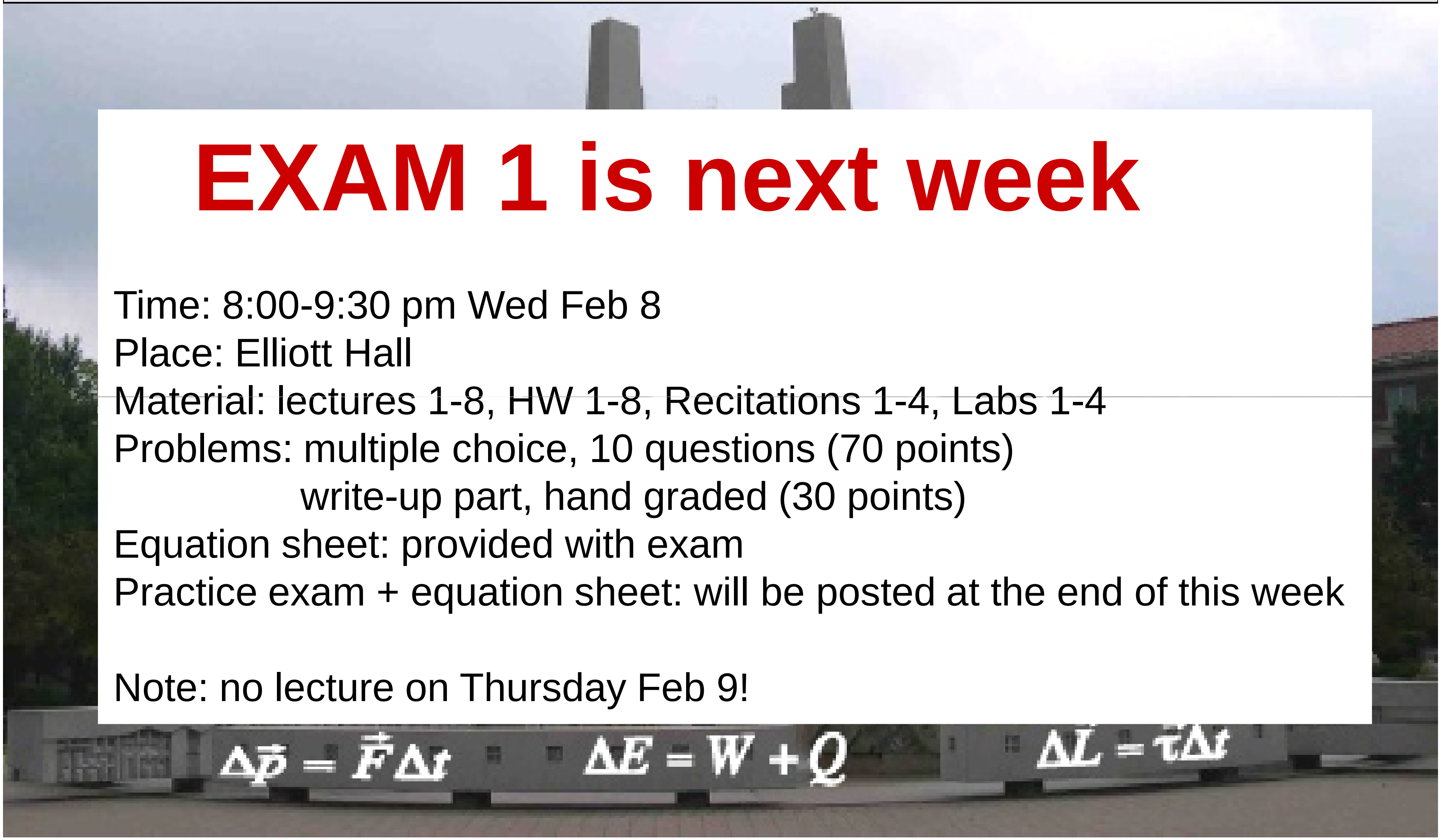
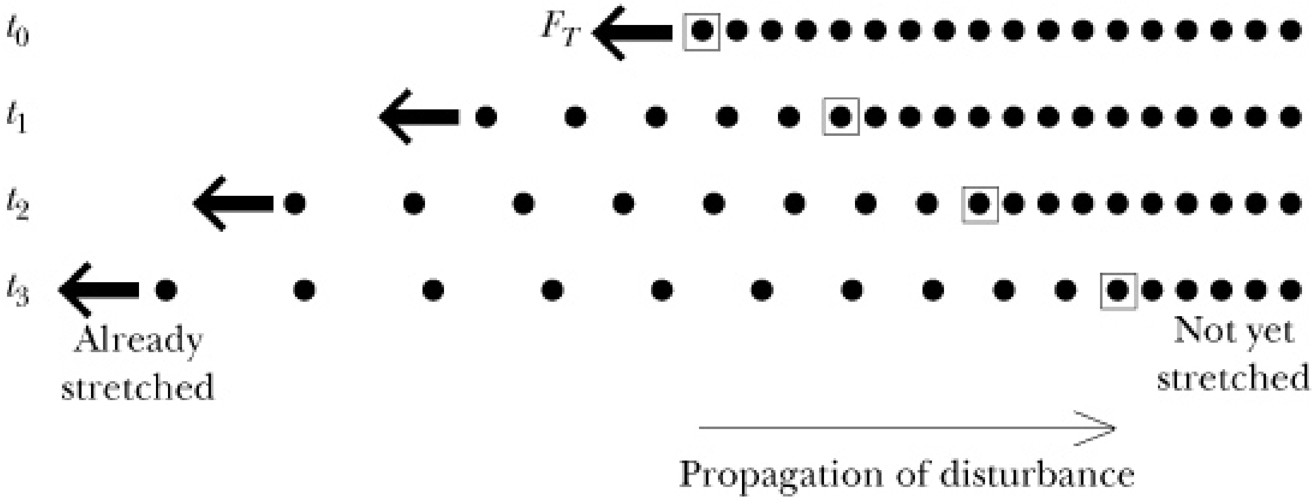


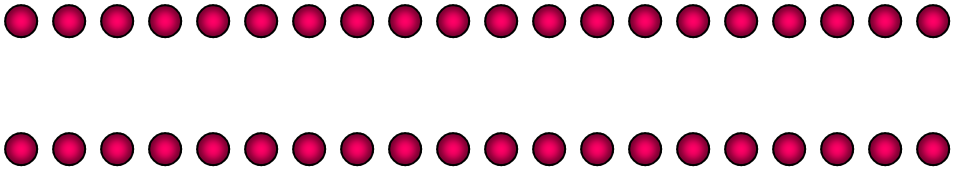
EXAM 1 is next week

Time: 8:00-9:30 pm Wed Feb 8
Place: Elliott Hall
Material: lectures 1-8, HW 1-8, Recitations 1-4, Labs 1-4
Problems: multiple choice, 10 questions (70 points)
write-up part, hand graded (30 points)
Equation sheet: provided with exam
Practice exam + equation sheet: will be posted at the end of this week

Note: no lecture on Thursday Feb 9!

Speed of sound in solids



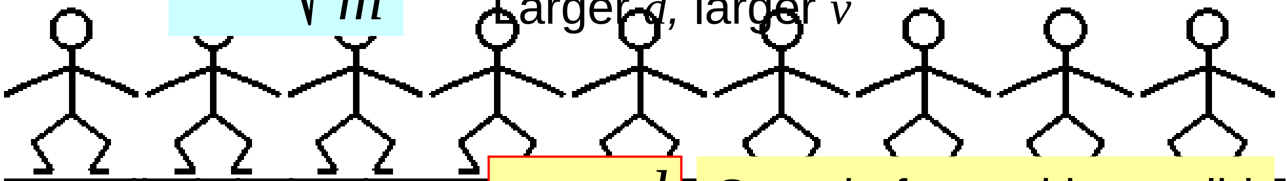


$$\omega = \sqrt{\frac{k_s}{m}}$$

Qualitatively:

Larger ω, larger v

Larger d, larger v



Detailed derivation: $v = \omega d$ Speed of sound in a solid

Derivative form of the Momentum Principle

The Momentum Principle

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

$$\frac{\Delta\vec{p}}{\Delta t} = \vec{F}_{net}$$

Works only if *force* is constant during Δt

The rate of the momentum change is equal to force

If force changes introduce instantaneous rate of change: $\frac{d\vec{p}}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{p}}{\Delta t}$

The momentum principle

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

Newton’s Second Law

Newton’s original formulation:

The rate of change of *amount of body’s motion* is proportional to *force*

momentum

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

Momentum principle is the second Newton’s law

Assume nonrelativistic case: $\vec{p} = m\vec{v}$

$$\frac{d\left(m\vec{v}\right)}{dt} = \vec{F}_{net} \qquad m\frac{d\vec{v}}{dt} = \vec{F}_{net}$$

(Assume m = const)

$\frac{d\vec{v}}{dt} \equiv \vec{a}$ **(definition of acceleration)**

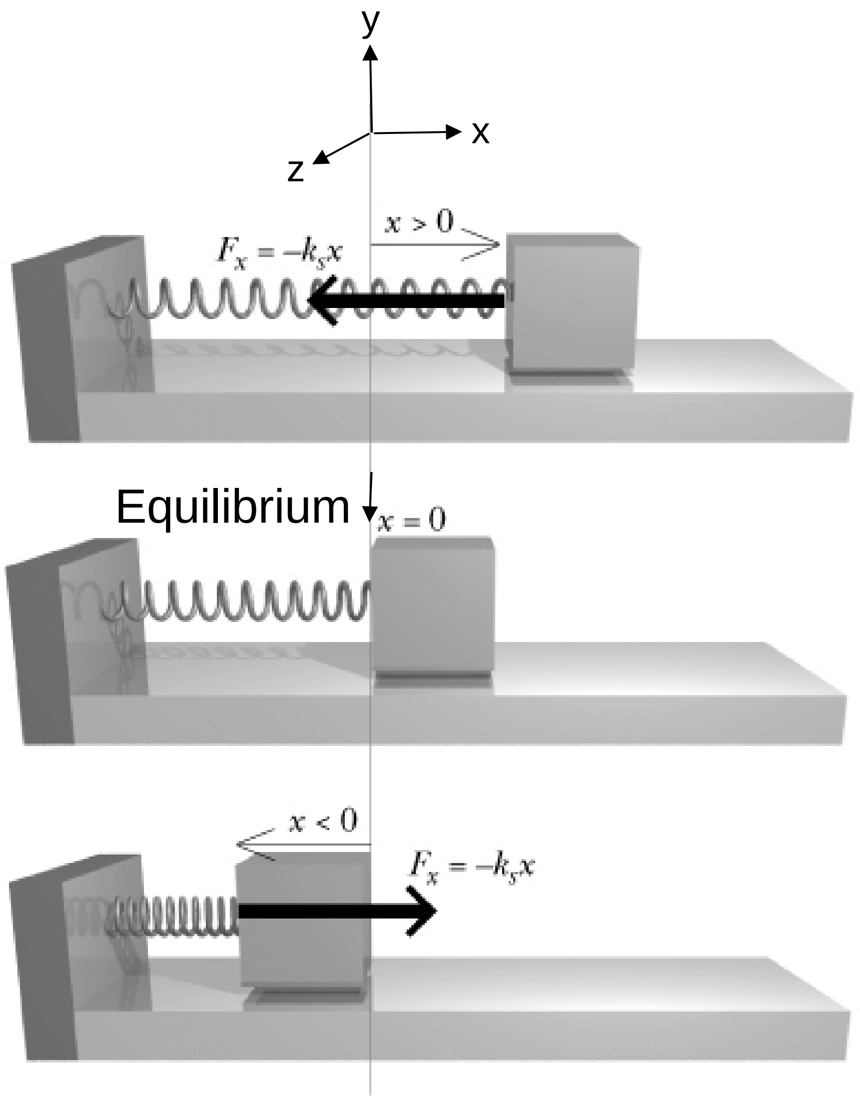
Newton’s second law

$$\vec{F}_{net} = m\vec{a}$$

Traditional form of 2nd Newton’s law

Spring-mass system: horizontal

$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}_{net}}$$



1. System: block

2. Apply momentum principle:

$$\frac{d\vec{p}}{dt} = \vec{F}_{spring} + \boxed{\vec{F}_{Earth} + \vec{F}_{table}} = 0$$

$$\left|\vec{F}_{spring}\right| = k_s \left|s\right|$$

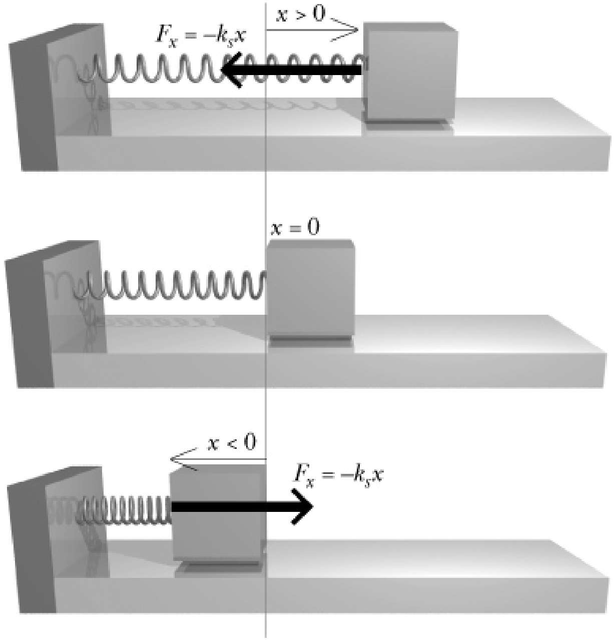
$$\downarrow$$

$$\boxed{F_x = -k_s x}$$

$$\left\langle \frac{dp_x}{dt}, 0, 0 \right\rangle = \left\langle -k_s x, 0, 0 \right\rangle$$

$$\boxed{\frac{dp_x}{dt} = -k_s x}$$

Spring-mass system: Analytical solution



$$\boxed{\frac{dp_x}{dt} = -k_s x}$$

Motion along *x*:
$p_x = p$

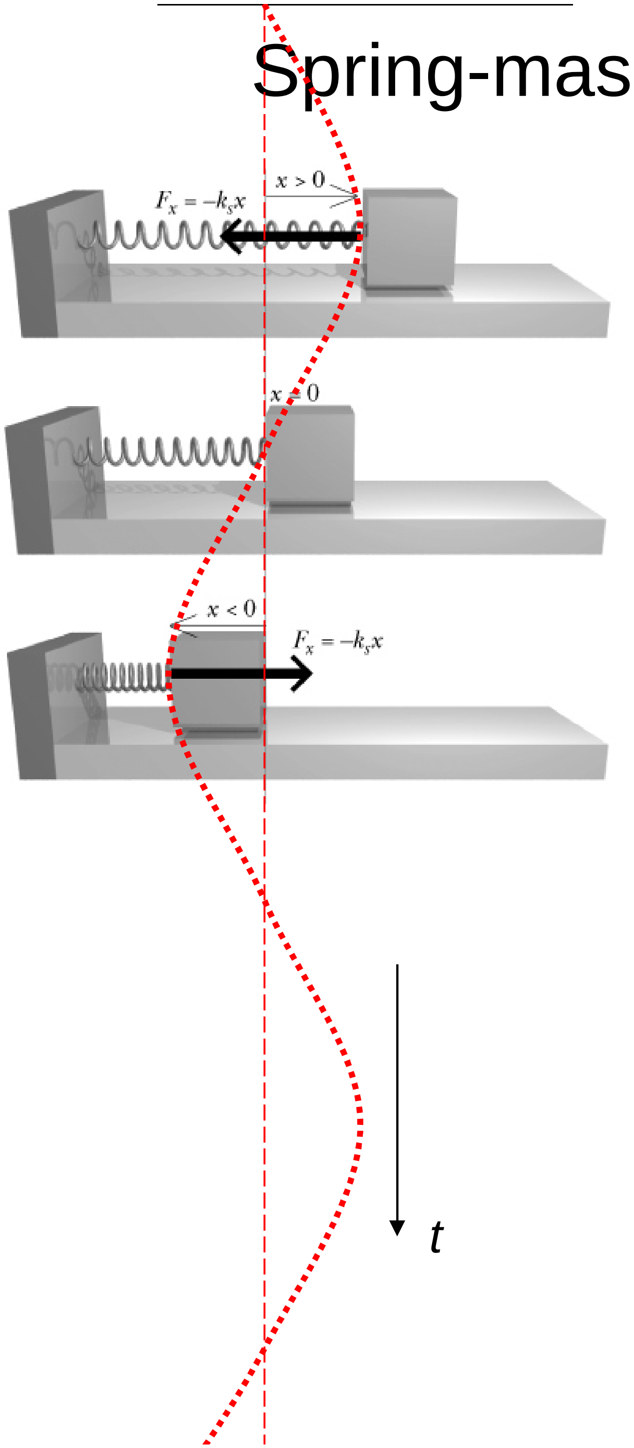
nonrelativistic

$$\frac{dp}{dt} = \frac{d\left(\overset{\downarrow}{m}v\right)}{dt} = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{dx}{dt}\right) = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x(t)}{dt^2} = -kx(t)$$

Differential equation: $\ddot{x}\left(t\right) = -\frac{k}{m}x\left(t\right)$

Spring-mass system: Analytical solution



$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$$

amplitude

Search solution in form: $x(t) = A\cos(\omega t)$

Angular frequency

$$-A\omega^2\cos(\omega t) = -\frac{k}{m}A\cos(\omega t)$$

$$\omega^2 = \frac{k}{m}$$

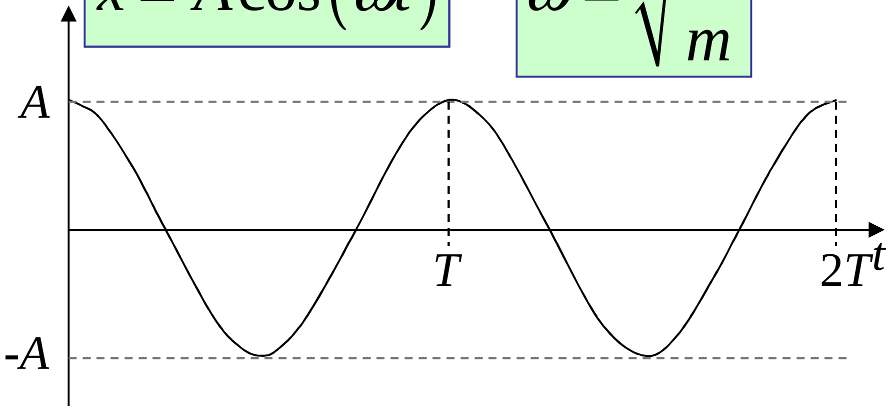
$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}t\right)$$

Spring-mass system: period and frequency

$$\boxed{x = A\cos(\omega t)}$$

$$\boxed{\omega = \sqrt{\frac{k_s}{m}}}$$

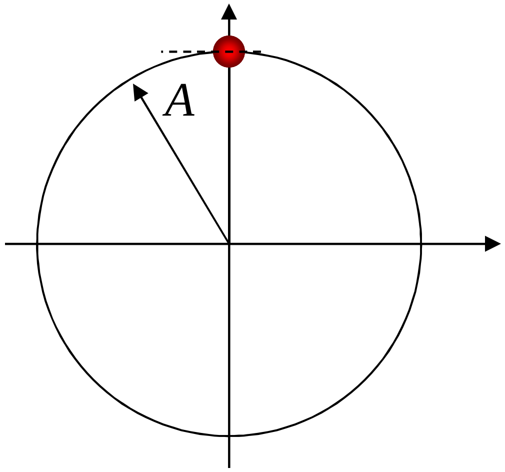


Period T: $\omega T = 2\pi$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_s}}}$$ [s]

Frequency: $f = 1/T$

$$\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}}}$$ [s^{-1}]≡[Hz]



The meaning of *angular* frequency:

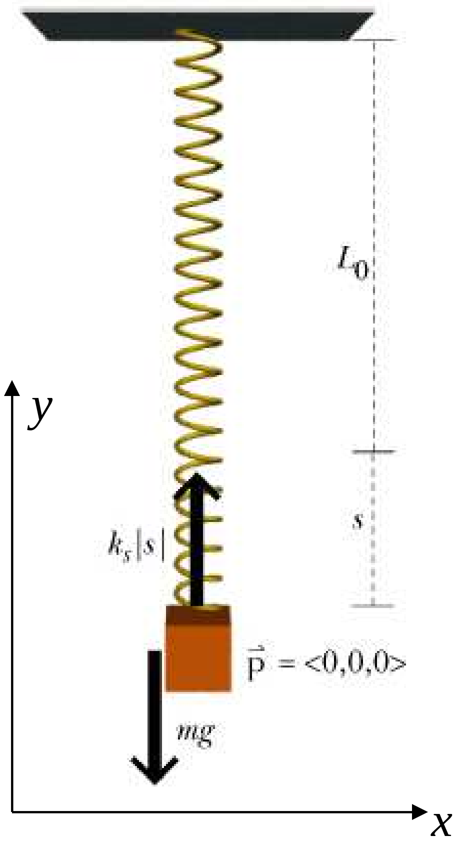
$$\omega = \frac{2\pi}{T}$$ [radian/second]

Static equilibrium

(*system never moves*)

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

System is at rest: $\vec{p} = \vec{0}$



$$\frac{d\vec{p}}{dt} = \vec{0}$$

$$\vec{0} = \vec{F}_{spring} + \vec{F}_{Earth}$$

$$\langle 0,0,0 \rangle = \langle 0, k_S s, 0 \rangle + \langle 0, -mg, 0 \rangle$$

$$\langle 0,0,0 \rangle = \langle 0, k_S s - mg, 0 \rangle$$

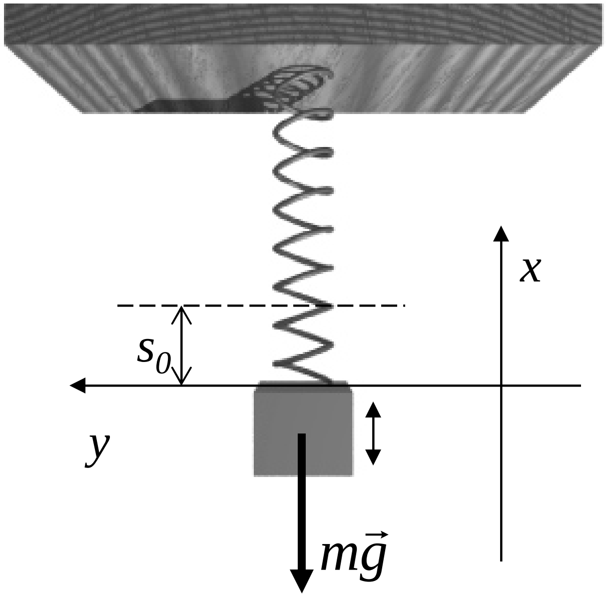
$$k_S s - mg = 0$$

Can predict *s*:

$$s = \frac{mg}{k_s}$$

Spring-mass system: vertical

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$



Choose origin at equilibrium position

Apply momentum principle:

$$\left\langle \frac{dp_x}{dt}, 0, 0 \right\rangle = \left\langle -k_s\left(x - s_0\right) - mg, 0, 0 \right\rangle$$

$$\frac{dp_x}{dt} = -k_s\left(x - s_0\right) - mg$$

$$\frac{dp_x}{dt} = -k_s x + k_s s_0 - mg$$

$$\boxed{\left|\vec{F}_{spring}\right| = k_s\left|s\right|}$$

$$k_s s_0 = mg$$

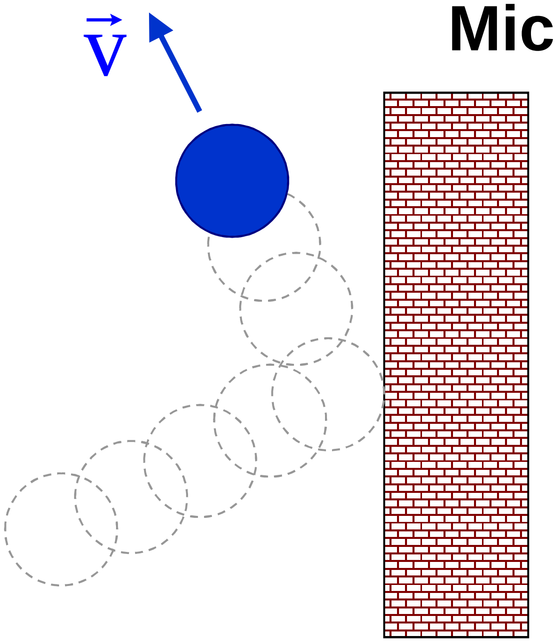
Details: 4.14 (p. 167)

$$\frac{dp_x}{dt} = -k_s x$$

The same equation and motion in the presence of gravity if you choose origin at equilibrium!

Buoyancy

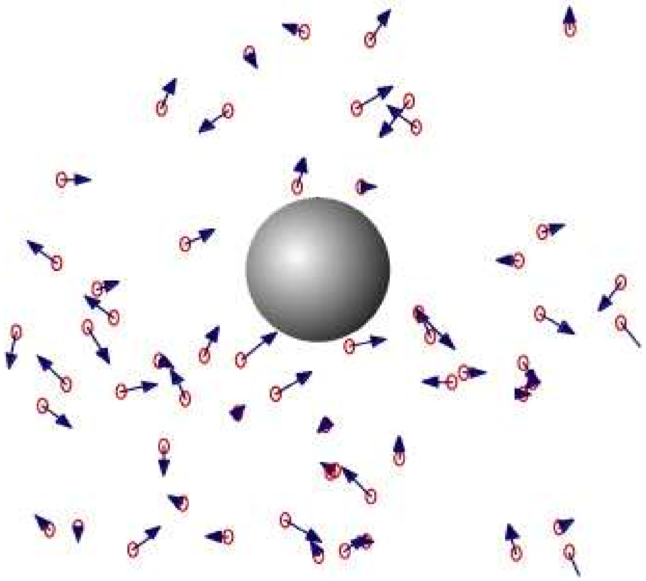
Microscopic view: Pressure



$$\Delta\vec{p}_{ball} = \vec{F}_{\text{on ball due to wall}}\Delta t$$

$$\vec{F}_{\text{on ball due to wall}} = -\vec{F}_{\text{on wall due to ball}}$$

Colliding ball exerts force on wall.



Balloon:

Air molecules constantly hit surface and bounce off, exerting forces on surface.

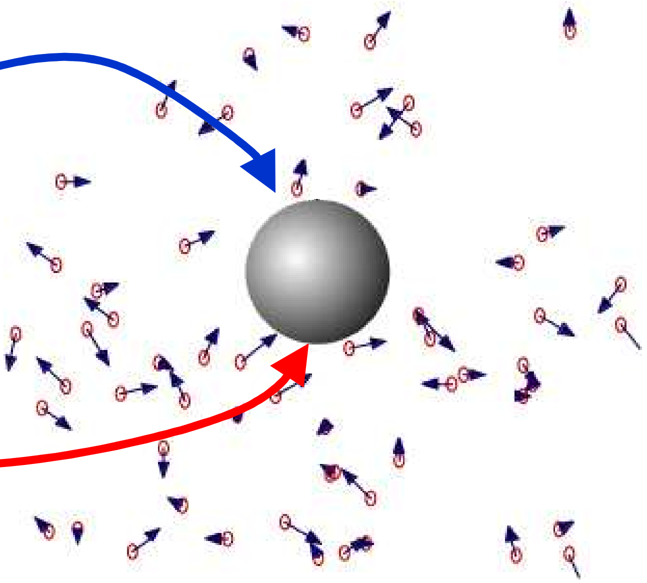
Pressure: force per unit area:

$$P = F/A$$

Buoyancy: microscopic view

Air is denser closer to the earth. (Why? We'll see in Chapter 12!)

There are more air molecules here than here

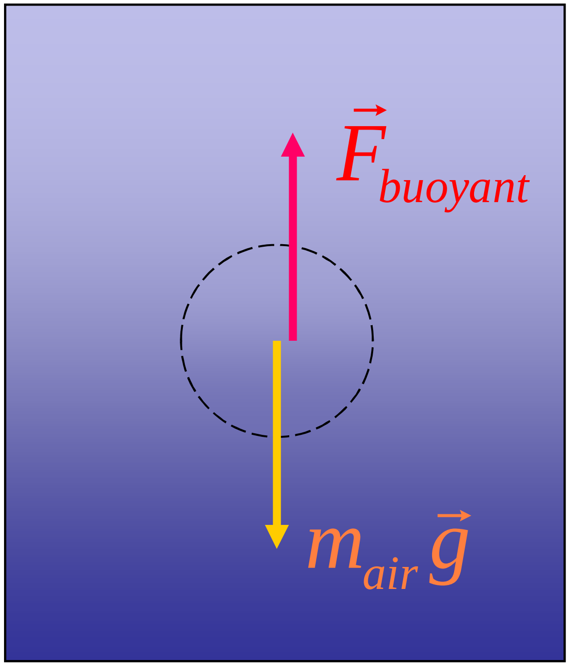


Thus $P_{bottom} > P_{top}$ OR

Net force pushing up > Net force pushing down

buoyancy

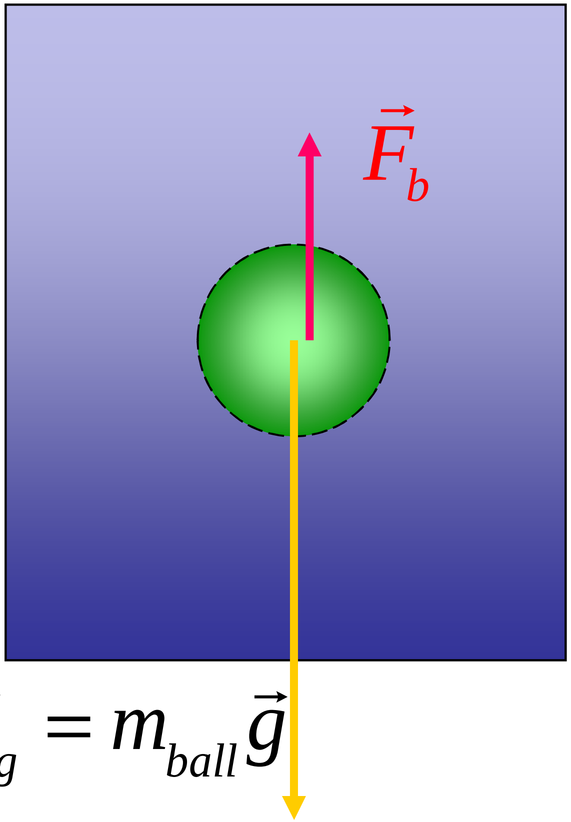
Buoyancy: macroscopic view



$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}_{net}}$$

$$\vec{F}_{buoyant} + m_{air}\vec{g} = 0$$

$$\vec{F}_{buoyant} = -m_{air}\vec{g}$$



$$\vec{F}_g = m_{ball}\vec{g}$$

Archimedes principle:
any body partially or completely submerged in a fluid or gas is buoyed up by a force equal to the weight of the fluid/gas displaced by the body.

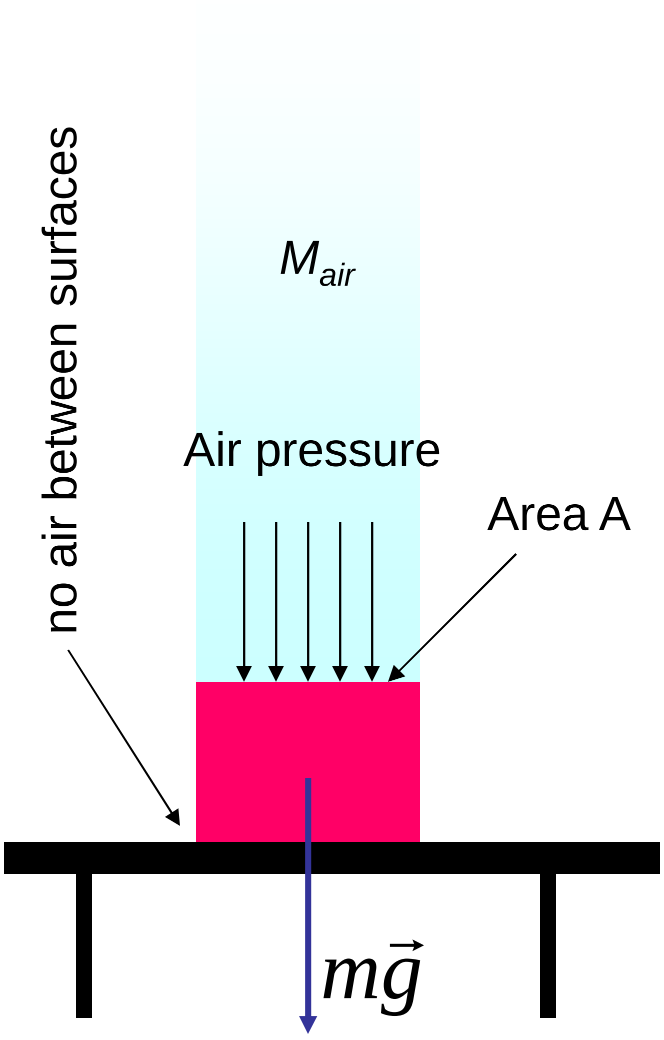
$$\frac{F_b}{F_g} = \frac{m_{air}g}{m_{ball}g} = \frac{V\rho_{air}}{V\rho_{ball}}$$

$$\boxed{\frac{F_b}{F_g} = \frac{\rho_{air}}{\rho_{ball}}}$$

$$\rho_{air} = 1.3 \text{ kg/m}^3$$

At STP:
0 °C and 101.325 kPa

Pressure and suction



Pressure: $P = \dfrac{M_{air}\,g}{A} \approx 10^5\ \text{N/m}^2$

~ 15 psi (pound/inch^2)

Example: brick on a table

mg = 1 kg × 9.8 N/kg =9.8 N

F_{air} = PA = 10^5 N/m^2 × 0.1 m × 0.2 m

F_{air} = 2 × 10^3 N (~450 lb)

? Why doesn't the table collapse?

Suction cup: how does it work?

1 cm^2 – F_{air} = 10 N