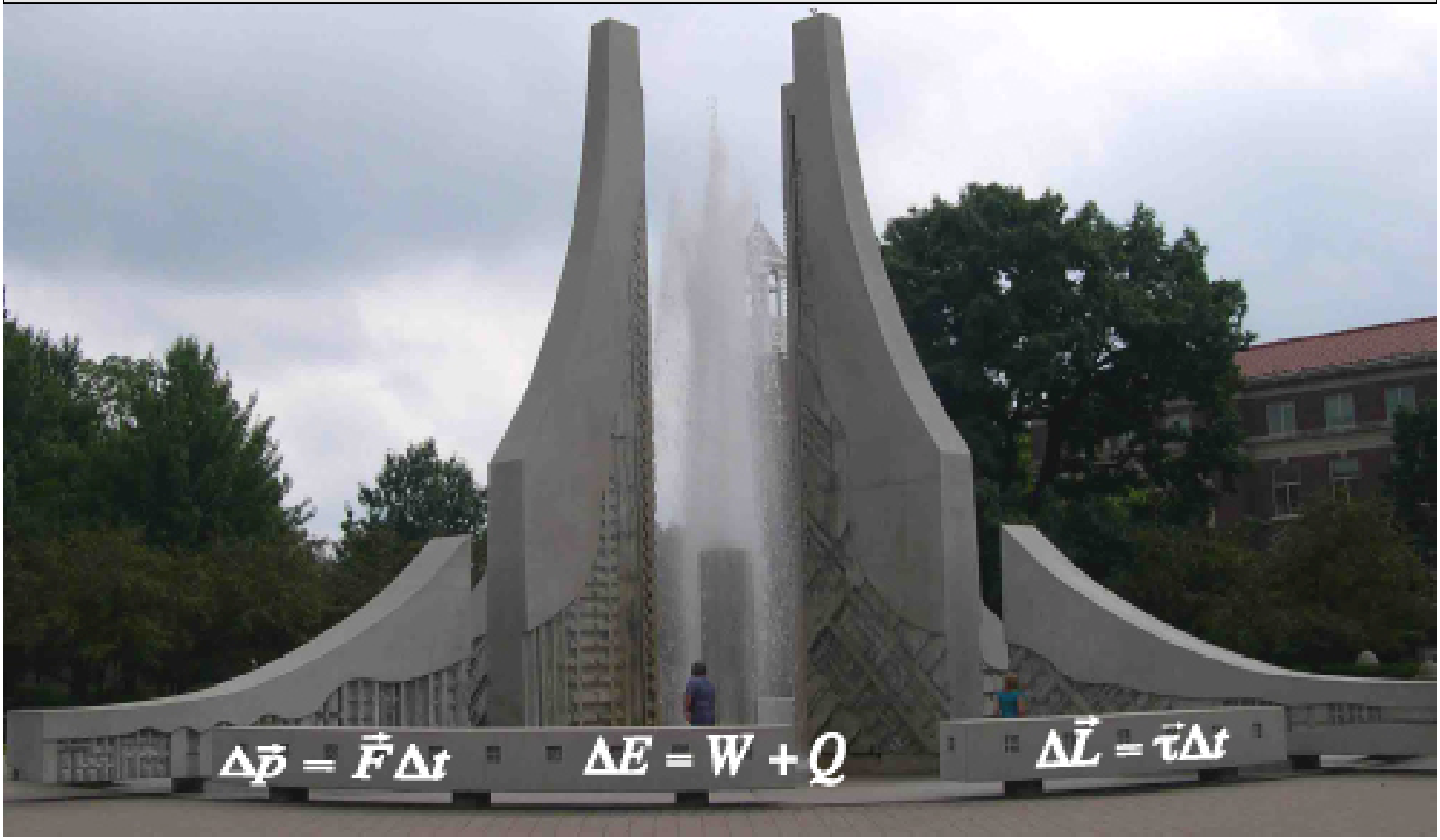
# PHYS 172: Modern Mechanics Spring 2012

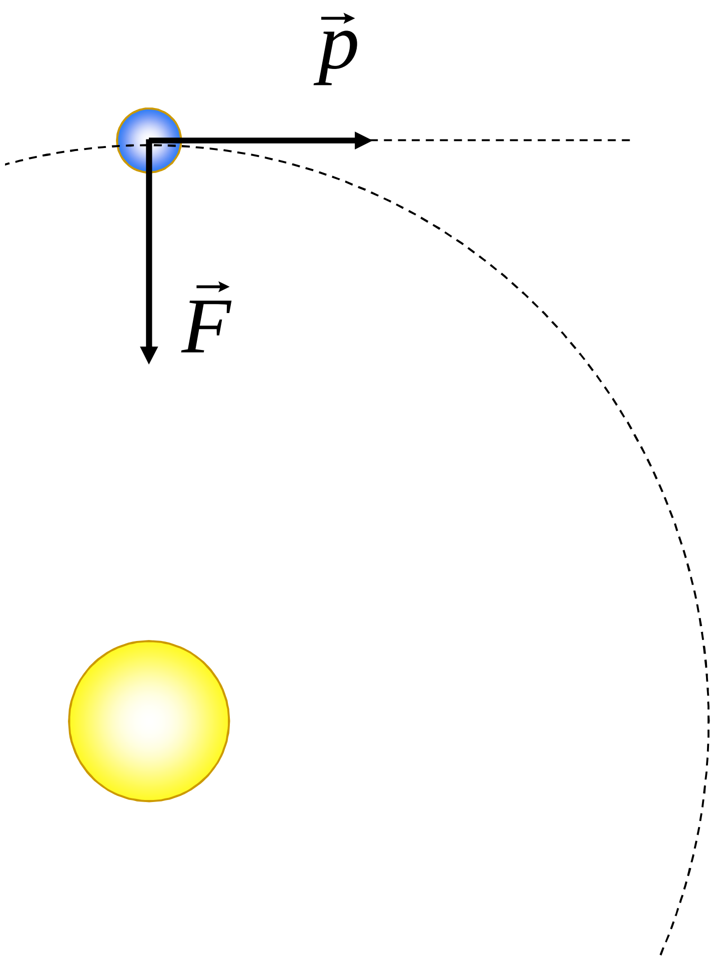



Lecture 5 – **Predicting Motion, Conservation of Momentum** Read 3.5 – 3.15

# Today

- Predicting Motion
- System of Many Objects
- Conservation of Momentum
- Electric Force – Large or Small?
- Can we REALLY predict The Future?

# Predicting motion of a planet




Where will the planet be after one month?

Use position update formula:

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

If we assume that velocity is constant

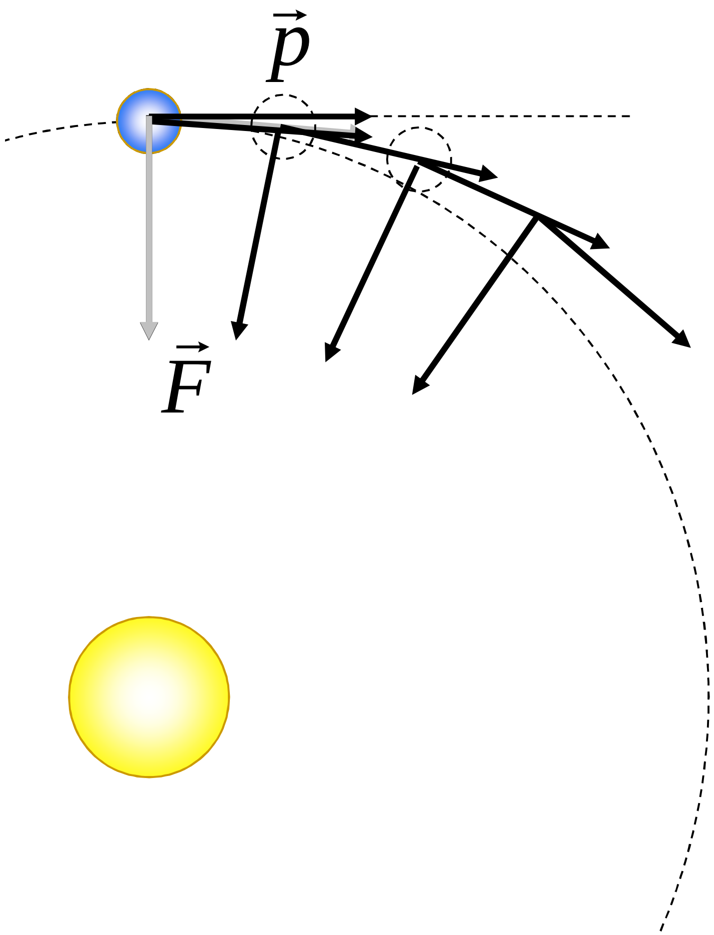
Does not work because the force is changing the velocity!

The direction of force on planet changes with position.
The momentum changes with position.

In general, there is no algebraic equation to predict the motion of more than 2 interacting objects.

# Iterative Prediction of Motion

Simple case: One Star (fixed in space) + One Planet




1. Calculate gravitational force:

$$\vec{F}_{grav\ \text{on}\,2\,\text{by}\,1} = -G\frac{m_2 m_1}{\left|\vec{r}_{2-1}\right|^2}\hat{r}_{2-1}$$

2. Update momentum $\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$

   Choose $\Delta t$ short enough

   ($F$ & $v$ do not change much)

3. Calculate $v$ and update position

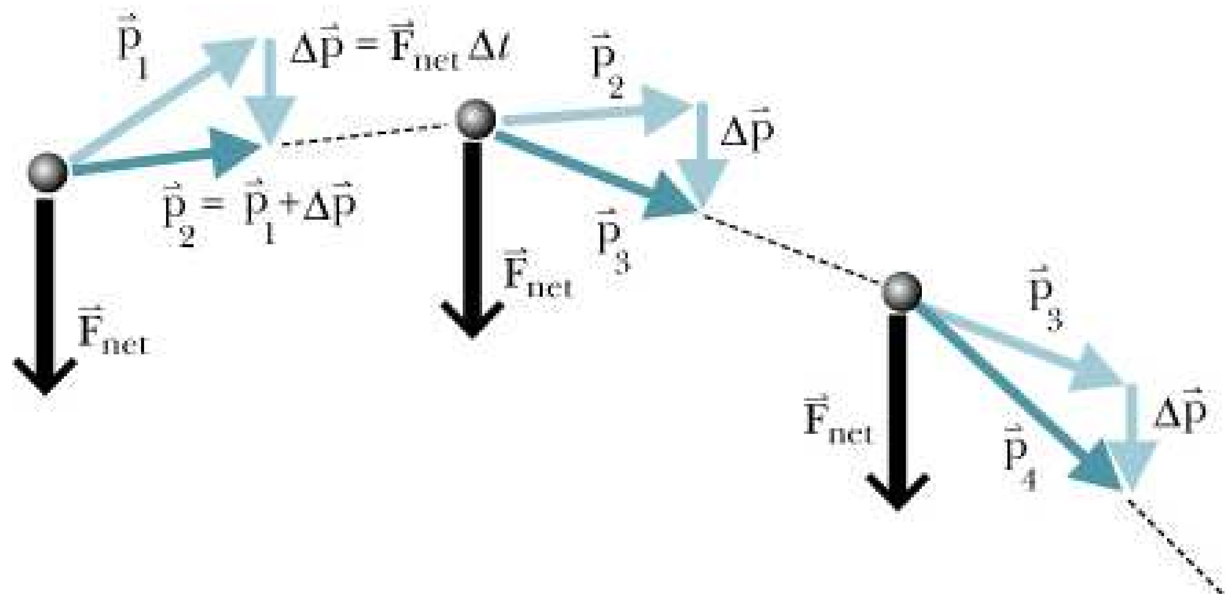
$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

4. Repeat



Critical parameter: $\Delta t$

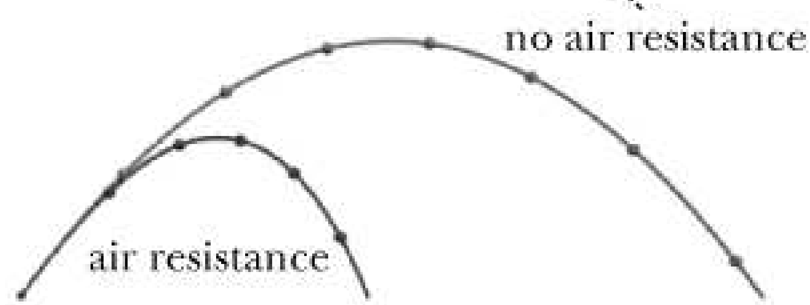
# Iterative Prediction of Motion




Constant Force (mg)

+ Momentum Principle

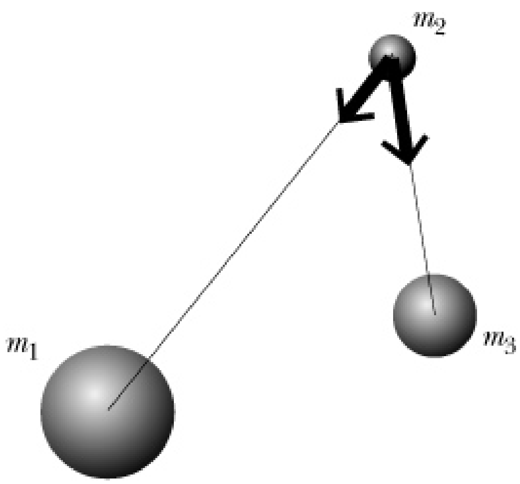
= Projectile Motion (curved path)




But, can also add air resistance = non-constant force

# Iterative prediction of motion

Real case: many objects objects are free to move




1. Calculate net force on each mass:

$$\vec{F}_{\text{on } m_i} = \sum_{i \neq j} \vec{F}_{m_j \text{ on } m_i}$$

2. Update momentum of each mass

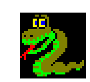
$$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$$

Choose $\Delta t$ short enough
(*F* & *v* do not change much)

3. Calculate *v* and update position of each mass

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} \Delta t$$

4. Repeat



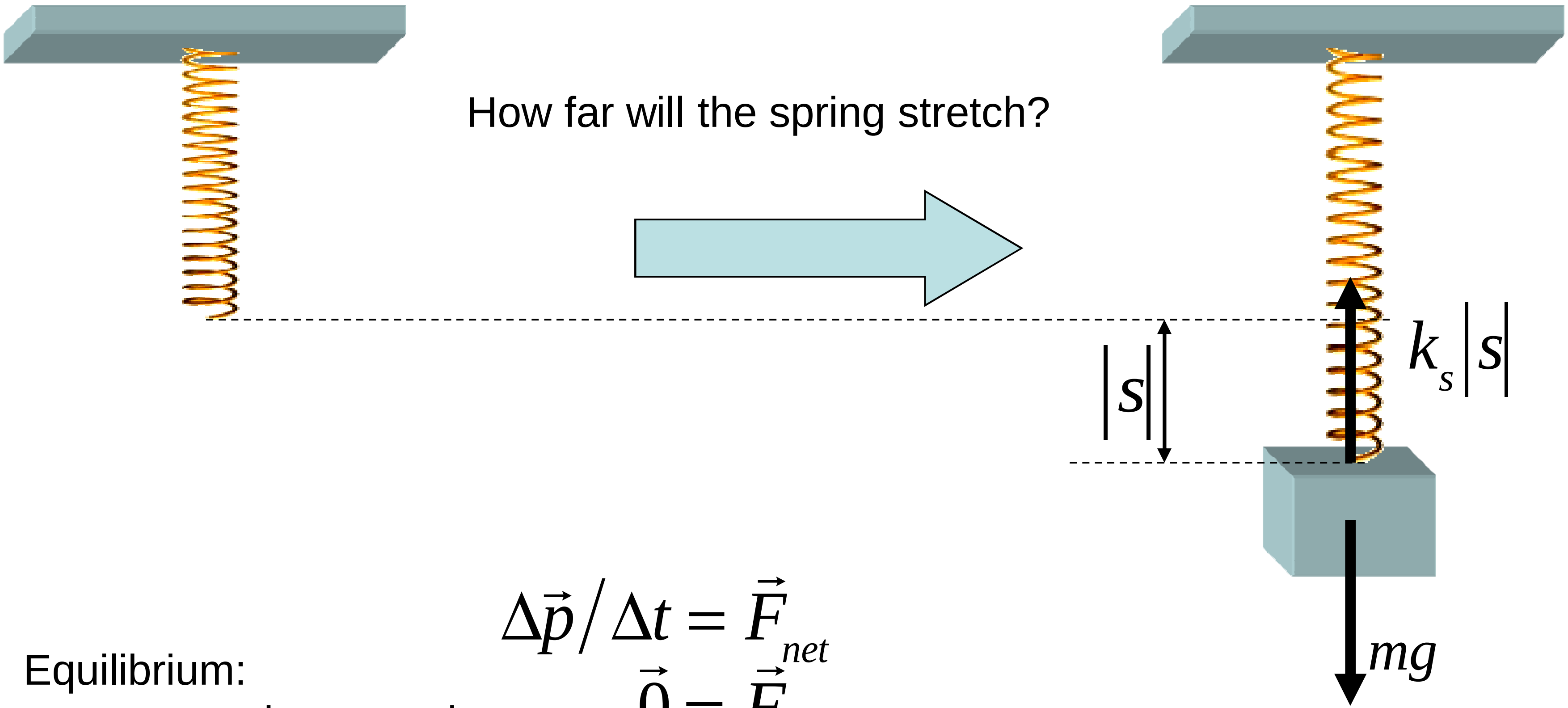

Iterative approach: works for any force, not just gravity!



$\Delta t$ is a critical parameter!

# Example: mass on spring, equilibrium

How far will the spring stretch?




Equilibrium:
momentum does not change

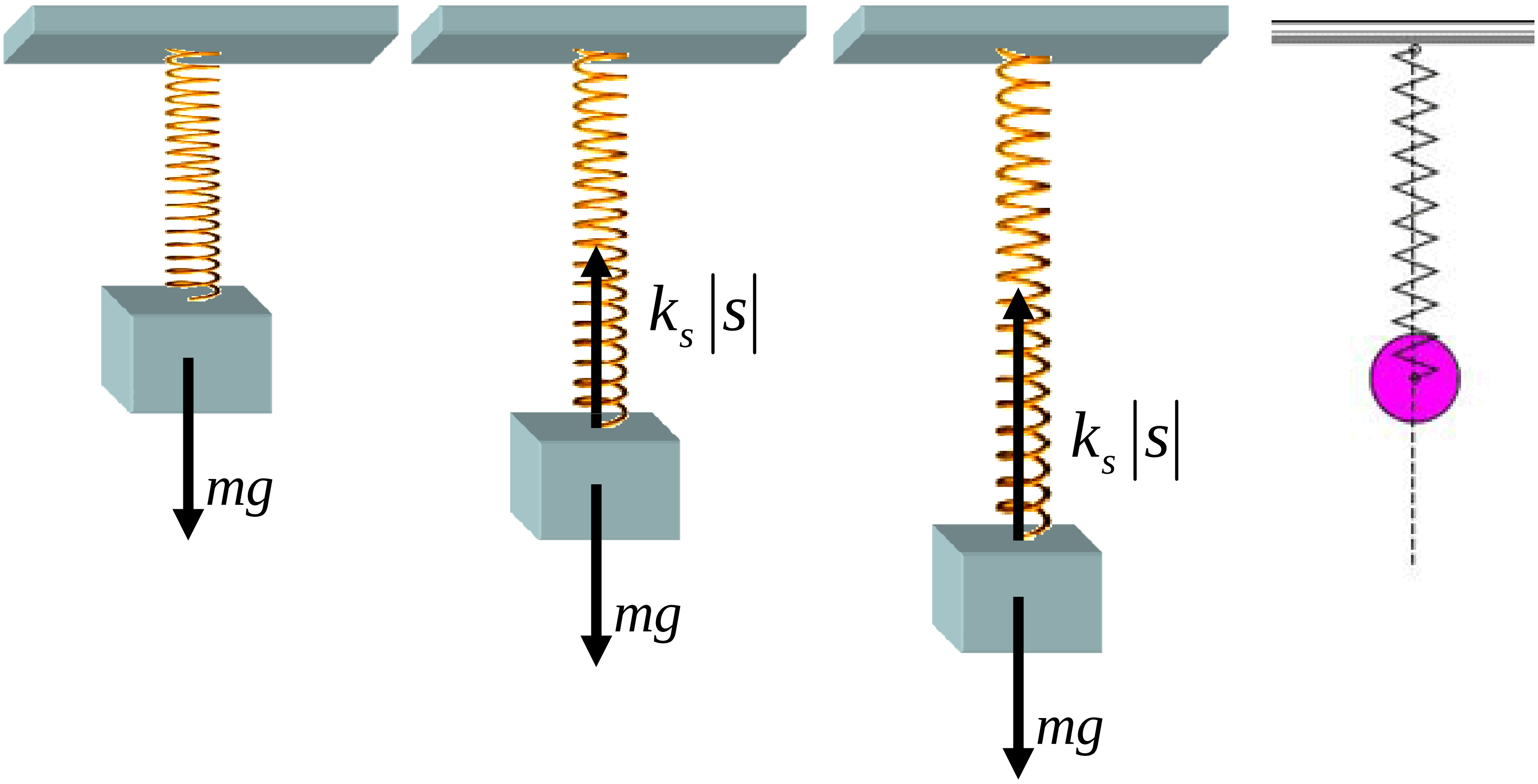
$$\Delta\vec{p}/\Delta t = \vec{F}_{net}$$

$$\vec{0} = \vec{F}_{net}$$

$$\vec{0} = \langle 0, k_s|s| - mg, 0 \rangle$$

$$0 = k_s|s| - mg \quad \Rightarrow \quad |s| = mg / k_s$$

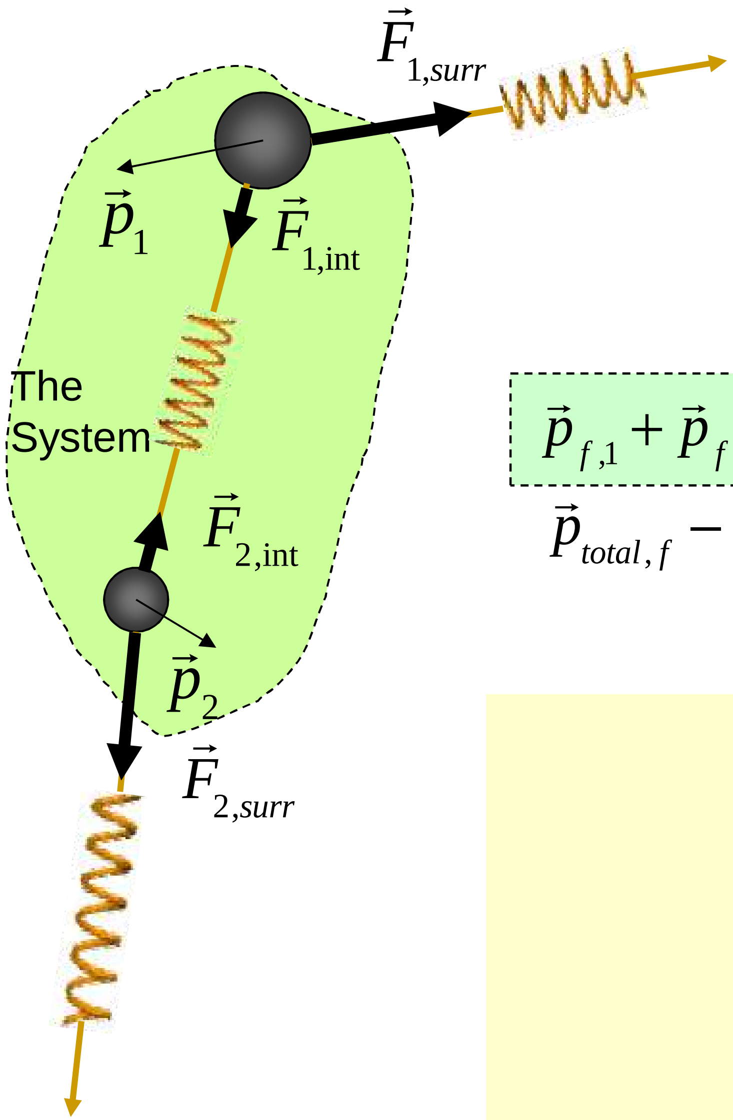
# Example: mass on spring, in motion




As spring stretches, force gets larger.

Block on spring example, Section 2.5 in the text.

# System of 2 Objects




Momentum principle:

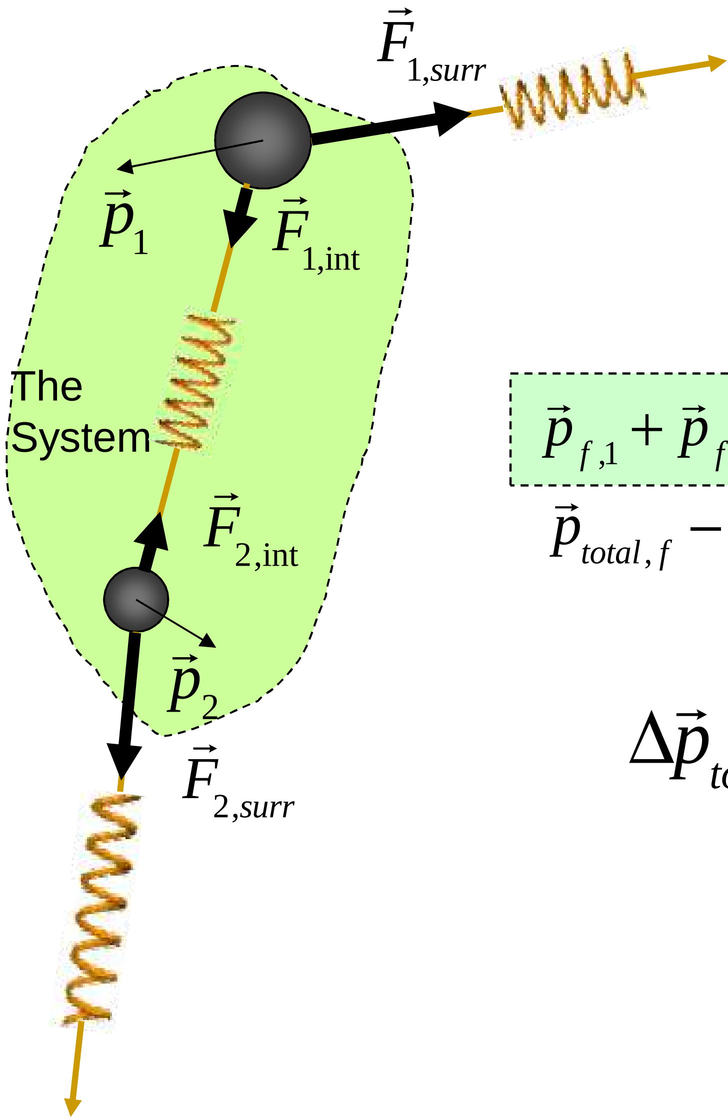
$$\Delta\vec{p}_1 \equiv \vec{p}_{f,1} - \vec{p}_{i,1} = \left(\vec{F}_{1,surr} + \vec{F}_{1,\text{int}}\right)\Delta t$$

$$+$$

$$\Delta\vec{p}_2 \equiv \vec{p}_{f,2} - \vec{p}_{i,2} = \left(\vec{F}_{2,surr} + \vec{F}_{2,\text{int}}\right)\Delta t$$

$$\underbrace{\boxed{\vec{p}_{f,1} + \vec{p}_{f,2}} - \boxed{\vec{p}_{i,1} - \vec{p}_{i,2}}}_{\vec{p}_{total,f} - \vec{p}_{total,i}} = \left(\underbrace{\boxed{\vec{F}_{1,surr} + \vec{F}_{2,surr}}}_{\vec{F}_{net,surr}} + \boxed{\vec{F}_{1,\text{int}} + \vec{F}_{2,\text{int}}}\right)\Delta t$$

# System consisting of two objects




Momentum principle:

$$\Delta\vec{p}_1 \equiv \vec{p}_{f,1} - \vec{p}_{i,1} = \left(\vec{F}_{1,surr} + \vec{F}_{1,\text{int}}\right)\Delta t$$

+

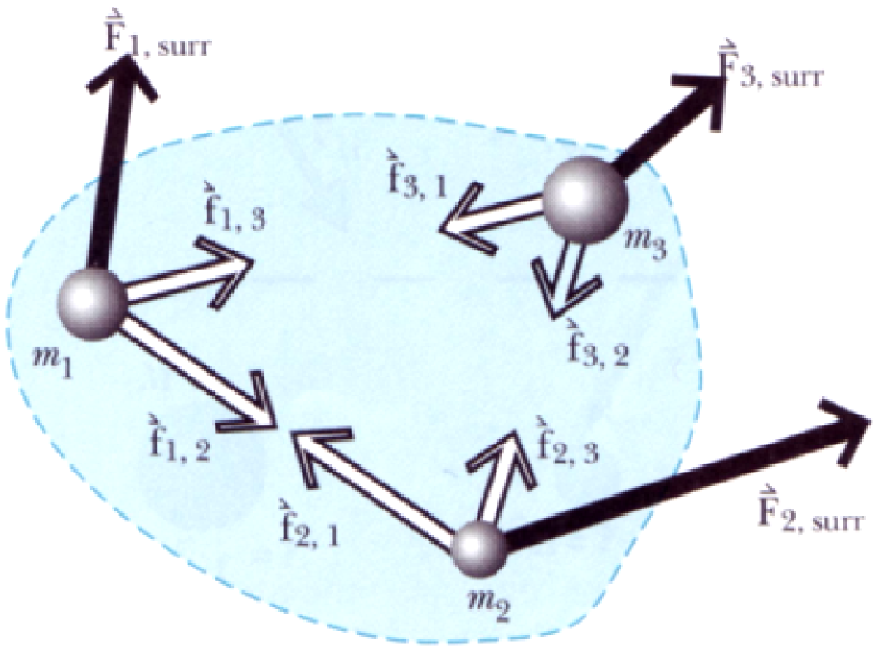
$$\Delta\vec{p}_2 \equiv \vec{p}_{f,2} - \vec{p}_{i,2} = \left(\vec{F}_{2,surr} + \vec{F}_{2,\text{int}}\right)\Delta t$$

$$\underbrace{\boxed{\vec{p}_{f,1} + \vec{p}_{f,2}} - \boxed{\vec{p}_{i,1} - \vec{p}_{i,2}}}_{\vec{p}_{total,f} - \vec{p}_{total,i}} = \left(\underbrace{\boxed{\vec{F}_{1,surr} + \vec{F}_{2,surr}}}_{\vec{F}_{net,surr}} + \underbrace{\boxed{\vec{F}_{1,\text{int}} + \vec{F}_{2,\text{int}}}}_{\vec{0}}\right)\Delta t$$

$$\Delta\vec{p}_{total} \equiv \vec{p}_{total,f} - \vec{p}_{total,i} = \vec{F}_{net}\Delta t$$

Only from surrounding!

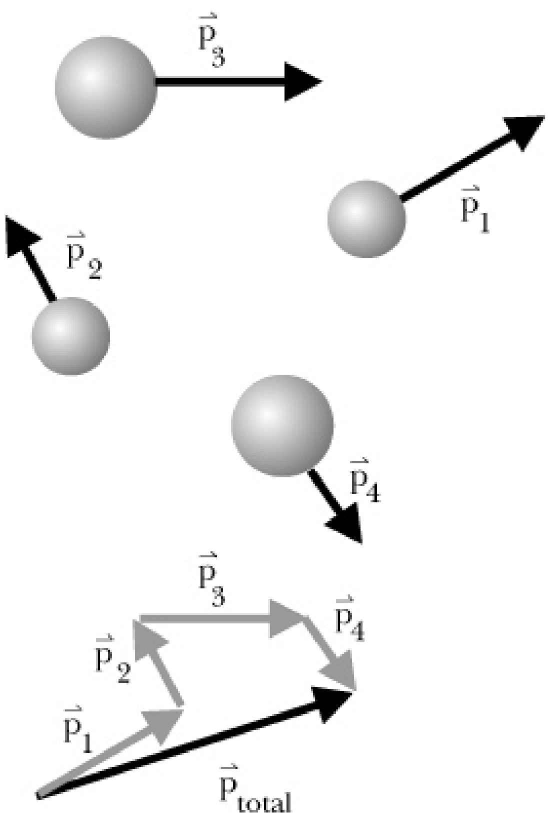
# System consisting of many objects




Because all the forces inside the system **<u>come in pairs</u>** they cancel out.

The only forces left over are forces from the surroundings!

# System consisting of several objects




Momentum principle for a system:

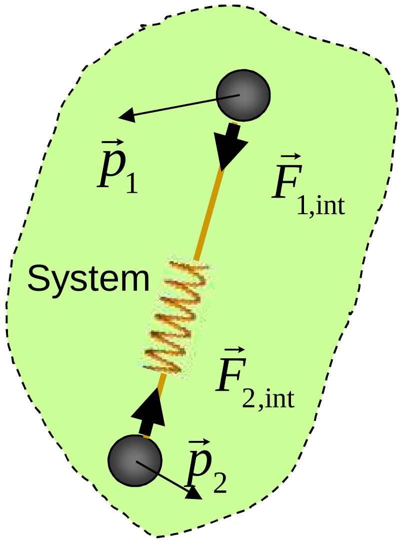
$$\boxed{\Delta\vec{p}_{total} \equiv \vec{p}_{total,f} - \vec{p}_{total,i} = \vec{F}_{net}\Delta t}$$

Total momentum of the system:

$$\boxed{\vec{p}_{total} \equiv \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots}$$

Sum of all *external* forces due to surrounding

# Conservation of momentum



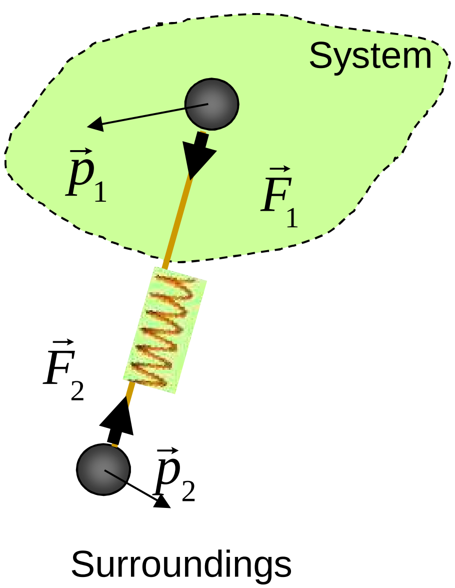

$$\Delta\vec{p}_1 \equiv \vec{p}_{f,1} - \vec{p}_{i,1} = \vec{F}_{1,\text{int}}\Delta t$$

+

$$\Delta\vec{p}_2 \equiv \vec{p}_{f,2} - \vec{p}_{i,2} = \vec{F}_{2,\text{int}}\Delta t$$

$$\underbrace{\boxed{\vec{p}_{f,1} + \vec{p}_{f,2}} - \boxed{\vec{p}_{i,1} - \vec{p}_{i,2}}}_{\vec{p}_{total,f} - \vec{p}_{total,i}} = \underbrace{\left(\boxed{\vec{F}_{1,\text{int}} + \vec{F}_{2,\text{int}}}\right)}_{\vec{0}}\Delta t$$

In the absence of external forces $\Delta\vec{p}_{total} = \vec{0}$

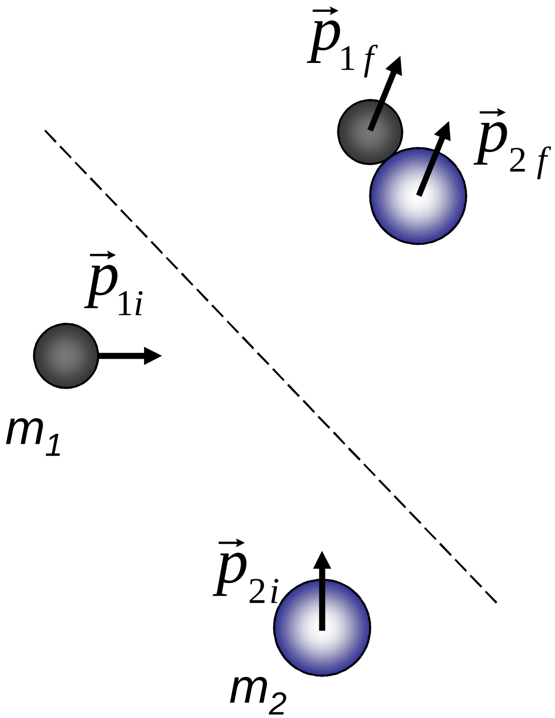



$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0}$$

**Conservation of momentum**

$$\Delta\vec{p}_{system} + \Delta\vec{p}_{surrounding} = \vec{0}$$

# Collisions: negligible external forces




1. Sticky ball

Momentum conservation:

$$\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$$

Assume $\gamma$=1: $$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_f + m_2\vec{v}_f$$

$$\boxed{\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}}$$

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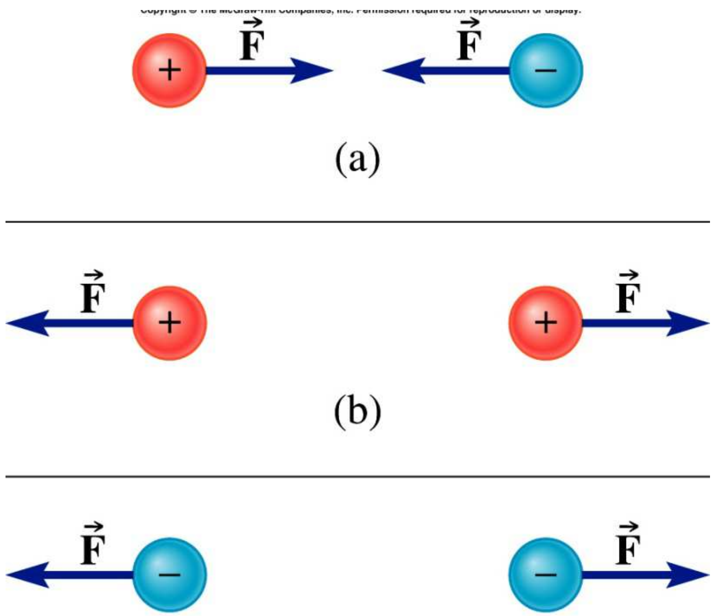
What if the balls bounce?

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Two unknowns, one equation

# Electric force: the electric charges

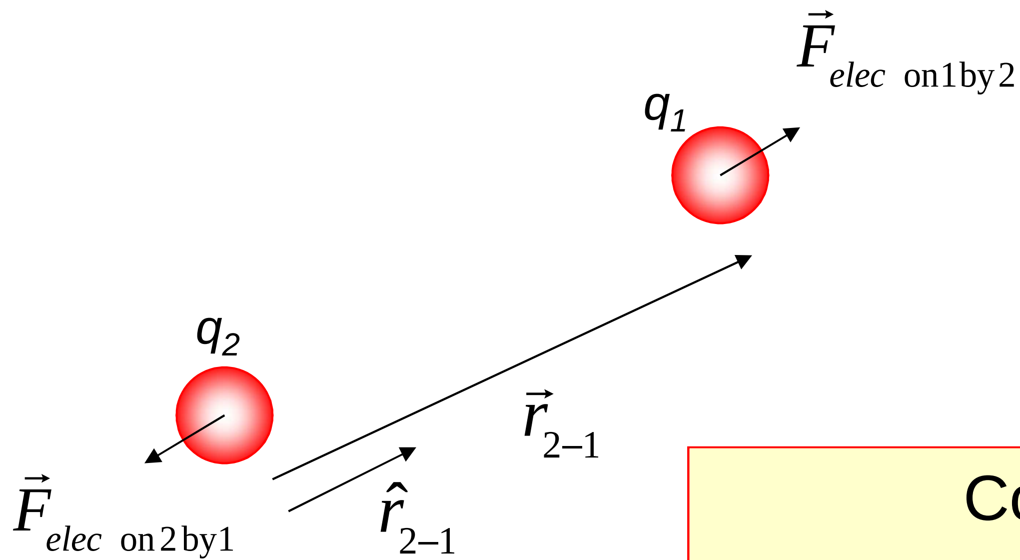



Charges: property of an object

- Two types: *positive (+)* and *negative (-)*
- Like charges: repel.
- Opposite charges: attract
- Net charge of a system:
  sum of all the charges
- Conservation of charge
- The force exerted by one point charge on another acts along the line joining the charges

Charge: measured in C (Coulomb)
Elementary charge: $e = 1.602{\times}10^{-19}$ C
Charge of electron is $-e$, of a proton $+e$

# The Electric Force Law (Coulomb's Law)





Coulomb's law

$$\vec{F}_{\mathrm{elec\ \ on\,2\,by\,1}} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_1}{\left|\vec{r}_{2-1}\right|^2}\hat{r}_{2-1}$$

$$\frac{1}{4\pi\varepsilon_0} = 9\times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C}^2}$$

# Electric force versus gravity

***Gravity***

$$F_g = G\frac{m_1 m_2}{r^2}$$

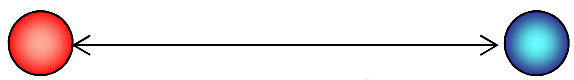
$G = 6.67\times10^{-11}\ \mathrm{m^3/(kg\cdot s^2)}$

***Electric force***

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

$1/(4\pi\varepsilon_0) = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$

Proton
$m_p=1.7 \times 10^{-27}$kg
$q_p = +1.6 \times 10^{-19}$C



$1\mathrm{cm}=10^{-2}\mathrm{m}$

Electron
$m_e=9.1 \times 10^{-31}$kg
$q_e = -1.6 \times 10^{-19}$C

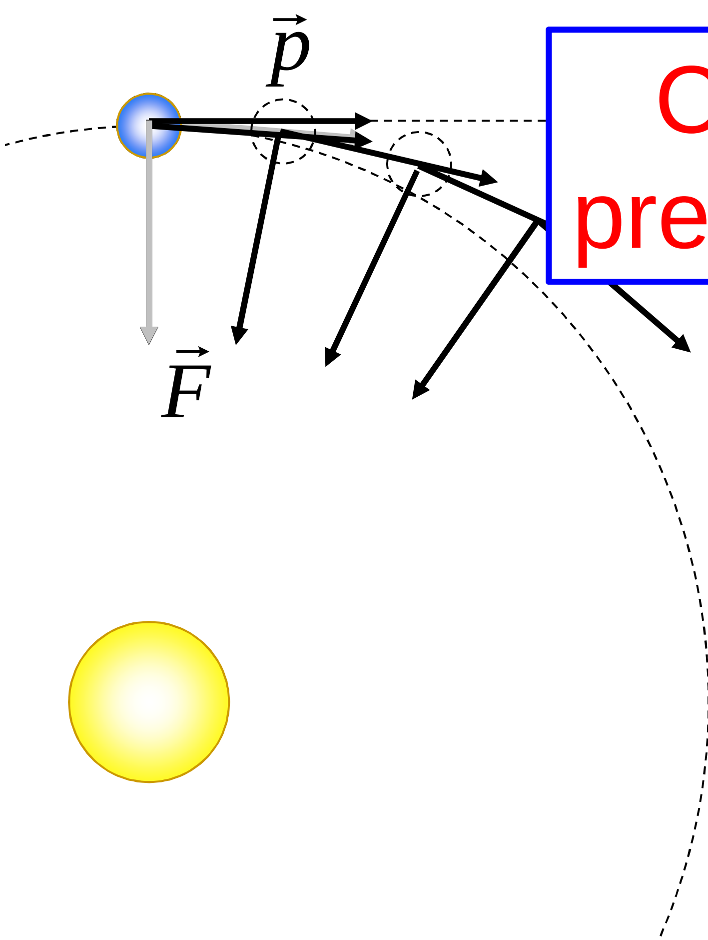
$$\vec{F}_{\text{grav}} = (6.7 * 1.7 * 9.1) \times \frac{10^{-11}10^{-27}10^{-31}}{10^{-4}}\mathrm{N} = \boxed{1.04 \times 10^{-63}\mathrm{N} = \vec{F}_{\text{grav}}}$$

$$\vec{F}_{\text{elec}} = -(9.0 * 1.6 * 1.6) \times \frac{10^{9}10^{-19}10^{-19}}{10^{-4}}\mathrm{N} = \boxed{2.3 \times 10^{-24}\mathrm{N} = \vec{F}_{\text{elec}}}$$

Electricity Wins!!

# Iterative Prediction of Motion

Simple case: One Star (fixed in space) + One Planet




1. Calculate gravitational force:

$$\vec{F}_{grav\ \ \mathrm{on\,2\,by\,1}} = -G\frac{m_2 m_1}{\left|\vec{r}_{2-1}\right|^2}\hat{r}_{2-1}$$

m $\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$

nough

(F & v do not change much)

**Can we really predict the future?**

3. Calculate *v* and update position

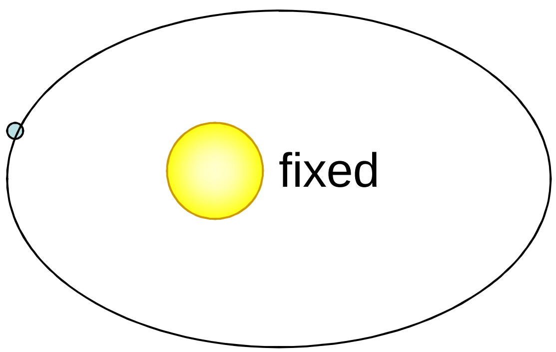
$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

4. Repeat

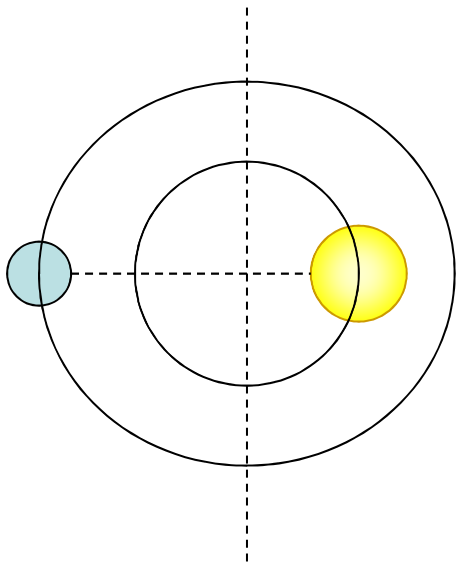
Blast From the Past (20 minutes ago)

parameter: $\Delta t$

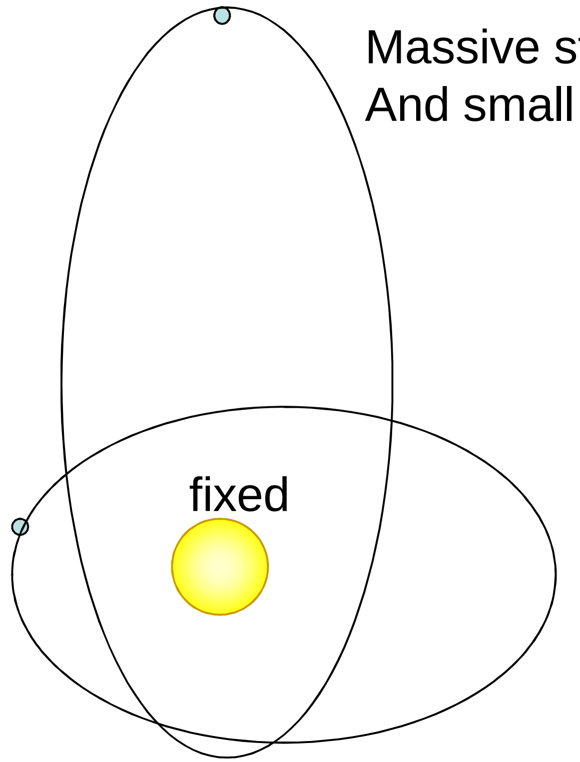
# Predicting the Future: Gravity




Two body: ellipse (or circle)



Massive star
And small planets

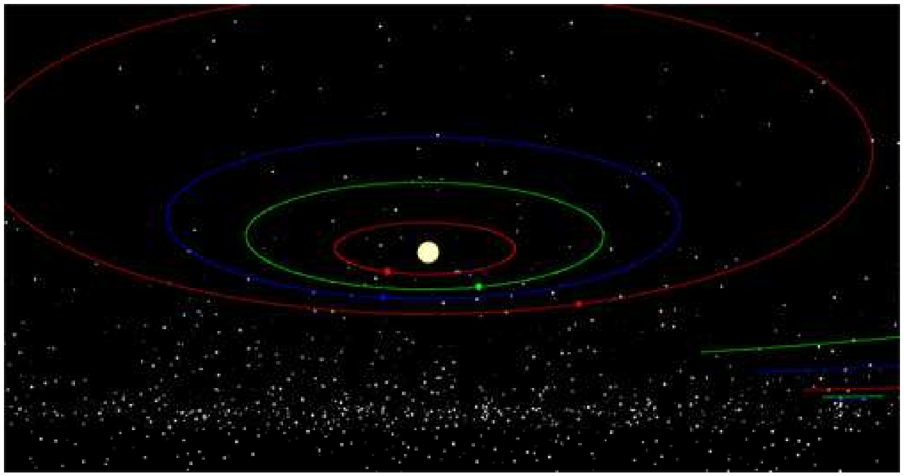



**Determinism:**
If we know the positions and momenta of all particles in the Universe we can predict the future

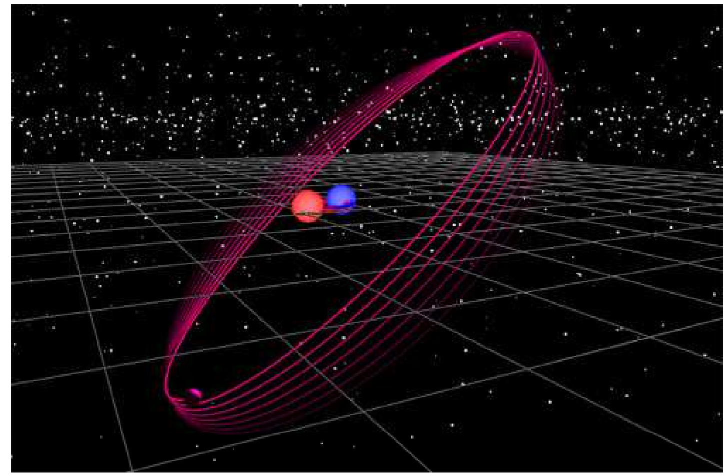
**Is there free will?**

# Can we really predict the future?

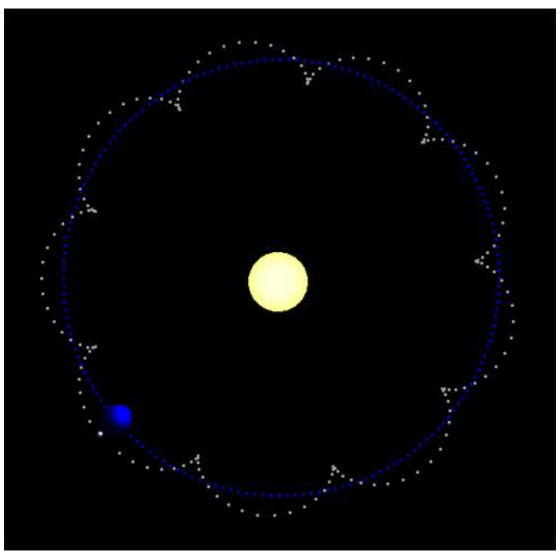
Solar system

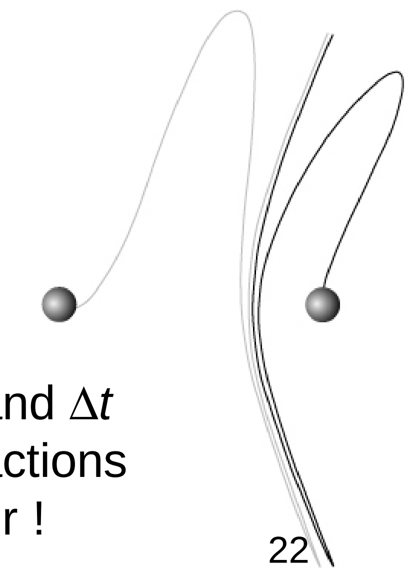


Binary star



Sun, Earth and Moon



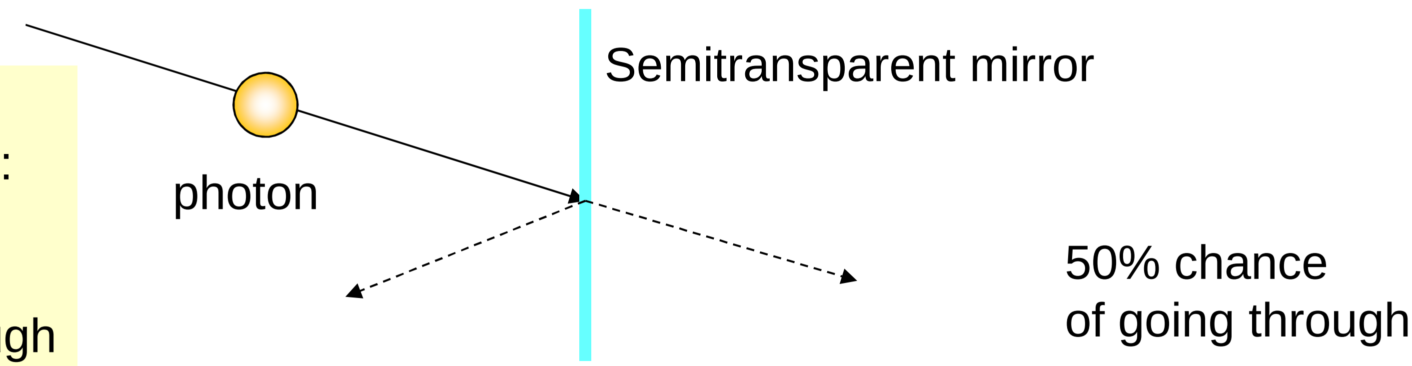


Problems:
Sensitivity to initial conditions and $\Delta t$
Inability to account for all interactions
$10^{25}$ molecules in glass of water !

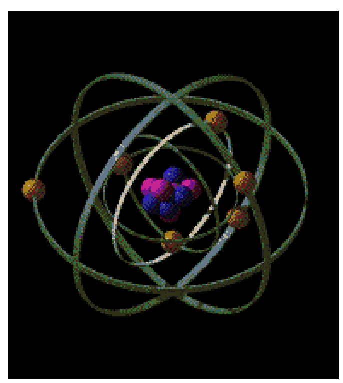
# Probability and uncertainty

**Clicker poll:**
The photon will:

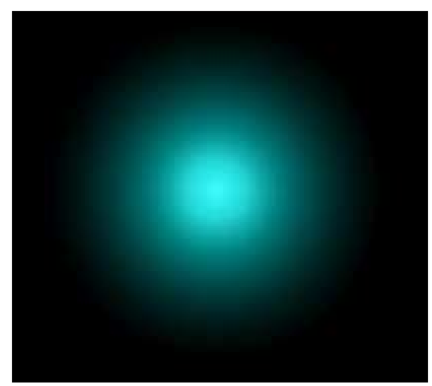
A) Reflect
B) Pass through




Where is the electron?



Classical



Quantum

# Heisenberg's Uncertainty Principle

$$\Delta x \Delta p_x \geq h$$

*Planck's constant*
$h = 6.6\times10^{-34}$ kg·m$^2$/s

The more we know about a particle's position
the less we know about its momentum

The more we know about a particle's momentum
the less we know about its position

But Planck's constant is SMALL.  For large objects, no big deal.

# Is The Whole World Random??

No.  Casinos still make money.

*Any one dice roll = random*

*1 million dice rolls = I know the outcome!*

# What We Did Today

- Predicting Motion
- System of Many Objects
- Conservation of Momentum
- Electric Force – Large or Small?
- Can we REALLY predict The Future?