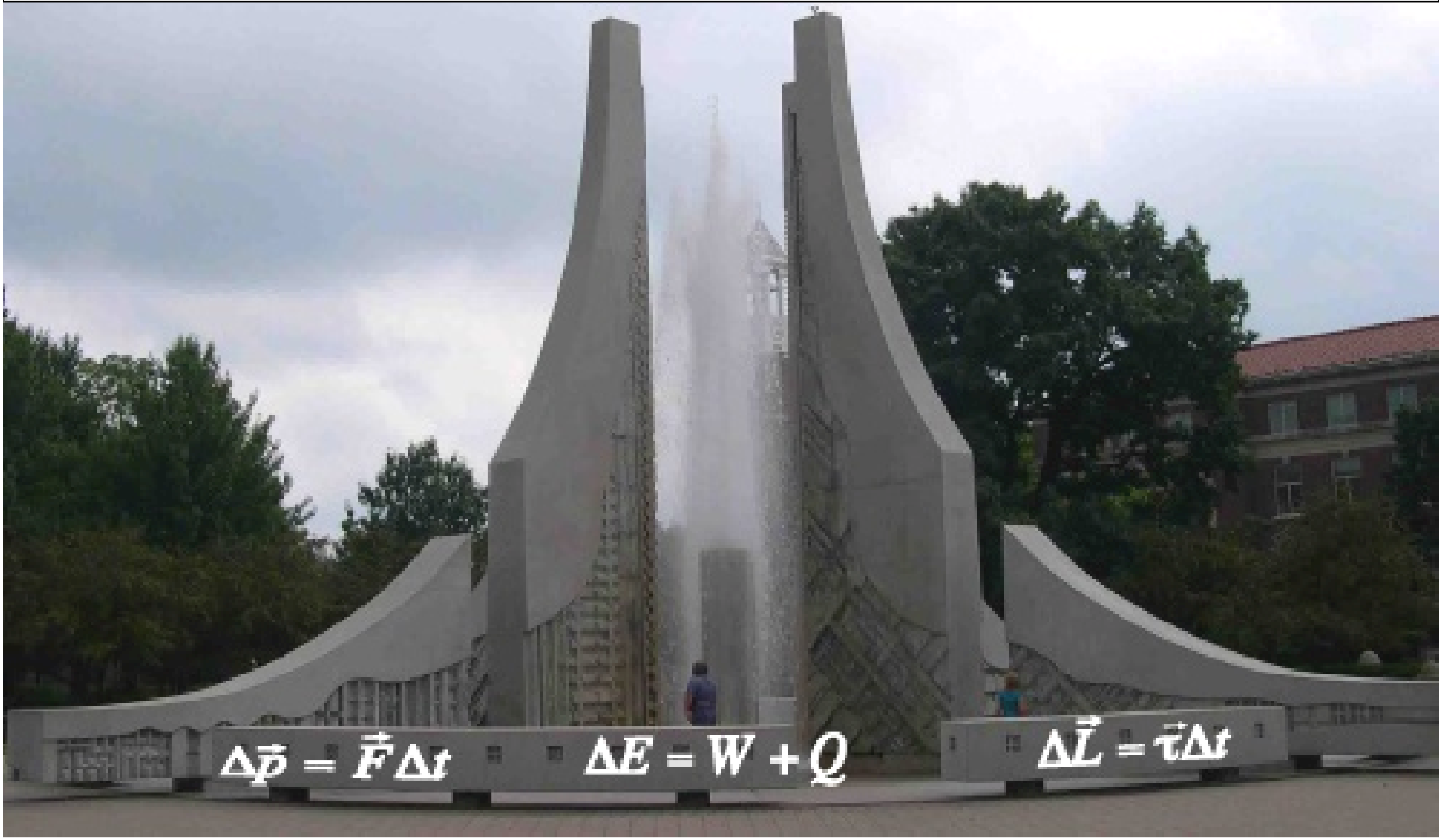
PHYS 172: Modern Mechanics Spring 2012



Lecture 6 – **Ball-Spring Model of Solids, Friction** Read 4.1-4.8

Can we *really* predict the future?

BASIC IDEA

We give you the initial positions, velocities, and the interactions.
You predict everything! .... Really Everything?

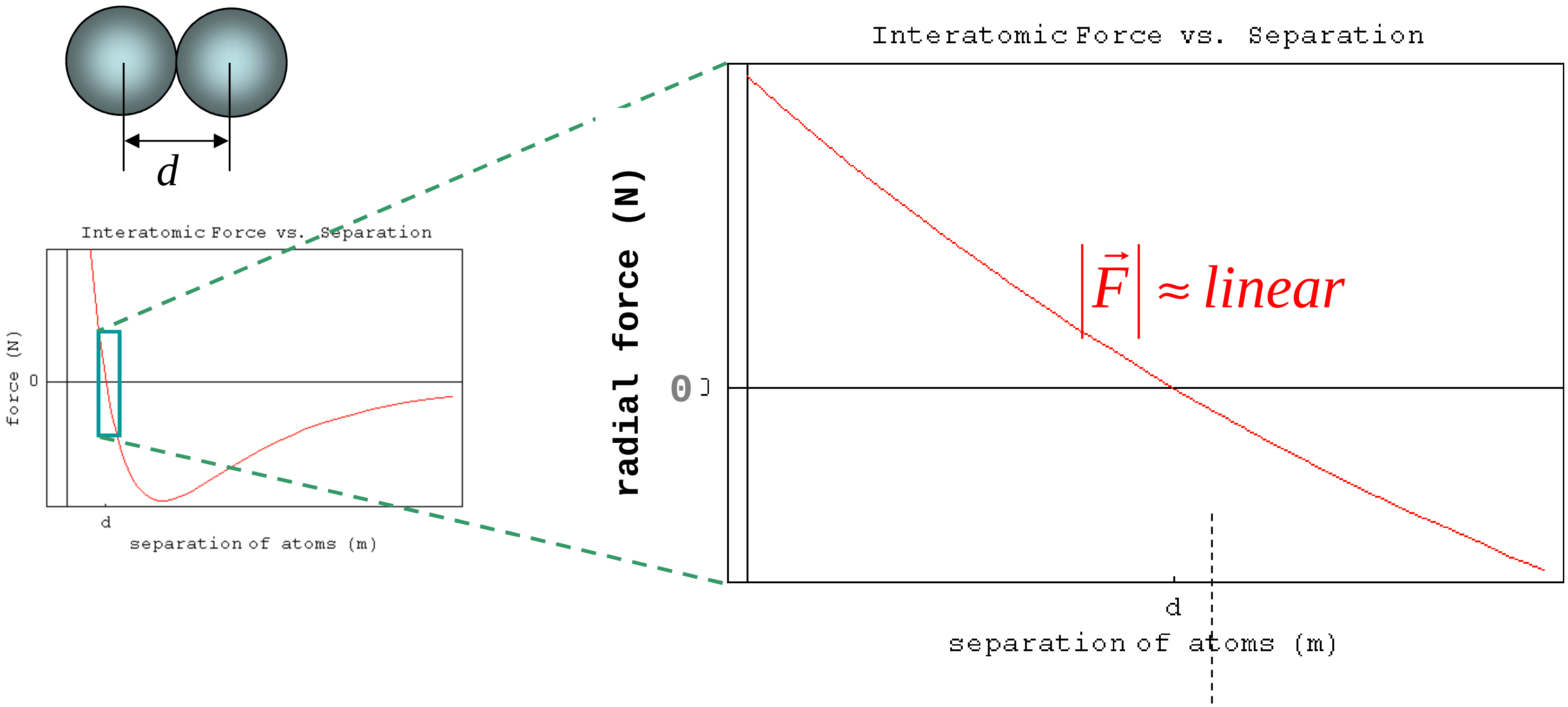
PHILOSOPHICAL PROBLEMS

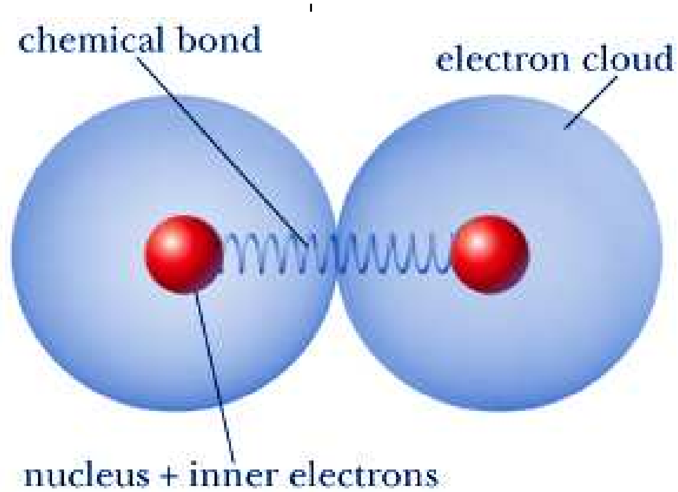
Is there free will?
Is there more than we can detect?
Emergence: some laws can only be discovered with 10^{23} particles.

PRACTICAL PROBLEMS

More than 10^{23} particles in a glass of water. Can't measure them all.
Sensitivity to initial conditions (chaos)
Quantum mechanics: *Probabilities* determine outcomes
Quantum mechanics: Heisenberg uncertainty principle

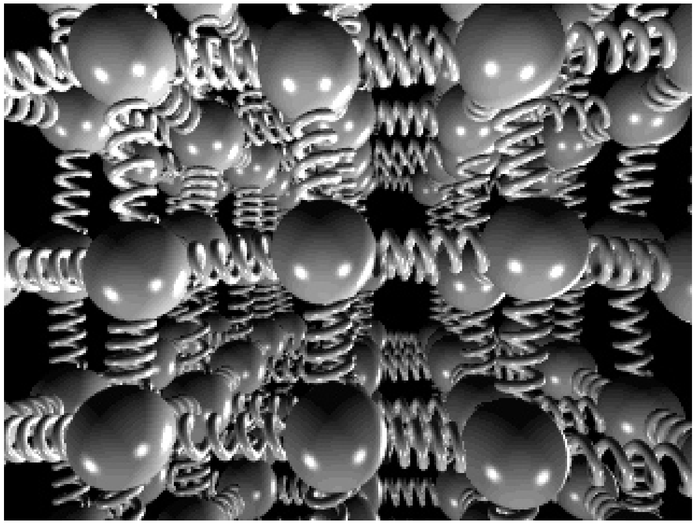
Model of solid: chemical bonds





If atoms don't move too far away from equilibrium, force looks like a spring force!

A ball-spring model of a solid



Ball-spring model of a solid

To model need to know:
- spring length s
- spring stiffness
- mass of an atom

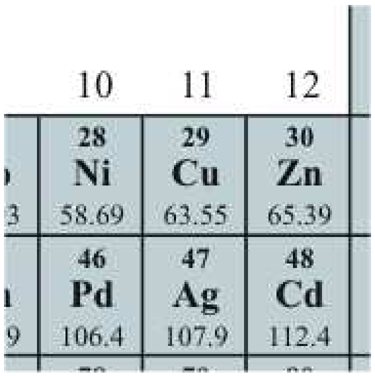
PERIODIC TABLE OF THE ELEMENTS

Transition Metals

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H 1.008																1 H 1.008	2 He 4.003
3 Li 6.941	4 Be 9.012											5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18
11 Na 22.99	12 Mg 24.30											13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.07	17 Cl 35.45	18 Ar 39.95
19 K 39.10	20 Ca 40.08	21 Sc 44.96	22 Ti 47.87	23 V 50.94	24 Cr 52.00	25 Mn 54.94	26 Fe 55.85	27 Co 58.93	28 Ni 58.69	29 Cu 63.55	30 Zn 65.39	31 Ga 69.72	32 Ge 72.61	33 As 74.92	34 Se 78.96	35 Br 79.90	36 Kr 83.80
37 Rb 85.47	38 Sr 87.62	39 Y 88.91	40 Zr 91.22	41 Nb 92.91	42 Mo 95.94	43 Tc (97.91)	44 Ru 101.1	45 Rh 102.9	46 Pd 106.4	47 Ag 107.9	48 Cd 112.4	49 In 114.8	50 Sn 118.7	51 Sb 121.8	52 Te 127.6	53 I 126.9	54 Xe 131.3
55 Cs 132.9	56 Ba 137.3	71 Lu 175.0	72 Hf 178.5	73 Ta 180.9	74 W 183.8	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.1	79 Au 197.0	80 Hg 200.6	81 Tl 204.4	82 Pb 207.2	83 Bi 209.0	84 Po (209.0)	85 At (210.0)	86 Rn (222.0)
87 Fr (223.0)	88 Ra (226.0)	103 Lr (262.1)	104 Rf (261.1)	105 Db (262.1)	106 Sg (263.1)	107 Bh (264.1)	108 Hs (265.1)	109 Mt (266.1)	110 (269.1)	111 (272.1)	112 (277.1)						

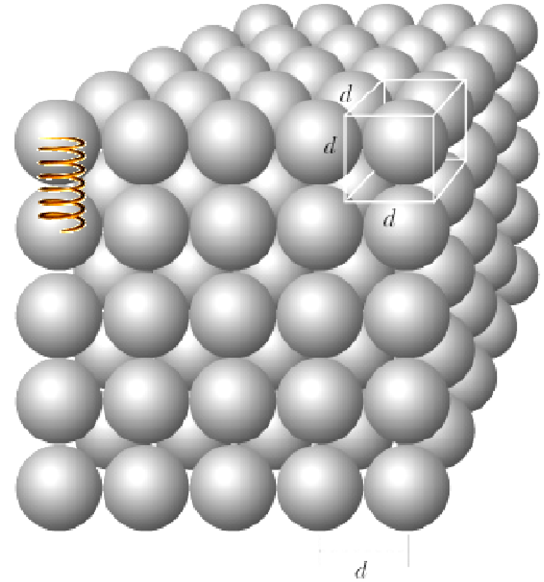
Lanthanides	57 La 138.9	58 Ce 140.1	59 Pr 140.9	60 Nd 144.2	61 Pm (144.9)	62 Sm 150.4	63 Eu 152.0	64 Gd 157.2	65 Tb 158.9	66 Dy 162.5	67 Ho 164.9	68 Er 167.3	69 Tm 168.9	70 Yb 173.0
Actinides	89 Ac (227.0)	90 Th 232.0	91 Pa 231.0	92 U 238.0	93 Np (237.0)	94 Pu (244.1)	95 Am (243.1)	96 Cm (247.1)	97 Bk (247.1)	98 Cf (251.1)	99 Es (252.1)	100 Fm (257.1)	101 Md (258.1)	102 No (259.1)

Length of a bond: diameter of copper atom



density ρ = 8.94 g/cm^3:
molecular weight = 63.55 g/mole

N_A molecules



1. Number of atoms in one cm^3

$$N = \frac{8.94\ \text{g/cm}^3}{63.55\ \text{g/mole}} \cdot 6.022\times10^{23}\ \frac{\text{atoms}}{\text{mole}} = 8.47\times10^{22}\ \frac{\text{atoms}}{\text{cm}^3}$$

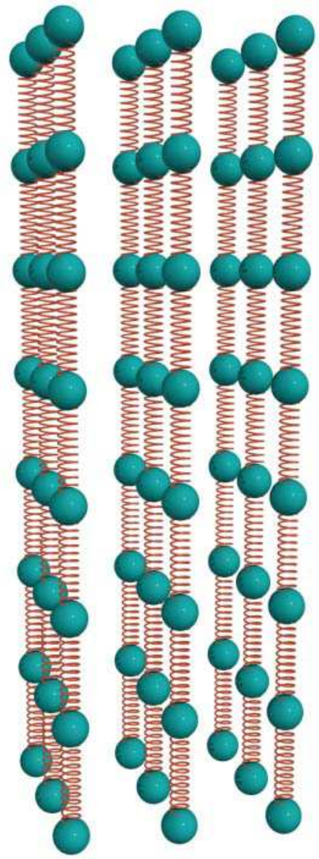
2. Volume per one atom:

$$V_{Cu} = \frac{1}{8.47\times10^{22}\ \text{atoms/cm}^3} = 1.18\times10^{-23}\ \frac{\text{cm}^3}{\text{atom}}$$

3. Bond length:

$$d_{Cu} = \sqrt[3]{1.18\times10^{-23}\text{cm}^3} = 2.27\times10^{-8}\text{cm} = 2.27\times10^{-10}\text{m} = 2.27\ \text{Å}$$

Ball-Spring Model of a Wire

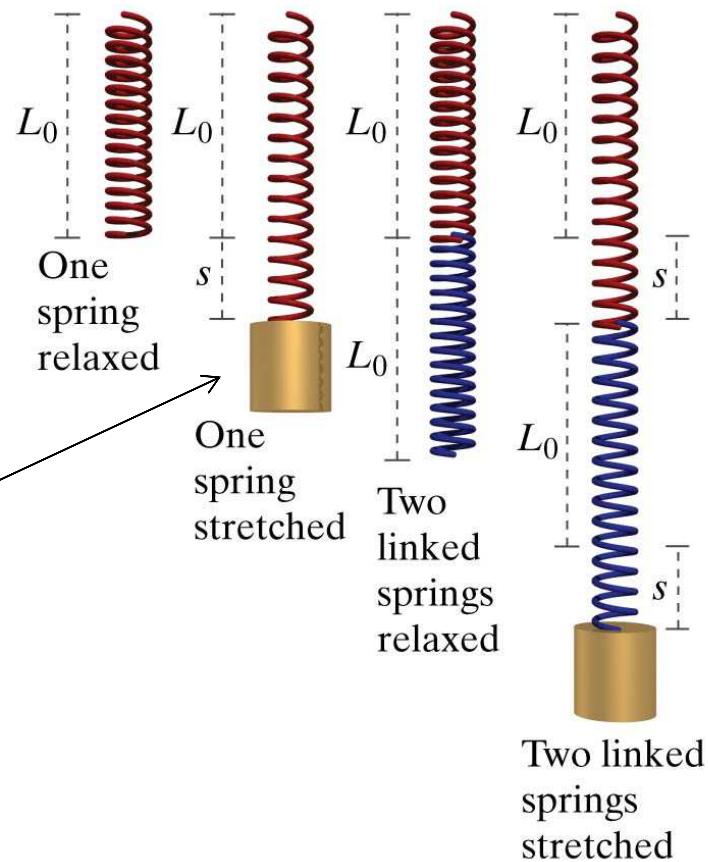


How is the stiffness of the wire related to the stiffness of one of the short springs (bonds)?

Two Springs in Series

Spring constant *k*

Mass M



Each spring must supply an upward force equal to Mg, thus, each stretches by *s* giving a total stretch of 2*s*, or an effective spring constant of *k*/2.

Two Springs in Parallel

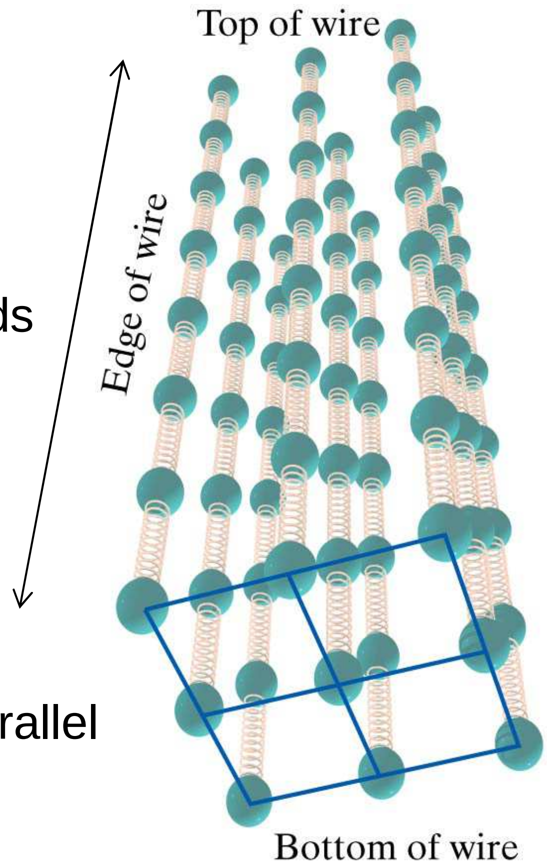
Mass M

Each spring provides an upward force of Mg/2, so each stretches *s*/2, giving an effective spring constant of 2*k*.

Stiffness of a Copper Wire

2-meter long Cu wire

8.77 x 10^9 bonds in series

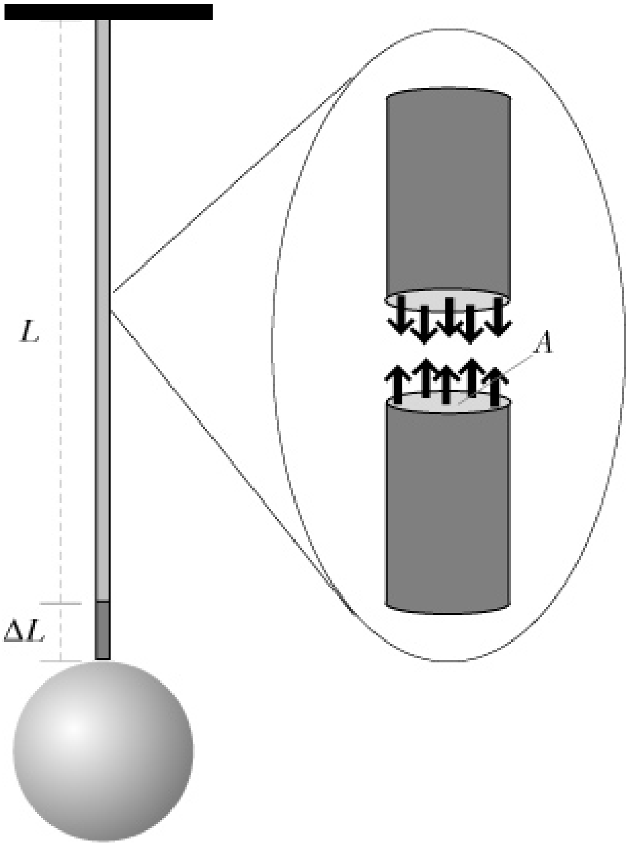


Each side = 1 mm

1.92 x 10^{13} chains in parallel

The stiffness of the wire is much greater than the effective spring stiffness between atoms due to the much greater number of chains in parallel than bonds in series.

Estimating interatomic “spring” stiffness



$$\text{strain} = \frac{\Delta L}{L}$$

$$\text{stress} = \frac{F_T}{A}$$

tension

$$\text{stress} = Y \cdot \text{strain}$$

Thomas Young (1773-1829)

$$\boxed{\frac{F_T}{A} = Y\frac{\Delta L}{L}}$$

Y - Young’s modulus
depends only on material

Compare:

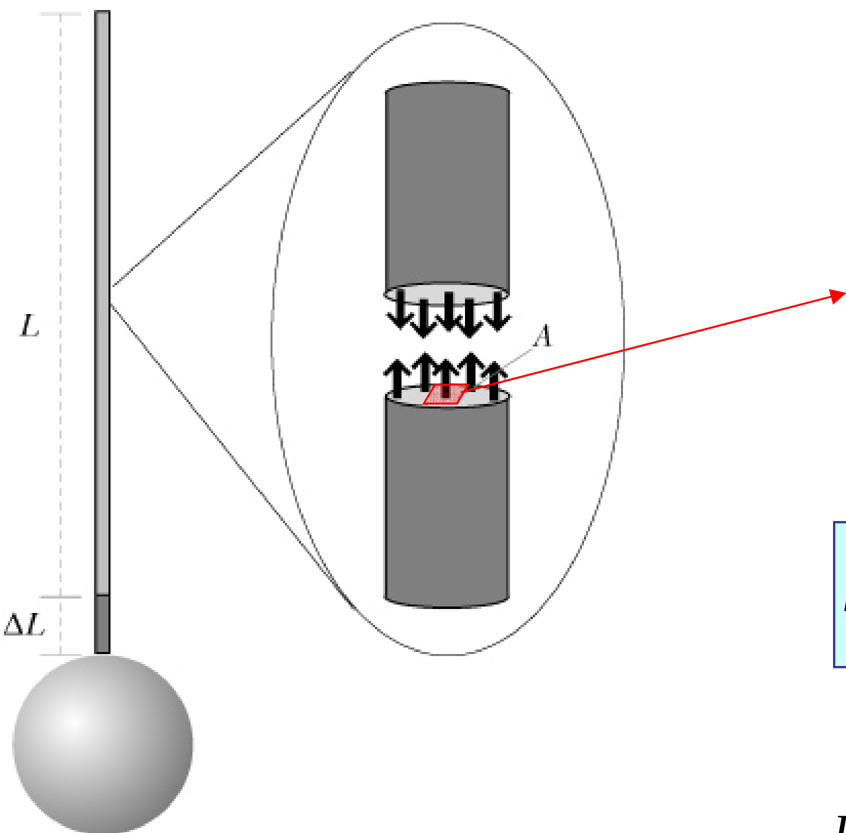
$$\left|\vec{F}_{spring}\right| = k_s\left|s\right|$$

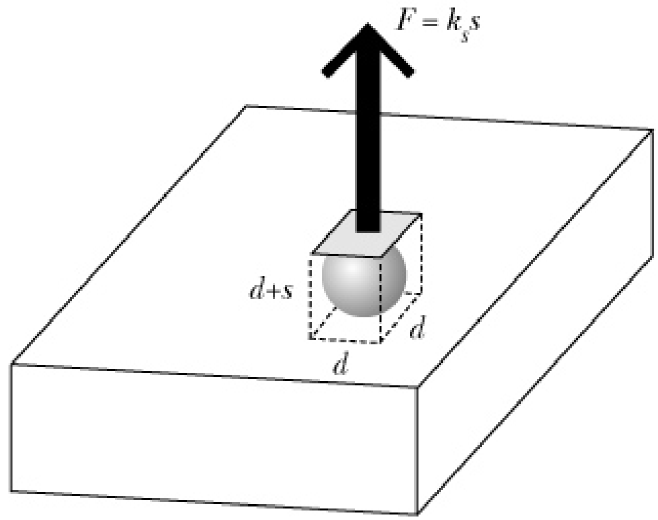
$$\frac{\left|\vec{F}_{spring}\right|}{A}A = k_s\frac{\left|s\right|}{L}L$$

$$\frac{\left|\vec{F}_{spring}\right|}{A} = \frac{L}{A}k_s\frac{\left|s\right|}{L}$$

$$\boxed{k_s = \frac{A}{L}Y}$$

Effective interatomic spring stiffness





$$\boxed{k_s = \frac{A}{L} Y}$$

$$k_s = \frac{d^2}{d} Y$$

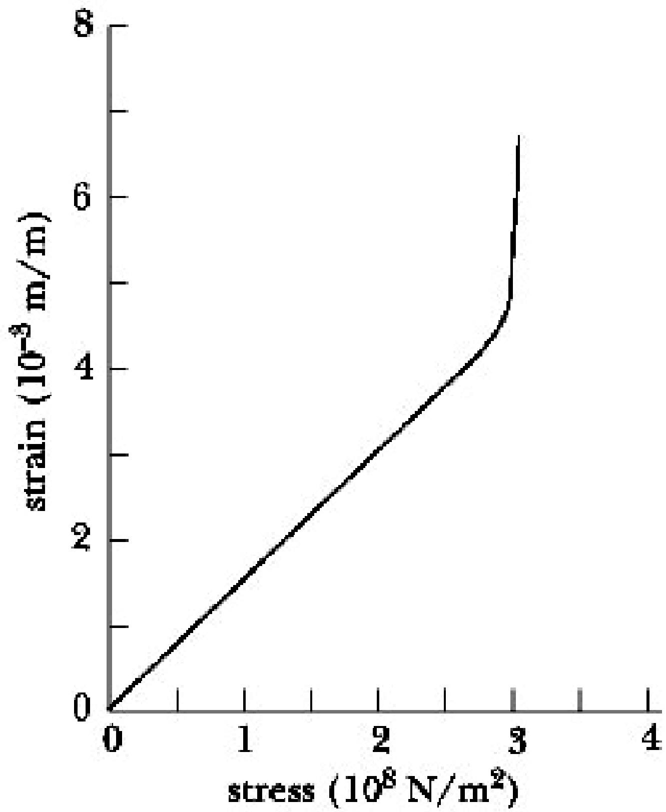
Interatomic spring stiffness

$$k_s = Yd$$

Limits of applicability of Young's modulus

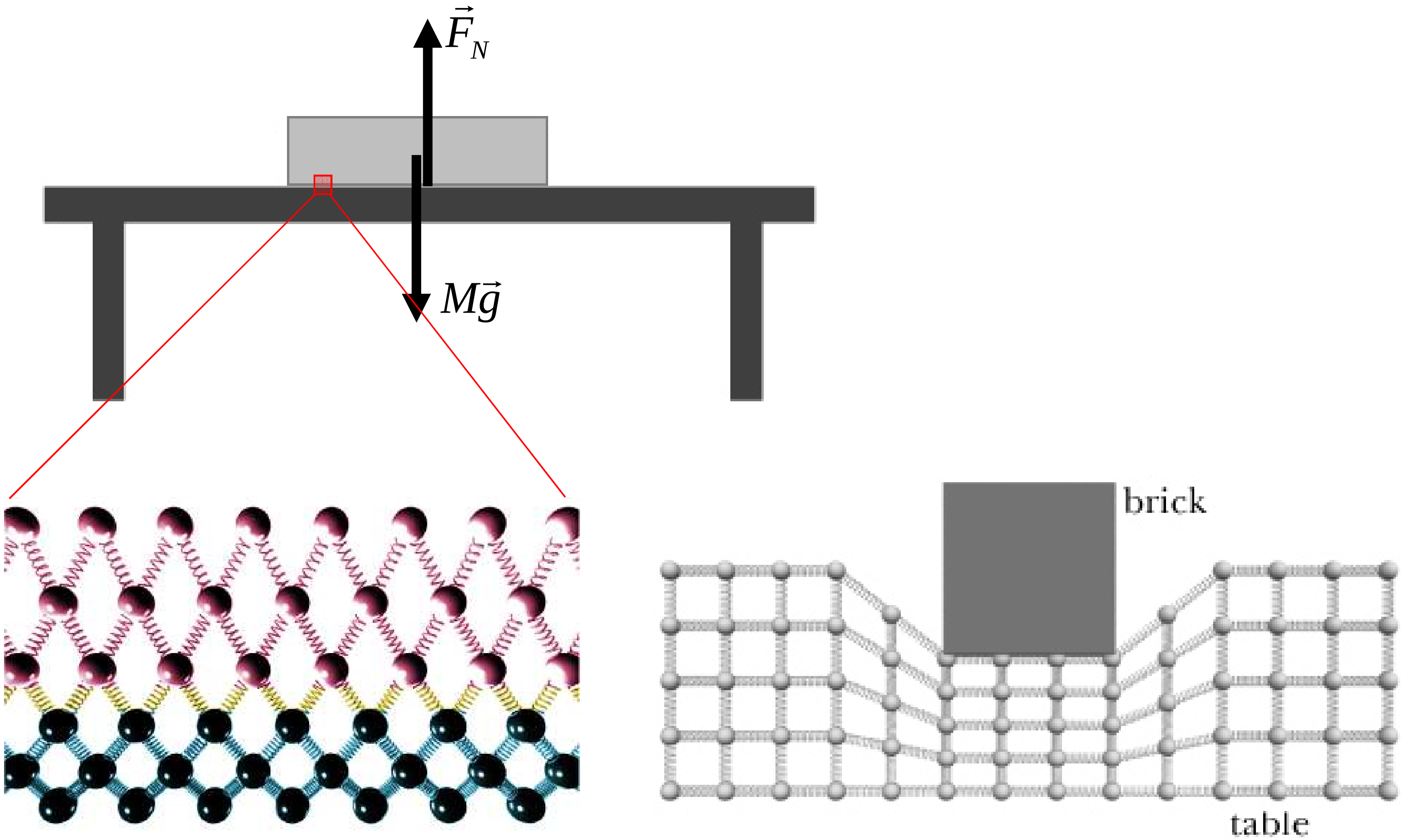
$$\text{stress} = Y \cdot \text{strain}$$

$$\boxed{\frac{F_T}{A} = Y\,\frac{\Delta L}{L}}$$

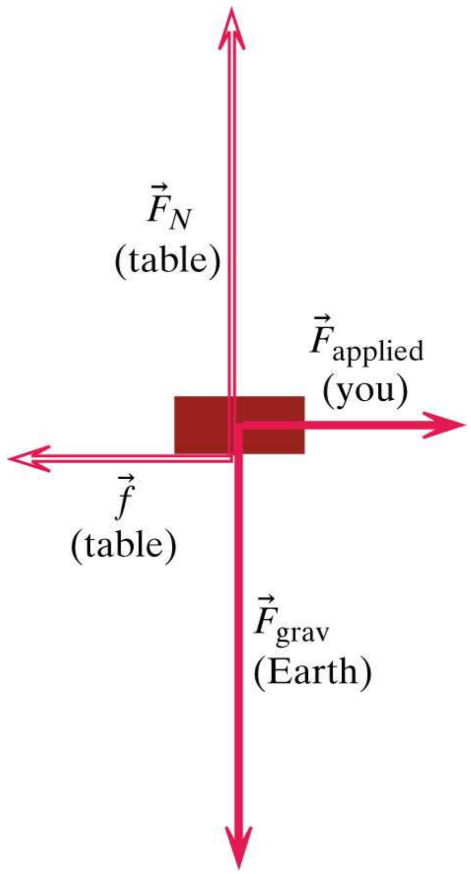


Aluminum alloy

Brick on a table: compression



Friction

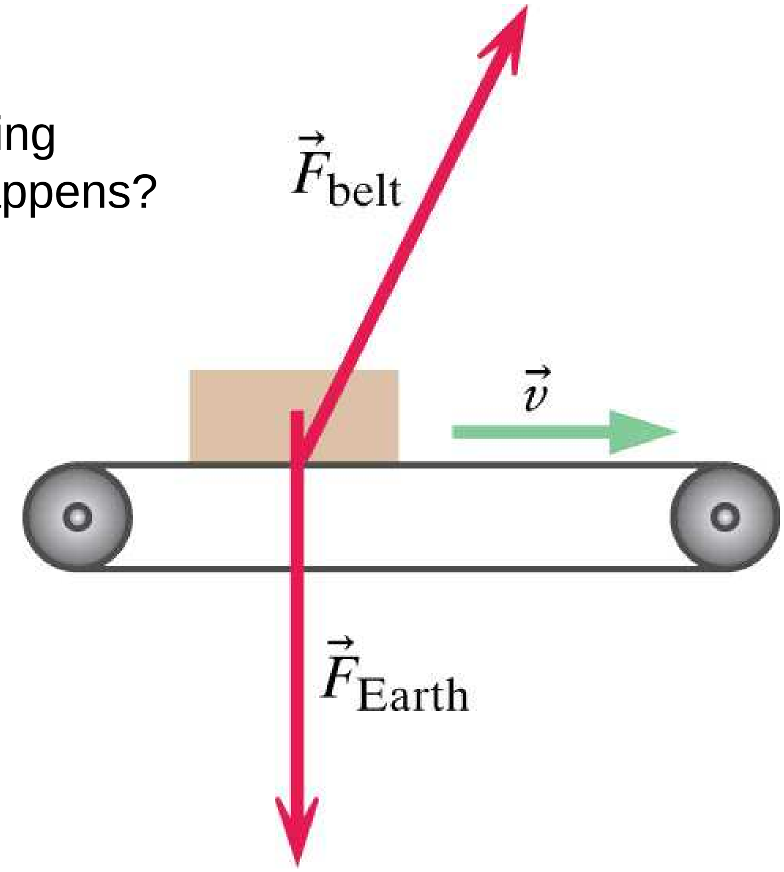


Exert a force so that the brick moves to the right at a constant speed.

What is the net force on the brick?

Friction Doesn't Always Oppose Motion

Box dropped onto moving conveyor belt. What happens?



How is it that a sprinter can accelerate?

Static Friction

- What happens when $F_{applied} < \mu_k F_N$?
- Block does not move due to static friction
- In general:

$$\mu_k \leq \mu_s$$