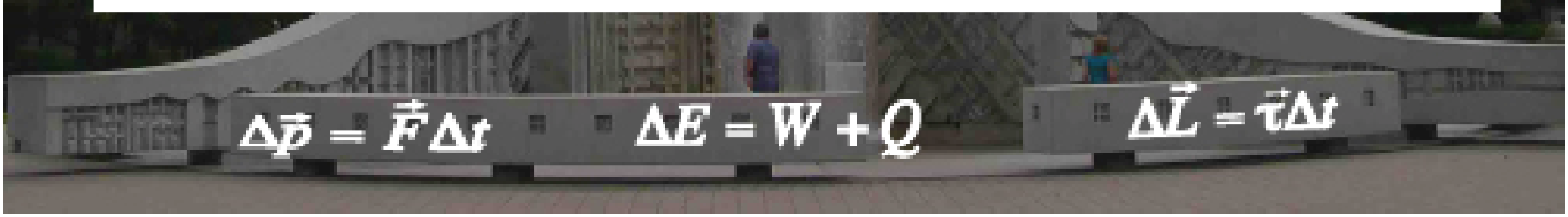
Exam 3 averages

Multiple-choice: **67.7%**

Hand-graded part**:** will be available **next week**

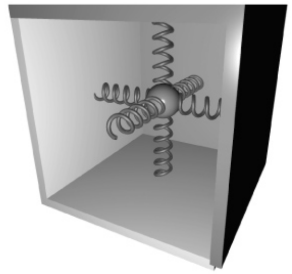
Finalizing iClicker scores 10-22

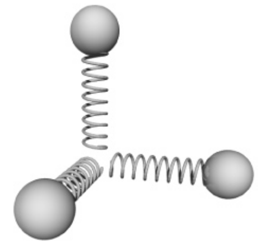
Scores for **Lectures 10-22** have been uploaded. Deadline for requesting corrections is **5 PM this Friday** (April 20).

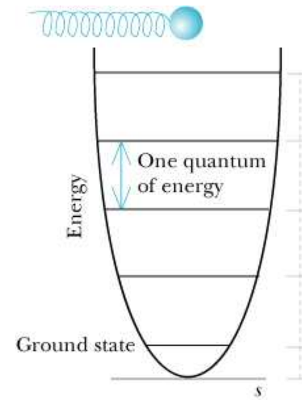


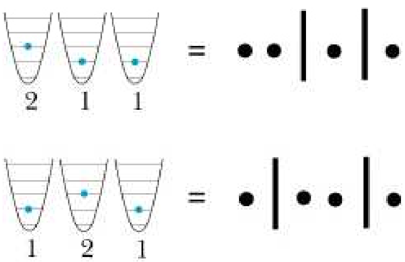
Last Time

Einstein Model of Solids (springs + balls)









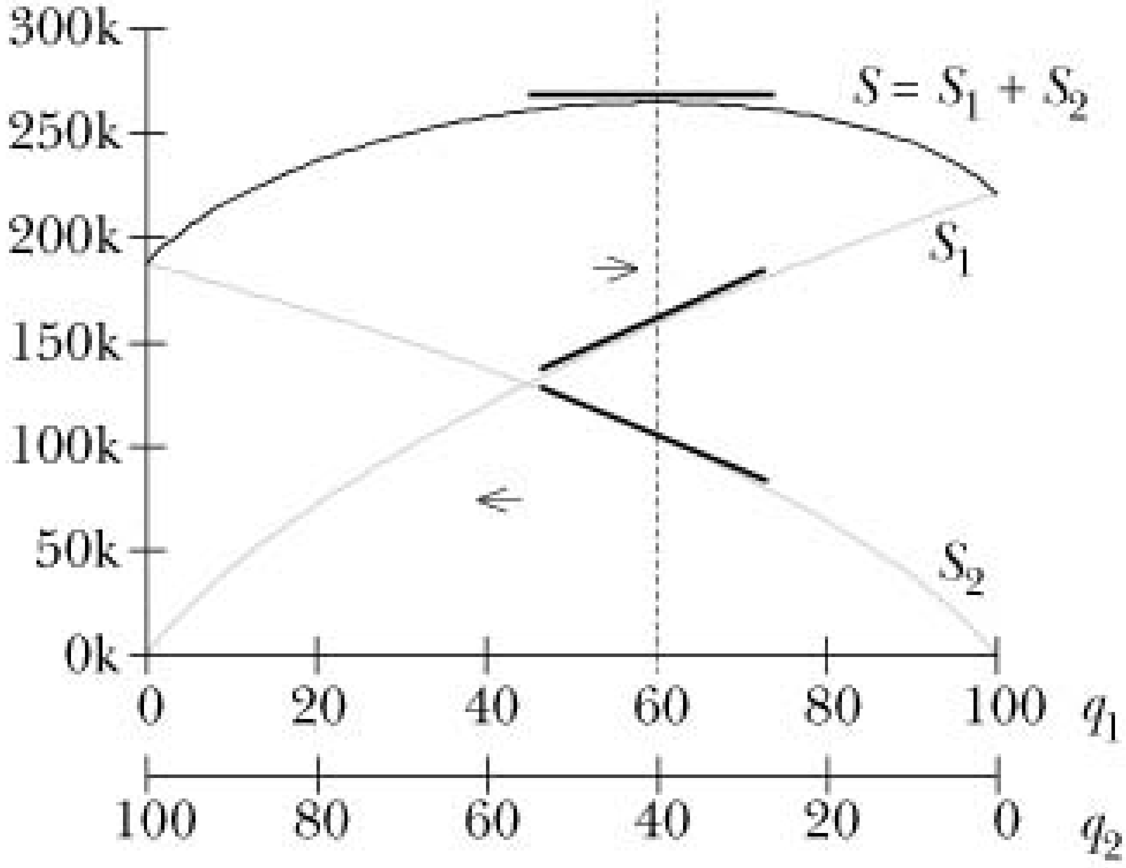
$$\Omega = \frac{(q+N-1)!}{q!(N-1)!}$$

 # microstates N oscillators & q quanta

Fundamental assumption of statistical mechanics

Over time, an isolated system in a given macrostate (total energy) is equally likely to be found in any of its microstates (microscopic distribution of energy).

Equilibrium = Most Probable Distribution



$$S \equiv k \ln \Omega$$

$$\frac{dS}{dq_1} = \frac{dS_1}{dq_1} + \frac{dS_2}{dq_1} = 0$$

$$\frac{dS_1}{dq_1} = \frac{dS_2}{dq_2}$$

$$\boxed{\frac{1}{T} \equiv \frac{dS}{dE_{\text{int}}}}$$

Energy is exchanged until the most probable distribution is reached.

Today: Heat Capacity

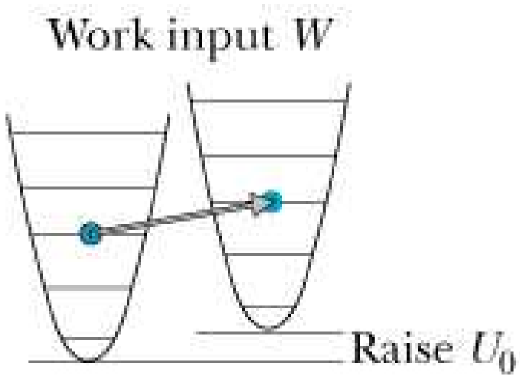
- Brief Review of Heat Capacity
- Pb vs Al: A Chain of Reasoning
- Quantum versus Classical

Note: How Do Heat and Work Differ?

WORK

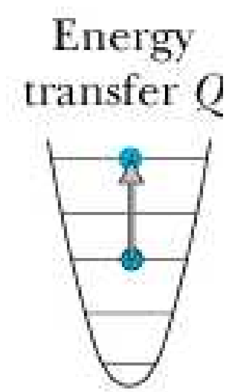
Compress a solid (force through a distance):

SHAPE changes $\rightarrow$ Energy Levels Change



HEAT

Energy levels don't change.
Transfer quanta from one place to another.



Heat Capacity

How much heat do you have to add to change the temperature by a certain amount?

a) Large amount $\rightarrow$ Large heat capacity
b) Small amount $\rightarrow$ Small heat capacity

Has to do with degrees of freedom -- where are all the microscopic places the system can store energy. More modes = higher heat capacity. (Energy will go into every mode it can...)

Heat Capacity

How much heat do you have to add

to change the temperature by a certain amount?

$$C = {}^{\partial E_{int}}\!\!/_{\partial T}$$

(Heat = energy transferred)

Water has a high heat capacity.
Live near water.

Heat Capacity of Solids

Which has the higher heat capacity,
Lead (Pb) or Aluminum (Al)?

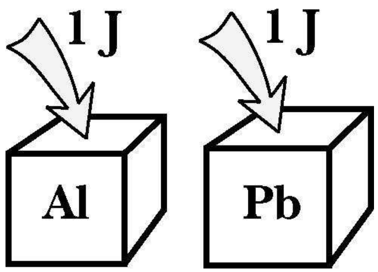
Compare for the <u>same number of atoms,</u> e.g., 6 x 10^{23} .

Lead

Aluminum

Heat Capacity for Pb and Al

Take a Pb and and Al block with <u>same</u> number of atoms 6 x 10^{23} . Initially both are at a temperature very near absolute zero (0 K). We will add 1 J of energy to the aluminum block, and 1 J of energy to the lead block, and see which block has the larger increase in temperature.



$$C_{atom} = \frac{\Delta E_{atom}}{\Delta T} \equiv \frac{\Delta E_{system} / N_{atoms}}{\Delta T}$$

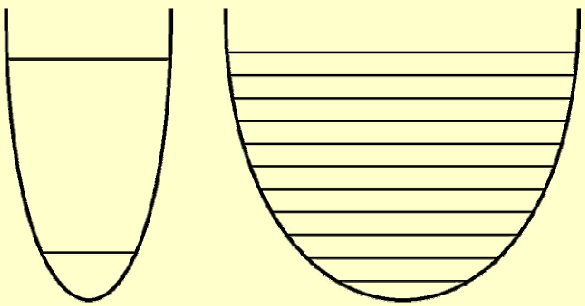
We will step through a chain of reasoning using statistical mechanics to answer this question, which will let us determine whether aluminum or lead has the higher heat capacity at low temperatures.

CLICKER QUESTION

k_{Al} = 16 N/m $\quad$ k_{Pb} = 5 N/m

m_{Al} = 27g $\quad$ m_{Pb} = 207g $\quad$ (1 mole of each)

Einstein model = independent quantum harmonic oscillators.
Which shows the right energy level diagram?



$$\Delta E = \hbar\omega = \hbar\sqrt{\frac{k_{atom}}{m_{atom}}}$$

A) Al $\quad$ Pb

B) Pb $\quad$ Al

ANSWER:

$$\sqrt{\frac{16}{27}} > \sqrt{\frac{5}{207}}$$

CLICKER QUESTION

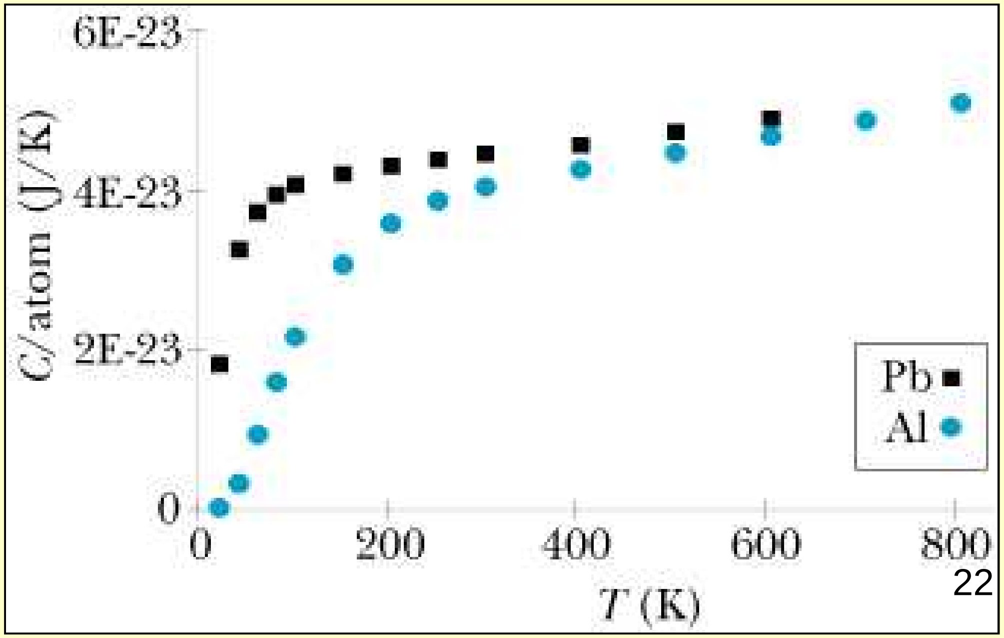
The original temperature was 0 K, and the final temperature of the Al block is higher than that of the Pb block, so the Al block has the larger *change* in temperature, ΔT. At low temperatures, which block has the greater heat capacity per atom, $C = (\Delta E / \Delta T) / 6e23$?

A) The low-temperature heat capacity per atom of Al is greater

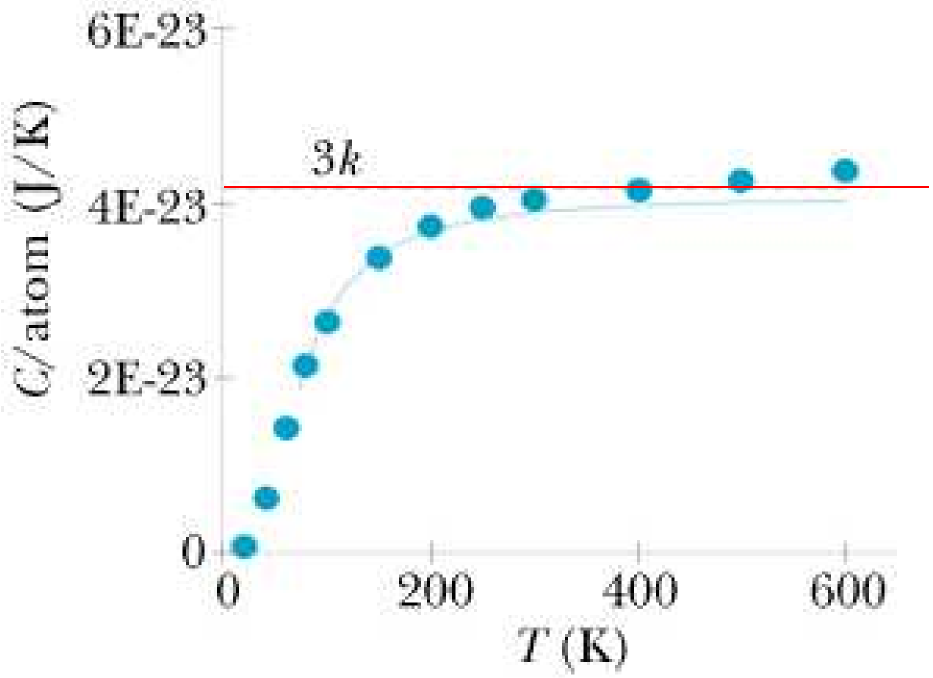
B) The low-temperature heat capacity per atom of Pb is greater

C) Same for both.

Measured heat capacities



Specific Heat and Quantization



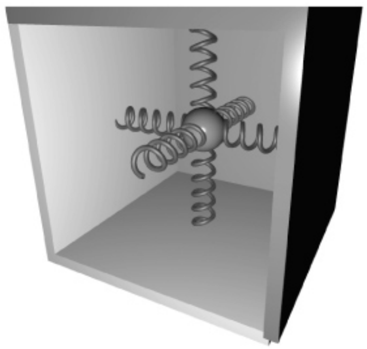
classical expectation C=3k

accurate when $k_B T \gg \hbar\omega$

(q is large)

NOTE: The classical expectation was known to disagree with the data long ago. Quantization of energy levels solved this paradox, one of the first signs of the need for quantum mechanics.

Improving Einstein's Model

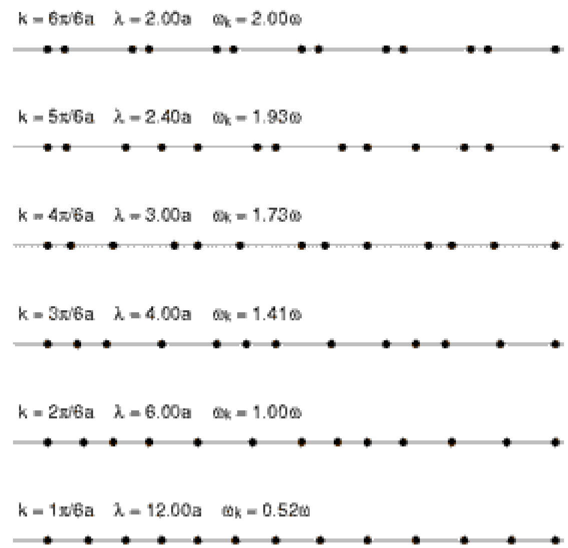


Are atoms in a solid really isolated from each other?   No!

Really: atoms interact with each other.

Atoms don't oscillate independently.

They have **waves** called **phonons.**



How to be better than Einstein:
Treat the **phonons** as harmonic oscillators!
You'll see this in Physics 416.

Today: Specific Heat Capacity

- Brief Review of Specific Heat
- Pb vs Al: A Chain of Reasoning
- Quantum versus Classical

Next Lecture: Boltzmann Distribution

- **Derivation**
- **Application: Kinetic Theory of Gasses**