EE-202
Exam I
February 5, 2013

Name: ________________________________
(Please print clearly)

Student ID: ________________

CIRCLE YOUR DIVISION

Section 2022, 8:30 MWF — Prof. DeCarlo

Section 2021, 12:30 MWF — Prof. DeCarlo

Section 2023 1:30 TuTh — Prof. Meyer

INSTRUCTIONS

There are 10 multiple choice worth 5 points each and there are 2 workout problems worth a total of 50 points.

This is a closed book, closed notes exam. No scrap paper or calculators are permitted. A transform table will be handed out separately as well as the property table.

Carefully mark your multiple choice answers on the scantron form. Work on multiple choice problems and marked answers in the test booklet will not be graded

Nothing is to be on the seat beside you. Scantrons are to be under exam.

When the exam ends, all writing is to stop. This is not negotiable.

No writing while turning in the exam/scantron or risk an F in the exam.

All students are expected to abide by the customary ethical standards of the university, i.e., your answers must reflect only your own knowledge and reasoning ability. As a reminder, at the very minimum, cheating will result in a zero on the exam and possibly an F in the course.

Communicating with any of your classmates, in any language, by any means, for any reason, at any time between the official start of the exam and the official end of the exam is grounds for immediate ejection from the exam site and loss of all credit for this exercise.

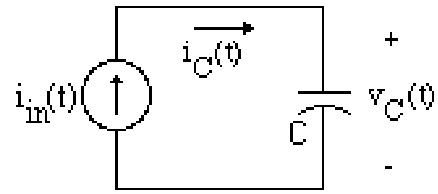
MULTIPLE CHOICE.

1. Consider the figure below in which $C = 0.1$ F and $v_C(0^-) = 0$. If $I_{in}(s) = \frac{0.2s}{s+a}$ then $v_C(t) =$ (in V):

(1) $e^{-at}u(t)$ **(2)** $2e^{-at}u(t)$ **(3)** $20e^{-at}u(t)$

(4) $2\delta(t) - 2ae^{-at}u(t)$ **(5)** $0.2\delta(t) - 0.2ae^{-at}u(t)$ **(6)** $20\delta(t) - 20ae^{-at}u(t)$

(7) $2e^{at}u(t)$ (8) None of above



Solution 1. (Ohm's law, capacitor impedance, inverse transform)

$V_C(s) = \frac{I_{in}(s)}{Cs} = \frac{0.2s}{(0.1s)(s+a)} = \frac{2}{s+a}$. $v_C(t) = 2e^{-at}u(t)$ V.

2. A circuit characterizing the decay of the ordinary attention span of a student in a DeCarlo circuit's class is $H(s) = \frac{24}{(s+2)(s+4)}$. The step response of the associated transfer function has a term of the form $Ae^{-2t}u(t)$ where $A =$:

(1) 12 (2) –12 (3) 3 (4) –3
(5) 4 (6) –4 (7) 6 (8) –6
(9) None of above

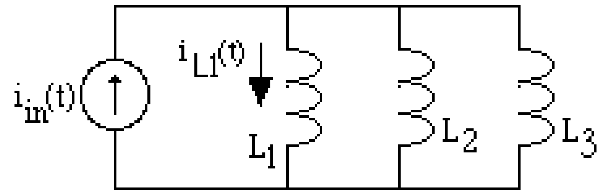
Solution 2. (Transfer function, step response, partial fraction expansion distinct poles, inverse transform)

$L[\text{StepResponse}] = \frac{H(s)}{s} = \frac{24}{s(s+2)(s+4)} = \frac{3}{s} - \frac{6}{s+2} + \frac{3}{s+4}$.

$\text{StepResponse} = \left[3 - 6e^{-2t} + 3e^{-4t}\right]u(t)$.

3. Consider the figures below in which $L_1 = 1$ H, $L_2 = 2$ H, and $L_3 = 2$ H. All initial conditions are zero. Suppose $I_{in}(s) = \frac{10s}{s^2+25}$. Then $i_1(t) =$ (in A):

(1) $5\cos(25t)u(t)$ **(2)** $5\sin(25t)u(t)$ **(3)** $10\cos(25t)u(t)$
(4) $5\cos(5t)u(t)$ **(5)** $5\sin(5t)u(t)$ **(6)** $10\sin(5t)u(t)$
(7) $10\cos(5t)u(t)$ **(8)** None of above



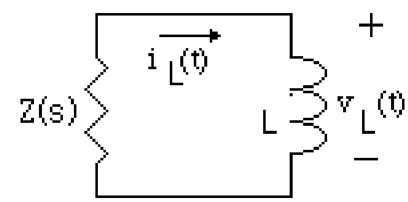
Solution 3. (Current division, ohms law) $I_{L1}(s) = \dfrac{\frac{1}{s}}{\left[\frac{2}{2s}+\frac{1}{2s}+\frac{1}{2s}\right]} \times I_{in}(s) = \dfrac{\frac{1}{s}}{\frac{2}{s}} \times I_{in}(s) = 0.5 I_{in}(s) = \dfrac{5s}{s^2+25}$.

4. Consider the circuit below in which $L = 2$ H, $i_L(0^-) = 2$ A, and $Z(s) = \dfrac{4s+2}{s}$. Note that we have used a resistor symbol to denote a general impedance. Then $i_L(t)$ has a term of the form $Ke^{-at}u(t)$ where $(K, a) =$:

(1) $(2,2)$ **(2)** $(2,-2)$ **(3)** $(2,1)$ **(4)** $(2,-1)$

(5) $(-2,2)$ **(6)** $(-2,-2)$ **(7)** $(-2,1)$ **(8)** $(-2,-1)$

(**9**) None of above



Solution 4.(Initial inductor current model, parallel impedance, ohm's law, inductor impedance) Using the series voltage source model of the inductor, we have that

$$I_L(s) = \frac{Li_L(0^-)}{Ls+Z(s)} = \frac{2\times 2}{2s+\frac{4s+2}{s}} = \frac{2s}{s^2+2s+1} = \frac{2s}{(s+1)^2} = \frac{2}{s+1} - \frac{2}{(s+1)^2}.$$ Hence

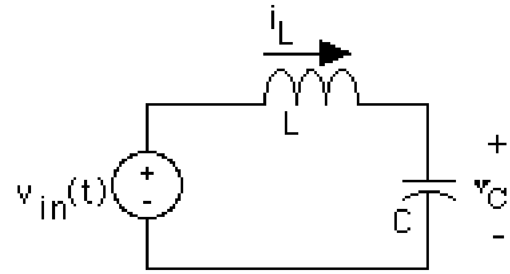
$i_L(t) = 2e^{-t}u(t) - 2te^{-t}u(t)$ A.

5. Consider the circuit below in which $C = 0.5$ F and $L = 0.5$ H. Suppose $v_{in}(t) = 20e^{-2t}u(t)$ and all initial conditions are ZERO. Then $v_C(t)$ has a term of the form $K\sin(\omega t)$ where $(K,\omega) =$ in (volts, rad/s):

(1) $(20,4)$ (2) $(20,2)$ (3) $(-10,4)$ (4) $(-10,2)$

(5) $(10,4)$ (6) $(10,2)$ (7) $(5,4)$ (8) $(5,2)$

(9) none of above



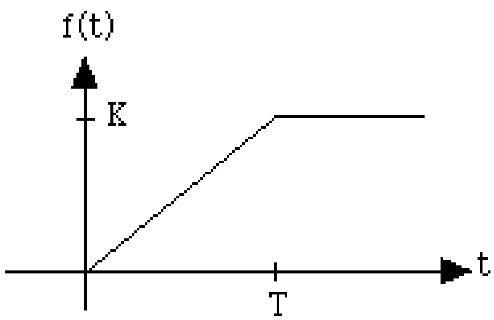
Solution 5. (Impedance, Voltage Division, Response calculation, partial fraction expansion with complex poles)

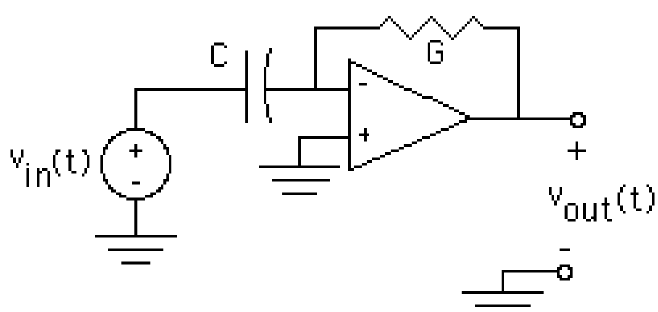
$$V_C(s)=\frac{\frac{1}{Cs}}{Ls+\frac{1}{Cs}}V_{in}(s)=\frac{\frac{1}{LC}}{s^2+\frac{1}{LC}}V_{in}(s)=\frac{20}{(s+2)}\times\frac{4}{s^2+4}$$

$=\frac{80}{(s+2)(s^2+4)}=\frac{10}{s+2}+\frac{-10s+20}{s^2+4}$. Thus $v_C(t)=10e^{-2t}u(t)-10\cos(2t)+10\sin(2t)$ V.

6. Consider the circuit below, driven by the input voltage $v_{in}(t)=f(t)$ V where $C=1$ F, $G=2\,\text{mho}$ (S), $K=12$, $T=2$, and the initial capacitor voltage is zero. Then $v_{out}(t)=$ (in V):

(1) $3u(t-2)-3u(t)$	**(2)** $3r(t-2)-3r(t)$	**(3)** $-3u(t-2)+3u(t)$
(4) $6u(t-2)-6u(t)$	**(5)** $6r(t-2)-6r(t)$	**(6)** $-3r(t-2)+3r(t)$
(7) $12r(t-2)-12r(t)$	**(8)** $12u(t-2)-12u(t)$	**(9)** None of above





Solution 6. (op amp transfer function, input decomposition in terms of ramps, inverse transform) The transfer function of the op amp circuit is $H(s)=-0.5s$. The input $v_{in}(t)=6r(t)-6r(t-2)$.

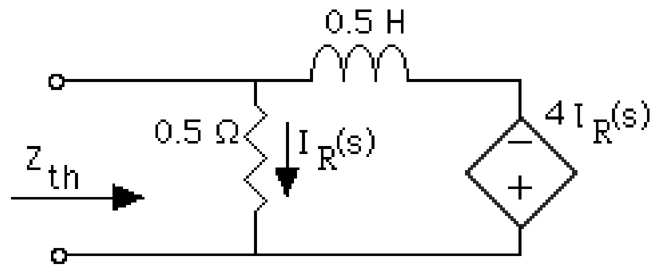
$V_{in}(s)=6\frac{1-e^{-2s}}{s^2}$. $V_{out}(s)=-3\frac{(1-e^{-2s})}{s}$. $v_{out}(s)=3u(t-2)-3u(t)$.

7. The Thevenin equivalent impedance seen by the circuit below is:

(1) $\dfrac{2s^2+18}{s}$ (2) $\dfrac{s}{2s+18}$ (3) $\dfrac{2s-14}{s}$ (4) $\dfrac{s}{2s-18}$

(5) $\dfrac{0.5s}{0.5s-1}$ (6) $\dfrac{2s+18}{s}$ (7) $\dfrac{s}{2s-6}$ (8) $\dfrac{2s^2-6}{s}$

(9) none of above



Solution 7. (Thevenin equivalent, node analysis, impedance, admittance)

$$I_{in}(s) = 2V_{in}(s) + \frac{V_{in}(s)+4I_R(s)}{0.5s} = 2V_{in}(s) + \frac{V_{in}(s)+8V_{in}(s)}{0.5s} = \left[2+\frac{2}{s}+\frac{16}{s}\right]V_{in}(s) = \frac{2s+18}{s}V_{in}(s)$$

Hence $Z_{in}(s) = \dfrac{s}{2s+18}$.

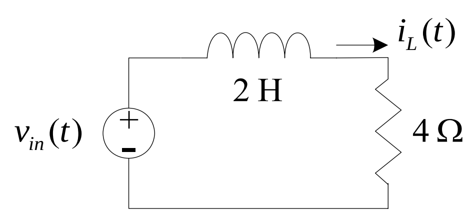
8. An impedance $Z(s) = \dfrac{20s}{2s+10}$. This impedance is known to be achieved by a parallel RL circuit. A possible choice for R and L is (in Ω and H):

(1) 0.1, 0.5 (2) 0.1, 2 (3) 10, 0.5 (4) 10, 2
(5) 0.5, 0.1 (6) 2, 0.1 (7) 0.5, 10 (8) 2, 10
(9) none of above

Solution 8. (Impedance and circuit realization.)

$$Z(s) = \frac{20s}{2s+10} = \frac{1}{0.1+\frac{1}{2s}} = \frac{1}{Y(s)}.$$ Thus $R = 10\ \Omega$ and $L = 2$ H.

9. Consider the circuit below in which $v_{in}(t) = \int_{0^-}^{t} 16e^{-2q}\cos(2q)u(q)dq$ V and all initial conditions are ZERO. Then $i_L(t)$= (in A):



(1) $[0.5-0.5(\cos(2t)+\sin(2t))e^{-2t}]u(t)$ (2) $8(\cos(2t)-\sin(2t))e^{-2t}u(t)$

(3) $[1-\left(\cos(2t)+\sin(2t)\right)e^{-2t}]u(t)$ (4) $[2-2(\cos(2t)+\sin(2))e^{-2t}]u(t)$

(5) $[16+16\left(\cos(2t)+\sin(2t)\right)e^{-2t}]u(t)$ (6) $4\sin(2t)e^{-2t}u(t)$

(7) $[2-2\cos(2t)]e^{-2t}u(t)$ (8) $[0.5-(0.5\cos(2t)-0.5\sin(2t)-1)e^{-2t}]u(t)$

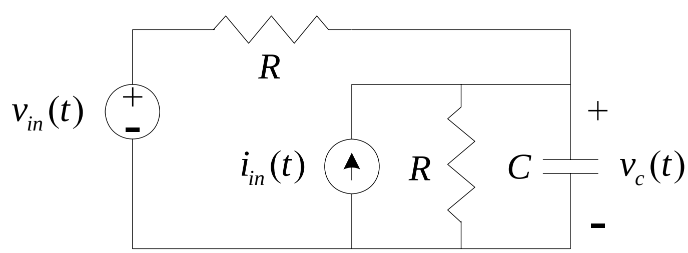
(9) None of above

Solution 9. (Integration property, Ohm's Law, partial fraction expansion)

$$V_{in}(s)=\frac{1}{s}\left(\frac{16(s+2)}{(s+2)^2+2^2}\right),\ I_L(s)=\frac{1}{s}\left(\frac{16(s+2)}{(s+2)^2+2^2}\right)\cdot\frac{1}{2(s+2)}=\frac{1}{s}\left(\frac{8}{(s+2)^2+2^2}\right),$$

$I_L(s)=\frac{1}{s}+\frac{-(s+2)}{(s+2)^2+2^2}+\frac{-2}{(s+2)^2+2^2}$. Thus $i_L(t)=[1-e^{-2t}\cos(2t)-e^{-2t}\sin(2t)]u(t)$

10. Consider the circuit below in which $R=0.2\ \Omega$, $C=1$ F, $v_{in}(t)=8u(t)$, $i_{in}(t)=2\delta(t)$, and all initial conditions are ZERO. Then $v_C(t)=$ (in V):



(1) $[4-4e^{-10t}]u(t)$ (2) $\left[\frac{4}{3}+2e^{-10t}-\frac{4}{3}e^{-6t}\right]u(t)$

(3) $[4+2e^{-5t}-4^{-10t}]u(t)$ (4) $42e^{-10t}$

(5) $[4+6e^{-10t}]u(t)$ (6) $[4-2e^{-10t}]u(t)$

(7) $[4+36e^{-10t}]u(t)$ (8) $[4-2e^{-5t}-4^{-10t}]u(t)$

(9) None of above

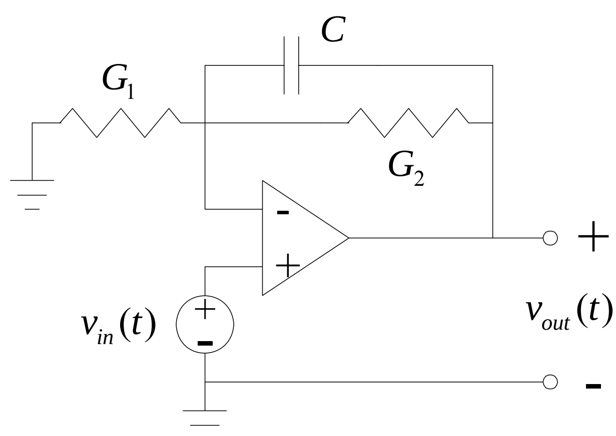
Solution 10. (Superposition, voltage division, Ohm's Law, partial fraction expansion)

Circuit I-source open: $V_c^1(s)=\frac{5}{s+10}V_{in}(s)=\frac{40}{s(s+10)}=\frac{4}{s}-\frac{4}{s+10}$

Voltage source shorted: $I_{in}(s)=2$, $Z_{eq}(s)=\frac{1}{s+10}$, $V_c^2(s)=Z_{eq}I_{in}(s)=\frac{1}{s+10}\cdot 2$,

$$V_c(s) = V_c^1(s) + V_c^2(s) = \frac{4}{s} - \frac{4}{s+10} + \frac{2}{s+10} = \frac{4}{s} - \frac{2}{s+10},$$
$$v_c(t) = [4 - 2e^{-10t}]u(t)$$

Workout problem 1. (20 pts)
In the circuit below, suppose $C = 2$ F, $G_2 = 6$ mho, and $G_1 = 4$ mho. Find the transfer function of the op amp circuit and determine the response to the input $v_{in}(t) = e^{-3t}u(t)$ V assuming ZERO initial conditions.



Solution Workout 1.
$Y_2(s) = Cs + G_2 = 2(s+3)$

KCL:
$$G_1 V_{in}(s) + Y_2(s)(V_{in}(s) - V_{out}(s)) = 0$$
$$H(s) = \frac{V_{out}}{V_{in}} = \frac{G_1}{Y_2} + 1 = \frac{4}{2(s+3)} + 1 = \frac{2}{s+3} + 1$$

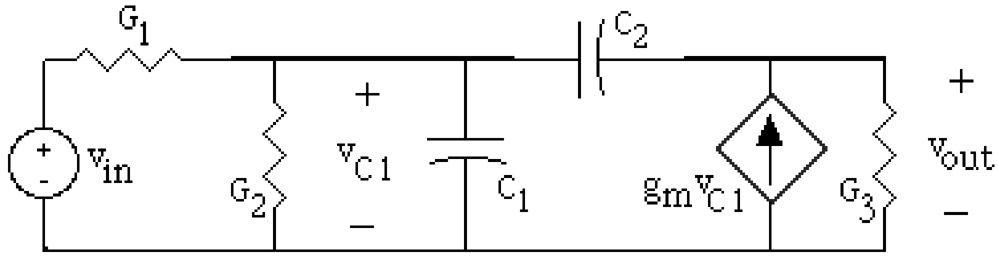
Take Laplace of input voltage: $V_{in}(s) = \dfrac{1}{s+3}$

Thus $V_{out}(s) = H(s)V_{in}(s) = \dfrac{2}{(s+3)^2} + \dfrac{1}{s+3}$

Inverse Laplace transform: $v_{out}(t) = (1+2t)e^{-3t}u(t)$ V

Workout problem 2. (30 pts)

Consider the circuit below with parameter values: $G_1 = 1$ mhos, $G_2 = 3$ mho, $G_3 = 6$ mhos, $C_1 = 2$ F, $C_2 = 1$ F, $g_m = 8$ mhos, $v_{C1}(0^-) \neq 0$, $v_{C2}(0^-) = 0$.



(a) (2 pts) Draw the equivalent circuit in the s-world.
(b) (7 pts) Wrtie a node equation in the s-world at the node with s-domain voltage $V_{C1}(s)$ in terms of $V_{C1}(s)$, $V_{out}(s)$, $V_{in}(s)$ and $v_{C1}(0^-)$.
(c) (6 pts) Wrtie a node equation in the s-world at the node with s-domain voltage $V_{out}(s)$ in terms of $V_{C1}(s)$, $V_{out}(s)$, and $V_{in}(s)$.
(d) (3 pts) Put the equations in matrix form with the unknown node voltages on the left and the input and IC on the right.
(e) (9 pts) Solve for $V_{out}(s)$ in terms of $V_{in}(s)$ and $v_{C1}(0^-)$ using Crammer's rule.
(f) (3 pts) Find the zero-input response in terms of $v_{C1}(0^-)$.

Solution Workout 2

Node 1: $\left[(C_1 + C_2)s + (G_1 + G_2)\right]V_{C1}(s) - C_2 s V_{out}(s) = G_1 V_{in}(s) + C_1 v_{C1}(0^-)$ or equivalently

$$(3s+4)V_{C1}(s) - sV_{out}(s) = V_{in}(s) + 2v_{C1}(0^-)$$

Node 2: $-(C_2 s + g_m)V_{C1}(s) + (C_2 s + G_3)V_{out}(s) = 0$ or equivalently
$-(s+8)V_{C1}(s) + (s+6)V_{out}(s) = 0$

Matrix Form:

$$\begin{bmatrix} 3s+4 & -s \\ -(s+8) & (s+6) \end{bmatrix} \begin{bmatrix} V_{C1}(s) \\ V_{out}(s) \end{bmatrix} = \begin{bmatrix} V_{in}(s) + 2v_{C1}(0^-) \\ 0 \end{bmatrix}$$

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>> M = [3*s+4  -s;-(s+8)  (s+6)]

M =

[ 3*s + 4,    -s]
[ - s - 8, s + 6]
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>>b = [Vin+2*vc10; 0]

b =

 Vin + 2*vc10
           0

>>vec = M\b

vec =

 ((Vin + 2*vc10)*(s + 6))/(2*s^2 + 14*s + 24)
 ((Vin + 2*vc10)*(s + 8))/(2*s^2 + 14*s + 24)

>> Vout = vec(2)

Vout =

((Vin + 2*vc10)*(s + 8))/(2*s^2 + 14*s + 24)

>>vout = ilaplace(Vout)

vout =

((5*Vin)/2 + 5*vc10)/exp(3*t) - (2*Vin + 4*vc10)/exp(4*t)
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So if Vin = 0, then $v_{out,zi}(t) = 5v_{C1}(0^-)e^{-3t}u(t) - 4v_{C1}(0^-)e^{-4t}u(t)$.