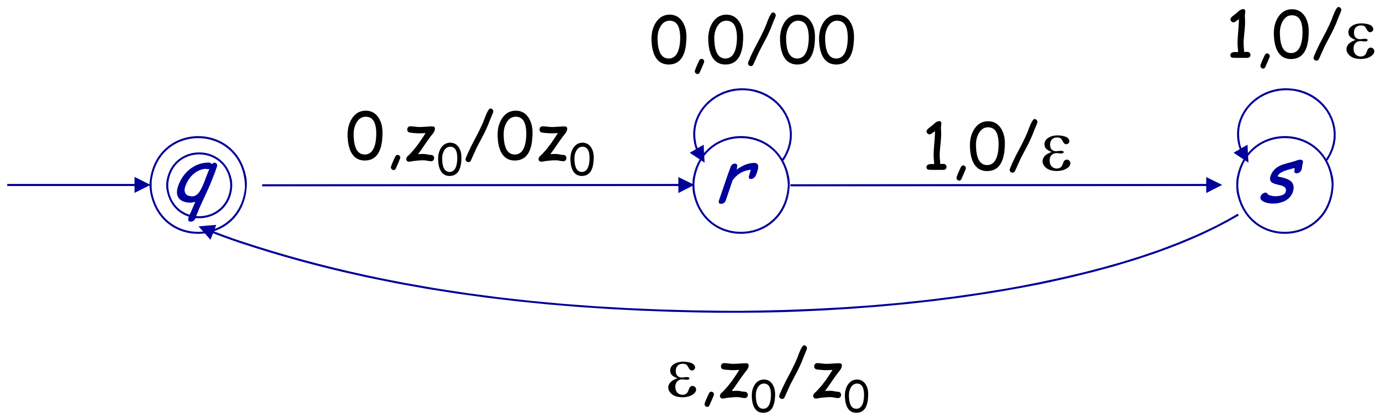


$L_{wwr}=\{ww^R \mid w \in \{0,1\}^*\}$



$L_{wcwr}=\{wcw^R \mid w \in \{0,1\}^*\}$

Example 8.5 DPDA for $L = \{ 0^n 1^n \mid n \geq 0 \}$

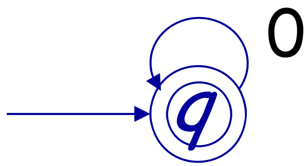


Question ?

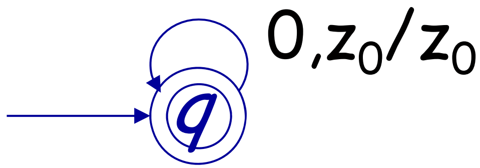
Two acceptance of DPDA

$L = \{ 0^n \mid n \geq 0 \} = \{ 0 \}^*$

FA :



DPDA :



---- by final state

by empty stack ?

Two acceptance of DPDA

- prefix property of language

➢ There are no two distinct string x and y in the language such that x is a prefix of y.

➢ yes : wcw^R . no : 0^*

- L is accepted by DPDA P by empty stack $\Leftrightarrow$

L is accepted by DPDA P' by final state and L has prefix property.

FA & DPDA

FA $A = (Q, \Sigma, \delta, q_0, F)$

DPDA $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

If L is accepted by a FA, then it must be accepted by a DPDA .

$$\delta_A(q, a) = p \Rightarrow \delta(q, a, z_0) = (p, z_0)$$

The stack is never used .

DPDA & PDA

Equivalent ?

$$L(FA) \subset L(DPDA) \subset L(PDA)$$

Good good study

day day up !