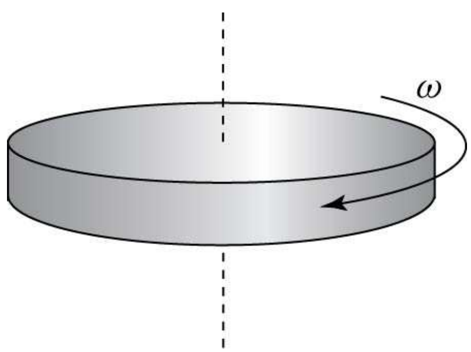
PHYS 172: Modern Mechanics Spring 2012

Lecture 17 – Multiparticle Systems, Moment of Inertia

Read 9.3 – 9.5

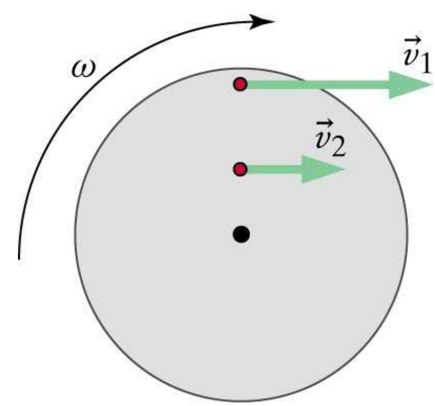
Rotational Kinetic Energy

- Consider a rigid system rotating on an axis
- All atoms are rotating at the same "angular speed"

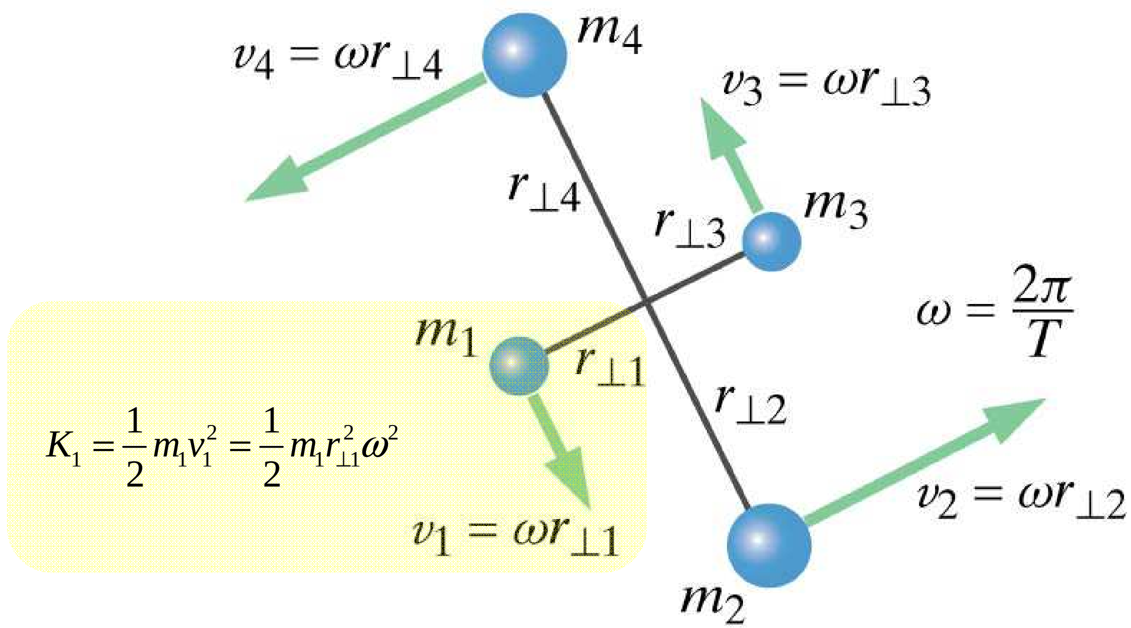


$$\omega = \frac{2\pi}{T}$$

$$v = \omega r$$



Moment of Inertia

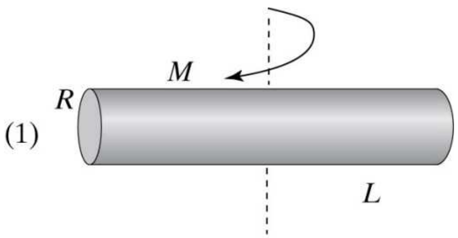


$$K_{rot} = K_1 + K_2 + \ldots = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \ldots = \frac{1}{2} m_1 r_{\perp 1}^2 \omega^2 + \frac{1}{2} m_2 r_{\perp 2}^2 \omega^2 + \ldots$$

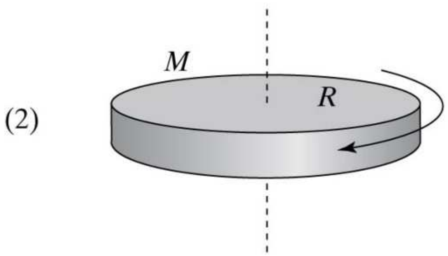
$$K_{rot} = \frac{1}{2} [\underbrace{m_1 r_{\perp 1}^2 + m_2 r_{\perp 2}^2 + \ldots}_{= \boldsymbol{I} \text{ (moment of inertia)}}] \omega^2 = \frac{1}{2} I \omega^2$$

Some Moments of Inertia

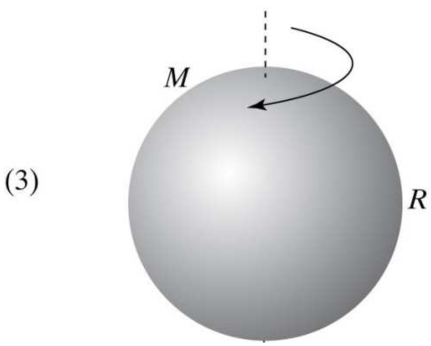
$$K_{rot} = \frac{1}{2} I \omega^2$$
$$I = m_1 r_{\perp 1}^2 + m_2 r_{\perp 2}^2 + \ldots$$



$$I_{cylinder} = \frac{1}{12} ML^2 + \frac{1}{4} MR^2$$

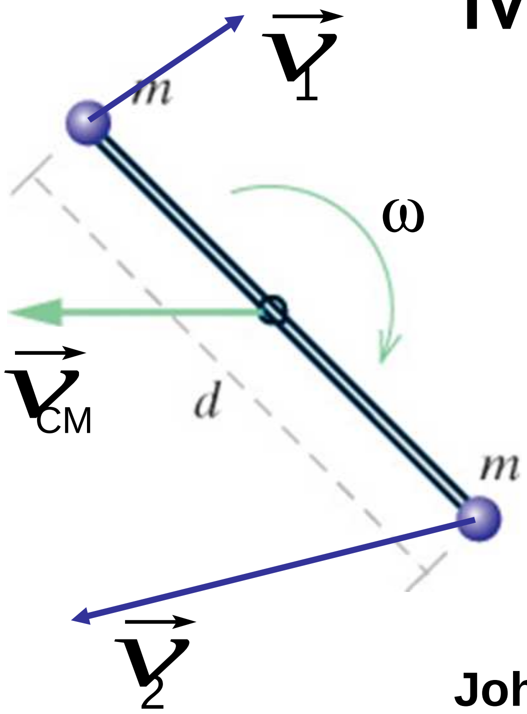


$$I_{disk} = \frac{1}{2} MR^2$$



$$I_{sphere} = \frac{2}{5} MR^2$$

Moment of Inertia



A barbell consists of two massive balls of mass *m* connected by a massless rod. The barbell slides across a low-friction icy surface.
The center of mass moves at speed *v* and the balls rotate around CM with angular velocity ω and have moment of inertia *I*. The velocity of each ball in the same reference frame is also given as v_1 and v_2

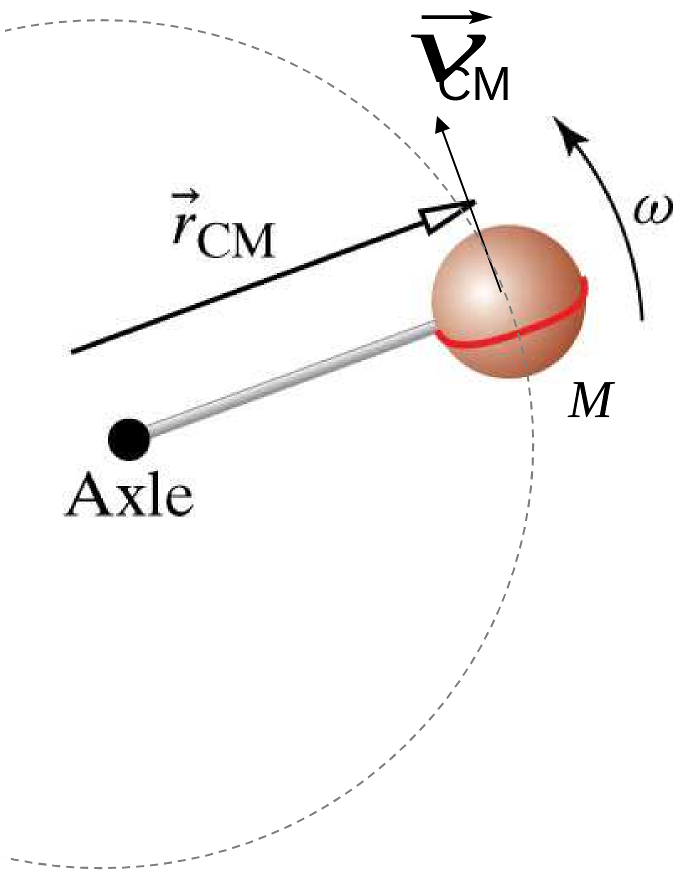
John insists that total kinetic energy is: $K_{tot} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

Mary insists that total kinetic energy is: $K_{tot} = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega_{CM}^2$

Clicker: Who is right?

A) John only
B) Mary only
C) Both are right
D) Both are wrong

Rigid Rotation about a Point Not the Center of Mass



In General: $K_{tot} = K_{trans} + K_{rel}$

Solution:

1) calculate K_{trans}

$$K_{trans} = \frac{1}{2} M v_{CM}^2 = \frac{1}{2} M(\omega r_{CM})^2 = \frac{1}{2}(M r_{CM}^2)\omega^2$$

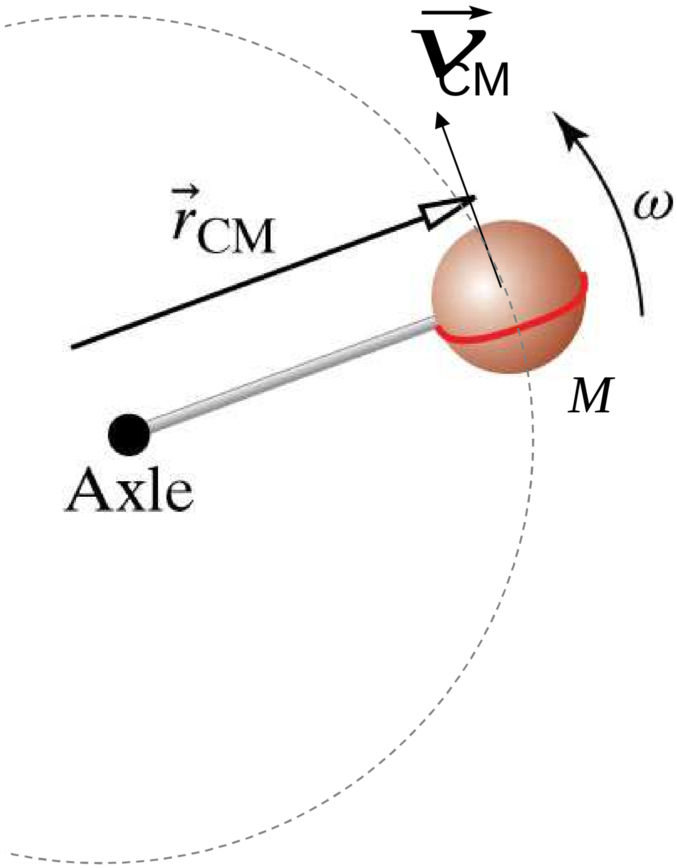
2) calculate K_{rel}

$$K_{rel} = K_{rot} = \frac{1}{2} I_{CM} \omega^2$$

3) calculate K_{tot}

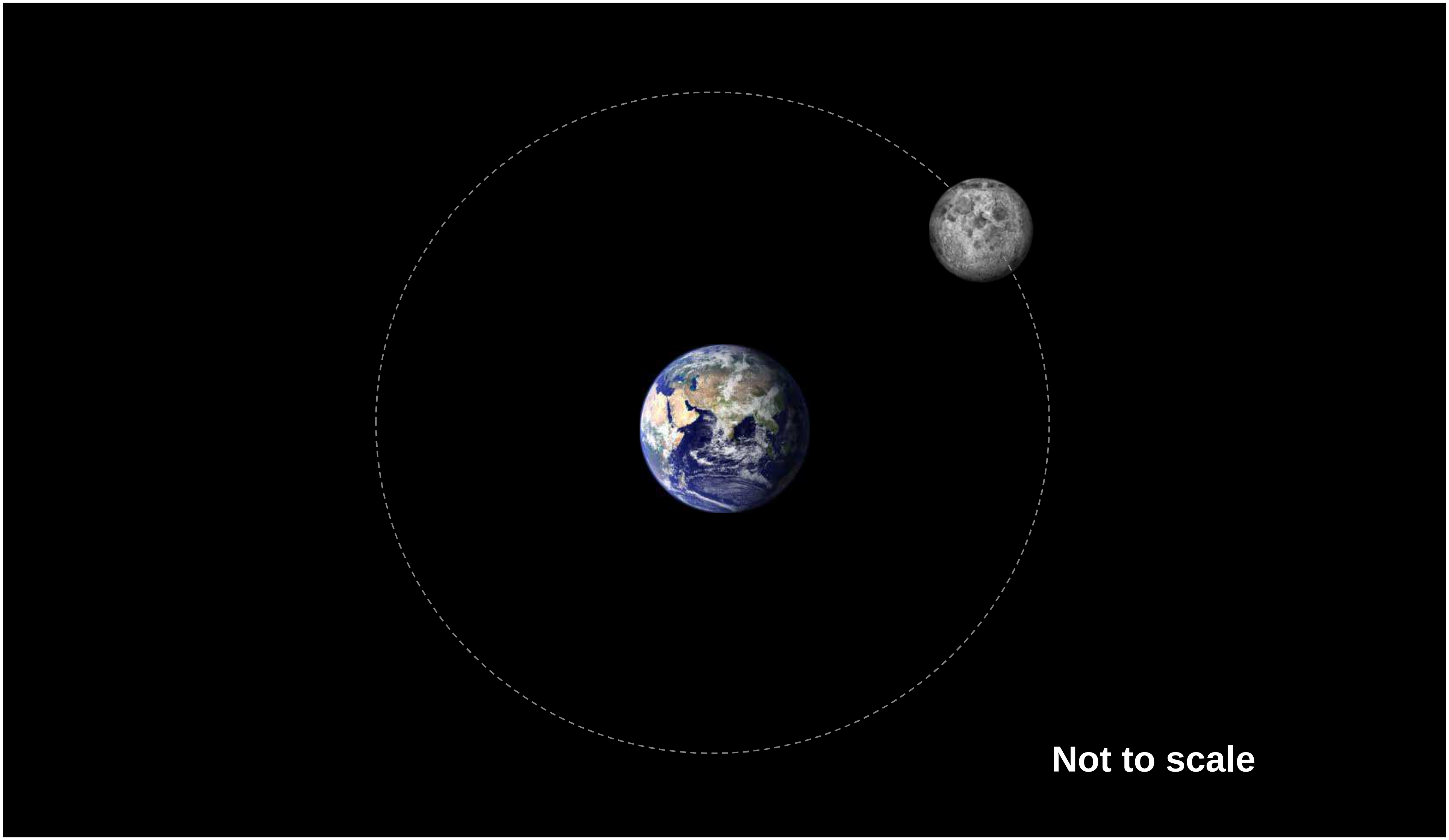
$$K_{tot} = \frac{1}{2}\left(M r_{CM}^2\right)\omega^2 + \frac{1}{2} I_{CM}\omega^2 = \frac{1}{2}\left(M r_{CM}^2 + I\right)\omega^2$$

Rigid Rotation about a Point Not the Center of Mass

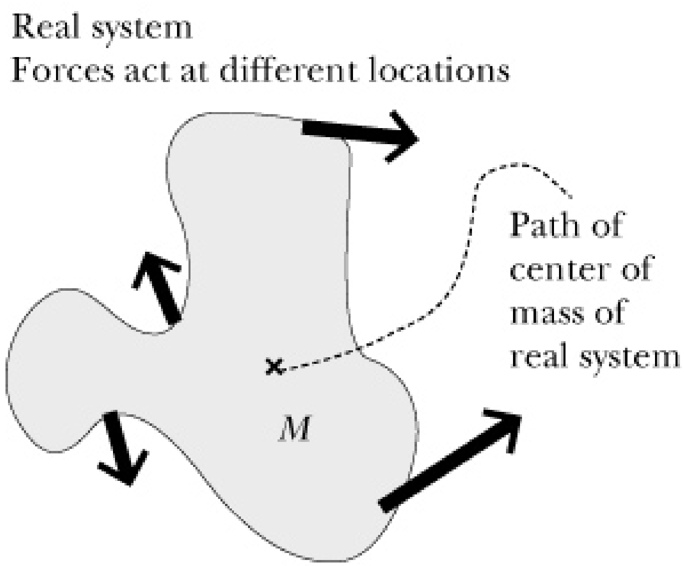


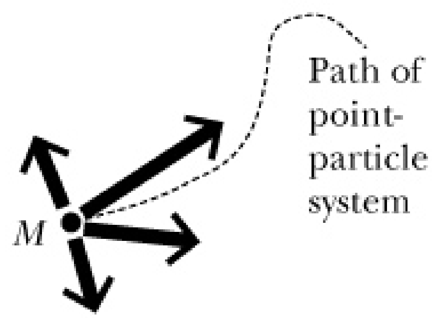
$$\boxed{K_{tot} = \frac{1}{2}(Mr_{CM}^2 + I_{CM})\omega^2}$$

Rotation about a Point Not the Center of Mass



Point particle system





For both, real and point system:

$$K_{trans} = \frac{1}{2} M v_{cm}^2 = \frac{P_{tot}^2}{2M}$$

Point particle system:

$$\frac{d\vec{P}_{tot}}{dt} = \vec{F}_{net,ext}$$

$$\Delta K_{trans} = \Delta\left(\frac{P_{tot}^2}{2M}\right) = \int_i^f \vec{F}_{net,ext} \cdot d\vec{r}_{cm}$$

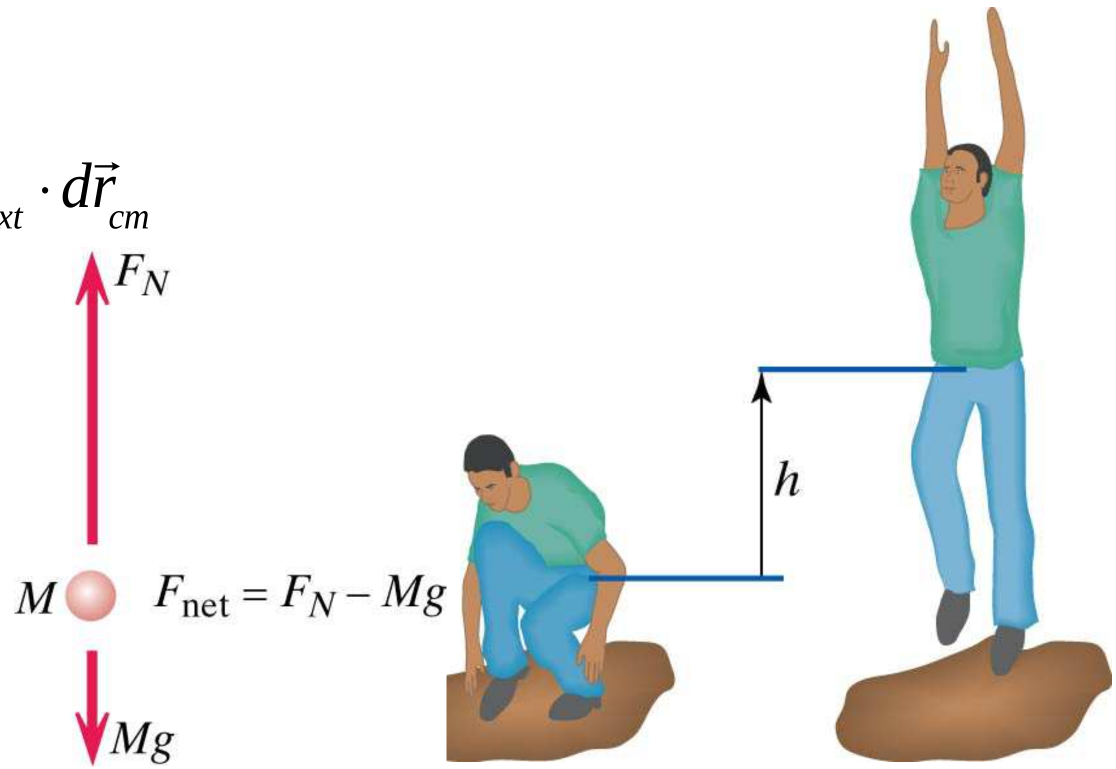
See derivation in the book

Application: Jumping up

Point particle system:

$$\Delta K_{trans} = \frac{P_{tot}^2}{2M} - 0 = \int_i^f \vec{F}_{net,ext} \cdot d\vec{r}_{cm}$$

$$\frac{P_{tot}^2}{2M} = \left(F_N - Mg\right)h$$



Real system: F_N is pushing your feet, but it is not doing any work!

$$\Delta K_{trans} + \Delta K_{hands,legs} + \Delta E_{thermal} + \Delta E_{chemical} = -Mgh$$

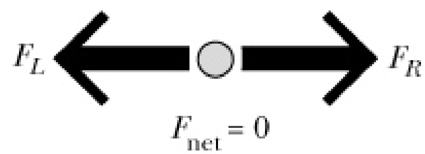
Difference: point particle does not change shape

Application: Stretching a spring

Real system: $W_L = F_L \Delta r_L$ $\qquad$ $W_R = F_R \Delta r_R$

$$\Delta\left(k_s s^2\right) = W_L + W_R$$

Point particle system:



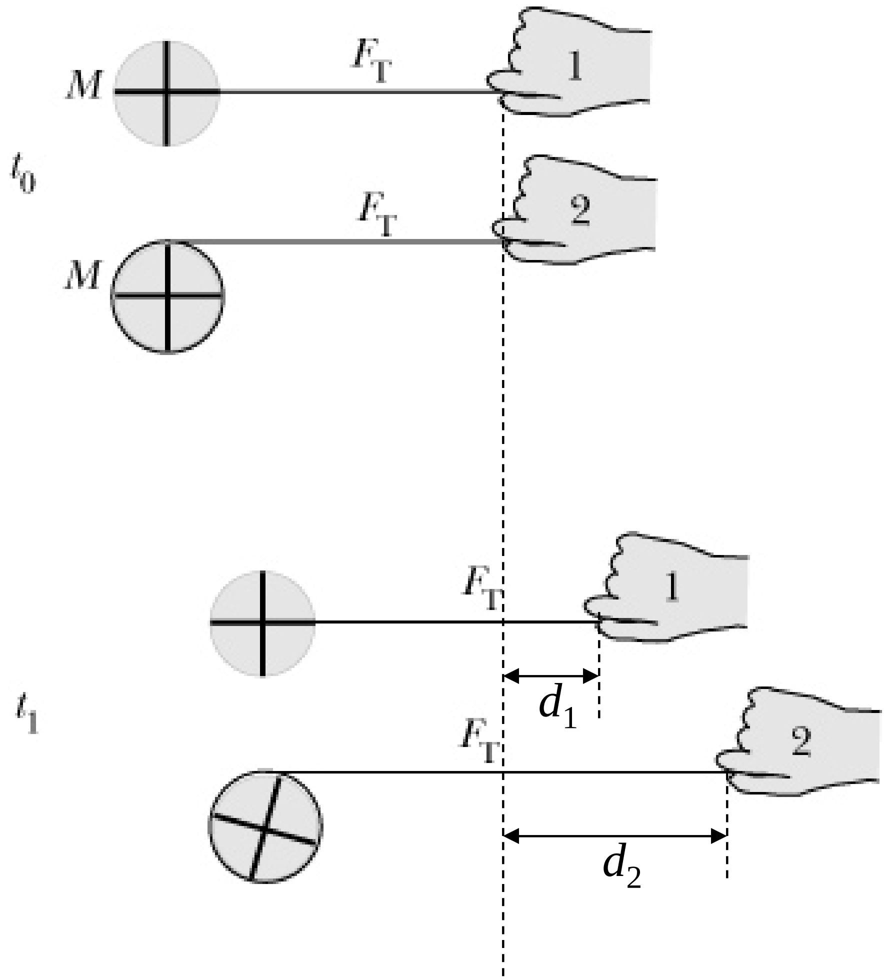
$$\Delta K_{trans} = \int_i^f \vec{F}_{net,ext} \cdot d\vec{r}_{cm}$$

$$\Delta K_{trans} = 0$$

In real system: *each* force does work,
involves displacement of the point to which the force is applied

Example: hockey pucks

$$\frac{d\vec{P}_{tot}}{dt} = \vec{F}_{net,ext}$$

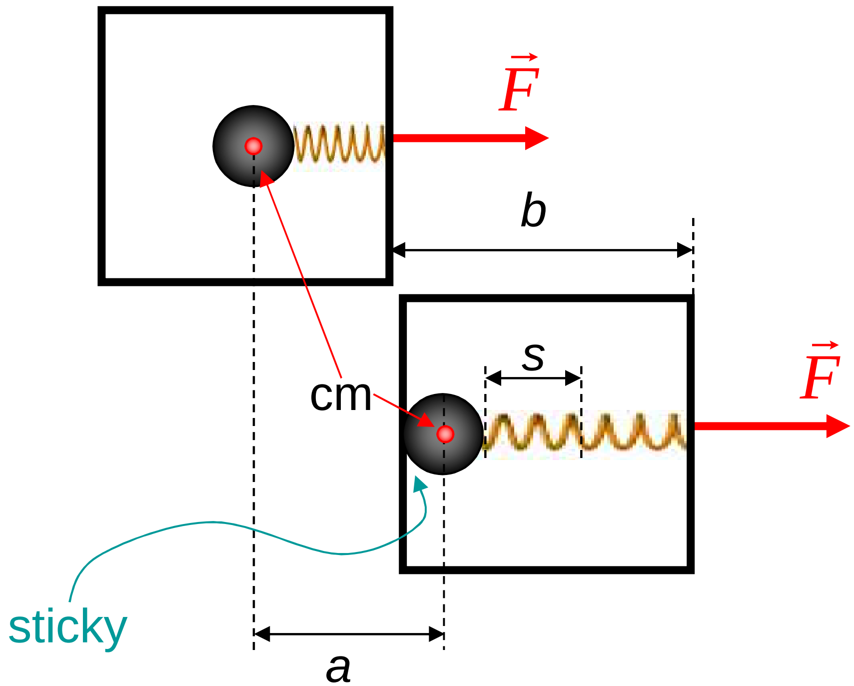


$$\vec{P}_{tot} = \vec{F}_T \Delta t \qquad W_1 = F_T d_1$$

$$\vec{P}_{tot} = \vec{F}_T \Delta t \qquad W_2 = F_T d_2$$

$$\Delta K_{trans} = \frac{P_{tot}^2}{2M} - 0 = \underbrace{\int_i^f \vec{F}_{net,ext} \cdot d\vec{r}_{cm}}_{\boxed{\text{Same in both cases}}}$$

Example: a box containing a spring



System:
Ball with mass m_{ball}
Box with mass $m_{box} << m_{ball}$
Spring – massless
$M \approx m_{ball}$

a) *How fast will the ball move immediately after it sticks to a box?*

$$\Delta K_{trans} = Fa$$

$$\tfrac{1}{2} M v_{cm}^2 - 0 = Fa \qquad v = \sqrt{\frac{2Fa}{M}}$$

b) *What is the increase in thermal energy of the ball?*

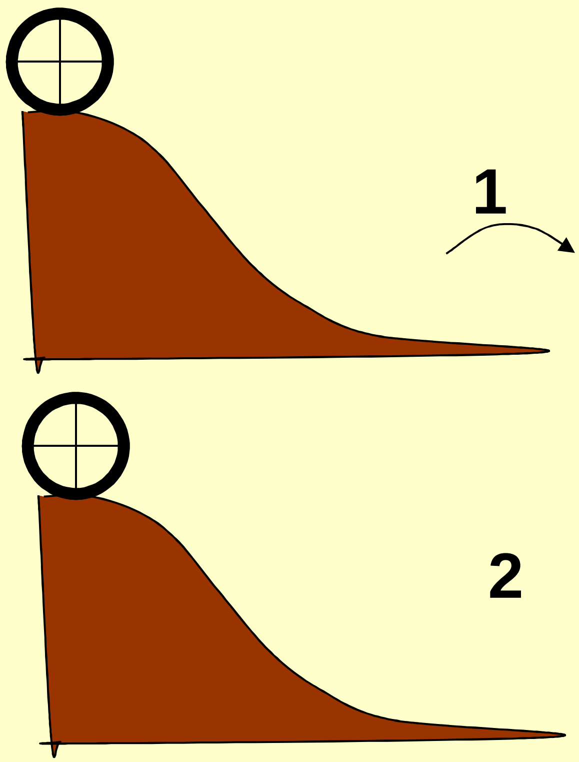
$$\Delta E_{system} = W + Q \qquad \text{assume } Q=0$$

$$\Delta\left(K_{trans} + U_{spring} + E_{thermal}\right) = Fb$$

$$Fa + \tfrac{1}{2} k_s s^2 + \Delta E_{thermal} = Fb$$

$$\boxed{\Delta E_{thermal} = F(b-a) - \tfrac{1}{2} k_s s^2}$$

Clicker question 3



Wheel 1 of mass M *rolls* down a slope. Wheel 2 of the same mass M *slides* down the same slope (ignore friction)

Which of the wheels will get down first?

A) Wheel 1 (rolling)
B) Wheel 2 (sliding)
C) Both will get down in the same time