a. (2 pts) Evaluate  $-17 \mod 5$ .

3

b. (2 pts) What is the prime factorization of 12! (factorial of 12)?

12 x 11 x 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2

$$2^{10} \times 3^{5} \times 2^{2} \times 7 \times 11$$
 $2^{10} \times 3^{5} \times 5^{2} \times 7 \times 11$ 
 $2^{10} \times 3^{5} \times 5^{2} \times 7 \times 11$ 

c. (3 pts) What is the LCM (least common multiple) of the following integers  $2^2 \cdot 3 \cdot 5$  and  $2^3 \cdot 7$ ?

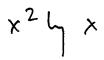
$$2^{3} \times 3 \times 5 \times 7$$
  
=  $8 \times 15 \times 7$  =  $120 \times 7$  =  $860$ 

d. (3 pts) What is the GCD (greatest common divisor) of the following integers  $2^3 \cdot 3^2 \cdot 5 \cdot 13$  and  $2^2 \cdot 3^3 \cdot 7 \cdot 11$ ?

$$2^2 \times 3^2 = 4 \times 9 = 36$$

Give the big-O estimate for each of the following functions. Provide a simple function g(x) of the smallest order.

**a.** (2 pts)  $f(x) = x^2 \log(x^3 - 1) + x^{1.5}$ .



b. (2 pts)  $f(x) = \lfloor (x^2 + 3)/2 \rfloor$ .



c. (2 pts)  $f(x) = 2^x + x^6$ .



d. (4 pts) Show that  $f(x) = 3x^2 + 4x$  is  $O(x^2)$  by providing constants C and  $x_0$  as evidence.

a. (5 pts) Compute the following:

$$\sum_{i=1}^{n} (2i-1).$$

- b. (2 + 3 pts) Determine if the following functions are bijections from  $\mathbb{R}$  to  $\mathbb{R}$ .
- i. f(x) = 2x + 1.

ii. 
$$f(x) = 2x^2 - 1$$
.



a. (6 pts) Show using set identities that

$$\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}.$$

$$\frac{AU(Bnc)}{An(Bnc)} = \frac{An(Bnc)}{An(Buc)} = \frac{An(Buc)}{An(Buc)} = \frac{Buc)nA}{EuB)nA}$$

**b.** (4 pts) What is the Cartesian product  $A \times B$  for sets  $A = \{\text{sunny}, \text{rainy}\}$  and  $B = \{\text{nights}, \text{days}\}$ .

a. (4 pts) Use a direct proof to show that the sum of two odd integers is even.

Suppose we have 2 odd integers 0, and 02. then  $O_1 = 2m + 1$  for some integer m  $O_2 = 2n + 1$  for some integer n Thus.  $O_1 + O_2 = 2m + 1 + 2n + 1 = 2(m + n) + 2$ , which is even.

**b.** (6 pts) Show using rules of inference that the premise "Everyone who exercises has a sore muscle" and "Jimmy exercises" imply that "Jimmy has a sore muscle." Use E(x) to denote "x exercises" and S(x) to denote that "x has a sore muscle."

E(Jimmy)

n E(Jimmy) → S(Jimmy)

i. S (Jimmy)

Let S(x) denote "x is a student," F(x) denote "x is on the faculty," and A(x,y) denote "x asked y a question." Let x be drawn from the universe of all people in the world (i.e., x need not necessarily be a student or faculty). Translate the following into logical statements.

a. (4 pts) Every student has asked Professor Smith a question.

$$\forall x (S(x) \rightarrow A(x, Smith))$$

b. (6 pts) Some student has not asked any faculty member a question.

(5 pts) Show that  $[(p \to q) \land (q \to r)] \to (p \to r)$  is a tautology.

(5 pts) Show using identities that  $\neg(p \lor (\neg p \land q))$  and  $(\neg p \land \neg q)$  are logically equivalent.