PHYS 172 Problem of the Week 5 - Solution (Spring 2012)

An automobile (and its occupants) of total mass M = 2200 kg, is moving through a curved dip in the road of radius R = 18 m at a constant speed v = 12 m/s. For this analysis, you can neglect air resistance. Consider the automobile (and its occupants) as the system of interest.

Calculate the force exerted by the road on the system (car and its occupants). Start from the derivative form of the momentum principle and show all your work.

We will use the conventional coordinate system. At the instant shown, the +x direction points to the right and the +y direction points towards the center of the curved dip.

Let us first identify all of the forces acting on the system.

Force of Earth (gravity) on automobile (and its occupants): <0, -Mg, 0>.

Force of road on automobile (tires): <0, F_r, 0>.

What can we say about the relative magnitude of these two forces? The momentum principle provides some guidance.

The momentum principle states that $\frac{d\vec{p}}{dt} = \vec{F}_{net}$. The right hand side of this equation consists of all of the forces acting on the system. The left hand side describes the instantaneous dynamical situation, i.e., the rate of change of the (vector) momentum.

We can express the rate of change of momentum in terms of a component parallel to the instantaneous momentum (given by $\hat{p}$) and a component perpendicular to the $\hat{p}$ direction. Likewise, we can express components of the net force along these same two directions.

$$\frac{d\vec{p}}{dt} = \frac{d\left|\vec{p}\right|}{dt}\hat{p} + \left|\vec{p}\right|\frac{d\hat{p}}{dt} = \vec{F}_{net\|} + \vec{F}_{net\perp} \quad .$$

In the case under consideration, the magnitude of the momentum is not changing but its direction is. This statement is consistent with the component form of the forces we expressed above, i.e. only y-components are non-zero.

We know that $\frac{d\hat{p}}{dt}$ points in the +y-direction at the instant under consideration. Thus, $\left|\vec{F}_r\right| \geq \left|F_g\right|$. The difference in magnitudes depends, of course, on the speed of the car. The equality holds for $v = 0$.

The rate of change of the unit vector momentum is given by $\frac{v}{R}$, so

$$Mv\frac{v}{R}\hat{y} = F_{net\perp}\hat{y}$$

$$M\frac{v^2}{R} = F_r - Mg$$

$$F_r = M\frac{v^2}{R} + Mg = 1.76\times10^4\,N + 2.16\times10^4\,N = 3.92\times10^4\,N$$