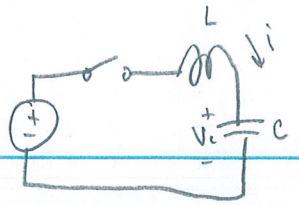
9.7



KVL: $-V_s + V_L + V_c = 0 \quad = -V_s + L\, di/dt + V_c = 0$

$i = C\, dV_c/dt \rightarrow di/dt = C\, d^2V_c/dt^2$

$-V_s + LC\, d^2V_c/dt^2 + V_c = 0$

$d^2V_c/dt^2 = \frac{V_s - V_c}{LC}$

General solution is $V_c(t) = A\cos\omega t + B\sin\omega t + V_s$, with $\omega = 1/\sqrt{LC}$.

$V_c(0) = 0 = A + V_s \rightarrow A = -V_s$

$C\,V_c'(0) = i(0) = 0 \rightarrow V_c'(0) = 0$

$V_c'(0) = 0 = \omega B(\cos 0) \rightarrow B = 0$

$$\boxed{V_c(t) = V_s\left(1 - \cos\tfrac{1}{\sqrt{LC}}t\right)}$$

$i_c(t) = C\, dV_c/dt = C\left[\frac{V_s}{\sqrt{LC}}\sin\tfrac{1}{\sqrt{LC}}\right] = \boxed{\frac{\sqrt{C}}{\sqrt{L}}V_s\sin\tfrac{1}{\sqrt{LC}}t}$

9.7 Part 2

$$P_{cap} = V_c \cdot i_L = V_s\left[1 - \cos\left(\frac{1}{\sqrt{LC}}t\right)\right]\frac{V_s}{\sqrt{L/C}}\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

$$= V_s^2\sqrt{\frac{C}{L}}\left[\sin\left(\frac{1}{\sqrt{LC}}t\right) - \frac{1}{2}\sin\left(\frac{2}{\sqrt{LC}}t\right)\right]$$

$$P_L = (V_s - V_c)i_L = \left[V_s - V_s + V_s\cos\left(\frac{1}{\sqrt{LC}}t\right)\right]V_s\sqrt{\frac{C}{L}}\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

$$= V_s^2\sqrt{\frac{C}{L}}\left[\frac{1}{2}\sin\left(\frac{2}{\sqrt{LC}}t\right)\right]$$

$$P_{tot} = P_c + P_L = V_s^2\sqrt{\frac{C}{L}}\sin\left(\frac{1}{\sqrt{LC}}t\right)$$