Fall 2013 HW#2 Solution

2.10

electron velocity $V_e = -\mu_e \mathcal{E} = (700\,cm^2/Vs)(2500\,\frac{V}{cm}) = \boxed{-1.75\times10^6\,\frac{cm}{s}}$

hole velocity $V_h = \mu_h \mathcal{E} = (250\,cm^2/Vs)(2500\,\frac{V}{cm}) = \boxed{625\times10^3\,\frac{cm}{s}}$

electron current density $j_n = -q \cdot n \cdot V_e = (1.602\times10^{-19}C)(10^{17}\,cm^{-3})(-1.75\times10^6\,\frac{cm}{s}) = \boxed{28000\,\frac{A}{cm^2}}$

hole current density $j_p = q \cdot p \cdot V_h = (1.602\times10^{-19}C)(10^3\,cm^{-3})(625\times10^3\,\frac{cm}{s}) = \boxed{1\times10^{-10}\,\frac{A}{cm^2}}$

2.16

$$j_p = q V_h p = (1.602\times10^{-19}C)(10^{19}\,cm^{-3})(10^7\,cm/s)$$
$$\boxed{= 1.60\times10^7\,A/cm^2}$$

$$I_p = j_p \cdot A = (1.60\times10^7\,A/cm^2)(1\mu m)(25\mu m)\left(\frac{1.00\,cm}{10^6\mu m}\right)^2$$

$$\boxed{I_p = 4\,A}$$

2.31

$N_A > N_D$, therefore p-type

use equation 2.12

$$p = \frac{(N_A - N_D) + \sqrt{(N_A - N_D)^2 + 4n_i^2}}{2}$$

$$p = \frac{(9\times10^{15}\,cm^{-3}) + \sqrt{(9\times10^{15}\,cm^{-3})^2 + 4(5\times10^{13}\,cm^{-3})^2}}{2}$$

$$p = \boxed{9\times10^{15}\,cm^{-3}}, \quad n = \frac{n_i^2}{p} = \frac{(5\times10^{13}\,cm^{-3})^2}{9\times10^{15}\,cm^{-3}} = \boxed{2.78\times10^{11}\,cm^{-3}}$$

Above always works.

Alternative solution:

use equation 2.10

$$N_D + p - N_A - n = 0$$
$$N_A - N_D - p + \frac{n_i^2}{p} = 0$$

However $\frac{n_i^2}{p} \ll N_A, N_D, p$, therefore neglect it
true if $N_A - N_D \gg n_i$

$$N_A - N_D - p \approx 0$$
$$p \approx N_A - N_D = \boxed{9\times10^{15}\,cm^{-3}}$$

$$n = \frac{n_i^2}{p} = \boxed{2.78\times10^{11}\,cm^{-3}}$$

2.34

Because the silicon is doped with acceptors, it is $\boxed{\text{p-type.}}$

using equation 2.10

$$N_A - N_D + n - p = 0$$

$N_A >> n_i \approx 10^{10}\,cm^{-3}$ @ 300K, therefore $N_A, p >> n$
and $\boxed{p \approx N_A = 2.5 \times 10^{18}\,cm^{-3}}$

$$n = \frac{n_i^2}{p} = \frac{(1 \cdot 10^{10}\,cm^{-3})^2}{2.5 \times 10^{18}\,cm^{-3}} = \boxed{40\,cm^{-3} = n}$$

from Fig. 2.8:

$$\mu_n = 92 + \frac{1270}{1 + \left(\frac{N_A + N_D}{1.3 \times 10^{17}}\right)^{0.91}} = \boxed{173\,cm^2/Vs}$$

$$\mu_p = 48 + \frac{447}{1 + \left(\frac{N_A + N_D}{6.3 \times 10^{16}}\right)^{0.76}} = \boxed{73\,cm^2/Vs}$$

$$N_T = N_A + N_D = N_A = 2.5 \times 10^{18}\,cm^{-3}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p} \approx \frac{1}{qp\mu_p} = \frac{1}{(1.602 \times 10^{-19}\,C)(2.5 \times 10^{18}\,cm^{-3})(73\,\frac{cm^2}{Vs})}$$

because $n << p$ $\quad \boxed{\rho = 30.5 \times 10^{-3}\,\Omega \cdot cm}$

2.45

Part A

- Since sample is only doped w/ acceptors, most likely $p \approx N_A >> n$. Assume this and check assumption later at end.

this is greek "rho" for resistivity

① $\rho = \frac{1}{\sigma} \approx \frac{1}{q p \mu_p} = \frac{1}{q\, p \cdot \mu_p(N_T = N_A = p)}$

this is hole concentration

② $\mu_p(N_T = p) = 48 + \frac{447\ cm^2/Vs}{1 + \left(\frac{p}{6.3\times10^{16}\ cm^{-3}}\right)^{0.76}}$ from figure 2.8

- Solve equation ① substituting equation ② for $\mu_p(N_T)$ w/ $\rho = 1\,\Omega.cm$ and $N_T = N_A = p$
this gives $p = 1.67\times10^{16}\ cm^{-3} = N_A$ for $\rho = 1\,\Omega.cm$

- because $p = 1.67\times10^{16}\ cm^{-3}$, our assumption of $p \approx N_A >> n$ is valid

- Now, solve equation ① again for $\rho = 0.25\,\Omega.cm$
this gives $p = 1.12\times10^{17}\ cm^{-3} = N_A$ for $\rho = 0.25\,\Omega.cm$

- subtract the two values of N_A for the two resistivities

$N_{A2} - N_{A1} = 1.12\times10^{17}\ cm^{-3} - 1.67\times10^{16}\ cm^{-3} = 9.50\times10^{16}\ cm^{-3}$

- Therefore $9.50\times10^{16}\ cm^{-3}$ boron dopant atoms must be added to change the resistivity as desired.

2.45 part B

- From part A, the p-type doping of the origional sample is $N_A = 1.67 \times 10^{16}\ cm^{-3}$

- To reduce the resistivity using doners, the sample must be counter-doped w/ $N_D > N_A$. Therefore the sample will change to $\boxed{\text{n-type.}}$
Also, therefore, $N_D - N_A >> n_i$ and $n \approx N_D - N_A >> p$, so we solve the following:

(3) $$\rho \approx \frac{1}{qn\mu_n(N_T)} = \frac{1}{q\, n\, \mu_n(N_T = N_D - N_A = n)}$$

(4) $$\mu(n) = 92 + \frac{1270}{1 + \left(\frac{N_T = N_D - N_A = n}{1.3 \times 10^{17}\ cm^{-3}}\right)}\ cm^2/Vs$$ from figure 2.8

- Solve equation (3) for n, substituting for equation (4) with $\rho = 0.25\ \Omega.cm$, this gives
$n = N_D - N_A = 2.16 \times 10^{16}\ cm^{-3}$

- Therefore $N_D = n + N_A = 2.16 \times 10^{16}\ cm^{-3} + 1.67 \times 10^{16}\ cm^{-3}$
$$\boxed{N_D = 3.82 \times 10^{16}\ cm^{-3}}$$

$\boxed{\text{$N_D = 3.82 \times 10^{16}\ cm^{-3}$ must be added to the silicon to achieve 0.25 Ω.cm resistivity using donor atoms}}$

2.49

$$j_n = -q D_n \left(-\frac{dn}{dx}\right)$$

equation 2.15: $D_n = \frac{KT}{q} \mu_n$ (for non-degenerate semiconductor)

$$j_n = -q \frac{KT}{q} \mu_n \left(-\frac{dn}{dx}\right) = -KT \mu_n \left(\frac{-dn}{dx}\right)$$

$$\frac{dn}{dx} = \frac{10^{18}\,cm^{-3}}{W_B} = \frac{10^{18}\,cm^{-3}}{0.5\,\mu m} \cdot \left(\frac{10^6\,\mu m}{100\,cm}\right) = 2\times10^{22}\,cm^{-4}$$

$\mu_n = 350\,cm^2/Vs$ (given)

$KT = 0.0259\,eV \cdot \frac{1.602\times10^{-19}\,J}{1\,eV}$ at $T = 300K$ (Room temp)

$$j_n = 0.0259\,eV \cdot \left(\frac{1.602\times10^{-19}\,J}{1\,eV}\right)\left(350\,cm^2/Vs\right)\left(2\times10^{22}\,cm^{-4}\right)$$

$\boxed{j_n = 29000\ A/cm^2}$ (units $\cancel{eV} \cdot \frac{\cancel{V} \cdot C}{\cancel{eV}} \cdot \frac{cm^2}{\cancel{V} s} \cdot cm^{-4} = \frac{A}{cm^2}$)

2.51

- $N_A(x) = 10^{14} + 10^{18}\exp(-10^4 x)$ x in cm

- Because $N_A >> n_i$, $p \approx N_A >> n$

- Total current must equal zero in equlibrium

- $J_{n,diffusion} + J_{p,diffusion} + J_{n,drift} + J_{p,drift} = 0$ (eq. 2.17)

because $n << p$, $J_{n,diffusion}$ and $J_{n,drift} \approx 0\,A$.

so $J_{p,diffusion} + J_{p,drift} = 0$ (eq. 2.17)

$J_{p,diffusion} = -q\mu_p V_T \frac{dp}{dx}$, $J_{p,drift} = qp\mu_p\mathcal{E}$

- Because $\frac{dp}{dx} \approx \frac{dN_A}{dx} \neq 0$, there is a diffusion current. Therefore there must be a built in electric field $\mathcal{E}$ so that $J_{total} = 0$.

- What is $\mathcal{E}$ at $x = 0\,\mu m$?

- $\frac{dp}{dx} \approx \frac{dN_A}{dx} = -10^{18} \cdot 10^4 \cdot \exp(-10^4 x) = -10^{22}\exp(-10^4 x)\ cm^{-4}$

$\left.\frac{dN_A}{dx}\right|_{x=0} = -10^{22}\ cm^{-4}$

- $J_{p,diffusion} - J_{p,drift} = 0 \Rightarrow -q\mu_p V_T \frac{dp}{dx} = qp\mu_p\mathcal{E}$

$\Rightarrow V_T\frac{dp}{dx} = p\mathcal{E} \Rightarrow \mathcal{E} = \frac{V_T}{p}\frac{dp}{dx}$

- so $\mathcal{E} = \frac{V_T}{p}\frac{dp}{dx} \approx \frac{V_T}{N_A(x)}\frac{dN_A}{dx} = \frac{0.0259\,V}{10^{18}\,cm^{-3}} \cdot -10^{22}\,cm^{-4} = -259\,\frac{V}{cm}$

- $\boxed{\text{At } x = 0\,\mu m \text{, there is a built in electric field of } -259\,\frac{V}{cm}}$

2.51 (continued)

- What is electric field $\mathcal{E}$ at $x = 5\mu m$?

$$\frac{dp}{dx} \approx \left.\frac{dN_A}{dx}\right|_{x=5\mu m} = -10^{22} \exp\left(-10^4\left(5\mu m \times \frac{100\,cm}{10^6\,\mu m}\right)\right) cm^{-4}$$

$$= -6.74 \times 10^{-21}\ cm^{-4}$$

$$N_A(x=5\mu m) = 10^{14} + 10^{18} \exp\left(-10^4\left(5\mu m \times \frac{100\,cm}{10^6\,\mu m}\right)\right)$$

$$= 6.84 \times 10^{15}\ cm^{-3}$$

$$\mathcal{E} = \frac{V_T}{p} \cdot \frac{dp}{dx} \approx \frac{V_T}{N_A(5\mu m)} \left.\frac{dN_A}{dx}\right|_{x=5\mu m}$$

$$= \frac{0.0259\,V}{6.84 \times 10^{15}\,cm^{-3}} \left(-6.74 \times 10^{-21}\ cm^{-4}\right)$$

$$\mathcal{E}\Big|_{x=5\mu m} = -25500\ \frac{V}{cm}$$

> At $x = 5\mu m$, there is a built in electric field of $-25500 \frac{V}{cm}$ (minus sign indicates electric field is in minus x direction.)