1. Evaluate $\int_0^{\pi/2} \cos^3 x \sin^2 x \, dx$.

$$\int \cos^3 x \sin^2 x \, dx = \int \cos^2 x \sin^2 x \cos x \, dx$$

$$= \int (1-\sin^2 x)\sin^2 x \cos x \, dx$$

$u = \sin x \quad du = \cos x \, dx$

$$= \int (1-u^2)u^2 \, du = \int (u^2 - u^4) du = \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

$$\int_0^{\pi/2} \cos^3 x \sin^2 x \, dx = \left. \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} \right]_0^{\pi/2} = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

A. $\frac{2}{15}$

B. $\frac{7}{10}$

C. $\frac{15}{24}$

D. $\frac{1}{8}$

E. $\frac{4}{9}$

2. Evaluate $\int_0^{\pi/4} \tan x \sec^4 x \, dx$.

$$\int \tan x \sec^4 x \, dx = \int \tan x \sec^2 x \sec^2 x \, dx$$

$$= \int \tan x (1+\tan^2 x)\sec^2 x \, dx \qquad u = \tan x, \quad du = \sec^2 x \, dx$$

$$= \int u(1+u^2) \, du = \int (u + u^3) du$$

$$= \frac{u^2}{2} + \frac{u^4}{4} + C = \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + C$$

$$\int_0^{\pi/4} \tan x \sec^4 x \, dx = \left. \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} \right]_0^{\pi/4} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

A. $\frac{\pi}{8}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{1}{2}$

E. $\frac{\pi}{4}$

3. When one makes a suitable trigonometric substitution to evaluate

$$\int \frac{x^3}{\sqrt{x^2-9}}\,dx,$$

which integral arises?

$x = 3\sec\theta \qquad dx = 3\sec\theta\tan\theta\,d\theta$

$$\int \frac{x^3}{\sqrt{x^2-9}}\,dx = \int \frac{3^3\sec^3\theta\, 3\sec\theta\tan\theta\,d\theta}{\sqrt{9\sec^2\theta-9}}$$

$$= 3^3\int \frac{\sec^4\theta\tan\theta\,d\theta}{\sqrt{\sec^2\theta-1}}$$

$$= 27\int \frac{\sec^4\theta\tan\theta\,d\theta}{\tan\theta} = 27\int \sec^4\theta\,d\theta$$

A. $27\int \sec^4\theta\,d\theta$

B. $\frac{1}{27}\int \sec^4\theta\tan\theta\,d\theta$

C. $9\int \frac{\sec^3\theta}{\tan\theta}\,d\theta$

D. $27\int \sin^3\theta\,d\theta$

E. $9\int \frac{\sin^3\theta}{\cos\theta}\,d\theta$

4. Evaluate $\displaystyle\int_0^{1/\sqrt{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx$.

$x = \sin\theta \qquad x = 0 \quad \theta = 0$

$dx = \cos\theta\,d\theta \qquad x = 1/\sqrt{2} \quad \theta = \pi/4$

$$\int_0^{1/\sqrt{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx = \int_0^{\pi/4} \frac{\sin^2\theta\cos\theta\,d\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \int_0^{\pi/4} \sin^2\theta\,d\theta = \int_0^{\pi/4} \frac{1-\cos(2\theta)}{2}\,d\theta$$

$$= \frac{\theta}{2} - \frac{\sin(2\theta)}{4}\Big]_0^{\pi/4} = \frac{\pi}{8} - \frac{1}{4}$$

A. $\frac{\pi}{2} - \frac{1}{8}$

B. $\frac{\pi}{2} - \frac{\sqrt{3}}{2}$

C. $\frac{\pi}{8} - \frac{\sqrt{2}}{2}$

D. $\frac{\pi}{8} - \frac{1}{4}$

E. $\frac{\pi}{3} + \sqrt{2}$

5. Compute $\displaystyle\int_{-2}^{0} \frac{dx}{x^2+4x+8}$.

$$\int \frac{dx}{x^2+4x+8} = \int \frac{dx}{(x+2)^2+4}$$

① $u = x+2 \quad du = dx$

$$= \int \frac{du}{u^2+4}$$

② $u = 2\tan\theta \quad du = 2\sec^2\theta\, d\theta$

$$= \int \frac{2\sec^2\theta\, d\theta}{4\tan^2\theta+4} = \frac{1}{2}\int \frac{\sec^2\theta\, d\theta}{\tan^2\theta+1}$$

$$= \frac{1}{2}\int d\theta = \frac{\theta}{2} + C = \frac{\tan^{-1}(u/2)}{2} + C$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right). \quad \int_{-2}^{0} \frac{dx}{x^2+4x+8} = \frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)\Bigg]_{-2}^{0} = \frac{\pi}{8}$$

A. $\frac{\pi}{16}$

B. $\frac{\pi}{8}$

C. $1+\frac{\pi}{2}$

D. $\frac{\pi}{4}$

E. $\frac{\pi}{2}$

6. Find the correct form of the partial fraction decomposition of

$$\frac{x-5}{(x-1)^2(x^2-9)(x^2+9)}.$$

A. $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-3} + \frac{D}{x+3} + \frac{Ex+F}{x^2+9}$

B. $\frac{A}{(x-1)^2} + \frac{B}{x-3} + \frac{C}{x+3} + \frac{Dx+E}{x^2+9}$

C. $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x^2-9} + \frac{Dx+E}{x^2+9}$

D. $\frac{A}{(x-1)^2} + \frac{Bx+C}{x^2-9} + \frac{Dx+E}{x^2+9}$

E. $\frac{A}{(x-1)^2} + \frac{B}{x^2-9} + \frac{C}{x^2+9}$

7. Evaluate $\displaystyle\int_0^2 \frac{1}{(x+1)(x+2)}\,dx$.

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$1 = A(x+2) + B(x+1) = (A+B)x + (2A+B)$$

$A+B=0$ $\quad A=1,\ B=-1.$

$2A+B=1$

$$\int_0^2 \frac{1}{(x+1)(x+2)}\,dx = \int_0^2 \left(\frac{1}{x+1} - \frac{1}{x+2}\right)dx$$

$$= \ln|x+1| - \ln|x+2| \Big]_0^2 = \ln 3 - \ln 4 + \ln 2$$

A. $\ln 2 - \ln 4$

B. $\ln 2 + \ln 4 + \ln 3$

C. $\dfrac{\ln 3}{2} + \dfrac{\ln 4}{2} + \ln 2$

D. $\ln 3 - \ln 4$

E. $\ln 3 - \ln 4 + \ln 2$

8. Given that $\displaystyle\int_1^2 \frac{1}{x^2-2x+2}\,dx = \frac{\pi}{4}$, evaluate

$$\int_1^2 \frac{3x+5}{x^2-2x+2}\,dx.$$

$$= \frac{3}{2}\int_1^2 \frac{2x-2}{x^2-2x+2}\,dx + 8\int_1^2 \frac{1}{x^2-2x+2}\,dx$$

$$= \frac{3}{2}\ln|x^2-2x+2|\Big]_1^2 + 8\left(\frac{\pi}{4}\right)$$

$$= \frac{3}{2}\ln 2 + 2\pi$$

A. $\ln 2 + \dfrac{\pi}{4}$

B. $2\ln 2 - \dfrac{\pi}{2}$

C. $\dfrac{3}{2}\ln 2 + 2\pi$

D. $\dfrac{1}{2}\ln 2 + \dfrac{\pi}{2}$

E. $\dfrac{3}{4}\ln 2 + \dfrac{\pi}{6}$

9. Which of the following improper integrals converge.

$$(1)\ \int_1^\infty \frac{x^2+2x+1}{x^5+1}\,dx, \qquad (2)\ \int_{-1}^1 \frac{1}{x^3}\,dx, \qquad (3)\ \int_1^\infty e^{-x}\cos^2 x\,dx.$$

A. (1) and (2) converge. (3) diverges.

B. (1) and (3) converge. (2) diverges.

C. (2) and (3) converge. (1) diverges.

D. (1) converges. (2) and (3) diverge.

E. (1), (2) and (3) converge.

10. Find the arclengh of the curve

$$y = \frac{2}{3}(x+1)^{3/2}, \qquad -1 \le x \le 2.$$

$$\frac{dy}{dx} = (x+1)^{1/2}$$

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{2+x}\,dx$$

$$l = \int_{-1}^{2} \sqrt{2+x}\,dx = \frac{2}{3}(2+x)^{3/2}\Big]_{-1}^{2}$$

$$= \frac{2}{3}\left(4^{3/2}-1\right) = \frac{2}{3}(8-1)$$

$$= \frac{14}{3}$$

A. $\frac{2}{3}$

B. $\frac{7}{6}$

C. $\frac{8}{3}$

D. $\frac{14}{3}$

E. $\frac{20}{3}$

11. Which integral gives the surface area of the surface obtained by rotating the curve

$$y = 1 + 2x^2, \qquad 0 \le x \le 1,$$

about the y–axis.

$$\frac{dy}{dx} = 4x$$

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$ds = \sqrt{1+16x^2}\,dx$$

$$S = 2\pi\int_0^1 x\sqrt{1+16x^2}\,dx$$

A. $2\pi \int_0^1 (1+2x^2)\sqrt{1+16x^2}\,dx$

B. $2\pi \int_0^1 x\sqrt{1+16x^2}\,dx$

C. $2\pi \int_0^1 x(1+2x^2)\,dx$

D. $2\pi \int_0^1 x(1+16x^2)\,dx$

E. $2\pi \int_0^1 (1+2x^2)(1+16x^2)\,dx$

12. The substitution $u = \sqrt{1+x}$ transforms the integral

$$\int_3^8 \frac{1}{x\sqrt{1+x}}\,dx$$

into which integral?

$u = \sqrt{1+x}$ $\qquad u^2 = 1+x$

$x = u^2 - 1$

$dx = 2u\,du$

$x = 3 \quad u = 2.$

$x = 8 \quad u = 3.$

$$\int_3^8 \frac{1}{x\sqrt{1+x}}\,dx = \int_2^3 \frac{1}{(u^2-1)u}\,2u\,du$$

$$= \int_2^3 \frac{2}{u^2-1}\,du$$

A. $\int_3^8 \frac{1}{(u^2-1)u}\,du$

B. $\int_2^3 \frac{1}{(u^2-1)u}\,du$

C. $\int_2^3 \frac{2u}{u^2-1}\,du$

D. $\int_3^8 \frac{1}{u^2-1}\,du$

E. $\int_2^3 \frac{2}{u^2-1}\,du$