

# Announcements

---

EXAM 2 is Wednesday, Nov. 7

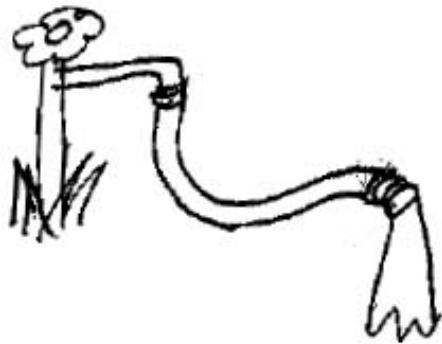
@ 8:00pm-9:30pm in ELLIOT HALL of MUSIC.

Practice Exam is on Blackboard Learn.

# Hall Effect gives sign of charge carriers

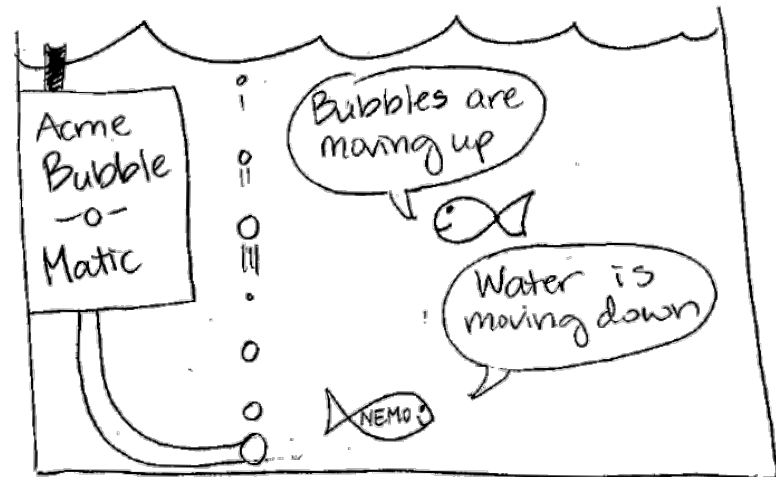
In some metals and semiconductors,  
current is *not* carried by electrons,  
but is carried by "*holes*" = bubbles in the electron sea.

## Water from a hose



“Holes” not useful here

## Bubbles in water



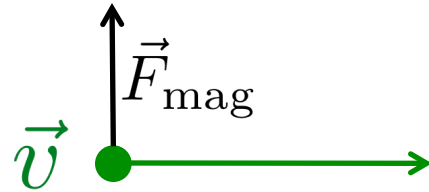
“Holes” useful here

# iClicker

$$\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B}$$

**MAGNETIC FORCE**  
**point charge**

**Proton**



$\otimes$   $B_{\text{applied}}$

In which direction does  $\vec{F}_{\text{mag}}$  point for the **proton**?

- A) Up
- B) Down
- C) Into the Board
- D) Out of the Board

# iClicker

$$\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B}$$

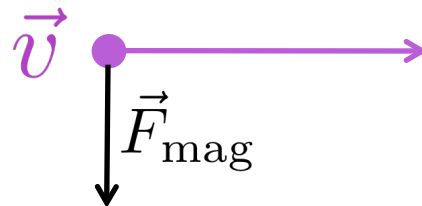
**MAGNETIC FORCE**  
**point charge**

**Proton**



$\otimes$   $B_{\text{applied}}$

**Electron**



- A) Up
- B) Down
- C) Into the Board
- D) Out of the Board

In which direction does  $\vec{F}_{\text{mag}}$  point for the **electron**?

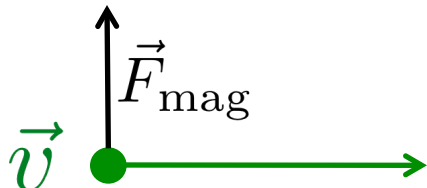
# Hall Effect

$$\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B}$$

**MAGNETIC FORCE**  
point charge

A **hole** in the electron sea  
behaves like a **proton**.

**Proton**  
or "Hole"



⊗  $B_{\text{applied}}$

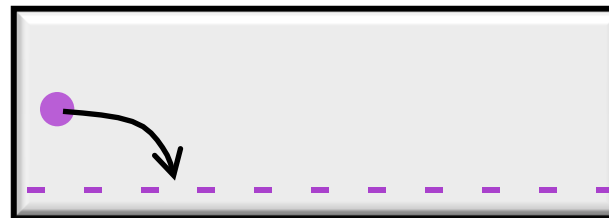
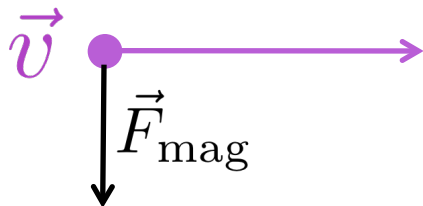
**Inside a Material:**



Moving **holes**  
get pushed  
to the top

⊗  $B_{\text{applied}}$

**Electron**



Moving **electrons**  
get pushed  
to the bottom

# Hall Effect

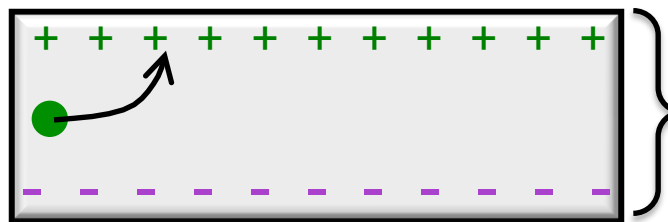
$$\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B}$$

**MAGNETIC FORCE**  
point charge

A **hole** in the electron sea  
behaves like a **proton**.

Inside a Material:

Moving **holes**  
get pushed  
to the top



$\Delta V = \text{Hall Voltage}$

$\otimes$   $B_{\text{applied}}$

How long does this go on?

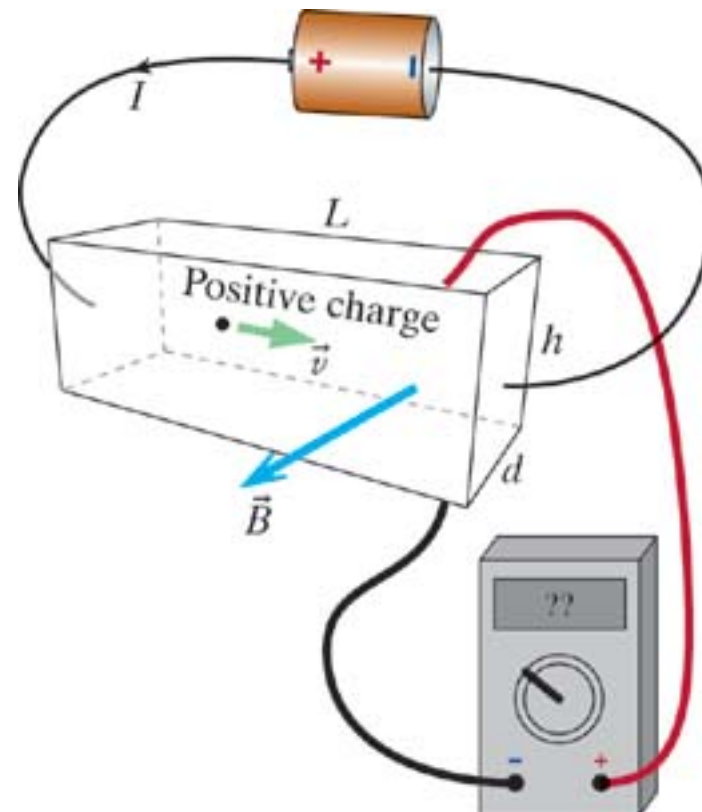
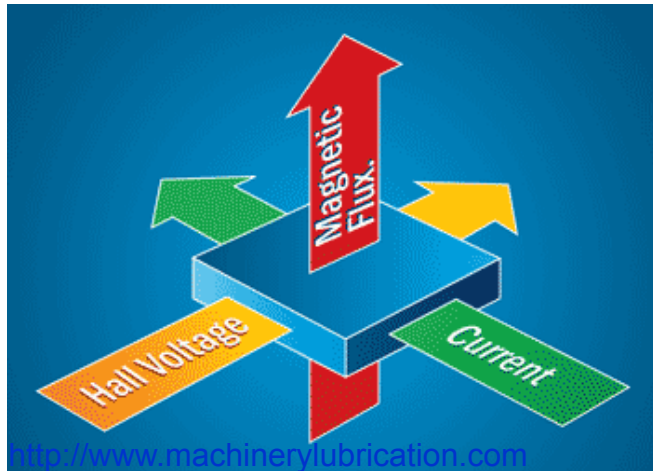
→ Until the Hall Voltage  
is strong enough to balance  
the magnetic force  $\vec{F}_{\text{mag}}$

Moving **electrons**  
get pushed  
to the bottom



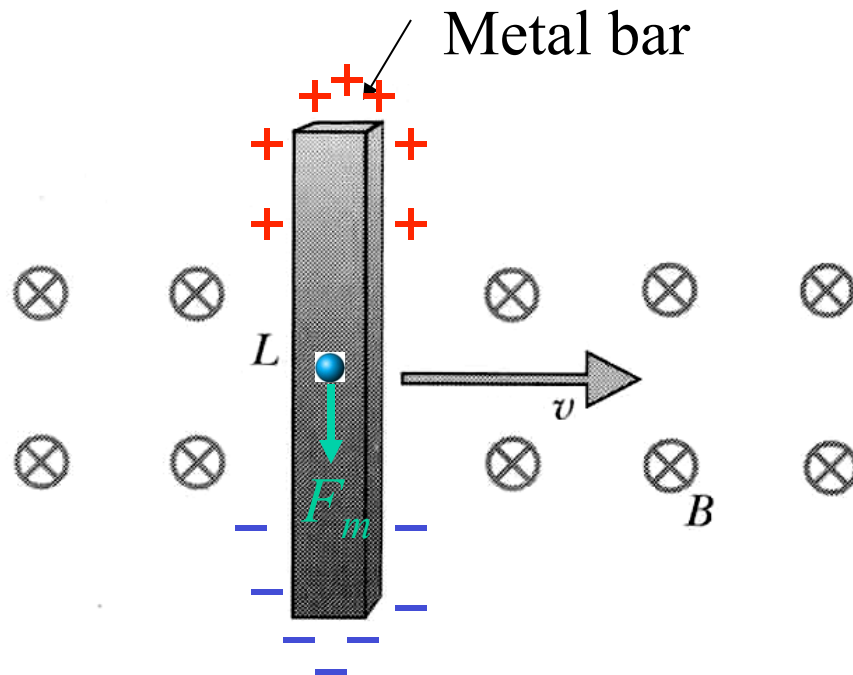
$\Delta V = \text{Hall Voltage}$

# Measuring the Hall Effect



1. Apply B-Field
2. Apply Current  $I$
3. Measure "Hall Voltage"

# Currents Due to Magnetic Forces



$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$\vec{F}_m = (-e)\vec{v} \times \vec{B}$$

↓  
polarization

How much force do we need to apply to keep the bar moving at constant speed?



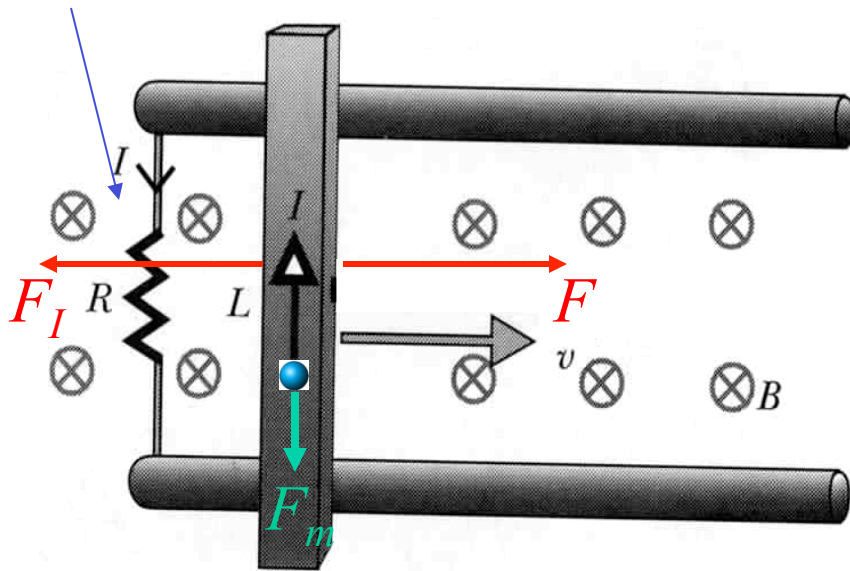
Does this polarized bar remind you anything we've already studied?



# Moving Bar and Energy Conservation

$$P = I\Delta V = I(\text{emf})$$

Are we getting something for nothing?



$$\text{emf} = vBL$$

→ x

Bar – current  $I$ :

$$\vec{F}_I = I\Delta\vec{l} \times \vec{B} = -\vec{F}$$

$$F_I = I L B$$

Work:  $W = F\Delta x = I L B \Delta x$

Power:  $P = \frac{W}{\Delta t} = I L B \frac{\Delta x}{\Delta t}$

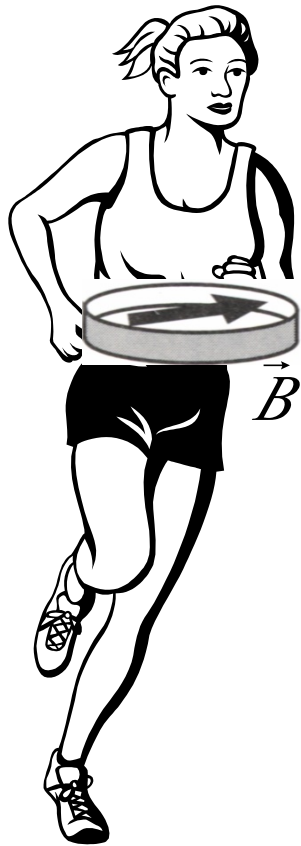
$$P = I L B v$$

$$P = I(\text{emf})$$

Main principle of electric generators:

Mechanical power is converted to electric power

# Reference Frame



$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \neq 0$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} = 0$$

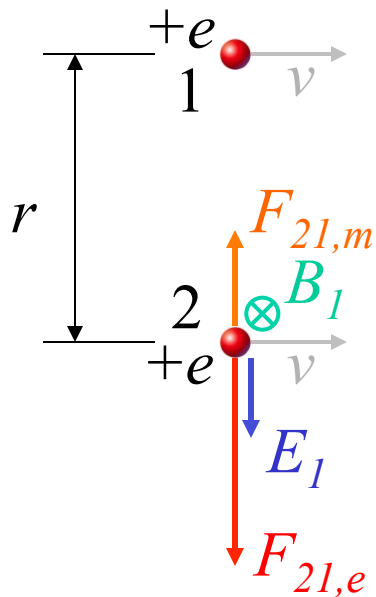
Any magnetic field?



charged tape

# Magnetic Forces in Moving Reference Frames

Two protons



Electric force:

$$\vec{F}_{21,e} = q_2 \vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \hat{r}$$

Magnetic field:

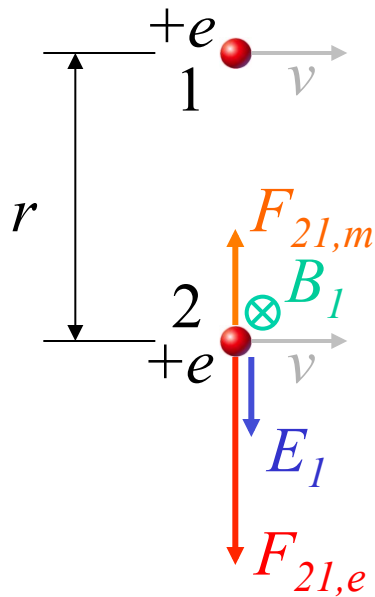
$$\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{q_1 \vec{v}_1 \times \hat{r}}{r^2}$$

Magnetic force:

$$\vec{F}_{21,m} = q_2 \vec{v}_2 \times \vec{B}_1$$

$$F_{21,m} = q_2 v B_1 = \frac{\mu_0}{4\pi} \frac{e^2 v^2}{r^2}$$

# Magnetic Forces in Moving Reference Frames



Electric force:  $F_{21,e} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$

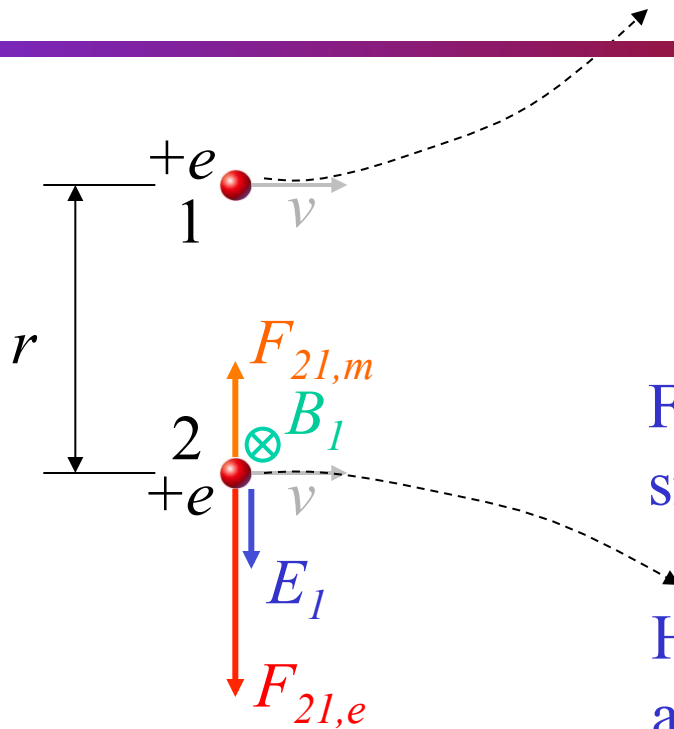
Magnetic force:  $F_{21,m} = \frac{\mu_0}{4\pi} \frac{e^2 v^2}{r^2}$

Ratio:  $\frac{F_{21,m}}{F_{21,e}} = \left( \frac{\mu_0}{4\pi} \frac{e^2 v^2}{r^2} \right) / \left( \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \right)$

$$\frac{F_{21,m}}{F_{21,e}} = (\mu_0 \epsilon_0) v^2$$

$$\frac{F_{21,m}}{F_{21,e}} = \frac{v^2}{c^2}$$

# Magnetic Forces in Moving Reference Frames



$$\frac{F_{21,m}}{F_{21,e}} = \frac{v^2}{c^2}$$

For  $v \ll c$  the magnetic force is much smaller than electric force

How can we detect the magnetic force on a current carrying wire?

Full Lorentz force:

$$F = F_{21,e} - F_{21,m} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \left( 1 - \frac{v^2}{c^2} \right)$$

downward

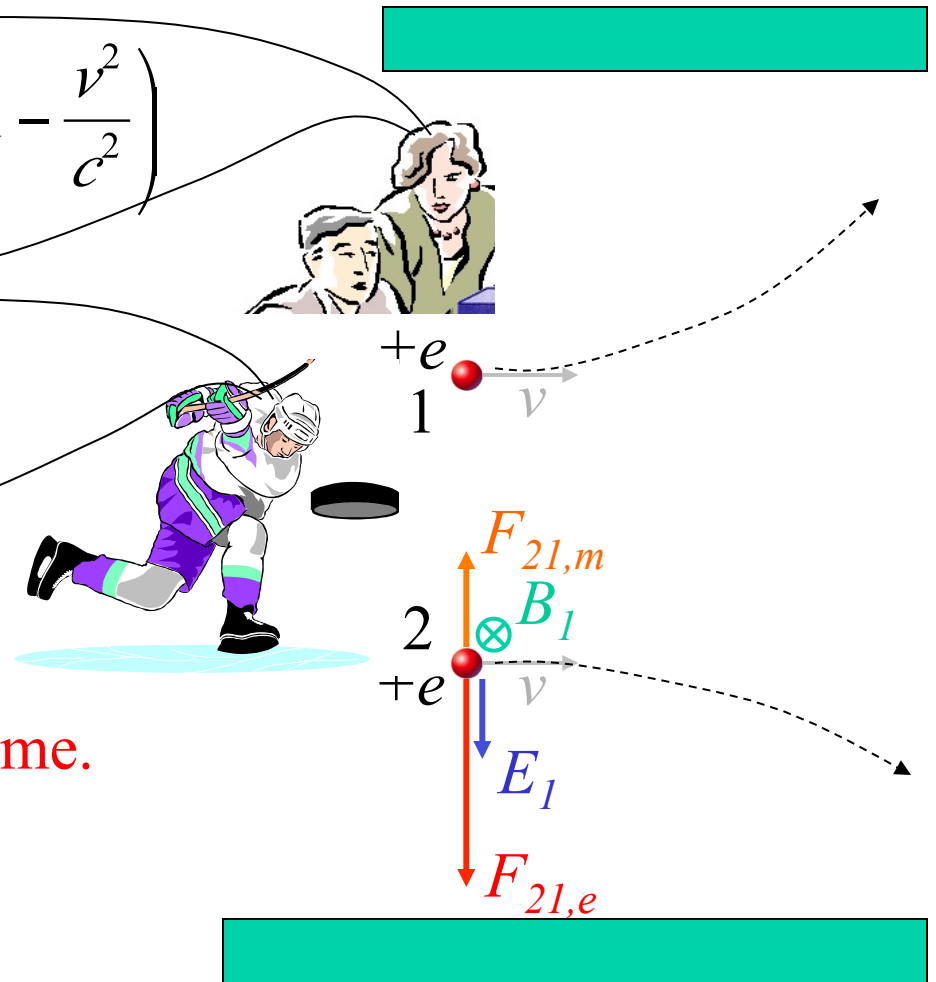
# Magnetic Forces in Moving Reference Frames

$$20 \text{ ns} \quad F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \left( 1 - \frac{v^2}{c^2} \right)$$

$$15 \text{ ns} \quad F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

Who will see protons hit floor and ceiling first?

Time must run slower in moving frame.



Einstein 1905:

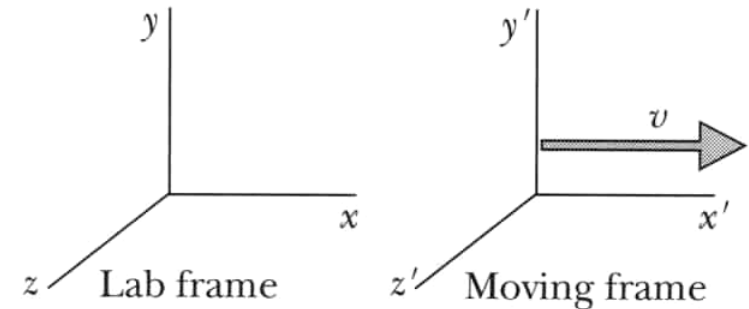
“On the electrodynamics of moving bodies”

# Relativistic Field Transformations

Our detailed derivations are not correct for relativistic speeds, but the ratio  $F_m/F_e$  is the same for any speed:

$$\frac{F_m}{F_e} = \frac{v^2}{c^2}$$

According to the theory of relativity:

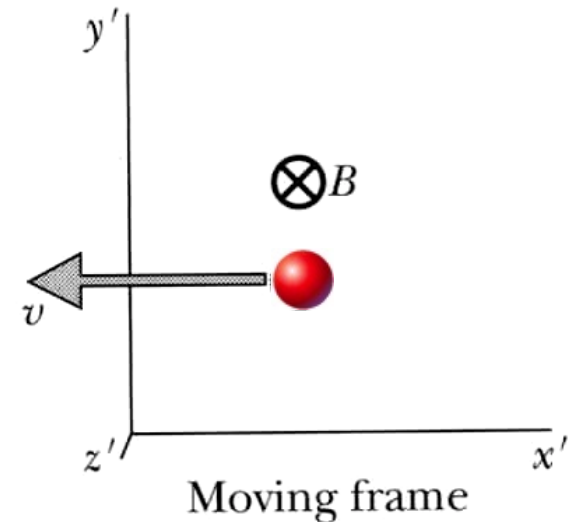


$$\begin{aligned} E'_x &= E_x & E'_y &= \frac{(E_y - vB_z)}{\sqrt{1 - v^2 / c^2}} & E'_z &= \frac{(E_z + vB_y)}{\sqrt{1 - v^2 / c^2}} \\ B'_x &= B_x & B'_y &= \frac{\left(B_y + \frac{v}{c^2} E_z\right)}{\sqrt{1 - v^2 / c^2}} & B'_z &= \frac{\left(B_z - \frac{v}{c^2} E_y\right)}{\sqrt{1 - v^2 / c^2}} \end{aligned}$$

# Magnetic Field of a Moving Particle

Still:  $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad B = 0$

Moving:  $B'_z = \frac{\left( B_z - \frac{v}{c^2} E_y \right)}{\sqrt{1 - v^2 / c^2}} = \frac{-\frac{v}{c^2} E_y}{\sqrt{1 - v^2 / c^2}}$



Slow case:  $v \ll c \rightarrow B'_z = -\frac{v}{c^2} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

$$\frac{1}{\mu_0\epsilon_0} = c^2$$

$$B'_z = -\frac{\mu_0}{4\pi} \frac{qv}{r^2}$$

Field transformation is consistent with Biot-Savart law

Electric and magnetic fields are interrelated

Magnetic fields are relativistic consequence of electric fields

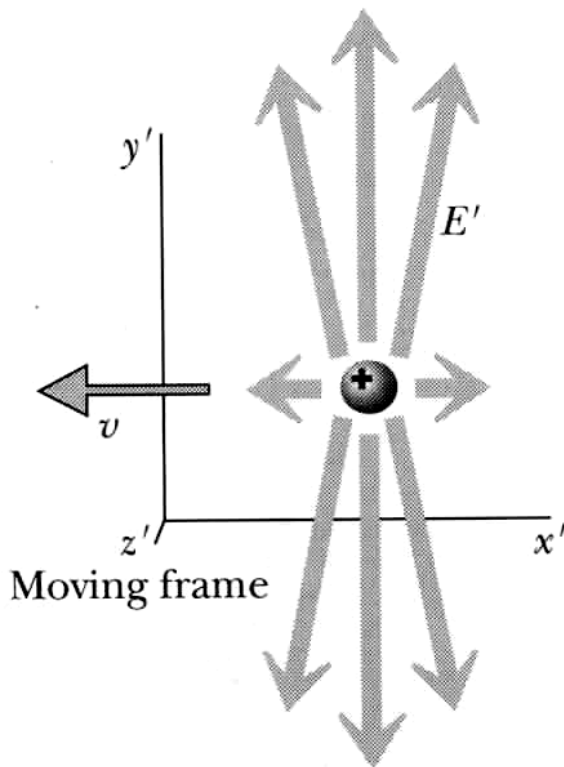


# Electric Field of a Rapidly Moving Particle

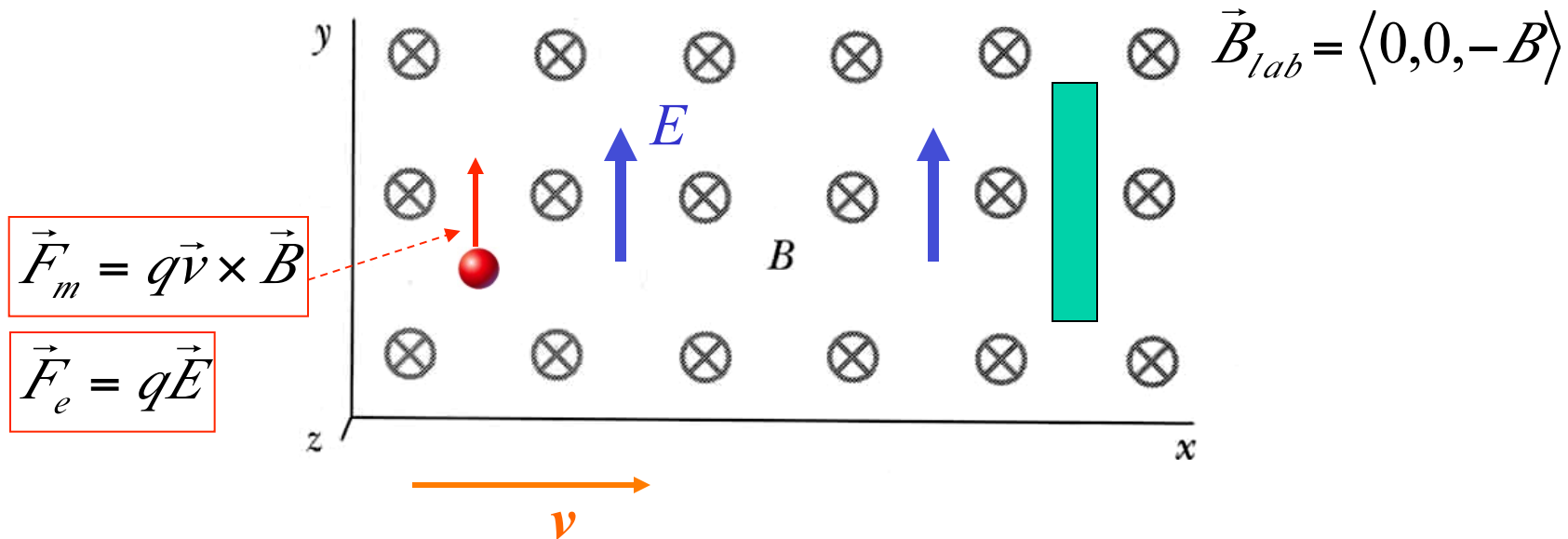
---

$$\vec{E}_x = E_x \quad \vec{E}_y = \frac{(E_y - vB_z)}{\sqrt{1 - v^2 / c^2}} \quad \vec{E}_z = \frac{(E_z + vB_y)}{\sqrt{1 - v^2 / c^2}}$$

$$\vec{E}_y = \frac{E_y}{\sqrt{1 - v^2 / c^2}} \quad \vec{E}_z = \frac{E_z}{\sqrt{1 - v^2 / c^2}}$$



# Moving Through a Uniform Magnetic Field



$$E'_x = E_x \quad E'_y = \frac{(E_y - vB_z)}{\sqrt{1 - v^2/c^2}} \quad E'_z = \frac{(E_z + vB_y)}{\sqrt{1 - v^2/c^2}}$$

$$\downarrow$$

$$E'_y = \frac{-vB_z}{\sqrt{1 - v^2/c^2}} \approx vB \quad \text{if } v \ll c$$

# The Principle of Relativity

---

E and B look different for different observers in different reference frames, but all observers can correctly predict what will happen in their own frames, using the same relativistically correct physical laws.

E and B are "unified" – two sides of the same coin.