REVIEW QUESTIONS EE-202 Exam I

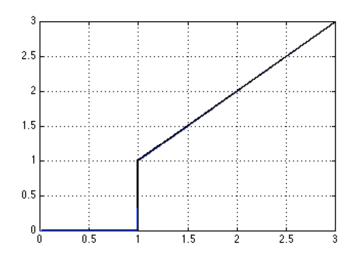
MULTIPLE CHOICE.

- 1. The mathematical expression of the time function (note that the function continues as an increasing straight line beyond the right edge of the graph) given below is:
- (a) r(t)
- (b) tu(t-1)

(c) r(t-1) + u(t)

- (d) (t-1)u(t-1)
- (e) r(t) r(t-1)
- (f) r(t) u(t-1)

(g) Something else



- 2. Recall that $e \approx 2.7$. The Laplace Transform of $e^{-t}u(t-2)$ has the form $\frac{Ae^{-2s}}{s+1}$ where A is closest to:
- (a) 3
- (b) 1
- (c) 1/3
- (d) 1/9
- (e) 9

- (f) -1/9
- (g) Something else
- 3. The Laplace transform of $\sin(\frac{\pi}{6}t)\delta(t-1)$ is $A(s)e^{-s}$ where A(s) is:
- (a) $\sin\left(\frac{\pi}{6}\right)e^{-s}$

- (b) $\sin\left(\frac{\pi}{6}\right)$ (c) $\frac{\frac{\pi}{6}}{s^2 + (\frac{\pi}{6})^2}$ (d) $\frac{\frac{\pi}{6}}{(s+1)^2 + (\frac{\pi}{6})^2}$
- (e) $\sin(0)$
- (f) $\sin\left(\frac{\pi}{6}\right) \frac{e^{-s}}{s}$

(g) Something else

4. A partial fraction expansion is given by

$$\frac{8}{(s+2)^2(s+4)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{s+4}$$

Then A is:

- (a) -2
- 2 (b)
- (c) -4
- (d) 4
- (e) 0

- (f) 0.5
- (g) Something else

5. The inverse Laplace Transform of $\frac{3s^2 + 4s + 5}{s^2 + 2s + 5}$ is $K\delta(t) + \left[Ae^{-t}\cos(2t) + Be^{-t}\sin(2t)\right]u(t)$ where

B is:

- (a) 5
- (b) 2.5
- (c) -2
- (d) -4

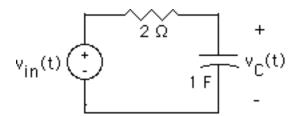
- (e) -5
- (f) 2
- (g) Something else
- The Laplace Transform of f(t) is given as $F(s) = \frac{1 e^{-s}}{s}$. Then, the Laplace Transform of

 $\frac{df(t)}{dt}$ with $f(0^-) = 3$ is:

- (a) $-3-e^{-s}$ (b) $-1-e^{-s}$ (c) $-4-e^{-s}$ (d) $-2-e^{-s}$ (e) $\frac{1-e^{-s}}{s^2}-3$ (e) $\frac{1-e^{-s}}{s^2}-\frac{3}{s}$ (g) Something else
- 7. For the same f(t) as Problem 6, the Laplace Transform of tf(t) is $\frac{A + B(s)e^{-s}}{c^2}$ where B(s) is:
- (a) 1
- (b) -s
- (c) s
- (d) 1 s
- (e) 1+s

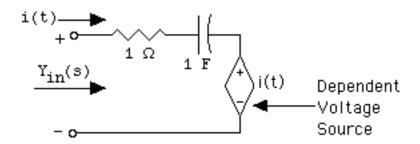
- (f) (1+s)
- (g) Something else
- 8. The circuit given below has differential equation $\frac{d}{dt}v_C(t) + 0.5v_C(t) = 0.5v_{in}(t)$ with $v_{in}(t) = 10\delta(t)$ and $v_C(0^-) = 5$ V. Then $v_C(t) = Ae^{-Bt}u(t)$ where A is:
- (a) 20
- (b) 15
- (c) 10
- (d) 5
- (e) 0

- (f) -10
- (g) Something else



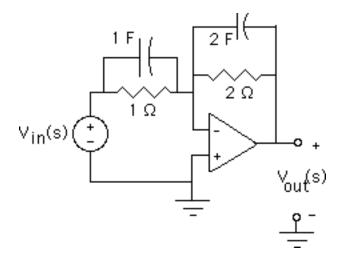
- 9. For the circuit given below, the input admittance is $K + \frac{A}{s+B}$ where A is:
- (a) 1
- (b) -0.25
- (c) -1
- (d) -1.25

- (e) -0.5
- (f) 0.5
- (g) Something else



- 10. The transfer function of the circuit below is:
- (a) $-\frac{s+1}{2s+0.5}$ (b) $-\frac{s+1}{2s+2}$ (c) $-\frac{2s+0.5}{s+1}$ (d) $\frac{2s+2}{s+1}$

- (e) $\frac{s+1}{2s+0.5}$ (f) $-\frac{2s+2}{s+1}$
- (g) Something else



11. The transfer function of the circuit below is:

$$s/(s+1)$$
 (c)

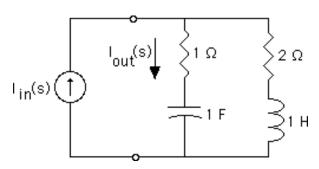
(c)
$$\frac{s}{(s+1)} + \frac{1}{(s+2)}$$

(a) 1 (b)
$$\frac{s}{(s+1)}$$
 (c) $\frac{s}{(s+1)} + \frac{1}{(s+2)}$ (d) $\frac{s}{s(s+2)+(s+1)}$ (e) $\frac{s(s+2)}{s(s+2)+(s+1)}$

(e)
$$\frac{s(s+2)}{s(s+2)+(s+1)}$$

(f)
$$\frac{(s+1)}{s(s+2)+(s+1)}$$

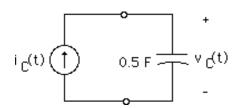
(g) Something else



12. For the circuit below, recall that $v_C(t) = \frac{1}{C} \int_{-\infty}^{t} i_C(q) dq$. If $I_C(s) = \frac{0.5}{s}$ and $V_C(s) = \frac{1}{s^2} + \frac{2}{s}$, then

$$v_C(0^-) = :$$

- (a) 1
- (b) 2
- (c) -1
- (d) -2
- (e) 0.5



13. For the circuit below, the values of R (in Ω) and C (in F) that make the input admittance

$$Y_{in}(s) = \frac{4}{2s+1} + \frac{4s}{2s+1}$$

are:

(1)
$$R = 1, C = 4$$

(2)
$$R = 4, C = 0.25$$
 (3) $R = 4, C = 4$

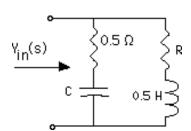
(3)
$$R = 4, C = 4$$

(4)
$$R = 0.25, C = 0.25$$

(5)
$$R = 0.5, C = 2$$

(5)
$$R = 0.5, C = 2$$
 (6) $R = 0.25, C = 4$

(7) none of above



14. The transfer function for the following op amp circuit is:

$$(1) - \frac{s + 0.5}{0.5s + 1}$$

$$(2) - \frac{0.5s + 1}{s + 0.5}$$

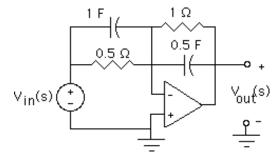
(3)
$$\frac{s+0.5}{0.5s+1}$$

$$(4) \ \frac{s+1}{0.5s+1}$$

$$(4) \frac{s+1}{0.5s+1} \qquad (5) -\frac{s+1}{0.5s+1}$$

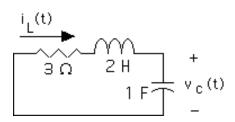
$$(6) -2$$

(7) none of above



- 15. In a circuit shown below, the initial conditions are $i_L(0^-) = 1$ amp and $v_C(0^-) = 2$ volt. If $i_L(t) = ae^{-0.5t}u(t) + be^{-dt}u(t)$ then a is:
- (1) 1
- (2) 2
- (3) 3
- (4) 4

- (5) -4
- (6) -3
- (7) -1



16. The Laplace transform of $f(t) = 2\cos(t+1)\delta(t-1) + 2\cos(t-1)\delta(t+1)$ is:

(1)
$$2e^{s} \frac{s}{s^2 + 1}$$

(1)
$$2e^{s} \frac{s}{s^2 + 1}$$
 (2) $2\left[e^{s} + e^{-s}\right] \frac{s}{s^2 + 1}$

(3)
$$2\cos(2)e^{-s} + 2\cos(-2)e^{s}$$

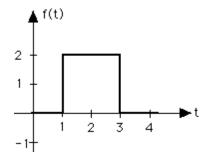
(4)
$$2\cos(2)e^{-s}$$
 (5) $2\cos(-2)e^{s}$

$$(5) 2\cos(-2)e^{s}$$

(6)
$$2e^{-(s+1)} \frac{s+1}{(s+1)^2+1}$$

(7) None of above

17. The Laplace transform of



is:

$$(1) u(t) - u(t-2)$$

(2)
$$e^{-s}u(t-1) - e^{-3s}u(t-3)$$

(3)
$$e^{-s} - e^{-3s}$$

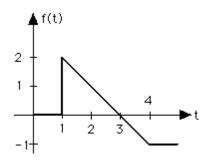
(4)
$$\frac{1-e^{-3s}}{s}$$

(5)
$$\frac{e^{s}-e^{3s}}{s}$$

(1)
$$u(t) - u(t-2)$$
 (2) $e^{-s}u(t-1) - e^{-3s}u(t-3)$ (3) $e^{-s} - e^{-3s}$
(4) $\frac{1 - e^{-3s}}{s}$ (5) $\frac{e^{s} - e^{3s}}{s}$ (6) $\frac{e^{-s} - e^{-3s}}{s}$

(7) None of above

18. The Laplace transform of



is:

(1)
$$F(s) = \frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{e^{-4s}}{s}$$

(1)
$$F(s) = \frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{e^{-4s}}{s}$$
 (2) $F(s) = \frac{2e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{e^{-4s}}{s}$

(3)
$$2\left[\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{e^{-4s}}{s}\right]$$
 (4) $2\left[\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{e^{-4s}}{s^2}\right]$

(4)
$$2\left[\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{e^{-4s}}{s^2}\right]$$

(5)
$$F(s) = \frac{2e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{e^{-4s}}{s^2}$$
 (6) $\frac{2e^{-s}}{s} - \frac{2e^{-s}}{s^2} + \frac{e^{-4s}}{s^2}$

(6)
$$\frac{2e^{-s}}{s} - \frac{2e^{-s}}{s^2} + \frac{e^{-4s}}{s^2}$$

- (7) None of above
- 19. The output of a circuit has Laplace transform $V_{out}(s) = \frac{16}{s^3(s+2)}$ then

$$v_{out}(t) = Ae^{-2t}u(t) + Bu(t) + other terms$$

where A =:

$$(3) -2$$

$$(3) -2$$
 $(4) 4$ $(5) -4$

(7) None of above

- 20. Referring again to problem 19 above, the value of B is:
- (1) 8
- (2)

- (3) -2 (4) 4 (5) -4

- (6) 16
- (7) None of above
- 21. The transfer function $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$ associated with the integro-differential equation

$$v_{out}'(t) + 4v_{out}(t) + 4\int_{0^{-}}^{t} v_{out}(q)dq = 2v_{in}'(t) - 8\int_{0^{-}}^{t} v_{in}(q)dq$$

- is:
- (1) $\frac{s^2 + 4s + 4}{s^2 4}$ (2) $\frac{s^2 + 4s + 4}{2s^2 8}$
- (3) $\frac{2s-8}{s+4+4}$

- (4) $\frac{s+4+\frac{4}{s}}{2s-8}$ (5) $\frac{2s-8}{s^2+4s+4}$ (6) $\frac{2s^2-8}{s^2+4s+4}$
- (7) None of above
- 22. For a > 0, the Laplace transform of $f(t) = 2e^{(t+a)}\delta(t-a) + 2e^{(t-a)}\delta(t+a)$ is:

- (1) $2e^{2a}$ (2) $2e^{2a} + 2e^{-2a}$ (3) $2e^{-2a}$ (4) $2e^{-a(s-2)}$ (5) $2e^{a(s-2)}$ (6) $2e^{-a(s-2)} + 2e^{a(s-2)}$
- (7) None of above
- 23. The output of a circuit has Laplace transform $V_{out}(s) = \frac{64}{s^3(s+2)}$ with

$$v_{out}(t) = Ae^{-2t}u(t) + Bu(t) + other - terms$$

- the value of B is:
- (1) 8
- (2) 2 (3) -4 (4) 4 (7) 32
- (5)8

- (6) -16
- 24. Suppose $L\{f(t)\} = -\ell n \left[\frac{s}{s+2} \right]$. Then $L\{tf(t)\}$ is:
- $(1) \frac{1}{s} + \frac{1}{s+2}$ $(2) \frac{1}{s+2} \frac{s}{(s+2)^2}$ $(3) \frac{-1}{s+2} + \frac{s}{(s+2)^2}$

- (4) $\frac{1}{s} \frac{1}{s+2}$ (5) $-\frac{1}{s} + \frac{s}{(s+2)^2}$ (6) $\frac{1}{s} \frac{s}{(s+2)^2}$

- $(7) \frac{s+2}{s}$
- (8) None of above

25. The Thevenin equivalent admittanace $Y_{in}(s)$ of the circuit below is:

(1)
$$s+2+\frac{3}{s}$$
 (2) $s+2-\frac{1}{s}$ (3) $s+2+\frac{1}{s}$

(2)
$$s+2-\frac{1}{s}$$

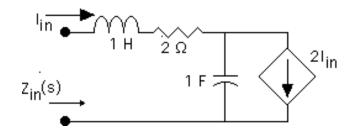
(3)
$$s+2+\frac{1}{s}$$

(3)
$$\frac{1}{s} + 0.5 - s$$

(3)
$$\frac{1}{s} + 0.5 - s$$
 (4) $\frac{1}{s} + 0.5 + 3s$ (5) $\frac{1}{s} + 0.5 + s$ (6) $s + 2 - \frac{3}{s}$ (7) None of Above

(5)
$$\frac{1}{s} + 0.5 + s$$

(6)
$$s+2-\frac{3}{s}$$



26. The transfer function for the following op amp circuit is:

(1)
$$\frac{(s+2)}{s(0.5s+1)}$$
 (2) $\frac{s(0.5s+1)}{s+2}$ (3) $\frac{0.5s+1}{s(s+2)}$ (4) $-\frac{0.5s+1}{s(s+2)}$ (5) $\frac{-(s+2)}{s(0.5s+1)}$ (6) $\frac{s(s+2)}{2s+1}$

(2)
$$\frac{s(0.5s+1)}{s+2}$$

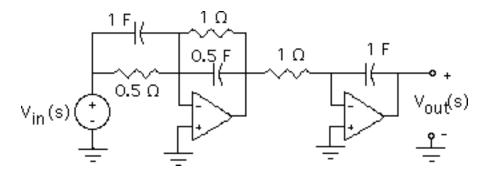
(3)
$$\frac{0.5s+1}{s(s+2)}$$

$$(4) - \frac{0.5s + 1}{s(s+2)}$$

$$(5) \frac{-(s+2)}{s(0.5s+1)}$$

(6)
$$\frac{s(s+2)}{2s+1}$$

$$(7) \ \frac{s(s+2)}{0.5s+1}$$



27. The Laplace Transform of f(t) is given as $F(s) = \frac{1 - e^{-(s-a)}}{s-a}$. Then the Laplace Transform of

 $e^{-at}tf(t)$ is:

(1)
$$\frac{1 - e^{-(s-a)}}{(s-a)^2} - \frac{e^{-(s-a)}}{s-a}$$
 (2) $\frac{1 - e^{-s}}{s}$ (3) $\frac{1 - e^{-s}}{s^2} - \frac{e^{-s}}{s}$

$$(2) \ \frac{1-e^{-s}}{s}$$

$$(3) \ \frac{1 - e^{-s}}{s^2} - \frac{e^{-s}}{s}$$

$$(4) \ \frac{1 - e^{-s}}{s^2}$$

$$(5) - \frac{1 - e^{-s}}{s^2}$$

(5)
$$-\frac{1-e^{-s}}{s^2}$$
 (6) $\frac{1-e^{-(s-2a)}}{(s-2a)^2} - \frac{e^{-(s-2a)}}{s-2a}$

(7) None of above

28. For the circuit below, the admittance $Y_{in}(s) =$:

$$(1) \ \frac{2s^2 + 3s + 1}{s^2}$$

$$(2) \quad 0.5 + s + \frac{s^2}{2s+1}$$

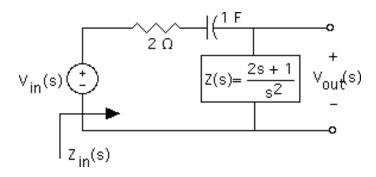
(2)
$$0.5 + s + \frac{s^2}{2s+1}$$
 (3) $\frac{1}{2+s+\frac{s^2}{2s+1}}$

$$(4) \ \frac{s^2}{2s^2 + 3s + 1}$$

(5)
$$\frac{s^2 + s}{2s + 1}$$
 (6) $\frac{2s + 1}{s^2 + s}$

$$(6) \quad \frac{2s+1}{s^2+s}$$

(7) None of above



29. For the circuit of problem 7, if $v_{in}(t) = 4\delta(t)$ V, then $v_{out}(t)$ equals (in volts):

(1)
$$\left(4e^{-t} - 2e^{-0.5t}\right)u(t)$$
 (2) $2u(t) + tu(t)$ (3) $2\delta(t) + u(t)$

$$(2) \quad 2u(t) + tu(t)$$

(3)
$$2\delta(t) + u(t)$$

(4)
$$4e^{-t}u(t)$$

(5)
$$\delta(t) + 0.5e^{-0.5t}u(t)$$

(6)
$$16\delta(t) + 8u(t)$$

30. The Thevenin equivalent admittanace $Y_{in}(s)$ of the circuit below is:

(1)
$$s+2+\frac{0.5}{s}$$

(2)
$$s+2+\frac{1.5}{s}$$

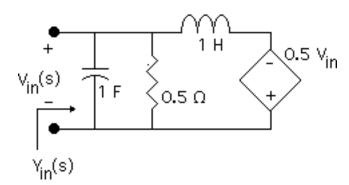
(3)
$$s+2+\frac{1}{s}$$

(3)
$$\frac{1}{s} + 0.5 + s$$

(4)
$$\frac{1}{s} + 0.5 + 1.5s$$

(2)
$$s+2+\frac{1.5}{s}$$
 (3) $s+2+\frac{1}{s}$
(4) $\frac{1}{s}+0.5+1.5s$ (5) $\frac{1}{s}+0.5-0.5s$

(6)
$$s+2-\frac{1.5}{s}$$



31. The transfer function for the following op amp circuit is:

(1)
$$\frac{s+0.5}{s(0.5s+1)}$$
 (2) $\frac{0.5s+1}{s(s+0.5)}$ (3) $\frac{-s-0.5}{s(0.5s+1)}$

(2)
$$\frac{0.5s+1}{s(s+0.5)}$$

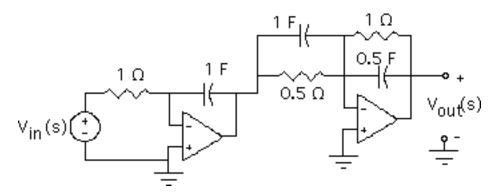
(3)
$$\frac{-s-0.5}{s(0.5s+1)}$$

(4)
$$\frac{s+1}{s(0.5s+1)}$$
 (5) $\frac{s+1}{s(0.5s+1)}$ (6) $\frac{2}{s}$

$$(5) \ \frac{s+1}{s(0.5s+1)}$$

(6)
$$\frac{2}{s}$$

$$(7) \frac{-2}{s}$$



32. Suppose

$$L\{f(t)\} = \ell n \left[\frac{s+3}{s+5} \right].$$

Then $L\{tf(t)\}$ is:

$$(1) - \frac{s+5}{s+3}$$

$$(2) \ \frac{1}{s+5} - \frac{s+3}{s+5}$$

(1)
$$-\frac{s+5}{s+3}$$
 (2) $\frac{1}{s+5} - \frac{s+3}{s+5}$ (3) $\frac{1}{s+5} - \frac{s+3}{(s+5)^2}$

$$(4) - \frac{1}{s+3} + 1$$

$$(4) -\frac{1}{s+3} + 1 \qquad (5) -\frac{1}{s+3} + \frac{1}{s+5} \qquad (6) -\frac{1}{s+3} - \frac{1}{s+5}$$

$$(6) -\frac{1}{s+3} - \frac{1}{s+5}$$

(7) None of above

33. The transfer function of the circuit below is:

$$(1) \ \frac{s+2}{s+1}$$

$$(2) \quad \frac{s(s+2)}{s+1}$$

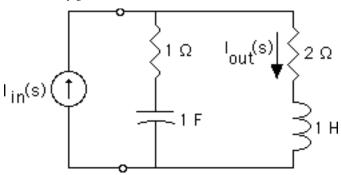
(2)
$$\frac{s(s+2)}{s+1}$$
 (3) $\frac{s}{(s+1)} + \frac{1}{(s+2)}$ (5) $\frac{s(s+2)}{s(s+2)+(s+1)}$

(4)
$$\frac{s}{s(s+2)+(s+1)}$$

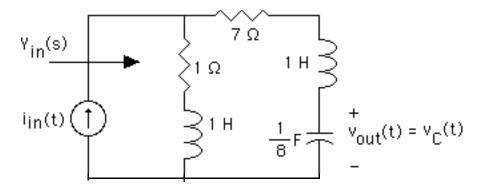
(5)
$$\frac{s(s+2)}{s(s+2)+(s+1)}$$

(6)
$$\frac{(s+1)}{s(s+2)+(s+1)}$$

(7) None of the above



Workout Problem (40 points): Consider the circuit

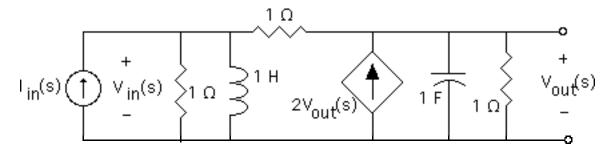


- (a) (7 points) Execute a source transformation in the s-domain, so that the circuit in the s-domain is a pure series circuit. Simplify and clearly draw the new circuit.
- **(b)** (11 points) Find a simplified expression for the transfer function $H(s) = \frac{V_{out}(s)}{I_{in}(s)}$; express the

denominator polynomial as a product of its factors.

- (c) (3 pts) Suppose all initial conditions on the circuit are zero and $i_{in}(t) = 2te^{-t}u(t)$ A (possibly generated by a lightening spike). Compute $I_{in}(s)$.
- (d) (15 points) Find an expression for $V_{out}(s)$ and compute the associated partial fraction expansion.
- (e) (4 points) Compute $v_{out}(t)$.

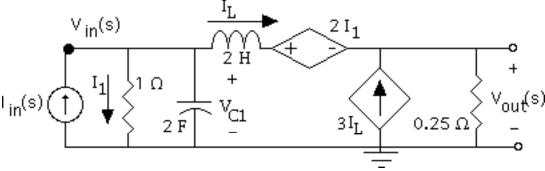
Workout Problem (50 points): Consider the circuit



This problem is to be solved using nodal analysis.

- (a) (5 pts) Draw the equivalent frequency domain circuit assuming $i_L(0^-) = 1$ A and $v_C(0^-) = 1$ V.
- (b) (10 pts) Write two nodal equations for the circuit of part (a) in terms of the voltages $V_{in}(s)$ and $V_{out}(s)$ and of course the input and initial conditions. SIMPLIFY. Put equations in Matrix form.
- (c) (8 pts) Determine the transfer function $H(s) = \frac{V_{out}(s)}{I_{in(s)}}$ of the circuit.
- (d) (3 pts) Find the impulse response h(t) of the circuit.
- (e) (10 pts) If $i_{in}(t) = 4te^{-t}u(t)$ A find the zero-state response $v_{out,zs}(t)$.
- (f) (5 pts) Find the response due only to the initial condition on the inductor.
- (g) (7 pts) Find the response due only to the initial condition on the capacitor.
- **(h)** (2 pts) Find the complete response. State this in words. There is no need to write out all the equations.

Workout Problem (40 points): Consider the circuit



This problem is to be solved using nodal analysis.

- (a) (3 pts) Draw the equivalent frequency domain circuit assuming $i_L(0^-) = 0$ and $v_{C1}(0^-) = 2$ V.
- (b) (11 pts) Write three nodal equations for the circuit of part (a) only in terms of the voltages $V_{in}(s)$, $V_{out}(s)$, $I_L(s)$, and of course the input and initial conditions. SIMPLIFY EACH EQUATION.

(4 pts)
$$I_{in} + Cv_{C1}(0^-) = (2s+1)V_{in} + I_L$$

(3 pts)
$$4V_{out}(s) - 3I_L - I_L = 4V_{out}(s) - 4I_L = 0$$

(4 pts) $V_{in} - V_{out} = 2sI_L + 2V_{in} \implies 0 = V_{in} + V_{out} + 2sI_L$

(c) (3 pts) Put equations in Matrix form.

$$\begin{bmatrix} (2s+1) & 0 & 1 \\ 0 & 4 & -4 \\ 1 & 1 & 2s \end{bmatrix} \begin{bmatrix} V_{in} \\ V_{out} \\ I_L \end{bmatrix} = \begin{bmatrix} I_{in} + Cv_{C1}(0^-) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} I_{in} + 4 \\ 0 \\ 0 \end{bmatrix}$$

(d) (8 pts) Using Crammer's rule, find the transfer function $H(s) = \frac{V_{out}(s)}{I_{in(s)}}$ of the circuit. (If

you use something other than Crammer's rule, maximum points is 6.)

- (e) (4 pts) Find the impulse response h(t) of the circuit.
- (f) (8 pts) Find the response of the circuit to $i_{in}(t) = -8u(t)$ A assuming the initial conditions are zero.
- (g) (3 pts) Find the response due only to the initial condition on the capacitor. A simple observation leads to the answer directly.

SOLUTION: (b) and (c)

$$\begin{aligned} \textbf{(4 pts)} \ \ I_{in} + Cv_{C1}(0^{-}) &= (2s+1)V_{in} + I_{L} \\ \textbf{(3 pts)} \ \ 4V_{out}(s) - 3I_{L} - I_{L} &= 4V_{out}(s) - 4I_{L} &= 0 \\ \textbf{(4 pts)} \ \ V_{in} - V_{out} &= 2sI_{L} + 2V_{in} \implies 0 = V_{in} + V_{out} + 2sI_{L} \\ & \begin{bmatrix} (2s+1) & 0 & 1 \\ 0 & 4 & -4 \\ 1 & 1 & 2s \end{bmatrix} \begin{bmatrix} V_{in} \\ V_{out} \\ I_{L} \end{bmatrix} = \begin{bmatrix} I_{in} + Cv_{C1}(0^{-}) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} I_{in} + 4 \\ 0 \\ 0 \end{bmatrix}$$

(d)

$$H(s) = \frac{V_{out}}{I_{in}} = \frac{\det \begin{bmatrix} (2s+1) & 1 & 1\\ 0 & 0 & -4\\ 1 & 0 & 2s \end{bmatrix}}{\det \begin{bmatrix} (2s+1) & 0 & 1\\ 0 & 4 & -4\\ 1 & 1 & 2s \end{bmatrix}} = \frac{-4}{(2s+1)(8s+4)-4} = \frac{-4}{16s^2 + 16s} = \frac{-1}{4s(s+1)}$$

Alternately, $V_{out} = I_L$ from second equation. Therefore, $-(1+2s)V_{out} = V_{in}$ (third equation) implies

$$I_{in} + Cv_{C1}(0^-) = (2s+1)V_{in} + I_L = -(2s+1)^2V_{out} + V_{out} = -(4s^2+4s)V_{out}$$

The same result follows immediately.

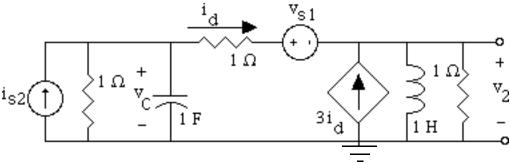
(e)

$$H(s) = \frac{-1}{4s(s+1)} = \frac{-0.25}{s} + \frac{0.25}{s+1} \implies h(t) = 0.25 \left(e^{-t} - 1\right) u(t)$$
(f)

$$V_{out}(s) = H(s)I_{in}(s) = \frac{2}{s^2(s+1)} = \frac{-2}{s} + \frac{2}{s^2} + \frac{2}{s+1}$$
. Hence $V_{out}(t) = 2(t-1+e^{-t})u(t)$

(g) The transfer function from the IC to V_{out} is the same as from I_{in} to V_{out} . Hence, the response due to the IC is simply $Cv_C(0^-)h(t) = \left(e^{-t} - 1\right)u(t)$

Workout Problem (40 points): Full credit requires a clear organized solution. Consider the circuit



This problem is to be solved using nodal analysis.

- (a) (5 pts) Draw the best equivalent frequency domain circuit for nodal analysis accounting for the as yet unknown initial conditions $i_L(0^-)$ and $v_{C1}(0^-)$.
- (b) (11 pts) Following the procedure explained in class, write three nodal equations for the circuit of part (a) only in terms of the variables $V_C(s)$, $V_2(s)$, $I_d(s)$, $V_{s1}(s)$, $I_{s2}(s)$, and the initial conditions. SIMPLIFY EACH EQUATION.
- (c) (4 pts) Put equations in Matrix form.
- (d) (14 pts) Assuming $i_L(0^-) = 0$, $v_{C1}(0^-) = 5$ V, $i_{s2}(t) = 5\delta(t)$ A, and $v_{s1}(t) = 10\delta(t)$ V, use Crammer's rule to find the current $I_d(s)$ and then $i_d(t)$. (If you use something other than Crammer's rule, maximum points are 7.)
- (e) (7 pts) Now suppose that $i_L(0^-) = 0$, $v_{C1}(0^-) = 0$ V, $i_{s1}(t) = 0$ A, and $v_{s1}(t) = 10u(t)$ V. Find $v_2(t)$.

Solution: (b)

(3 pts)
$$I_{s2} + Cv_{C1}(0^-) = (s+1)V_C + I_d$$

(4 pts)
$$-\frac{i_L(0^-)}{s} = \left(\frac{s+1}{s}\right)V_2 - 4I_d$$

(4 pts)
$$V_C - V_2 = I_d + V_{s1} \implies V_C - V_2 - I_d = V_{s1}$$

(c)

$$\begin{bmatrix} (s+1) & 0 & 1 \\ 0 & \frac{s+1}{s} & -4 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} V_C \\ V_2 \\ I_d \end{bmatrix} = \begin{bmatrix} I_{s2}(s) + Cv_C(0^-) \\ -\frac{i_L(0^-)}{s} \\ V_{s1}(s) \end{bmatrix}$$

(d) The equations become

$$\begin{bmatrix} (s+1) & 0 & 1 \\ 0 & \frac{s+1}{s} & -4 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} V_C \\ V_2 \\ I_d \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 10 \end{bmatrix}$$

Hence

$$I_d(s) = \frac{\det\begin{bmatrix} (s+1) & 0 & 10 \\ 0 & \frac{s+1}{s} & 0 \\ 1 & -1 & 10 \end{bmatrix}}{\det\begin{bmatrix} (s+1) & 0 & 1 \\ 0 & \frac{s+1}{s} & -4 \\ 1 & -1 & -1 \end{bmatrix}} = \frac{\frac{10(s+1)^2}{s} - \frac{10(s+1)}{s}}{(s+1)\left[-\frac{s+1}{s} - 4\right] - \frac{s+1}{s}} = \frac{10(s+1)}{-(s+1)\left[\frac{5s+2}{s}\right]}$$
$$= \frac{-10s}{5s+2} = \frac{-2s}{s+0.4} = -2 + \frac{0.8}{s+4} \implies i_d(t) = -2\delta(t) + 0.8e^{-0.4t}u(t)$$

(e)

$$V_2(s) = \frac{\det \begin{bmatrix} (s+1) & 0 & 1 \\ 0 & 0 & -4 \\ 1 & \frac{10}{s} & -1 \end{bmatrix}}{\det \begin{bmatrix} (s+1) & 0 & 1 \\ 0 & \frac{s+1}{s} & -4 \\ 1 & -1 & -1 \end{bmatrix}} = \frac{(s+1)4\frac{10}{s}}{-(s+1)\left[\frac{5s+2}{s}\right]} = \frac{-40}{5s+2} = \frac{-8}{s+0.4}$$

Thus $v_2(t) = -8e^{-0.4t}u(t)$ V.