Solution to Assignment 2: Operational Semantics

CS - 456 – Programming Languages

Exercise 10

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\dfrac{
\dfrac{x \in \mathsf{dom}(\rho) \qquad \dfrac{}{\langle \texttt{LITERAL}(3), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \rangle}\,\textsc{Literal}}{\langle \texttt{SET}(x, \texttt{LITERAL}(3)), \xi, \phi, \rho\{x \mapsto 3\} \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\} \rangle}\,\textsc{FormalAssign}
\qquad
\dfrac{x \in \mathsf{dom}(\rho\{x \mapsto 3\})}{\langle \texttt{VAR}(x), \xi, \phi, \rho\{x \mapsto 3\} \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\} \rangle}\,\textsc{FormalVar}
}{
\langle \texttt{BEGIN}(\texttt{SET}(x, \texttt{LITERAL}(3))\ \texttt{VAR}(x)), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\} \rangle
}\,\textsc{Begin}
$$

Exercise 11

We will consider 4 cases:

1. Consider the case where $x \in \mathsf{dom}(\rho)$ and $\rho(x) = 0$. We then have that:

$$\dfrac{\dfrac{x \in \mathsf{dom}(\rho) \qquad \rho(x) = 0}{\langle \mathtt{VAR}(x), \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi, \phi, \rho\rangle}\textsc{FormalVar} \qquad 0 = 0 \qquad \dfrac{}{\langle \mathtt{LITERAL}(0), \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi, \phi, \rho\rangle}\textsc{Literal}}{\langle \mathtt{IF}(\mathtt{VAR}(x), \mathtt{VAR}(x), \mathtt{LITERAL}(0)), \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi, \phi, \rho\rangle}\textsc{IfFalse}$$

   is the only possible derivation, and hence $v_1 = 0$, $\xi = \xi'$, and $\rho = \rho'$. We have to find out the value of $\mathtt{VAR}(x)$ in the same environments. But that obviously is the left branch of the derivation tree above. That is:

$$\dfrac{x \in \mathsf{dom}(\rho) \qquad \rho(x) = 0}{\langle \mathtt{VAR}(x), \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi, \phi, \rho\rangle}\textsc{FormalVar}$$

   From this we conclude that $v_2 = 0$, $\xi = \xi''$, and $\rho = \rho''$. Evidently $v_1 = v_2 = 0$.

2. Consider the case where $x \in \mathsf{dom}(\rho)$ and $\rho(x) \neq 0$. We then have that:

$$\dfrac{\dfrac{x \in \mathsf{dom}(\rho)}{\langle \mathtt{VAR}(x), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle}\textsc{FormalVar} \qquad \rho(x) \neq 0 \qquad \dfrac{x \in \mathsf{dom}(\rho)}{\langle \mathtt{VAR}(x), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle}\textsc{FormalVar}}{\langle \mathtt{IF}(\mathtt{VAR}(x), \mathtt{VAR}(x), \mathtt{LITERAL}(0)), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle}\textsc{IfTrue}$$

   is the only possible derivation and therefore, $v_1 = \rho(x)$, $\xi = \xi'$, and $\rho = \rho'$. Once more, evaluating $\mathtt{VAR}(x)$ is the left (and right) branch(es) of the derivation above:

$$\dfrac{x \in \mathsf{dom}(\rho)}{\langle \mathtt{VAR}(x), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle}\textsc{FormalVar}$$

   We conclude that $v_2 = \rho(x)$, $\xi = \xi''$, and $\rho = \rho''$. We finalize the case observing that $v_1 = v_2 = \rho(x)$.

3. Consider the case where $x \notin \mathsf{dom}(\rho)$, $x \in \mathsf{dom}(\xi)$ and $\xi(x) = 0$. In this case, which is omitted, we have a derivation similar to the case 1. above, where we use GlobalVar instead of FormalVar. We reach the exact same conclusion.

4. Consider the case where $x \notin \mathsf{dom}(\rho)$, $x \in \mathsf{dom}(\xi)$ and $\xi(x) \neq 0$. In this case, which is omitted, we have a derivation similar to the case 2. above, where we use GlobalVar instead of FormalVar. Again, we reach the exact same result.

Exercise 13

(a) Awk-like semantics for unbound variables:

$$\textsc{UnboundVar}\quad \frac{x \notin \mathsf{dom}(\rho) \qquad x \notin \mathsf{dom}(\xi)}{\langle \mathtt{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi\{x \mapsto 0\}, \phi, \rho \rangle}$$

$$\textsc{UnboundAssign}\quad \frac{x \notin \mathsf{dom}(\rho) \qquad x \notin \mathsf{dom}(\xi)}{\langle \mathtt{SET}(x, v), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi\{v \mapsto v\}, \phi, \rho \rangle}$$

(b) Icon-like semantics for unbound variables:

$$\textsc{UnboundVar}\quad \frac{x \notin \mathsf{dom}(\rho) \qquad x \notin \mathsf{dom}(\xi)}{\langle \mathtt{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho\{x \mapsto 0\} \rangle}$$

$$\textsc{UnboundAssign}\quad \frac{x \notin \mathsf{dom}(\rho) \qquad x \notin \mathsf{dom}(\xi)}{\langle \mathtt{SET}(x, v), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho\{v \mapsto v\} \rangle}$$

(c) Any answer to this question is correct if properly argumented. Generally the effects of the Icon-like semantics are preferable since usage of unbounded variables in one piece of code will not affect other functions, preserving some degree of modularity, easing debugging, among other advantages.

Exercise 20

(Students are expected to provide all the cases of the proof following the samples of IFTRUE and GLOBALVAR).

The answer to this exercise is hinted in the samples that are provided. All the cases that have a premise of the form $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$ require using the **Induction Hypothesis** (as in the provided IFTRUE example). In some cases (BEGIN, APPLYUSER, and all the other primitive applications, eg. APPLYADD) one has to use the inductive hypothesis repeatedly and in order.

The only interesting case, which actually modifies the environment ρ is FORMALASSIGN.

$$\textsc{FormalAssign}\quad \frac{x \in \mathsf{dom}(\rho) \qquad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle \mathtt{SET}(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho'\{x \mapsto v\} \rangle}$$

In this case we have that by the Induction Hypothesis ρ can be safely discarded. There is only one copy of ρ' at the end of the evaluation of e and it's at the top of the stack. To conclude the proof we can pop ρ', make a copy of it with x assigned to v (resulting in $\rho'\{x \mapsto v\}$), discharge it, and then push the updated copy in the stack to obtain the resulting configuration $\langle v, \xi', \phi, \rho'\{x \mapsto v\} \rangle$. It is easy from this step to observe that if instead of poping ρ', copying it and updating it, throwing away the ρ' and pushing the updated copy ($\rho'\{x \mapsto v\}$), we just update *in-place* the original ρ' we would obtain the same configuration. This justifies the in-place operations carried out by `bind` in the implementation of the interpreter.

For all the cases that do not have a hypothesis of the form $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$, a simple observation (as in the provided FORMALVAR) about the fact that the formal environment does not change, and therefore can be pushed back is enough.