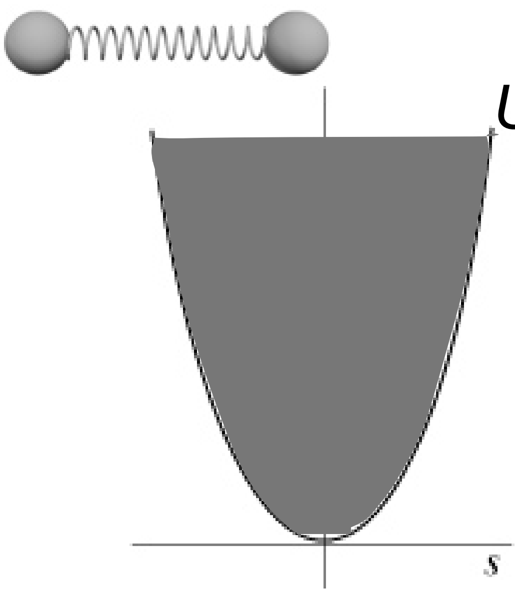
EXAM 2 is tomorrow

Time: 8:00-9:30 pm Wed Mar 7
Place: Elliott Hall
Material: lectures 1-15, HW 1-15, Recitations 1-8, Labs 1-8
focus will be on second half of material (not on Exam 1)
Problems: multiple choice, 10 questions (70 points)
write-up part, hand graded (30 points)
Equation sheet: provided with exam
Practice exam + eqn sheet + solutions: already posted

Note: **no lecture this Thursday (March 8)** !

Quantizing two interacting atoms

Classical harmonic oscillator:



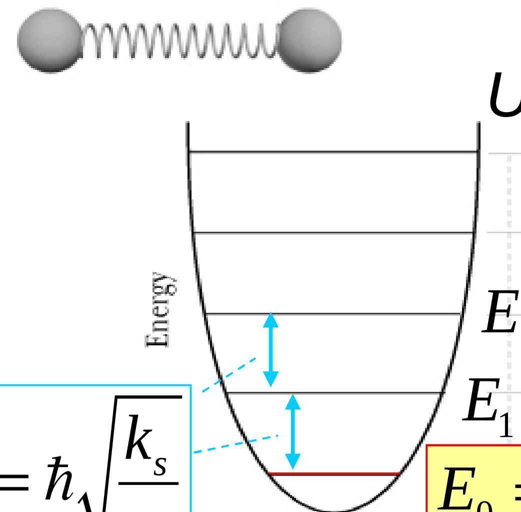
$U = k_s s^2$

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA_{\max}^2$$

Any value of A is allowed

$\rightarrow$ any E is possible.

Quantum harmonic oscillator:



$U = k_s s^2$

$$E_2 = 2\hbar\omega_0 + E_0$$

$$E_1 = \hbar\omega_0 + E_0$$

$$\boxed{E_0 = \tfrac{1}{2}\hbar\omega_0}$$

ground state

$$\boxed{\hbar\omega_0 = \hbar\sqrt{\frac{k_s}{m}}}$$

equidistant spacing

$$\omega_0 = \sqrt{k_s / m}$$

$$\hbar \equiv \frac{h}{2\pi} = 1.05\times10^{-34}\ \mathrm{J\cdot s}$$

Energy levels:

$$E_N = N\hbar\omega_0 + \tfrac{1}{2}\hbar\omega_0$$

Time to Throw Things

BALL

BATON

We need to understand Center of Mass

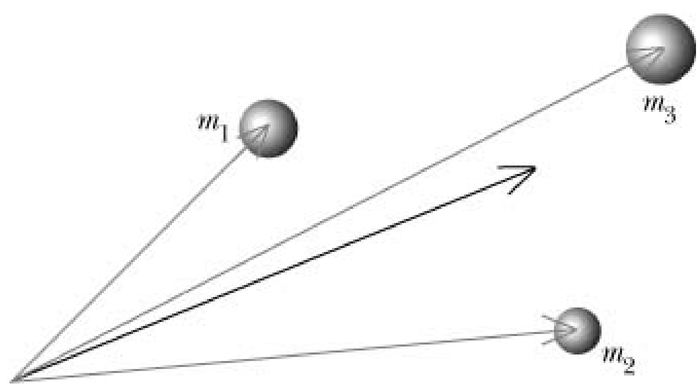
The Center of Mass (definition)

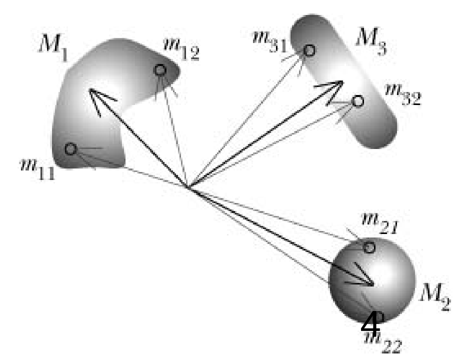
$$\vec{r}_{\mathrm{cm}} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$= \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{M}$$

This is a **weighted average** of the positions -- each position appears in **proportion** to its mass

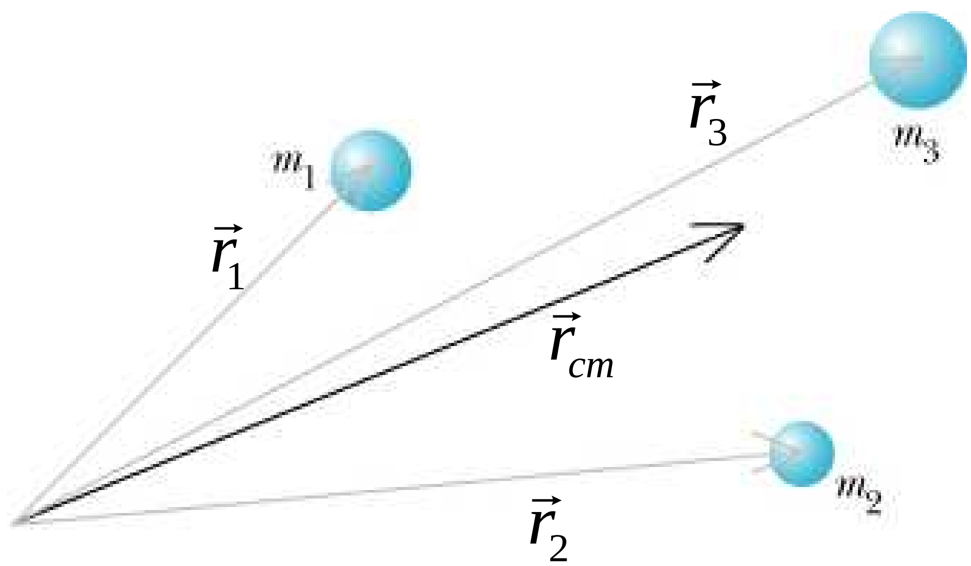
where $M = m_1 + m_2 + m_3 + \ldots$

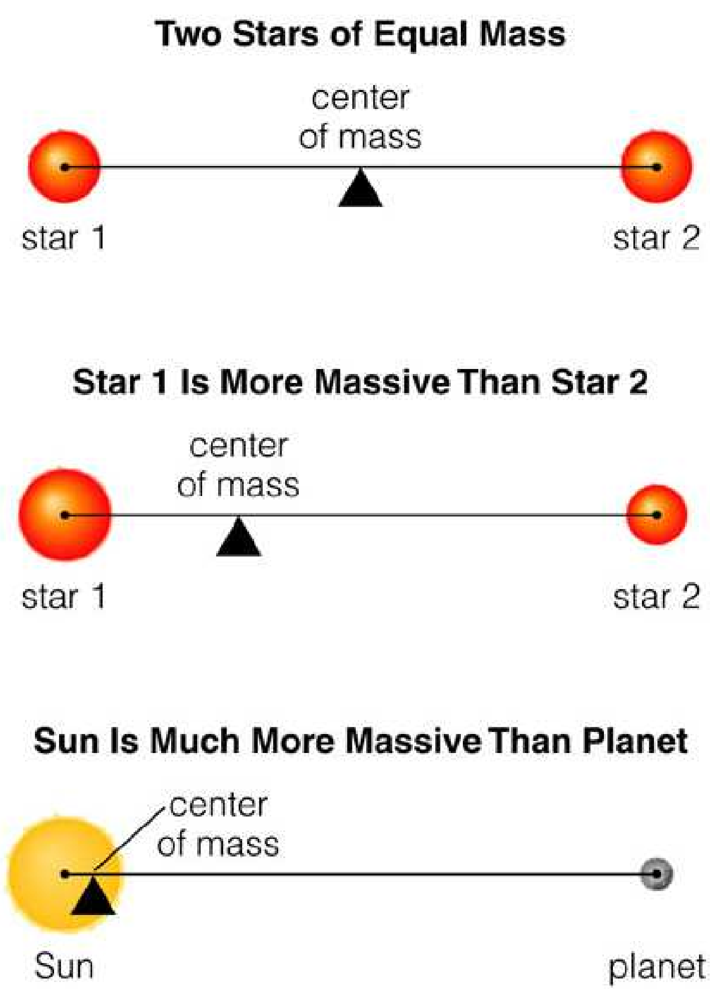




The Center of Mass

$$\vec{r}_{cm} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$





Motion of the Center of Mass

$$\vec{r}_{\mathrm{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$\Rightarrow M\vec{r}_{\mathrm{cm}} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots$$

1) Take one time derivative:

$$\Rightarrow M\frac{d\vec{r}_{\mathrm{cm}}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + \ldots$$

$$\Rightarrow M\vec{v}_{\mathrm{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots$$

If motion is **nonrelativistic** (p=mv), this is same as:

$$\vec{P}_{\mathrm{tot}} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$$ (Good!)

Motion of the Center of Mass

$$M\vec{r}_{\mathrm{cm}} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots$$

$$\vec{P}_{\mathrm{tot}} = M\frac{d\vec{r}_{\mathrm{cm}}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + \ldots$$

1) Take a second time derivative:

$$\frac{d\vec{P}_{\mathrm{tot}}}{dt} = M\frac{d^2\vec{r}_{\mathrm{cm}}}{dt^2} = m_1\frac{d^2\vec{r}_1}{dt^2} + m_2\frac{d^2\vec{r}_2}{dt^2} + m_3\frac{d^2\vec{r}_2}{dt^2} + \ldots$$

$$= M\vec{a}_{\mathrm{cm}} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots$$

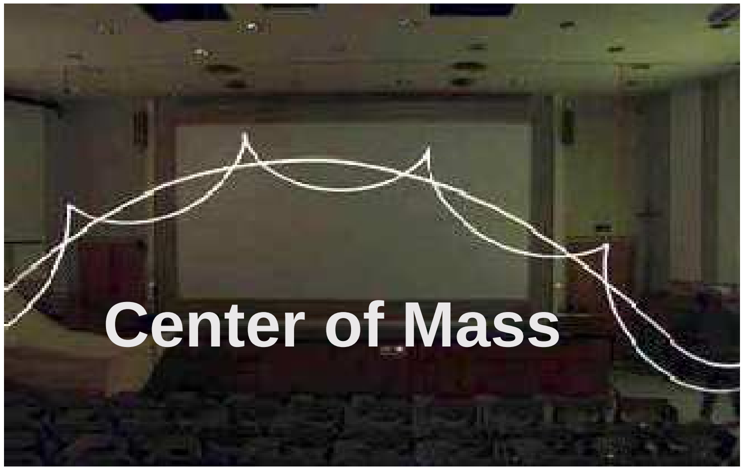
$$\frac{d\vec{P}_{\mathrm{tot}}}{dt} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots = \boxed{\vec{F}_{\mathrm{net}} = \frac{d\vec{P}_{\mathrm{tot}}}{dt}}$$

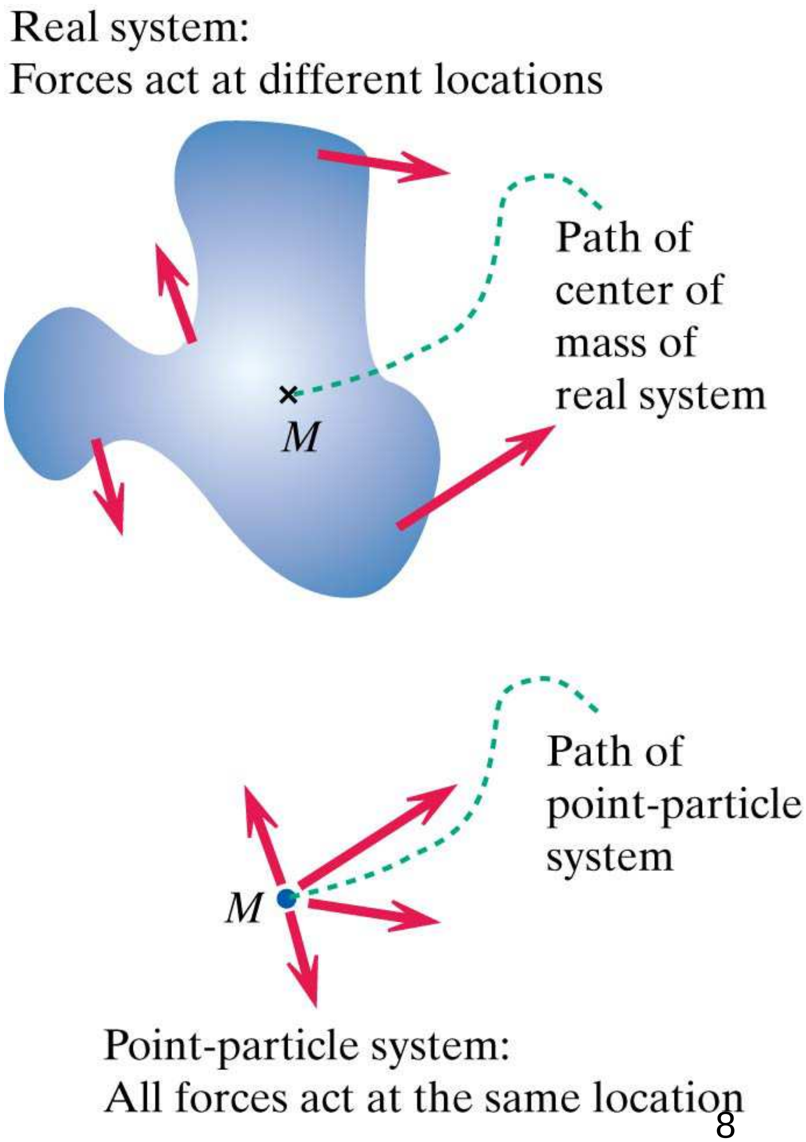
This says that the motion of the center of mass looks just like what would happen if all forces were applied to the total mass, as a point particle located at the center of mass position!

Motion of the Center of Mass

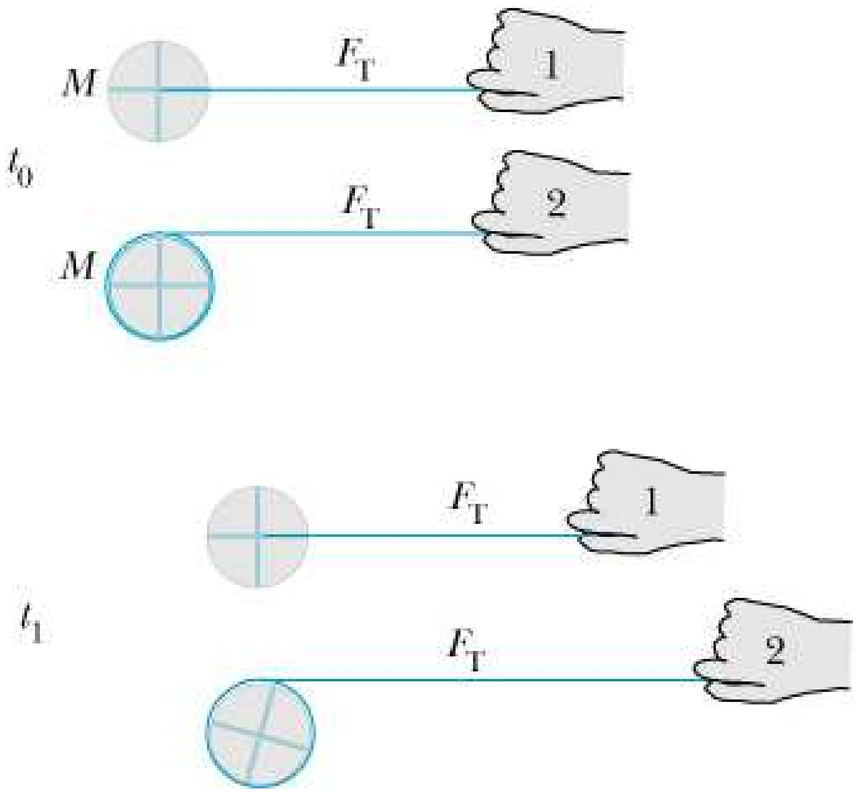
This says that the motion of the center of mass looks just like what would happen if all forces were applied to the total mass, as a **<u>point particle located at the center of mass position</u>.** Note, this result only holds for nonrelativistic motion.

$$\vec{F}_{net,ext} = M_{total}\vec{a}_{cm} = \frac{d\vec{P}_{total}}{dt}$$





Center of Mass Motion



Same Tension.

Which puck will move faster?

$$\frac{d\vec{P}_{tot}}{dt} \approx M\,\vec{a}_{cm} = \vec{F}_{net,surr}$$

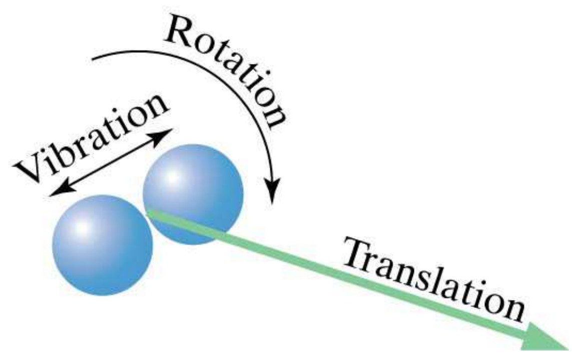
The centers of mass experience the same acceleration!

HOWEVER: Hand #2 has to pull the string farther: $W_2 > W_1$.

Where does this energy go?

Rotational energy. The bottom spool is spinning.

Kinetic energy of a multiparticle system



Translational, motion of center of mass

Motion of parts relative to center of mass

$$K_{tot} = K_{trans} + K_{rel}$$

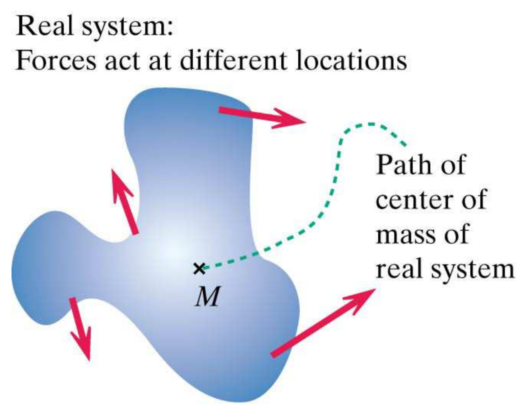
$$K_{rel} = K_{vib} + K_{rot}$$

Rotation about center of mass

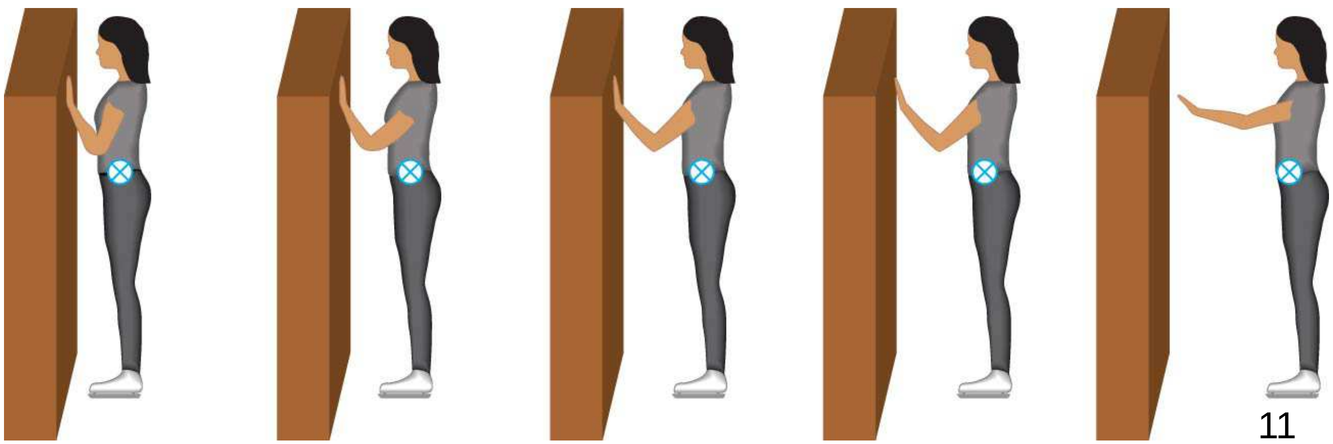
Vibration

Question for Discussion

Q6: A skater pushes straight away from a wall. She pushes on the wall with a force whose magnitude is *F*, so the wall pushes on her with a force *F* (in the direction of her motion). As she moves away from the wall, her center of mass moves a distance *d*. What is the correct form of the energy principle for the real system consisting of the skater?



1) $\Delta K_{\text{trans}} + \Delta E_{\text{internal}} = Fd$
2) $\Delta K_{\text{trans}} + \Delta E_{\text{internal}} = -Fd$
3) $\Delta K_{\text{trans}} + \Delta E_{\text{internal}} = 0$
4) $\Delta K_{\text{trans}} = Fd$
5) $\Delta K_{\text{trans}} = -Fd$



Question for Discussion

Q7: A skater pushes straight away from a wall. She pushes on the wall with a force whose magnitude is *F*, so the wall pushes on her with a force *F* (in the direction of her motion). As she moves away from the wall, her center of mass moves a distanc *d*. What is the correct form of the energy principle for the point particle system for the skater?

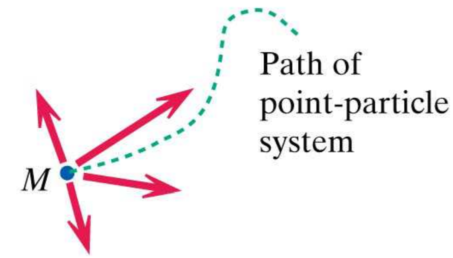
1) $\Delta K_{\text{trans}} + \Delta E_{\text{internal}} = Fd$

2) $\Delta K_{\text{trans}} + \Delta E_{\text{internal}} = -Fd$

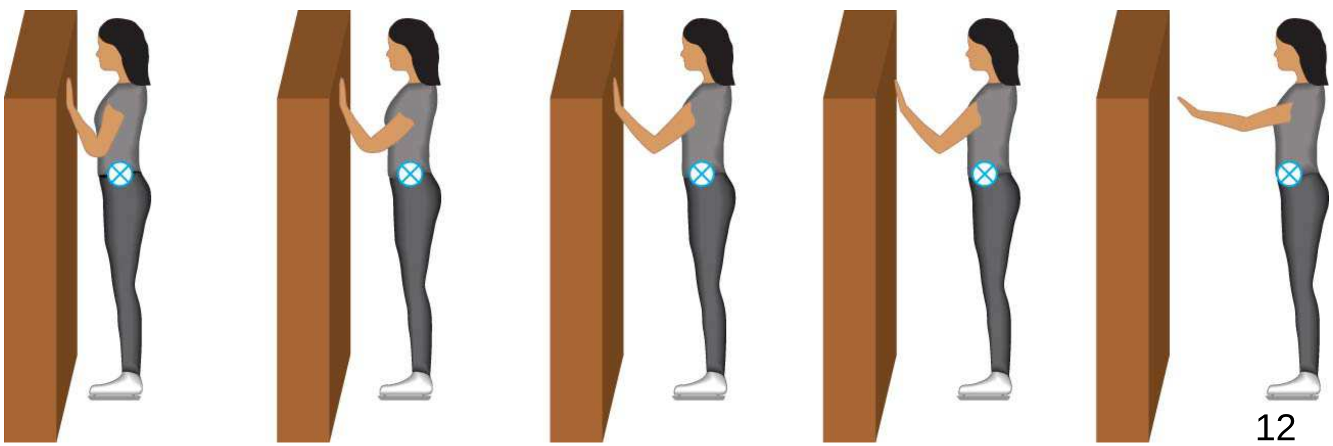
3) $\Delta K_{\text{trans}} + \Delta E_{\text{internal}} = 0$

4) $\Delta K_{\text{trans}} = Fd$

5) $\Delta K_{\text{trans}} = -Fd$



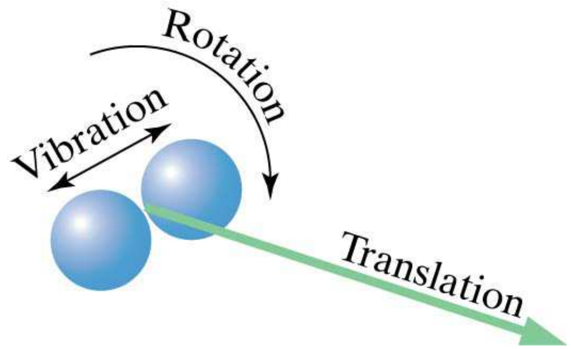
Point-particle system:
All forces act at the same location



Translational kinetic energy

$$\boxed{K_{tot} = K_{trans} + K_{rel}}$$

$$\boxed{K_{rel} = K_{vib} + K_{rot}}$$



Translational kinetic energy:
(motion of center of mass)

$$\boxed{K_{trans} = \frac{Mv_{CM}^2}{2} = \frac{P_{tot}^2}{2M}}$$ (nonrelativistic case)

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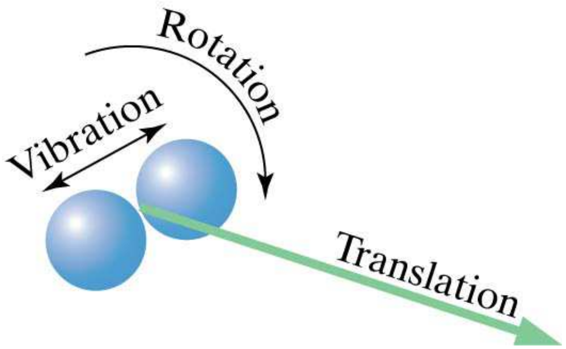
Translational kinetic energy

$$\boxed{K_{tot} = K_{trans} + K_{rel}}$$

$$\boxed{K_{rel} = K_{vib} + K_{rot}}$$



Translational kinetic energy:
(motion of center of mass)

$$\boxed{K_{trans} = \frac{Mv_{CM}^2}{2}}$$

(nonrelativistic case)

http://www.punchstock.com/asset_images/95652058

Vibrational kinetic energy

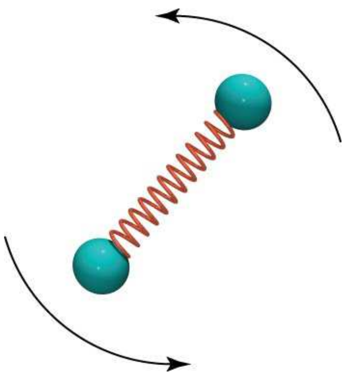
- Net momentum = 0
- Energy is constant (sum of elastic energy and kinetic energy)

$$\boxed{E_{vib} = K_{vib} + U_{spring}}$$

Rotational kinetic energy

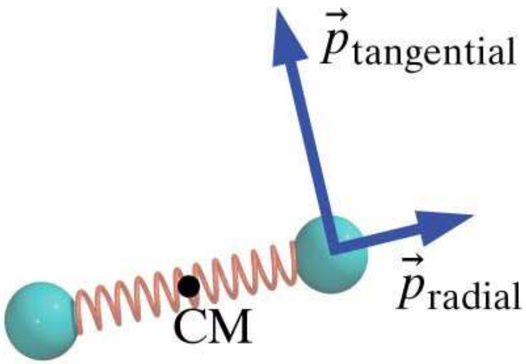
- Net momentum = 0
- Energy is constant

$$\boxed{E_{rot} = K_{rot}}$$



Motion around of center of mass

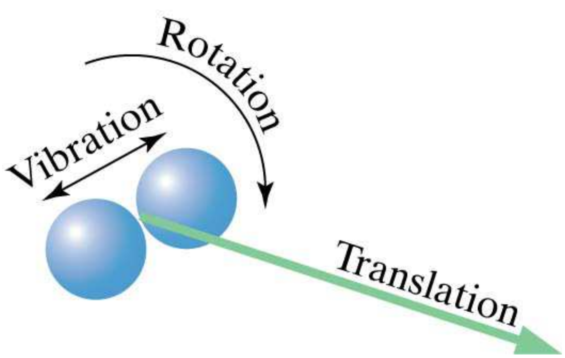
Rotation and vibration



$$K_{rel} = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} = \sum_{i=1}^{N} \left(\frac{p_{\tan,i}^2 + p_{rad,i}^2}{2m_i} \right)$$

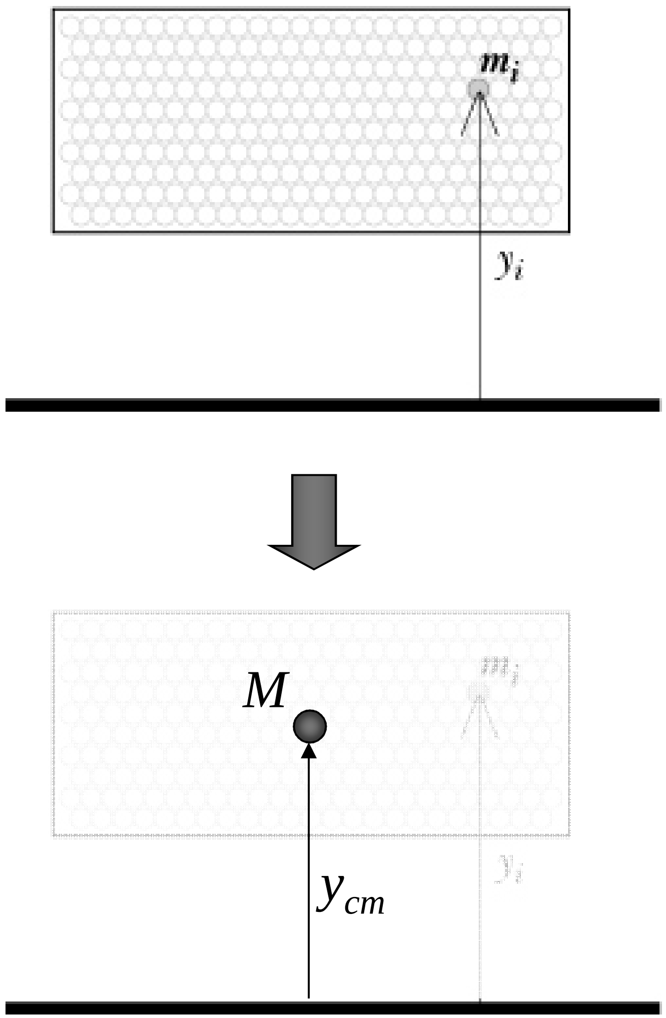
$$K_{rel} = \underbrace{\sum_{i=1}^{N} \left(\frac{p_{\tan,i}^2}{2m_i} \right)}_{K_{rot} \equiv} + \underbrace{\sum_{i=1}^{N} \left(\frac{p_{rad,i}^2}{2m_i} \right)}_{K_{vib} \equiv}$$

Rotation and vibration and translation



$$E_{tot} = \frac{1}{2} M v_{cm}^2 + K_{rot} + K_{vib} + \frac{1}{2} k s^2 + 2mc^2$$

Gravitational potential energy of a multiparticle system



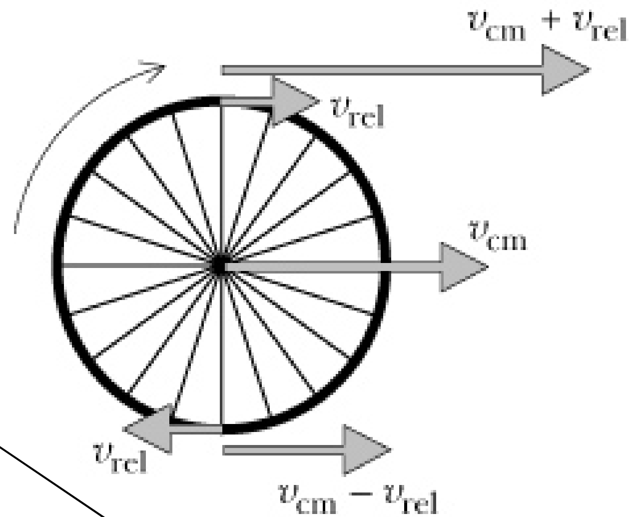
$$U_g = m_1 g y_1 + m_2 g y_2 + m_3 g y_3 + \ldots$$

$$U_g = \left(\underbrace{m_1 y_1 + m_2 y_2 + m_3 y_3 + \ldots}_{= M y_{cm}}\right) g$$

$$\boxed{U_g = M g y_{cm}}$$

Gravitational energy near the Earth's surface

Example: Rotation and translation



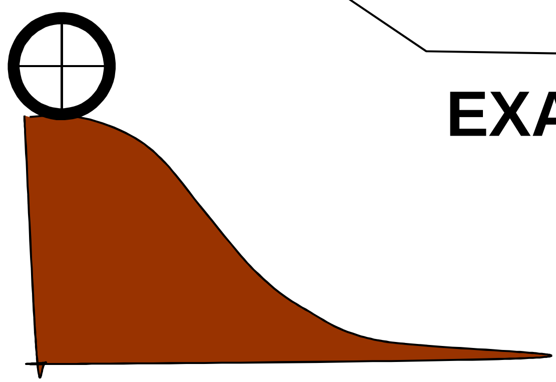
$$K_{tot} = K_{trans} + K_{rot}$$

$$K_{trans} \rightarrow \frac{1}{2}Mv_{cm}^2$$

Assume all mass is in the rim

$$K_{rot} \rightarrow \frac{1}{2}Mv_{rel}^2$$

EXAMPLE:



$$K_{tot} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}Mv_{rel}^2$$

Energy principle:

$$\underbrace{\frac{1}{2}Mv_{cm,i}^2}_{=0} + \underbrace{\frac{1}{2}Mv_{rel,i}^2}_{=0} + Mgy_{cm,i} = \frac{1}{2}Mv_{cm,f}^2 + \frac{1}{2}Mv_{rel,f}^2 + Mgy_{cm,f}$$

$$v_{cm} = v_{rel}$$

$$Mg\Delta y_{cm} = Mv_{cm,f}^2$$