Recitation 14 – Entropy and Temperature

Problem 1

a) List explicitly all of the different ways that 2 quanta of vibrational energy can be arranged over 4 one-dimensional oscillators, that is, list <u>all</u> microstates of a 4 oscillator system in the macrostate with total vibrational energy $2\hbar\omega$ (above ground state).

b) Verify that the total number of microstates, $\Omega(q,N)$, is equal to $\frac{(q+N-1)!}{q!(N-1)!}$, the general formula for this number derived in the text.

c) If you make a sequence of 10,000 independent observations of the microstate of this system, about how many times do you expect to find it in the 0020 microstate? About how many times do you expect to find it in a microstate with 2 quanta in the same oscillator? About how many times do you expect to find it in a state in which no oscillator has more than one quantum of vibrational energy?

[Checkpoint 1]

Problem 2

Consider a nanoparticle consisting of 27 copper atoms. Recall that the molar mass of copper is about 63 grams and that the Young's modulus of copper implies an inter-atomic bond stiffness of about 25 N/m.

A crude model of this system can be visualized as a cube with 3 copper atoms on a side. Because atoms on the cube's surface interact with less than six neighboring atoms, the Einstein model provides only rough, but reasonable estimates of the system's properties.

a) What is the quantum of vibrational energy for the Einstein model of this system? How many oscillators does the model of this nanoparticle have? What is the general expression for the number of ways that this nanoparticle system can share *q* quanta?

b) The table on the next page is a template that you may find helps you to organize information about the copper nanoparticle in states with 0, 1 and 2 quanta of energy on your whiteboard. Verify that the entries in the "Number of Ways" column are consistent with your general expression for the number of ways that this nanoparticle system can share *q* quanta. [Recall that $\hbar = 1.05\times10^{-34}$J sec and that Boltzmann's constant is $k = 1.38\times10^{-23}$J/K.]

On your whiteboard, show your work as you compute the information that completes this table. Work to 4 or 5 significant digits to avoid losing precision when computing the necessary entropy and temperature differences.

q	#ways	E (energy)	S (entropy)	ΔE	ΔS	T
0	1	J	0 J/K			
				J	J/K	K
1	81	J	J/K			
				J	J/K	K
2	3321	J	J/K			

[Checkpoint 2]

c) Use information from the table above to estimate the copper nanoparticle's heat capacity when it has one quantum of energy. What is the corresponding heat capacity per copper atom? How does it compare to the classical estimate of the heat capacity per atom, 3 times Boltzmann's constant?

[Checkpoint 3]