CS182
HW #8

Yinglai Wang

Q1. $f(n) = 3f(n/3) + 1 = 3[3f(n/3^2) + 1] + 1$

$= 3^3 f(n/3^3) + 3^2 + 3^1 + 1 = \cdots\cdots$

$f(n) = 3^k f(n/3^k) + \underbrace{3^{k-1} + 3^{k-2} + \cdots + 1}_{k}$

$= 3^k f(n/3^k) + \frac{3^k - 1}{2}$

let $n = 3^k$

we have $f(n) = nf(1) + \frac{n-1}{2}$

Q2. $f(n/3) = 3f(n/3^2) + \frac{n}{3}$ $\quad f(n/3^2) = 3f(n/3^3) + (\frac{n}{3^2})$

$f(n) = 3f(n/3) + n = 3[3f(n/3^2) + \frac{n}{3}] + n$

$= 3^2 f(n/3^2) + 2n = 3^2[3f(n/3^3) + \frac{n}{3^2}] + 2n$

$= 3^3 f(n/3^3) + 3n = 3^k f(n/3^k) + kn$

Let $n = 3^k$, $\Rightarrow k = \log_3 n$

we have $f(n) = nf(1) + \log_3 n \cdot n$

$= n(1 + \log_3 n)$

Q3. $P(A) = \frac{\text{valid combinations}}{\text{size of set}} = \frac{N(N-1)(N-2)(N-3)\cdots(N-n+1)}{N^n}$

$= \frac{N!}{N^n (N-n)!}$

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Q4 (i) round 1 $= -2^0 \cdot 100$

round 2 $= -2^1 \cdot 100$

round 3 $= -2^2 \cdot 100$

round 4 $= -2^3 \cdot 100$

round k $= -2^{k-1} \cdot 100$

round k+1 $= 2^k \cdot 100$ (because this round he wins)

$$\text{Net gain} = 100 \cdot \left[2^k - \left[2^{k-1} + 2^{k-2} + 2^{k-3} + \cdots 2^0\right]\right]$$
$$= 100 \cdot \left(2^k - \frac{2^k - 1}{2-1}\right)$$
$$= 100.$$

So, the net gain is 100 \$.

(ii) expected gain

$$E = \sum p(x) \cdot x = \sum \left(\tfrac{1}{2}\right)^k \left(100 \cdot 2^{(k+1)2} - 100 \cdot 2^{k+1} - 100 \cdot 2^k\right)$$
$$= \sum \left(\tfrac{1}{2}\right)^k \cdot 2^k (\cdots\cdots).$$