WebAssign
CH12-HW03-SP12 (Homework)

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PHYS 172-SPRING 2012, Spring 2012
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Current Score : 25 / 25 **Due :** Thursday, April 19 2012 11:59 PM EDT

1. 14/14 points | Previous Answers MI3 12.6.P.040

(You may have done these temperature calculations before, but it is useful to practice with these concepts, and this time you'll use the temperatures to determine the heat capacity.)

The interatomic spring stiffness for gold is determined from Young's modulus measurements to be 20 N/m. The mass of one mole of gold is 0.197 kg. If we model a block of gold as a collection of atomic "oscillators" (masses on springs), what is one quantum of energy for one of these atomic oscillators? Note that since each oscillator is attached to *two* "springs", and each "spring" is half the length of the interatomic bond, the effective interatomic spring stiffness for one of these oscillators is 4 times the calculated value given above.

Use these precise values for the constants:

$\hbar = 1.0546 \times 10^{-34}$ J · s (Planck's constant divided by 2π)

Avogadro's number = 6.0221×10^{23} molecules/mole

$k_B = 1.3807 \times 10^{-23}$ J/K (the Boltzmann constant)

In order for WebAssign to be able to give you sensible feedback, you must give 6 figures in your answers for E and S. Otherwise there is a build-up of round-off errors when you take differences of E and S, which could result in correct answers being marked wrong.

one quantum = 1.64920e-21 ✓ joules

Here is a table containing the number of ways to arrange a given number of quanta of energy in a particular block of gold. Complete the remainder of the table, including calculating the temperature of the block. The energy E is measured from the ground state. **Be sure to give the temperature to the nearest 0.1 degree.**

q	#ways	E (energy)	S (entropy)	ΔE	ΔS	T
20	2.66e+31	3.29840e-20 ✓ J	9.99053e-22 ✓ J/K			
				1.64920e-21 ✓ J	3.76270e-23 ✓ J/K	43.8302 ✓ K
21	4.06e+32	3.46332e-20 ✓ J	1.03668e-21 ✓ J/K			
				1.64920e-21 ✓ J	3.70000e-23 ✓ J/K	44.5730 ✓ K
22	5.92e+33	3.62824e-20 ✓ J	1.07368e-21 ✓ J/K			

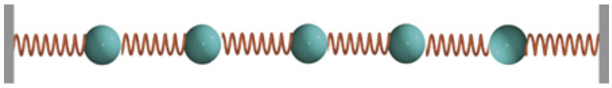
There are 100 atoms in this object. What is the heat capacity on a per-atom basis? (Note that at high temperatures the heat capacity on a per-atom basis approaches the classical limit of $3k_B = 4.2 \times 10^{-23}$ J/K/atom.)

Heat capacity per atom = 2.22025e-23 ✓ J/K/atom

- *Read the eBook*
- Section 12.6

2. 11/11 points | Previous Answers MI3 12.6.P.047

The diagram below shows a one-dimensional row of 5 microscopic objects each of mass 5 × 10^{-26} kg, connected by forces that can be modeled by springs of stiffness 14 N/m (so each object can be modeled as if it were connected to a single spring of effective stiffness $4k_s$ = 56 N/m -- neglect any possible differences for objects near the ends). These 5 objects can move only along the *x* axis.



Use these precise values for the constants:

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What is one quantum of energy for one of these objects?

ΔE = 3.5294e-21 J

Using the Einstein model, calculate the entropy of this system for total energy of 0, 1, 2, 3, 4, and 5 quanta.

q = 0: S = 0 J/K

q = 1: S = 2.2222e-23 J/K

q = 2: S = 3.7390e-23 J/K

q = 3: S = 4.9089e-23 J/K

q = 4: S = 5.8659e-23 J/K

q = 5: S = 6.6775e-23 J/K

Calculate to the nearest degree the average absolute temperature of the system when the total energy is in the range from 3 to 4 quanta. (You can think of this as the temperature when there would be 3.5 quanta of energy in the system, if that were possible.)

$T_{3 \text{ to } 4}$ = 368.80 K

Calculate to the nearest degree the average absolute temperature of the system when the total energy is in the range from 4 to 5 quanta. (You can think of this as the temperature when there would be 4.5 quanta of energy in the system, if that were possible.)

$T_{4 \text{ to } 5}$ = 434.87 K

Calculate the heat capacity per object when the total energy is 4 quanta. (Think of this in terms of increasing from 3.5 quanta of energy in the system to 4.5 quanta of energy in the system, if that were possible.)

C_4 = 1.0684e-23 J/K/object

If the temperature were raised very high, classically what would we expect the heat capacity per object to be for this one-dimensional system? Give a numerical value.

$C_{\text{high T}}$ = 1.3807e-23 J/K/object

(One reason for the discrepancy is that the high-temperature limit assumes that the number of oscillators is large ($N >> 1$), which is not the case in this tiny system.)

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