EE-202
Exam I
February 3, 2011

Name: ________________________________

(Please print clearly)

Student ID: ________________

CIRCLE YOUR DIVISION

Section 01, 7:30 MWF	Section 02, 3:30 MWF
Prof. Furgason	Prof. DeCarlo

INSTRUCTIONS

There are 12 multiple choice worth 5 points each and there is 1 workout problem worth 40 points.

This is a closed book, closed notes exam. No scrap paper or calculators are permitted. A transform table will be handed out separately as well as the property table.

Carefully mark your multiple choice answers on the scantron form. Work on multiple choice problems and marked answers in the test booklet will not be graded

Nothing is to be on the seat beside you. Scantrons are to be under exam.

When the exam ends, all writing is to stop. This is not negotiable.

No writing while turning in the exam/scantron or risk an F in the exam.

All students are expected to abide by the customary ethical standards of the university, i.e., your answers must reflect only your own knowledge and reasoning ability. As a reminder, at the very minimum, cheating will result in a zero on the exam and possibly an F in the course.

Communicating with any of your classmates, in any language, by any means, for any reason, at any time between the official start of the exam and the official end of the exam is grounds for immediate ejection from the exam site and loss of all credit for this exercise.

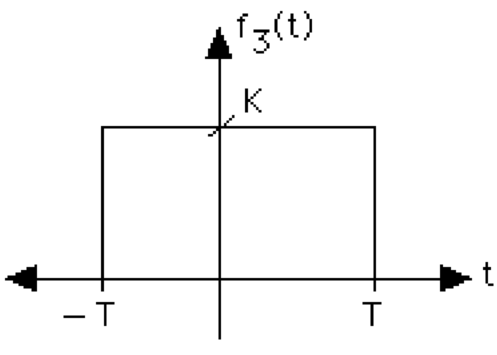
MULTIPLE CHOICE.

1. For the signal sketched below, $\mathcal{L}[f_3(t)] = F_3(s) =$:

(1) $\frac{e^s}{s} - \frac{e^{-s}}{s}$ **(2)** $\frac{e^{sT}}{s} - \frac{e^{-sT}}{s}$ **(3)** $\frac{K}{s} - \frac{Ke^{-sT}}{s}$

(4) $\frac{1}{s} - \frac{e^{-s}}{s}$ **(5)** $\frac{1}{s} - \frac{e^{-sT}}{s}$ **(6)** $\frac{Ke^{sT}}{s} - \frac{Ke^{-sT}}{s}$

(7) None of above



2. A voltage, $v(t)$, has Laplace transform $V(s) = \frac{s}{(s+1)^2}$. Then $v(t)$ has a term of the form $Ae^{-t}u(t)$ where $A =$:

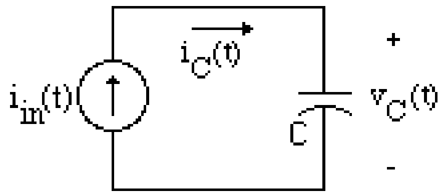
(1) –1 **(2)** 2 **(3)** 1 **(4)** –2

(5) –0.5 **(6)** 0 **(7)** 0.5 **(8)** None of above

3. For the circuit below, $C = 0.25$ F, $v_C(0^-) = 0$, and $I_{in}(s) = \dfrac{s(s+3)}{(s+1)^2}$ A. Then $v_C(t)$ has a term of the form $Kte^{-t}u(t)$ where $K =$:

(1) –4 **(2)** 2 **(3)** –2 **(4)** 4

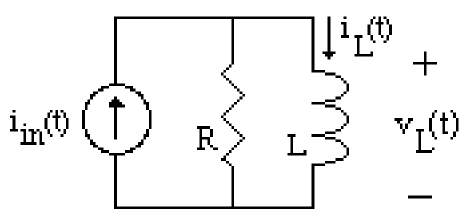
(5) 8 **(6)** –8 **(7)** None of above



4. For the circuit below, $i_{in}(t) = e^{-2t}\cos(4t)u(t)$ A, $i_L(0^-) = 0$, $R = 2$ Ω, and $L = 1$ H. . Then $v_L(t)$ has a term of the form $Ke^{-2t}\cos(4t)u(t)$ where $K =$:

(1) 1 **(2)** 2 **(3)** 3 **(4)** 4

(5) 8 **(6)** 12 **(7)** 16 **(8)** None of above



5. The differential equation associated with a circuit given on the 202 HW is

$$\frac{d^2 v_C}{dt^2} + \frac{1}{RC}\frac{dv_C}{dt} + \frac{1}{LC}v_C = \frac{1}{LC}v_{\text{in}}$$

Suppose $v_{in}(t) = 0$, $R = 0.5\ \Omega$, $C = 0.1$ F, $L = 0.1$ H, $\frac{dv_C}{dt}(0^-) = 10$ V, but $v_C(0^-) = 0$. Then $v_C(t)$ (in V) has a term of the form:

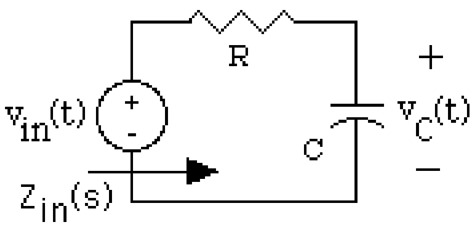
(1) $10e^{-10t}u(t)$ (2) $10e^{-100t}u(t)$ (3) $10e^{-20t}u(t)$
(4) $10te^{-100t}u(t)$ (5) $10te^{-10t}u(t)$ (6) $10te^{-20t}u(t)$
(7) two of above (8) none of above.

6. Suppose $F(s) = \frac{s+2a}{(s+a)^2}$, $a > 0$. If $g(t) = e^{a(t-T)} f(t-T)u(t-T)$, then $G(s) =$:

(1) $\frac{s+a}{s^2}$ (2) $\frac{s+3a}{(s+2a)^2}$ (3) $e^{-sT}\frac{s+a}{s^2}$
(4) $e^{-(s+a)T}\frac{s+a}{s^2}$ (5) $e^{-sT}\frac{s+3a}{(s+2a)^2}$ (6) $e^{-(s+a)T}\frac{s+3a}{(s+2a)^2}$
(7) none of above.

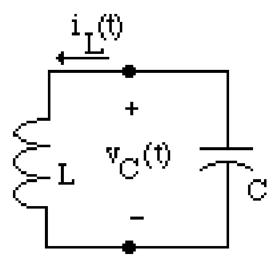
7. For the circuit sketched below, $v_C(0^-) = 0$ V, $v_{in}(t) = 5e^{-5t}u(t)$ V, $C = 0.1$ F and $R = 1\ \Omega$. The capacitor voltage, $v_C(t)$ has a term of the form $Ke^{-5t}u(t)$ where $K =$:

(1) 1	**(2)** –1	**(3)** –10	**(4)** 10
(5) 5	**(6)** –5	**(7)** 15	**(8)** None of above



8. For the circuit below, $L = \frac{2}{5}$ H, $C = \frac{1}{10}$ F, $i_L(0^-) = 2$ A and $v_C(0^-) = 0$. Then $v_C(t) =$ (inV):

(1) $20\cos(5t)u(t)$	(2) $4\sin(5t)u(t)$	(3) $-20\sin(5t)u(t)$
(4) $-4\cos(5t)u(t)$	(5) $-20\cos(5t)u(t)$	(6) $-4\sin(5t)u(t)$
(7) $4\cos(5t)u(t)$	(8) none of above.	

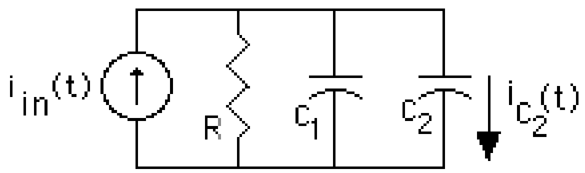


9. In the circuit below, $R = 1$, $C_1 = C_2 = 0.5$ F, and $i_{in}(t) = 4u(t)$ A. Then $i_{C_2}(t) =$ (in A):

(1) $4u(t)$ **(2)** $0.5\delta(t) - e^{-t}u(t)$ **(3)** $0.5\delta(t) + e^{-t}u(t)$

(4) $0.5e^{-2t}u(t)$ **(5)** $e^{-t}u(t)$ **(6)** $2e^{-t}u(t)$

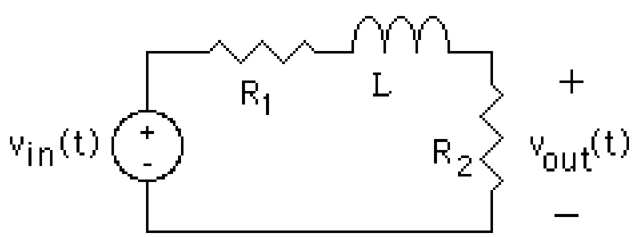
(7) $2e^{-2t}u(t)$ **(8)** none of above



10. In the circuit below, suppose $V_{in}(s) = \dfrac{5}{(s+1)}$. Suppose $i_L(0^-) = 0$ A, $L = 1$ H, $R_1 = 1$ Ω, and $R_2 = 8$ Ω. Then $v_{out}(t)$ has a term of the form $Ke^{-9t}u(t)$ where $K =$:

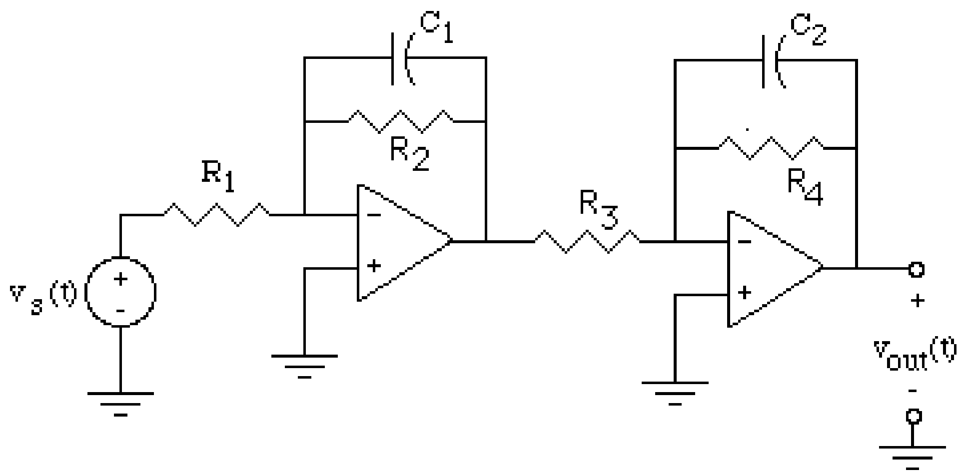
(1) 1 **(2)** 8 **(3)** –8 **(4)** 9

(5) –5 **(6)** 5 **(7)** –9 **(8)** None of above



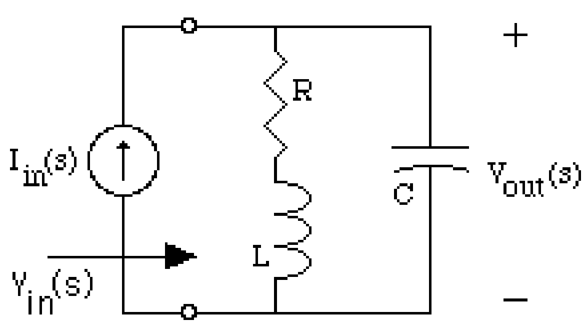
11. In the circuit below $C_1 = C_2 = 1$ F and $R_1 = R_3 = 1\ \Omega$. If the transfer function $H(s) = \frac{V_{out}(s)}{V_s(s)} = H_1(s)H_2(s) = \frac{-1}{s+4} \times \frac{-1}{s+2}$. We assume that $H_1(s) = \frac{-1}{s+4}$ corresponds to the first stage (first op amp circuit). The values for R_2 and R_4 (in Ω) respectively are:

(1) 4, 2 (2) 0.5, 0.25 (3) 0.25, 0.5 (4) 0.4, 0.2
(5) 2.5, 0.5 (6) two of above (7) none of above



12. In the circuit below, $L = 1$ H, $R = 3\ \Omega$, and $C = 0.5$ F. Let $Z_{in}(s)$ be the input impedance seen by the current source. THEN $Z_{in}(s)\big]_{s=1} =$:

(1) 0.75 (2) 2/3 (3) 1.5 (4) 0 (5) 4 / 3
(6) 0.5 (7) 0.25 (8) none of above



WORKOUT PROBLEM. (40 points) Consider the circuit below in which $R_1 = 1\ \Omega$, $R_2 = 3\ \Omega$, $L = 1$ H, and $C = 0.25$ F.

(a) (5 pts) Find $Y_{in}(s)$. You must express your answer as: $Y_{in}(s) = \dfrac{?}{s+?} + \dfrac{?}{s+?}$.

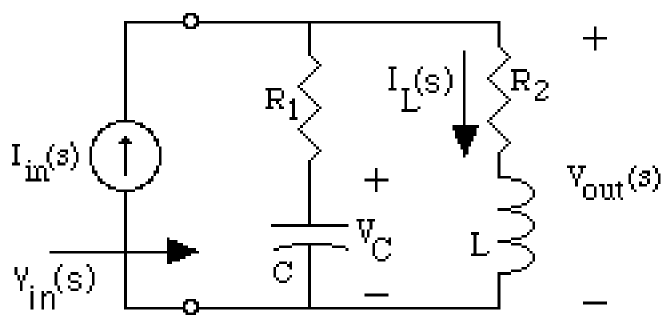
(b) (8 pts) Find $Z_{in}(s)$. Your answer must be a rational function with second order denominator having leading coefficient 1.

(c) (17 pts) Find the step response, i.e., find $v_{out,step}(t)$ when the input is a step function and all initial conditions are zero using the following steps:

(i) Find $V_{out,step}(s)$ and its partial fraction expansion;

(ii) Find $v_{out,step}(t)$.

(d) (10 pts) If $v_C(0^-) = 10$ V, and $i_{in}(t) = 0$, find $v_{out}(t)$.



Solution 1. One sided L-transform does not see parts of functions for t<0, therefore, as far as the one-sided L-transform is concerned, $f_3(t) = Ku(t) - Ku(t-T)$ making

$F_3(s) = \frac{K}{s} - \frac{Ke^{-sT}}{s}$.

ANSWER: 3.

Solution 2. $V(s) = \frac{s}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2}$. $B = s]_{s=-1} = -1$. Thus,

$\frac{s}{(s+1)^2} = \frac{A}{s+1} - \frac{1}{(s+1)^2}$. Thus

$\left.\frac{s}{(s+1)^2}\right]_{s=0} = 0 = \left.\frac{A}{s+1}\right]_{s=0} - \left.\frac{1}{(s+1)^2}\right]_{s=0} = A - 1 \Rightarrow A = 1$.

ANSWER: (3)

Solution 3. $V_C(s) = \frac{1}{0.25s} \times \frac{s(s+3)}{(s+1)^2} = \frac{4(s+3)}{(s+1)^2} = \frac{4}{s+1} + \frac{8}{(s+1)^2}$.

ANSWER: (5)

Solution 4.

$V_L(s) = \frac{LRs}{(Ls+R)} I_{in}(s) = \frac{2s}{s+2} \times \frac{s+2}{(s+2)^2+4^2} = 2\frac{s}{(s+2)^2+4^2} = 2\frac{s+2}{(s+2)^2+4^2} - \frac{4}{(s+2)^2+4^2}$

.

Thus $v_L(t) = 2e^{-2t}\cos(4t)u(t) - e^{-2t}\sin(4t)u(t)$ V.

ANSWER: (2)

Solution 5. Laplace transforming the equation yields

$s^2V_C(s) - sv_C(0^-) - \frac{dv_C}{dt}(0^-) + 20sV_C(s) - 20v_C(0^-) + 100V_C(s) = 0$. Plugging in numbers yields

$(s+10)^2V_C(s) = 10$ implies $V_C(s) = \frac{10}{(s+10)^2}$ which implies $v_C(t) = 10te^{-10t}u(t)$ V.

ANSWER (5).

Solution 6. $L\left[e^{at}f(t)u(t)\right] = F(s-a) = \frac{s+a}{s^2}$ because the sign on a in e^{at} is positive.

Thus $L\left[e^{a(t-T)}f(t-T)u(t-T)\right] = G(s) = e^{-sT}\frac{s+a}{s^2}$.

ANSWER: (3)

Solution 7. $V_C(s)=\dfrac{\frac{1}{Cs}}{R+\frac{1}{Cs}}V_{in}(s)=\dfrac{\frac{1}{RC}}{s+\frac{1}{RC}}V_{in}(s)=\dfrac{10}{s+10}\times\dfrac{5}{s+5}=\dfrac{10}{s+5}-\dfrac{10}{s+10}$.

Thus

$v_C(t)=10e^{-5t}u(t)-10e^{-10t}u(t)$ V.

ANSWER: (4)

Solution 8. Using the parallel current source model,

$$V_C(s)=-\frac{i_L(0^-)}{s}\times\frac{\frac{Ls}{Cs}}{Ls+\frac{1}{Cs}}=-\frac{i_L(0^-)}{C}\times\frac{1}{s^2+\frac{1}{LC}}=-4\frac{5}{s^2+25}.$$ Thus

$v_C(t)=-4\sin(5t)u(t)$ V.

ANSWER: (6)

Solution 9. $I_{C_2}(s)=\dfrac{C_2 s}{(C_1+C_2)s+G}I_{in}(s)=\dfrac{0.5s}{s+1}\times\dfrac{4}{s}=\dfrac{2}{s+1}$. Thus $i_{C_2}(t)=2e^{-t}u(t)$.

ANSWER: (6)

Solution 10.

$$\begin{aligned}V_{out}(s)&=\frac{R_2}{Ls+R_1+R_2}\times\frac{5}{(s+1)}=\frac{8}{s+9}\times\frac{5}{s+1}\\&=\frac{40}{(s+1)(s+9)}=\frac{5}{(s+1)}-\frac{5}{(s+9)}\end{aligned}.$$

ANSWER (5).

Solutioin 11. Refer to class notes. ANSWER: (3)

Solution 12. $Y_{in}(s)=\dfrac{1}{s+3}+0.5s=\dfrac{0.5s^2+1.5s+1}{s+3}$. $Z_{in}(s)=2\dfrac{s+3}{s^2+3s+2}$.

$Z_{in}(1)=2\dfrac{1+3}{1+3+2}=2\times\dfrac{4}{6}=\dfrac{4}{3}$.

ANSWER: (5)

Solution Workout.

(a) $Y_{in}(s)=\dfrac{1}{R_1+\frac{1}{Cs}}+\dfrac{1}{Ls+R_2}=\dfrac{\frac{s}{R_1}}{s+\frac{1}{R_1C}}+\dfrac{\frac{1}{L}}{s+\frac{R_2}{L}}=\dfrac{s}{s+4}+\dfrac{1}{s+3}$

(b) $Z_{in}(s)=\frac{1}{Y_{in}(s)}=\frac{1}{\frac{s}{s+4}+\frac{1}{s+3}}=\frac{(s+3)(s+4)}{s(s+3)+s+4}=\frac{(s+3)(s+4)}{s^2+4s+4}$

(c) $Z_{in}(s)=\frac{(s+3)(s+4)}{(s+2)^2}$.

(i)

$V_{out,step}(s)=Z_{in}(s)I_{in}(s)=\frac{Z_{in}(s)}{s}=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{(s+2)^2}=\frac{3}{s}-\frac{2}{s+2}-\frac{1}{(s+2)^2}$

(ii) $v_{out,step}(t)=3u(t)-2e^{-2t}u(t)-te^{-2t}u(t)$

(d) Using the series capacitor model and voltage division,

$$V_{out}(s)=\frac{10}{s}\frac{s+3}{s+3+1+\frac{4}{s}}=\frac{10(s+3)}{(s+2)^2}=\frac{10}{s+2}+\frac{10}{(s+2)^2}$$

Hence $v_{out}(t)=10(t+1)e^{-2t}u(t)$ V.