CS251 Home

Homework 1

Homework 1: Proofs, Performance Analysis, and Basic Data Structures

Handed out: **August 30, 2012**
Due: **September 6, 2012 at 11:59pm**

Last updated: *August 30, 2012 3:42 PM*

Submission

Submit your answers to the questions below **as a PDF document**. Failure to do so will result in points loss. If you do not know how to generate a PDF document, do ask! You will use turnin for this submission and name your PDF file <your_first_name>_<your_last_name>.pdf, as you did for the first project. The turnin command will therefore be:

```
% turnin -c cs251 -p homework1 <your_first_name>_<your_last_name>.pdf
```

Grading

There are a total of 100 points for this homework, distributed as follows.

Question 1. (10 points)

Use mathematical induction to prove the following statement:

For $n \geq 1$, $1^3 + 2^3 + ... + n^3 = (1 + 2 + ... + n)^2$

Use for that the known result (seen in class): $1 + 2 + ... + n = n(n+1)/2$.

Question 2. (10 points)

Let $p(x)$ be a polynomial of degree n, that is, $p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$.

(1) Describe a simple $O(n^2)$ time method for computing $p(x)$. *(5 points)*

(2) Now consider a rewriting of $p(x)$ as $p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + ... + x(a_{n-1} + xa_n) ...)))$, which is known as *Horner's method*. Using big-Oh notation, characterize the number of arithmetic operations executed by this method. *(5 points)*

Question 3. (15 points)

Indicate for each of the following statements what must be disproven in a proof by contradiction.

(1) $\sqrt{7}$ is irrational. *(5 points)*

(2) If five sisters split up 2000 grams of chocolate, then at least one of the sisters receives 400 or more grams of chocolate. *(5 points)*

(3) Given four non-collinear points in the plane, there exist three points which form an angle measuring 90º or more. *(5 points)*

Question 4. (13 points)

(1) Describe in detail how to swap two nodes *x* and *y* (the nodes themselves, not just their contents) in a singly list *L* given references only to *x*, *y*, and the first node in the list. *(5 points)*

(2) Repeat this exercise for the case when *L* is a doubly linked list. *(5 points)*

(3) Which algorithm takes more time? *(3 points)*

Question 5. (12 points)

For the following expressions:

(1) find the big-Oh expression *(6 points)*

(2) list in increasing big-Oh order. *(6 points)*

(a) $1723198\ n^3 + 0.0078\ n^4 + 1023$

(b) 2^{1440}
(c) $\log(\log(n^2)^4)$
(d) $2^{\log(n2)}$
(e) $n! + 2^n$
(f) $n^{\log(n)} + n$

Question 6. (20 points)

Find the big-Oh notation for the expression computed by the following codes.

(1) *(10 points)*

```
int sum=0
for (int i=0 ; i<N ; ++i) {
   for (int j=0; j<N ; ++j) {
      sum++;
   }
}
```

(2) *(10 points)*

```
int prod = 2;
for (int i=0 ; i<N2 ; i++) {
   for (int j=i+1 ; j<N2 ; j*=2) {
      prod *= prod;
   }
}
```

Question 7. (20 points)

Suppose you have a stack *S* containing *n* elements and a queue *Q* that is initially empty. Describe how you can use *Q* to scan *S* to see if it contains a certain element *x*, with the additional constraint that your algorithm must return the elements back to *S* in their original order. You may not use an array or linked list, only *S* and *Q* and a constant number of reference variables.