MA 162 Exam 3 Spring 2005

Name: ______________________________

Student ID: ______________________________

Lecturer: ______________________________

Recitation Instructor: ______________________________

Recitation Time: ______________________________

Instructions:

1. This package contains 11 problems worth 9 points each.
2. Please supply all information requested above. You get 1 point for supplying all information correctly.
3. Work only in the space provided, or on the backside of the pages. Circle your choice for each problem in this booklet.
4. No books, notes, or calculator, please.

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\, x^n, \quad |x| < 1$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\, x^{2n+1}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\, x^{2n}$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n, \; |x| < 1$$

1. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$.

   A. Both series are conditionally convergent.

   B. Both series are absolutely convergent.

   C. The first is conditionally convergent, the second is absolutely convergent.

   D. The first is absolutely convergent, the second is conditionally convergent.

   E. The first is divergent, the second is absolutely convergent.

2. $\sum_{k=1}^{\infty} \frac{3^k}{2k^2 \cdot 2^k}$ is

   A. convergent by comparison with $\sum_{k=1}^{\infty} \frac{3^k}{2^k}$

   B. divergent by comparison with $\sum_{k=1}^{\infty} \frac{3^k}{2^k}$

   C. divergent by ratio test

   D. convergent by ratio test

   E. the ratio test, applied to the series, is inconclusive

3. What is the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{2 \cdot 4 \cdot 6 \cdot (2n)}$?

A. -1

B. 0

C. 1

D. 2

E. ∞

4. Given that the series $\sum_{k=1}^{\infty} \frac{x^k}{k 2^k}$ has radius of convergence 2, what is its interval of convergence?

A. $(-2, 2)$

B. $[-2, 2)$

C. $(-2, 2]$

D. $[-2, 2]$

E. None of the above.

5. The function $\dfrac{x^2}{1+2x}$ is represented by the power series

A. $\sum\limits_{n=0}^{\infty} (-1)^n 2^n x^{n+2}$

B. $\sum\limits_{n=1}^{\infty} n^2 x^n$

C. $\sum\limits_{n=1}^{\infty} (-1)^n 2n^2 x^n$

D. $\sum\limits_{n=0}^{\infty} (-\frac{1}{2})^n x^{2n}$

E. $\sum\limits_{n=0}^{\infty} n(-\frac{1}{2})^n x^{2n}$

6. If $f(x) = \sum\limits_{n=1}^{\infty} \dfrac{x^{2n+1}}{2n+1}$, then $\int_0^{\frac{1}{2}} f(x)dx =$

A. $\sum\limits_{n=1}^{\infty} \dfrac{(-\frac{1}{2})^n}{2n+1}$

B. $\sum\limits_{n=1}^{\infty} \dfrac{1}{(2n+1)(2n+2)2^{2n+2}}$

C. $\sum\limits_{n=1}^{\infty} \dfrac{1}{(n^2+n)2^{2n+1}}$

D. $\sum\limits_{n=1}^{\infty} \ln(2n+1)x^{2n+1}$

E. The integral is divergent

7. If e^x is expanded as a power series of the form $\sum_{n=0}^{\infty} c_n(x-1)^n$, then $c_4 =$

A. $\dfrac{1}{4!}$

B. $\dfrac{e^4}{4!}$

C. $\dfrac{e}{4!}$

D. $\dfrac{-1}{4!}$

E. $\dfrac{e^{-1}}{4!}$

8. $x\sin(x^2) =$

A. $\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+2}}{(2n+1)!}$

B. $\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+2}}{(2n)!}$

C. $\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{4n+2}}{(2n+1)!}$

D. $\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{4n+2}}{(2n)!}$

E. $\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{4n+3}}{(2n+1)!}$

9. Find the first three terms of the Maclaurin series of $f(x) = \sqrt{4+x^2}$.

A. $1 + \frac{1}{4}x + \frac{1}{64}x^2$

B. $2 + \frac{1}{4}x - \frac{1}{64}x^2$

C. $1 + \frac{1}{4}x^2 + \frac{1}{64}x^4$

D. $2 + \frac{1}{4}x^2 - \frac{1}{64}x^4$

E. $2 + \frac{1}{4}x^2 + \frac{1}{64}x^4$

10. How many terms of the Maclaurin series for $\ln(1+x)$ do you need to use to estimate $\ln(1.2)$ to within 0.001?

A. 2

B. 3

C. 4

D. 5

E. 6

11. Find a Cartesian equation of the curve with parametric equations $x = 2(\cos\theta - 1)$, $y = \sin\theta + 1$.

A. $(x+2)^2 + 4(y-1)^2 = 4$

B. $(x+2)^2 + 2(y-1)^2 = 1$

C. $(x+2) + (y-1)^2 = 1$

D. $(x+2) + 2(y-1)^2 = 2$

E. $(x+2)^2 + 2(y-1) = 4$