PHYS 172 Problem Of the Week - Solution #2 (Spring 2012)

A falling rubber ball bounces off the floor. The velocity just before it hits the floor is <2.7,−5.2,0> m/s. Just after it hits the floor, the ball's velocity is <2.7,5.2,0> m/s (the y axis is the vertical axis). The ball's mass is 0.038 kg. The ball is in contact with the floor for only 1.8×10^{-3} seconds.

(a) What is the vector change in momentum of the ball from just before to just after it is in contact with the floor? Express your result as a vector.

Let us call the velocity just prior to hitting the floor $\vec{v}_{initial}$ and the velocity just after hitting the floor $\vec{v}_{final}$. Then we can express the change in momentum as:

$$\Delta\vec{p} = \vec{p}_{final} - \vec{p}_{initial} = m(\vec{v}_{final} - \vec{v}_{initial}) = 0.038 \cdot \langle (2.7-2.7), (5.2-(-5.2)), 0 \rangle = \langle 0, 0.40, 0 \rangle \ kg \cdot m/s$$

Alternatively, we could find the initial and the final momentum of the ball explicity and then take the difference to find the change in momentum:

$$\begin{aligned}
\vec{p}_{initial} &= m \cdot \vec{v}_{initial} = \langle (0.038 \cdot 2.7), (0.038 \cdot -5.2), 0 \rangle = \langle 0.1026, -0.1976, 0 \rangle \ kg \cdot m/s \\
\vec{p}_{final} &= m \cdot \vec{v}_{final} = \langle (0.038 \cdot 2.7), (0.038 \cdot 5.2), 0 \rangle = \langle 0.1026, 0.1976, 0 \rangle \ kg \cdot m/s \\
\Delta\vec{p} &= \vec{p}_{final} - \vec{p}_{initial} = \langle 0, 0.40, 0 \rangle \ kg \cdot m/s
\end{aligned}$$

[Notice that we have used the non-relativistic momentum. This is justified because the speed of the ball is much less than the speed of light.]

(b) What is the net force exerted on the ball during the time it is in contact with the floor? (You may assume the net force is approximately constant.) Express your result as a vector.

The change in momentum of the ball occurs while it is in contact with the floor for 1.8×10^{-3} seconds. According to the Momentum Principle:

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t} = \left\langle 0, \frac{0.4}{1.8 \times 10^{-3}}, 0 \right\rangle = \langle 0, 222.2, 0 \rangle \ N$$

(c) Calculate the ratio of the magnitude of the net force on the ball to the magnitude of the gravitational force (due to Earth) on the ball.

There are two forces (neglecting air resistance) acting on the ball – the force due to the floor and the force of gravity.

$$\vec{F}_{net} = (\vec{F}_{gravity} + \vec{F}_{floor})$$

We've already found $\vec{F}_{net}$.

$$\vec{F}_{gravity} = \langle 0, -0.038 \cdot 9.8, 0 \rangle = \langle 0, -0.372, 0 \rangle \ N$$

Therefore the ratio of the magnitude of the net force on the ball to the magnitude of the gravitational force on the ball due to the earth is:

$$\frac{\left|\vec{F}_{net}\right|}{\left|\vec{F}_{gravity}\right|} = \frac{222.2}{.372} = 597$$

Just for fun, let's find the force on the ball due to the floor.

$$\vec{F}_{floor} = \vec{F}_{net} - \vec{F}_{gravity} = \langle 0, 222.2, 0 \rangle - \langle 0, -0.038 \cdot 9.8, 0 \rangle = \langle 0, 222.6, 0 \rangle \ N$$

As expected, the magnitude of the force due to the floor is slightly greater than the net force. Why?

(d) How far did the ball move horizontally during the time it was in contact with the floor? (That is, what is the x-component of the ball's change in position during this time?)

$$\Delta x = v_{x,average} \Delta t$$

where $v_{x,average}$ is the average velocity in the x-direction over the time interval Δt. Since the x-component of ball's velocity did not change over the time interval, $v_{x,average} = v_x = 2.7 \ m/s$. Thus, over the time interval, the ball moved

$$\Delta x = v_x \Delta t = 2.7 \cdot 1.8 \times 10^{-3} = 0.005 \ m$$

along the positive x-axis.