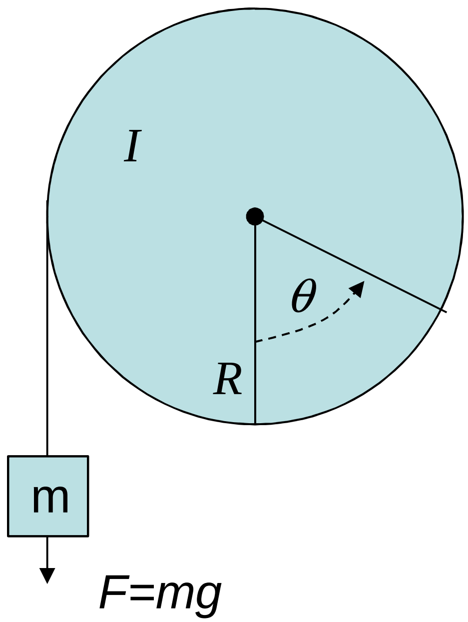
EXAM 3 is next week

Time: 8:00-9:30 pm Wed Apr 11
Place: Elliott Hall
Material: lectures 1-22, HW 1-22, Recitations 1-12, Labs 1-12
focus will be on last 3rd of material (not on Exams 1 & 2)
Problems: multiple choice, 10 questions (70 points)
write-up part, hand graded (30 points)
Equation sheet: provided with exam
Practice exam + equation sheet: will be posted at the end of this week

Note: no lecture on Thursday Apr 12!

Predicting Position with Rotation



A light string is wrapped around disk of radius *R* and moment of inertia *I* that can freely spin around its *fixed* axis. The string is pulled with force *F* during time Δt. Assume that the disk was initially at rest (ω_i=0)

1) What will be the angular speed ω_f ?

$$\frac{d\vec{L}_{tot}}{dt} = \vec{\tau}_{net}$$

Solution:

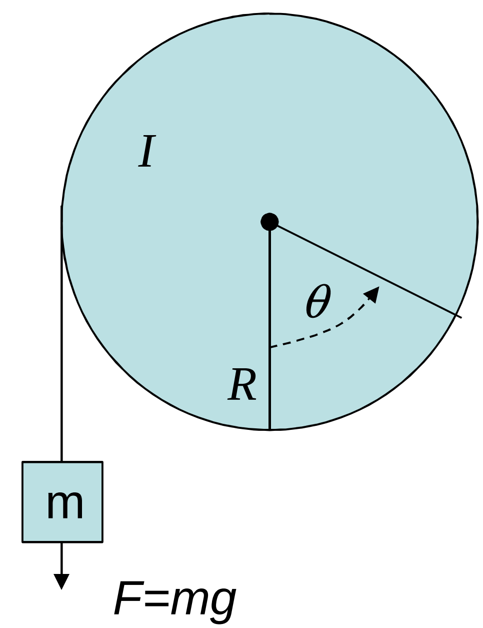
$$\Delta\vec{L}_{tot} = \vec{\tau}_{net}\Delta t$$

$$I\vec{\omega}_f - I\vec{\omega}_i = I\vec{\omega}_f = \vec{R} \times \vec{F} \cdot \Delta t$$

$$I\omega_f = RF\Delta t$$

$$\boxed{\omega_f = \frac{RF\Delta t}{I}}$$

Predicting Position with Rotation



A light string is wrapped around disk of radius *R* and moment of inertia *I* that can freely spin around its *fixed* axis. The string is pulled with force *F* during time Δt. Assume that the disk was initially at rest ($\omega_i=0$)

1) What will be the angular speed ω_f ?
2) How far (Δx) will the end of string move?

Solution:

$$\Delta\theta = \omega_{aver}\Delta t \longleftarrow \omega_{aver} \equiv \frac{\Delta\theta}{\Delta t}$$

ω changes linearly with time:

$$\omega_{aver} = \frac{\omega_i + \omega_f}{2} = \frac{\omega_f}{2}$$

$$\Delta\theta = \frac{\omega_f}{2}\Delta t = \frac{RF\left(\Delta t\right)^2}{2I}$$

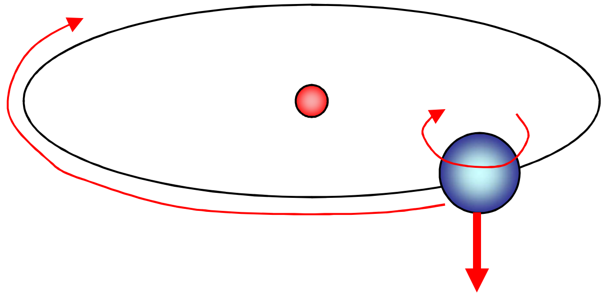
$$\boxed{\Delta x = R\Delta\theta = \frac{F\left(R\Delta t\right)^2}{2I}}$$

$$\frac{d\vec{L}_{tot}}{dt} = \vec{\tau}_{net} \quad \boxed{\omega_f = \frac{RF\Delta t}{I}}$$

See also examples in Section 11.8

Angular momentum quantization

Many elementary particles behave as if they posses
***intrinsic* rotational angular momentum**



Electron can have translational (orbital around nucleus), and intrinsic rotational angular momenta

Strange but true: **Angular momentum is quantized**

$[\mathrm{J\ s}] = [\mathrm{kg\ m^2 s^{-1}}]$

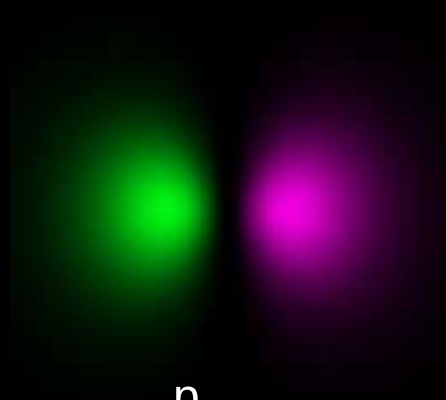
Angular momentum quantum = $\hbar = \dfrac{h}{2\pi} = 1.05\times10^{-34}\ \mathrm{J\ s}$

Whenever you measure a vector component of angular momentum you get either half-integer or integer multiple of $\hbar$

Orbital angular momentum comes in integer multiples, but intrinsic spin of "Fermions" (building blocks) is ½ unit of $\hbar$

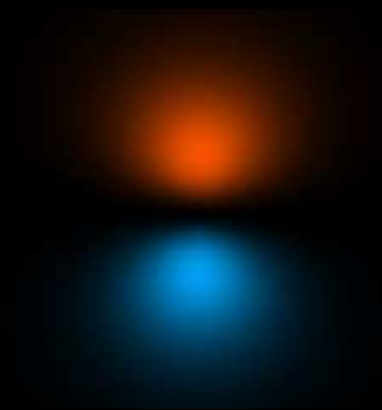
Orbital Angular Momentum

Where is the orbital angular momentum in a hydrogen orbital?



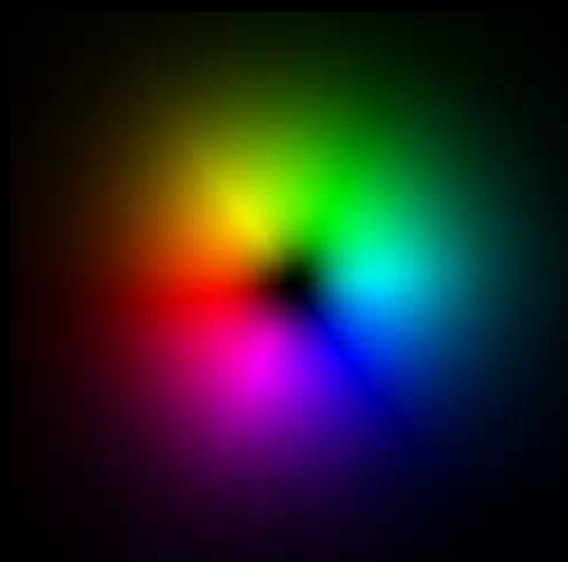
p_x

+



$i\ p_y$

Electron "current" circles around the atom.



$= |L=1, L_z=1>$

Quantized because these are 3D standing electron waves around the nucleus.

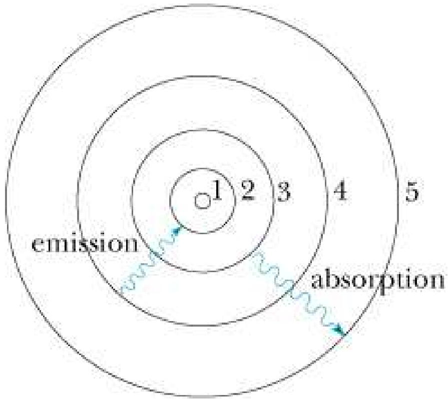
See Atom in a Box
www.daugerresearch.com

Bohr's Atomic Model

Niels Bohr

$$\left|\vec{L}_{A,trans,electron}\right| = \sqrt{mrkq_e^2}$$

1913: IDEA: Electron can only take orbits where its translational angular momentum is integer multiple of $\hbar$



Allowed radii: $$r = N^2 \frac{\hbar^2}{kq_e^2 m_e}$$

$$\hbar = 1.05 \times 10^{-34}\, J \cdot s$$

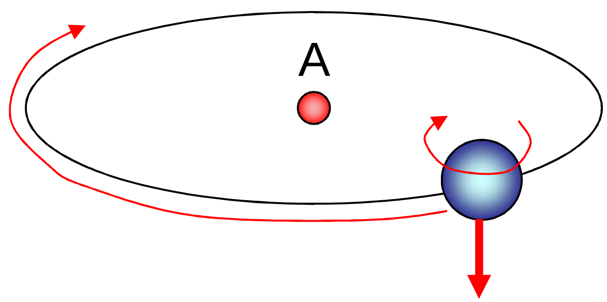
$$N = 1, 2, 3, \ldots$$

This implies that only certain values of $L_{A,trans,electron}$ are allowed:

$$\left|\vec{L}_{A,trans,electron}\right| = N\hbar \quad \text{where } N=1,2,3,\ldots$$

NOTE: Because K and U are functions of r and v, energy levels are quantized also.

Bohr Model



Consider an electron in circular orbit about a proton. What are the possible values of $L_{A,trans,electron}$?

Assume circular motion:

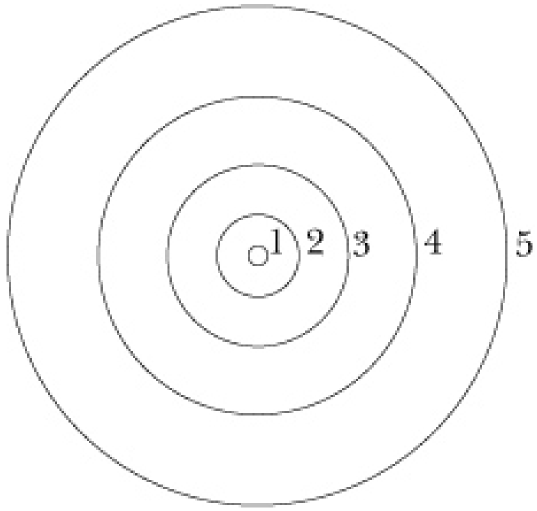
$$\left|\vec{F}_e\right| = \frac{mv^2}{r} \Rightarrow \frac{kq_e^2}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{kq_e^2}{mr}}$$

Thus, $$\left|\vec{L}_{A,trans,electron}\right| = mvr = \sqrt{mrkq_e^2}$$

If any orbital radius r is allowed, $L_{A,trans,electron}$ can be anything.

However, only certain values of r are allowed . . .

The Bohr model: allowed radii and energies



See derivation on page 444-446

Allowed Bohr radii for electron orbits:

$$r_N = N^2 \frac{h^2}{\frac{1}{4\pi\varepsilon_0} e^2 m} \approx N^2 \left(0.53\times10^{-10}\ \text{m}\right)$$

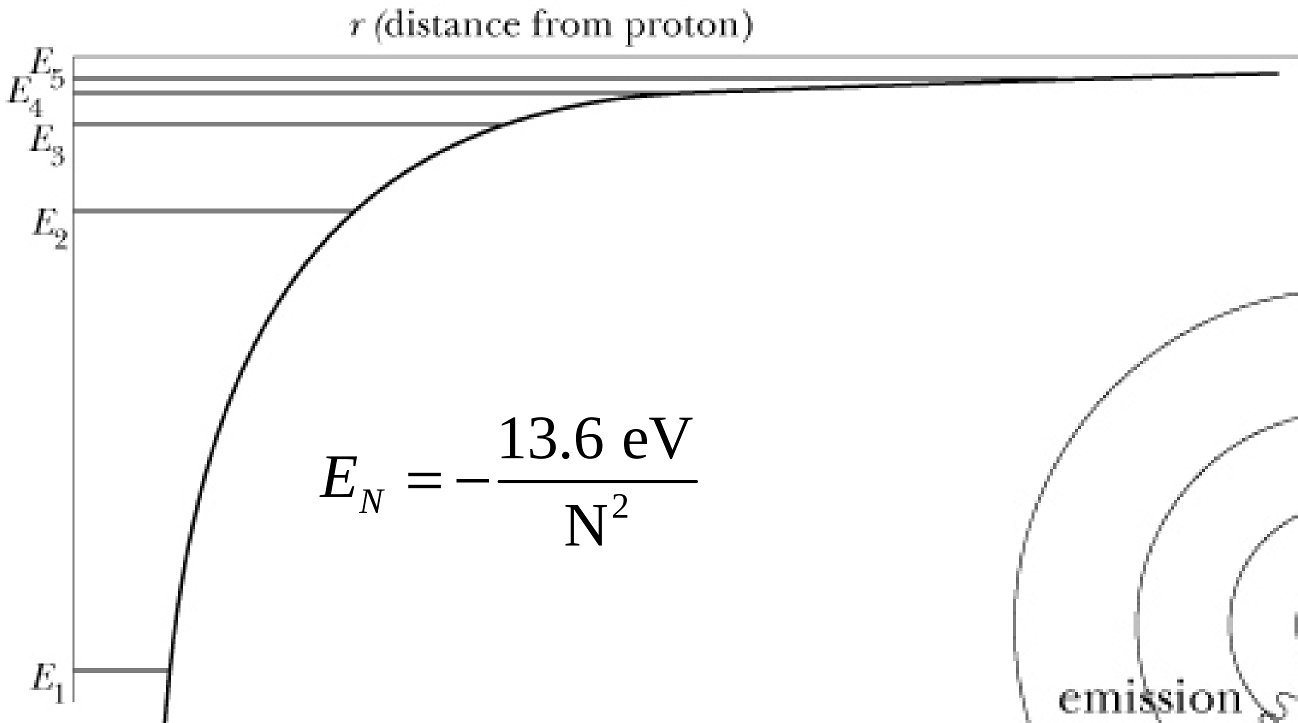
This is 2K

Use E_N = K+U and $\left|\vec{F}_e\right| = \frac{m\nu^2}{r} = \frac{kq_e^2}{r^2}$ $k = \frac{1}{4\pi\epsilon_o}$

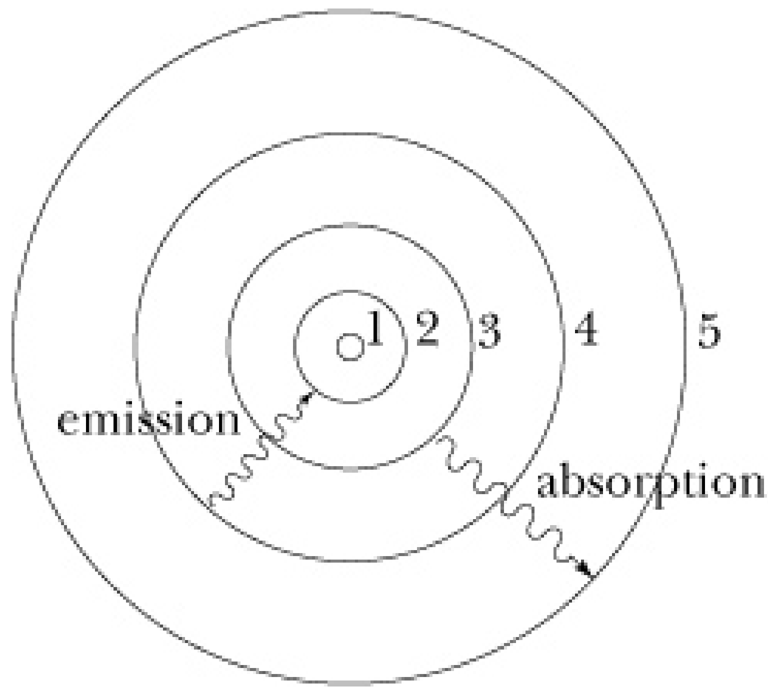
Bohr model energy levels:

$$E = -\frac{\left(\frac{1}{4\pi\epsilon_o}\right)^2 e^4 m}{2N^2\hbar^2} = -\frac{13.6\ \text{eV}}{\text{N}^2}, \quad \text{N=1,2,3,...}$$

The Bohr model: and photon emission



$$E_N = -\frac{13.6\ \text{eV}}{\text{N}^2}$$



Particle spin

Rotational angular momentum

Electron, muon, neutrino have spin 1/2 :
mesurements of a component of their angular momentum yields $\pm\frac{1}{2}\hbar$

Quarks have spin ½
Protons and neutrons (three quarks) have spin ½

Mesons: (quark+antiquark) have spin 0 or 1

Macroscopic objects: quantization of *L* is too small to notice!

Two lowest energy electrons in any atom have total angular momentum 0

Fermions: spin ½, Pauli exclusion principle ← Cooper pairs: superconductivity
Bosons: integer spin

Rotational energies of molecules are quantized

Quantum mechanics: L_x, L_y, L_z can only be integer or half-integer multiple of $\hbar$

Quantized values of $L^2 = l(l+1)\hbar^2$ where l is integer or half-integer

Gyroscopic Stability

Edmund Scientifics

In 1917, the Chandler Company of Indianapolis, Indiana,
created the "Chandler gyroscope," a toy gyroscope with a pull string and pedestal.
It has been in continuous production ever since and is considered a classic American toy.
-- Wikipedia

Best Trick in the Book

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{net}}$$

$$\vec{p} = |p|\hat{p}$$

Vectors have *direction* and *magnitude*.

Vector Notation and the Momentum Principle:

$$\frac{d\vec{p}}{dt} = |p|\frac{d\hat{p}}{dt} + \hat{p}\frac{d|p|}{dt}$$

$$= \vec{F}_{\perp} + \vec{F}_{||}$$

Use the chain rule

$\vec{F}_{\perp}$ causes changes in the *direction* of $\vec{p}$

$\vec{F}_{||}$ causes changes in the *magnitude* of $\vec{p}$

Blast from the Past
Section 5.5

Best Trick *Not* in the Book

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}}$$

$$\vec{\tau} = |\tau|\hat{\tau}$$

Vectors have *direction* and *magnitude*.

Vector Notation and the Angular Momentum Principle:

$$\frac{d\vec{L}}{dt} = |L|\frac{d\hat{L}}{dt} + \hat{L}\frac{d|L|}{dt}$$

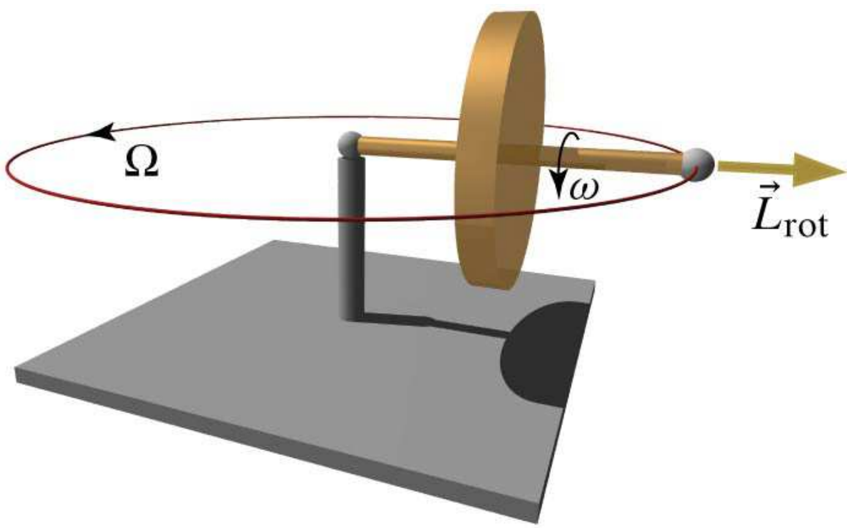
$$= \vec{\tau}_{\perp} + \vec{\tau}_{||}$$

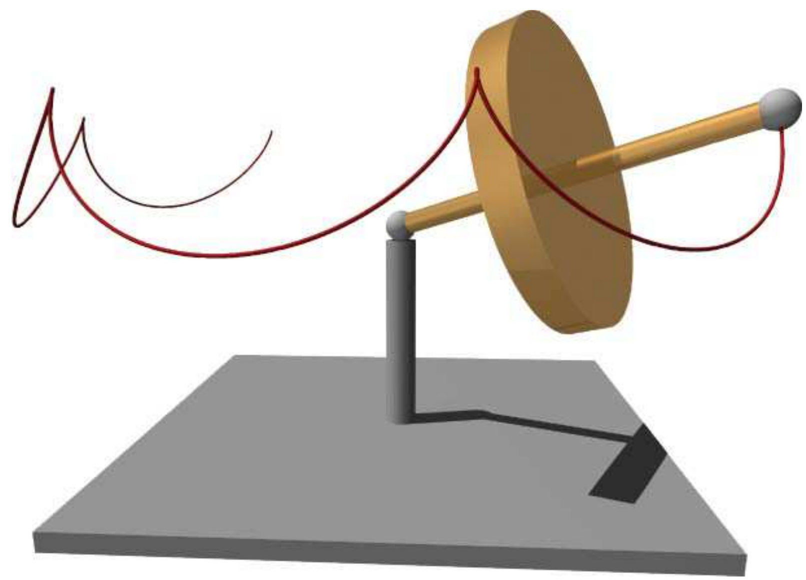
Use the chain rule

$\vec{\tau}_{\perp}$ causes changes in the *direction* of $\vec{L}$

$\vec{\tau}_{||}$ causes changes in the *magnitude* of $\vec{L}$

Gyroscopes





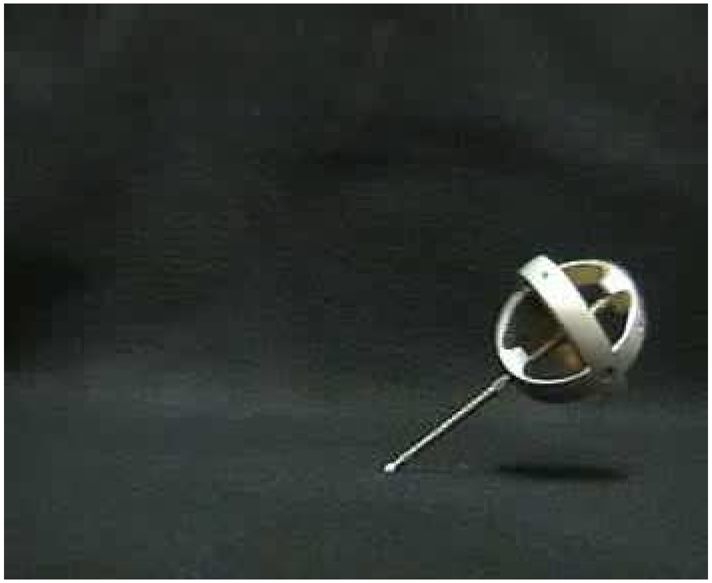
Precession

Precession and nutation

i>clicker

$$\Omega = \frac{RMg}{I\omega}$$

A



B

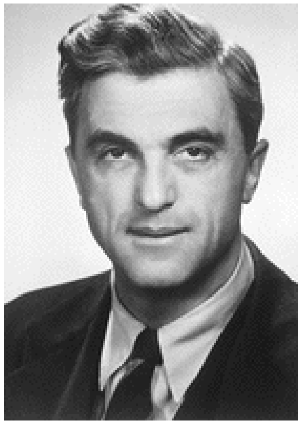
In which of the two gyroscopes is the disk spinning faster?

Precession phenomena (see book)

Magnetic Resonance Imaging (MRI)

Precession of spin axes in astronomy

Tidal torques



Felix Bloch
1905-1983

Edward Mills Purcell
1912-1997

B.S.E.E. from **Purdue electrical engineering**

NMR - nuclear magnetic resonance

Independently discovered (1946)

Nobel Price (1952)

NMRI = MRI