Question 1. Consider a depth-first search tree T of the graph, rooted at r. We count the number of non-tree edges by looking, for each such edge (v, w), at its lower end-point w: No other non-tree edge (x, w) can have w as lower endpoint, because otherwise one of the two edges (v, w),(x, w) could be removed without creating a bridge (a contradiction). Therefore to each vertex w corresponds at most one non-tree edge whose lower vertex is w. If w is the root or a child of the root, then there is no non-tree edge whose lower vertex is w. This implies that there are at most $n - 2$ non-tree edges (the "-2" is because the root and its children cannot be lower endpoints of a non-tree edge). This, and the fact that the number of tree edges is $n - 1$, together imply that the total number of edges is not more than $2n - 3$.

Question 2. Let G be the directed graph with vertex (rather than edge) costs, and let $c(v)$ denote the cost of vertex v. Create from G the following graph G' that has costs associated with its edges (not its vertices).

1. For every vertex of G create two vertices v^- and v^+ in G', and create a directed edge of cost $c(v)$ from v^- to v^+.
2. For every directed edge (x, y) in G create in G' an edge of zero cost from x^+ to y^-.

Note that if G has n vertices and e edges then G' has $2n$ vertices and $e + n$ edges. Use the existing software with G' as its input, and let D' be the matrix it produces. The length of a shortest path from v to w in G is simply $D'[v^-, w^+]$.

Question 3. The proof is by contradiction: Suppose there is a path $P_{x,y}$ between x and y that is cheaper than the path along T; if there are many such paths then we choose $P_{x,y}$ to be the one that has the smallest number of edges not in T. Suppose that $P_{x,y}$ uses an edge (i, j) that is not in T, and let $H_{i,j}$ be the path in T between i and j. We obtain a contradiction in each of the following two cases:

- Case 1: Edge (i, j) is cheaper than some edge μ on $H_{i,j}$. Then by adding edge (i, j) to T and removing μ from T we obtain a spanning tree cheaper than T, a contradiction.
- Case 2: Edge (i, j) is not cheaper than any edge on $H_{i,j}$. Then replacing in $P_{x,y}$ edge (i, j) by $H_{i,j}$ results (perhaps after "trimming" some created cycles) in a path between x and y that is at least as cheap as $P_{x,y}$ but has a smaller number of edges not in T, a contradiction with the definition of $P_{x,y}$.

Question 4. Let T be the string obtained by concatenating X with itself, that is, $T = XX = a_1 \cdots a_n a_1 \cdots a_n$. Run the pattern matching algorithm using T as text and Y as pattern: If Y occurs in T then Y is a circularly rotated version of X, otherwise it is not.

Question 5. Use pattern matching with XX as text (the concatenation of X with itself), and with the reverse of X (denoted X^R) as pattern. This gives all occurrences of X^R in XX. Every such occurrence of X^R in XX overlaps with a suffix of the first X in XX: If we let ℓ denote the length this overlap, then that particular occurrence of X^R in XX implies a "yes" answer if at least one of $\{\ell, n-\ell\}$ is even (in which case the amount of circular rotation is half the length of that even number). If no occurrence of X^R in XX exists for which one of $\{\ell, n-\ell\}$ is even, then the answer is "no" (that is, no circular rotation exists that turns X into a palindrome).

For example, if $X = amaamanaplanacanalpan$ then $X^R = naplanacanalpanamaama$ occurs starting at position 7 in $XX = amaamanaplanacanalpanamaamanaplanacanalpan$, hence $\ell = 15$ and $n-\ell = 6$, and a rotation by 3 ($= 6/2$) positions indeed turns X into a palindrome.

To see that the "even overlap" condition is needed, consider $X = acabdb$ for which the answer should be "no" yet it would erroneously be "yes" without the even-overlap condition: $X^R = bdbaca$ does occur in $XX = acabdbacabdb$. Of course, if n is odd then one of $\{\ell, n-\ell\}$ is guaranteed to be even, and hence there is no need to worry about the even-overlap issue (i.e., it is enough to check whether X^R occurs in XX).