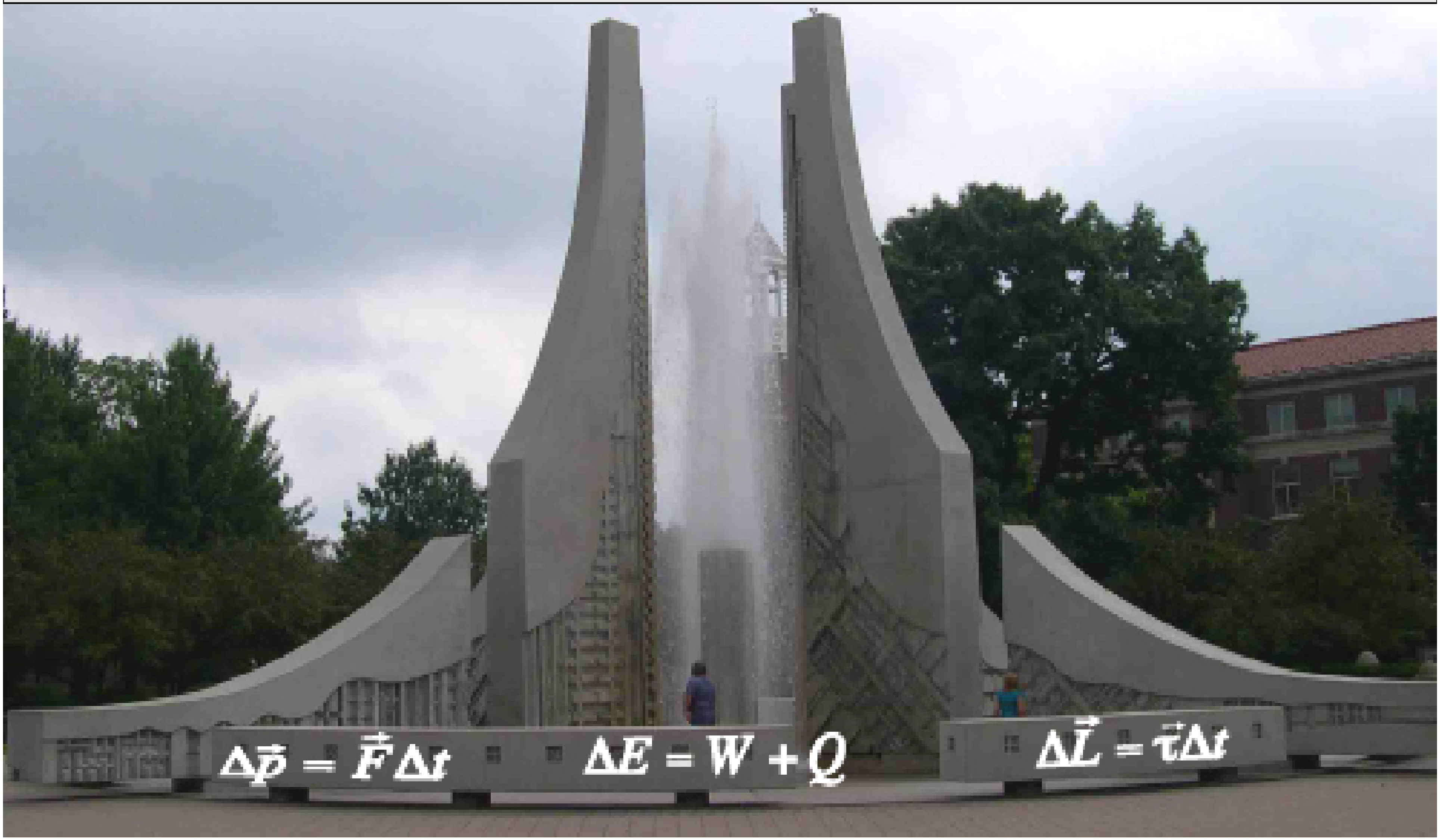
PHYS 172: Modern Mechanics

Spring 2012



Lecture 2 – Vectors Momentum & Relativity

Read: 1.6-1.11

Today

- General Course Information
- Newton's First Law of Motion
- Vectors
- Velocity
- Principle of Relativity
- Special Relativity

Syllabus & General Information:

Read the syllabus carefully.

This is a 4 credit hour course.

The rule of thumb at this University is that you should spend twice as many hours (on average) studying for this course outside of class as you do in class.

That is, eight hours a week spent reading, doing the homework, recitation, and laboratory assignments, and exams would be a typical investment of time for most students in this course.

Syllabus & General Information:

Evening Exams (3)	30%
Final Exam	20%
Labs	15%
Homework (WebAssign)	15%
Clicker questions	5%
Recitation	15%
TOTAL	100%

We will use an absolute grading scale with values
as given on the syllabus. 89% or better to get an A-.

QUESTIONS? Email Prof. Denes Molnar **molnar@purdue.edu**

Syllabus & General Information:

Reminders:

For help with homework problems, go to the Physics Learning Center in Room 12 (PHYS Bldg) during the assigned hours. Schedule can be found on the home page. It is staffed by trained teaching assistants assigned to this course. Use it!

1. Extensions can be granted for Homework assignments, if you have a good reason and don't abuse the privilege. Download form under "HW Extension request" from the course web site.
2. To request an Excused Grade for an exam due to a valid reason (illness, etc), download "Absentee Report" from the course web site. Do this in advance if at all possible. Advance notification is required for Evening Exams and the Final.
3. Do NOT notify the instructor if you will miss a lecture. We drop the 3 lowest clicker quiz grades to accommodate occasional absences.

See the Syllabus for more details.

Syllabus & General Information:

Academic Honesty

You are encouraged to work on homework together – discussing ideas and concepts reinforces the material for everyone involved in the conversation. Just be sure that what you turn in is your own work that you fully understand.

The following are examples of cheating:

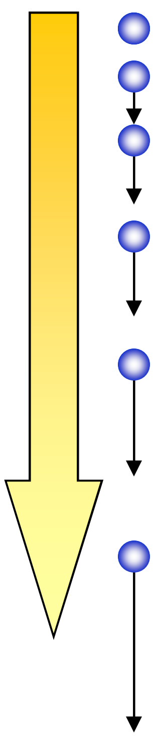
- Any effort to represent somebody else's work as your own, or allowing your work to be represented as somebody else's
- Having somebody else solve assigned problems for you
- Entering iClicker responses for anybody else
- Being in possession of more than one iClicker in lecture

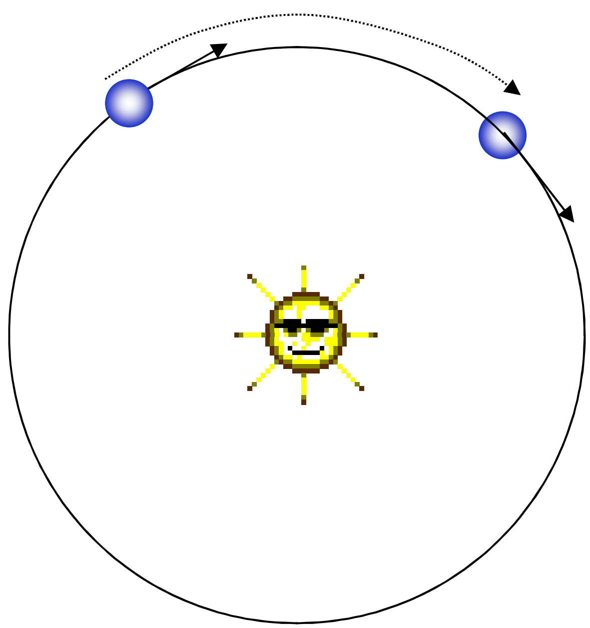
Read the syllabus.

QUESTIONS? Email Prof. Denes Molnar **molnar@purdue.edu**

Indicators of interaction

- Change of velocity
- Change of identity — $H_2 + O_2 \rightarrow H_2O$
- Change of shape — *bending a wire*
- Change of temperature — *heating pot of water on a hot stove*
- Lack of change when change is expected — *balloon floating in sky*





Uniform motion: velocity is constant

Newton's first law of motion

≡ uniform (constant) velocity

An object moves in a straight line and at constant speed except to the extent that it interacts with other objects

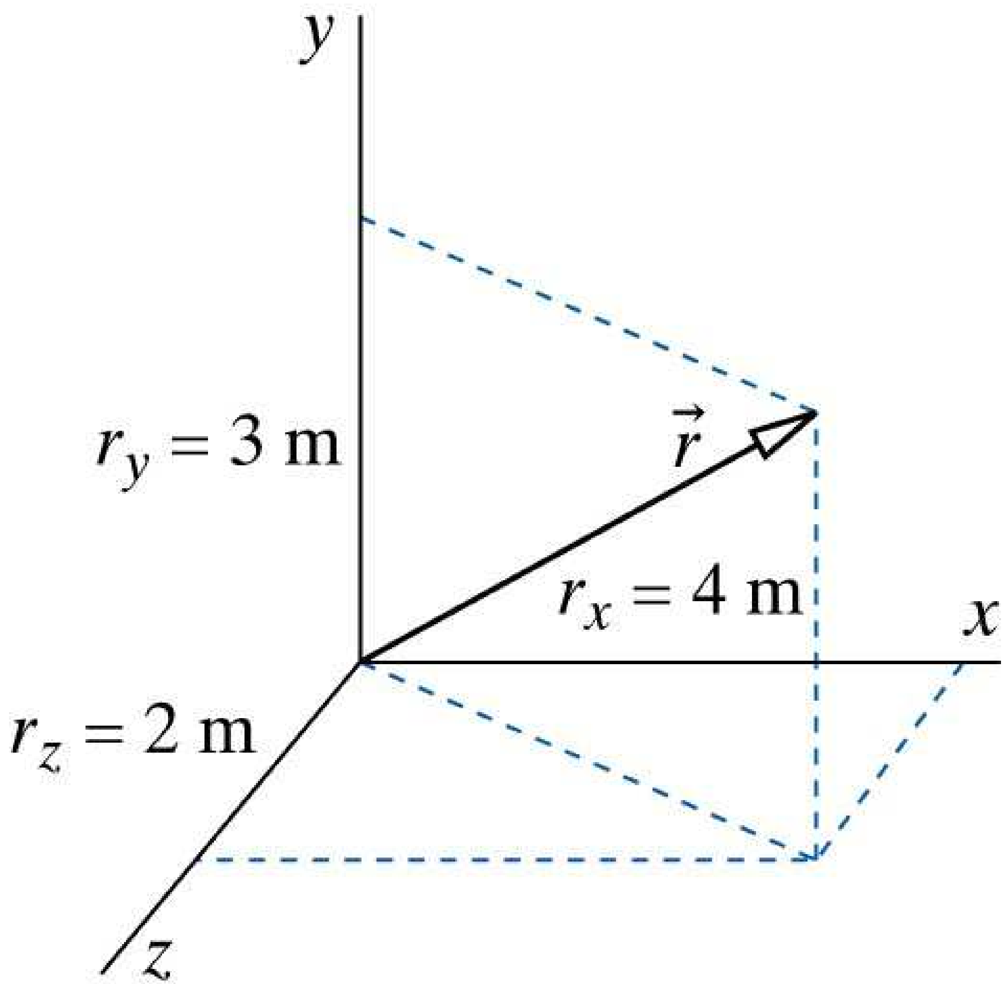
The stronger the interaction, the faster the change in velocity

- (Object at rest is a case of uniform velocity)

Does this match what you see in the REAL WORLD??

Vectors have Magnitude and Direction

In three dimensions and Cartesian coordinates:



Vectors

$$\vec{r} = \langle x, y, z \rangle = \langle r_x, r_y, r_z \rangle = r_x\hat{i} + r_y\hat{j} + r_z\hat{k} = |\vec{r}|\hat{r}$$

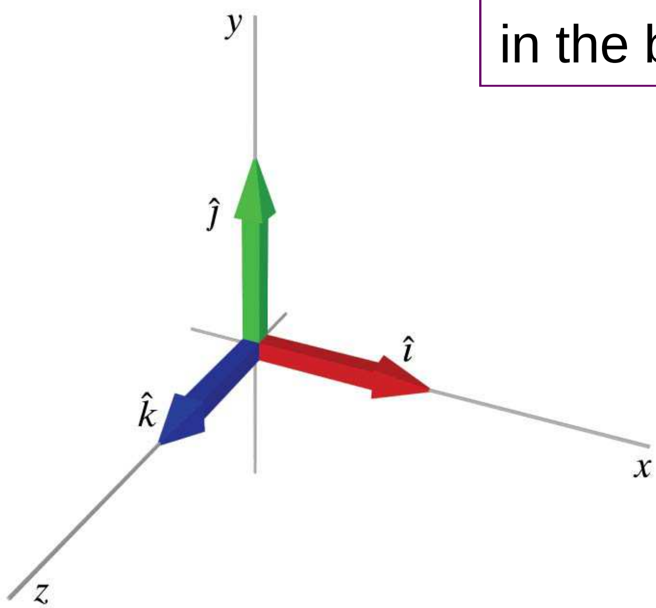
coolest trick in the book!

Unit vectors in the direction of the axes:

$$\hat{i} = \langle 1,0,0 \rangle$$

$$\hat{j} = \langle 0,1,0 \rangle$$

$$\hat{k} = \langle 0,0,1 \rangle$$

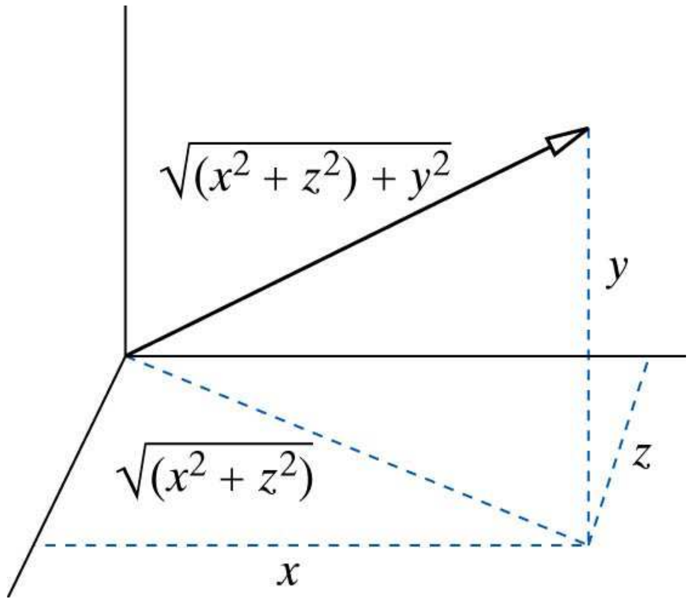


General unit vector:

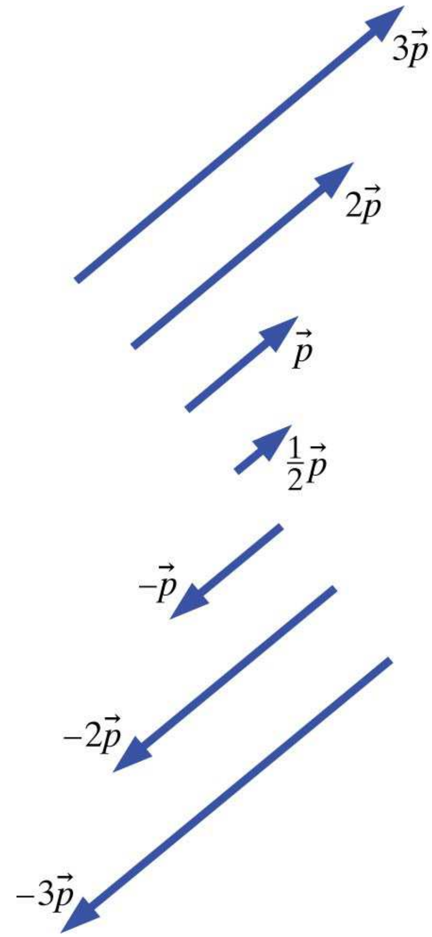
$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\langle x, y, z \rangle}{\sqrt{(x^2 + y^2 + z^2)}}$$

HOME READING: OPERATIONS WITH VECTORS

Vectors



vector magnitudes



scalar multiplication

Velocity has Magnitude and Direction

Magnitude of Velocity = **Speed** (a scalar)

100 m in 10 s

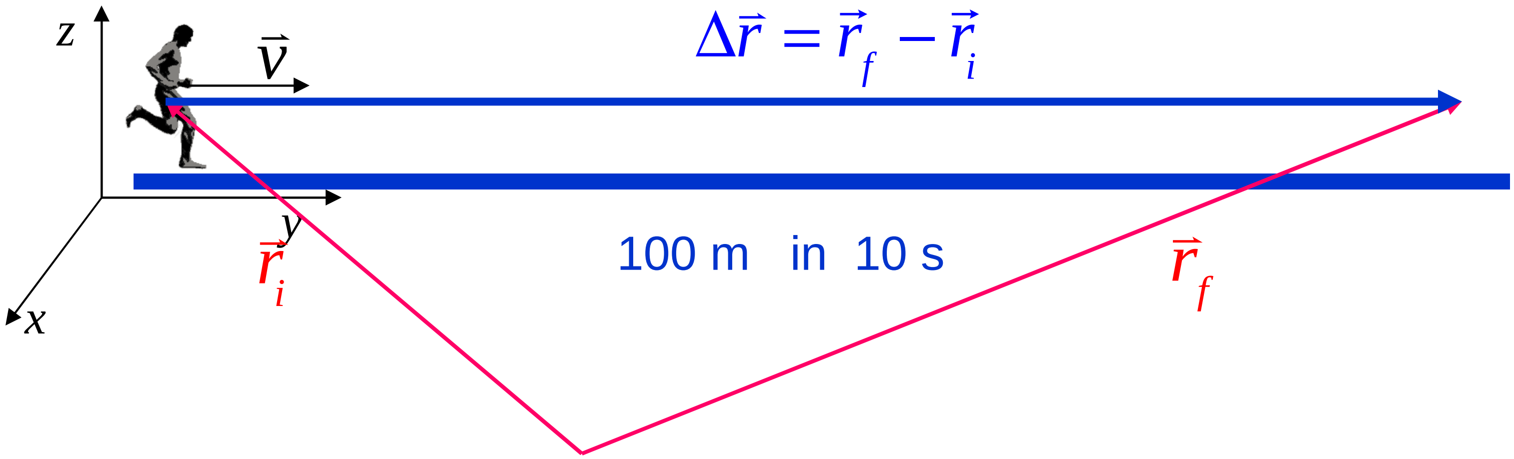
Average speed: $v_{avg} = \dfrac{d}{t} = \dfrac{100\text{ m}}{10\text{ s}} = 10\text{ m/s}$

If we know speed we can predict future: $d = v_{avg}t = 10\text{ m/s} \cdot 10\text{ s} = 100\text{ m}$

If we know speed we can reconstruct past: $t = \dfrac{d}{v_{avg}} = \dfrac{100\text{ m}}{10\text{ m/s}} = 10\text{ s}$

Velocity has Magnitude and Direction

Velocity is a Vector

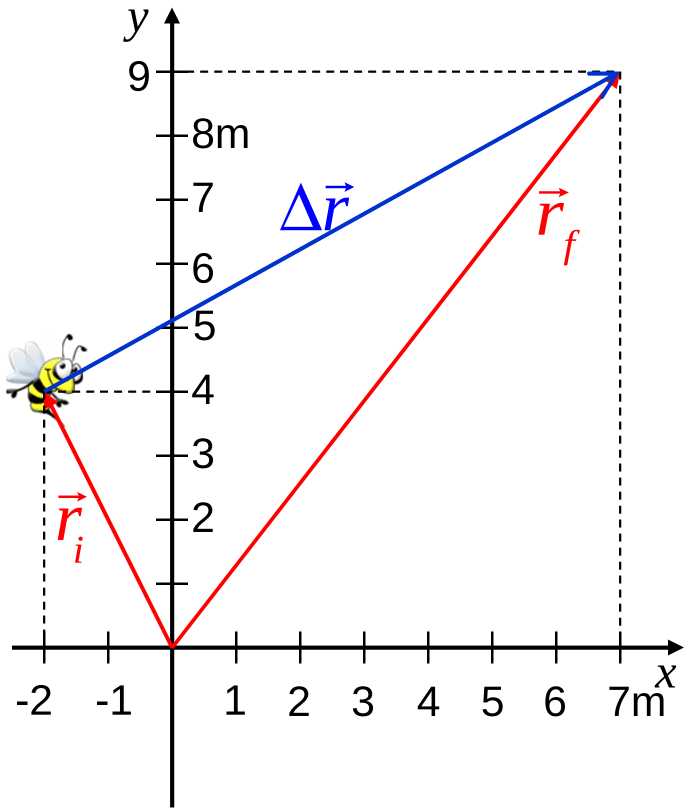


$$v_{avg} = \frac{d}{t} \longrightarrow$$

Definition:
average velocity

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} \equiv \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \equiv \left\langle \frac{\Delta r_x}{\Delta t}, \frac{\Delta r_y}{\Delta t}, \frac{\Delta r_z}{\Delta t} \right\rangle$$

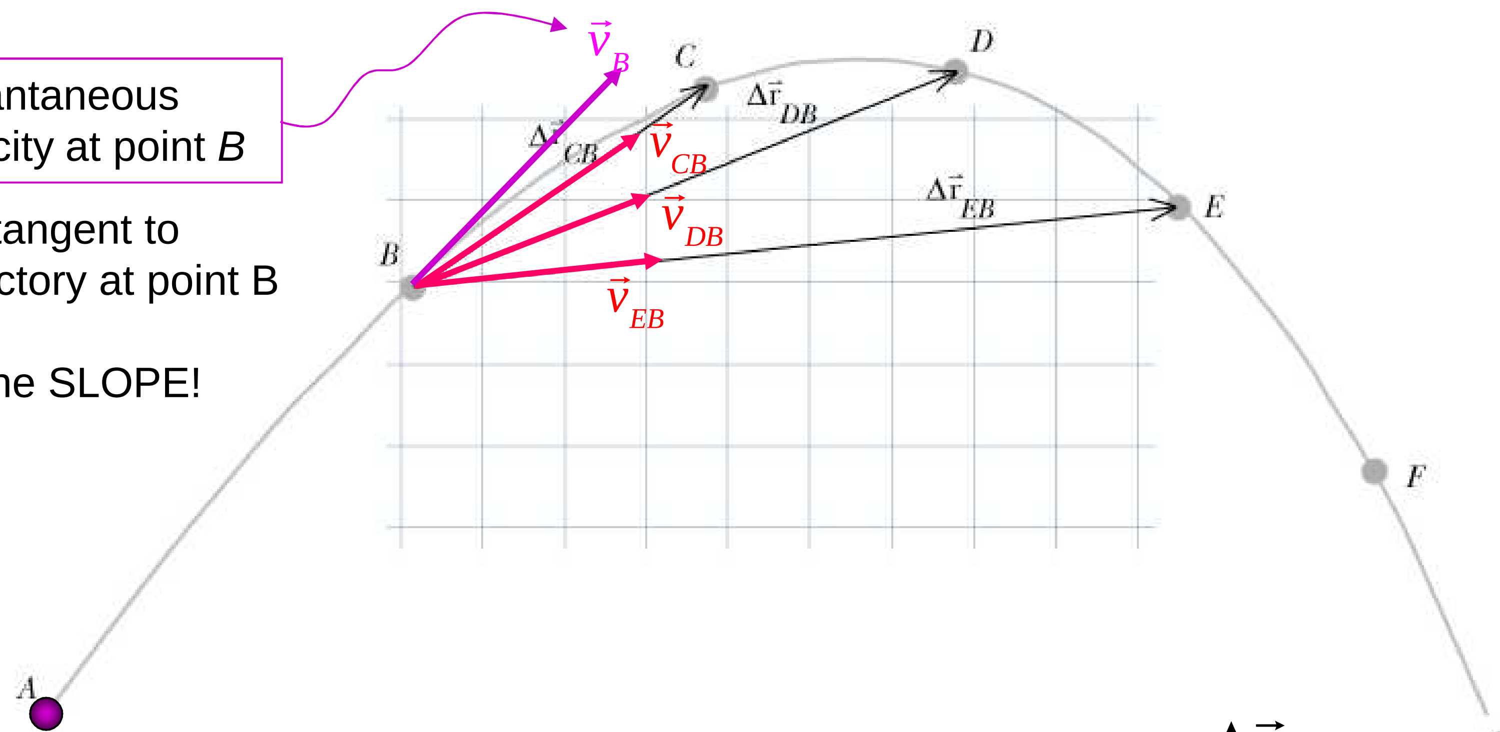
Example



$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} \equiv \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

Instantaneous vs. average velocity

The trajectory of a ball through air:



Instantaneous velocity at point *B*

It is tangent to trajectory at point B

It's the SLOPE!

The average velocity will depend on the choice of $\Delta\vec{r}$ and Δt

Instantaneous velocity:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} \equiv \frac{d\vec{r}}{dt}$$

derivative

Acceleration = Change in Velocity

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Express $\vec{v}$ as $\vec{v} = |\vec{v}|\hat{v}$

Now use the chain rule to take the derivative:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(|\vec{v}|\hat{v}\right)$$

Rate of change of direction

$$= \left(\frac{d|\vec{v}|}{dt}\right)\hat{v} + |\vec{v}|\left(\frac{d\hat{v}}{dt}\right)$$

Rate of change of magnitude of velocity (speed) is parallel to the velocity.

Predicting new position

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} \equiv \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

$$\downarrow$$

$$\vec{r}_f - \vec{r}_i = \vec{v}_{avg}\left(t_f - t_i\right)$$

$$\downarrow$$

The position update formula

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\left(t_f - t_i\right)$$

Interactions: changing velocity

Newton's first law of motion is qualitative:

An object moves in a straight line and at constant speed except to the extent that it interacts with other objects

Interactions can change velocity!

? What factors make it difficult to change an object velocity? $\rightarrow$ Mass!

Introduce new parameter that involves product of mass and velocity: *momentum*

The stronger the interaction, the larger the change in *momentum*

The simplest way: $\vec{p} = m\vec{v}$

(Legal Disclaimer: there's more to momentum for objects near the speed of light! Void in NH.)

Average rate of change of momentum

The stronger the interaction, the faster is the change in the momentum

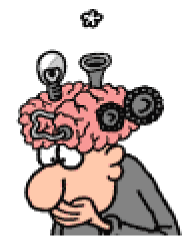
Average rate of change of momentum: $$\boxed{\frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{t_f - t_i}}$$

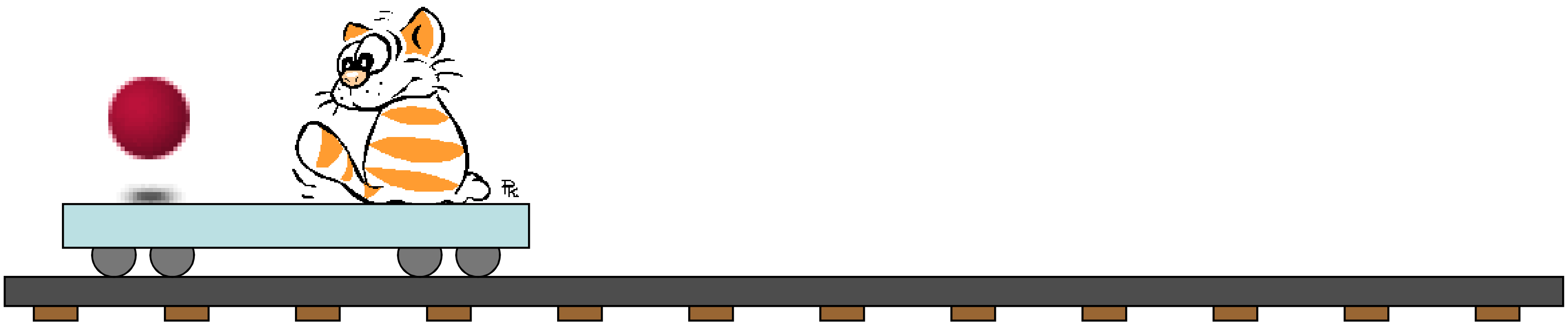
Units: $$\frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2}$$

Instantaneous rate of change of momentum: $$\boxed{\frac{d\vec{p}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}}$$

The principle of relativity

Physical laws work in the same way for observers in uniform motion as for observer at rest





Inertial reference frame

Inertial frame moves at constant velocity.

Physical laws work in the same way in any inertial frame

Are you in an inertial reference frame right now?

Special theory of relativity

Inertial frame moves at constant velocity.

Speed of light = constant in all inertial reference frames!

SPACE AND TIME WARP
TO ENSURE THIS STAYS TRUE

Time dilation: time runs slower in moving reference frames
Length contraction: object length becomes shorter in moving reference frame

Momentum – The Whole Story

Definition of momentum:

$$\vec{p} = \gamma m \vec{v}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

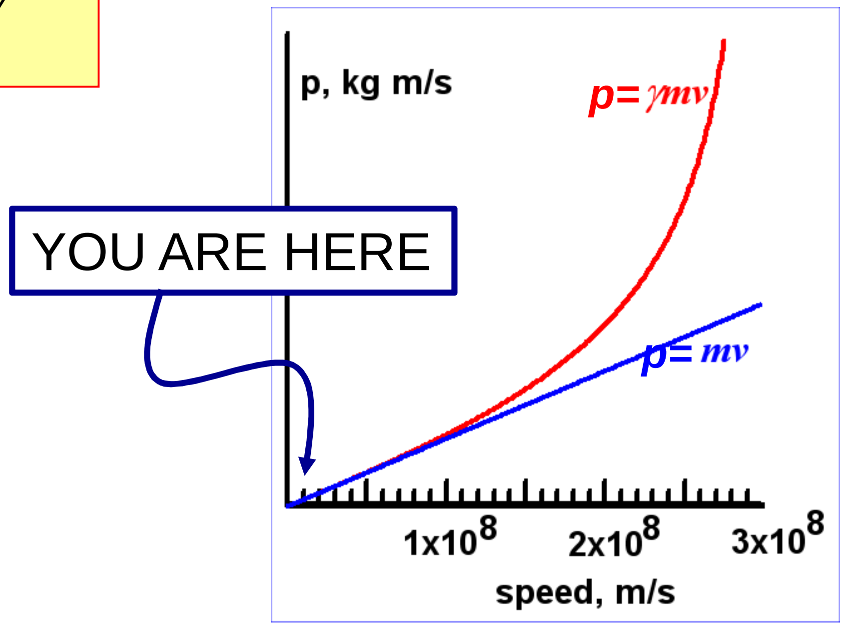
(Lorentz factor)

$c = \text{speed of light} \approx 3\times10^8 m/s$

For $v << c$, $\gamma \approx 1$, approximation: $\vec{p} = m\vec{v}$

v, m/s	γ
0	1
300	1.0000000000005
30,000	1.000000005
3×10^7	1.005
0.9999c	70.7

No mass can reach speed of light!



What We Did Today

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