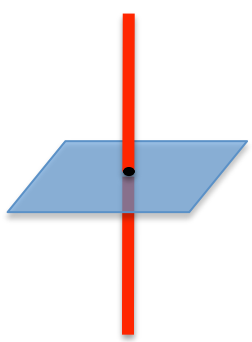
Last Time

- Charge Density
- Electric Field of a Charge Distribution
- Electric Field of a Charged Rod

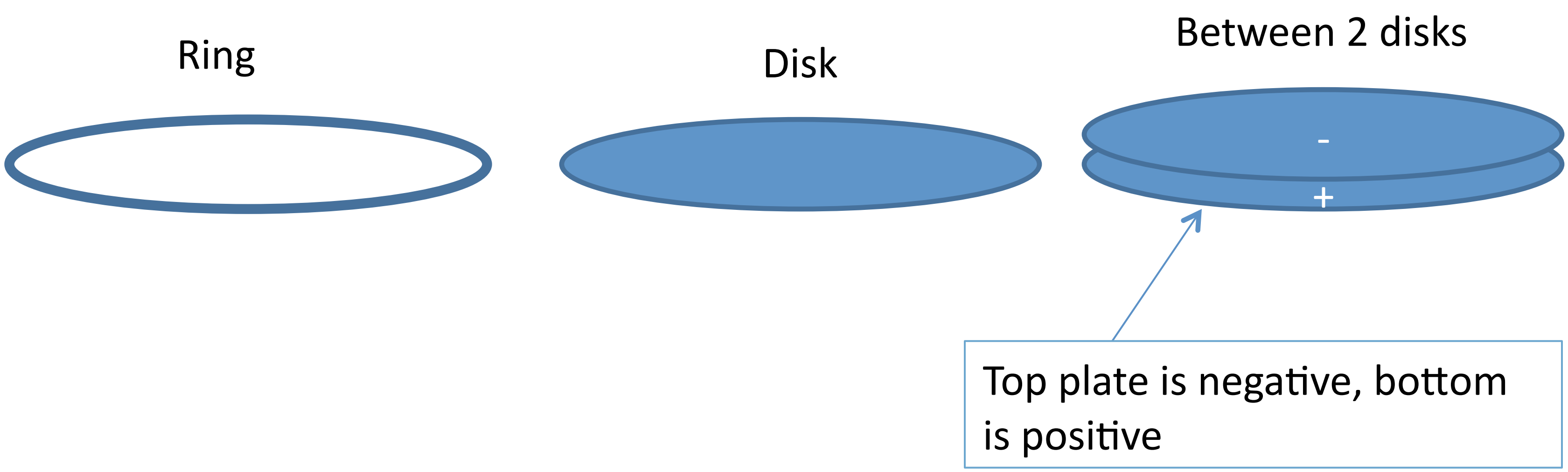


On the bisecting plane, $E \propto \dfrac{1}{r}$ close to the plane

For an infinite rod, $E \propto \dfrac{1}{r}$ everywhere

Today

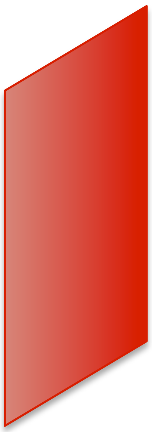
- Find the fields of:



Points, Lines, and Planes

Point Charge $E \propto \frac{1}{r^2}$

∞ Line Charge $E \propto \frac{1}{r}$



∞ Plane Charge

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Points, Lines, and Planes

Point Charge $$E \propto \frac{1}{r^2}$$

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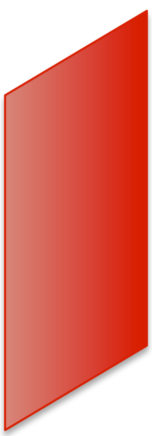
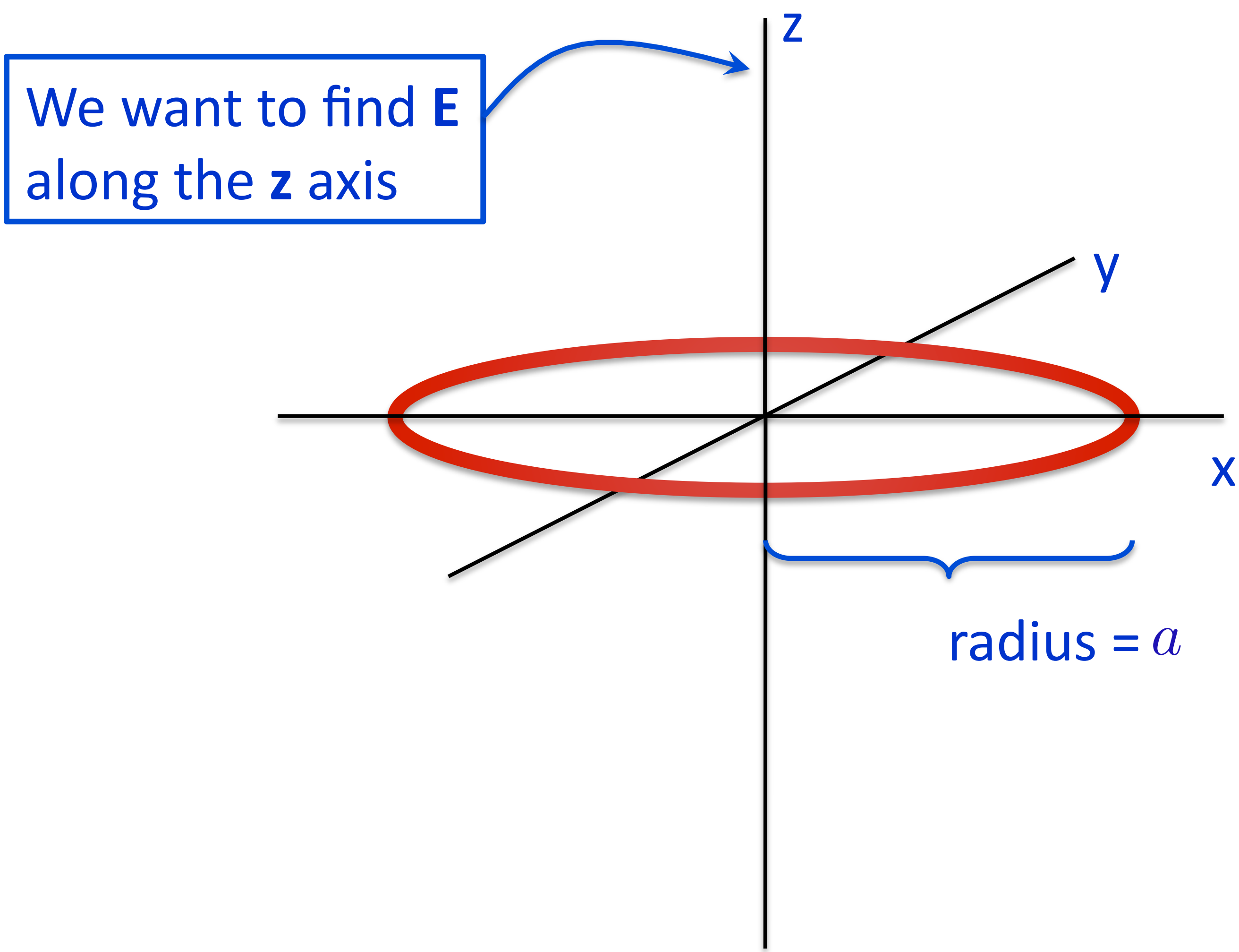


∞ Plane Charge $$\boxed{E \propto \text{constant!}}$$

We will show this

GOAL: E along *axis* of a ring



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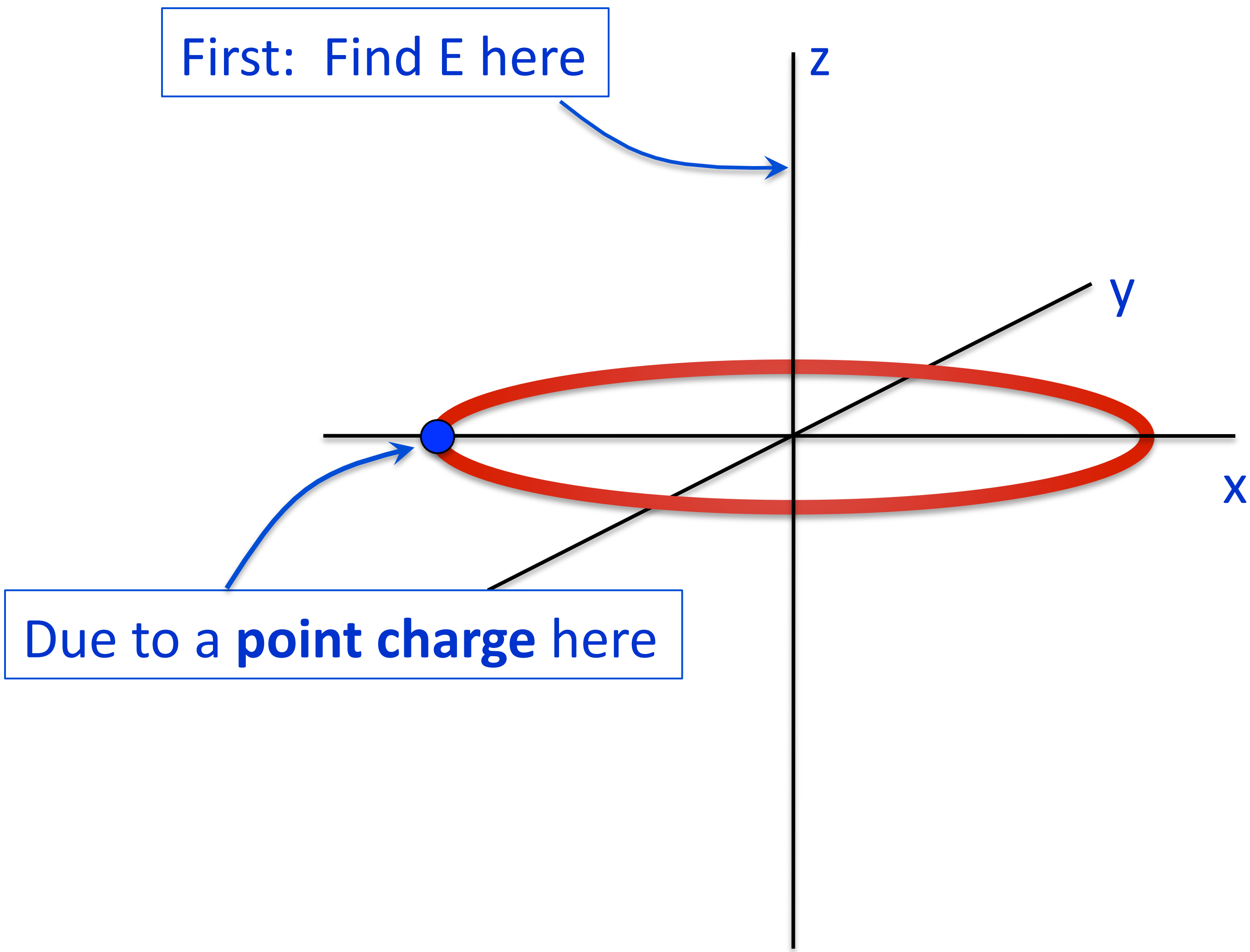

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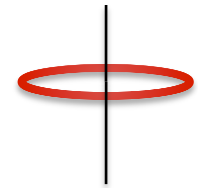
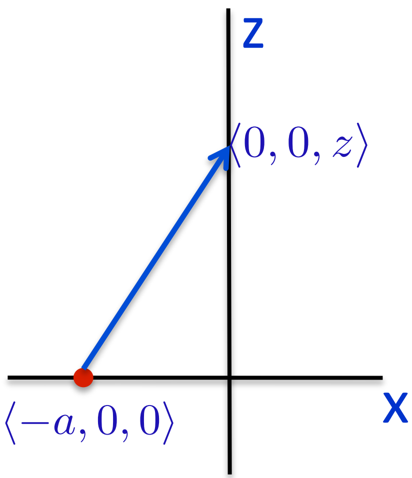


- Point Charge $\left.\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{|r|^2}\hat{r}\right\}$ $\vec{E}$ at $\vec{r}$ due to q at $\langle 0,0,0\rangle$



Place the charge at $\langle -a,0,0\rangle$

What is **E** at $\langle 0,0,z\rangle$?

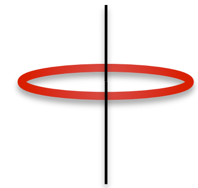
First find $|r|,\ \hat{r}$ for $\vec{r} = \langle a,0,z\rangle$

$$|r| = (a^2+z^2)^{1/2} \qquad \hat{r} = \frac{\vec{r}}{|r|} = \frac{\langle a,0,z\rangle}{(a^2+z^2)^{1/2}}$$

E at $\langle 0,0,z\rangle$: $\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{|r|^2}\hat{r}$

$$= \frac{1}{4\pi\epsilon_o}\frac{q}{(a^2+z^2)}\frac{\langle a,0,z\rangle}{(a^2+z^2)^{1/2}} = \frac{1}{4\pi\epsilon_o}\frac{q}{(a^2+z^2)^{3/2}}\langle a,0,z\rangle$$

GOAL: E along *axis* of a ring



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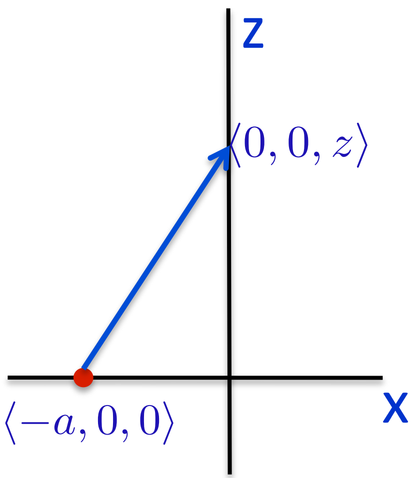

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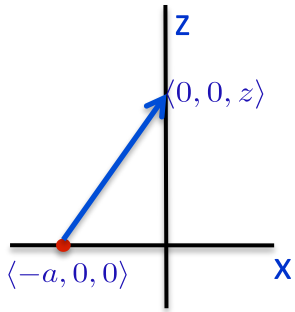
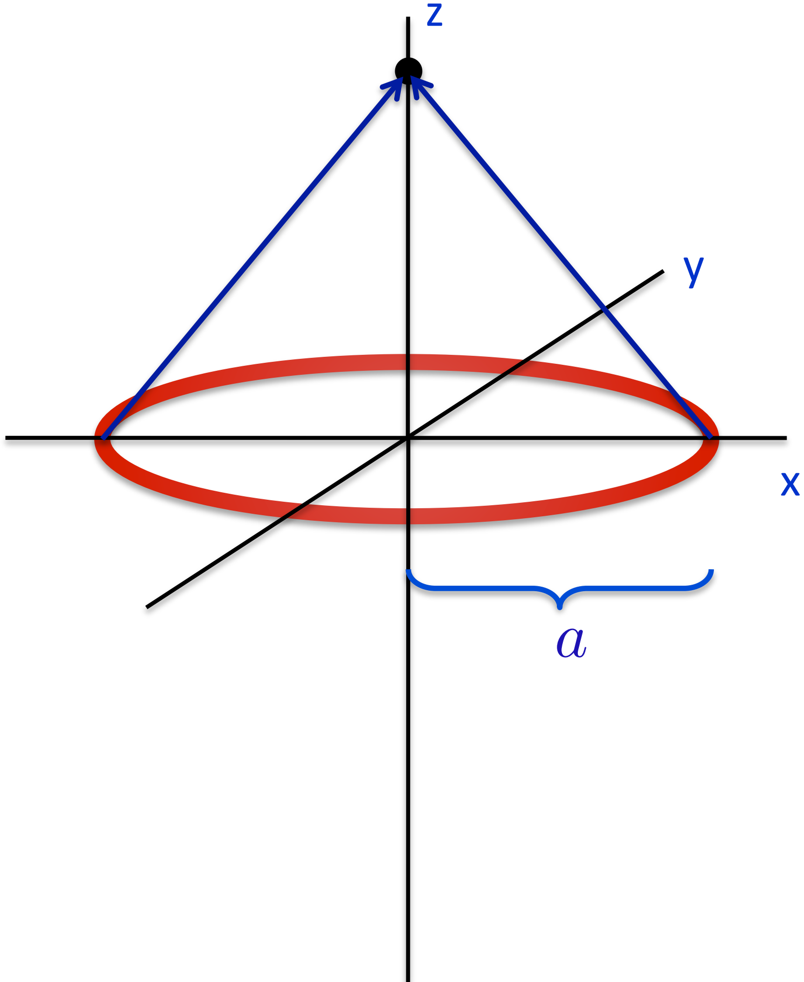
$$= \frac{1}{4\pi\epsilon_o}\underbrace{\frac{q}{(a^2+z^2)}\frac{\langle a,0,z\rangle}{(a^2+z^2)^{1/2}}}_{1+\frac{1}{2}=\frac{3}{2}} = \boxed{\frac{1}{4\pi\epsilon_o}\frac{q}{(a^2+z^2)^{3/2}}\langle a,0,z\rangle = \vec{E}}$$



GOAL: E along *axis* of a ring

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{(a^2+z^2)^{3/2}}\langle a, 0, z\rangle$$



We have point charges all around the ring

Symmetry: only the z component survives!

$$\Delta E_z = \frac{1}{4\pi\epsilon_o}\frac{z\Delta Q}{(a^2+z^2)^{3/2}}$$

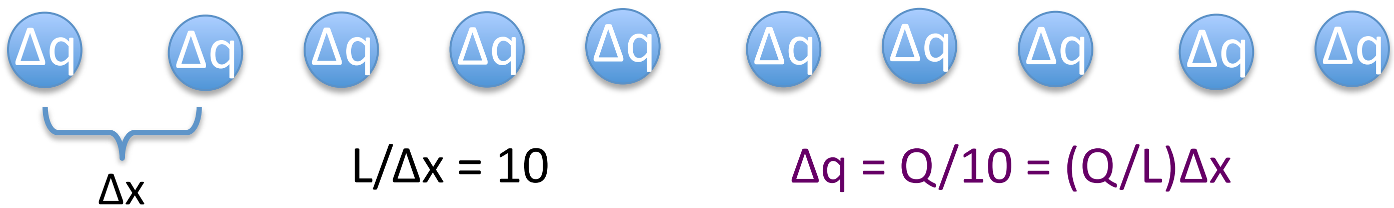
How do we set up the integral?

$$\sum \Delta Q \;\rightarrow\; \int d\theta$$

How do we set up the integral?

We need $\sum \Delta Q \;\to\; \int d\theta$

Calculus We Will Need



$L/\Delta x = 10$ $\Delta q = Q/10 = (Q/L)\Delta x$

Recall how to convert a sum to an integral:

UNITS = [Length] → $\sum \Delta x \rightarrow \int dx$ ← UNITS = [Length]

We will need to sum over all charges:

$$\boxed{\sum \Delta q = \frac{Q}{L} \sum \Delta x \rightarrow \frac{Q}{L} \int dx}$$

UNITS = [Charge]

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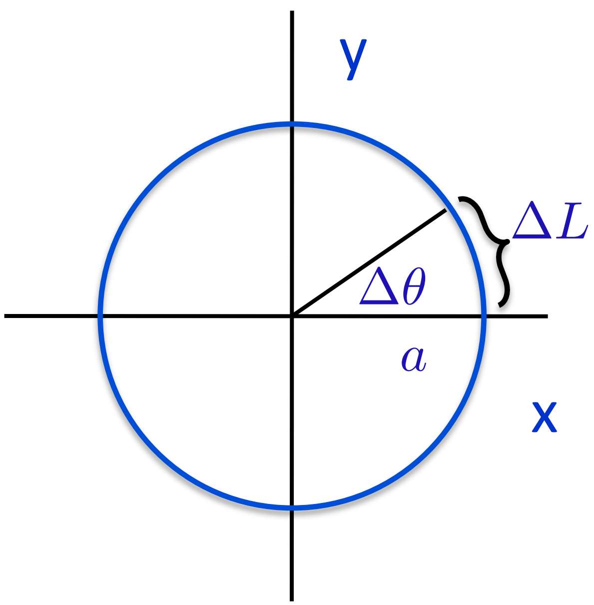
How do we set up the integral?

We need $\sum \Delta Q \rightarrow \int d\theta$

Ring of radius a has total "length" $L = 2\pi a$

We need this in terms of θ

Charge q uniformly distributed as $\Delta Q = \frac{q}{L}\Delta L = \frac{q}{2\pi a}\Delta L$



Circumference: $L = a2\pi$ All the way around

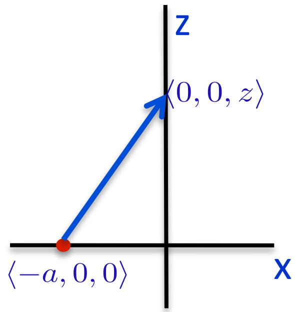
Arclength: $\Delta L = a\Delta\theta$ Partway around the circle

Doublecheck: $\Sigma\Delta L \rightarrow a\int_0^{2\pi}\theta d\theta = 2\pi a = L$

$$\Rightarrow \Delta Q = \frac{q}{2\pi a}a\Delta\theta = \frac{q}{2\pi}\Delta\theta$$

$$\boxed{\Sigma\Delta Q \rightarrow \frac{q}{2\pi}\int_0^{2\pi} d\theta}$$

GOAL: E along *axis* of a ring



$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{(a^2+z^2)^{3/2}}\langle a,0,z\rangle$$

$$\Sigma\Delta Q \ \to\ \frac{q}{2\pi}\int_0^{2\pi} d\theta$$

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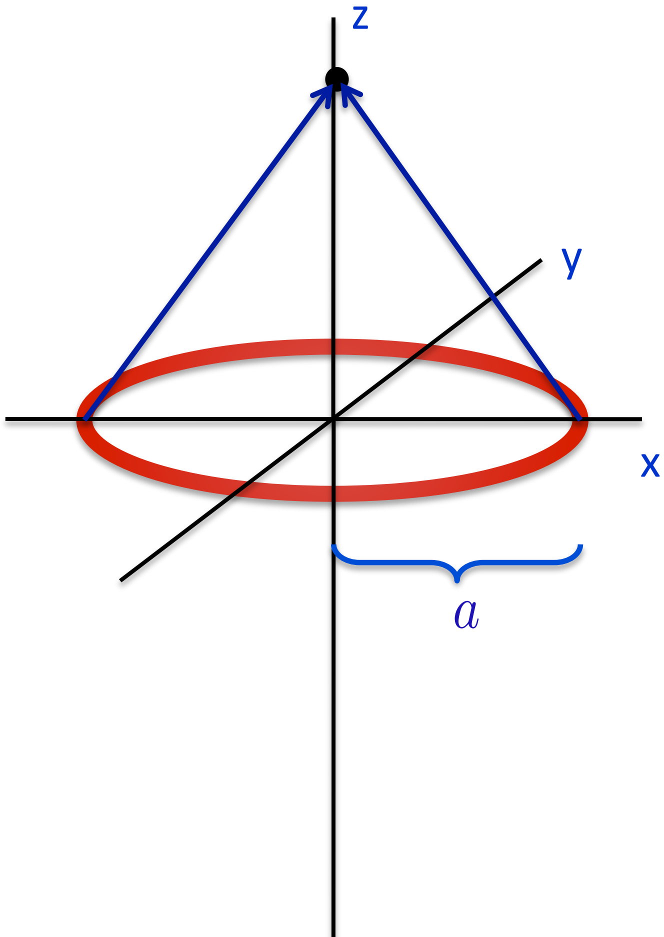
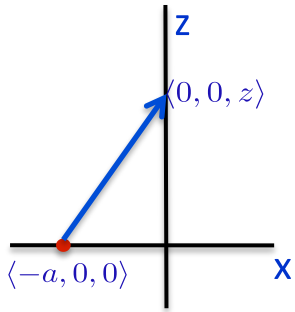


$$\Delta E_z = \frac{1}{4\pi\epsilon_o}\frac{z\Delta Q}{(a^2+z^2)^{3/2}}$$

$$E_z^{\text{tot}} = \frac{1}{4\pi\epsilon_o}\sum\frac{\Delta Q z}{(a^2+z^2)^{3/2}} \ \to\ \frac{1}{4\pi\epsilon_o}\frac{q}{2\pi}\int_0^{2\pi}\frac{z\,d\theta}{(a^2+z^2)^{3/2}}$$

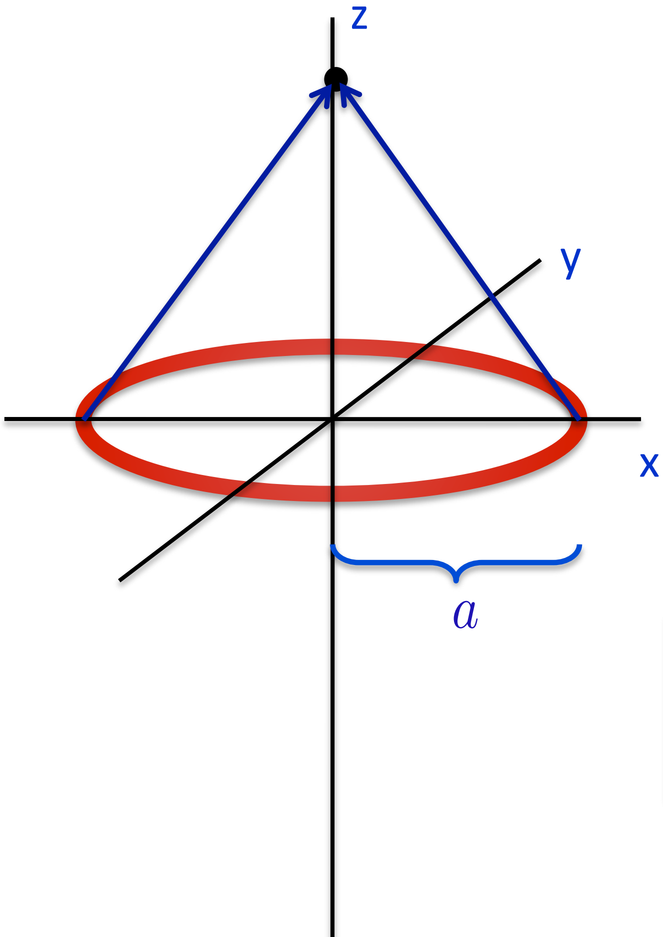
constant or θ-dependent?

GOAL: E along *axis* of a ring



$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{(a^2+z^2)^{3/2}}\langle a, 0, z\rangle$$

$$\Sigma\Delta Q \;\rightarrow\; \frac{q}{2\pi}\int_0^{2\pi} d\theta$$



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$$E_z^{\text{tot}} = \frac{1}{4\pi\epsilon_o}\sum\frac{\Delta Q z}{(a^2+z^2)^{3/2}} \;\rightarrow\; \frac{1}{4\pi\epsilon_o}\frac{q}{2\pi}\int_0^{2\pi}\frac{z\,d\theta}{(a^2+z^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_o}\frac{q}{2\pi}\frac{z}{(a^2+z^2)^{3/2}}\int_0^{2\pi} d\theta$$

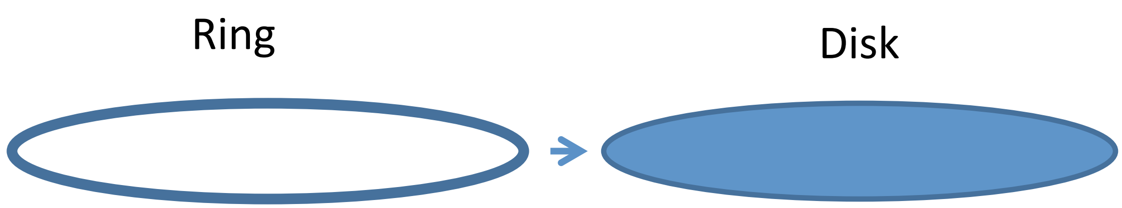
$$\boxed{E_z^{\text{tot}} = \frac{1}{4\pi\epsilon_o}\frac{qz}{(a^2+z^2)^{3/2}}}$$

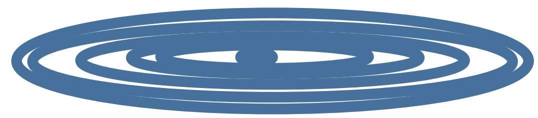
Field along
axis of a ring

iClicker question

Going from a ring to a disk

- How can we use the field of a ring to find the field of a disk?



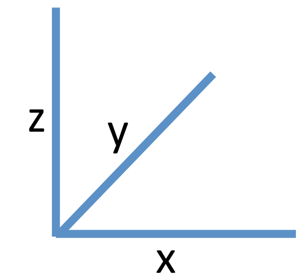


- We could build up the disk by adding the field a series of concentric rings. How do we do this addition?
- Integration!

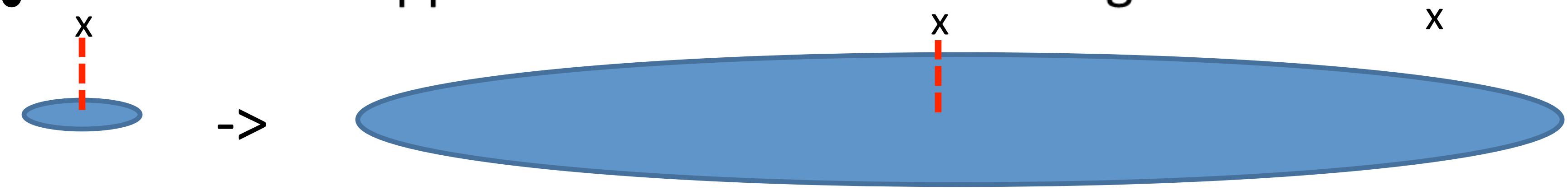
Field from a disk

- We can carry out the integration of the field from a ring for the radius of the ring going from 0 to R
- The (uniform) charge density of the disk is Q/A
- Therefore, the charge on each ring is 2πr* Q/A
- $\vec{E}_{disk} = \int_0^R \frac{2Qrz}{4A\epsilon_0(r^2+z^2)^{3/2}}\hat{z}\, dr = \frac{Q}{2A\epsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]\hat{z}$
- Use mathematica or something for the integral
- When R is big compared to z, $\sqrt{R^2+z^2} \approx R$
- $\vec{E}_{disk} \approx \frac{Q}{2A\epsilon_0}\left[1 - \frac{z}{R}\right]\hat{y}$

Infinite disk



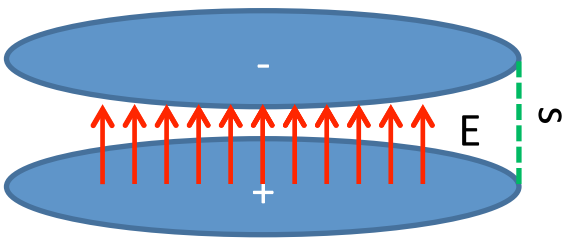
- Note what happens if we make the disk large



- Close up, it starts to look more and more like and infinite plane
- For an infinite plane, there is no center, motion along the x and y axes is meaningless
- It becomes a 1d problem (z axis), so as in lecture 1, E is constant
- $\lim_{r\to\infty} \frac{Q}{2A\epsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]\hat{z} = \frac{Q}{2A\epsilon_0}\hat{z}$

Field between two disks

- The hard part is done—we just have to add the field from two individual disks
- Between the disks the fields point the same way, if they have opposite charge (up)
- For a small separation, s, the field is very uniform (close to the field at center)
- Consider a point a distance z from the bottom plate. Distance from top = s-z

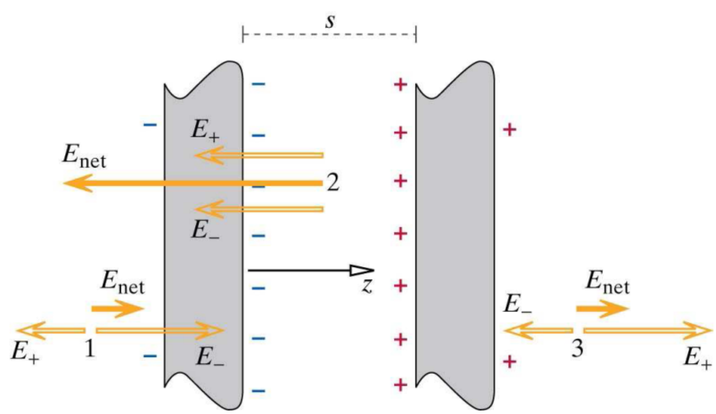


- $\vec{E} = \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{R}\right]\hat{z} + \frac{Q/A}{2\epsilon_0}\left[1 - \frac{s-z}{R}\right]\hat{z}$
- $= \frac{Q/A}{\epsilon_0}\left[1 - \frac{s}{2R}\right]\hat{z} \approx \frac{Q/A}{\epsilon_0}\hat{z}$

iClicker question

Capacitor

- A capacitor is just two oppositely charged plates next to each other, like on previous slide
- $E \approx \frac{Q/A}{\epsilon_0}$ inside the capacitor, perpendicular to plates
- Fringe field at locations 1 and 3, due to the fact that the two plates are different distances away: fields don't quite cancel
- $E_{Fringe} = \frac{Q/A}{\epsilon_0}\frac{s}{R}$



Today

- Find the fields of: