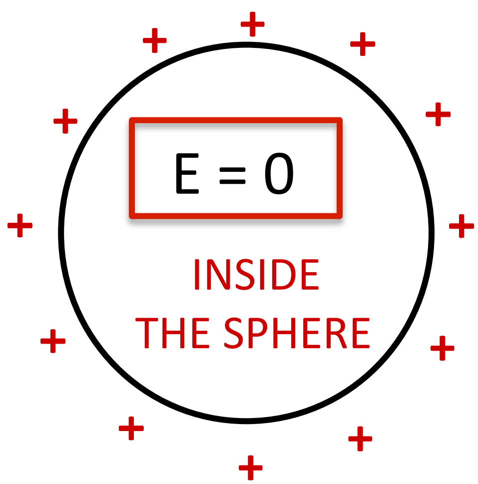
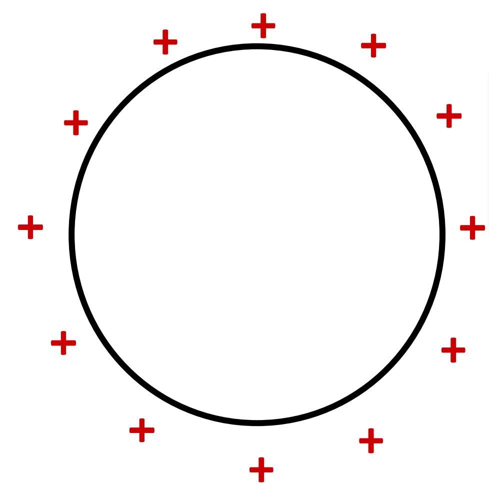
Last Time

- Electric field of a hollow sphere





$$\vec{E} = \frac{Q}{4\pi\epsilon_o|r|^2}\hat{r}$$

OUTSIDE
THE SPHERE

- Electric field of a solid sphere

$$\vec{E} = \frac{Q|r|}{4\pi\epsilon_o R^3}\hat{r}$$

INSIDE
THE SPHERE

$$\vec{E} = \frac{Q}{4\pi\epsilon_o|r|^2}\hat{r}$$

OUTSIDE
THE SPHERE

Today

- Review of potential energy
- Electric potential
- Potential due to charges

Review: Single Particle Energy

Energy of a Single Particle:

$$E_{\text{particle}} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = mc^2 + K \approx \underbrace{mc^2}_{\text{Rest energy}} + \underbrace{\frac{1}{2}mv^2}_{\text{Kinetic Energy (v<<c)}}$$

The Energy Principle for a Particle:

$$\Delta E_{\text{particle}} = W = \int \vec{F} \cdot d\vec{r}$$

W = Work done ON the particle

If the rest energy does not change,

$$\Delta E_{\text{particle}} = \boxed{\Delta K = W}$$

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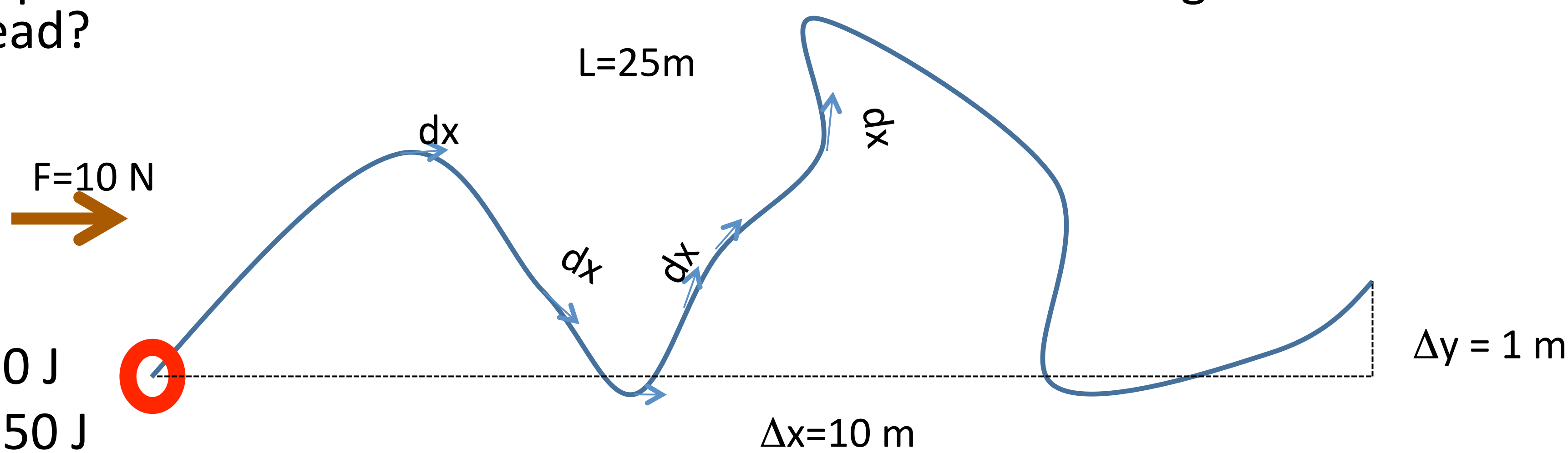
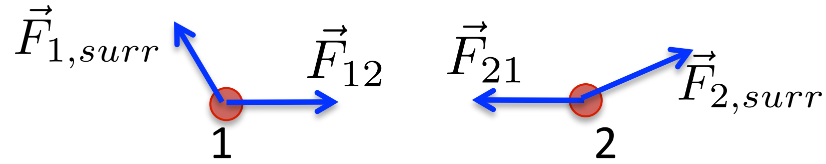
- A horizontal force of 10 N pushes a bead along a wire. The wire has a length of 25 m. The horizontal displacement of the bead when it reaches the end of the wire is 10m. The vertical displacement is 1m. How much work was done moving the bead?



a) 10 J
b) 250 J
c) 100 J
d) 100 N

Review: Multiparticle Energy

Energy Principle for Each Particle:



$$\Delta E_1 = W_{\text{on }1} = \int \vec{F}_{1,surr} \cdot d\vec{r} + \int \vec{F}_{12} \cdot d\vec{r}$$

$$= \quad W_{1,surr} \quad + \quad W_{1,2}$$

$$\Delta E_2 = \quad W_{2,surr} \quad + \quad W_{2,1}$$

Multiparticle Energy Principle:

$$\Delta(E_1 + E_2) = \quad W_{surr} \quad + \quad W_{internal}$$

$$\Delta(E_1 + E_2) - W_{internal} = \quad W_{surr}$$

$$\Delta(E_1 + E_2) \quad + \quad \Delta U \quad = \quad W_{surr}$$

$$\boxed{\Delta U \equiv -W_{internal}}$$

Potential Energy is Meaningless for a Single Particle

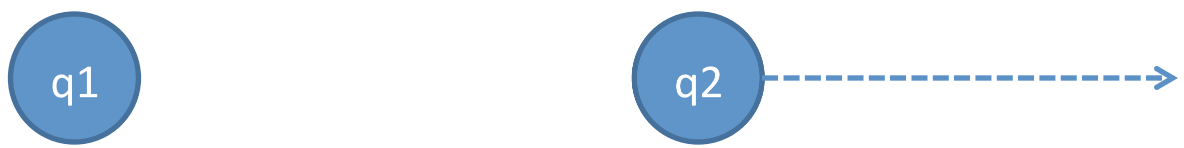
Assuming Rest Mass Unchanged

$$\boxed{\Delta K \quad + \quad \Delta U \quad = \quad W_{surr}}$$

MULTIPARTICLE ENERGY PRINCIPLE

Potential energy of charges

- Remember: potential energy comes from interaction of **TWO** objects
- We can find potential energy by checking the interaction of 2 particles



Hold q1 fixed and move q2.
How much work do we have to do?

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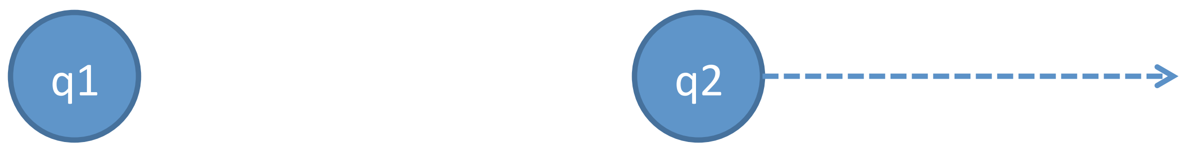
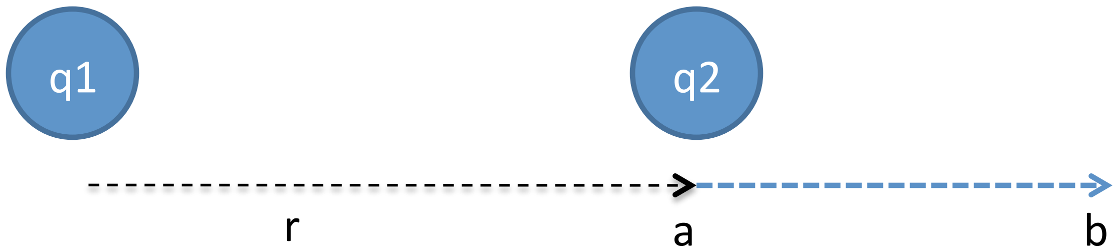
- We wish to find the work that it requires to move q2 along the horizontal path away from q1. Which F appears in the expression $W = \int \vec{F} \cdot d\vec{x}$?



a) The force exerted on q2 by q1, F_{q2q1}
b) The force need to counteract the force exerted on q2 by q1, - F_{q2q1}
c) Any constant, horizontal force

Work to move q_2

$$\vec{F}_{1 \text{ on } 2} = \frac{q_1 q_2}{4\pi\epsilon_o r^2}\hat{r} \qquad W = \int_a^b \vec{F} \cdot d\vec{x}$$

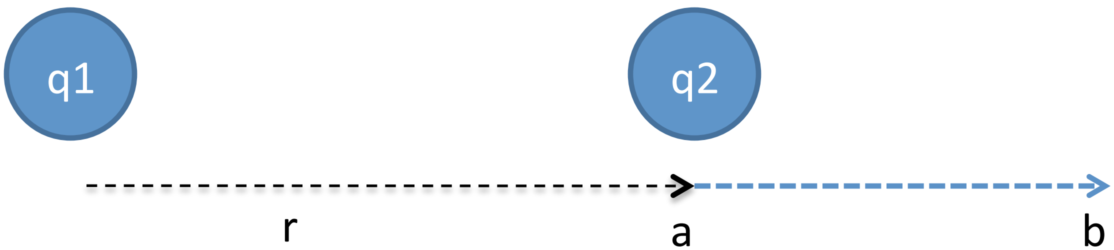


$$W = \int_a^b \vec{F} \cdot d\vec{x} = -\int_a^b \frac{q_1 q_2}{4\pi\epsilon_o r^2}\hat{r} \cdot d\vec{r}$$

$$= \frac{q_1 q_2}{4\pi\epsilon_o}\frac{1}{r}\Big]_a^b \quad = \frac{q_1 q_2}{4\pi\epsilon_o}\left(\frac{1}{b} - \frac{1}{a}\right)$$

Where did the Energy go?

$$\Delta K \quad + \quad \Delta U \quad = \quad W_{surr} \qquad\qquad W = \frac{q_1 q_2}{4\pi\epsilon_o}\left(\frac{1}{b} - \frac{1}{a}\right)$$



Assume $v_f = v_i$ -- Then $\Delta K = 0$.
Work **always** changes E_{sys}, so the potential energy must have changed:

$$\Rightarrow \quad \Delta U \quad = \quad W_{surr} = \frac{q_1 q_2}{4\pi\epsilon_o}\left(\frac{1}{b} - \frac{1}{a}\right) \qquad \equiv U_b - U_a$$

$$\boxed{U = \frac{q_1 q_2}{4\pi\epsilon_o r}}$$

ELECTRIC
POTENTIAL
ENERGY

Angular motion

- We’ve shown what the work is required to move a charge along the radial direction.
- The work required to move a charge in the angular direction (to circle q2 around q1) is zero.
- This is because we are moving perpendicular to field lines.
- We’ll discuss this in greater detail in next lecture.

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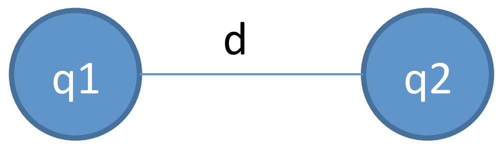
$$U = \frac{q_1 q_2}{4\pi\epsilon_o r}$$

- Two particles with charge q sit a distance d apart. What is the potential energy of the system, including both particles?



a) $2q1q2/4\pi\varepsilon_0 d$

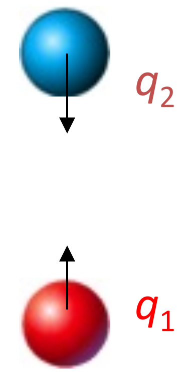
b) $q1q2/4\pi\varepsilon_0 d$

c) $2q1q2/4\pi\varepsilon_0 d^2$

d) $q1q2/4\pi\varepsilon_0 d^2$

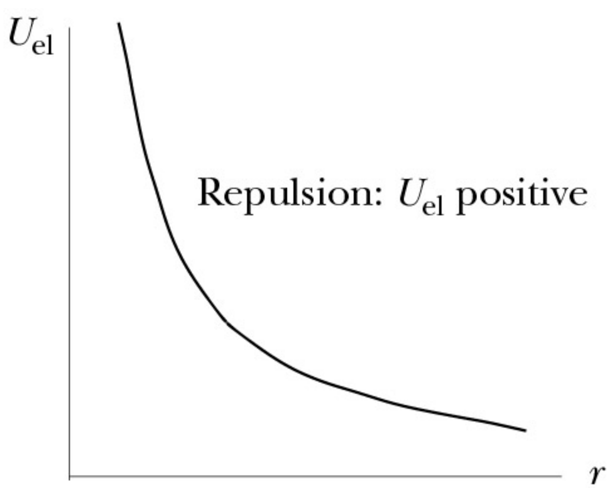
Electric Potential Energy of Two Particles

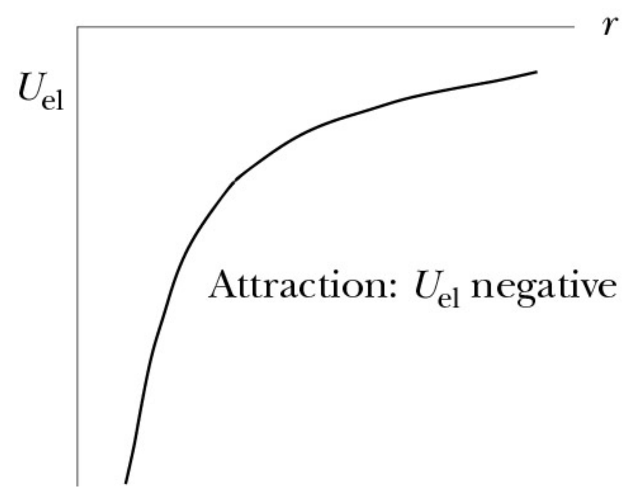
$$U_{el} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} \text{ (joules)}$$



$U_{el} > 0$ for two like charges (repulsion)

$U_{el} < 0$ for two opposite charges (attraction)



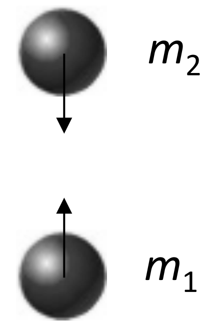


Electric and Gravitational Potential Energy

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

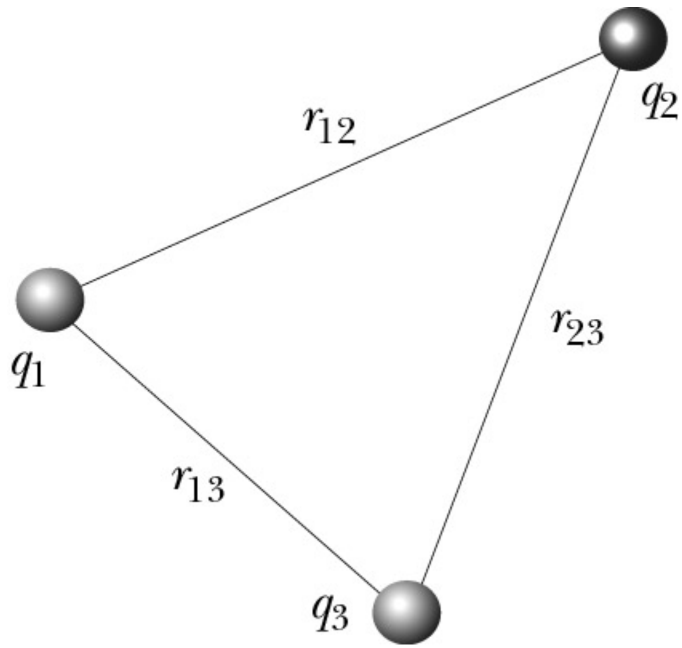
$$U_{el} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$\vec{F} = -G\frac{m_1 m_2}{r^2}\hat{r}$$



$$U_{grav} = -G\frac{m_1 m_2}{r}$$

Three Electric Charges



Interaction between q_1 and q_2 is independent of q_3

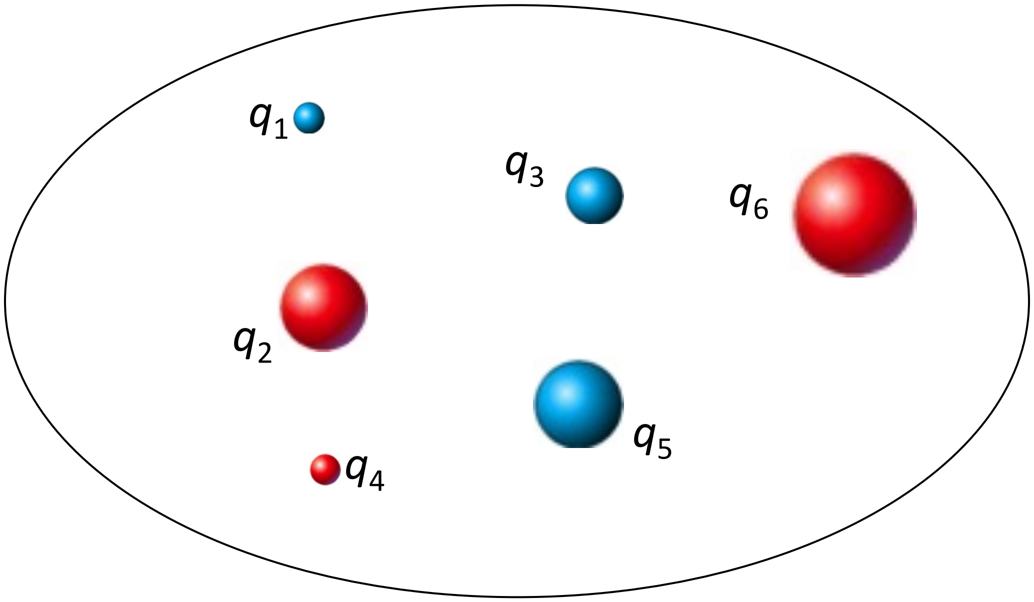
There are three interacting pairs:

$q_1 \leftrightarrow q_2$	U_{12}
$q_2 \leftrightarrow q_3$	U_{23}
$q_3 \leftrightarrow q_1$	U_{31}

$U = U_{12} + U_{23} + U_{31}$

$$U_{el} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}}$$

Multiple Electric Charges



Each (i,j) pair interacts: potential energy U_{ij}

$$U_{el} = \sum_{i<j} U_{ij} = \sum_{i<j} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{r_{ij}}$$

Electric Potential

Electric potential ≡ electric potential energy per unit charge

$$V = \frac{U_{el}}{q}$$

Units: J/C = V (Volt)

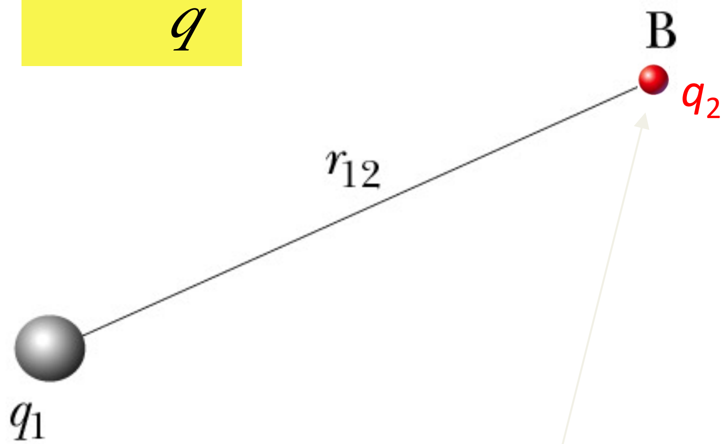
Volts per meter = Newtons per Coulomb

Electric potential – often called potential

Electric potential difference – often called voltage

V due to One Particle

$$V = \frac{U_{el}}{q}$$



Single charge has *no* electric potential energy

Single charge has *potential* to interact with other charge –
it creates electric potential

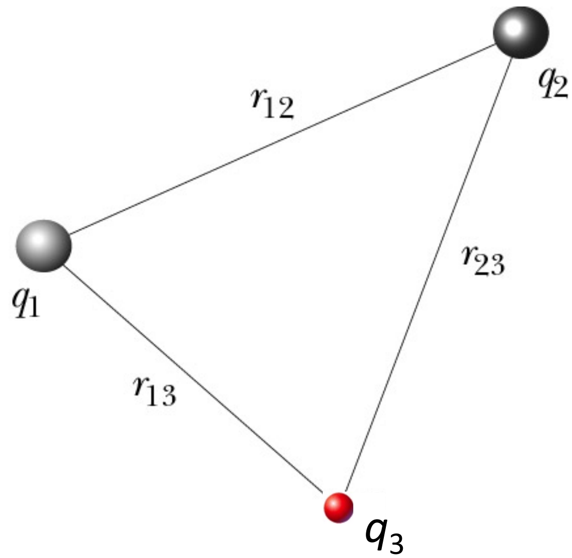
probe charge

$$U_{el} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

J/C, or Volts

V due to Two Particles



Electric potential is scalar:

$$V_C = V_{C,1} + V_{C,2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}$$

Electric potential energy of the system:

$$U_{sys} = U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

If we add one more charge q_3:

$$U_{sys} = U_{12} + V_C q_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \left(\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}\right) q_3$$

$$U_{sys} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}} = U_{12} + U_{13} + U_{23}$$

Today

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