Question 1.

Proof:

Basic Step:

for $n=1$, $1^3 = 1^2$

$n=2$, $1^3+2^3 = (1+2)^2$

Inductive Step:

Because of $1+2+\cdots+n = n(n+1)/2$

Assume $1^3+2^3+\ldots+n^3+(n+1)^3 = (n(n+1)/2)^2$

We need prove, for n+1, $1^3+2^3+\ldots+n^3+(n+1)^3 = ((n+1)(n+2)/2)^2$

$1^3+2^3+\ldots+n^3+(n+1)^3$

$= (n(n+1)/2)^2 + (n+1)(n+1)^2$

$= n^2(n+1)^2/4 + (n+1)(n+1)^2$

$= (n+1)^2(n^2/4 + n + 1)$

$= (n+1)^2(n+2)^2/4$

$= ((n+1)(n+2)/2)^2$

So, for $n\geq1$, $1^3 + 2^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2$ is true.

Question 2.

(1)
```
sum=0    T=1
for(i=0; i<n; i++){
        for(m=0; m≤I; m++){
                T *= x;
        }
        T *= a[i];
        Sum += T;
}
```

(2) n times additions and n times multiplications.

So, the complexity is O(n).

Question 3.

(1) Assume √7 is rational.

So there must have two integers k and m such that √7 = k/m, and k and m has no common factor.

Square both side we get $7 = k^2/m^2$, therefore $7m^2 = k^2$

Hence, we get k^2 has a factor 7. As k is an integer, so k also has a factor 7. We set k = 7n.

Use this replace k in $7m^2 = k^2$, we get $7m^2 = 49n^2$, $m^2 = 7n^2$.

So, m has a factor 7.

Now, we can see k and m has a common factor 7, which is contradictive with our assumption.

So, √7 is irrational.

(2) Assume no one will receive 400 or more than 400 grams of chocolate.

They receive chocolate in the following order.

Sisters	chocolate
1	399

2	399
3	399
4	399

Now, there are 404 grams chocolate remaining and only one person left, therefore there is no commination whereby no one will receive 400 or more than 400 grams chocolate. This is contradictive with our assumption. So, it is true that at least one of them will receive 400 or more than 400 grams chocolate.

(3) Assume there is no angle equal or more than 90^o.
These four points are in the same plane, so we connect them and get a quadrilateral.
For this shape, we know that the sum of its four angles is 360^o.
Based on the assumption, we can get:

Angle 1	89^o
Angle 2	89^o
Angle 3	89^o

Then the last one is 93^o, which is more than 90^o. So there is no way all of them less than 90^o. In the worst case, at least one of them is equal or more than 90^o.
This is contradictive with the assumption we made.
It is true that given four non-collinear points in the plane, there exist three points which form an angle measuring 90^o or more.

Question 4.

(1) Declare two node **prex** and **prey** to save the previous node of x and y. And a temporary pointer **temp=head** to go through the linked list.
Through two while loop while(temp!=x) and while(temp!=y) to find the previous node of x and y.
If(temp -> next == x) prex = temp;
If(temp -> next == y) prey = temp;
If x! = first. prex = y, prey = x, temp = y -> next, y -> next = x -> next, x-> next = temp.
If x == first. prey -> x, temp = y -> next, y -> next = x -> next, x -> next = temp.

(2) If(x!=head)   (x->previous)->next=y;
If(y->next!=NULL)   (y->next)->previous=x;
(x->next)->previous = y;
(y->previous)->next = x;
temp = x->next;
x->next = y->next;
y->next = temp;
temp = x->previous;
x->previous = y->previous;
y->previous = temp;

(3) Because of the twice loops in the single linked list, the second one obviously will take less time.

Question 5.

(1) (a) $O(n^4)$

(b) O(1)
(c) O(log(log n))
(d) $O(4^{\log n})$
(e) O(n!)
(f) $O(n^{\log n})$

(2) $1 < \log(\log n) < 4^{\log n} < n^4 < n^{\log n} < n!$
b < c < d < a < f < e

Question 6.

(1) N^2
(2) N logN

Question 7.

Set a boolean variable found = false

Execute pop() n times for stack S, after every pop(), inqueue() the value to queue Q. At the same time check if pop() equals x or not. If pop()==x, then set found = true.

Now, we have n elements in queue Q.

Then execute dequeue(), inqueue() n times for queue Q.

Now, push all the elements from queue Q to stack S. Execute dequeue() and push() n times.