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Summer 2012

ECE 20200 : Linear Circuit Analysis II
School of ECE, Purdue University

LECTURE 12

- H(s) : poles, zeros, s-plane
- Stability

Reference: Decarlo/Lin PP 685-693

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H(s) : poles , zeros, s-plane

Transfer functions

General structure

$$H(s) = \frac{n(s)}{d(s)} = K \frac{(s-z_1)(s-z_2)(s-z_3)\cdots(s-z_m)}{(s-p_1)(s-p_2)(s-p_3)\cdots(s-p_n)}$$

$$= K \frac{s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n}$$

$\boxed{m \leq n \text{ in ECE 20200}}$

(a) z_i are finite zeros of $H(s)$: $H(z_i) = 0$

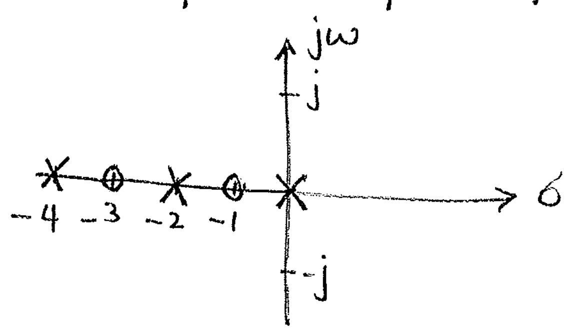
(b) p_i are finite poles of $H(s)$: $H(p_i) = \infty$

(c) $n - m$ = # of infinite zeros

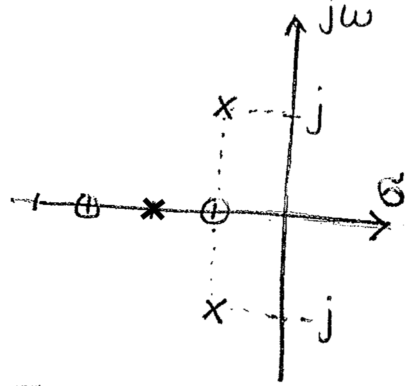
(d) D.C gain = $H(0) = (-1)^{m-n} \dfrac{\prod z_i}{\prod p_i}$ provided $p_i \neq 0$ for all i.

Example 1. $H(s) = 2.5 \dfrac{(s+1)(s+3)}{s(s+2)(s+4)}$

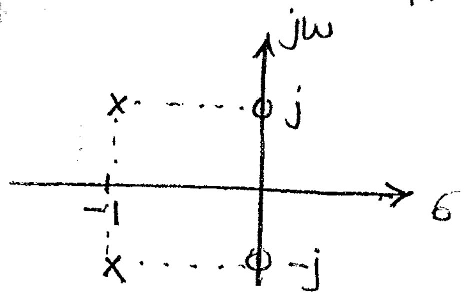
Draw the pole-zero plot of H(s).



Example 2. Draw the pole-zero plot of $H(s) = \dfrac{(s+1)(s+3)}{((s+1)^2+1)(s+2)}$



Example 3. Find $H(s)$ from the pole-zero plot given below. Suppose $H(0) = 5$.



$$H(s) = K \frac{(s-j)(s+j)}{(s+1-j)(s+1+j)} = K \frac{(s^2+1)}{(s+1)^2+1^2}$$

$$H(0) = 5 = \frac{K\ (1)}{(1)^2+1^2} = \frac{K}{2} \Rightarrow K = 10$$

$$\therefore\ H(s) = 10 \frac{s^2+1}{(s+1)^2+1} \leftarrow$$

Stability

1. Bounded function $\quad |f(t)| \leq K < \infty$ for $\forall t$.
2. BIBO stability (Bounded Input Bounded Output stability)

$\Rightarrow$ Bounded input must ALWAYS map to Bounded output for BIBO stability.

3. Test for BIBO stability.

If p_i is a pole of $H(s)$, then the circuit/system is BIBO stable if and only if $Re\{p_i\} < 0$ for all poles p_i.

In other words, all poles must be in the left half-plane (complex s-plane)

4. If $Re\{p_i\}$ is zero, $p_i = \sigma_i + j\omega_i = j\omega_i$, or $p_i = 0$, then any sinusoid of the form $K_1 \sin(\omega_i t) + K_2 \cos(\omega_i t)$ where K_1 or K_2 is non-zero, or if $p_i = 0$, a step input will give unbounded output and thus the system/circuit is unstable.

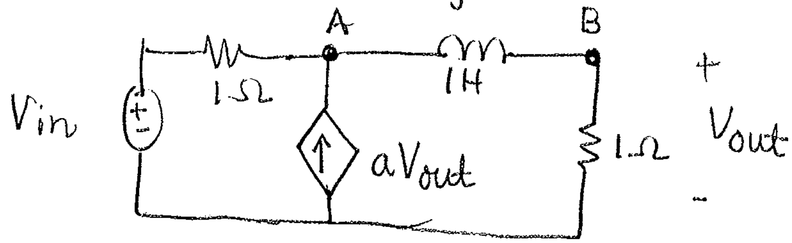
5. If $H(s) = \frac{1}{s}$, then the step response is

$$\mathcal{L}^{-1}\left[\frac{1}{s}\cdot\frac{1}{s}\right] = \mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = t\,u(t) \to \infty \text{ as } t \to \infty$$

6. If $H(s) = \frac{K}{(s+\sigma)^2+\omega^2} = \frac{K}{s^2+\omega^2}$, then for the input $\frac{1}{s^2+\omega^2}$, we obtain the response to be

$$\mathcal{L}^{-1}\left[\frac{K}{(s^2+\omega^2)^2}\right] = Kt\sin(\omega t) \to \infty \quad \text{as } t \to \infty$$

Example: Find the range of 'a' for instability.



Use nodal analysis to obtain $H(s)$

at A:
$$V_A - V_{in} - aV_{out} + \frac{V_A - V_{out}}{s} = 0 \quad —①$$

at B:
$$\frac{V_{out} - V_A}{s} + V_{out} = 0$$

$$V_{out} - V_A + sV_{out} = 0$$

$$(s+1)V_{out} = V_A \quad —②$$

Substitute ② into ①

$$(s+1)V_{out} - V_{in} - aV_{out} + \frac{V_{out}(s+1)}{s} - \frac{V_{out}}{s}$$

$$(s+1-a)V_{out} + V_{out} = V_{in}$$

$$(s+2-a)V_{out} = V_{in}$$

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{1}{s+2-a}$$

pole: $s+2-a=0 \Rightarrow$ pole at $s = -2+a$

For the system to be unstable,

$$-2+a \geq 0$$

$$a \geq 2 \leftarrow$$

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WORKSHEET 1

Qualitative Analysis of an H(s).

$$H(s) = 36 \frac{(s-1)^2 (s+2)^2}{(s+1)^2 (s+4)(s-3)^2}$$

1. How many finite zeros? ______
2. Multiplicities of finite zeros? ______
3. Locations of finite zeros? ______
4. How many finite poles? ______
5. Multiplicities of finite poles? ______
6. Locations of finite poles? ______
7. How many infinite zeros? ______
8. T/F The impulse response contains a term of the form $K\delta(t)$. ______
9. The circuit/system described by the above transfer function is BIBO stable. T/F ______
10. Pole/zero plot of H(s).

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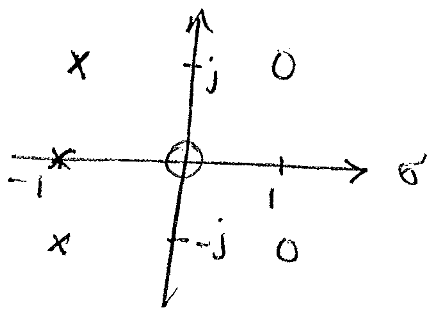
Worksheet 2

Given $H(1) = 15$ and the pole-zero plot below.

Find $H(s)$.



1) $H(s) = K$ ____________________

2) $H(1) = 15 = K \times \dfrac{[\qquad\qquad]}{[\qquad\qquad]}$

$\therefore K =$ ____

3) T-F The above transfer function is BIBO stable. ____