

EXAM 3 is tomorrow

Time: 8:00-9:30 pm Wed Apr 11

Place: Elliott Hall

Material: lectures 1-21, HW 1-21, Recitations 1-12, Labs 1-12

focus will be on last 3rd of material (not on Exams 1 & 2)

Problems: multiple choice, 10 questions (70 points)

write-up part, hand graded (30 points)

Equation sheet: provided with exam

Practice exam + equation sheet: will be posted at the end of this week

Note: no lecture this Thursday (Apr 12)!

$$\Delta \vec{p} = \vec{F} \Delta t$$

$$\Delta E = W + Q$$

$$\Delta \vec{L} = \vec{\tau} \Delta t$$

Clicker Question:

Consider the energy principle, $\Delta E_{\text{system}} = W + Q$. Choose the correct statement:

- A. If Q is added to the system, the system temperature must increase.
- B. If Q is added, the temperature must increase if $W=0$.
- C. If Q is added, the temperature might not change at all.
- D. None of the above.

First, an important reminder about multiplying probabilities:

A deck of 52 cards has 4 aces. Starting from a randomly-shuffled deck, what is the probability that an ace is the first card dealt?

$$P_1 = \frac{4}{52} = 0.0769$$

Once this happens, what is the probability that the second card is an ace?

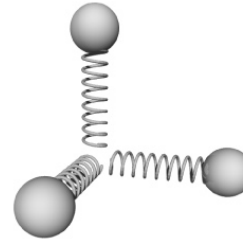
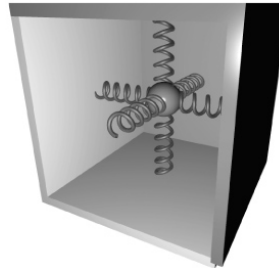
$$P_2 = \frac{3}{51} = 0.0588$$

So, what is the probability that the first two cards are aces?

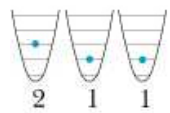
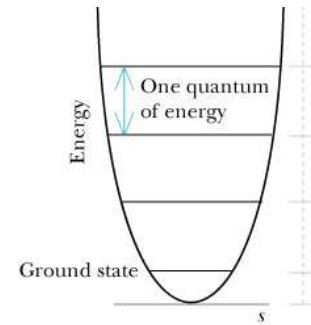
$$P_{two\ aces} = P_1 \times P_2 = \frac{4}{52} \times \frac{3}{51} = (0.0769) \times (0.0588) = 0.00452$$

Last Time

Einstein Model of Solids

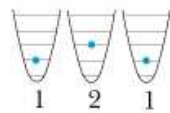


$$\Delta E = \hbar \sqrt{\frac{k}{m}} = \hbar \omega$$



= $\bullet \bullet \mid \bullet \mid \bullet$ # microstates

$$\equiv \Omega = \frac{(q + N - 1)!}{q!(N - 1)!}$$

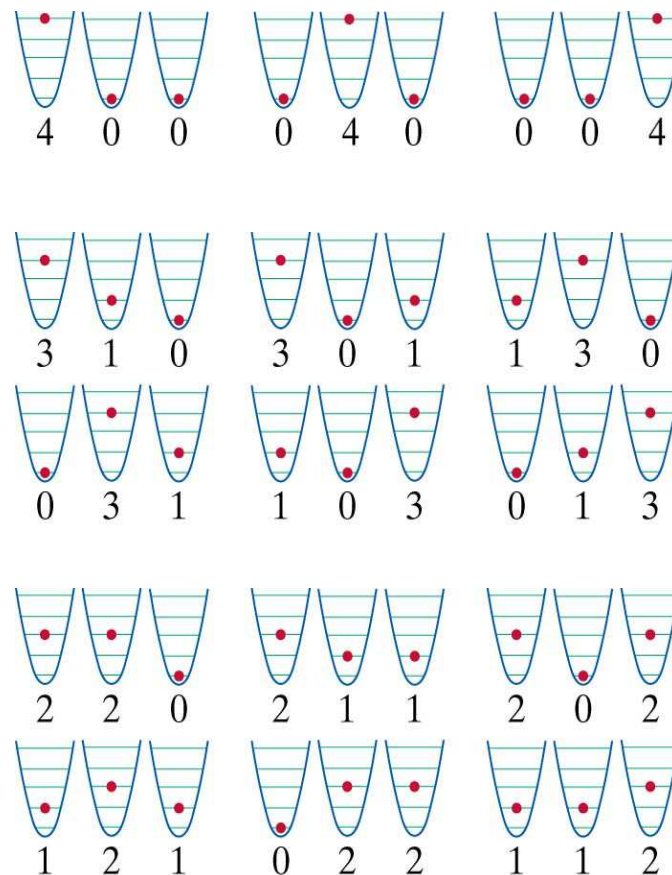


= $\bullet \mid \bullet \bullet \mid \bullet$ (N oscillators, q quanta)

Fundamental assumption of statistical mechanics

Over time, an isolated system in a given macrostate (total energy) is equally likely to be found in any of its microstates (microscopic distribution of energy).

These show the 15 different microstates where 4 quanta of energy are distributed among 3 oscillators (1 atom).

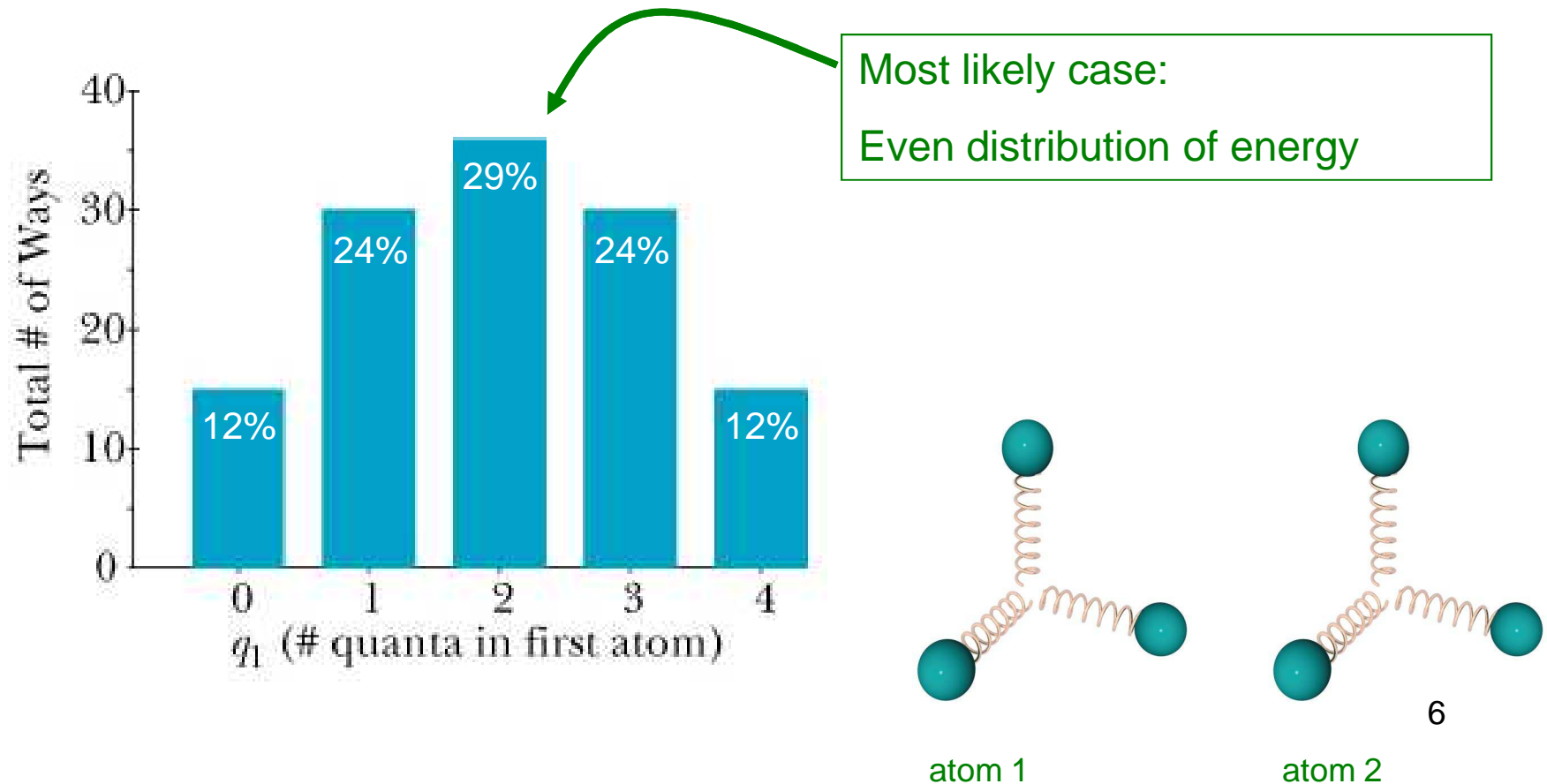


The Fundamental Assumption of Statistical Mechanics

A fundamental assumption of statistical mechanics is that in our state of macroscopic ignorance, each microstate (microscopic distribution of energy) corresponding to a given macrostate (total energy) is equally probable.

Fundamental Assumption of Statistical Mechanics

The fundamental assumption of statistical mechanics is that, over time, an isolated system in a given macrostate is equally likely to be found in any of its microstates.



The entire chart is about a single **macrostate** of the 2-atom system

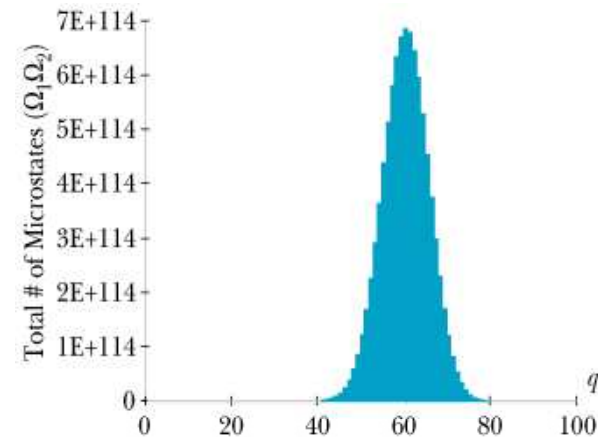
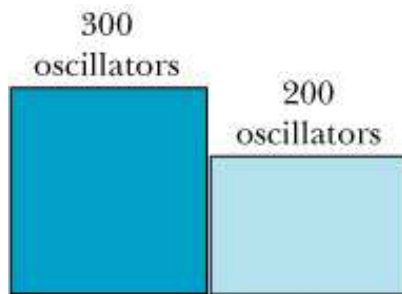
					q_1	q_2
					Ω_1	Ω_2
					Ω_1	Ω_2
Each line is about a different energy distribution	0	4	1	15	15	
	1	3	3	10	30	
	2	2	6	6	36	
	3	1	10	3	30	
	4	0	15	1	15	
					Total = 126	

Number of **microstates** available for the $q_1=2$ **energy distribution**

This is the total number of **microstates** accessible to the entire 2-atom system

Two Big Blocks Interacting

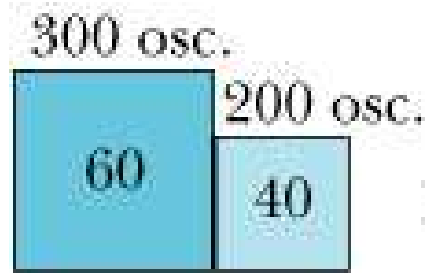
Add 100 quanta of energy.
Most microstates distribute
the energy evenly.



q_1	$q_2 = 100 - q_1$	Ω_1	Ω_2	$\Omega_1 \Omega_2$
0	100	1	$2.78 E+81$	$2.77 E+81$
1	99	300	$9.27 E+80$	$2.78 E+83$
2	98	$4.52 E+4$	$3.08 E+80$	$1.39 E+85$
3	97	$4.55 E+6$	$1.02 E+80$	$4.62 E+86$
4	96	$3.44e E+8$	$3.33 E+79$	$1.15 E+88$
...

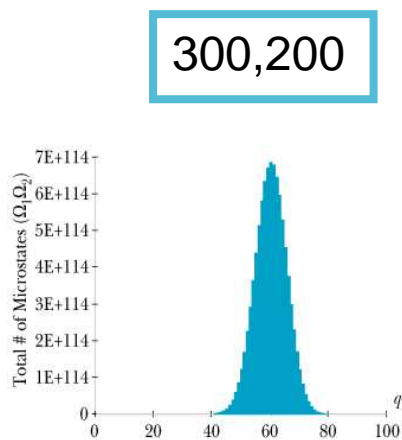
Two Important Observations

1. The most likely circumstance is that the energy is divided in proportion to the # of oscillators in each object:

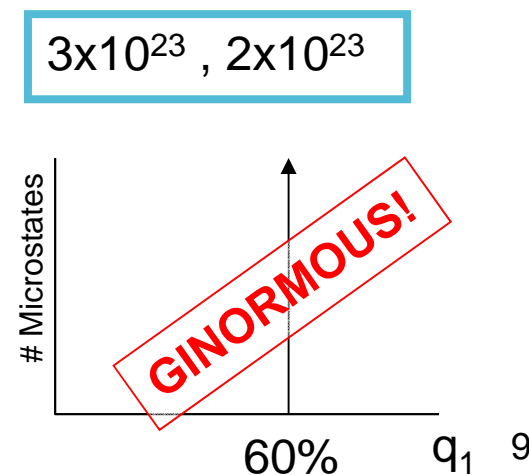


most likely state has 60% of quanta in block with 60% of the oscillators.

2. As the # of oscillators gets larger, **the curve gets narrower**:



add more
oscillators →



Definition of Entropy

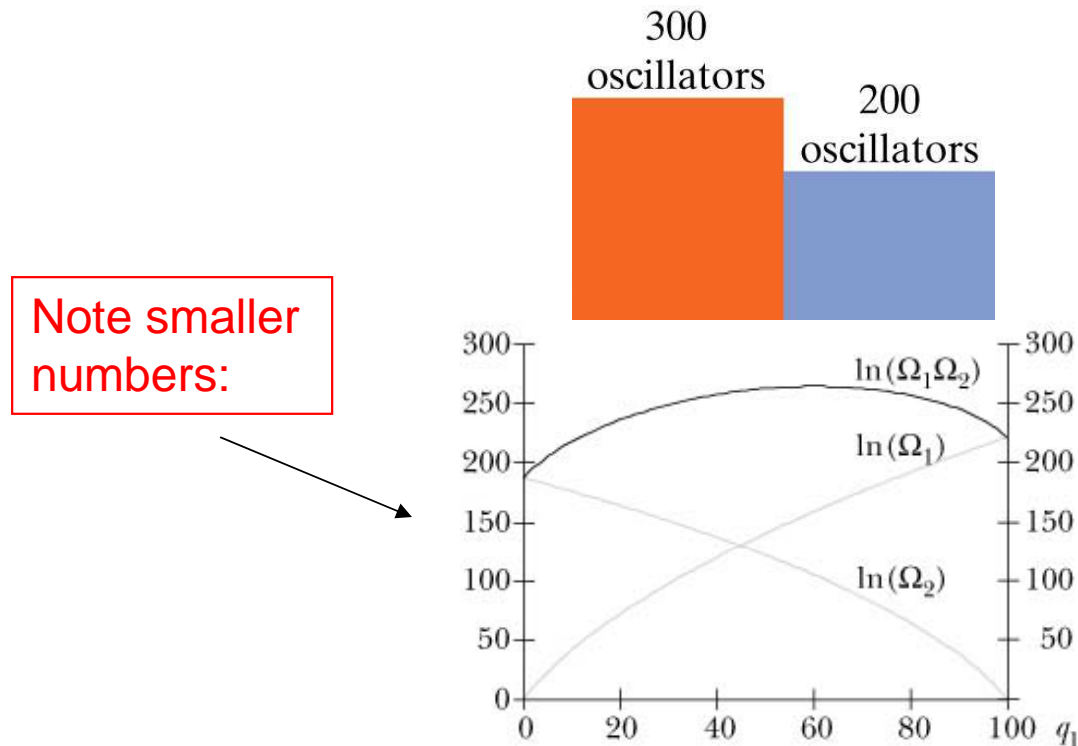
$$\text{Entropy: } S \equiv k \ln \Omega$$

where Boltzmann's constant is $k = 1.38 \times 10^{-23} \text{ J/K}$

It is convenient to deal with $\ln \Omega$ instead of Ω for several reasons:

1. Ω is typically huge, and $\ln(\Omega)$ is a smaller number to deal with
2. $\Omega_{12} = \Omega_1 \Omega_2$ (multiplies), whereas $\ln \Omega_{12} = \ln \Omega_1 + \ln \Omega_2$ (adds).
Double the size, double the entropy. ("Extensive property")

Entropy: $S \equiv k \ln \Omega$, where $k = 1.4 \times 10^{-23} \text{ J / K}$



We will see that in equilibrium, the most probable distribution of energy quanta is that which maximizes the total entropy. (Second Law of Thermodynamics).

$$\text{Entropy: } S \equiv k \ln \Omega, \quad \text{where } k = 1.4 \times 10^{-23} \text{ J / K}$$

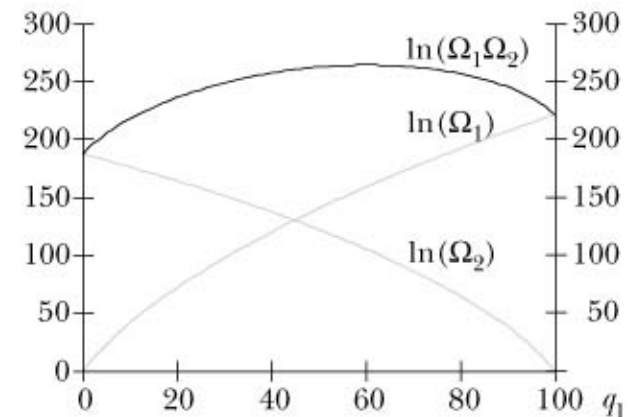
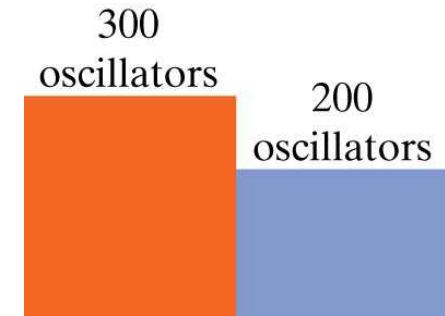
Why entropy?

$$k \ln(\Omega_1 \Omega_2) = k \ln(\Omega_1) + k \ln(\Omega_2)$$

(property of logarithms)

When there is more than one “object” in a system, we can add entropies just like we can add energies:

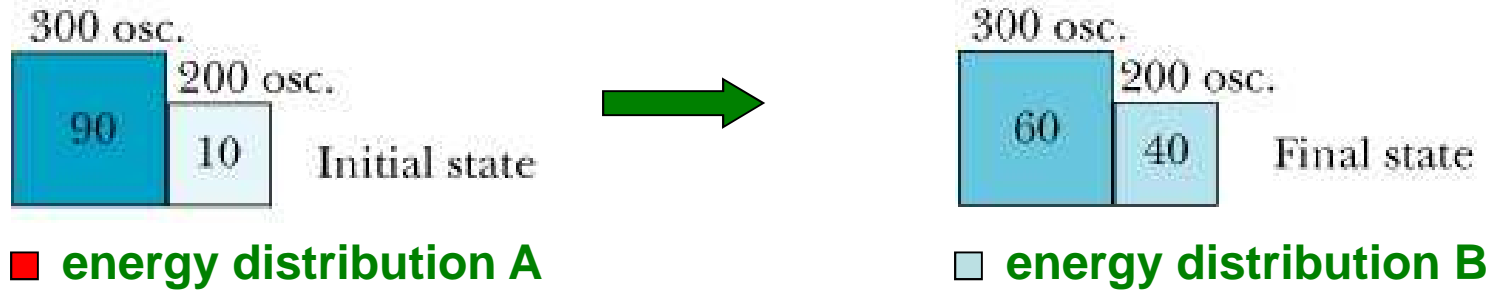
$$S_{\text{total}} = S_1 + S_2, \quad E_{\text{total}} = E_1 + E_2.$$



Remember:

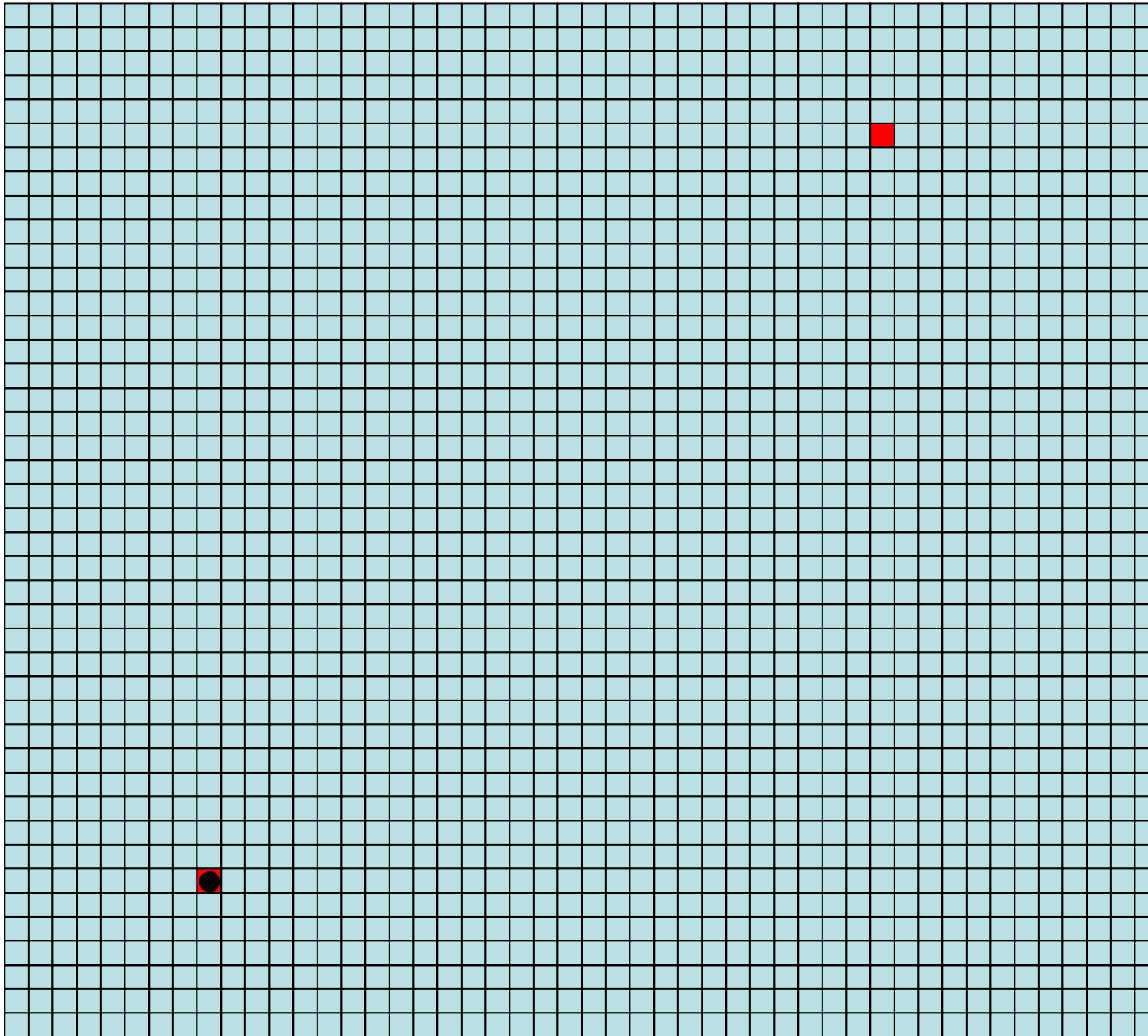
$$\Omega = \frac{(q + N - 1)!}{q!(N - 1)!}$$

Approaching Equilibrium



- Assume system starts in an energy distribution A far away from equilibrium (most likely state).
- What happens as the system evolves?
- Crucial point: consider our system to contain both blocks. The macrostate of the system is specified by the total # quanta: 100.
- All microstates compatible with $q_{\text{total}}=100$ are equally likely, **but not all energy distributions (values of q_1) are equally likely!**

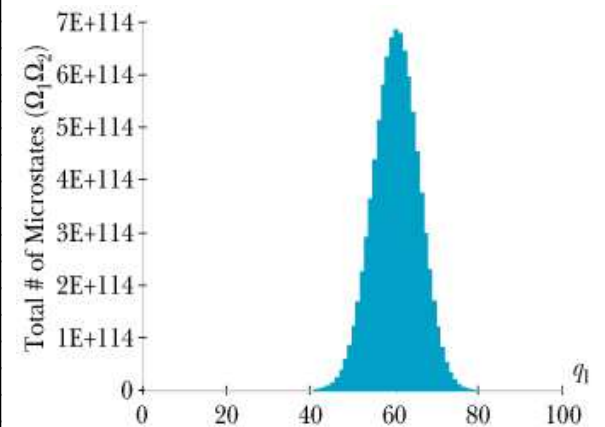
Approaching Equilibrium



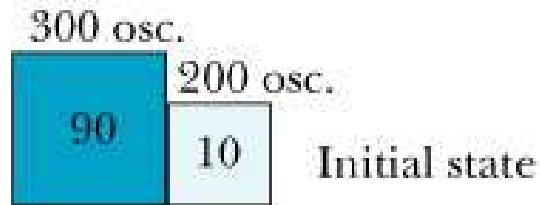
Each square represents $\approx 10^{112}$ microstates.

■ near peak

■ far away from peak



Entropy Increased!



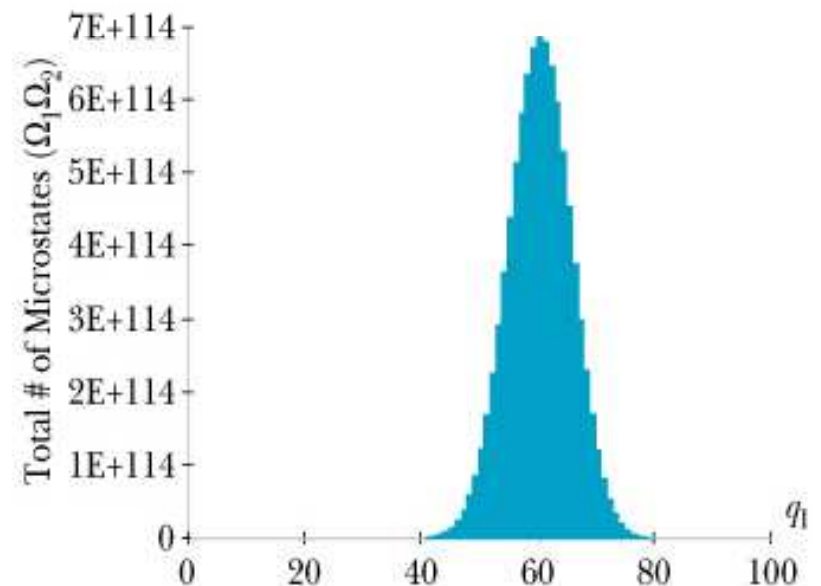
■ energy distribution A



■ energy distribution B

There are many more microstates compatible with 60/40 than with 90/10. Hence, the entropy $S = k \ln \Omega$ has increased.

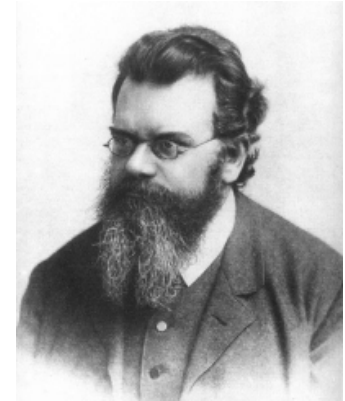
This behavior is very general . . .



Second Law of Thermodynamics

A mathematically sophisticated treatment of these ideas was developed by Boltzmann in the late 1800's.

The result of such considerations, confirmed by all experiments, is



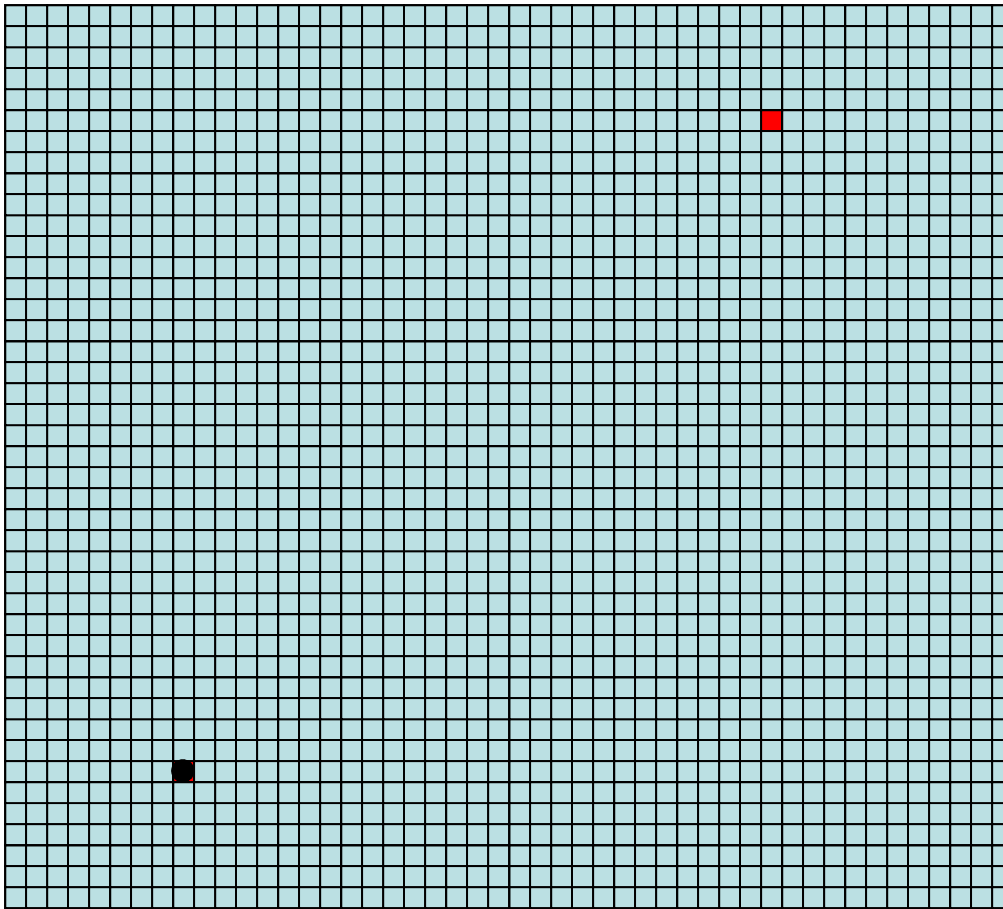
THE SECOND LAW OF THERMODYNAMICS

If a closed system is not in equilibrium, the most probable consequence is that the entropy of the system will increase.

By “most probable” we don’t mean just “slightly more probable”, but rather “so ridiculously probable that essentially no other possibility is ever observed”. Hence, even though it is a statistical statement, we call it the “2nd Law” and not the “2nd Likelihood”.

Irreversibility

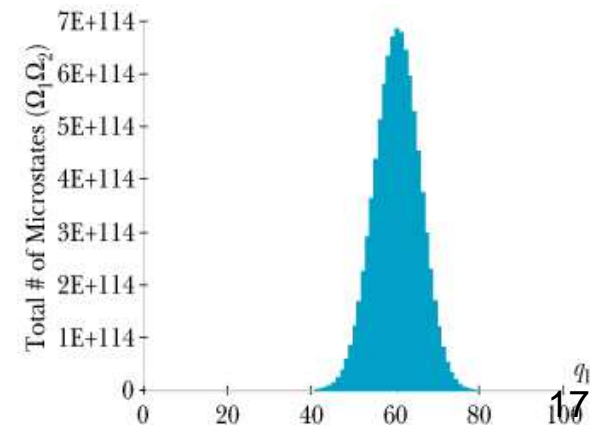
An isolated system can go from a low entropy (■) to high entropy (■), but never from high to low.



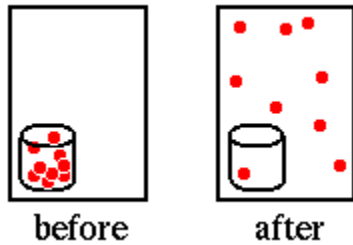
Each square represents $\approx 10^{112}$ microstates.

■ near peak
(high S)

■ far away from peak
(low S)



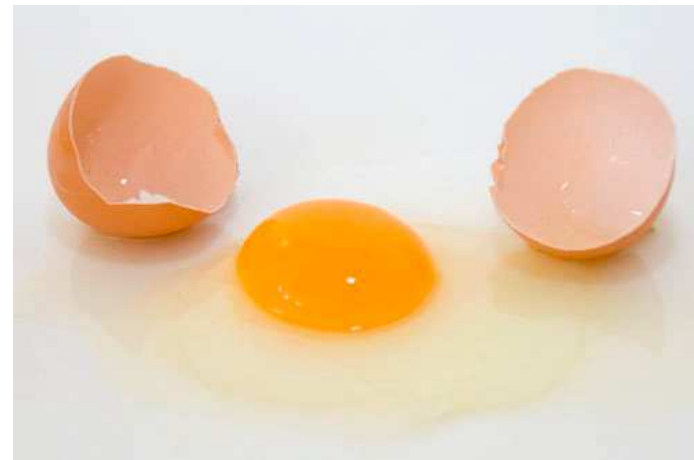
Order, Disorder, and the 2nd Law



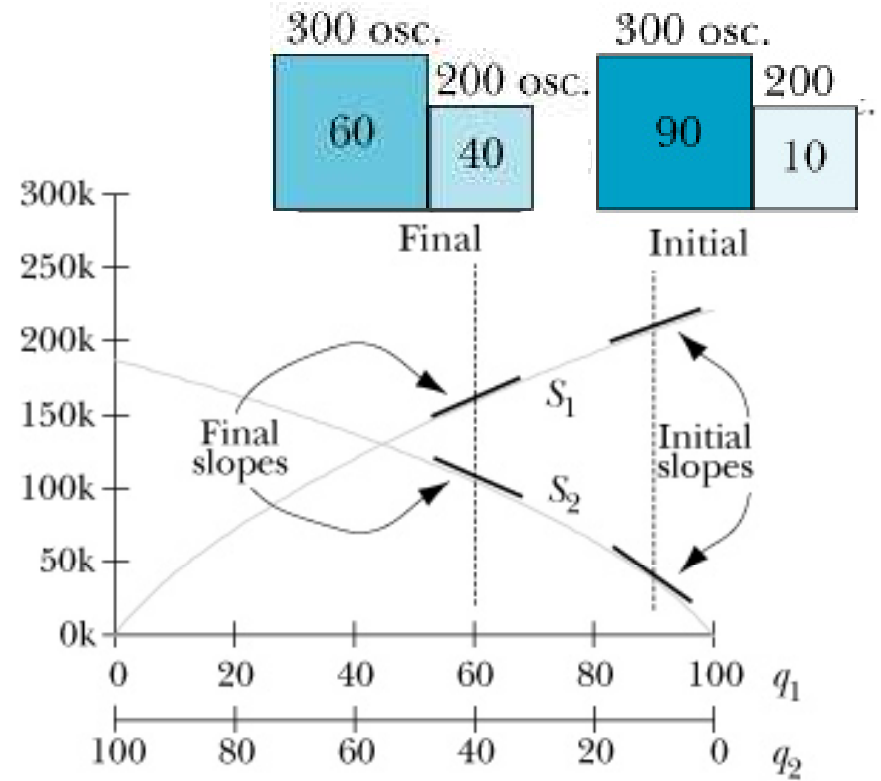
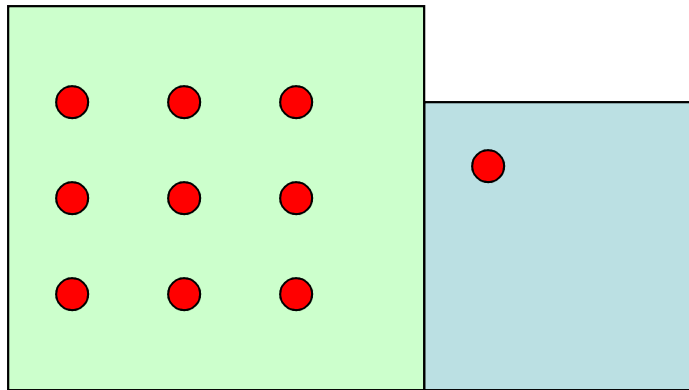
There are many more ways to distribute the perfume molecules throughout the room than there are ways to distribute them in the bottle.

The natural tendency is $S \uparrow$, meaning the perfume diffuses.

This final, higher entropy state is more “disordered” than the initial state.



Thermal transfer of energy



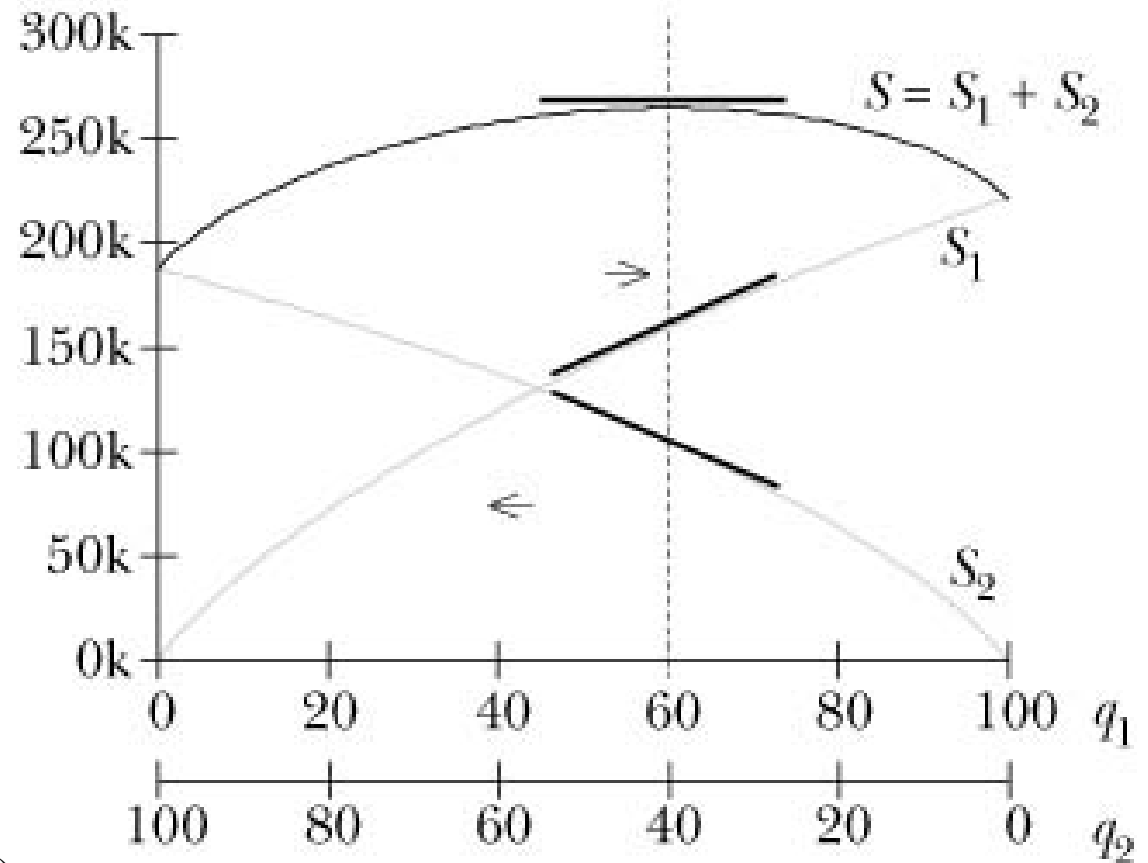
$$S = S_1 + S_2$$

Note: highest entropy S is the most likely macrostate (equilibrium). Remember 2nd Law!

$$\frac{dS}{dq_1} = \frac{dS_1}{dq_1} + \frac{dS_2}{dq_1} = 0$$

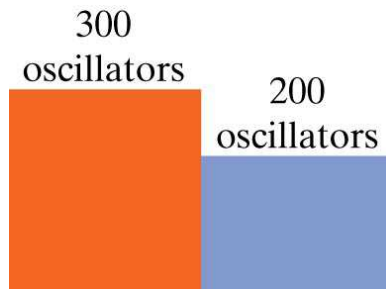
$$\frac{dS_1}{dq_1} - \frac{dS_2}{dq_2} = 0 \text{ (at equilibrium)}$$

$$\frac{dS_1}{dq_1} = \frac{dS_2}{dq_2} \text{ (at equilibrium)}$$



At equilibrium, these two slopes are equal. What else is equal here?

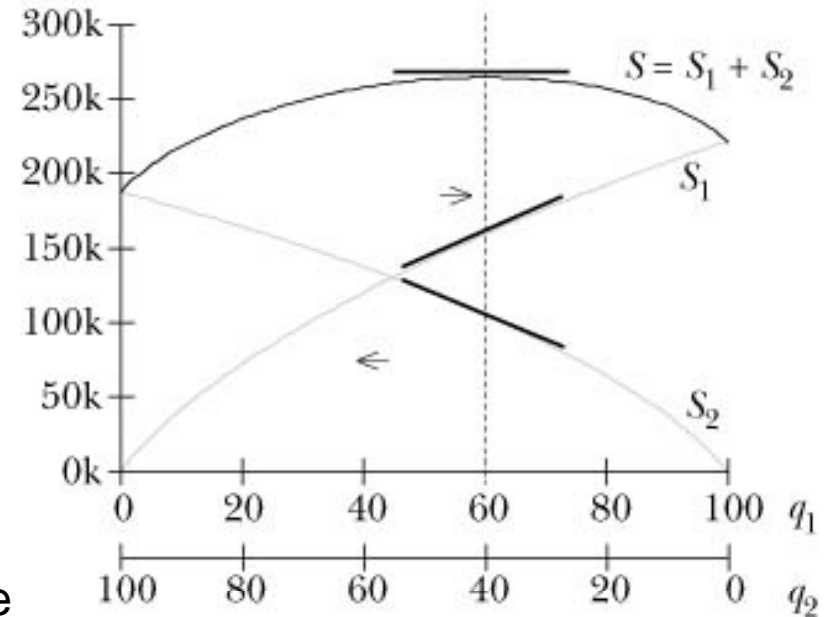
At equilibrium, these two slopes are equal. What else is equal here?



The temperature of the two objects that are in contact and have come to equilibrium.

This leads to the definition of temperature:

Temperature: $\frac{1}{T} \equiv \frac{\partial S}{\partial E_{\text{int}}}$

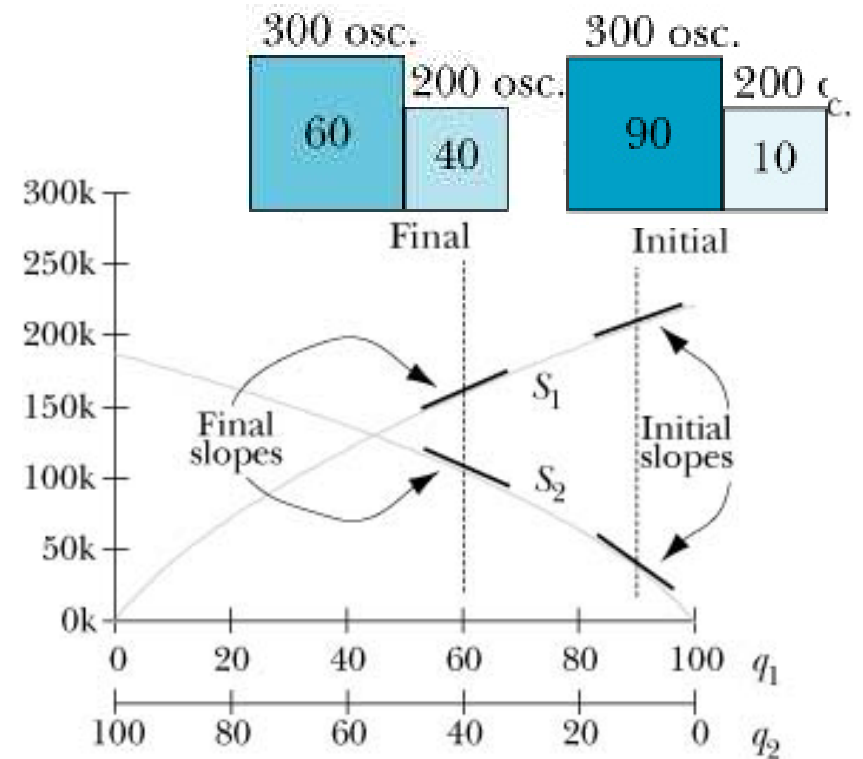


Remember: the box is hotter when the slope is smaller.

We use E_{int} instead of q_1 to be more general.

SUMMARY:

1. We learned to count the number of microstates in a certain macrostate, using the Einstein model of a solid.
2. We defined ENTROPY.
3. We observe that the 2nd Law of thermodynamics requires that interacting systems move toward their most likely macrostate (equilibrium reflects statistics!); explains irreversibility.
4. We defined TEMPERATURE to correspond to the observed behavior of systems moving into equilibrium, using entropy as a guide (plus 2nd law, statistics).



$$\text{Entropy: } S \equiv k \ln \Omega$$

$$\text{Temperature: } \frac{1}{T} \equiv \frac{\partial S}{\partial E_{\text{int}}}$$