FINAL EXAM

Friday, Dec. 14, 8am-10am in STEW 183, also known as Loeb Playhouse.

Material covered: Comprehensive (Ch. 14-24), emphasizing Ch. 21-24.
Format: 20 multiple choice questions. (No hand-graded question.)

1. Closed book exam: no notes, books, etc. Necessary equations will be provided.
2. BRING: Pencil, calculator, Purdue student ID card.
Make sure your calculator cannot access internet. Graphing calculator is okay.

Practice exam and equation sheet will be posted this week.
Solutions will be posted next week.

STUDY HARD AND GOOD LUCK!!

Last Time

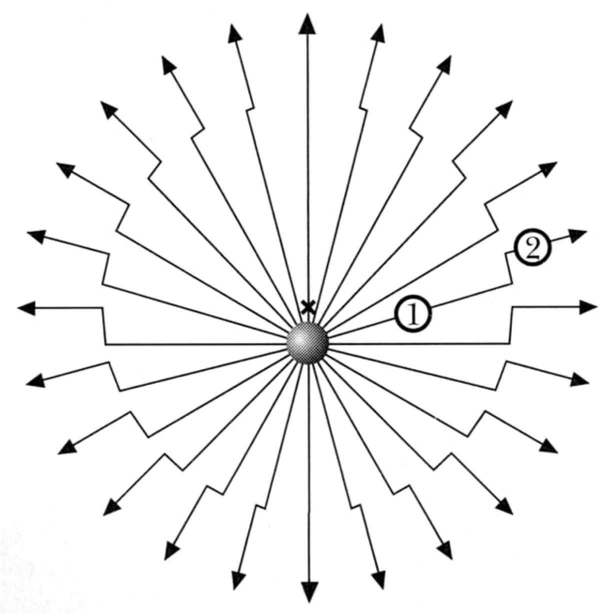
Maxwell Equations – complete!
Wave solutions

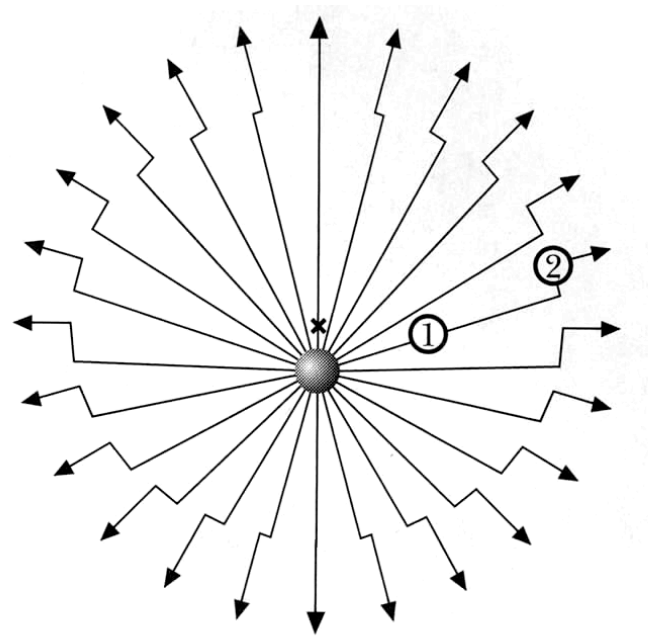
Today

Accelerated Charges
Energy and Poynting Vector
Momentum and Poynting Vector
Re-Radiation (scattering)
Polarized Light
Why the sky is blue

Accelerated Charges

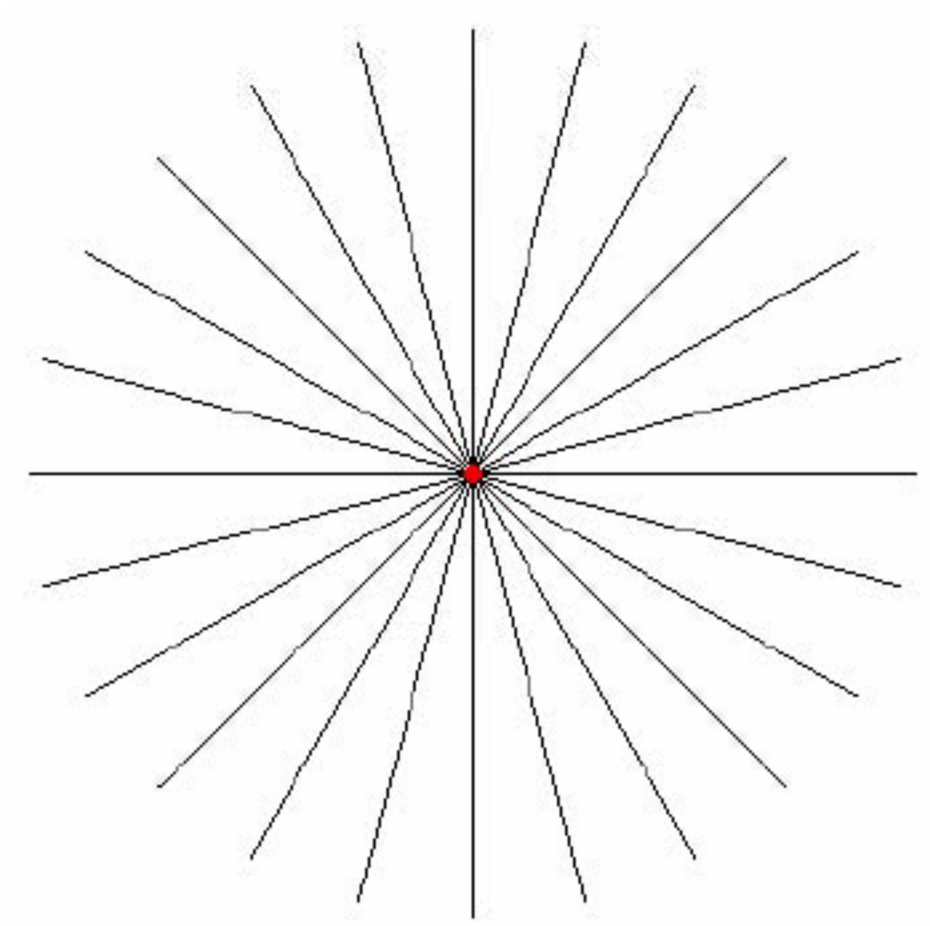
Electromagnetic pulse can propagate in space
How can we initiate such a pulse?



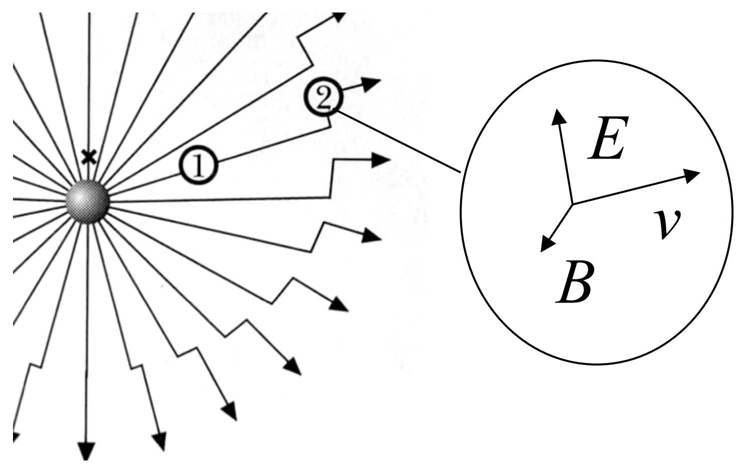


Short pulse of transverse electric field

Accelerated Charges



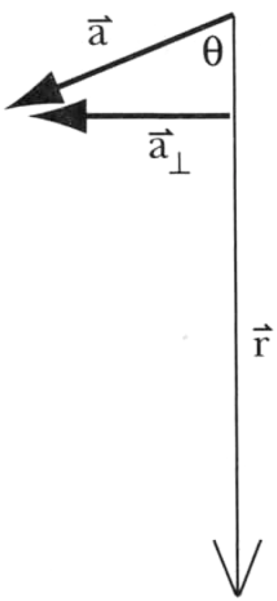
1. Transverse pulse propagates at speed of light
2. Since $E(t)$ there must be B
3. Direction of v is given by: $\vec{E} \times \vec{B}$



Magnitude of the Transverse Electric Field

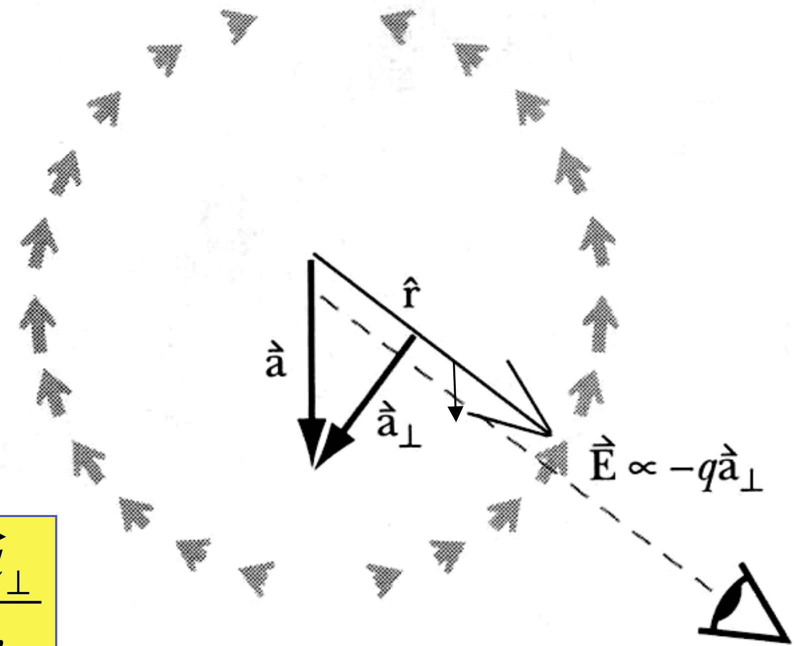
We got the direction.

Magnitude can be derived from Gauss's law*



Field ~ $-q\vec{a}_\perp$

$$\boxed{\vec{E}_{radiative} = \frac{1}{4\pi\varepsilon_0}\frac{-q\vec{a}_\perp}{c^2 r}}$$



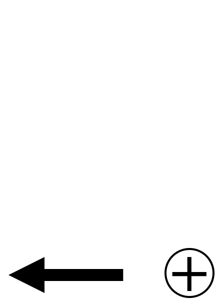
1. The direction of the transverse field is opposite to $q\vec{a}_\perp$

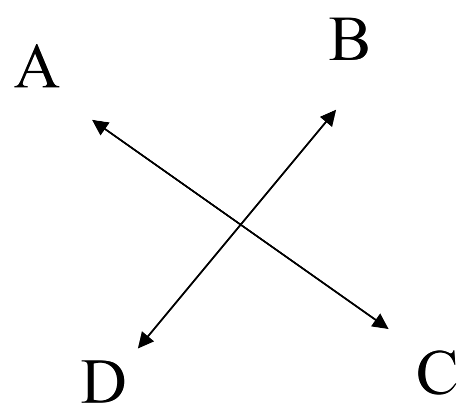
2. The electric field falls off at a rate $1/r$

* This was first shown by Edward Purcell, BSEE Purdue, Nobeal Laureate!

iClicker Question

A proton is briefly accelerated as shown below. What is the direction of the radiative electric field that will be detected at location A?





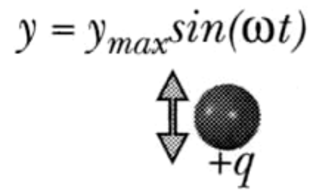
Sinusoidal Electromagnetic Radiation

Acceleration:

$$a = \frac{d^2 y}{dt^2} = -y_{max}\omega^2 \sin(\omega t)$$

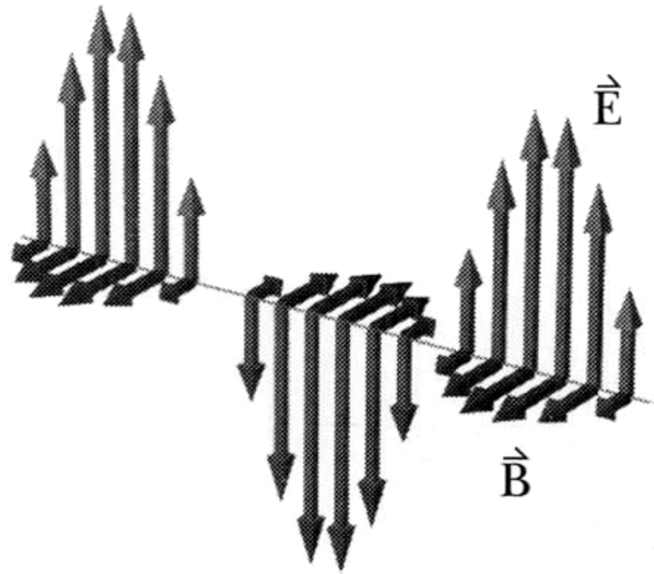
$$\boxed{\vec{E}_{radiative} = \frac{1}{4\pi\varepsilon_0}\frac{-q\vec{a}_\perp}{c^2 r}}$$

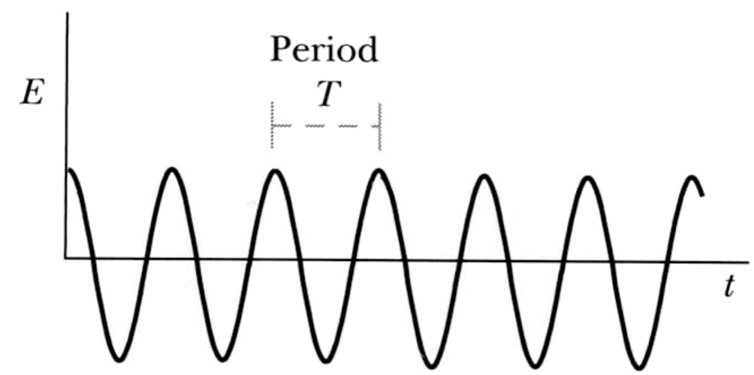
$$\vec{E}_{radiative} = \frac{1}{4\pi\varepsilon_0}\frac{q y_{max}\omega^2}{c^2 r}\sin(\omega t)\hat{j}$$



$$f = \frac{\omega}{2\pi}$$

$$T = 1/f$$



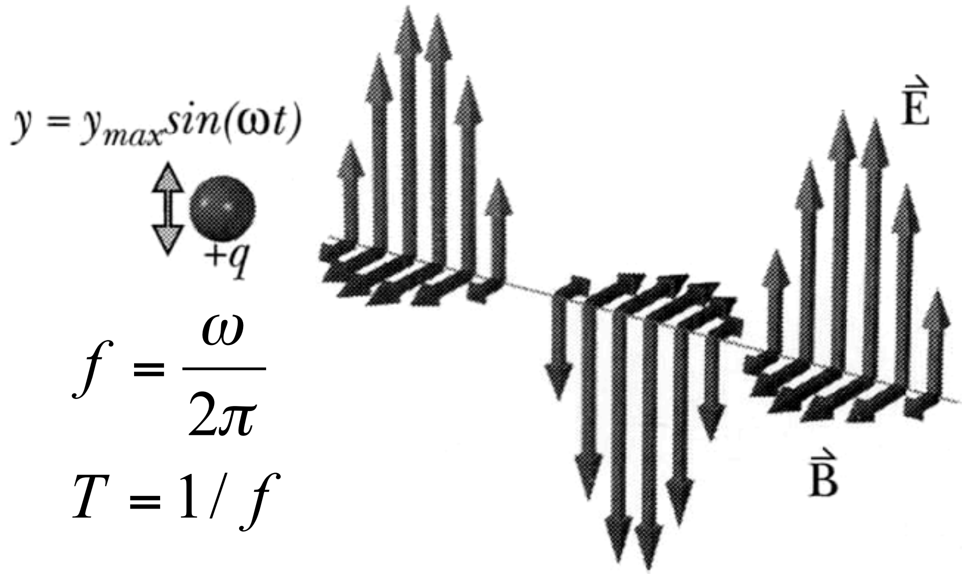


Sinusoidal E/M field

Sinusoidal E/M Radiation: Wavelength

Instead of period can use wavelength:

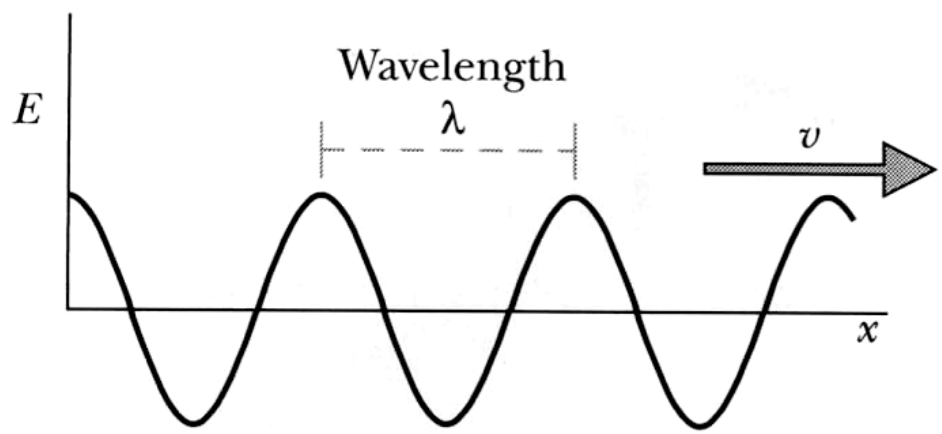
$$\lambda = cT = \frac{c}{f}$$



$$f = \frac{\omega}{2\pi}$$

$$T = 1/f$$

Freeze picture in time:



Example of sinusoidal E/M radiation:

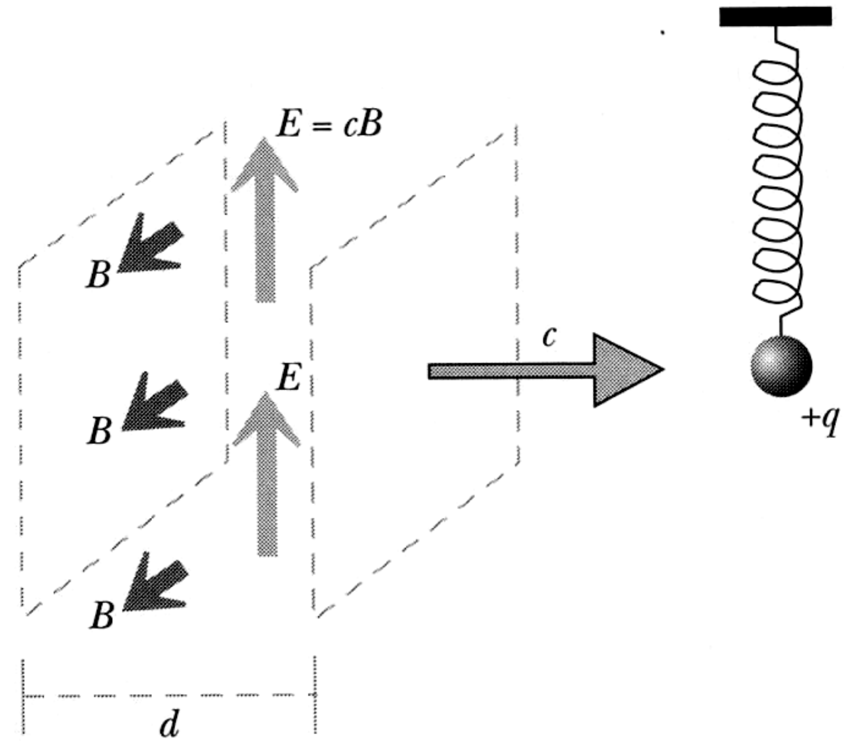
- atoms
- radio stations
- E/M noise from AC wires

Energy of E/M Radiation

A particle will experience electric force during a short time *d/c:*

$$F_{elec} = qE$$

$$\Delta p = p - 0 = F_{elec}\Delta t = \left(qE\right)\left(\frac{d}{c}\right)$$



What will happen to the ball?

It will oscillate

Energy was transferred from E/M field to the ball

$$\Delta K = K - 0 \approx \frac{p^2}{2m} = \left(\frac{qEd}{c}\right)^2\left(\frac{1}{2m}\right) \longrightarrow$$

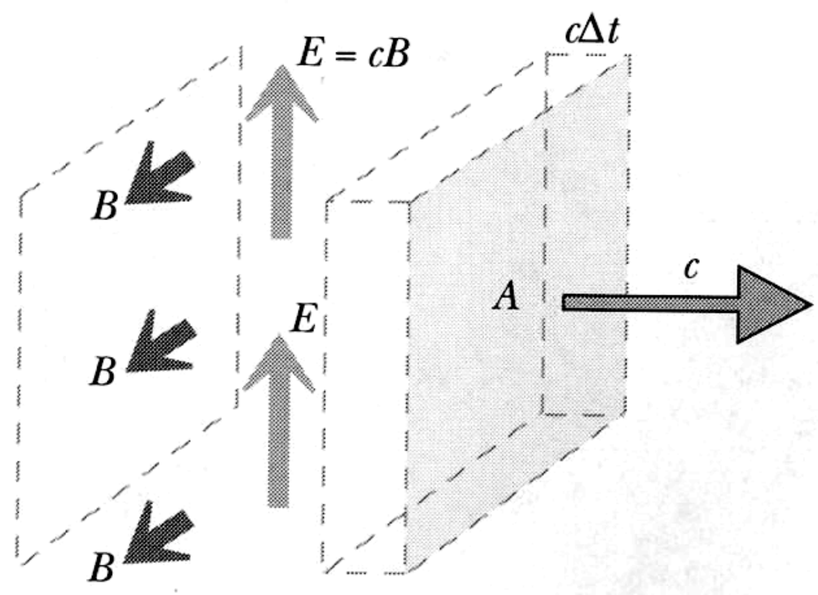
Amount of energy in the pulse is $\sim E^2$

Energy Flux

There is E/M energy stored in the pulse: $\dfrac{Energy}{Volume} = \varepsilon_0 E^2$ (J/m^3)

Pulse moves in space:

there is energy flux



Units: J/(m^{2}s) = W/m^2

During Δt:

$$Energy = \left(\varepsilon_0 E^2\right)\left(A \cdot c\Delta t\right)$$

$$flux = \frac{Energy}{\left(A \cdot \Delta t\right)} = c\varepsilon_0 E^2$$

$$flux = \frac{1}{\mu_0} EB$$ $\longleftarrow$ used: $E=cB$, $\mu_0\varepsilon_0=1/c^2$

Energy Flux: The Poynting Vector

$$flux = \frac{1}{\mu_0} EB$$

The direction of the E/M radiation was given by $\vec{E} \times \vec{B}$

Energy flux, the "Poynting vector":

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \qquad (\text{in W/m}^2)$$

John Henry Poynting (1852-1914)

- S is the rate of energy flux in E/M radiation
- It points in the direction of the E/M radiation

Note energy flux = [Energy/(time * Area)]
Energy flux = Intensity = [Power/Area] Same Thing!

Example

In the vicinity of the Earth, the energy intensity of radiation emitted by the sun is ~1400 W/m^2. What is the approximate magnitude of the electric field in the sunlight?

Solution:

$$flux = \frac{1}{\mu_0} EB = c\varepsilon_0 E^2$$

$$\downarrow$$

$$E = \sqrt{\frac{flux}{c\varepsilon_0}} = 725\ \mathrm{N/C}$$

Note: this is an average (rms) value

Momentum of E/M Radiation

- *E* field starts motion
- Moving charge in magnetic field:

$$\vec{F}_{mag} = q\vec{v} \times \vec{B}$$

What if there is negative charge?

$$\vec{F}_{mag} = -|q|\vec{v} \times \vec{B}$$

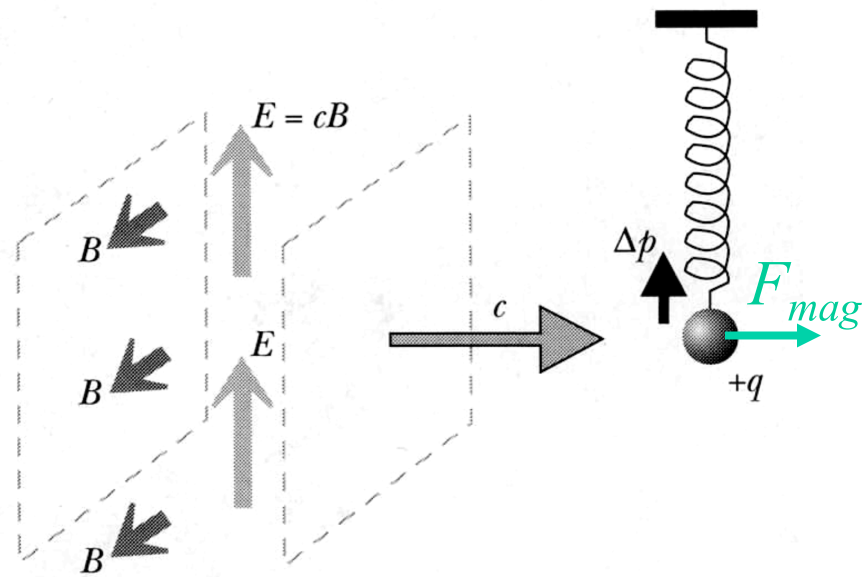
'Radiation pressure':

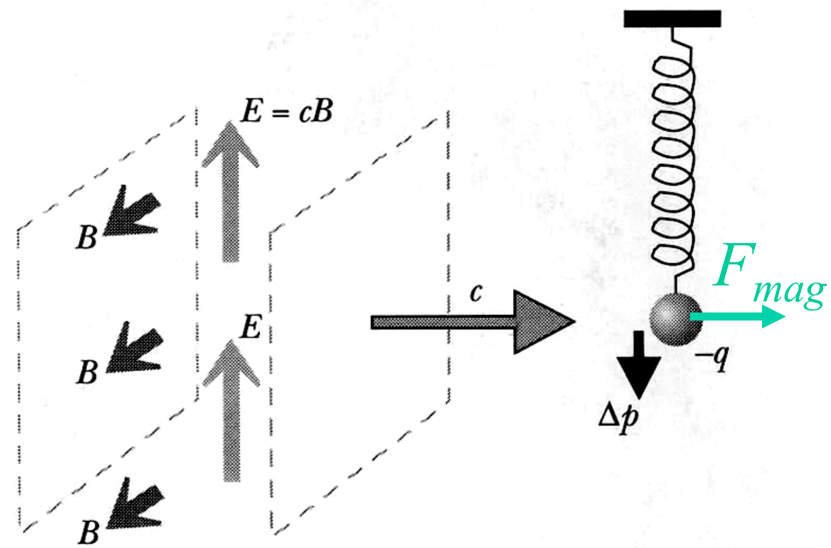
What is its magnitude?

Average speed: $v/2$

$$F_{mag} = q\frac{v}{2}B = q\frac{v}{2}\frac{E}{c}$$

$$\frac{F_{mag}}{F_{elec}} = q\frac{v}{2}\frac{E}{c}/(qE) = \frac{v}{2c} << 1$$



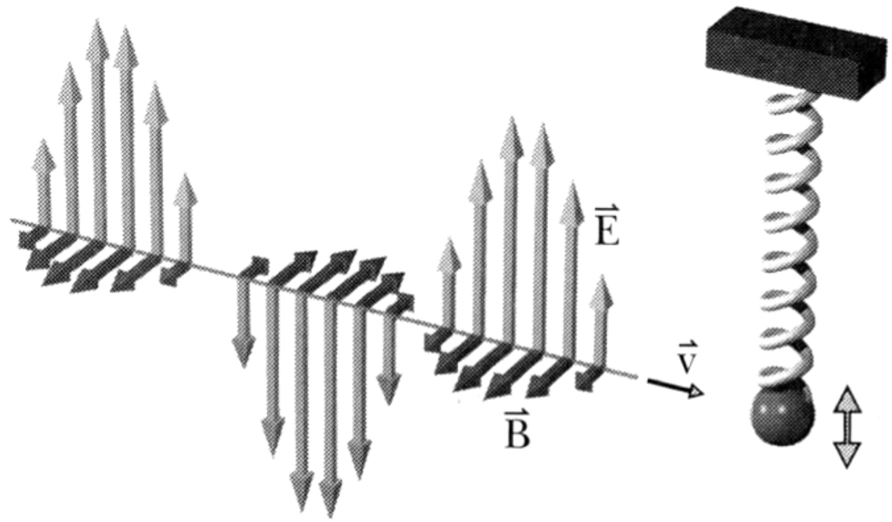


Momentum Flux

Net momentum:

in transverse direction: 0

in longitudinal direction: >0



Relativistic energy:

$$E^2 = (pc)^2 + (mc^2)^2$$

Quantum view: light consists of photons with zero mass: $E^2 = (pc)^2$

Classical (Maxwell): it is also valid, i.e. momentum = energy/speed

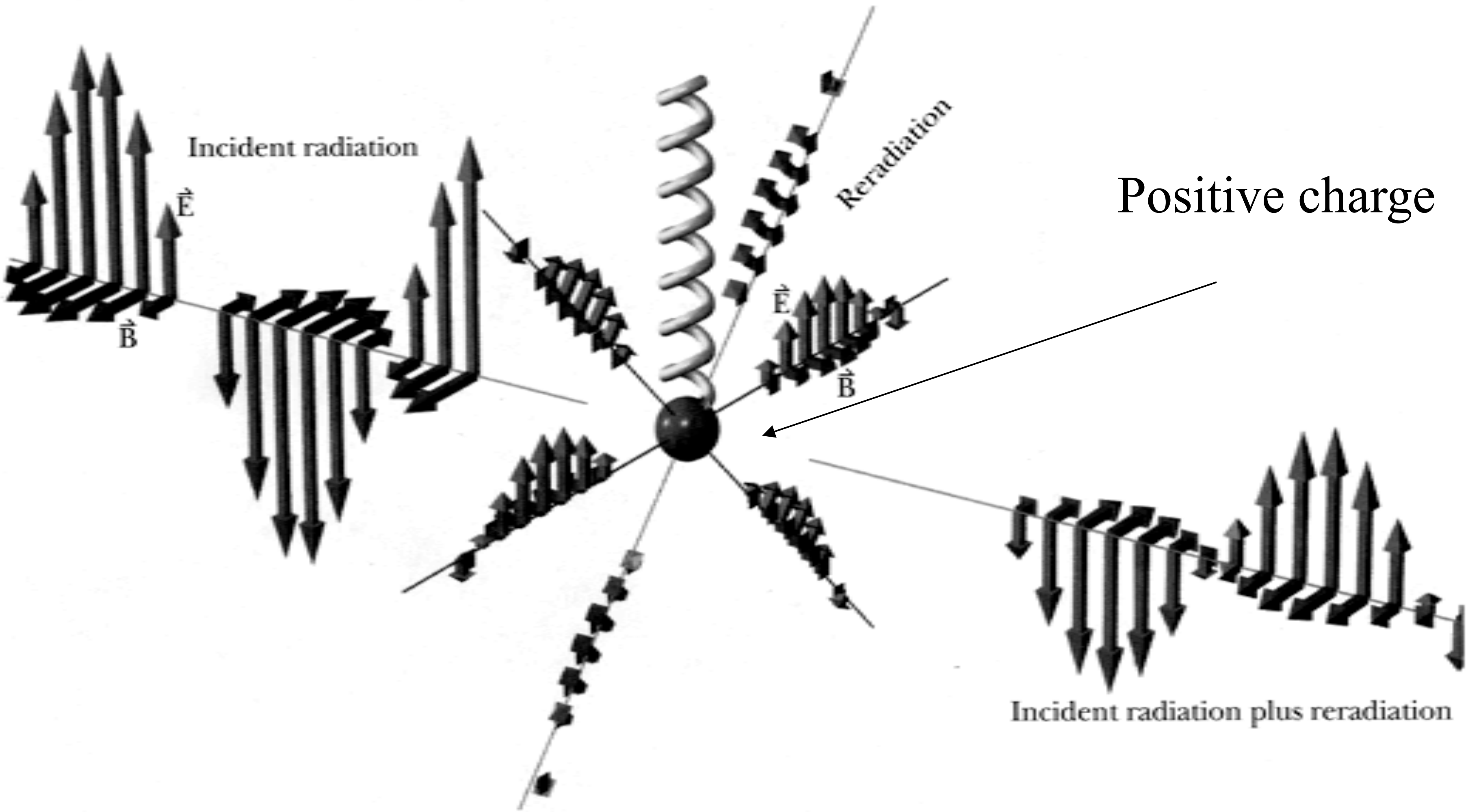
$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \longrightarrow$$

Momentum flux:

$$\frac{\vec{S}}{c} = \frac{1}{\mu_0 c}\vec{E}\times\vec{B} \quad (\text{in N/m}^2)$$

Units of Pressure

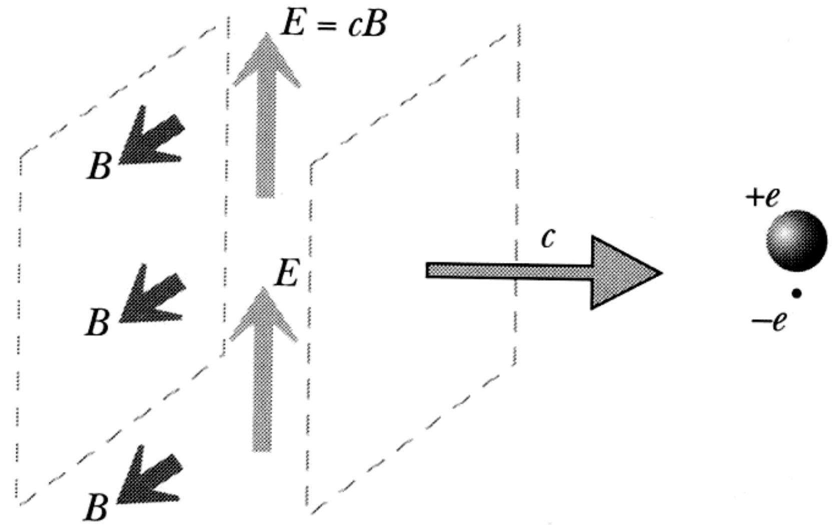
Re-radiation: Scattering



Effect of Radiation on a Neutral Atom

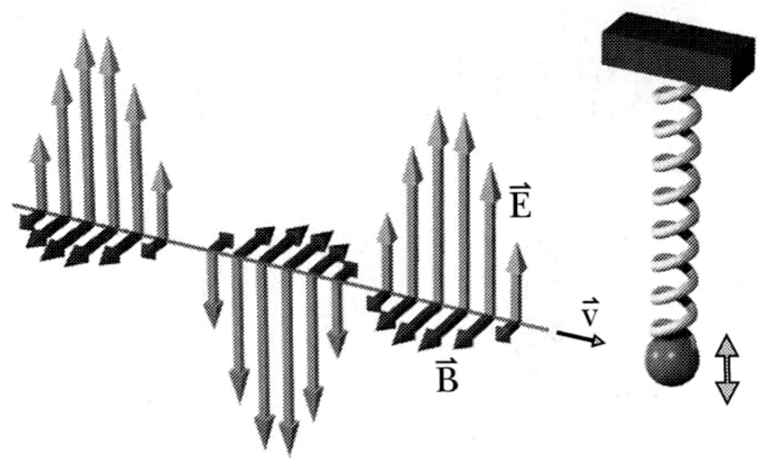
Main effect:

brief electric kick sideways



Neutral atom: polarizes

Electron is much lighter than nucleus:
can model atom as outer electron connected to the rest of the atom by a spring:

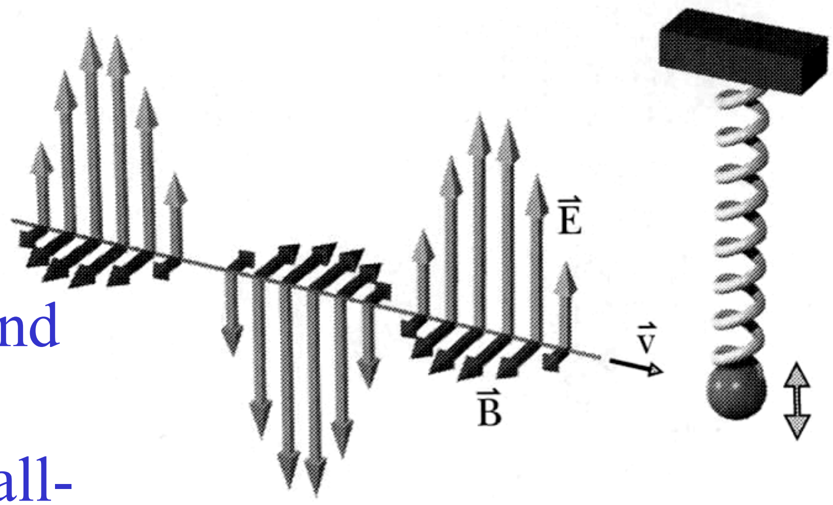


$$F=eE$$

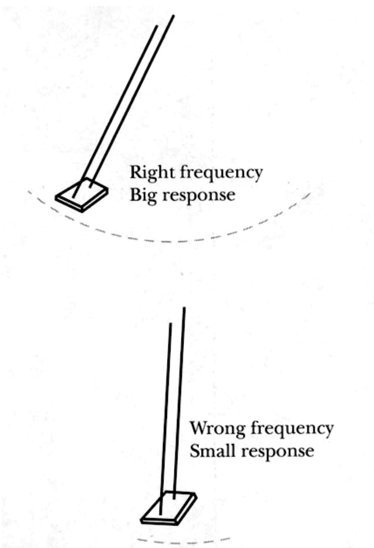
Radiation and Neutral Atom: Resonance

$$E_y = E_0 \sin(\omega t)$$

$$F_y = -eE_y = F_0 \sin(\omega t)$$



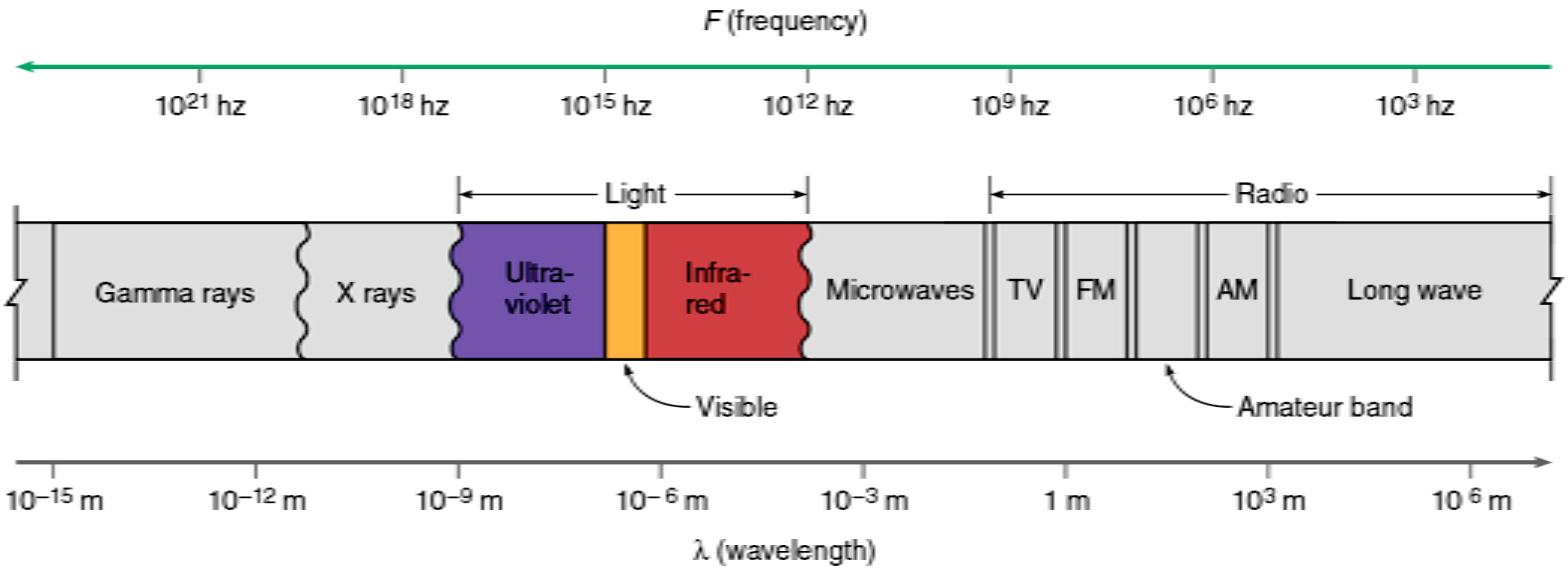
Amplitude of oscillation will depend on how close we are to the natural free-oscillation frequency of the ball-spring system

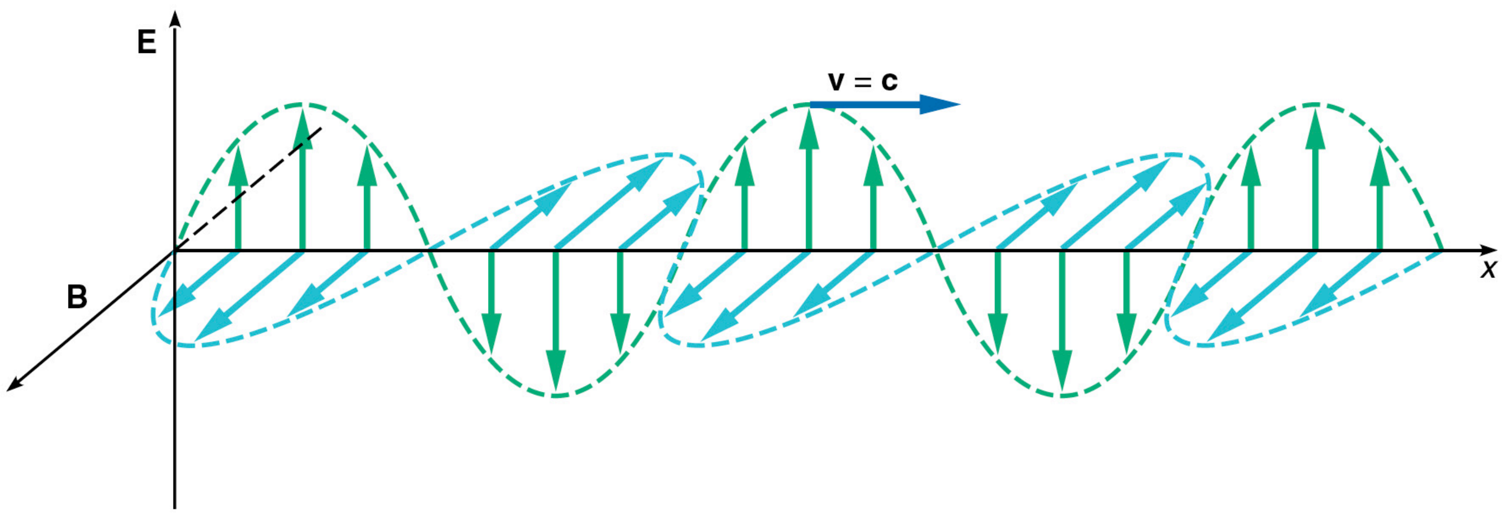


Resonance

This is why you can tune a radio

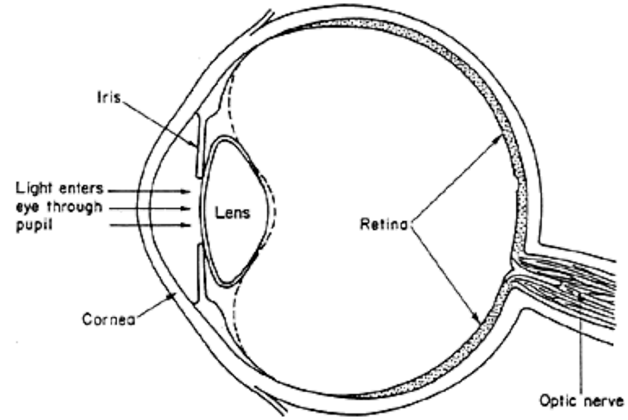
Electromagnetic Spectrum

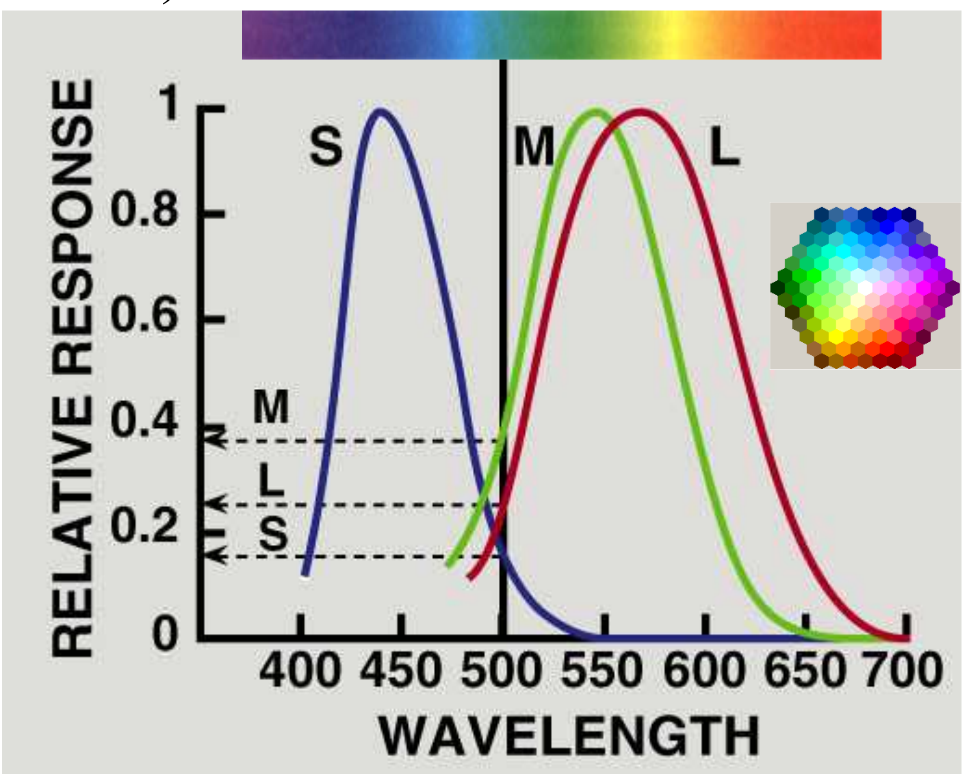




Color Vision

Three types of receptors (cones) in retina which incorporate three different organic molecules which are in resonance with red, green and blue light frequencies (RGB-vision):





Response spectra for three types of receptors

Max response wavelengths:

S – 440 nm ($6.81 \cdot 10^{14}$ Hz)

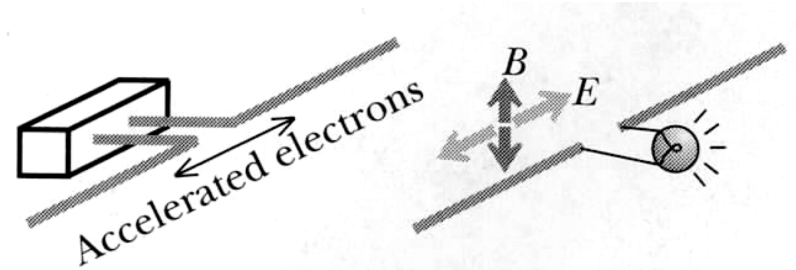
M – 540 nm ($5.56 \cdot 10^{14}$ Hz)

L – 560 nm ($5.36 \cdot 10^{14}$ Hz)

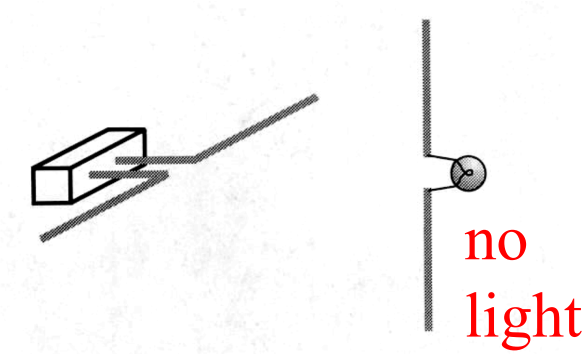
Refers to length of cone

Polarized E/M Radiation

AC voltage
(~300 MHz)

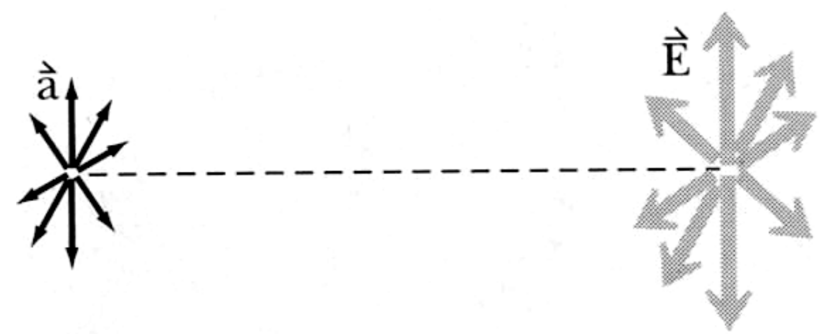


What will happen if distance is increased twice?



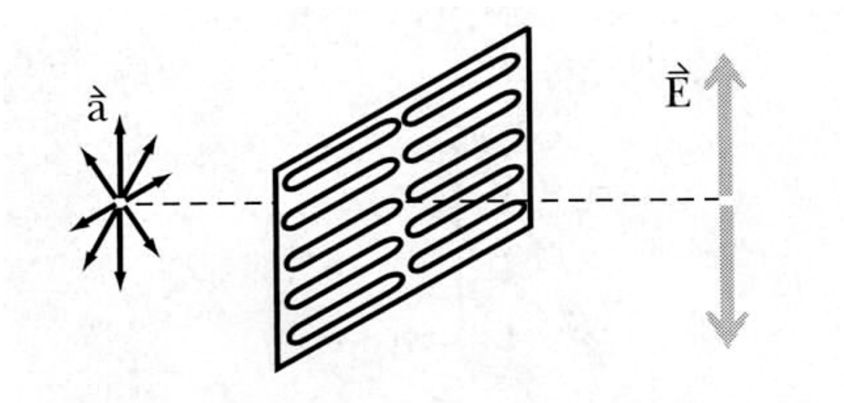
E/M radiation can be polarized along one axis…

…and it can be unpolarized:

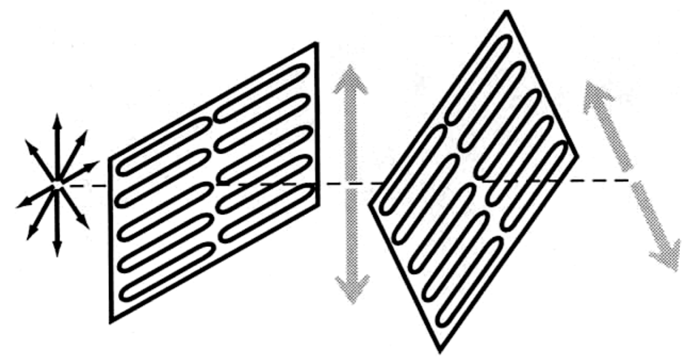


Polarized Light

Making polarized light



Turning polarization



Polaroid sunglasses and camera filters:

reflected light is highly polarized: can block it

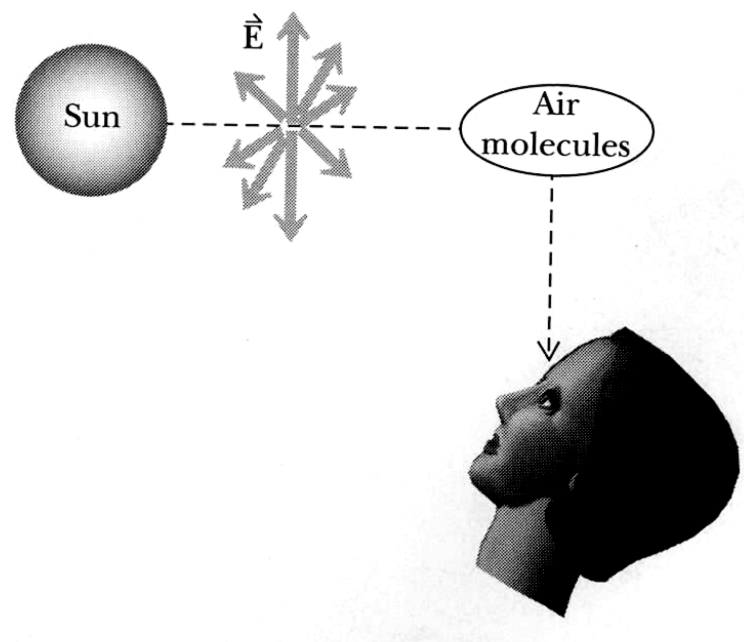
Considered: using polarized car lights and polarizers-windshields

Why the Sky is Blue

Why there is light coming from the sky?

Why is it polarized?

Why is it blue?



$$x = x_0 \sin(\omega t)$$

$$E \sim a = \frac{d^2 x}{dt^2} = -y_0 \omega^2 \sin(\omega t)$$

Energy flux: $\sim E^2 \sim \omega^4$

Ratio of blue/red frequency is ~2 $\Rightarrow$ scattering intensity ratio is 16

Why is sun red at sunset? Is its light polarized?

Why are distant mountains blue?

Today

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Energy and Poynting Vector
Momentum and Poynting Vector
Re-Radiation (scattering)
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