# Morning

# Deterministic Finite Automata(DFA)

1. Definition
2. Notation
3. Construction
4. Language accepted by a DFA
5. Regular language

# Formal Definition of DFA

Deterministic finite automaton is a five-tuple ,

*such as* $M = (Q, \Sigma, \delta, q_0, F)$

Where $Q$ is a finite set of *states* ,

$\Sigma$ is a finite set of *input symbols* ,

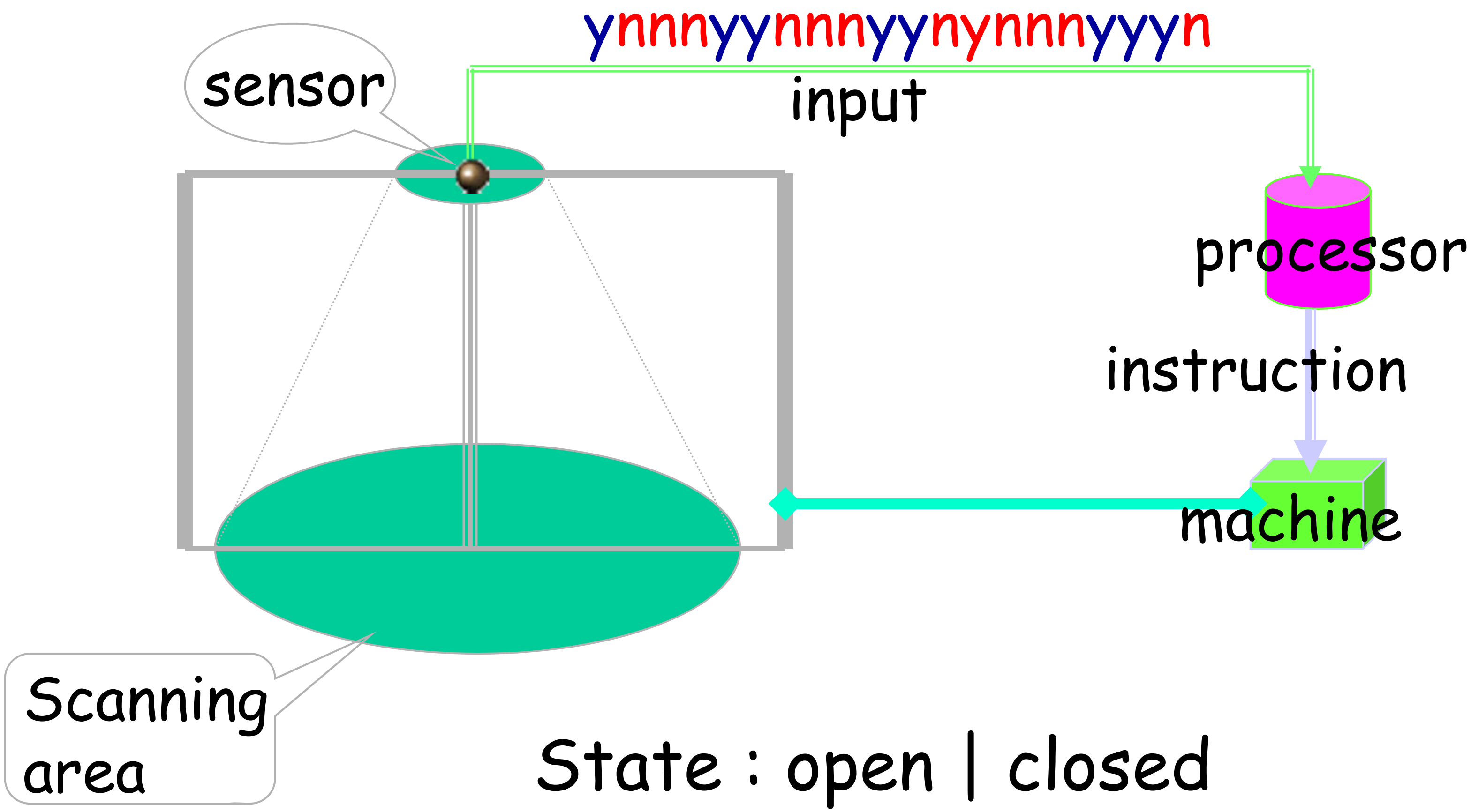
$q_0$ is a *start state* ,

$F$ is *a set of final state* ,

$\delta$ is *transition function* , which is a mapping

from $Q \times \Sigma$ to $Q$ .

# Example 2.1 DFA for Auto-gate




State : open | closed

Action : open | close

input : y | n

# Example 2.1 DFA for Auto-gate

Input symbols : { 0 , 1 } State : { Closed , Open }

State transition :

( Closed, 0 ) $\Rightarrow$ Closed

( Closed, 1 ) $\Rightarrow$ Open

( Open , 1 ) $\Rightarrow$ Open

( Open , 0 ) $\Rightarrow$ Closed

Start state : Closed

Final state : Closed

# Example 2.1 DFA for Auto-gate

Input symbols : { 0 , 1 } State : { $q$ , $p$ }

State transition function:

$$\delta(q, 0) \Rightarrow q$$

$$\delta(q, 1) \Rightarrow p$$

$$\delta(p, 1) \Rightarrow p$$

$$\delta(p, 0) \Rightarrow q$$

Start state : $q$

Final state : $q$

Automaton

# example 2.1 DFA for Auto-gate

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{p, q\}, \quad \Sigma = \{0, 1\}$

$q_0 = q, \quad F = \{q\}$

$\delta$ :

$\delta(q, 0) = q$

$\delta(q, 1) = p$

$\delta(p, 0) = q$

$\delta(p, 1) = p$

# example 2.1 DFA for Auto-gate

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{closed, open\}$, $\Sigma = \{n, y\}$

$q_0 = closed$, $F = \{closed\}$

$\delta$:

$\delta(closed, n) = closed$

$\delta(closed, y) = open$

$\delta(open, n) = closed$

$\delta(open, y) = open$

# example 2.1 DFA for Auto-gate

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1\}$, $\Sigma = \{0,1\}$

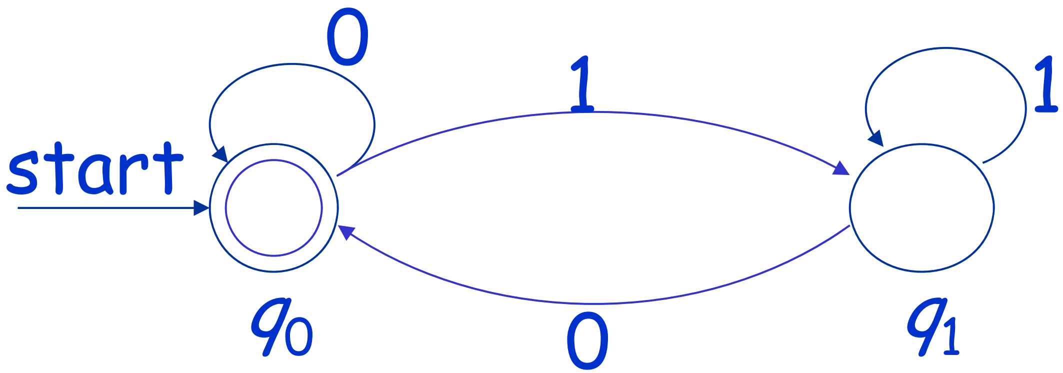
$q_0 = q_0$, $F = \{q_0\}$

$\delta$ :

$\delta(q_0, 0) = q_0$, $\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_0$, $\delta(q_1, 1) = q_1$

# Diagram Notation of DFA




$$M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_0\})$$

$\delta$ :

$$\delta(q_0, 0) = q_0, \ \ \delta(q_0, 1) = q_1$$
$$\delta(q_1, 0) = q_0, \ \ \delta(q_1, 1) = q_1$$

# Table Notation of DFA

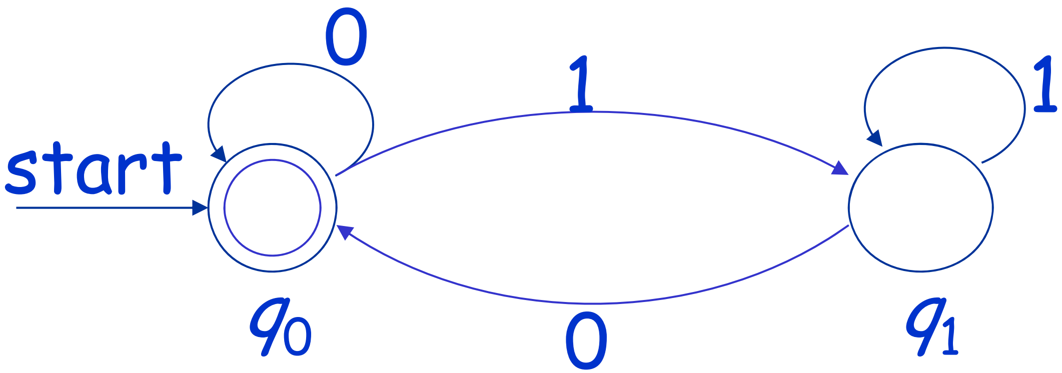
| | 0 | 1 |
|---|---|---|
| $\rightarrow * q_0$ | $q_0$ | $q_1$ |
| $q_1$ | $q_0$ | $q_1$ |

$$M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_0\})$$

$\delta$ :

$$\delta(q_0, 0) = q_0, \quad \delta(q_0, 1) = q_1$$
$$\delta(q_1, 0) = q_0, \quad \delta(q_1, 1) = q_1$$

# Partition Strings

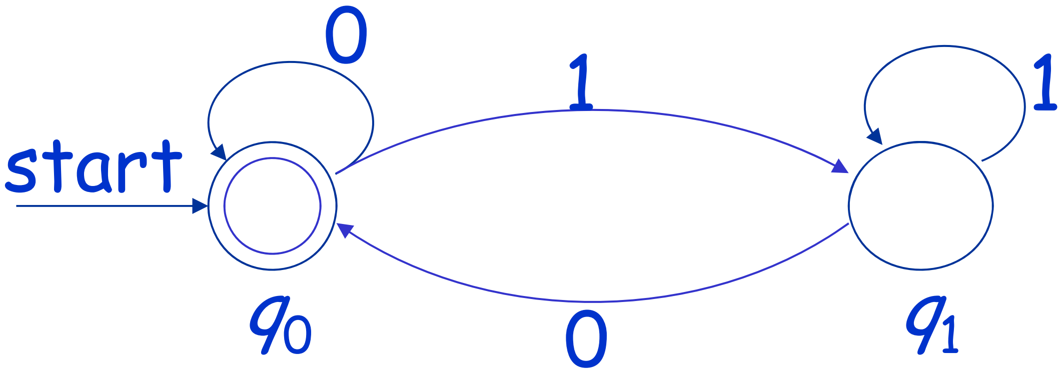



$M$ partitions all strings into two groups:

$L_1 = \{ w \in \{0,1\}^* \mid w \text{ end with } 0 \} \cup \{ \varepsilon \}$

$L_2 = \{ w \in \{0,1\}^* \mid w \text{ end with } 1 \}$

# DFA as a recognizer of language




$M$ "recognize" the following language :

$$L = \{ w \in \{0,1\}^* \mid w \text{ end with } 0 \} \cup \{ \varepsilon \}$$

With the language $L$, and a string $w \in \{0,1\}^*$

$M$ tell us whether $w$ belongs to $L$ , or not

# Decision problem

Given a language $L$, and a string $w$

Is $w$ belong to $L$ ?

# Example 2.2

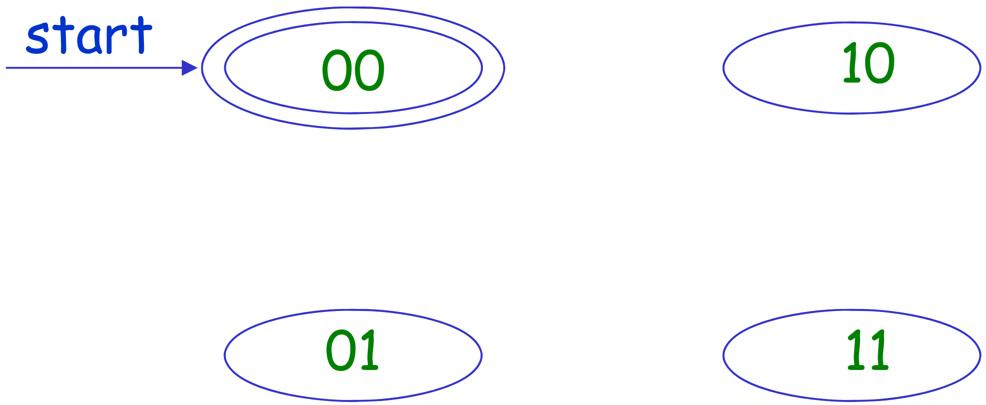
$L = \{ w \in \{0,1\}^* \mid w$ has both an even number of 0's and an even number of 1's $\}$

- Partition strings into four groups
  - 00 : even 0 and even 1
  - 01 : even 0 and odd 1
  - 10 : odd 0 and even 1
  - 11 : odd 0 and odd 1

# Example 2.2

$L = \{w \mid w$ has both an even number of 0's and an even number of 1's $\}$

- Set states corresponding to partitions

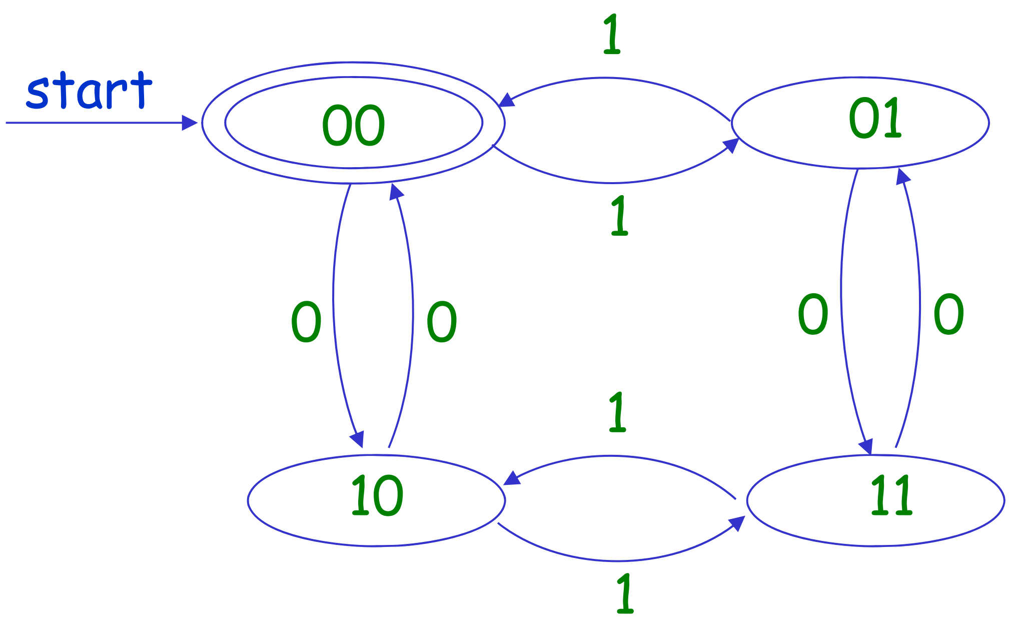


# Example 2.2

$L = \{w \mid w$ has both an even number of 0's

and an even number of 1's $\}$

- Put transition arcs between states



## Example 2.3

$L$ = {w | w consists of 0's and 1's , and contains sub-string 01}

or {x01y | x and y are consists of any number of 0's and 1's }

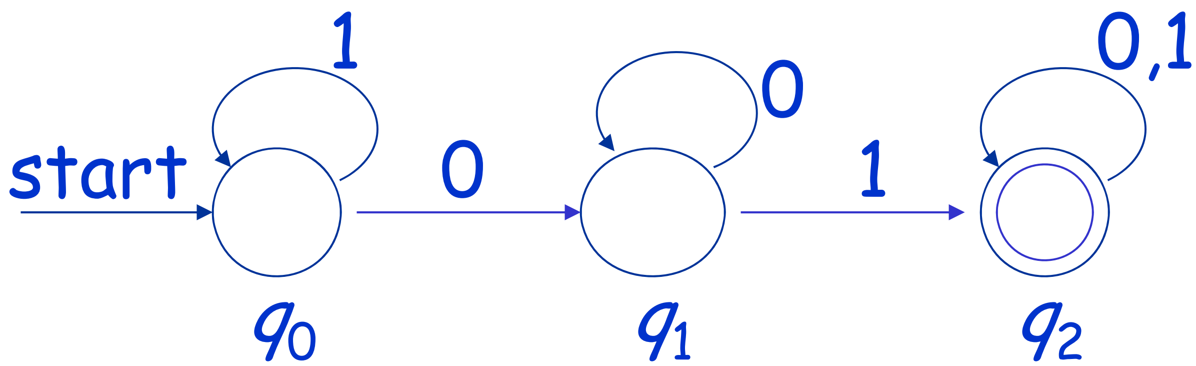
Problem :

How to decide whether a given string w belongs to L ?

# Construction of DFA

How to start our work ?

- What meaning of " w belongs to L "
- Partition strings by properties of L
- Set states which correspond to the partitions
- Put transition arcs between states



# Extending transition function to string

<u>BASIS</u>

$$\hat{\delta}(q, \varepsilon) = q.$$

<u>INDUCTION</u>

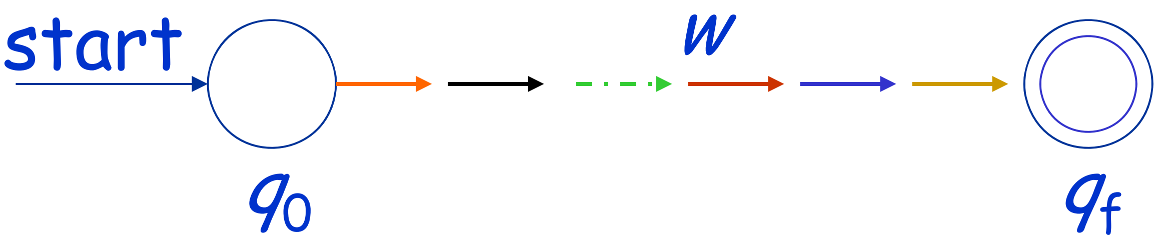
Suppose *w* is a string of the form *xa* , that is , *a* is the last symbol of *w*, and *x* is the string consisting of all but the last symbol. Then

$$\hat{\delta}(q, w) = \delta(\hat{\delta}(q, x), a)$$

# The language of a DFA

*Definition* *The* *language* of a DFA *A* is denoted *L*(*A*), and defined by

$$L(A) = \{w \mid \hat{\delta}(q_0, w) \ \ is \ \ in \ \ F\}$$




Note： one language accepted by one DFA

# Regular language

*Definition*

If $L$ is $L(A)$ for some DFA $A$, then we say $L$ is a *regular language.*

$$RL = \{ L \mid \text{There is a DFA to accept } L \}$$

Note : a kind of languages accepted by DFA's

# Example 2.4

Construct DFA for following languages :

a) $\{ 0 \}^*$

b) $\{w \mid w \in \{0,1\}^*$ and begin with 0 $\}$

c) $\{w \mid w$ consists of any number of 0's followed by any number of 1's $\}$

d) $\{ \varepsilon \}$

e) $\phi$

Good good study

day day up !