HW3 Solutions

ECE 202 Fall 2013

MK
MN

9.a

$$L\left\{2v'_{out}(t) + 7v_{out}(t) + 6\int_{-\infty}^{t} v_{out}(q)dq = 2v'_{in}(t) + 6v_{in}(t) + 5\int_{-\infty}^{t} v_{in}(q)dq\right\}$$

$$2sV_{out}(s) + 7V_{out}(s) + \frac{6}{s}V_{out(s)} = 2sV_{in}(s) + 6V_{in}(s) + \frac{5}{s}V_{in}(s)$$

$$\left(\frac{2s^2 + 7s + 6}{s}\right)V_{out}(s) = \left(\frac{2s^2 + 6s + 5}{s}\right)V_{in}(s)$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{2s^2 + 6s + 5}{2s^2 + 7s + 6}$$

Alternatively: Differentiate both sides first

$$L\{2v''_{out}(t) + 7v'_{out}(t) + 6v_{out}(t) = 2v''_{in}(t) + 6v'_{in}(t) + 5v_{in}(t)$$

$$2s^2V_{out}(s) + 7sV_{out}(s) + 6V_{out}(s) = 2s^2V_{in}(s) + 6sV_{in}(s) + 5V_{in}(s)$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{2s^2 + 6s + 5}{2s^2 + 7s + 6}$$

9.b

$$L\left\{2v'_{out}(t) + 7v_{out}(t) + 6\int_{-\infty}^{t} v_{out}(q)dq = 2v'_{in}(t) + 6v_{in}(t) + 5\int_{-\infty}^{t} v_{in}(q)dq\right\}$$

$$2sV_{out}(s) - 24 + 7V_{out}(s) + \frac{6}{s}V_{out(s)} = 2sV_{in}(s) + 6V_{in}(s) + \frac{5}{s}V_{in}(s)$$

$$\left(\frac{2s^2 + 7s + 6}{s}\right)V_{out}(s) - 24 = \left(\frac{2s^2 + 6s + 5}{s}\right)V_{in}(s)$$

$$V_{out}(s) = \frac{2s^2 + 6s + 5}{2s^2 + 7s + 6}V_{in}(s) + \frac{24s}{2s^2 + 7s + 6}$$

$$V_{in}(s) = \frac{1}{s+1}$$

In MATLAB:

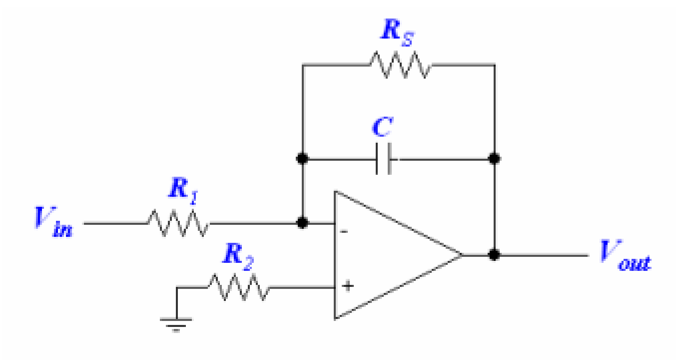
```
close all
clear all
clc
syms s t Vin H Vout vout vin
vin=exp(-1*t)*heaviside(t);
Vin=laplace(vin);
H=(2*s^2+6*s+5)/(2*s^2+7*s+6);
Vout=H*Vin+24*s/(2*s^2+7*s+6);
Vout=collect(Vout);
pretty(Vout)
```

$$V_{out}(s) = \frac{26s^2 + 30s + 5}{2s^3 + 9s^2 + 13s + 6}$$

```
vout=ilaplace(Vout);
pretty(vout)
```

$$v_{out}(t) = (e^{-t} + 49e^{-2t} - 37e^{-\frac{3t}{2}})u(t)$$

10. R_1 =100Ω, R_2 =R_s=200 Ω, C =5mF



Idead Op-Amp:

i_=0

i_+=0

v_+=v_=0

Capacitor Current: $i_c(t) = C\frac{dv_{c(t)}}{dt}$

KCL at v_: $\frac{v_- - v_{in}(t)}{R_1} + C\frac{d}{dt}\left(v_- - v_{out}(t)\right) + \frac{v_- - v_{out}(t)}{R_2} = 0$

$$-\frac{v_{in}(t)}{R_1} - C\frac{d}{dt}\, v_{out}(t) - \frac{v_{out}(t)}{R2} = 0$$

$$Plugging\ the\ values{:}\ -\frac{v_{in}(t)}{100} - 0.005\frac{d}{dt}v_{out}(t) - \frac{v_{out}(t)}{200} = 0$$

b) $v_c(0^-) = \pm v_{out}(0^-)$

$$L\left\{-\frac{v_{in}(t)}{100} - 0.005\frac{d}{dt}v_{out}(t) - \frac{v_{out}(t)}{200} = 0\right\}$$

$$-0.01V_{in}(s) - 0.005\left(sV_{out}(s) - v_{c_1}(0^-)\right) - \frac{V_{out}(s)}{200} = 0$$

$$V_{out}(s) = -\frac{2}{s+1}V_{in}(s) \pm \frac{1}{s+1}v_{c_1}(0^-)$$

c)

$$V_{in}(s) = \frac{2}{s+4}$$

In MATLAB

```
close all
clear all
clc
syms s t Vin Vout vout vin vc0
vin=2*exp(-4*t)*heaviside(t);
Vin=laplace(vin);
Vout=(-2/(s+1))*Vin+vc0/(s+1);
vc0=0;
Vout=collect(eval(Vout));
pretty(Vout)
```

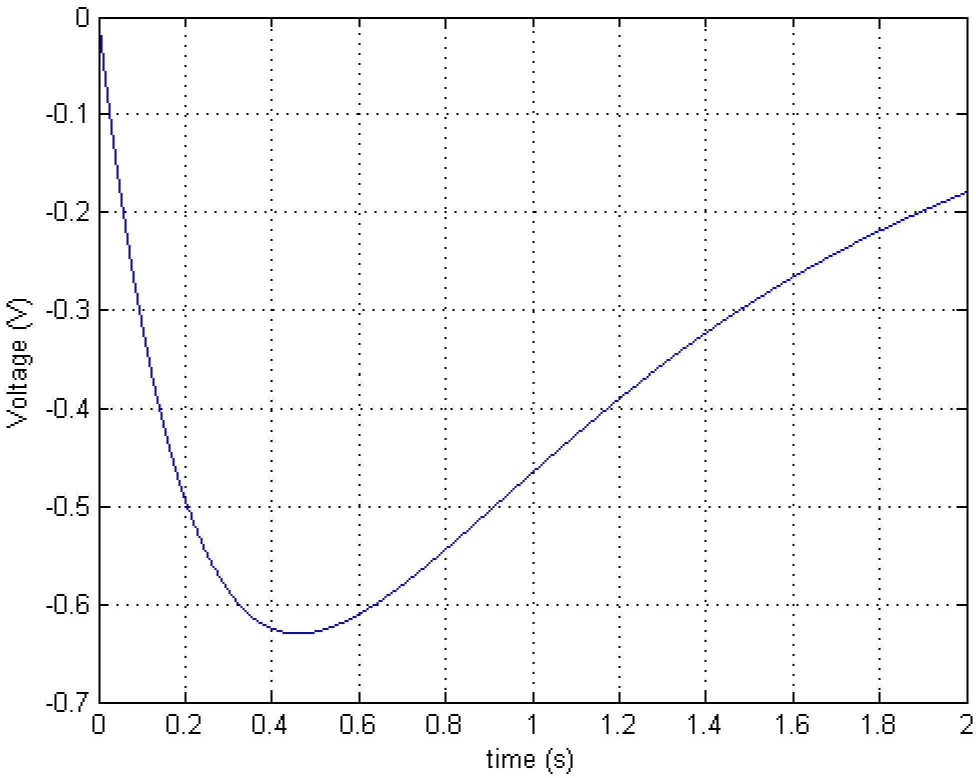
$$V_{out}(s) = -\frac{4}{s^2+5s+4}$$

```
vout=ilaplace(Vout);
pretty(vout)
```

$$v_{out}(t) = \left(\frac{4}{3}e^{-4t} - \frac{4}{3}e^{-t}\right)u(t)$$

```
t=0:0.00001:2;
```

```
vin=((4/3)./exp(4*t) - (4/3)./exp(t)).*heaviside(t);
figure(1)
plot(t,vin)
grid
xlabel('time (s)')
ylabel('Voltage (V)')
```



d) *Assuming* $v_c(0^-) = v_{out}(0^-)$

$$V_{in}(s) = \frac{2}{s+2}$$

In MATLAB:

```
close all
clear all
clc
syms s t Vin Vout vout vin vc0
vin=2*exp(-2*t)*heaviside(t);
Vin=laplace(vin);
Vout=(-2/(s+1))*Vin+vc0/(s+1);
vc0=4;
Vout=collect(eval(Vout));
```

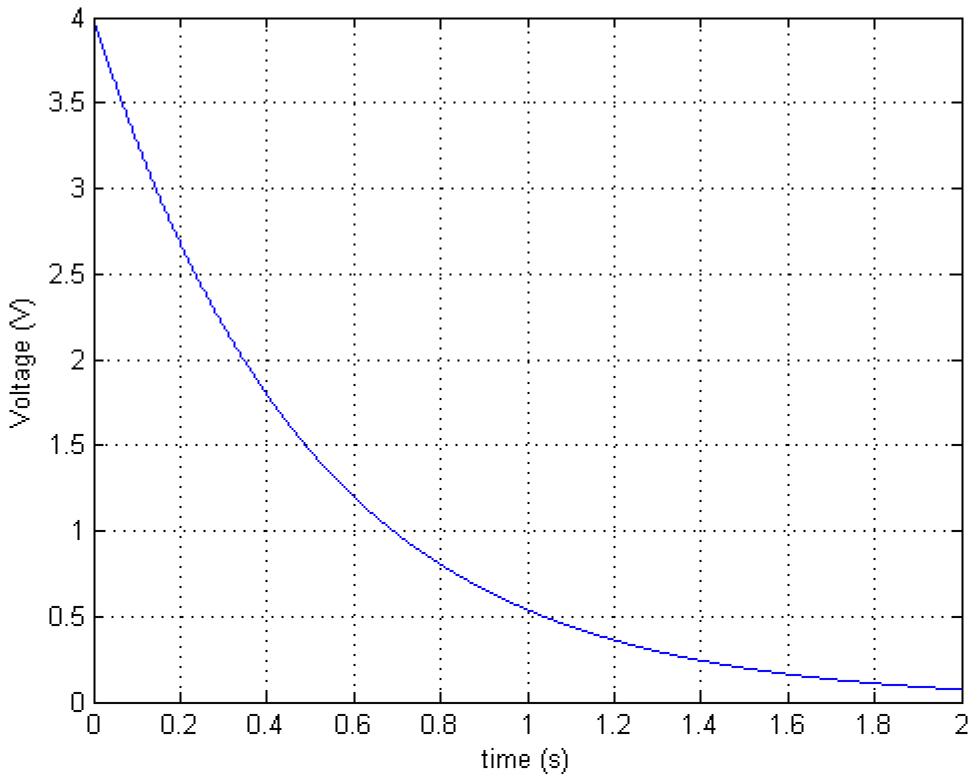
```
pretty(Vout)
```

$$V_{out}(s) = \frac{4}{s+2}$$

```
vout=ilaplace(Vout);
pretty(vout)
```

$$v_{out}(t) = 4e^{-2t}u(t)$$

```
t=0:0.00001:2;
vin=(4./exp(2*t)).*heaviside(t);
figure(1)
plot(t,vin)
grid
xlabel('time (s)')
ylabel('Voltage (V)')
```



$$Assuming\ v_c(0^-) = -v_{out}(0^-)$$

$$V_{in}(s) = \frac{2}{s+2}$$

In MATLAB:

```
close all
```

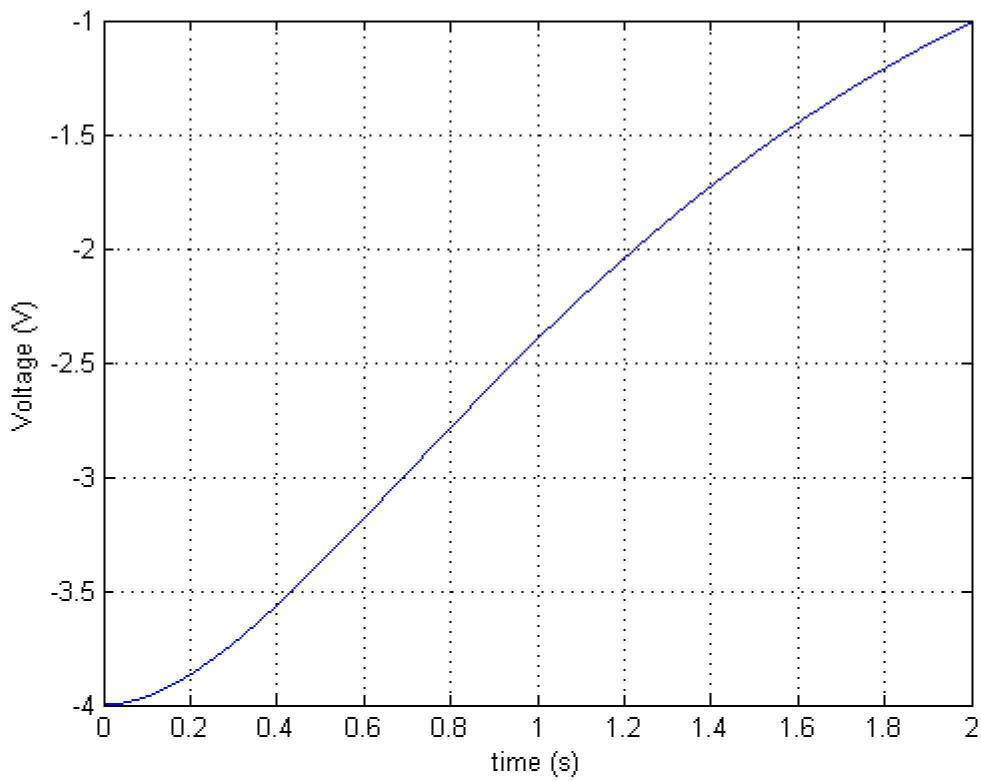
```
clear all
clc
syms s t Vin Vout vout vin vc0
vin=2*exp(-2*t)*heaviside(t);
Vin=laplace(vin);
Vout=(-2/(s+1))*Vin-vc0/(s+1);
vc0=4;
Vout=collect(eval(Vout));
pretty(Vout)
```

$$V_{out}(s) = \frac{-4s - 12}{s^2 + 3s + 2}$$

```
vout=ilaplace(Vout);
pretty(vout)
```

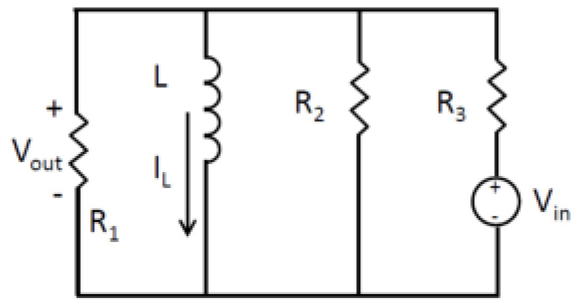
$$v_{out}(t) = (4e^{-2t} - 8e^{-t})u(t)$$

```
t=0:0.00001:2;
vin=(4./exp(2*t) - 8./exp(t)).*heaviside(t);
figure(1)
plot(t,vin)
grid
xlabel('time (s)')
ylabel('Voltage (V)')
```



11.

a)



$$v_{in}(t) = 10r(t) - 10r(t-2)$$

$$V_{in}(s) = \frac{10}{s^2} - \frac{10e^{-2s}}{s^2}$$

$$v_{out}(t) = \frac{5}{4}(1 - e^{-3t})u(t) + \frac{15}{12}\left(e^{-3(t-2)} - 1\right)u(t-2)$$

In MATLAB:

```
close all
clear all
clc
syms s L R1 R2 Vout R3 Vin Zin Zeq H
Zeq=1/((1/R1)+(1/R2)+1/(L*s));
Zin=R3+Zeq;
R1=120;R2=30;R3=40;L=5;
Zin=collect(eval(Zin));
pretty(Zin)
```

$$Z_{in}(s) = \frac{320s + 960}{5s + 24}$$

```
H=Zeq/(Zin);
H=collect(eval(H));
pretty(H)
```

$$H(s) = \frac{3s}{8s + 24}$$

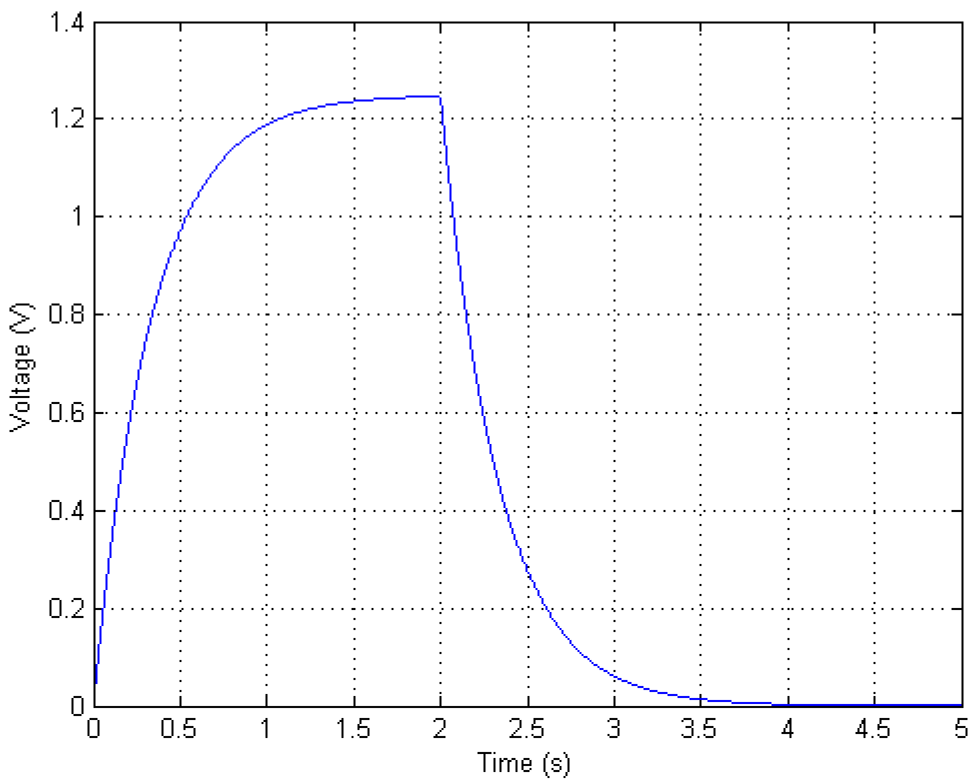
```
Vin=10/s^2-(10*exp(-2*s))/s^2;
Vout=collect(H*Vin);
pretty(Vout)
```

$$V_{out}(s) = \frac{15 - 15e^{-2s}}{4s(s+3)}$$

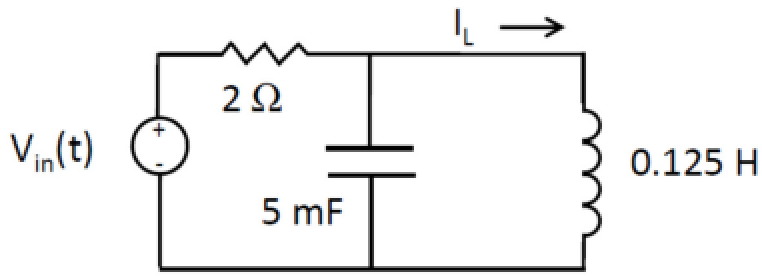
```
vout=ilaplace(Vout);
pretty(vout)
```

$$v_{out}(t) = \frac{5}{4}(1 - e^{-3t})u(t) + \frac{15}{12}\left(e^{-3(t-2)} - 1\right)u(t-2)$$

```
t=0:0.00001:5;
vout=  5/4*heaviside(t)- (5/4)*exp(-3*t).*heaviside(t)+15/12*exp(6 -
3*t).*heaviside(t - 2)- 15/12.*heaviside(t - 2);
figure(1)
plot(t,vout)
grid
xlabel('Time (s)')
ylabel('Voltage (V)')
```



b)



$$V_{in}(s) = \frac{10}{s+1}$$

```
close all
clear all
clc
syms s C R L Vin Vc IL H Zin Zeq Yin
Zeq=1/(C*s+1/(L*s));
Zin=R+Zeq;
R=2;C=0.005;L=0.125;
Zin=collect(eval(Zin));
pretty(Zin)
```

$$Z_{in}(s) = \frac{2s^2 + 200s + 3200}{s^2 + 1600}$$

```
Yin=1/Zin;
pretty(Yin)
```

$$Y_{in}(s) = \frac{s^2 + 1600}{2s^2 + 200s + 3200}$$

```
H=Zeq/Zin;
H=collect(eval(H));
pretty(H)
```

$$H(s) = \frac{100s}{s^2 + 100s + 1600}$$

```
Vin=10/(s+1);
Vc=H*Vin;
IL=Vc/(L*s);
IL=collect(eval(IL));
pretty(IL)
```

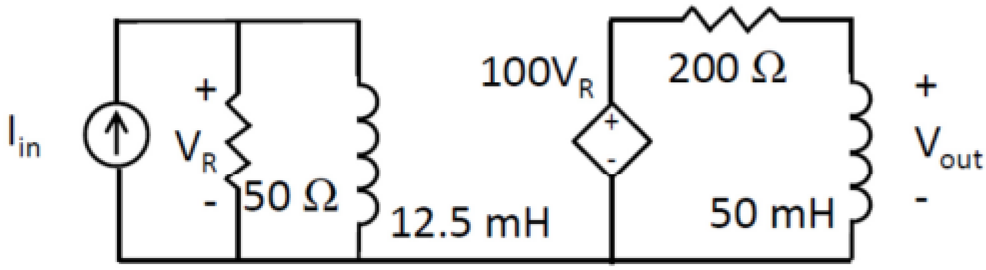
$$I_L(s) = \frac{8000}{s^3 + 101s^2 + 1700s + 1600}$$

```
iL=ilaplace(IL);
```

```
pretty(iL)
```

$$i_L(t) = \left(\frac{8000}{1501}e^{-t} - \frac{400}{57}e^{-20t} + \frac{400}{237}e^{-80t}\right)u(t)$$

12.



a)

$$Y_{in}(s) = \frac{1}{50} + \frac{1}{0.0125s}$$

$$Y_{in}(s) = \frac{s+4000}{50s}$$

$$Z_{in}(s) = \frac{50s}{s+4000}$$

b)

$$V_R = Z_{in}(s)I_{in}(s)$$

$$V_{out}(s) = \frac{0.05s}{200+0.05s} . 100V_R$$

$$V_{out}(s) = \frac{100s}{s+4000} . \frac{50s}{s+4000} I_{in}(s)$$

$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{5000s^2}{(s+4000)^2}$$

c)

$$I_{in}(s) = \frac{40}{s^2}$$

$$V_{out}(s) = H(s)I_{in}(s) = \frac{200000}{(s+4000)^2}$$

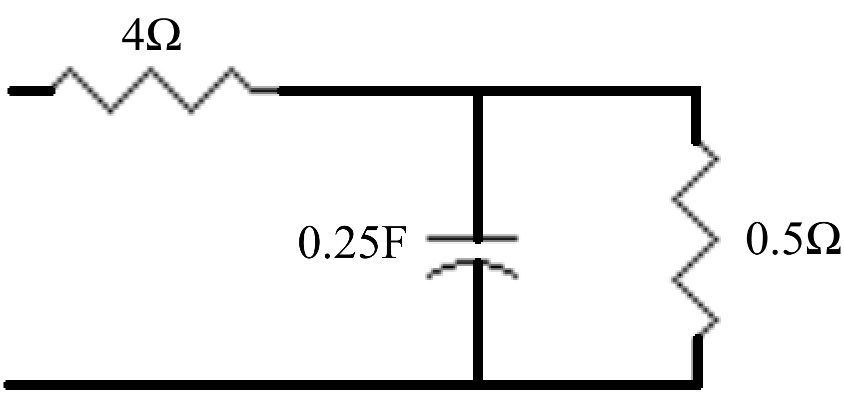
$$i_{in}(t) = 200000te^{-4000t}u(t)$$

NR:

a)

$Use\ long\ division: Z_{in}(s) = \dfrac{4s+36}{4s+8} = 4 + \dfrac{4}{s+8} = 4 + \dfrac{1}{0.25s+2}$

$This\ \dfrac{1}{\frac{s}{4}+2}\ term\ is\ the\ reciprocal\ of\ an\ admittance\ and\ thus\ represents\ a\ 0.25F\ cap\ in\ parallel\ with\ a\ 0.5\Omega\ resistor$



$b)$

$$Y_{in} = \frac{2s+12}{s+2} + \frac{2s+12}{s+4}$$

$There\ are\ two\ parallel\ branches\ represented\ by:$

$$Y_1 = \frac{2s+12}{s+2}, and\ Y_2 = \frac{2s+12}{s+4}$$

$$Solve\ Y_1$$

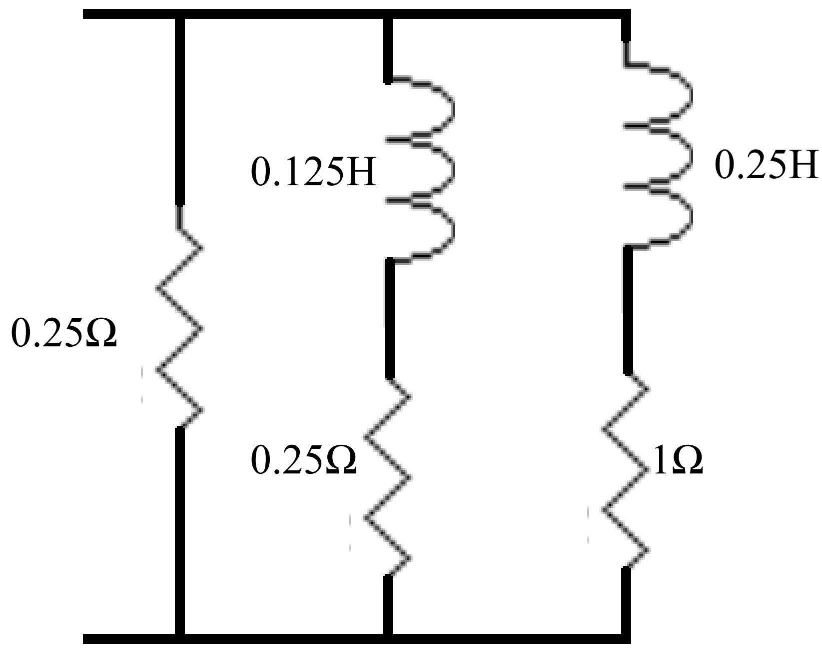
$$Y_1 = \frac{2s+12}{s+2} = 2 + \frac{8}{s+2} = 2 + \frac{1}{\frac{s}{8}+\frac{1}{4}}$$

$The\ \dfrac{1}{\frac{s}{8}+\frac{1}{4}}\ term\ is\ the\ reciprocal\ of\ an\ impedance\ and\ thus\ represent\ a\ 0.125H\ inductor\ in\ series\ with\ a\ 0.25\Omega\ resistor$

$Solve\ Y_2$

$$Y_2 = \frac{2s+12}{s+4} = 2 + \frac{4}{s+4} = 2 + \frac{1}{\frac{s}{4}+1}$$

$$\textit{Recombining } Y_1 + Y_2 = 4 + \frac{1}{\frac{s}{8}+\frac{1}{4}} + \frac{1}{\frac{s}{4}+1}$$



$c)$

$$Z_{in}(s) = \frac{s^2+4}{2s} + \frac{2s}{s^2+4}$$

$$Z_1 = \frac{s^2+4}{2s}, \ and \ Z_2 = \frac{2s}{s^2+4}$$

Solve for Z_1

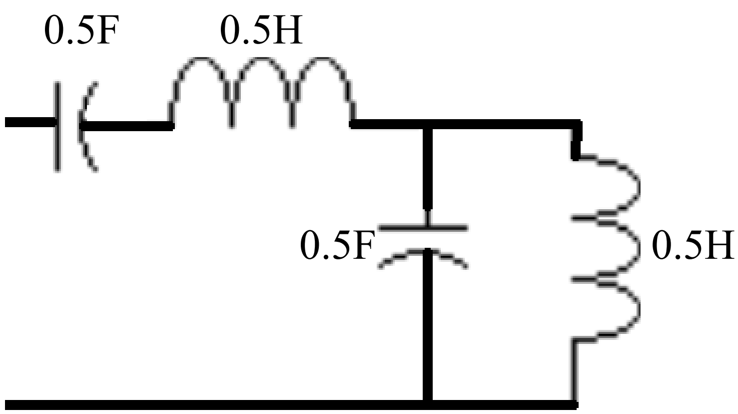
$$Z_1 = \frac{s^2+4}{2s} = \frac{s}{2} + \frac{2}{s} = 0.5s + \frac{1}{0.5s}$$

Z_1 *represents a* $0.5H$ *inductor which is in series with a* $0.5F$ *capacitor*

Solve for Z_2

$$Z_2 = \frac{2s}{s^2+4} = \frac{1}{\frac{s}{2}+\frac{2}{s}} = \frac{1}{0.5s + \frac{1}{0.5s}}$$

Z_2 *represents a* $0.5H$ *inductor which is in parallel with a* $0.5F$ *capacitor*



$d)$

$$Y_{in}(s) = \frac{s^2+1}{2s} + \frac{0.25s}{s^2+4} = Y_1 + Y_2 = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$Solve\ for\ Y_1$

$$Y_1 = \frac{s^2+1}{2s} = 0.5s + \frac{1}{2s}$$

$Y_1\ represents\ a\ 2H\ inductor\ which\ is\ in\ parallel\ with\ a\ 0.5F\ capacitor$

$Solve\ for\ Y_2$

$$Y_2 = \frac{0.25s}{s^2+4} = \frac{1}{4s + \frac{16}{s}}$$

$Y_2\ represents\ a\ 4H\ inductor\ which\ is\ in\ series\ with\ a\ \frac{1}{16}F\ capacitor$