Last Time

- Insulators: Electrons stay close to their own atoms
- Conductors: Charges are free to move
  - $E = 0$ inside conductor in equilibrium
  - Ionic solutions
  - Metals
- Charging and Discharging Objects
  - Why humidity matters!

Today

- Charge Density
- Electric Field of a Charge Distribution
- Electric Field of a Charged Rod

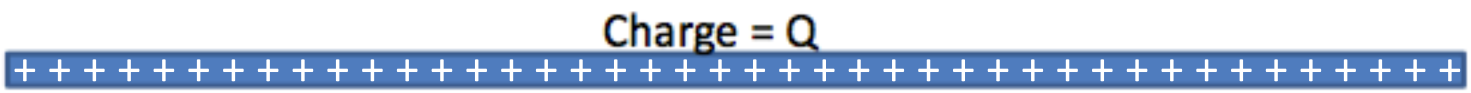
Superposition

- Recall that the net electric field is the sum of fields from individual objects
  - So: $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$
- A charged object of any shape can be thought of as a collection of point charges

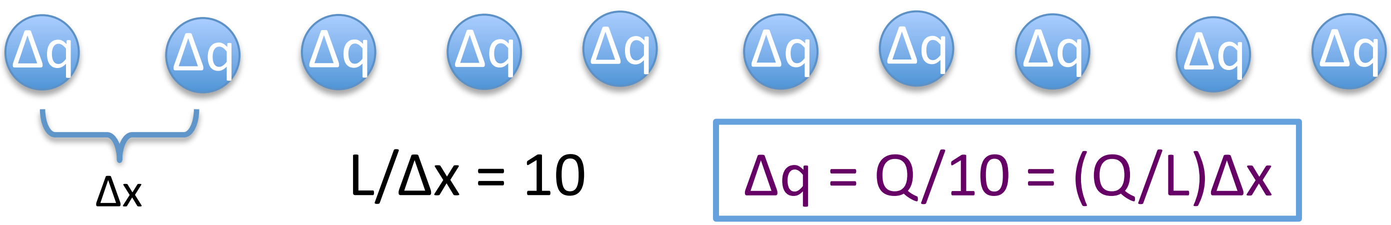
- So add up (integrate) the point charges to find field

Charge density

- Start simple with a 1d charge density.
- Rod has a charge Q, length L (in meters)



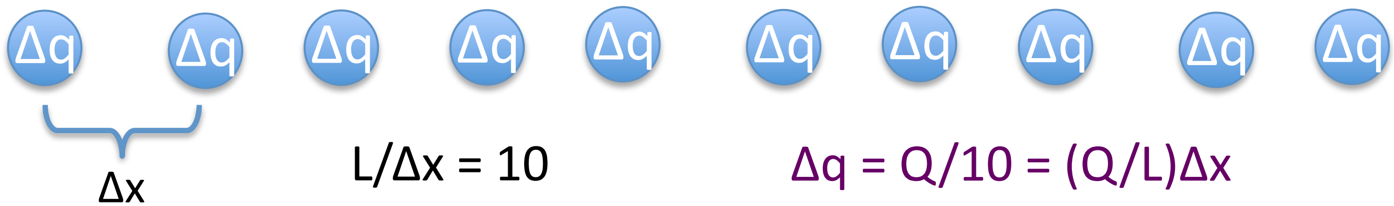
- Suppose we approximate it as 10 point charges in a row. How much charge, Δq,does each have?



$L/\Delta x = 10$

$\boxed{\Delta q = Q/10 = (Q/L)\Delta x}$

Calculus We Will Need



$L/\Delta x = 10$

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Recall how to convert a sum to an integral:

UNITS = [Length]

$$\sum \Delta x \rightarrow \int dx$$

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We will need to sum over all charges:

$$\boxed{\sum \Delta q = \frac{Q}{L} \sum \Delta x \rightarrow \frac{Q}{L} \int dx}$$

UNITS = [Charge]

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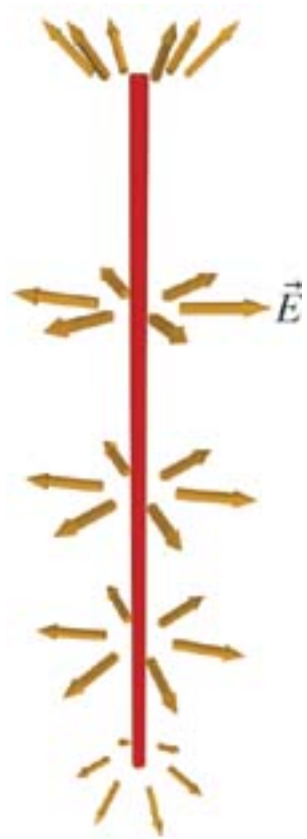
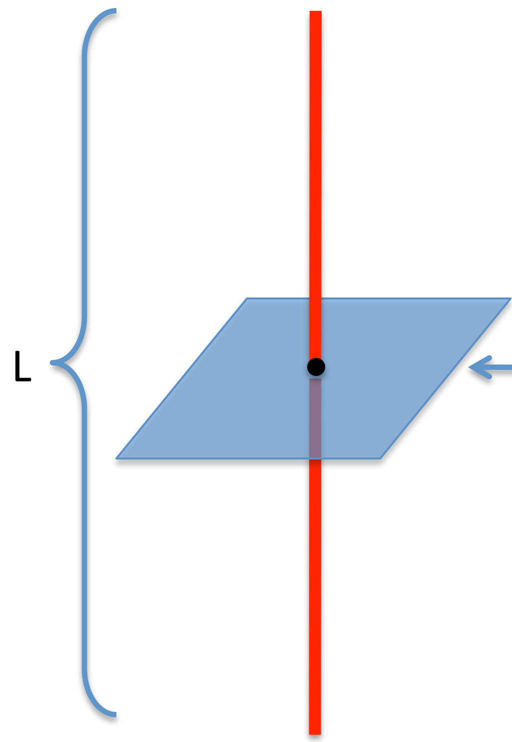
iClicker Question

iClicker Question

Finding the Rod's Electric Field in the Bisecting Plane

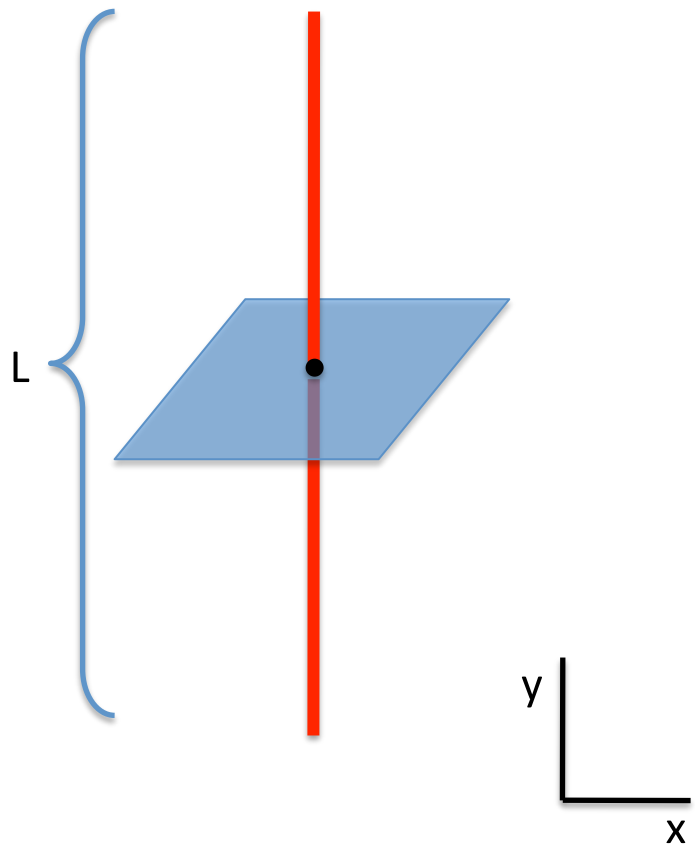


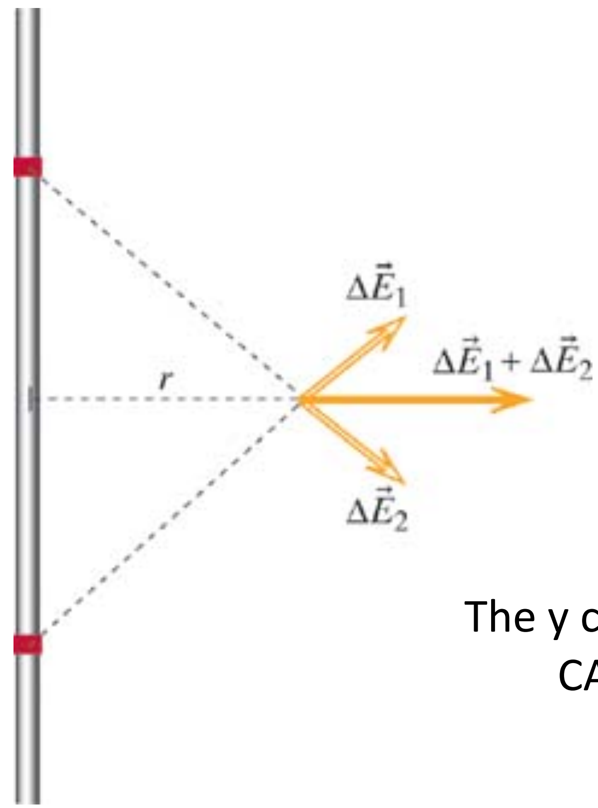


We will calculate the Electric Field only on this plane, which bisects the rod.

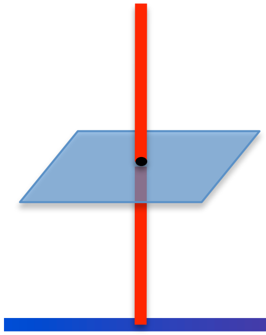
Reading Hint:
All of Section 16.2 pertains to the bisecting plane.

Finding the Rod's Electric Field in the Bisecting Plane



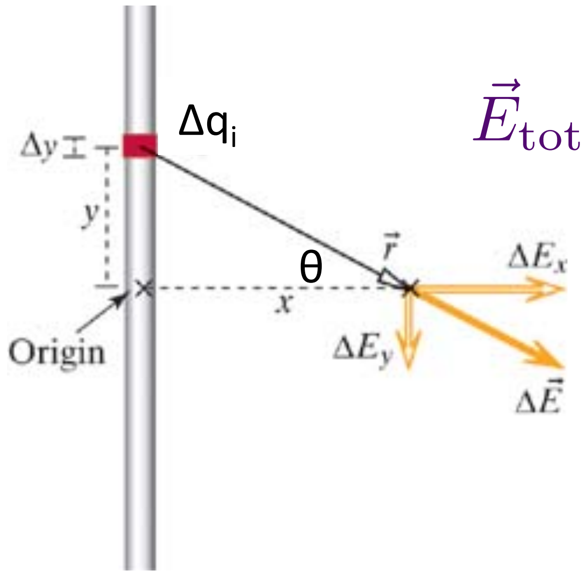


The y components CANCEL!



Electric Field on the Bisecting Plane

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}$$ for a point charge



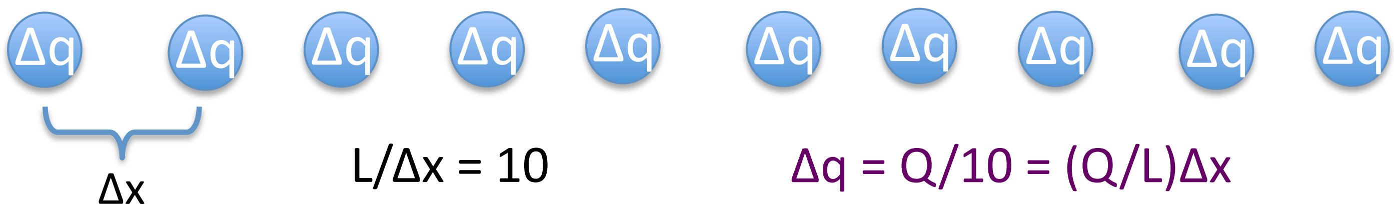
$$\vec{E}_{\rm tot} = \sum_i \vec{E}_i = \sum_i E_{i,x}\hat{x}$$ We only need the x component

$$E_{i,x} = |E_i| cos(\theta)$$

$$cos(\theta) = \frac{x}{r} = \frac{x}{(x^2+y^2)^{1/2}}$$

$$\vec{E}_{\rm tot} = \frac{1}{4\pi\epsilon_o}\sum_i \frac{\Delta q_i}{r^2}\frac{x}{(x^2+y^2)^{1/2}}\hat{x}$$

Calculus We Now Need



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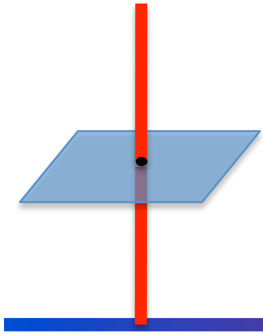

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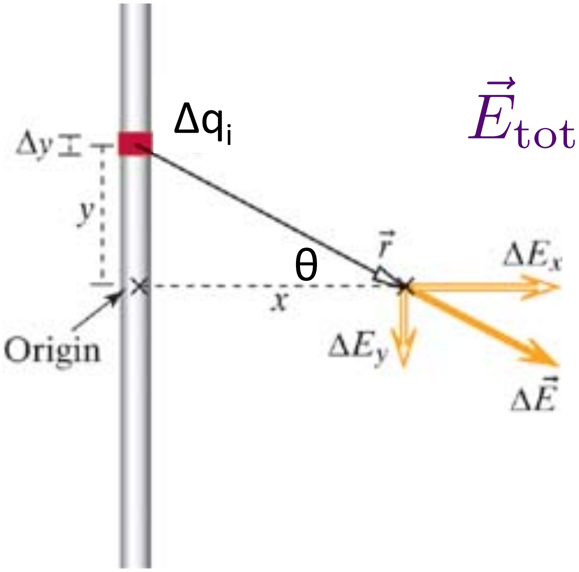

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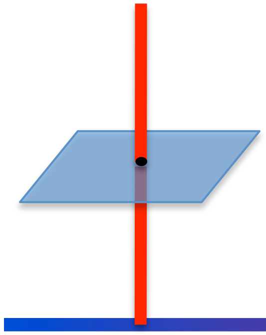
$$\vec{E}_{\text{tot}} = \frac{1}{4\pi\epsilon_o}\sum_i \frac{\Delta q_i}{r^2}\frac{x}{(x^2+y^2)^{1/2}}\hat{x} \rightarrow \frac{1}{4\pi\epsilon_o}\frac{Q}{L}\int \frac{dy}{r^2}\frac{x}{(x^2+y^2)^{1/2}}\hat{x}$$

CONVERT SUM $\sum \Delta q = \frac{Q}{L}\sum \Delta y \rightarrow \frac{Q}{L}\int dy$ INTO INTEGRAL



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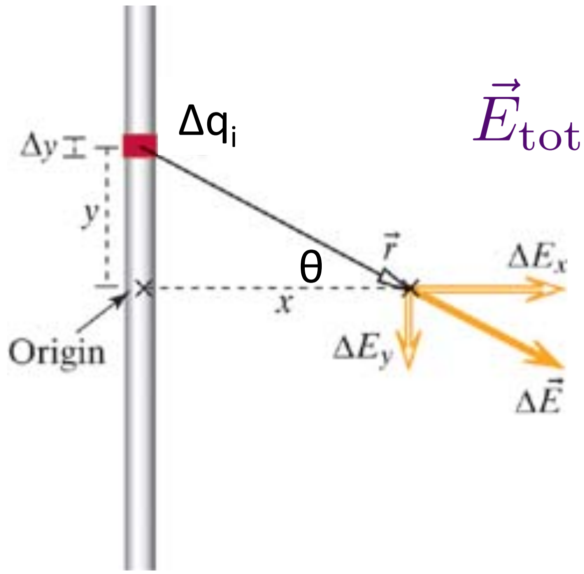

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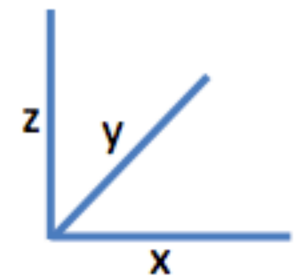
$$= \frac{1}{4\pi\epsilon_o}\frac{Qx}{L}\int_{-L/2}^{L/2}\frac{dy}{(x^2+y^2)^{3/2}}\hat{x} = \frac{1}{4\pi\epsilon_o}\frac{Q}{x\sqrt{x^2+(L/2)^2}}\hat{x}$$

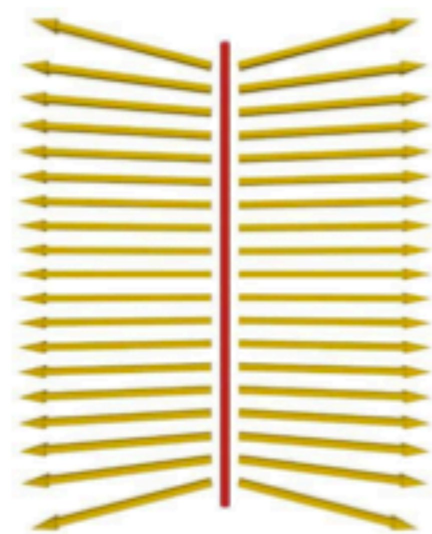
iClicker question

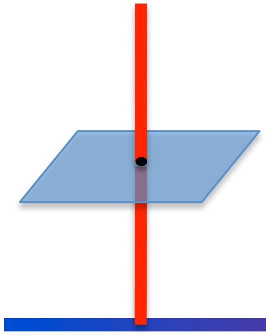
Symmetry of an infinite rod

- A rod has circular symmetry in the xy plane
- Also, an infinitely long rod has no distinction between different z values



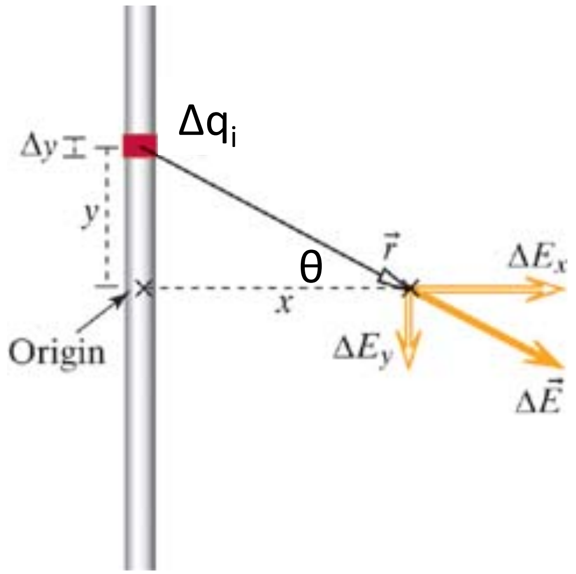
- Brain twister: An infinitely long rod s gives the same field that a point charge would give in a universe with only two dimensions!





Electric Field of an Infinite Rod

$$\vec{E}_{\text{tot}} = \frac{1}{4\pi\epsilon_o}\frac{Qx}{L}\int_{-L/2}^{L/2}\frac{dy}{(x^2+y^2)^{3/2}}\hat{x} = \frac{1}{4\pi\epsilon_o}\frac{Q}{x\sqrt{x^2+(L/2)^2}}\hat{x}$$



For an infinite rod,

$$L \to \infty$$

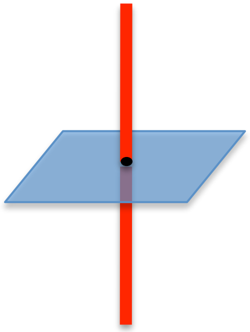
$$\vec{E}_{\text{tot}} = \frac{1}{4\pi\epsilon_o}\frac{Qx}{L}\int_{-\infty}^{\infty}\frac{dy}{(x^2+y^2)^{3/2}}\hat{x}$$

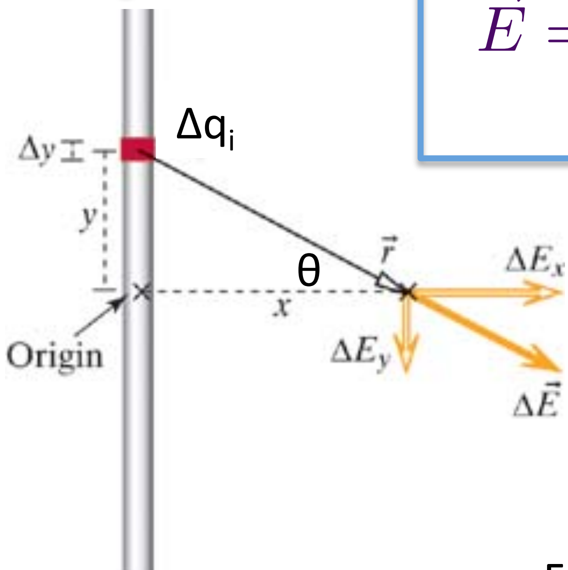
$$= \frac{1}{4\pi\epsilon_o}\frac{2(Q/L)}{x}\hat{x} \qquad = \frac{1}{4\pi\epsilon_o}\frac{2(\lambda)}{x}\hat{x}$$

What does Q/L mean for an infinite rod?

We can only define λ = charge per unit length

The Bottom Line: Charged Rods





$$\vec{E} = \frac{1}{4\pi\epsilon_o} \frac{Q}{x\sqrt{x^2 + (L/2)^2}} \hat{x}$$

FINITE ROD
Only on the
Bisecting Plane

For an infinite rod, $L \to \infty$

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \frac{2(Q/L)}{x} \hat{x}$$

INFINITE ROD
Everywhere in space!

Remember Q/L is not infinite – it's a linear charge density

General Procedure for Calculating Electric Field of Distributed Charges

1. Cut the charge distribution into pieces for which the field is known

2. Write an expression for the electric field due to one piece
   (i) Choose origin
   (ii) Write an expression for ΔE and its components

3. Add up the contributions of all the pieces
   (i) Try to integrate symbolically
   (ii) If impossible – integrate numerically

4. Check the results:
   (i) Direction
   (ii) Units
   (iii) Special cases

Today

- Charge Density
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