

3.1

Fall 2013 HW # 3 solution

Built in potential

$$\phi_i = V_T \ln \left(\frac{N_A N_D}{n_i^2} \right) = (0.025 \text{ V}) \ln \left(\frac{10^{19} \text{ cm}^{-3} 10^{18} \text{ cm}^{-3}}{10^{20} \text{ cm}^{-6}} \right)$$

$$= 0.979 \text{ V}$$

Depletion Width

$$W_{do} = \sqrt{\frac{2 \epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \phi_i}$$

$$= \sqrt{\frac{2 \cdot 11.7 \cdot 8.85 \times 10^{-14} \text{ F/cm}}{1.60 \times 10^{-19} \text{ C}} \left(\frac{1}{10^{19} \text{ cm}^{-2}} + \frac{1}{10^{18} \text{ cm}^{-2}} \right) (0.979 \text{ V})}$$

$$= 3.73 \times 10^{-6} \text{ cm}$$

$$= 37.3 \text{ nm} \text{ or equivalent}$$

from example 3i2:

$$X_n = \frac{W_{do}}{1 + \frac{N_D}{N_A}} = \frac{37.3 \text{ nm}}{1 + \frac{10^{18}}{10^{19}}} = \frac{37.3 \text{ nm}}{1.1} = 33.9 \text{ nm}$$

$$X_p = \frac{W_{do}}{1 + \frac{N_A}{N_D}} = \frac{37.3 \text{ nm}}{1 + \frac{10^{19}}{10^{18}}} = \frac{37.3}{11} = 3.39 \text{ nm}$$

$$E_{\max} = \frac{2 \phi_i}{W_{do}} = \frac{2 (0.979 \text{ V})}{3.73 \times 10^{-6} \text{ cm}} = 5.2 \times 10^5 \text{ V/cm}$$

$$= \frac{q N_A X_p}{\epsilon_s} = \frac{q N_D X_n}{\epsilon_s} = 5.2 \times 10^5 \text{ V/cm}$$

3.2 Part A

$$p_p \approx N_A = 10^{18} \text{ cm}^{-3}$$

$$n_p = \frac{n_i^2}{p_p} = \frac{10^{20}}{10^{18}} = 100 \text{ cm}^{-3}$$

$$n_n \approx N_D = 10^{15} \text{ cm}^{-3}$$

$$p_n = \frac{n_i^2}{n_n} = \frac{10^{20}}{10^{15}} = 10^5 \text{ cm}^{-3}$$

3.12

$$N_A(x) = N_0 \cdot \exp(-x/L) = p(x)$$

$$j_p = \underbrace{q \mu_p p E}_{\text{Drift term}} - \underbrace{q D_p \frac{\partial p}{\partial x}}_{\text{Diffusion term}} = 0 \quad (*)$$

Because $\frac{\partial p}{\partial x} \neq 0$, there must be an electric field $E \neq 0$ to satisfy the requirement that $j_p = 0$ in equilibrium.

What is $E(x)$?

$$E(x) = \frac{V_T}{p(x)} \frac{\partial p}{\partial x} \quad \text{using equation } (*) \text{ above and recalling that } \frac{D}{\mu} = \frac{kT}{q} = V_T \text{ for non-degenerate semiconductor.}$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial x} \left(N_0 \exp\left(-\frac{x}{L}\right) \right) = -\frac{N_0}{L} \exp\left(-\frac{x}{L}\right)$$

$$E(x) = \frac{V_T}{N_0 \exp(-x/L)} \cdot -\frac{N_0}{L} \exp\left(-\frac{x}{L}\right) = -\frac{V_T}{L}$$

$$E(x) = \frac{-0.0250 \text{ V}}{1 \times 10^{-6} \text{ m}} = 25000 \frac{\text{V}}{\text{m}} = \boxed{250 \frac{\text{V}}{\text{cm}}}$$

For this special doping profile, the electric field is position independent. It is also independent of N_0 .

3.21

Because the diode current increases exponentially with bias voltage, it is in forward bias. Therefore we use

$$i_D = I_S \left[\exp\left(\frac{V_D}{nV_T}\right) - 1 \right] \approx I_S \exp\left(\frac{V_D}{nV_T}\right)$$

take \log_{10} of Both sides

$$\log_{10}(i_D) = \log_{10}(I_S) + \frac{1}{\ln(10)} \cdot \frac{V_D}{nV_T}$$

Now, from the graph, we use the points

$(0.2V, 10^{-9}A)$ and $(0.6V, 10^{-4}A)$ to

determine the slope $\frac{1}{nV_T} \cdot \frac{1}{\ln(10)}$ and intercept $\log_{10}(I_S)$ of the log plot.

$$\text{slope} = \frac{-4 - (-9)}{0.6V - 0.2V} = \frac{5}{0.4V} = 12.5$$

$$\begin{aligned} \text{intercept} = \log_{10}(I_S) &= \log_{10}(I_1) + V_1 \cdot (-\text{slope}) \\ &= -9 - 0.2(12.5) \\ &= -11.5 \end{aligned}$$

Now

$$\eta = \frac{1}{\ln(10)} \cdot \frac{1}{\text{slope} \cdot V_T} = \frac{1}{12.5 \cdot 0.025} (0.434) = \boxed{1.39}$$

$$I_S = 10^{(-11.5)} = \boxed{3.2 \text{ pA}}$$

3.24

$$I = I_s \left(\exp\left(\frac{V}{nV_T}\right) - 1 \right) \\ = 10^{-16} \left(\exp\left(\frac{0.675V}{0.025V}\right) - 1 \right)$$

$$I = 53.2 \mu A$$

$$I = 10^{-16} \left(\exp\left(\frac{3 \cdot 0.675V}{0.025V} - 1\right) \right) =$$

$$I = 1.5 \times 10^{19} A$$

3.37

$$W_d = W_{d0} \sqrt{1 + \frac{V_R}{\phi_j}} \quad \text{eq. 3.18}$$

$$= 1 \mu m \sqrt{1 + \frac{5V}{0.8V}} =$$

$$I = 2.7 \mu m \quad \text{for } V_R = 5V$$

$$V_R = 5V \\ V_D = -10V \\ \phi_j = 0.8 \\ W_{d0} = 1 \mu m$$

$$V_d = -10V \Rightarrow V_R = 10V$$

$$W_d = 1 \mu \sqrt{1 + \frac{10V}{0.8V}}$$

$$I = 3.7 \mu m \quad \text{for } V_D = -V_R = -10V$$

$$\frac{3.44}{C_{j0}} = \frac{\epsilon_s}{W_{d0}}$$

$$W_{d0} = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \phi_j}$$

$$\phi_j = V_t \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

$$\phi_j = 0.025 \ln \left(\frac{10^{18} \cdot 10^{15}}{10^{20}} \right) = 0.748 \text{ V}$$

$$W_{d0} = \sqrt{\frac{2 \cdot 11.7 \cdot 8.854 \times 10^{-14} \text{ F/cm}}{1.602 \times 10^{-19} \text{ C}} \left(\frac{1}{10^{18} \text{ cm}^{-3}} + \frac{1}{10^{15} \text{ cm}^{-3}} \right) (0.748 \text{ V})}$$

$$W_{d0} = 9.85 \times 10^{-5} \text{ cm}$$

$$C_{j0} = \frac{\epsilon_s}{W_{j0}} = \frac{11.7 \cdot 8.854 \times 10^{-14} \text{ F/cm}}{9.85 \times 10^{-5} \text{ cm}} = \boxed{1.05 \times 10^{-8} \text{ F/cm}^2}$$

for $V_R = 9 \text{ V}$

$$C_j = \frac{C_{j0} \cdot A}{\sqrt{1 + \frac{V_R}{\phi_j}}} = \frac{(1.05 \times 10^{-8} \text{ F/cm}^2)(0.02 \text{ cm}^2)}{\sqrt{1 + \frac{9 \text{ V}}{0.748 \text{ V}}}}$$

$$= 5.83 \times 10^{-11} \text{ F}$$

$$= 58.3 \text{ pF}$$