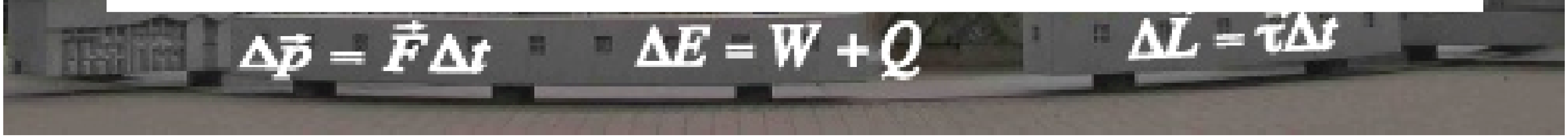
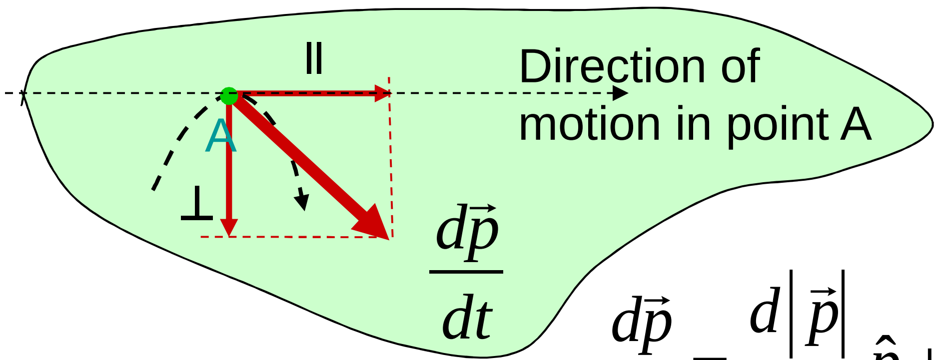
EXAM 1 tomorrow

Time: 8:00-9:30 pm tomorrow (Wed Feb 8)
Place: Elliott Hall
Material: lectures 1-8, HW 1-8, Recitations 1-4, Labs 1-4
Problems: multiple choice, 10 questions (70 points)
write-up part, hand graded (30 points)
Equation sheet: provided with exam
Practice exam + equation sheet: have been posted over the weekend

Note: no lecture this Thursday (Feb 9)!



The Momentum Principle for components



$$\frac{d\vec{p}}{dt}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

$$\frac{d\vec{p}}{dt} = \frac{d\left|\vec{p}\right|}{dt}\hat{p} + \left|\vec{p}\right|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$

Parallel component:

$$\frac{d\left|\vec{p}\right|}{dt}\hat{p} = \vec{F}_{\parallel}$$

Perpendicular component:

$$\left|\vec{p}\right|\frac{d\hat{p}}{dt} = \vec{F}_{\perp}$$

$$p\left|\frac{d\hat{p}}{dt}\right| = F_{\perp}$$

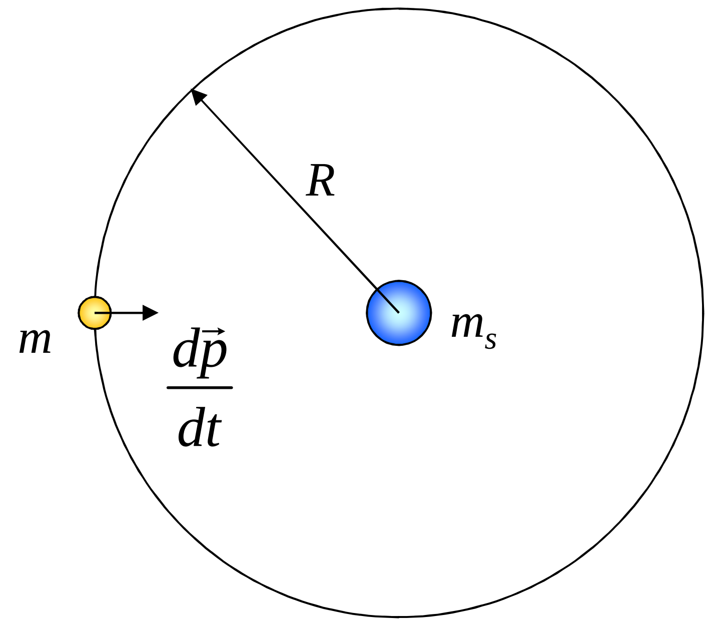
$$\left(\frac{d\vec{p}}{dt}\right)_{\parallel} = \frac{dp}{dt} = F_{\parallel} \qquad \left(\frac{d\vec{p}}{dt}\right)_{\perp} = p\frac{v}{R} = F_{\perp}$$

Circular motion: conditions

Only perpendicular component for circle!

Condition for circular motion:

$$\left(\frac{d\vec{p}}{dt}\right)_{\perp} = p\frac{v}{R} = F_{\perp} = F$$



<u>For planets (assume sun is fixed)</u>

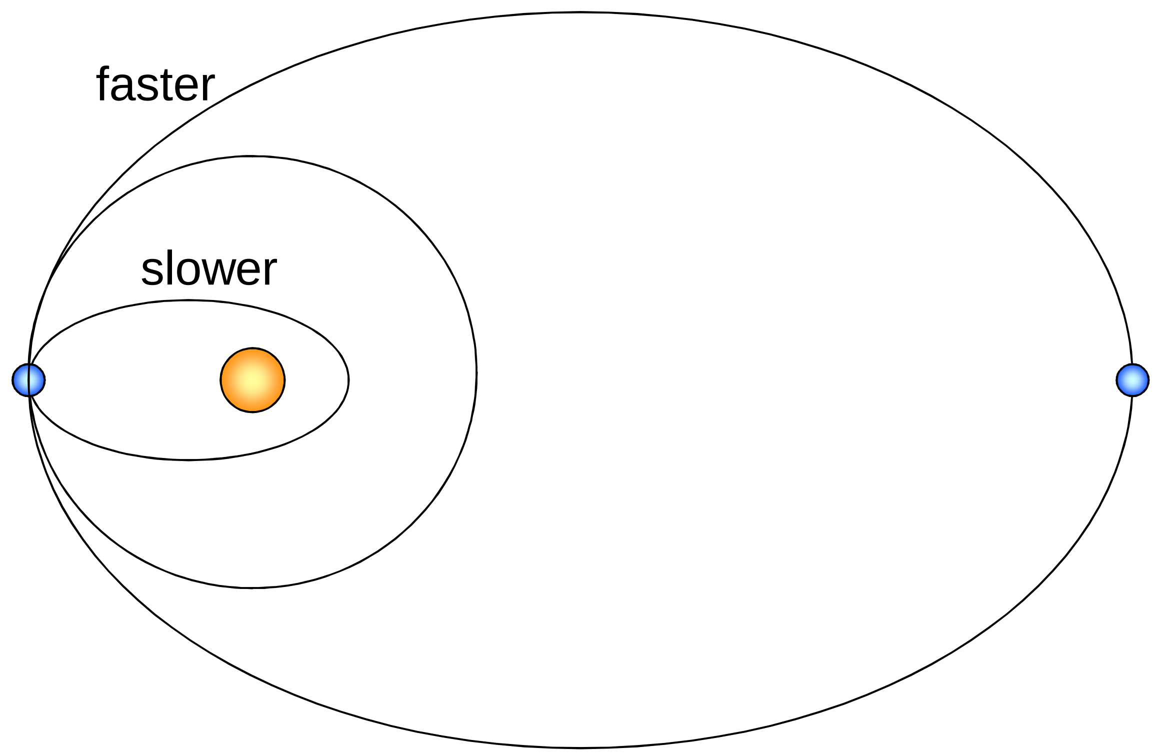
$$p\frac{v}{R} = G\frac{mm_s}{R^2}$$

Nonrelativistic: $p \approx mv$

$$mv\frac{v}{R} = G\frac{mm_s}{R^2}$$

$$v^2 = G\frac{m_s}{R}$$ Condition for circular motion for planets

Initial conditions for circular motion



$$v^2 = G\frac{m_s}{R}$$

Condition for circular motion for planets

Energy Principle

What Is Energy?

Energy is just some number you can compute that obeys a conservation law. It's helpful book-keeping.

(See Feynman Lectures on Physics, Vol. 1, Sec. 4-1)

Why Introduce Energy?

Without following the details of a complicated process, we can:

- say something about initial and final states of a system
- predict whether some process CAN occur

Basically, **for some problems, using energy is much simpler than using the momentum principle**.

Often, however, we'll use both principles in combination.

The Energy Principle

$$\boxed{\Delta E_{system} = \underbrace{W_{surr} + Q}_{\text{“cause” (due to interactions)}}}$$

“effect”

* We mean the Work done ON a System by a Force in the Surroundings*

Compare With Momentum Principle:

$$\boxed{\Delta\vec{p}_{system} = \underbrace{\text{impulse}}_{\text{“cause” (due to interactions)}} = \vec{F}_{net}\Delta t}$$

“effect”

NOTE: Momentum is a vector. Energy is a scalar (no direction).

Energy is A Conserved Quantity

Energy is a useful thing to consider because energy can't be destroyed: it can only change forms.

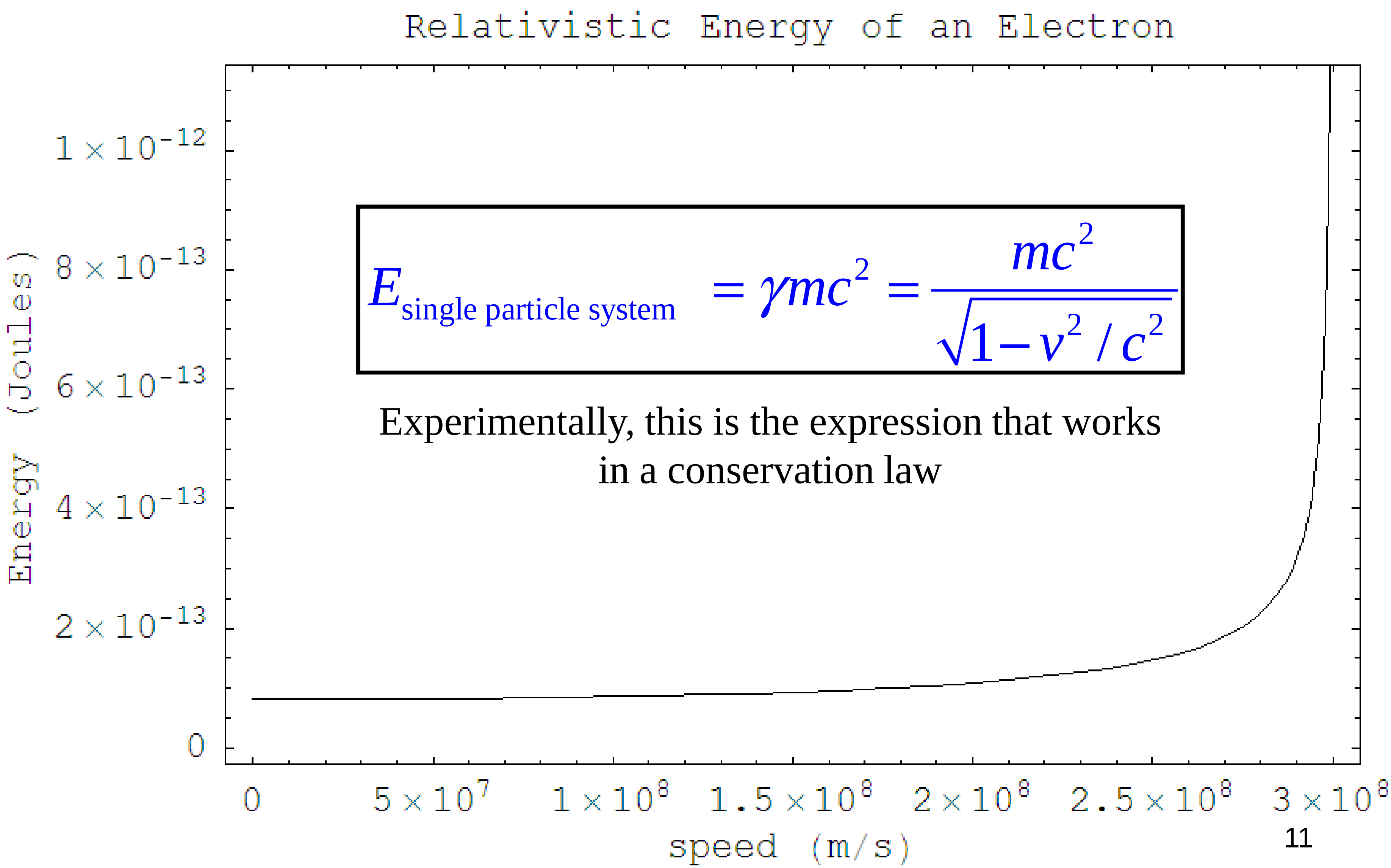
$$\boxed{\Delta E_{system} + \Delta E_{surroundings} = 0}$$

Compare with Conservation of Momentum:

$$\boxed{\Delta \vec{p}_{system} + \Delta \vec{p}_{surroundings} = 0}$$

Reminder: Momentum is a vector. Energy is a scalar (no direction).

Energy of a Single Particle System

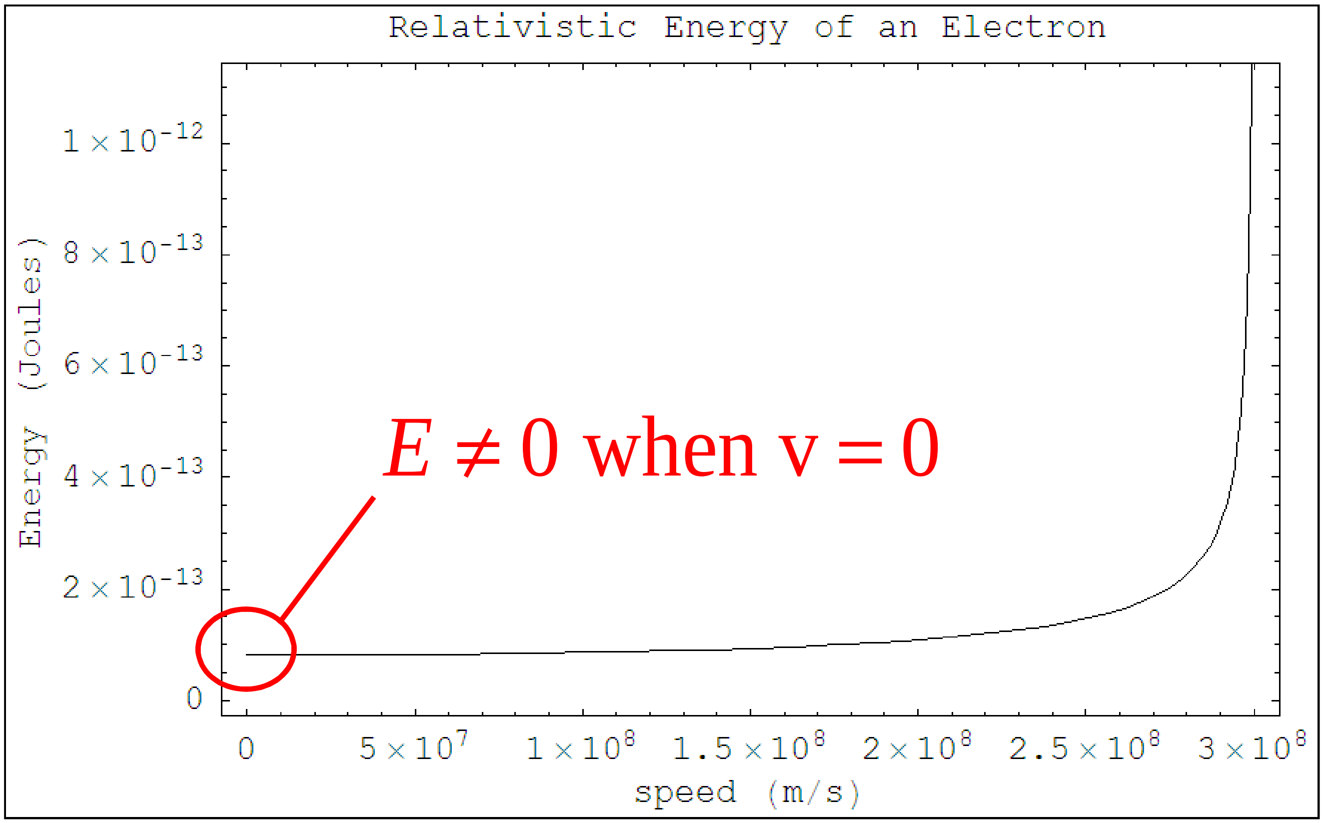


$$\boxed{E_{\text{single particle system}} = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}}$$

Experimentally, this is the expression that works in a conservation law

Rest Energy

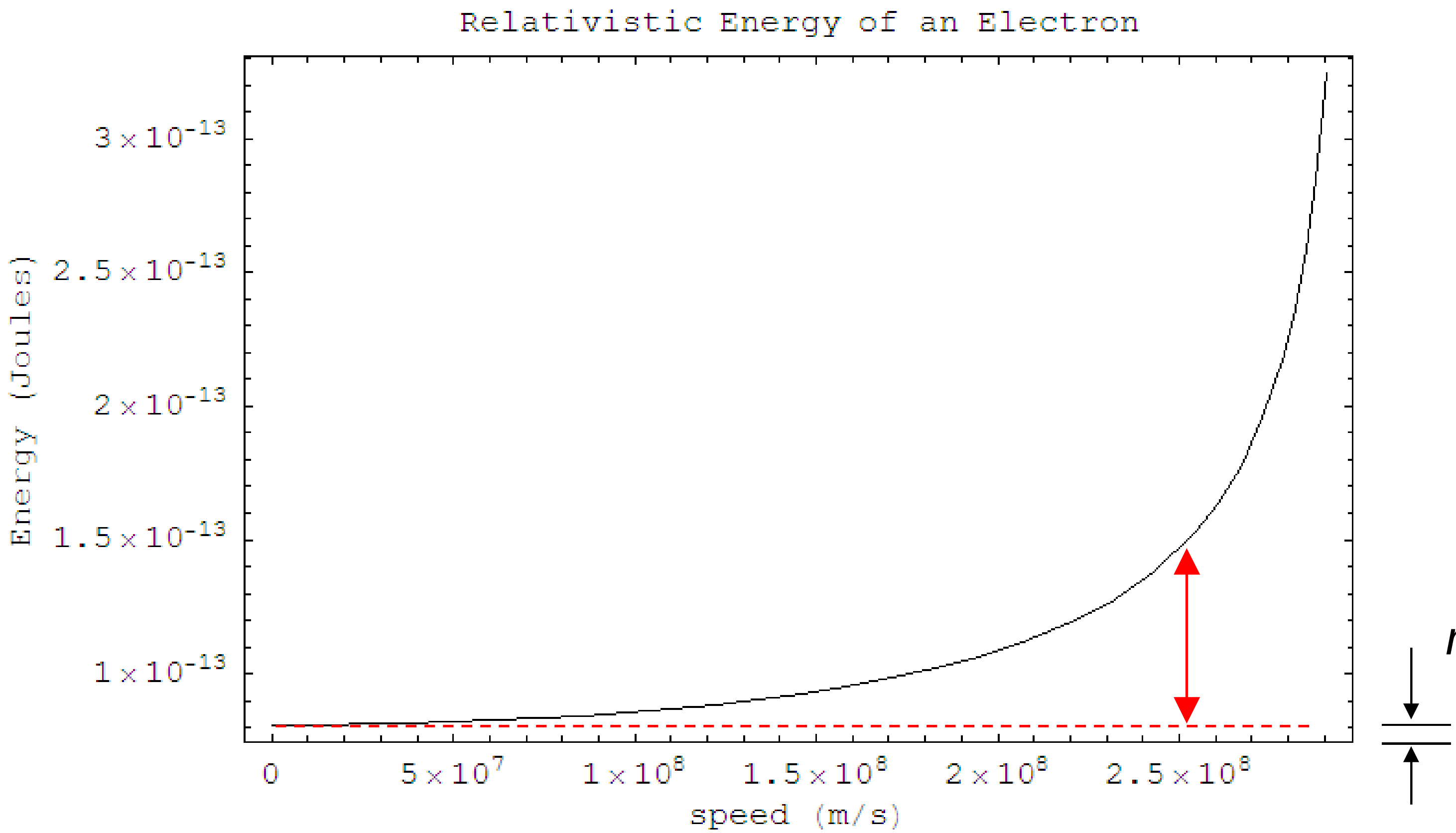
Note: $v = 0 \;\rightarrow\; \gamma = 1 \;\rightarrow\; E_{\text{single particle}} = \gamma mc^2 = mc^2$



A particle at rest has some energy just because it exists!

Rest energy = mc^2.

Kinetic Energy



Excess energy above rest energy (mc^2) is due to motion (ignoring internal degrees of freedom):

$$\boxed{\text{Kinetic energy } K \equiv \gamma mc^2 - mc^2}$$

Kinetic Energy

$$K \equiv \gamma mc^2 - mc^2$$

$$\boxed{K \approx \frac{1}{2}mv^2 \quad \text{for} \quad v/c \ll 1}$$

What?? What happened to $K = \frac{1}{2}mv^2$?

At low velocities, $v/c \ll 1$

Use a Taylor expansion (power series): $(1+x)^n \approx 1 + nx + \dots$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \left(1-(v/c)^2\right)^{-1/2}$$

$$\approx 1 - \frac{1}{2}\left(-(v/c)^2\right) + \dots = 1 + \frac{1}{2}(v/c)^2$$

$$\Rightarrow K = mc^2(\gamma - 1) \approx mc^2\left(1 + \frac{1}{2}(v/c)^2 - 1\right) = \frac{1}{2}mv^2$$

Energy And Momentum

$$E = \gamma m c^2 \qquad p = \gamma m v \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$E^2 = \gamma^2 m^2 c^4 \qquad (pc)^2 = \gamma^2 m^2 c^2 v^2$$

$$E^2 - (pc)^2 = \gamma^2 m^2 c^2 (c^2 - v^2)$$

Use $\gamma^2 = \dfrac{1}{1 - v^2/c^2} = \dfrac{c^2}{c^2 - v^2}$

$$= \left(\frac{c^2}{\cancel{c^2 - v^2}}\right) m^2 c^2 \cancel{(c^2 - v^2)} \quad = m^2 c^4$$

$$\boxed{E^2 - (pc)^2 = m^2 c^4}$$

ALWAYS TRUE.
EVERY REFERENCE FRAME.