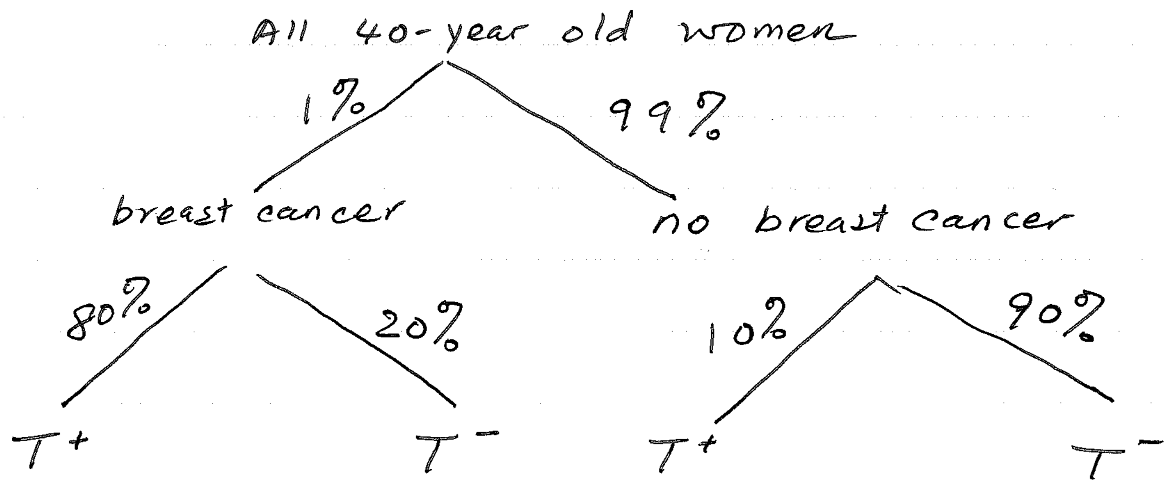
1.



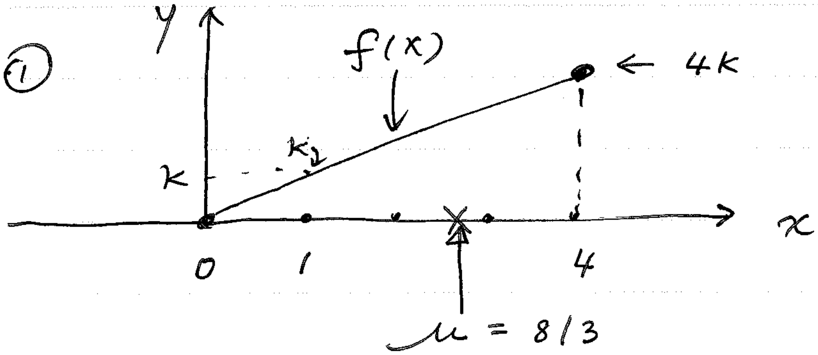
want to find $P(\text{Cancer} \mid T^+)$.

$$P(\text{cancer} \mid T^+) = \frac{P(\text{Cancer} \cap T^+)}{P(T^+)}$$

$$= \frac{0.01 \times 0.8}{0.01 \times 0.8 + 0.99 \times 0.1}$$

$$= \frac{0.008}{0.107} = \boxed{0.075}$$

2. ①



② area = 1

$$1 = \int_0^4 k\,x\,dx = k \cdot \left(\tfrac{1}{2}x^2\right)\Big|_0^4 = 8k$$

$$k = \boxed{\tfrac{1}{8}}$$

③

$$\begin{aligned}
\mu &= \int_0^4 x\, f(x)\, dx \\
&= \int_0^4 x \left[\tfrac{1}{8} x\right] dx \\
&= \tfrac{1}{8} \times \tfrac{1}{3} x^3 \Big|_0^4 \\
&= \boxed{8/3}
\end{aligned}$$

$$\begin{aligned}
\sigma^2 &= \int_0^4 x^2 f(x)\, dx - \mu^2 \\
&= \int_0^4 x^2 \left(\tfrac{1}{8} x\right) dx - \left(\tfrac{8}{3}\right)^2 \\
&= \tfrac{1}{8} \times \tfrac{1}{4} x^4 \Big|_0^4 - \tfrac{64}{9} \\
&= 8 - 64/9 \\
&= 8/9
\end{aligned}$$

$$\sigma = \sqrt{8/9} = \boxed{2\sqrt{2}/3}$$

3. Poisson p.m.f:

$$P(X = k) = \frac{e^{-\mu} \cdot \mu^k}{k!}$$

we are given $\mu = 2$ (per yd^2)

① $P(X = 5)$

$$= \frac{e^{-2} \times 2^5}{5!} = 0.036$$

② For a circular region of radius R,

$$\mu = \underbrace{(\pi R^2)}_{\text{area}} \times \underbrace{2}_{\text{per } yd^2} = 2\pi R^2$$

new mean for the circular area

$$0.99 = P(X \geq 1) \Rightarrow$$

$$0.01 = P(X = 0)$$

$$= \frac{e^{-\mu} \cdot \mu^0}{0!}$$

$$= e^{-\mu}$$

$$\Rightarrow \quad -\mu = \ln(0.01) = -4.605$$

$$2\pi R^2 = 4.605$$

$$R^2 = 0.733$$

$$\boxed{R = 0.86}$$

(yard)

4. X = # of passengers who show up

X has a Binomial distribution w.

$p = 0.9$

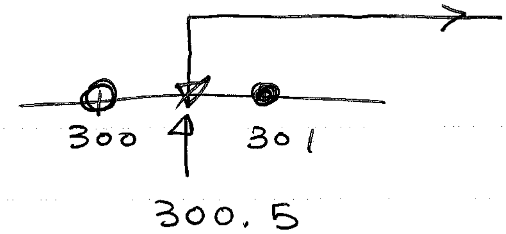
$n = 324$.

overbook $\iff X \geq 301$

Find :

$P(X \geq 301)$ *

$= P(X > 300.5)$



X is approximately normal since

$np > 5$, $n(1-p) > 5$

$\mu_X = np = 324 \times 0.9 = 291.6$

$\sigma_X = \sqrt{np(1-p)} = \sqrt{29.16} = 5.4$

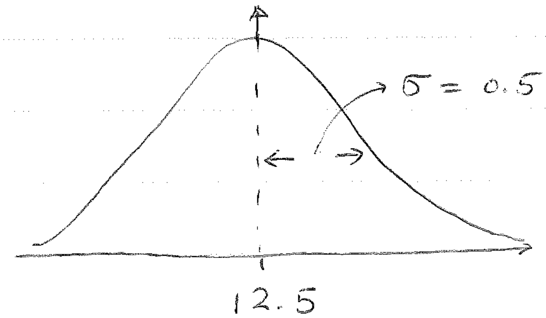
so Z-score for 300.5 is

$z = (300.5 - 291.6)/5.4$

$= 1.65$

$P(z > 1.65) = 1 - 0.95 = 0.05$

✓ 5.



1 cup. $\Rightarrow X$

(a)

$P(X < 12)$

$= P(Z < -1)$

$= \boxed{0.1587}$

z-score for 12:

$z = (12 - 12.5)/0.5$

$= -1$

(b)

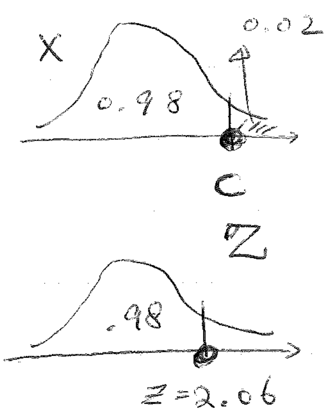
overflow $\Rightarrow X > c$.

$P(X > c) = 0.02$

$\dfrac{c - 12.5}{0.5} = 2.06$

z-score for c

inverse look up



$c = 12.5 + 2.06 \times 0.5$

$= 12.5 + 1.03 = \boxed{13.53}$

c) $\bar{X}$ is exactly Normal with

$\mu_{\bar{x}} = 12.5, \quad \sigma_{\bar{x}} = 0.5/\sqrt{25} = 0.1$

$P(\bar{X} < 12)$

$= P(Z < (12 - 12.5)/0.1)$

$= P(Z < -5) = \boxed{0}$

6. speed X is Normal $(\mu, \sigma = 4)$

$n = 8$

$\bar{X}$ is <u>exactly</u> Normal $(\mu, \sigma_{\bar{x}} = \frac{4}{\sqrt{8}})$

a). confidence intervals:

$$\bar{x} \pm z^* \cdot \sigma/\sqrt{n}$$

$z^* = 1.645$ for $C = 90\%$

1.96 for $C = 95\%$

2.576 for $C = 99\%$

90% CI: $102.2 \pm 1.645 \times \frac{4}{\sqrt{8}}$

$102.2 \pm 2.33 \rightarrow ($

95% CI: $102.2 \pm 1.96 \times \frac{4}{\sqrt{8}}$

102.2 ± 2.77

99% CI: $102.2 \pm 2.576 \times \frac{4}{\sqrt{8}}$

102.2 ± 3.64

6. b)

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

$$= \frac{102.2 - 100}{4/\sqrt{8}}$$

$$= 2.2/\sqrt{2} = 1.56$$

p-value $= P(Z < 1.56)$

$H_1: \mu < 100$

$= 0.9406 > \alpha$

conclusion: fail to reject H_0.

Think about ① testing $H_0: \mu = 100$ vs $H_1: \mu > 100$.

② what if $\bar{X} = 103$?

~~test~~ :

6. d) $H_0: \mu = 100, \quad H_1: \mu < 100$

with alternate $\mu_1 = 95$

power $= 1 - \beta$

$= P(\text{reject } H_0 \mid \mu = 95)$

desirable decision when true mean $\mu = 95$.

step 1. figure out when to reject H_0

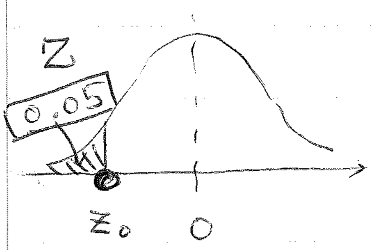
w. $\alpha = 0.05$

$n = 8$. (1

reject H_0 is p-value < 0.05

$\Rightarrow P(Z < z_0) = 0.05$

cut-off value z_0 for ~~test~~ statistic.



$z_0 = -1.645$

$$\frac{\bar{x} - 100}{4/\sqrt{8}} = -1.645$$

$\Rightarrow \bar{x} = 100 - 1.645 \times \sqrt{2}$

$\bar{x} = 100 - 2.33 = 97.67$.

cut-off value for $\bar{x}$.

$\Rightarrow$ if $\bar{x} < 97.67$, we would reject $H_0: \mu = 100$ in favor of $H_1: \mu < 100$

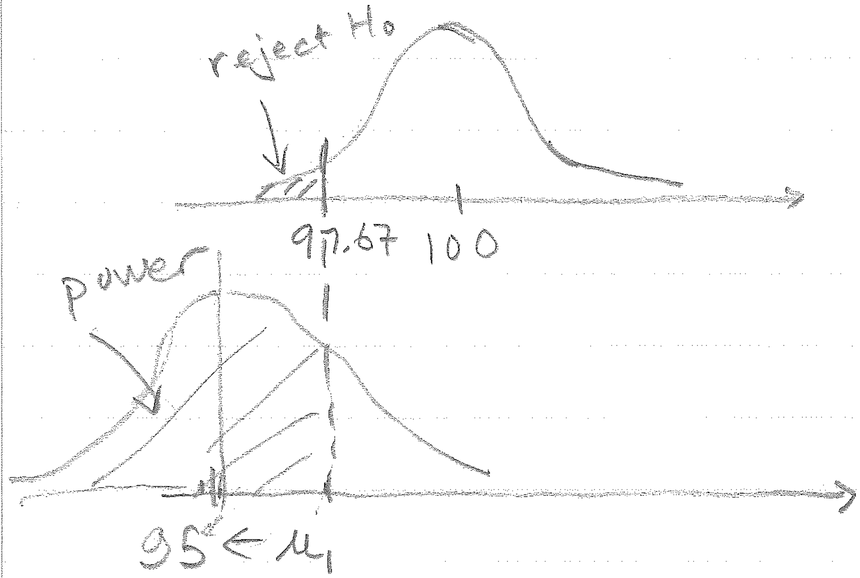
step 2: power $= P(\text{reject } H_0 \mid \mu = 95)$

step 2:

$$\begin{aligned} \text{power} &= P(\text{reject } H_0 \mid \mu = 95) \\ &= P(\bar{X} < 97.67 \mid \mu = 95) \\ &= P\left(Z < \frac{97.67 - 95}{8/\sqrt{4}}\right) \\ &= P(Z < 1.89) \\ &= 0.97. \end{aligned}$$

~~6 e) sample size is n~~

Graphs for step 1 & 2:



step 1

step 2

6. e) find n, so that $\beta = 0.85$.

step 1. Find rejection region similar to part (d).

$$\frac{\bar{X} - 100}{4/\sqrt{n}} = -1.645$$

↑ now n is unkown. no longer 8.

$$\bar{X} = 100 - \frac{4 \times 1.645}{\sqrt{n}}$$
$$= 100 - 6.58/\sqrt{n}$$

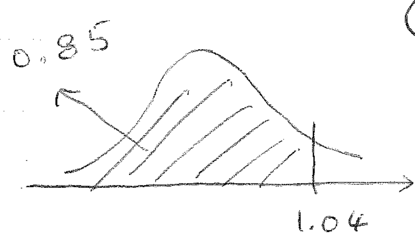
reject H_0 if $\bar{X} < 100 - 6.58/\sqrt{n}$

step 2. Power $= P(\text{reject } H_0 \mid \mu = 95)$

$$= P(\bar{X} < 100 - 6.58/\sqrt{n} \mid \mu = 95)$$
$$= P\left(Z < \frac{100 - 6.58/\sqrt{n} - 95}{4\ \cancel{1.645}/\sqrt{n}}\right)$$

(want to →) $= 0.85$

⟶ z-score $= 1.04$



$$\frac{100 - 6.58/\sqrt{n} - 95}{4\ \cancel{1.645}/\sqrt{n}} = 1.04.$$

$$5 - 6.58/\sqrt{n} = \cancel{1.71}\ 4.16/\sqrt{n}$$

$$5 = \cancel{8.29}\ 10.74/\sqrt{n} \Rightarrow n = \cancel{2.7}\ 4.6 \approx \cancel{\boxed{3}}\ \boxed{5}$$

minitab
↑
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