COS 226	Algorithms and Data Structures	Spring 2012

Midterm Solutions

1. **Analysis of algorithms.**

   (a) $\frac{3}{800000}N^3$

   (b) N^3

2. **Data structure and algorithm properties.**

   (a)

E	*Min* height of a binary heap with N keys.	A. ~ 1
E	*Max* height of a binary heap with N keys.	B. $\sim \frac{1}{2}\lg N$
C	*Min* height of a 2-3 tree with N keys.	C. $\sim \log_3 N$
E	*Max* height of a 2-3 tree with N keys.	D. $\sim \ln N$
E	*Min* height of left-leaning red-black BST with N keys.	E. $\sim \lg N$
F	*Max* height of left-leaning red-black BST with N keys.	F. $\sim 2\lg N$
A	*Min* height of a weighted quick union tree with N items.	G. $\sim 2\ln N$
E	*Max* height of a weighted quick union tree with N items.	H. $\sim N$

   (b) insertion sort and top-down mergesort are parsimonious

   Selection sort counterexample: `C B A`. *The keys* `B` *and* `C` *get compared twice, once in first iteration and once in second iteration.*

   Heapsort counterexample: `A B C`. *The keys* `B` *and* `C` *get compared twice, once in the heap construction phase (when sinking* `A`*) and once in the sortdown phase (when sinking* `C` *after* `A` *and* `C` *are exchanged).*

3. **Data structures.**

   (a)
   - Best case: $\sim 2N$
     When the array is full.
   - Worst case: $\sim 8N$
     When the array is one-quarter full.

   (b)

operation	*description*	*time*
`charAt(int i)`	*return the ith character in sequence*	1
`deleteCharAt(int i)`	*delete the ith character in the sequence*	N
`append(char c)`	*append c to the end of the sequence*	1
`set(int i, char c)`	*replace the ith character with c*	1

4. **8 sorting and shuffling algorithms.**

   7 9 3 5 4 2 6 8

5. **Red-black BSTs.**

   (a) U V W X

   (b) P S Y

   (c)

	`E`	`N`	`G`
`rotateLeft()`	1	1	1
`rotateRight()`	0	0	3
`flipColors()`	1	0	3

6. **Hashing.**

   (a)

0	1	2	3	4	5	6
G	D	B	F	E	A	C

   (b) I. Possible.
   Consider the order F D B G E C A.

   II. Impossible.
   No key is in the correct position.

   III. Impossible.
   We can assume B and G were inserted first since they are in correct position. But then third key inserted is guaranteed to be in correct position.

7. **Comparing two arrays of points.**

   (a)
   - Sort `a[]` using heapsort (using the point's natural order).
   - For each point `b[j]`, use binary search to search for it in the sorted array `a[]`, incrementing a counter if found.

   (b) $N \log M$.
   The running time is $M \log M$ for the sort and $N \log M$ for the N binary searches. Since $N \geq M$, the latter term is the bottleneck.

   (c) 1.
   Both heapsort and binary search use at most a constant amount of extra space.

8. **Randomized priority queue.**

- `sample()`: Pick a random array index `r` (between `1` and `N`) and return the key `a[r]`.
- `delRandom()`:
  - *Select:* pick a random array index r (between `1` and `N`) and save away the key `a[r]`, to be returned.
  - *Delete:* exchange `a[r]` and `a[N]` and decrement `N`.
  - *Restore heap order invariants:* call `sink(r)` and `swim(r)` to fix up any heap order violation at `r`. Note that `a[N]` in the original heap need not be the largest key, so the call to `swim(r)` is necessary.

```
public Key sample() {
    int r = 1 + StdRandom.uniform(N);  // between 1 and N
    return a[r];
}

public Key delRandom() {
    int r = 1 + StdRandom.uniform(N);  // between 1 and N
    Key key = a[r];                    // save away
    exch(r, N--);                      // to make deleting easy
    sink(r);                           // if a[N] was too big
    swim(r);                           // if a[N] was too small
    a[N+1] = null;                     // avoid loitering
    return key;
}
```