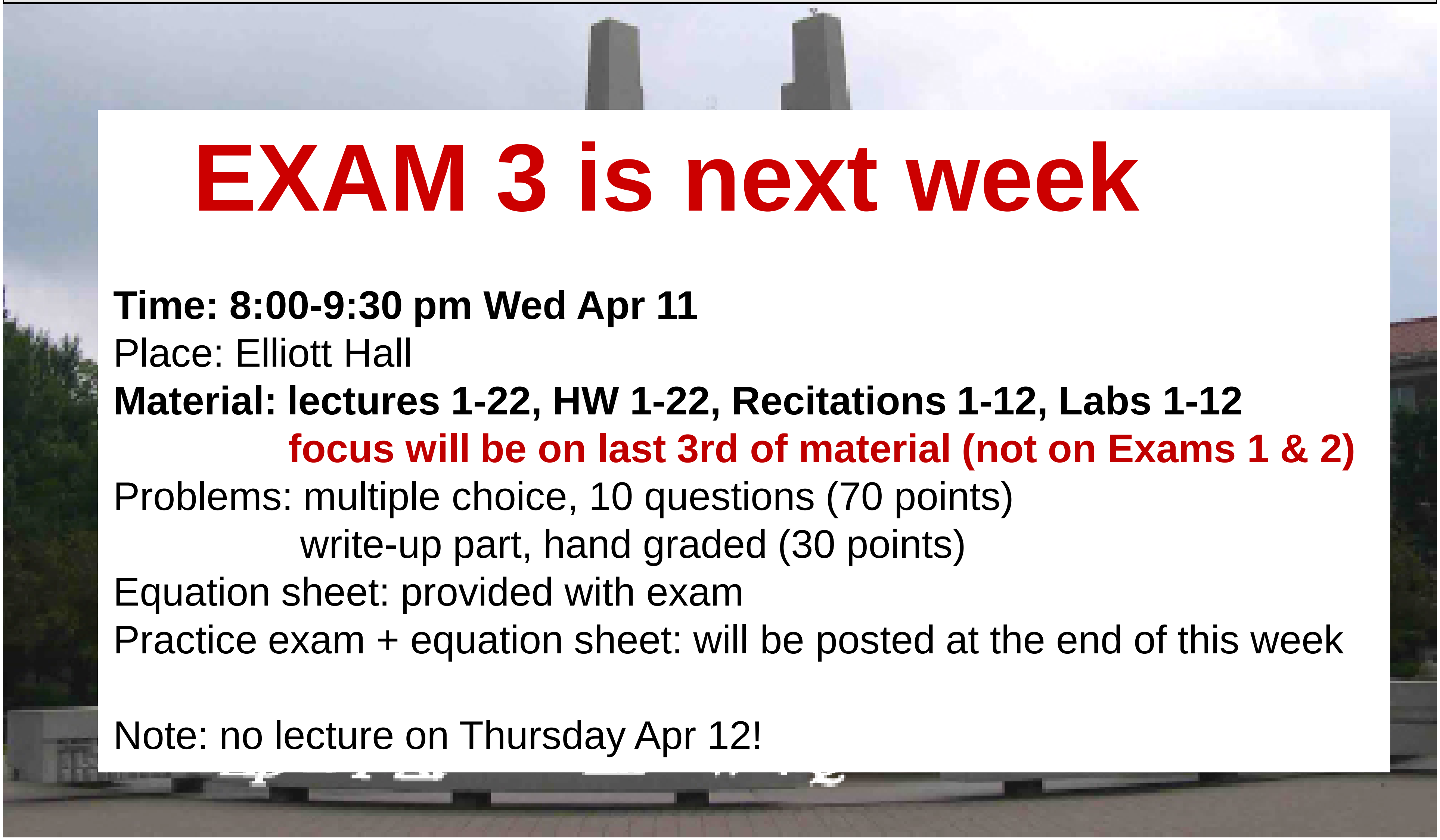


EXAM 3 is next week

Time: 8:00-9:30 pm Wed Apr 11
Place: Elliott Hall
Material: lectures 1-22, HW 1-22, Recitations 1-12, Labs 1-12
focus will be on last 3rd of material (not on Exams 1 & 2)
Problems: multiple choice, 10 questions (70 points)
write-up part, hand graded (30 points)
Equation sheet: provided with exam
Practice exam + equation sheet: will be posted at the end of this week

Note: no lecture on Thursday Apr 12!

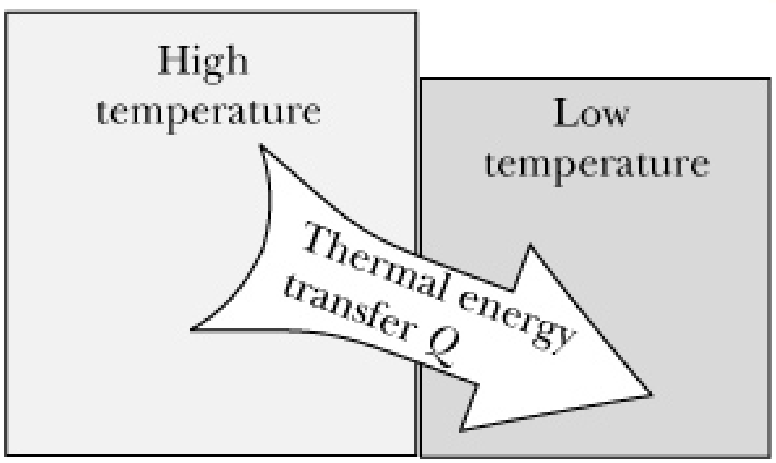
Reversibility

Movie taken and modified from: www.theclam.com/Video/JumpingIndex.asp

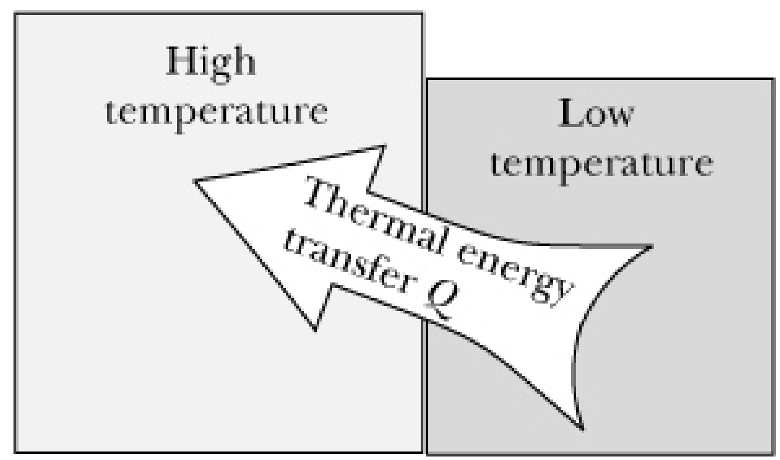
Today: Counting Statistics

- Need For Statistical Considerations
- Einstein Model and Counting Microstates
- Fundamental Assumption of Statistical Mechanics
- Entropy
  - 2^{nd} Law of Thermodynamics
  - Irreversibility

Statistical issue: Thermal energy flow



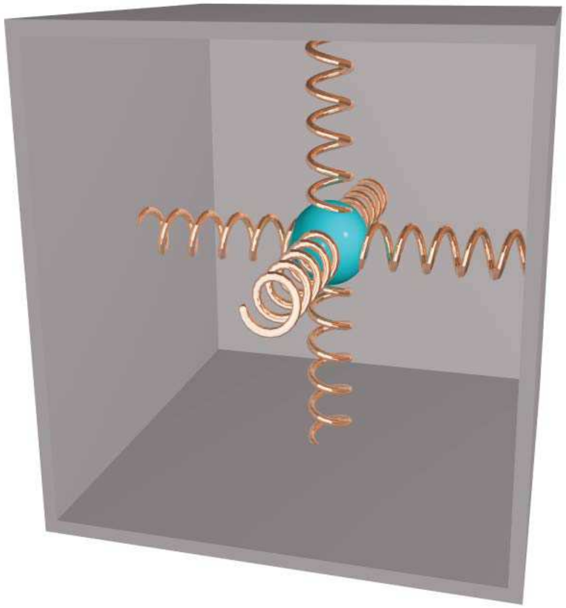
Happens all the time.



Will this ever happen?

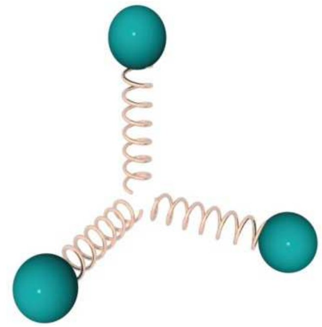
A statistical model of solids

Spring-ball model of solid



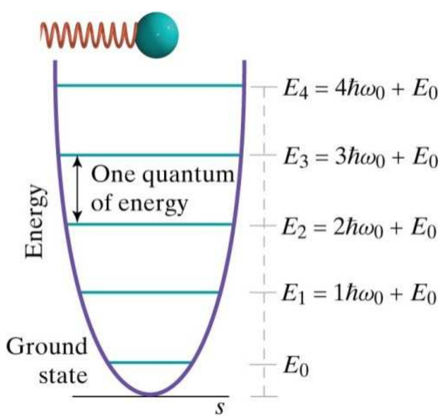
Einstein's model of solid (1907)

Each atom in a solid is connected to immovable walls



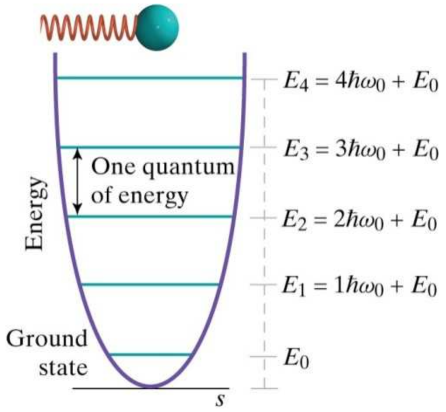
Energy:

$$K_{vib}+U_s=\left(\frac{p_x^2}{2m}+\tfrac{1}{2}k_s s_x^2+U_0\right)+\left(\frac{p_y^2}{2m}+\tfrac{1}{2}k_s s_y^2+U_0\right)+\left(\frac{p_z^2}{2m}+\tfrac{1}{2}k_s s_z^2+U_0\right)$$

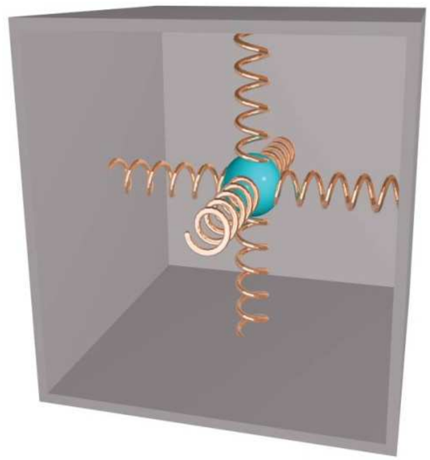


3D oscillator: can separate motion into *x,y,z* components
Each component: energy is quantized in the same way

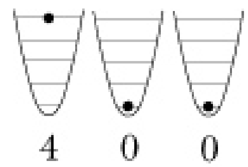
Distributing energy: 4 quanta



$$\hbar\sqrt{k_s/m}$$



1. Can have all 4 quanta in one oscillator

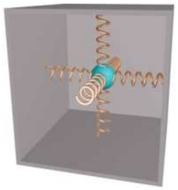


3 ways

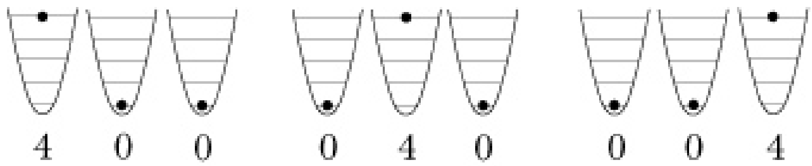
Analogy: distributing 4 dollars among 3 pockets:

1. Can have all 4$ in one pocket:

3 ways

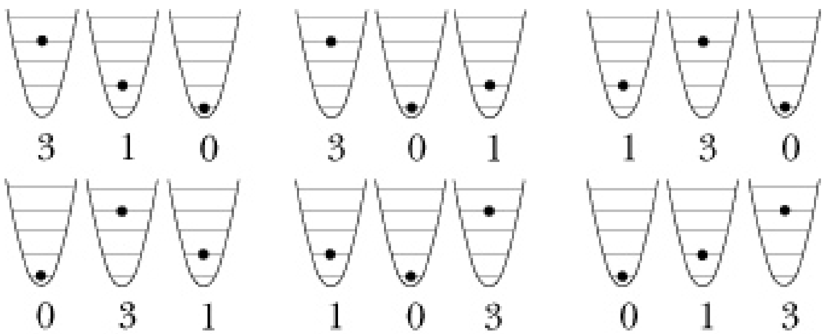


Distributing energy: 4 quanta



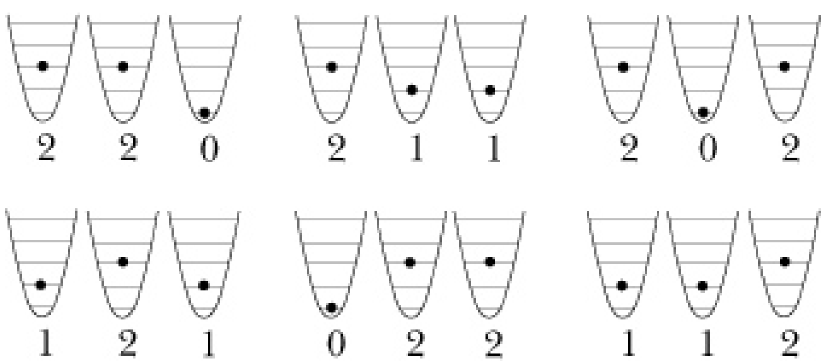
3 ways:
4-0-0 quanta

3



6 ways:
3-1-0 quanta

6



6 more ways:
2-2-0 quanta
1-1-2 quanta

3

3

15 microstates: The same macrostate

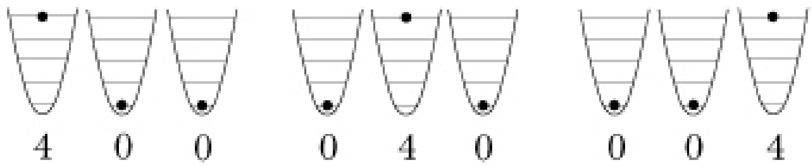
The fundamental assumption of statistical mechanics

Each microstate corresponding to a given macrostate is equally probable.

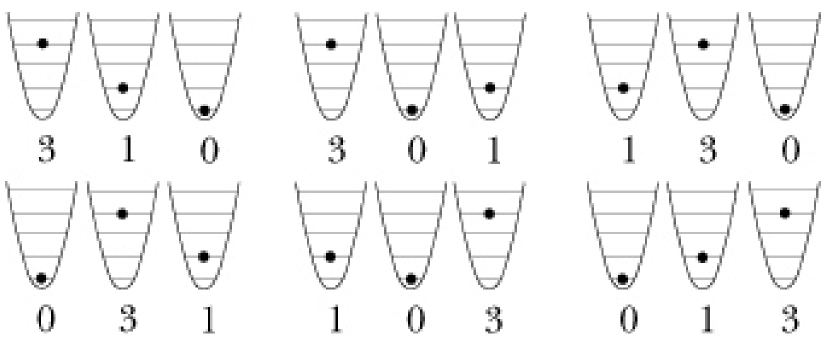
Macrostate = same total energy

Microstate = microscopic distribution of energy

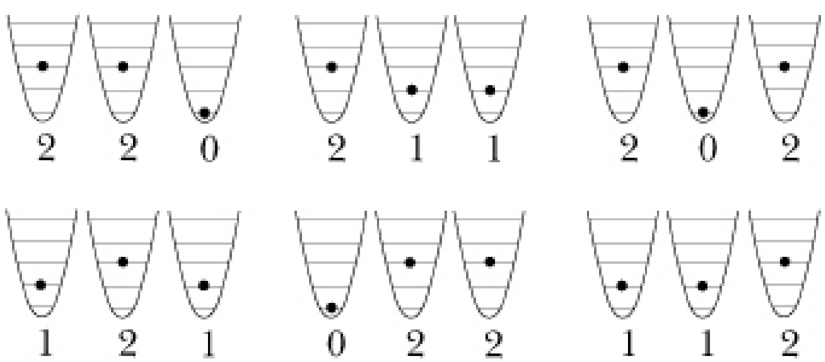
Distributing energy



3 ways:
4-0-0 quanta



6 ways:
3-1-0 quanta



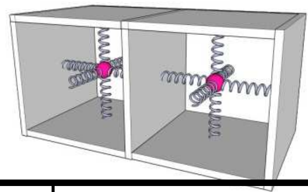
6 more ways:
2-2-0 quanta
1-1-2 quanta

15 *microstates*

1 *macrostate*
(I always have $4)

All microstates are equally probable

Interacting atoms: 4 quanta and 2 atoms



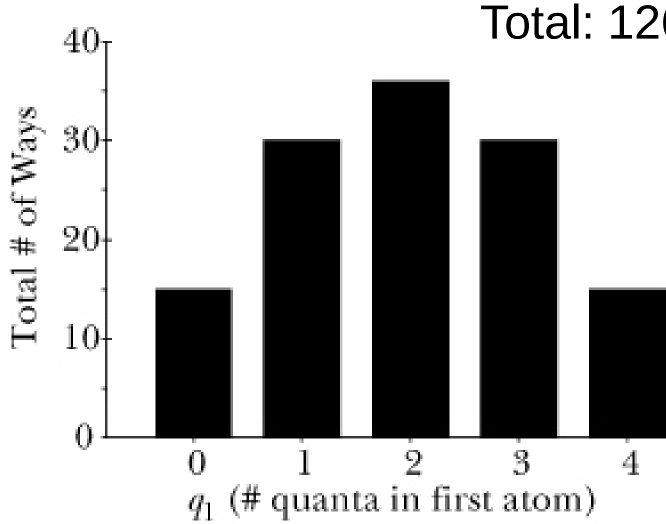
4 dollar bills

quanta in 1 (# of ways)	quanta in 2 (# of ways)	(# of ways 1) × (# of ways 2)
0 (1)	4 (15)	1 x 15 =15
1 (3)	3 (10)	3 x 10 = 30
2 (6)	2 (6)	6 x 6 = 36
3 (10)	1 (3)	10 x 3 = 30
4 (15)	0 (1)	15 x 1 = 15

3 POCKETS

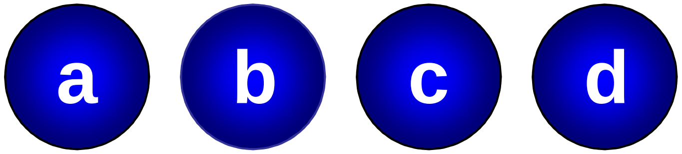
3 POCKETS

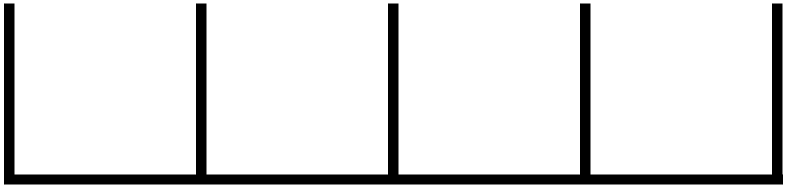
Total: 126



Which state is the most probable?

Counting Arrangements of Objects





arrangements = 4x3x2x1= 4!=24

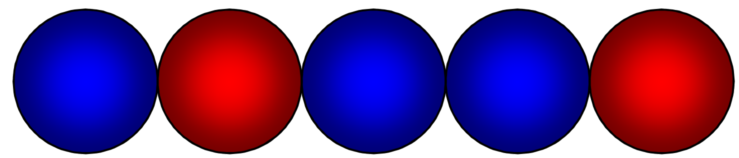
abcd	bacd	cabd	dabc	acdb	bcda	cbda	dbca
abdc	badc	cadb	dacb	adbc	bdac	cdab	dcab
acbd	bcad	cbad	dbac	adcb	bdca	cdba	dcba

Counting Arrangements of Objects

Five distinct objects $\rightarrow$ 5! = 120 arrangements.

What if the objects aren't distinct?

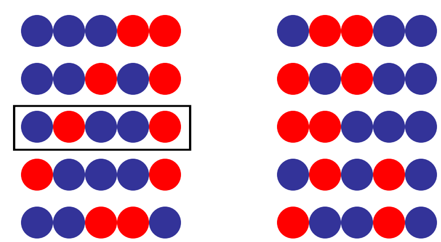
Consider the following arrangement of colored balls:



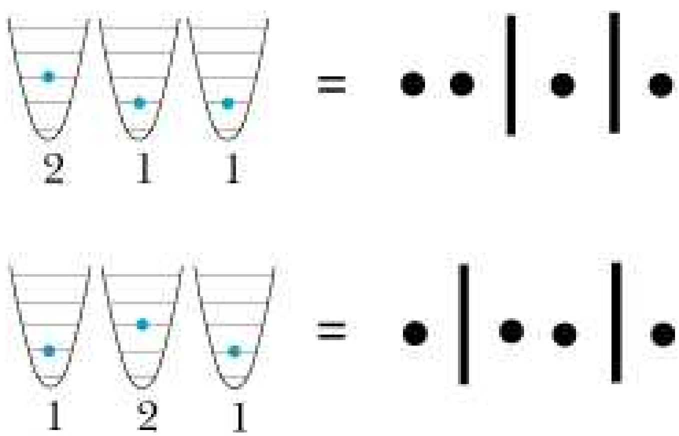
Interchanging the red balls gives an identical sequence: thus, the 5! overcounts by a factor of 2 = 2!.

Likewise, there are 3! = 6 ways of interchanging the blue balls. The 5! overcounts by a factor of 3!.

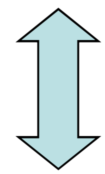
\# arrangements = $\dfrac{5!}{2!\times 3!} = 10$



A Formula for Counting Microstates



N oscillators and q quanta



N-1 bars and q dots

3 oscillators, 4 quanta $\leftrightarrow$ 2 bars, 4 dots: # microstates = $\dfrac{(2+4)!}{2!4!} = 15$ as before

Generally, # microstates (N oscillators, q quanta) $\equiv \Omega = \dfrac{(q+N-1)!}{q!(N-1)!}$

The fundamental assumption of statistical mechanics

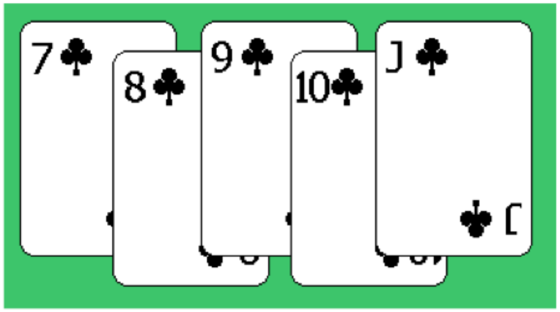
Each microstate corresponding to a given macrostate is equally probable.

Macrostate = same total energy

Microstate = microscopic distribution of energy

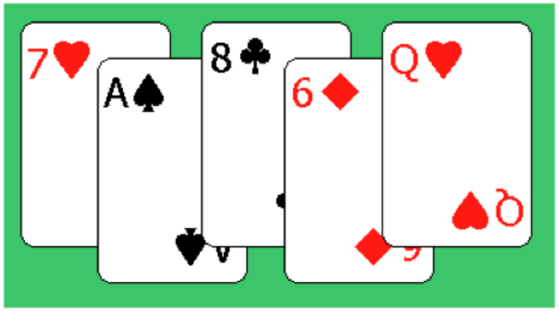
Poker

Your hand:



Straight flush

My hand:



I fold!

Probability of your hand: $\sim \dfrac{1}{2.6 \times 10^6}$

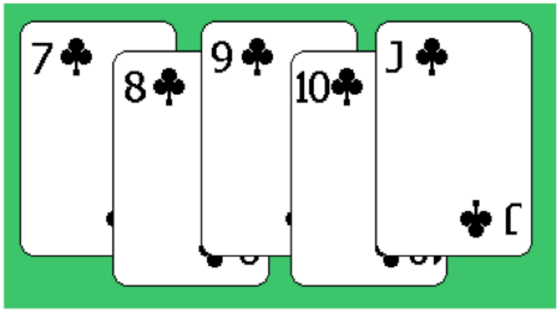
Probability of my hand: $\sim \dfrac{1}{2.6 \times 10^6}$

Why do we say a straight flush is rare?

My hand is rare too...

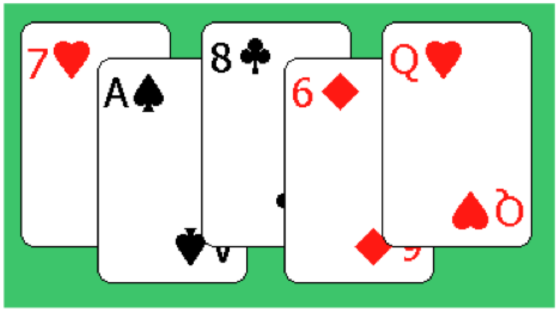
Poker

Your hand:



Straight flush

My hand:



I fold!

# of microstates producing macrostate "straight flush" = *40*

# of microstates producing macrostate "I fold" = $\sim 2.6 \times 10^6$

One of these macrostate outcomes is a lot more likely than the other.

Casinos

How do they make money?

They play games <u>many</u> <u>many</u> times.

Any one dice roll = random

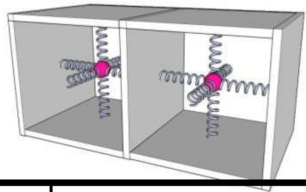
1 million dice rolls = I know the outcome!

If you own a casino, you need to know the

<u>most probable outcome</u>,

and the statistics of large numbers.

Interacting atoms: 4 quanta and 2 atoms



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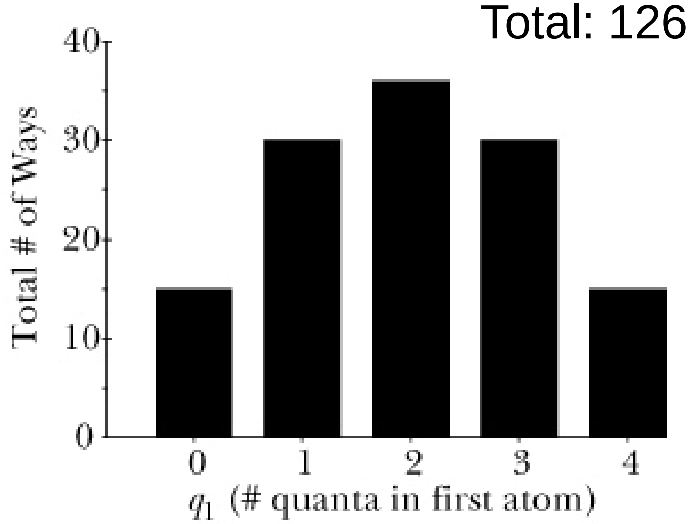
Total: 126

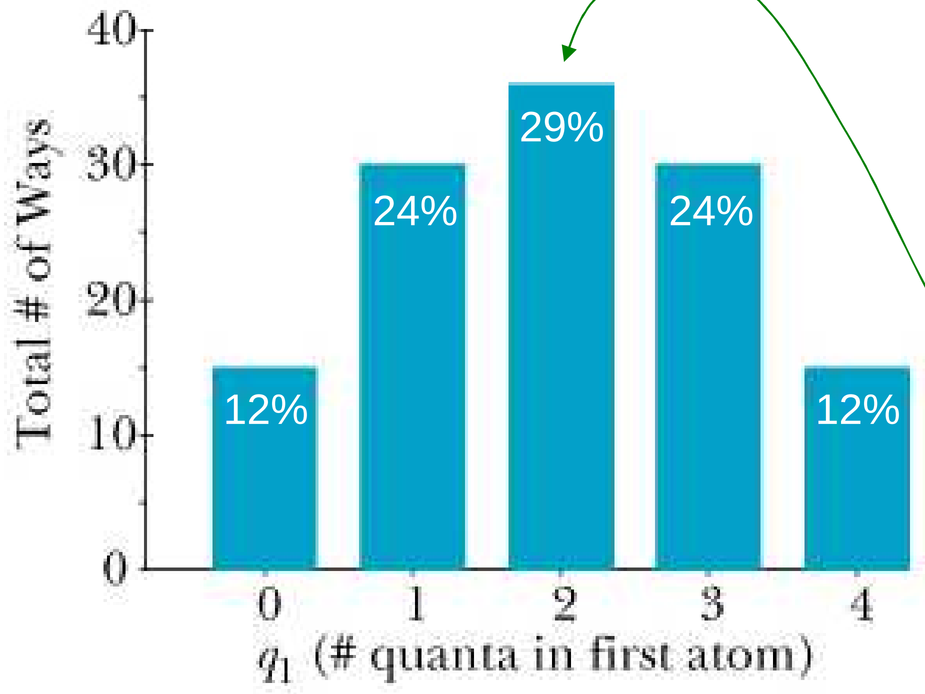


4 dollar bills

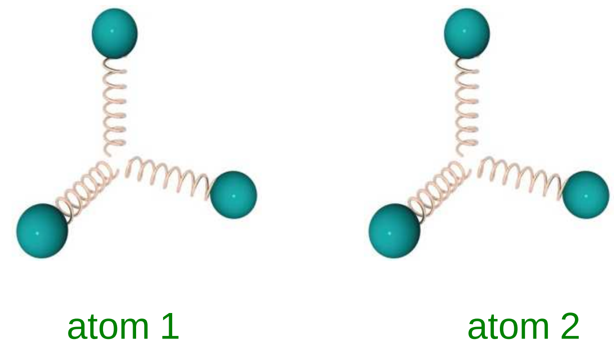
Which state is the most probable?

Fundamental Assumption of Statistical Mechanics

The fundamental assumption of statistical mechanics is that, over time, an isolated system in a given macrostate (total energy) is equally likely to be found in any of its microstates (microscopic distribution of energy).



Thus, our system of 2 atoms is most likely to be in a microstate where energy is split up 50/50.



Reversibility

Next time: Entropy will explain why some processes are not reversible.

Movie taken and modified from: www.theclam.com/Video/JumpingIndex.asp