Aung Kyi San
Summer 2012

ECE 20200 : Linear Circuit Analysis II
School of ECE, Purdue University

LECTURE 6

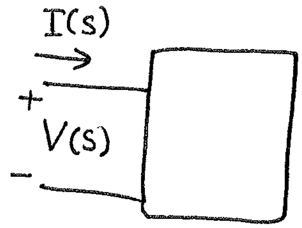
- Impedance $Z(s)$
- Admittance $Y(s)$

Reference: Decarlo/Lin PP 603-617

Impedance and Admittance

Impedance (s-domain)

$$\boxed{Z(s) = \frac{V(s)}{I(s)}}$$



- No independent source, zero initial condition
(no internal stored energy)

$$V(s) = Z(s)\, I(s)$$

unit of impedance : Ω (ohm)

Admittance

$$\boxed{Y(s) = \frac{I(s)}{V(s)}}$$

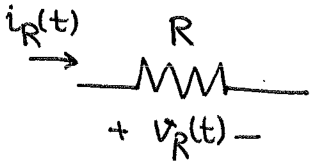
$$I(s) = Y(s)\, V(s)$$

unit of admittance : S (siemen) ($\mho$, mho)

Relationship between Impedance and Admittance

$$\boxed{Z(s) = \frac{1}{Y(s)}} \quad \text{or} \quad \boxed{Y(s) = \frac{1}{Z(s)}}$$

Resistor



$$v_R(t) = R\, i_R(t)$$

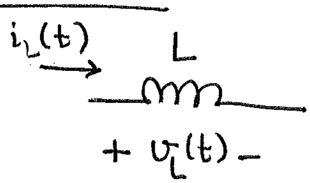
$$V_R(s) = R\, I_R(s)$$

$$V_R(s) = Z_R\, I_R(s) \Rightarrow \boxed{Z_R = R}$$

$$I_R(s) = \frac{1}{R} V_R(s) = Y_R(s) V_R(s)$$

$$\Rightarrow \boxed{Y_R = \frac{1}{R}}$$

Inductor



$$v_L(t) = L \frac{di_L(t)}{dt}$$

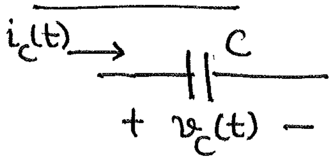
$$V_L(s) = L\, s\, I_L(s)$$

$$V_L(s) = Z_L\, I_L(s) \Rightarrow \boxed{Z_L = Ls}$$

$$I_L(s) = \frac{1}{Ls} V_L(s) = Y_L(s) V_L(s)$$

$$\Rightarrow \boxed{Y_L = \frac{1}{Ls}}$$

Capacitor



$$i_C(t) = C \frac{dv_C(t)}{dt}$$

$$I_C(s) = C\, s\, V_C(s) \Rightarrow \boxed{Y_C = Cs}$$

$$V_C(s) = \frac{1}{Cs} I_C(s) \Rightarrow \boxed{Z_C = \frac{1}{Cs}}$$

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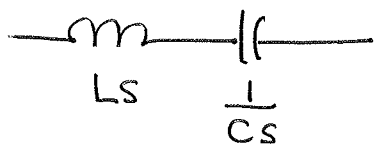
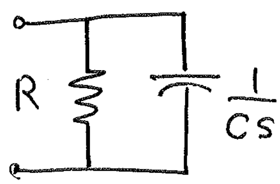
Manipulation Rules

(a) Impedances - manipulated as resistances



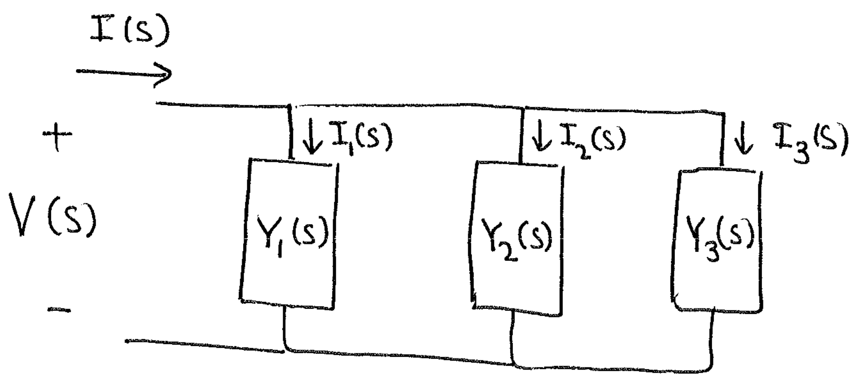
$$\Rightarrow Z_{eq}(s) = Ls + \frac{1}{Cs}$$

(b) Admittances - manipulated as conductances



$$\Rightarrow Y_{eq}(s) = Cs + \frac{1}{R}$$

Example: Parallel Admittances and Current Division



$$Y(s) = Y_1(s) + Y_2(s) + Y_3(s)$$

$$Z(s) = \frac{1}{Y(s)} = \frac{1}{Y_1(s) + Y_2(s) + Y_3(s)}$$

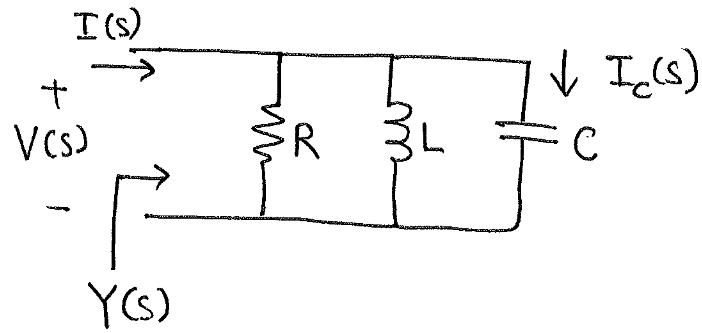
$$I_1(s) = \frac{Y_1(s)}{Y_1(s) + Y_2(s) + Y_3(s)} I(s)$$

$$I_2(s) = \frac{Y_2(s)}{Y_1(s) + Y_2(s) + Y_3(s)} I(s)$$

$$I_3(s) = \frac{Y_3(s)}{Y_1(s)+Y_2(s)+Y_3(s)} I(s)$$

$$Y(s) = \frac{I(s)}{V(s)} \Rightarrow V(s) = \frac{1}{Y(s)} \cdot I(s)$$

Example: Find $Y(s)$, $Z(s)$ and $I_c(s)$



$$Y(s) = \frac{1}{R} + \frac{1}{Ls} + \frac{1}{1/Cs} = \frac{1}{R} + \frac{1}{Ls} + Cs$$

$$= \frac{\frac{Ls}{R} + 1 + LCs^2}{Ls}$$

$$Z(s) = \frac{1}{Y(s)} = \frac{Ls}{\frac{Ls}{R} + 1 + LCs^2} = \frac{\frac{1}{C}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

$$I_c(s) = \frac{Y_c(s)}{Y(s)} \cdot I(s) = Z(s)\, Y_c(s)\, I(s)$$

$$= \frac{\frac{1}{C}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} \cdot Cs \cdot I(s)$$

$$I_c(s) = \frac{s^2}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} \cdot I(s)$$

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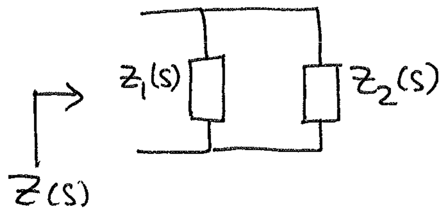
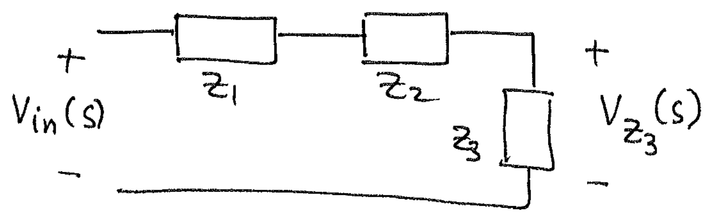
Product / Sum Rule



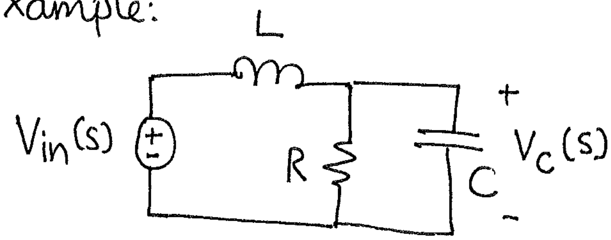
$$Z(s) = \frac{Z_1(s)\,Z_2(s)}{Z_1(s) + Z_2(s)}$$

Voltage Division



$$V_{Z_3}(s) = \frac{Z_3(s)}{Z_1(s)+Z_2(s)+Z_3(s)} \cdot V_{in}(s)$$

Example:



$L = 0.5\,H$

$R = 5\,\Omega$

$C = 0.1\,F$

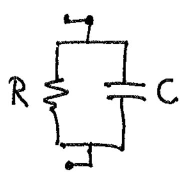
Find $V_c(s)$ if $v_{in}(t) = \delta(t)$

Solution:

$$V_{in}(s) = \mathcal{L}[v_{in}(t)] = \mathcal{L}[\delta(t)] = 1$$

$$V_c(s) = \frac{Z(s)}{Z(s) + Ls} \cdot V_{in}(s)$$

where $Z(s) \Rightarrow$



$$Y(s) = \frac{1}{R} + Cs$$

$$Y(s) = 0.2 + 0.1s$$

$$\therefore Z(s) = \frac{1}{Y(s)} = \frac{1}{0.1s + 0.2} = \frac{10}{s+2}$$

$$\therefore V_c(s) = \frac{\frac{10}{s+2}}{\frac{10}{s+2} + 0.5s} \quad (1)$$

$$= \frac{10}{10 + 0.5s(s+2)} = \frac{10}{10 + 0.5s^2 + s} = \frac{20}{s^2 + 2s + 20}$$

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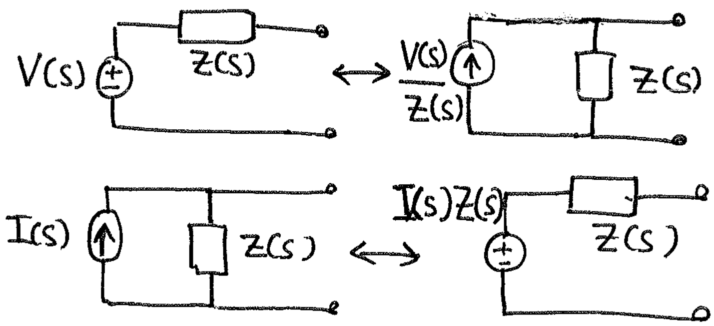
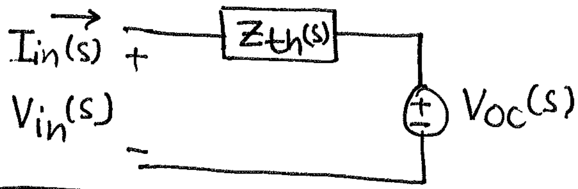
Source Transformation



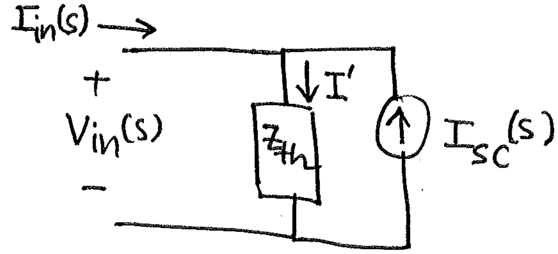
Thevenin and Norton Equivalents

Thevenin (s-domain)



$$\boxed{V_{in}(s) = Z_{th}(s)\, I_{in}(s) + V_{oc}(s)}$$

Norton (s-domain)

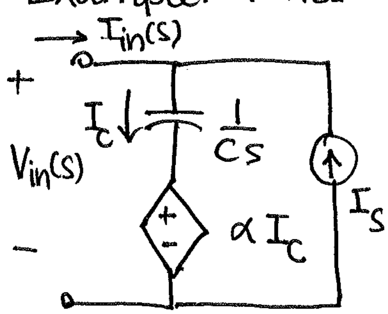


$$I_{in}(s) = I' - I_{sc}(s)$$

$$\boxed{I_{in}(s) = \frac{V_{in}(s)}{Z_{th}} - I_{sc}(s)}$$

Strategy: Find an equation of the circuit in one of the two forms. Use pattern recognition to identify $Z_{th}(s)$ and $V_{oc}(s)$ (or) $I_{sc}(s)$.

Example: Find Thevenin Equivalent



(a) $I_{in}(s) = I_c(s) - I_s$

(b) $I_c(s) = Cs\,[V_{in}(s) - \alpha I_c(s)]$

$= Cs\,V_{in}(s) - \alpha Cs\, I_c(s)$

$(1+\alpha Cs)\, I_c(s) = Cs\, V_{in}(s)$

$I_c(s) = \dfrac{Cs}{1+\alpha Cs} V_{in}(s)$

(c) $I_{in}(s) = \underbrace{\dfrac{Cs}{1+\alpha Cs}}_{1/Z_{th}} V_{in}(s) - \underbrace{I_s(s)}_{I_{sc}(s)}$

$$\boxed{\text{Note: } V_{oc}(s) = I_{sc}(s) \cdot Z_{th}(s)}$$