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Summer 2012

ECE 20200 : Linear Circuit Analysis II
School of ECE, Purdue University

LECTURE 5

- Integration Property
  - s domain interpretation of charged C/L
- Solution of Integro-Differential Equations using Laplace Transform

Reference: Decarlo/Lin PP 580-581, 585-590

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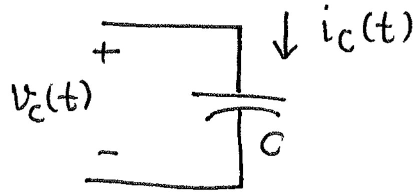
Example: A second s-domain interpretation of a charged capacitor by way of integration property

Time-domain



$t > 0$

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(q)\,dq$$

s-domain

Laplace transform the above equation

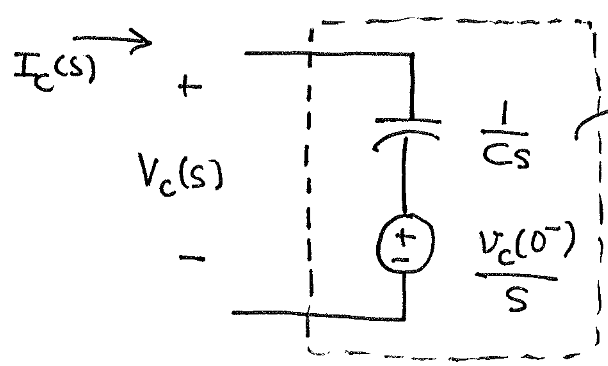
$$V_c(s) = \frac{1}{C}\left[\frac{I_c(s)}{s} + \frac{\int_{-\infty}^{0^-} i_c(q)\,dq}{s}\right]$$

$$= \frac{I_c(s)}{Cs} + \frac{\frac{1}{C}\int_{-\infty}^{0^-} i_c(q)\,dq}{s}$$

$$V_c(s) = \frac{I_c(s)}{Cs} + \frac{v_c(0^-)}{s}$$

Voltages

Circuit Interpretation



Equivalent circuit in s-domain

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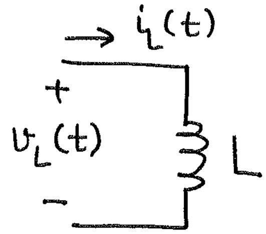
Example: A frequency domain interpretation of the inductor with $i_L(0^-) \neq 0$

Time-domain



$$\Longleftrightarrow \quad i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(\tau)\,d\tau$$

s-domain

Laplace transform the above equation

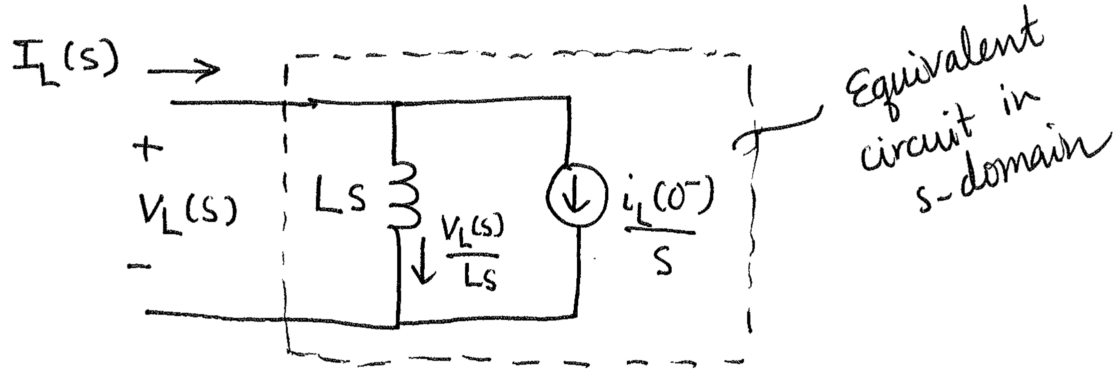
$$I_L(s) = \frac{1}{L}\left[\frac{V_L(s)}{s} + \frac{\int_{-\infty}^{0^-} v_L(\tau)\,d\tau}{s}\right]$$

$$= \frac{V_L(s)}{Ls} + \frac{\frac{1}{L}\int_{-\infty}^{0^-} v_L(\tau)\,d\tau}{s}$$

$$I_L(s) = \frac{V_L(s)}{Ls} + \frac{i_L(0^-)}{s}$$

currents

circuit Interpretation



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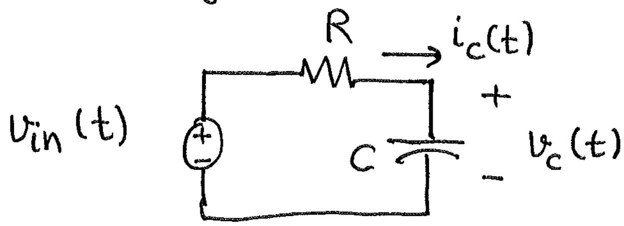
Solution of Integro-Differential Equations using Laplace Transform

Example: Find $v_c(t)$ for the series RC circuit assuming $v_c(0^-) \neq 0$ and $v_{in}(t) = 10e^{-4t}u(t)$



$R = 1\Omega$

$C = 0.5F$

Step 1: Construct differential equation in terms of R and C, then plug in numbers.

(a) $i_c = C\dfrac{dv_c}{dt}$

(b) $i_c = \dfrac{1}{R}\left[v_{in} - v_c\right]$

(c) $\therefore\; C\dfrac{dv_c}{dt} = \dfrac{1}{R}v_{in} - \dfrac{1}{R}v_c$

$$\frac{dv_c}{dt} = \frac{1}{RC}v_{in} - \frac{1}{RC}v_c$$

$$\frac{dv_c}{dt} + \frac{1}{RC}v_c = \frac{1}{RC}v_{in}$$

Plugging in R and C values, we get

$$\frac{dv_c}{dt} + 2v_c = 2v_{in}$$

Step 2: Take Laplace Transform on both sides of the differential equation

$$s V_c(s) - v_c(0^-) + \cancel{\frac{1}{RC}}^{\,2} V_c(s) = \cancel{\frac{1}{RC}}^{\,2} V_{in}(s)$$

$$(s+2) V_c(s) = 2 V_{in}(s) + v_c(0^-)$$

$$V_c(s) = \frac{2}{s+2} V_{in}(s) + \frac{v_c(0^-)}{s+2}$$

$$V_c(s) = \frac{2}{s+2} \cdot \frac{10}{s+4} + \frac{v_c(0^-)}{s+2}$$

$$V_c(s) = \frac{20}{(s+2)(s+4)} + \frac{v_c(0^-)}{s+2}$$

$$V_c(s) = \frac{10}{s+2} - \frac{10}{s+4} + \frac{v_c(0^-)}{s+2}$$

Step 3 : Find $v_c(t)$ by taking inverse Laplace transform

$$v_c(t) = \underbrace{10 e^{-2t} u(t) - 10 e^{-4t} u(t)}_{\substack{\text{due only to input} \\ \text{"zero-state response"}}} + \underbrace{v_c(0^-) e^{-2t} u(t)}_{\substack{\text{due only to IC} \\ \text{"zero-input response"}}}$$

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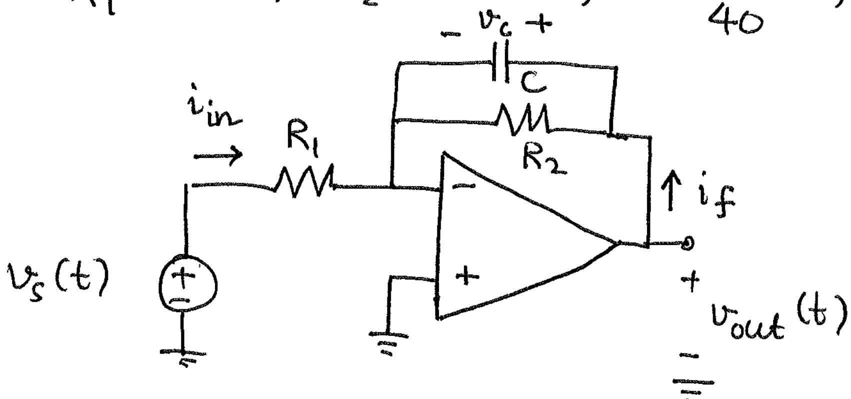
Example: Find $v_{out}(t)$ for the op-amp circuit below when $v_s(t) = e^{-4t}\cos(2t)\,u(t)$ V, $R_1 = 5\,\Omega$, $R_2 = 10\,\Omega$, $C = \frac{1}{40}$ F, $v_c(0^-) \neq 0$



Step 1: Construct a differential equation

(a) $i_{in} = -i_f$

(b) $i_{in} = \dfrac{v_s}{R_1}$

(c) $i_f = C\dfrac{dv_c}{dt} + \dfrac{v_c}{R_2}$

(d) $v_c = v_{out}$

(e) $\therefore\ -C\dfrac{dv_c}{dt} - \dfrac{v_c}{R_2} = \dfrac{v_s}{R_1}$

$$\frac{dv_c}{dt} + \frac{v_c}{R_2 C} = \frac{-v_s}{R_1 C}$$

Step 2: Take Laplace transform on both sides of the differential equation

$$s V_c(s) - v_c(0^-) + \frac{1}{R_2 C} V_c(s) = -\frac{1}{R_1 C} V_s(s)$$

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$$\left(s+\frac{1}{R_2C}\right)V_C(s) = -\frac{1}{R_1C}V_S(s) + v_C(0^-)$$

$$V_C(s) = \frac{-\frac{1}{R_1C}}{s+\frac{1}{R_2C}}V_S(s) + \frac{v_C(0^-)}{s+\frac{1}{R_2C}}$$

$$V_C(s) = -\frac{8}{\cancel{s+4}}\cdot\frac{\cancel{s+}4}{(s+4)^2+4} + \frac{v_C(0^-)}{s+4}$$

$$V_C(s) = \frac{-8}{(s+4)^2+4} + \frac{v_C(0^-)}{s+4}$$

Step 3: Find $v_{out}(t) = v_c(t)$ by taking inverse Laplace transform

$$v_{out}(t) = \underbrace{-4e^{-4t}\sin(2t)u(t)}_{\text{due only to input}} + \underbrace{v_c(0^-)e^{-4t}u(t)}_{\text{due only to IC}}$$

"zero-state response" "zero-input response"