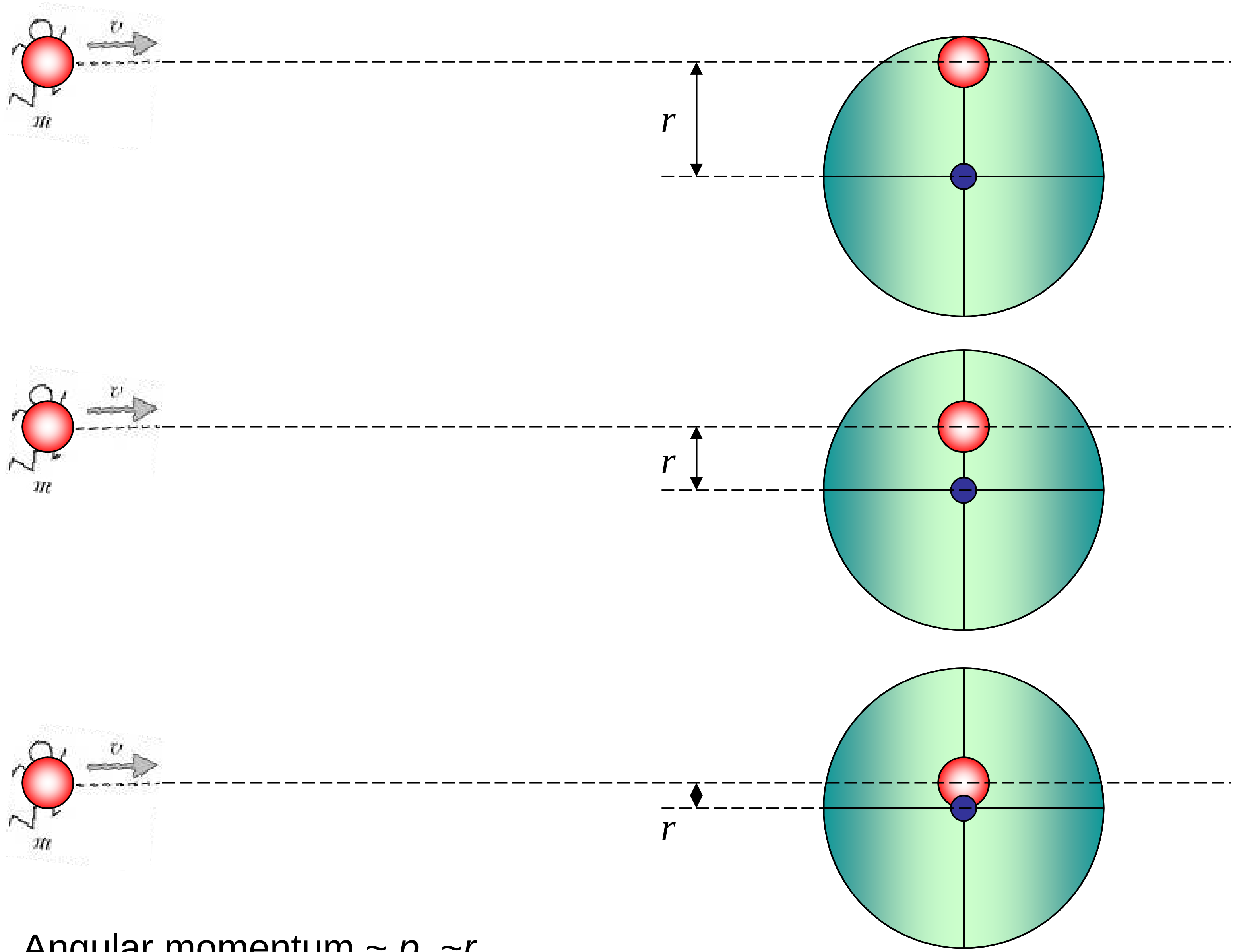
PHYS 172: Modern Mechanics **Spring 2012**

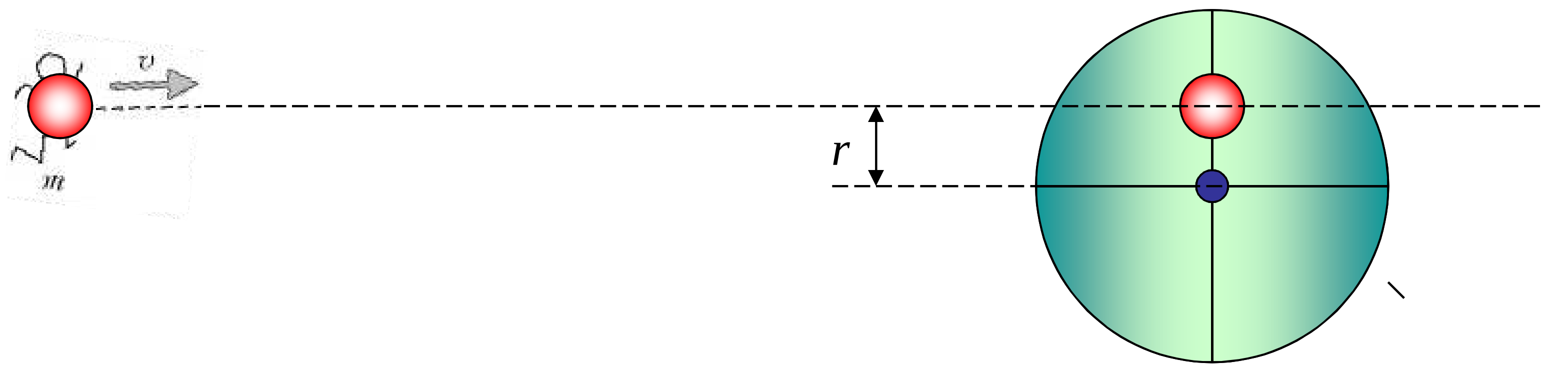
Lecture 19 – Angular momentum Read 11.1 – 11.3

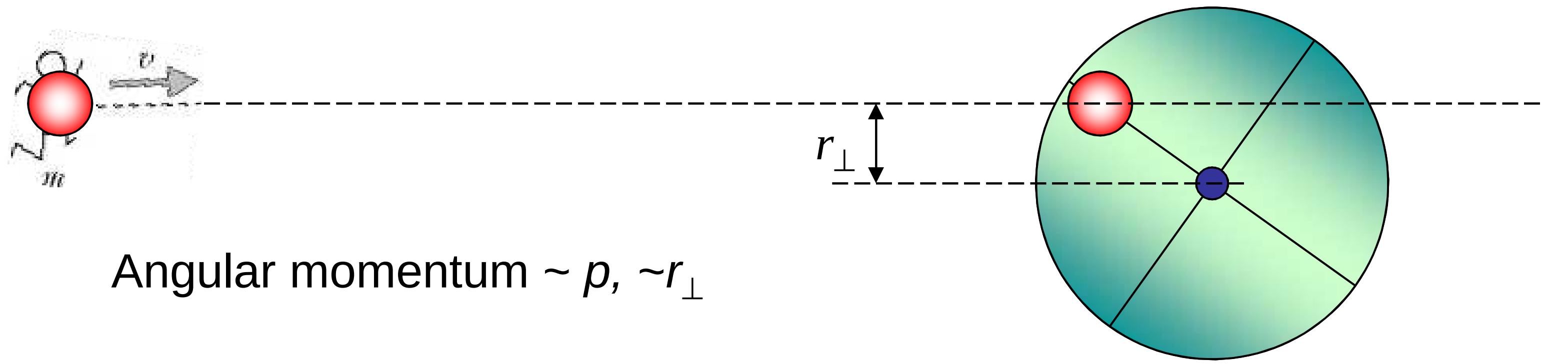
Angular momentum



Angular momentum ~ *p*, ~*r*

Angular momentum

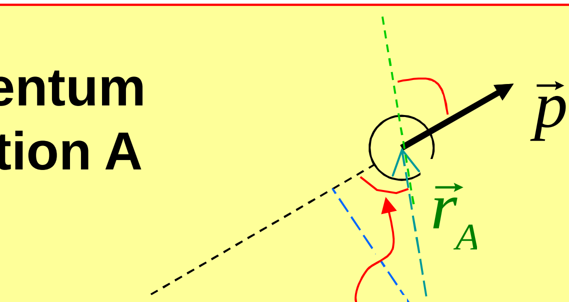




Angular momentum ~ *p*, ~$r_\perp$

Magnitude of angular momentum of a particle relative to location A

$$L_A = r_\perp p = r_A p \sin\theta$$



$$r_\perp = r_A \sin\theta$$

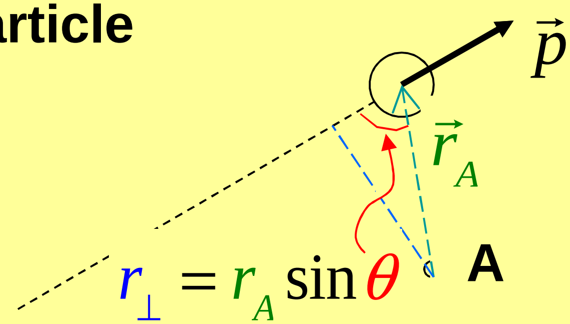
Cross-product

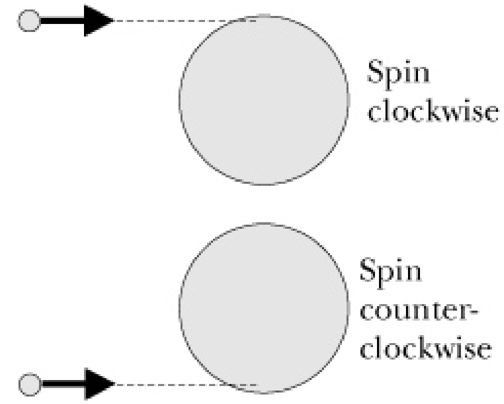
Angular momentum of a particle relative to location A

$$\vec{L}_A = \vec{r}_A \times \vec{p}$$

$$L_A = r_A p \sin\theta$$

$$\vec{L}_A = \left\langle \left(yp_z - zp_y\right), \left(zp_x - xp_z\right), \left(xp_y - yp_x\right) \right\rangle$$

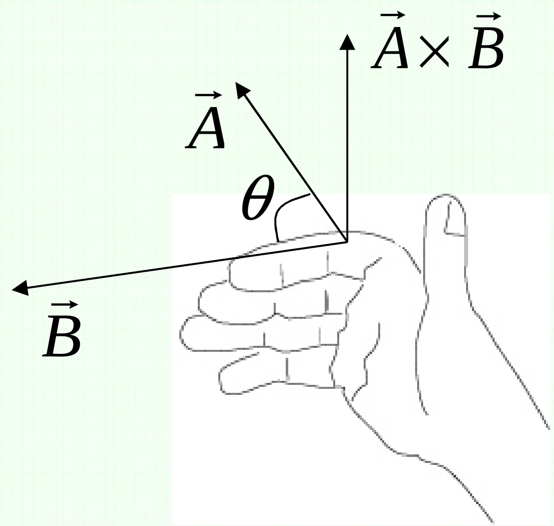




Cross-product:

$$\vec{A} \times \vec{B}$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$$

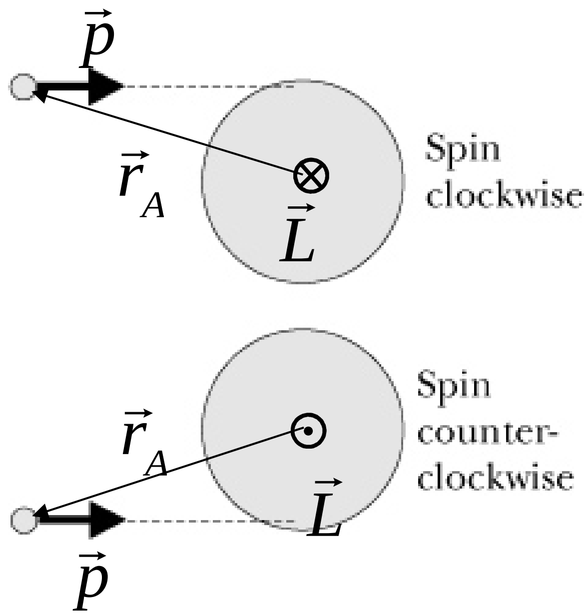


See book for more

$$\vec{A} \times \vec{B} = \left\langle \left(A_y B_z - A_z B_y\right), \left(A_z B_x - A_x B_z\right), \left(A_x B_y - A_y B_x\right) \right\rangle$$

Examples

$$\vec{L}_A = \vec{r}_A \times \vec{p}$$



See book for more examples

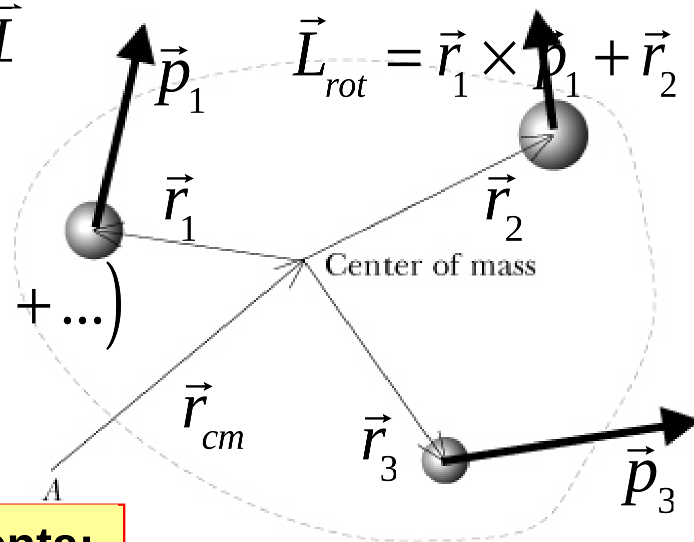
Multiparticle system

Split total kinetic energy: $K_{tot} = K_{trans} + \boxed{K_{rel}}$

$$K_{tot} = K_{trans} + \boxed{K_{rot} + K_{vib}}$$

Split angular momentum: $\vec{L}_A = \vec{L}_{trans,A} + \vec{I}$

$$\vec{L}_{rot} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 + \ldots$$



$$\vec{L}_A = (\vec{r}_{cm} + \vec{r}_1) \times \vec{p}_1 + (\vec{r}_{cm} + \vec{r}_2) \times \vec{p}_2 + \ldots$$

$$\vec{L}_A = \vec{r}_{cm} \times (\vec{p}_1 + \vec{p}_2 + \ldots) + (\vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 + \ldots)$$

$$\vec{L}_A = \vec{r}_{cm} \times \vec{P}_{tot} + (\vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 + \ldots)$$

Translational and rotational angular momenta:

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot}$$

$$\vec{L}_{trans} = \vec{r}_{cm} \times \vec{P}_{tot}$$

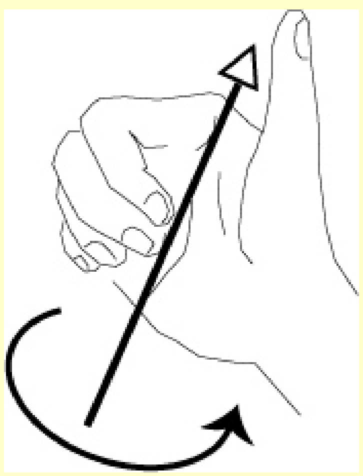
$$\vec{L}_{rot} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 + \ldots$$

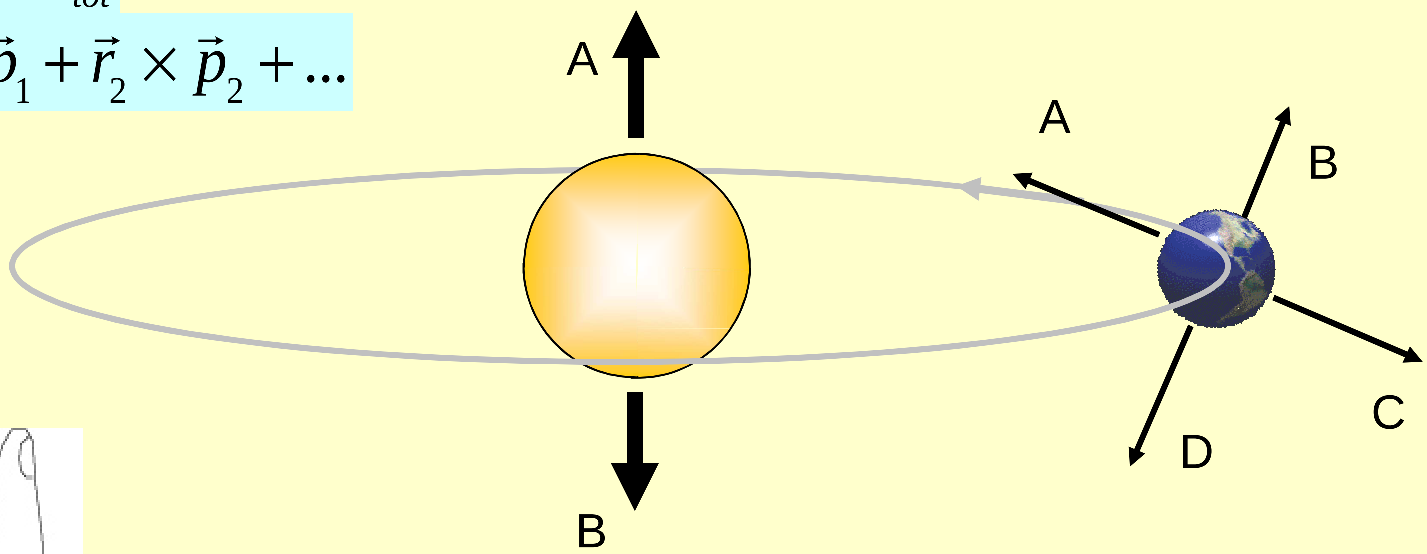
Clicker

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot}$$

$$\vec{L}_{trans} = \vec{r}_{cm} \times \vec{P}_{tot}$$

$$\vec{L}_{rot} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 + \ldots$$





System: Earth, angular momentum in respect to the Sun

1. What is the direction of L_{rot} ?

2. What is the direction of L_{trans} ?

Angular velocity

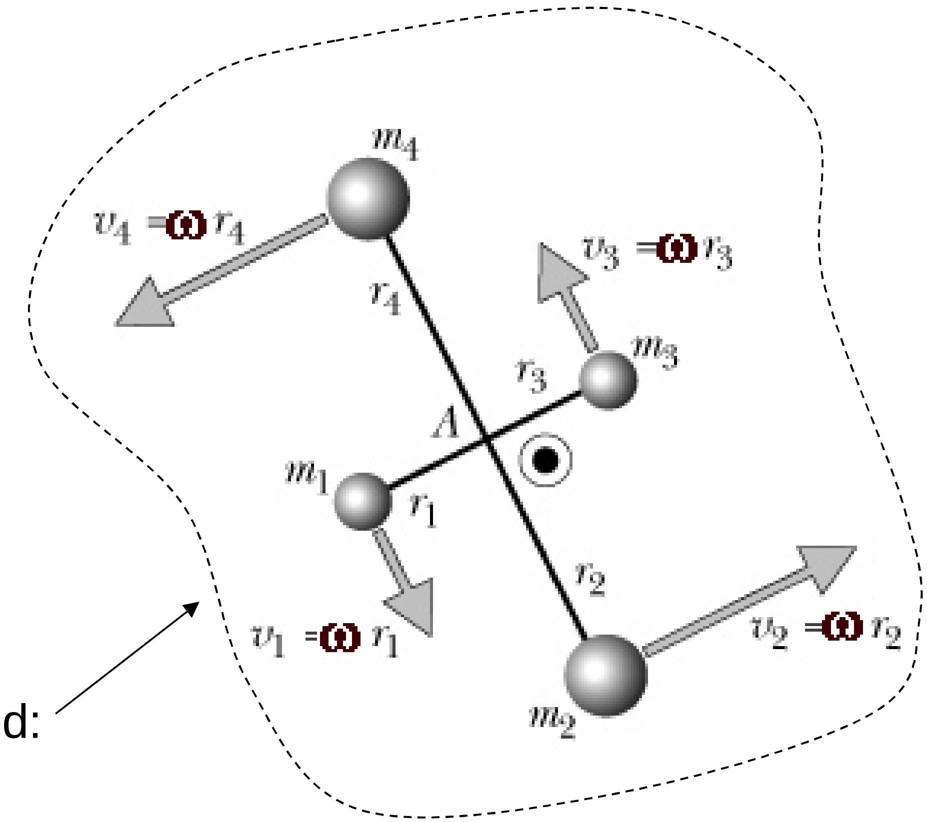
Angular speed: $\omega = 2\pi / T$
(chapter 2)

$\uparrow$ radians/second

For a rotating vector: $\left|\frac{d\vec{X}}{dt}\right| = \omega X$

For circular motion: $\left|\frac{d\vec{r}}{dt}\right| = v = \omega r$

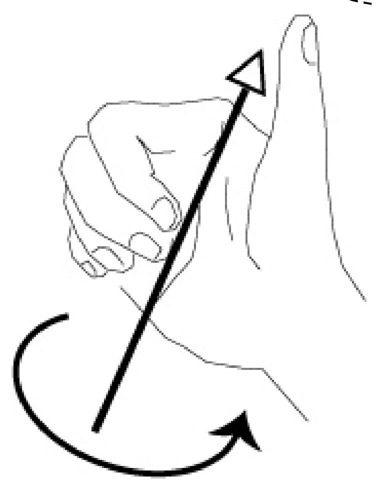
All parts share the same angular speed:
(rigid object)

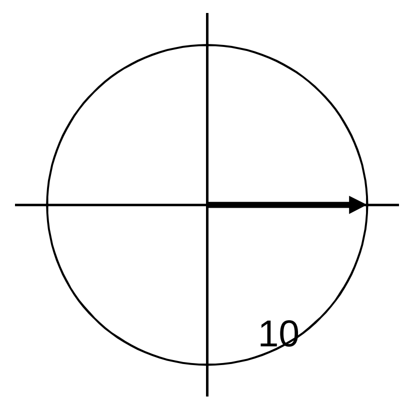


Angular velocity vector:

Magnitude: $|\vec{\omega}| = 2\pi / T$

Direction: right hand





Moment of inertia

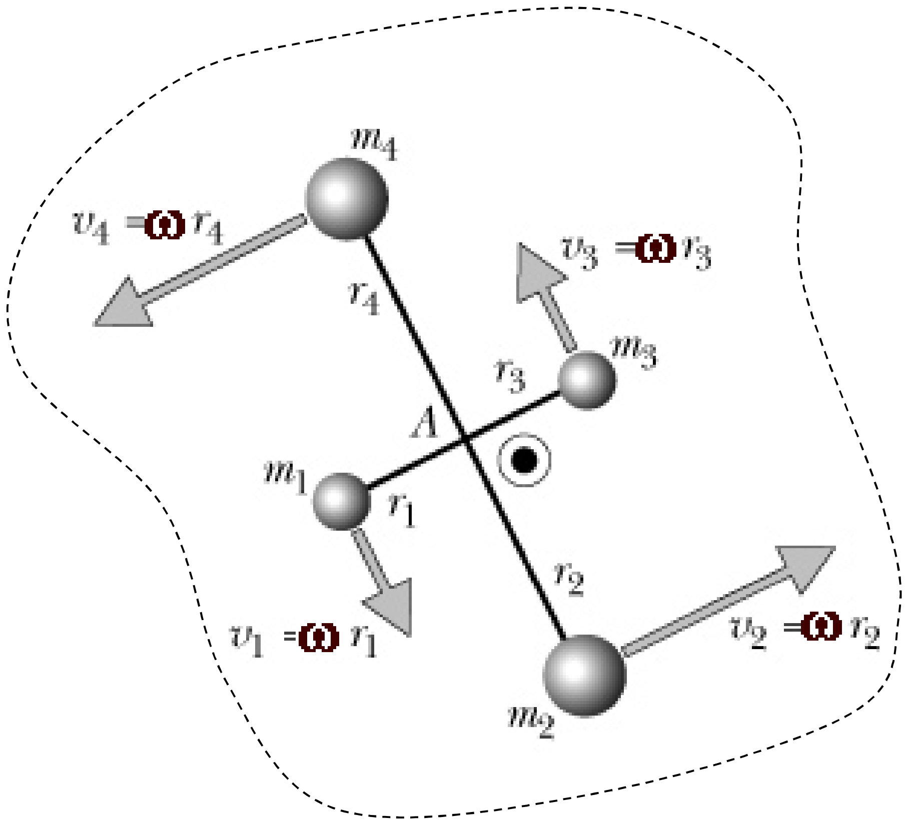
$\vec{r}_1$

$$\left|\vec{r}\times\vec{p}\right| = r_\perp mv = r_\perp m\left(\omega r_\perp\right) = mr_\perp^2\omega$$

$$\left|\vec{L}_{rot}\right| = \boxed{\left[m_1 r_{\perp 1}^2 + m_2 r_{\perp 2}^2 + \dots\right]}\omega$$

Moment of inertia

$$I = m_1 r_{\perp 1}^2 + m_2 r_{\perp 2}^2 + \dots$$



Rotational angular momentum

$$\vec{L}_{rot} = I\vec{\omega}$$

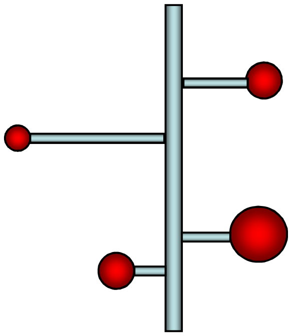
Analogy: $\vec{p} = m\vec{v}$

Moment of inertia: masses not in plane

Moment of inertia

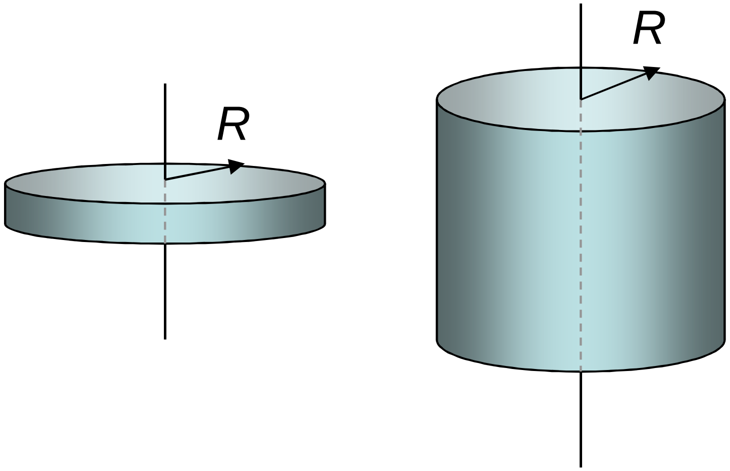
$$I = m_1 r_{\perp 1}^2 + m_2 r_{\perp 2}^2 + \dots$$

Masses not in plane:



$r_\perp$ is distance from axis of rotation

<u>Example</u>: solid disk of uniform density:



$$I = \tfrac{1}{2} M R^2$$

In respect to its axle

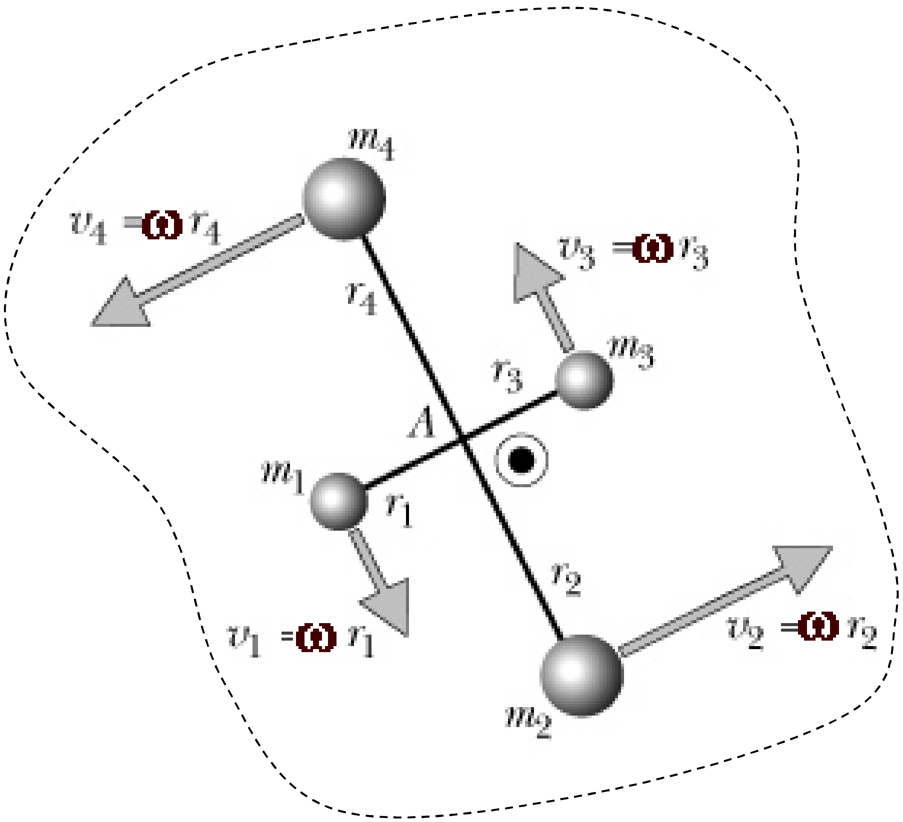
See table on page 359 (9.3) for moments of inertia of some simple shapes

Rotational kinetic energy

$$K_{rot} = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + \ldots$$

$$K_{rot} = \tfrac{1}{2} m_1 \left(r_{\perp 1}\omega\right)^2 + \tfrac{1}{2} m_2 \left(r_{\perp 2}\omega\right)^2 + \ldots$$

$$K_{rot} = \tfrac{1}{2}\left[m_1 r_{\perp 1}^2 + m_2 r_{\perp 2}^2 + \ldots\right]\omega^2$$



Rotational kinetic energy:

$$K_{rot} = \tfrac{1}{2} I\omega^2 = \frac{1}{2}\frac{\left(I\omega\right)^2}{I} = \frac{L_{rot}^2}{2I}$$

Analogy: $K = \tfrac{1}{2} m v^2 = \dfrac{p^2}{2m}$

In this course, we will only deal with rotation around center of mass
See exercises in chapter 11