Today

- Ammeters, Voltmeters, Ohmmeters, Oh my!
- Solving for Q(t) and I(t) in an RC circuit
- The "time constant" of an RC circuit is RC

Ammeters, Voltmeters and Ohmmeters

Ammeter: measures current I

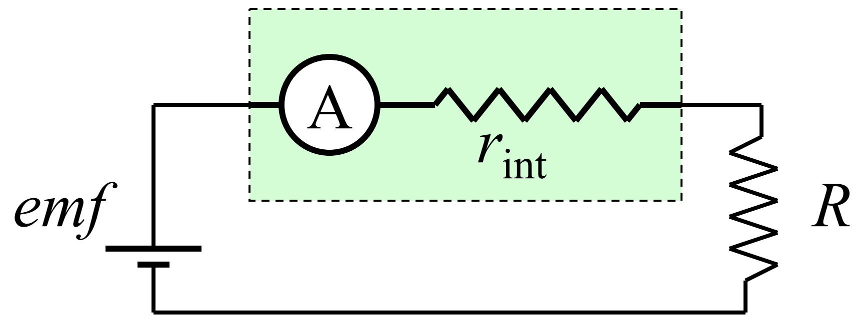
Voltmeter: measures voltage difference ΔV

Ohmmeter: measures resistance R

Ammeter Design: r_{int}

Ammeter is inserted in series into a circuit – measured current flows through it.

Process of measuring requires charges to do some work:
Internal resistance



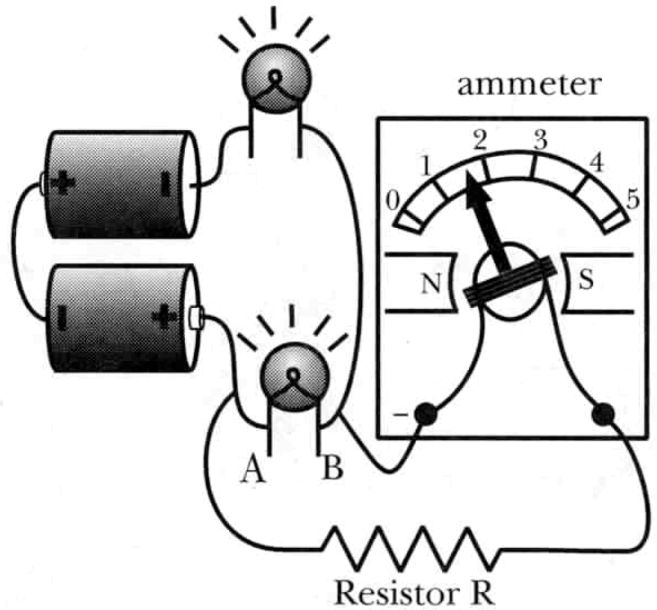
No ammeter: $emf - RI = 0 \longrightarrow I = \frac{emf}{R}$

With ammeter: $emf - r_{int}I - RI = 0 \longrightarrow I = \frac{emf}{R + r_{int}}$

Internal resistance of an ammeter must be very small

Voltmeter

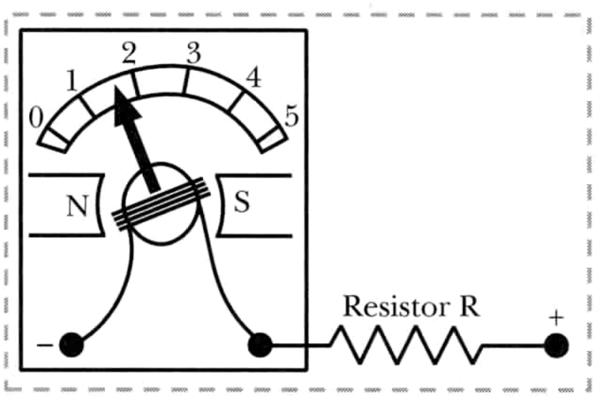
Voltmeters measure potential difference



ΔV_{AB} – add a series resistor to ammeter

$$\boxed{I = \frac{\Delta V}{R}}$$

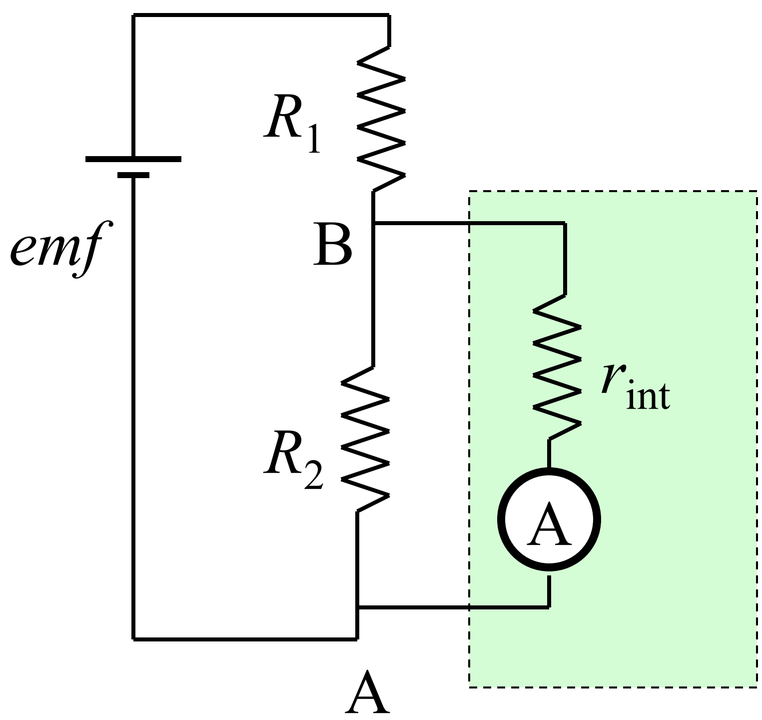
Measure I and convert to $\Delta V_{AB}=IR$



Connecting Voltmeter:

Higher potential must be connected to the ‘+’ socket and lower one to the ‘-’ socket to result in positive reading.

Voltmeter: Internal Resistance



ΔV_{AB} in absence of a voltmeter

$$\Delta V_{AB} = \frac{R_2}{R_1 + R_2} emf$$

ΔV_{AB} in presence of a voltmeter

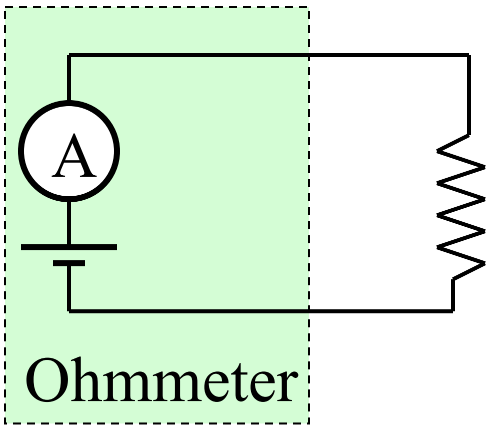
$$\Delta V_{AB} = \frac{R_{2\|\text{int}}}{R_1 + R_{2\|\text{int}}} emf$$

$$R_{2\|\text{int}} = \frac{R_2 r_{\text{int}}}{R_2 + r_{\text{int}}}$$

Internal resistance of a voltmeter must be very large

Ohmmeter

How would you measure R?



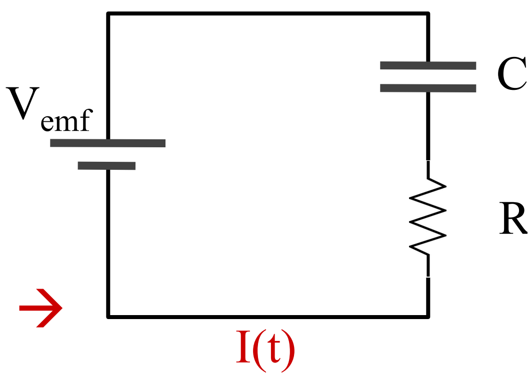
$$R = \frac{emf}{I}$$

$$\boxed{I = \frac{emf}{R}}$$

Ammeter with a small voltage source

GOAL: Find Q(t), I(t) in RC circuit

$$Q = CV$$



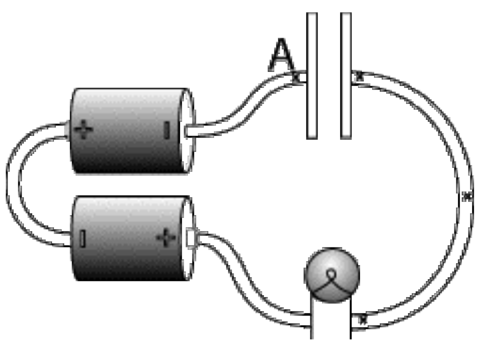
← Q is at the capacitor

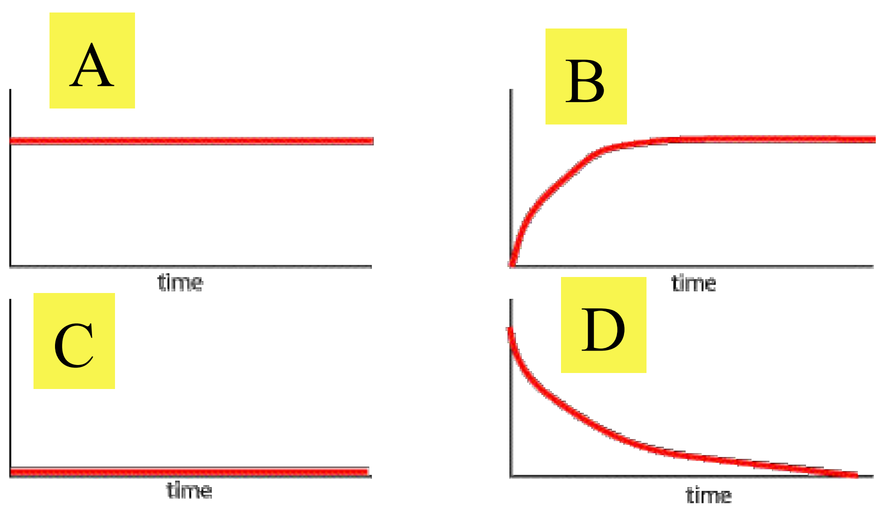
I is through all
circuit elements →

First: What do we expect?

Note: we use V_{emf}. Book calls it "emf".

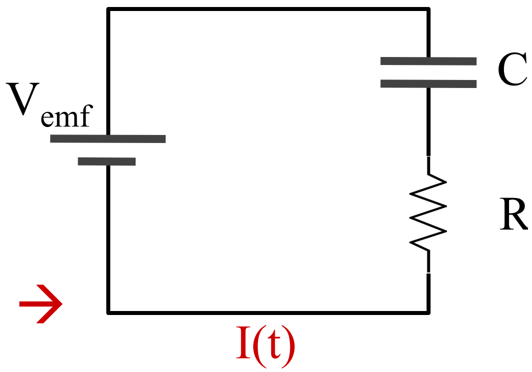
Capacitor initially uncharged. Which graph shows the magnitude of the
POTENTIAL DIFFERENCE across the LIGHT BULB FILAMENT while CHARGING?





GOAL: Find Q(t), I(t) in RC circuit

$$\mathbf{Q = CV}$$



← Q is at the capacitor

I is through all circuit elements →

First: What do we expect?

Just after we connect the circuit:

$Q = 0$

$V_{emf} = I\,R$

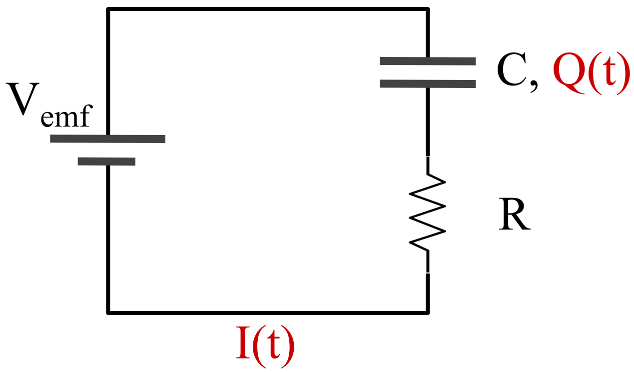
A long time after we connect it:

$Q = C\,V_{emf}$

$I = 0$

GOAL: Find Q(t), I(t) in RC circuit

Q = CV



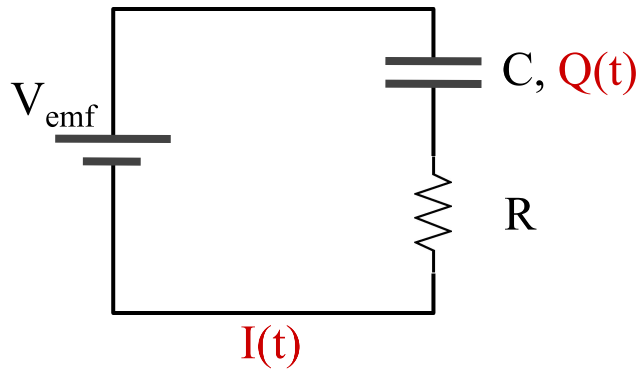
There's one more concept we need:

$$I = \frac{dQ}{dt}$$

Let's see how...

How are Q(t) and I(t) related?

Q = CV



In English: Current I = |q| n A v is:
How much charge (ΔQ) passes by per unit time (Δt).

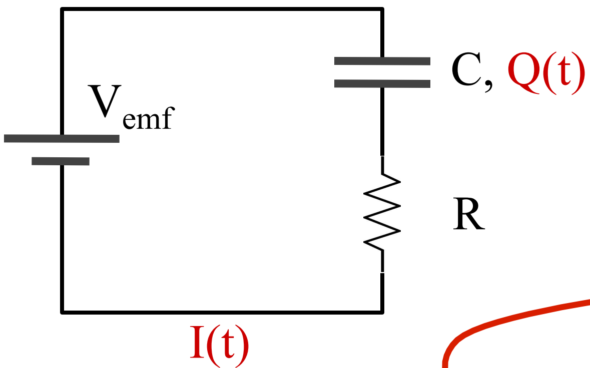
In Math:

$$I = \frac{\Delta Q}{\Delta t} \qquad lim\ \Delta t \to 0 \ \Rightarrow \boxed{I = \frac{dQ}{dt}}$$

Charge ΔQ per time Δt moves throughout the circuit,
but it **piles up** at C.
This is the same Q that is gathering on the capacitor.

GOAL: Find Q(t), I(t) in RC circuit

$Q = CV$ $\qquad I = \dfrac{dQ}{dt}$ $\qquad V = IR$



$$V_{\rm emf} = \frac{Q}{C} + IR$$ Apply Voltage Loop Rule

$$\boxed{V_{\rm emf} = \frac{Q}{C} + \frac{dQ}{dt}R}$$ Use $I = \dfrac{dQ}{dt}$

Solve this Differential Equation for Q(t).

TIP: How do you "solve" a differential equation? By already knowing the answer!

We have:

$$\frac{dQ(t)}{dt} = \frac{V_{\rm emf}}{R} - \frac{1}{RC}Q(t)$$

And the solution is:

$$\boxed{Q(t) = Ae^{-t/RC} + \text{constant}}$$

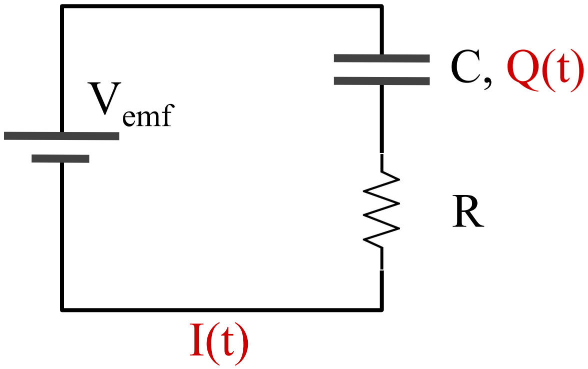
to be determined

GOAL: Find Q(t), I(t) in RC circuit

$Q = CV$ $\quad I = \dfrac{dQ}{dt}$ $\quad V = IR$

$$Q(t) = Ae^{-t/RC} + \text{constant}$$

$$\frac{dQ(t)}{dt} = \frac{V_{\text{emf}}}{R} - \frac{1}{RC}Q(t)$$



Verify solution:

Use this

$$\frac{dQ}{dt} = -\frac{1}{RC}Ae^{-t/RC} = -\frac{1}{RC}\Big(Q(t) - \text{constant}\Big)$$

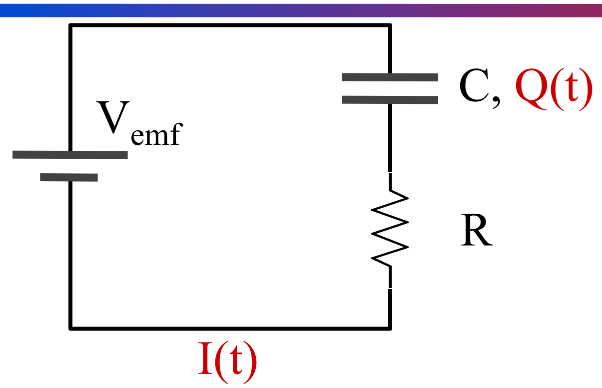
$$= \frac{1}{RC}\text{constant} - \frac{Q(t)}{RC}$$

GOAL: Find Q(t), I(t) in RC circuit

Q = CV $I = \dfrac{dQ}{dt}$ V = IR

$$Q(t) = Ae^{-t/RC} + \text{constant}$$

$$\frac{dQ(t)}{dt} = \frac{V_{\text{emf}}}{R} - \frac{1}{RC}Q(t)$$ Compare to this



Verify solution:

$$\frac{dQ}{dt} = -\frac{1}{RC}Ae^{-t/RC} = -\frac{1}{RC}\Big(Q(t) - \text{constant}\Big)$$

$$= \frac{1}{RC}\text{constant} - \frac{Q(t)}{RC}$$

$$\Rightarrow \boxed{\text{constant} = V_{\text{emf}}C}$$

"Boundary conditions"
(*i.e.* use physics – think about the extremes)

At t=0: $Q(t \to 0) = Ae^{o} + V_{\text{emf}}C = A + V_{\text{emf}}C = 0$

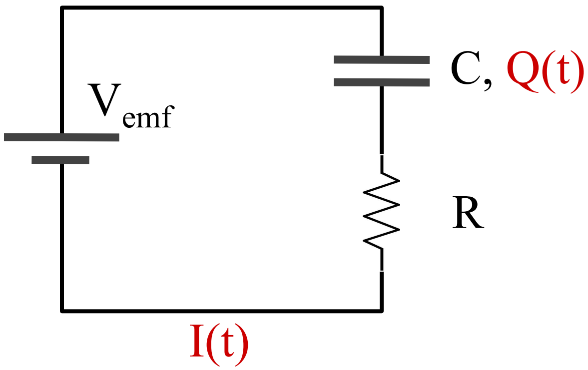
$$\Rightarrow \boxed{A = -V_{\text{emf}}C}$$

GOAL: Find Q(t), I(t) in RC circuit

$Q = CV$ $\quad I = \frac{dQ}{dt}$ $\quad V = IR$

$$Q(t) = Ae^{-t/RC} + \text{constant}$$

$$\frac{dQ(t)}{dt} = \frac{V_{\text{emf}}}{R} - \frac{1}{RC}Q(t)$$

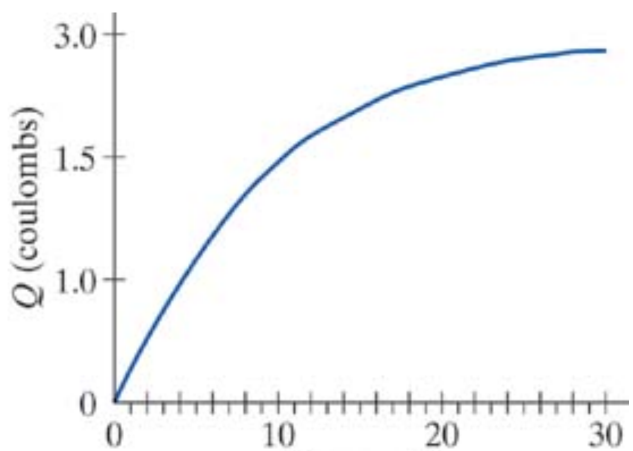


$$\text{constant} = V_{\text{emf}}C \qquad A = -V_{\text{emf}}C$$

$$Q(t) = -V_{\text{emf}}Ce^{-t/RC} + V_{\text{emf}}C$$

$$\boxed{Q(t) = V_{\text{emf}}C\left[1 - e^{-t/RC}\right]}$$

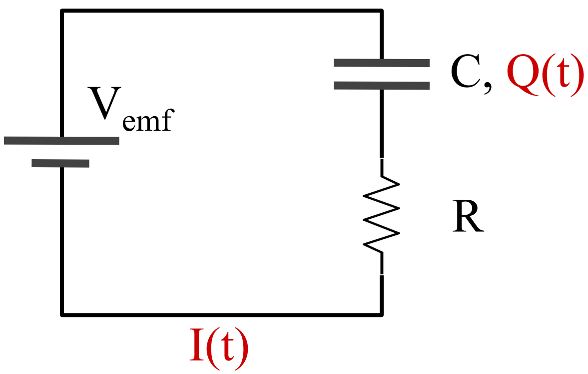
Doublecheck:
Is it what we expected?



✔ Yes, it is.

GOAL: Find Q(t), I(t) in RC circuit

$Q = CV \qquad I = \dfrac{dQ}{dt} \qquad V = IR \qquad Q(t) = V_{\mathrm{emf}} C \left[1 - e^{-t/RC} \right]$



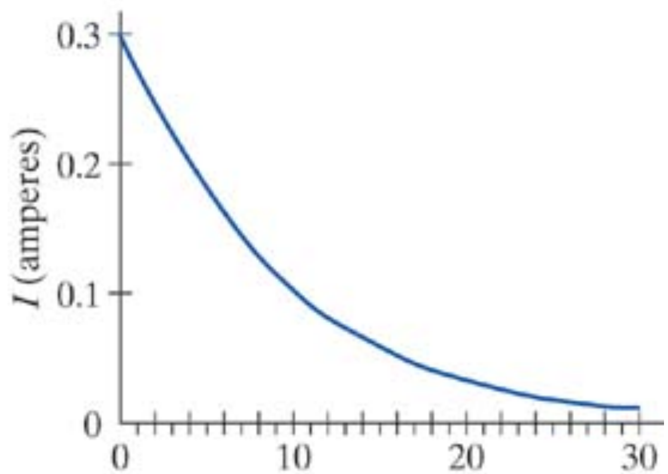
Now find I(t):

$$I(t) = \frac{dQ}{dt} = -V_{\mathrm{emf}} C \left[-\frac{1}{RC} e^{-t/RC} \right]$$

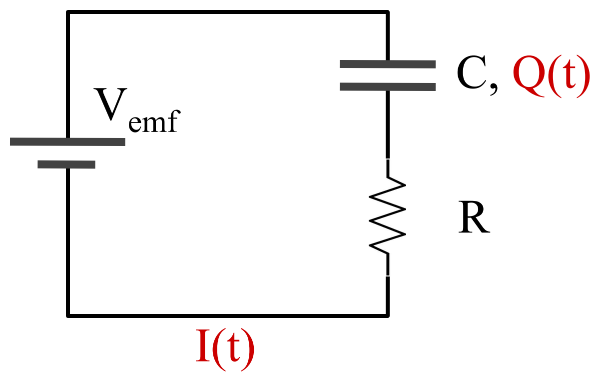
$$\boxed{I(t) = \frac{V_{\mathrm{emf}}}{R} e^{-t/RC}}$$

Doublecheck:
Is it what we expected?

✔ Yes, it is.



RC Circuit: Summary

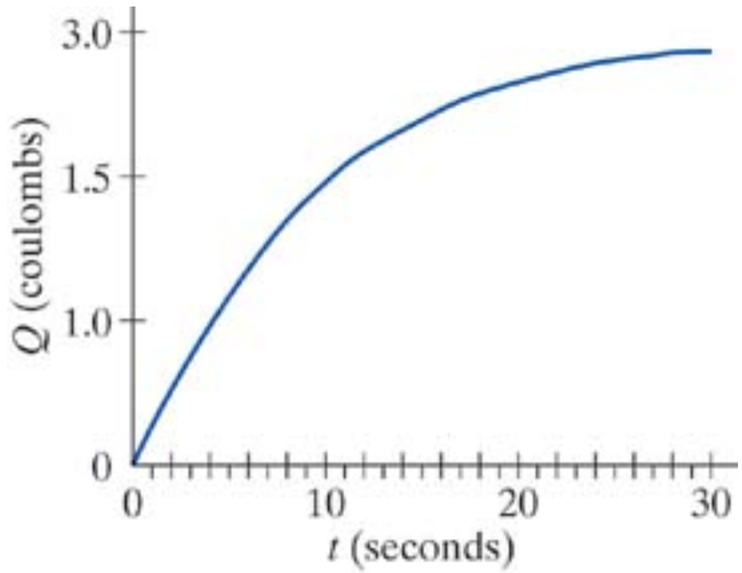


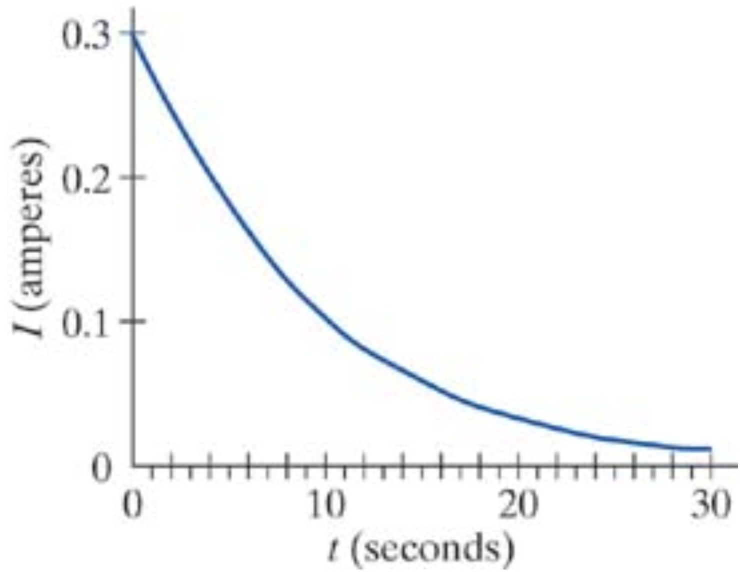
Charge in an RC circuit

$$Q = C(emf)\left[1 - e^{-t/RC}\right]$$

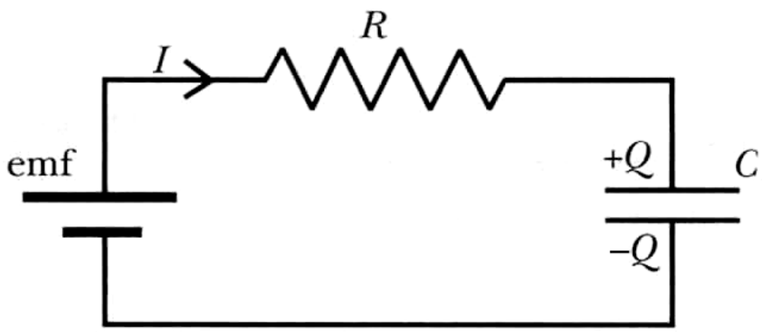
Current in an RC circuit

$$I = \frac{emf}{R} e^{-t/RC}$$





The RC Time Constant



Current in an RC circuit

$$I = \frac{emf}{R} e^{-t/RC}$$

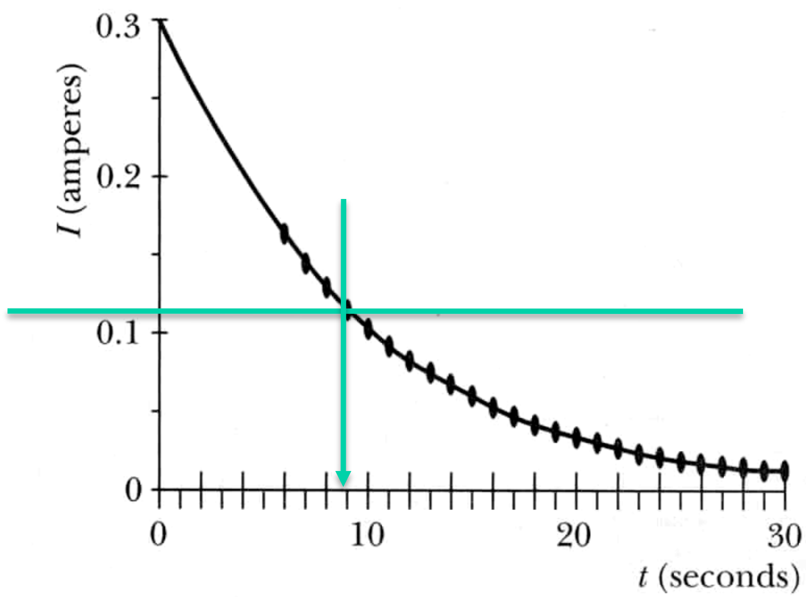
When time $t = RC$, the current I drops by a factor of e.

RC is the 'time constant' of an RC circuit.

$$e^{-t/RC} = e^{-1} = \frac{1}{2.718} = 0.37$$

A rough measurement of how long it takes to reach final equilibrium

What is the value of RC?



About 9 seconds

Today

- Ammeters, Voltmeters, Ohmmeters, Oh my!
- Solving for Q(t) and I(t) in an RC circuit
- The "time constant" of an RC circuit is RC