WebAssign
Lab #13: Entropy and Temperature (Homework)

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PHYS 172-SPRING 2012, Spring 2012
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Current Score : 20 / 20 **Due :** Thursday, April 19 2012 11:59 PM EDT

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1. 5/5 points | Previous Answers

Probability distribution of energy quanta in atoms

(a) Consider a system consisting of two atoms (three oscillators each), among which 4 quanta of energy are to be distributed. Write a program to display a histogram showing the total number of possible microstates of the two-atom system vs. the number of quanta assigned to atom 1. Compare your calculations and histogram to the one on page 378 (you should get the same distribution).

(b) Change your program to model a system consisting of two solid blocks, block 1 containing 300 oscillators and block 2 containing 200 oscillators. Find the possible distributions of 100 quanta among these blocks, and plot number of microstates vs. number of quanta assigned to block 1. Compare your calculations and histogram to those on page 381.

From your plot, determine the most probable distribution of energy between the two solid blocks. You may find a straightedge helpful in reading your plots.

For the most probable energy distribution between the two blocks:
How many microstates (ways of arranging the energy among all the oscillators) are there?

Number of ways = 6.903 ✓ $\times 10^{114}$
How many quanta of energy are in block 1? 60 ✓ quanta
How many quanta of energy are in block 2? 40 ✓ quanta

Together, the two blocks contain 500 oscillators.

What percentage of the total oscillators are in block 1? 60 ✓ %
What percentage of the total oscillators are in block 2? 40 ✓ %

(c) In the following calculations you will change the number of oscillators in each block, while keeping the total number of oscillators constant, and report on what you see.

Change your program so block 1 has 90 oscillators and block 2 has 410 oscillators:

For the most probable energy distribution between the two blocks:
How many microstates (ways of arranging the energy among all the oscillators) are there?

Number of ways = 8.754 ✓ $\times 10^{114}$
How many quanta of energy are in block 1? 17 ✓ quanta
How many quanta of energy are in block 2? 83 ✓ quanta
What percentage of the total oscillators are in block 1? 18 ✓ %
What percentage of the total oscillators are in block 2? 82 ✓ %

Change your program so block 1 has 150 oscillators and block 2 has 350 oscillators:

For the most probable energy distribution between the two blocks:

How many microstates (ways of arranging the energy among all the oscillators) are there?

Number of ways = 7.345 ✓ $\times 10^{114}$

How many quanta of energy are in block 1? 30 ✓ quanta

How many quanta of energy are in block 2? 70 ✓ quanta

What percentage of the total oscillators are in block 1? 30 ✓ %

What percentage of the total oscillators are in block 2? 70 ✓ %

Now consider a system too big for your program to handle: it has a total of 10000 oscillators, with 3000 quanta to be distributed between them. Block 1 has 4000 oscillators, and block 2 has 6000 oscillators.

Think about what you observed above. *For the most probable distribution of energy between the blocks:*

How many quanta would you expect to find in block 1?

1200 ✓ quanta in block 1

How many quanta would you expect to find in block 2?

1800 ✓ quanta in block 2

Now change your program back to the values given in part (b) (block 1 has 300 oscillators, block 2 has 200 oscillators, there are 100 quanta of energy available).

If you could measure the number of quanta in each block once every second for one month, more of your measurements would show 60 quanta of energy in block 1 and 40 quanta of energy in block 2 than any other values. However, some measurements would show other values, too.

Which of the following values are approximately half as probable as the most probable values? That is, which of these values would show up in half as many measurements as the most probable values do? You can find these values by looking at your plot.

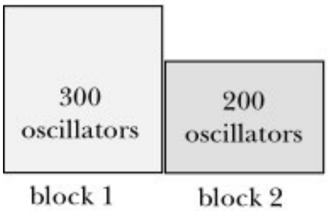
- [ ] q1 = 70 and q2 = 30
- [ ] q1 = 58 and q2 = 42
- [x] q1 = 53 and q2 = 47
- [ ] q1 = 50 and q2 = 50
- [x] q1 = 67 and q2 = 33
- [ ] q1 = 62 and q2 = 38

✓

2. 5/5 points | Previous Answers

Natural logarithm of ways to arrange energy

Start with your above solution. For the same system of two blocks, with N_1 = 300 oscillators and N_2 = 200 oscillators, plot ln(Ω_1), ln(Ω_2), and ln($\Omega_1\Omega_2$), for q_1 running from 0 to 100 quanta. Your graph should look like the one in Figure 11.24 shown below.



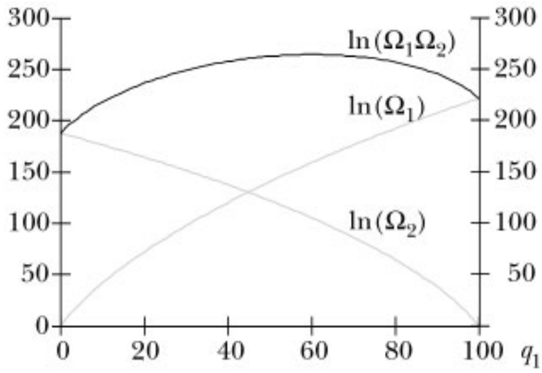


Figure 11.24

(a) Now change your program, so that block 1 has 375 oscillators, and block 2 has 125 oscillators.

What (approximately) is the maximum value of ln($\Omega_1\Omega_2$) for this system?

max ln(Ω_{12})= 266.7

How many quanta of energy are in block 1 when ln($\Omega_1\Omega_2$) is maximum?

Number of quanta of energy in block 1 = 75

The graphs of ln(Ω_1) and ln(Ω_2) cross. What is the significance of this crossing point?

- ○ The crossing point indicates the most probable energy distribution.
- ○ The crossing point occurs at the value of q_1 which produces a minimum of ln(Ω_{12})
- ◉ The crossing point has no physical significance.

At the value of q_1 for which ln($\Omega_1\Omega_2$) is a maximum, look at the slopes of the other two curves (ln(Ω_1) *vs.* q_1 and ln(Ω_2) *vs.* q_1. Which of the following statements is true? (This will turn out later to be important.)

- ◉ The slopes are equal in magnitude and opposite in sign.
- ○ The slopes are both zero.
- ○ One curve is almost flat, while the other is almost vertical.

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3. 5/5 points | Previous Answers

Calculate the temperature of a block as a function of its thermal energy.

A good way to organize your computation is this:

1) Inside the loop of your previous program create a new variable q1t = q1+1 and use this new variable to calculate the entropy with increased energy in block 1.

2)Then you can calculate the change in entropy (entropy for q1t - entropy for q1), and from this you can calculate the temperature.

3) You can do the same thing with q2; create another variable q2t = q2+1 and use it to calculate the entropy with increased energy in block 2.

4) Modify your program to plot the temperature in kelvins of block 1 as a function of the number of quanta q_1 present in the first block. On the same graph plot the temperature in kelvins of block 2 as a function of the *same* variable q_1 (where $q_2 = q_{tot} - q_1$), so in the graph of the temperature of block 2, the energy in block 2 runs from right to left. Note most of this graphing code is already in your program.

5) In order to plot the temperature in kelvins, you must determine the values of ΔE and ΔS that correspond to a one-quantum change in energy.

6) Consider the model we are using. **On page 390 we show that the effective k_s for oscillations in the Einstein solid is expected to be about 4 times the value obtained from measuring Young's modulus**. Therefore the increase in energy (in joules) corresponding to adding one quantum of energy is $\Delta E = \hbar(4k_{Al}/m)^{1/2}$, where we found in Problem 4.HW.76 that the interatomic spring constant k_{Al} for aluminum is about 16 N/m, based on measurements of Young's modulus. The increment in entropy corresponding to this increment ΔE in energy is $\Delta S = k\Delta(\ln\Omega)$. Assume that the blocks are made of aluminum; so the mass of one Al atom is given by multiplying the 27 nucleons in an Al atom by 1.7e-27 kg.

(a) Let the total number of oscillators in the aluminum blocks be 500 (approximately 167 atoms). In your program, make block 1 have 100 oscillators (approximately 33 atoms), and make block 2 have 400 oscillators (approximately 133 atoms). The total energy in the two blocks is 100 quanta.

On your graph, what are the numerical values of q_1 and q_2 where the temperature curves for the two blocks cross (that is, where the temperatures are equal)? For this to be physically meaningful, it is necessary that the curve for block 2 goes from right to left. That is, use $q_2 = q_{tot} - q_1$ to calculate T_2, but plot T_2 vs. q_1, not T_2 vs. q_2, so that at a particular value of q_1, you've correctly plotted the temperature of each block.

q_1 = 19 ✔

q_2 = 81 ✓

(b) Together, the two blocks contain 500 oscillators.

What percentage of the total oscillators are in block 1? 20 ✓ %

What percentage of the total oscillators are in block 2? 80 ✓ %

(c) Based on these observations, which of these statements is true?

- ○ When the temperatures reach the same value, the bigger block has more energy per atom than the smaller block.
- ○ When the temperatures reach the same value, the smaller block has more energy per atom than the bigger block.
- ◉ When the temperatures reach the same value, the average energy per atom is the same in both blocks.

✓

(d) What is the numerical value of the temperature when the two temperatures are equal? (Just estimate by eye from the graph; a straightedge such as a paper may be useful.)

$T_1 = T_2 =$ 153.6 ✓ K

(e) What is the physical significance of this temperature?

- [x] After reaching equilibrium, it is very unlikely to find either block with a temperature different than this.
- [ ] This is the initial temperature of block 2, and block 1 gets this temperature from block 2.
- [x] This is the final equilibrium temperature of both blocks.
- [ ] This is the initial temperature of block 1, and block 2 gets this temperature from block 1.

✓

(f) Change your program so that there are 300 oscillators (100 atoms) in block 1, there are 200 oscillators (about 67 atoms) in block 2, and the total number of quanta is 100.

On your graph, what are the numerical values of q_1 and q_2 where the temperature curves for the two blocks cross (that is, where the temperatures are equal)?

q_1 = 60 ✓

q_2 = 40 ✓

(g) Together, the two blocks contain 500 oscillators.

What percentage of the total oscillators are in block 1? 60 ✓ %

What percentage of the total oscillators are in block 2? 40 ✓ %

(h) Based on these observations, which of these statements is true?

- (selected) When the temperatures reach the same value, the average energy per atom is the same in both blocks.
- When the temperatures reach the same value, the smaller block has more energy per atom than the bigger block.
- When the temperatures reach the same value, the bigger block has more energy per atom than the smaller block.

(i) What is the numerical value of the temperature when the two temperatures are equal? (Just estimate by eye from the graph; a straightedge such as a paper may be useful. It is interesting to compare with your earlier result.)

$T_1 = T_2 =$ 151.1 K

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4. 5/5 points | Previous Answers

Please evaluate the teamwork experience of this lab by checking off your agreement/disagreement with the following statements.

1. Our group worked effectively as a team.

- strongly disagree
- disagree
- neutral
- agree
- (selected) strongly agree

2. Working as a team helped me stay focused on the problem.

- ( ) strongly disagree
- ( ) disagree
- ( ) neutral
- ( ) agree
- (•) strongly agree

3. Working as a team improved my understanding of physics.

- ( ) strongly disagree
- ( ) disagree
- ( ) neutral
- ( ) agree
- (•) strongly agree

4. One person did most of the work on my team.

- (•) strongly disagree
- ( ) disagree
- ( ) neutral
- ( ) agree
- ( ) strongly agree

5. Playing the different roles kept me engaged and helped me see different aspects of the problem.

- ◯ strongly disagree
- ◯ disagree
- ◯ neutral
- ◯ agree
- ◉ strongly agree

6. My team communicated well with each other.

- ◯ strongly disagree
- ◯ disagree
- ◉ neutral
- ◯ agree
- ◯ strongly agree

7. Disagreements on my team were resolved by discussion.

- ◯ strongly disagree
- ◯ disagree
- ◯ neutral
- ◯ agree
- ◉ strongly agree

These questions refer to the lab experience in general.

8. The lab is effective in teaching me important concepts of physics.

- ○ strongly disagree
- ○ disagree
- ○ neutral
- ○ agree
- ◉ strongly agree

9. The lab reinforces what I've learned in lecture and recitation.

- ○ strongly disagree
- ○ disagree
- ○ neutral
- ○ agree
- ◉ strongly agree

10. The visualization of motion provided by Vpython is a useful tool for teaching physics.

- ○ strongly disagree
- ○ disagree
- ○ neutral
- ○ agree
- ◉ strongly agree

Viewing Saved Work **Revert to Last Response**