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Summer 2012

ECE 20200 : Linear Circuit Analysis II
School of ECE, Purdue University

LECTURE 1

- Introduction to Laplace Transform Analysis
- Basic Signals

Reference: Decarlo/Lin PP 543-554

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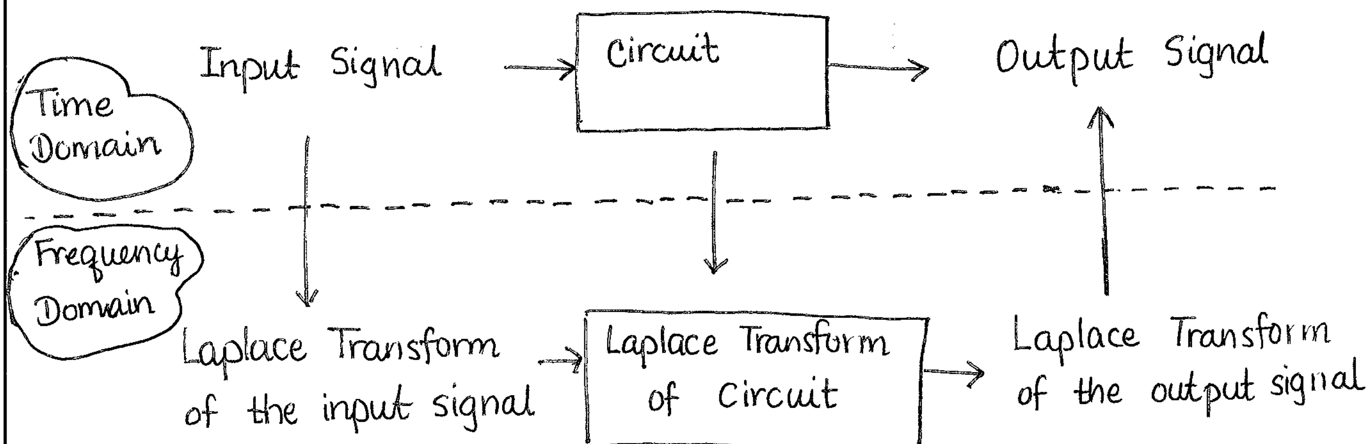
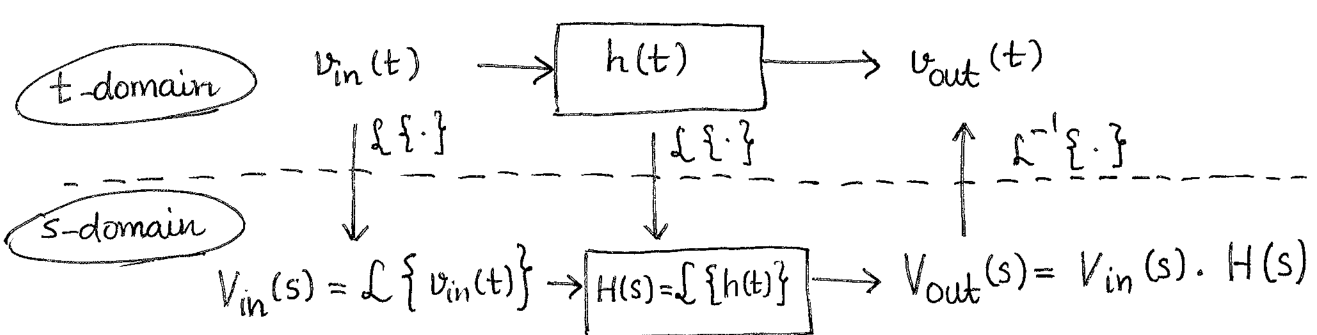
Laplace Transform Analysis

- A technique that transforms the time domain analysis of a circuit, system or differential equation to the so-called "frequency domain".
- Algebraic technique → easier

Circuit Problems and Laplace Transform



Example: Consider a circuit with input signal $v_{in}(t)$ and output signal $v_{out}(t)$. The circuit is represented by the impulse response $h(t)$.



$H(s)$ - transfer function

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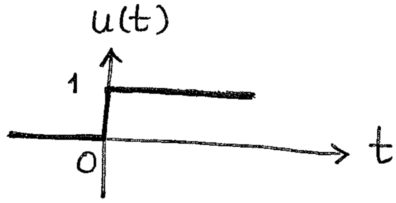
Basic Signals

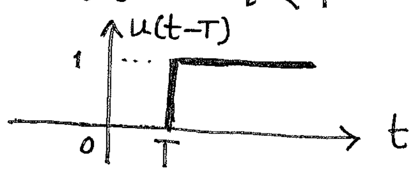
1) Unit step function

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

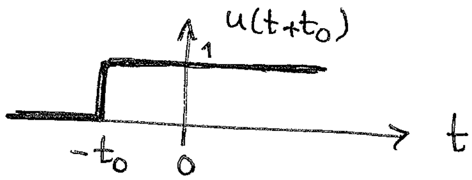


- Shifted step functions

$$u(t-T) = \begin{cases} 1 & t \geq T \\ 0 & t < T \end{cases} \qquad T > 0$$

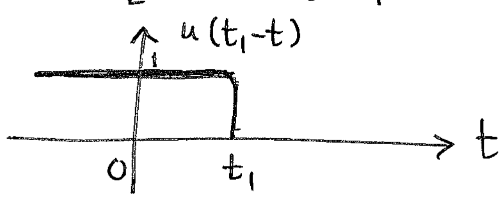


$$u(t+t_0) = \begin{cases} 1 & t \geq -t_0 \\ 0 & t < -t_0 \end{cases} \qquad t_0 > 0$$



- Flipped and shifted step functions

$$u(t_1 - t) = \begin{cases} 1 & t \leq t_1 \\ 0 & t > t_1 \end{cases} \qquad t_1 > 0$$

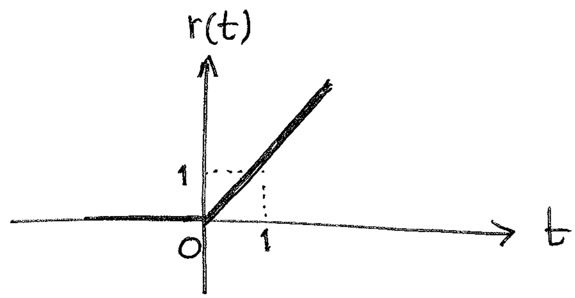


2) Ramp function

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

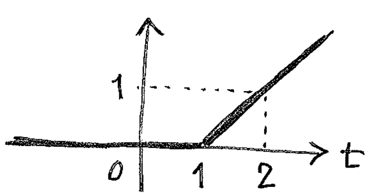
$$= t\,u(t)$$

$$= \int_{-\infty}^{t} u(\tau)\,d\tau$$

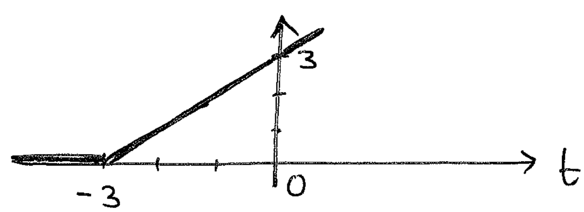


- Shifted and/or flipped ramp functions

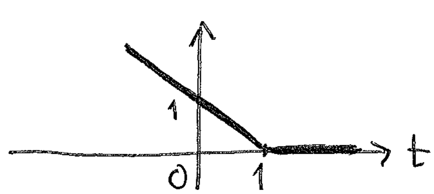
r(t−1)



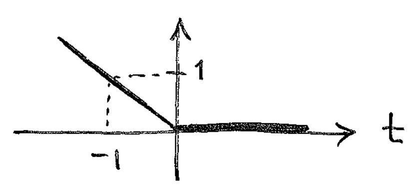
r(t+3)



r(1−t)



r(−t)



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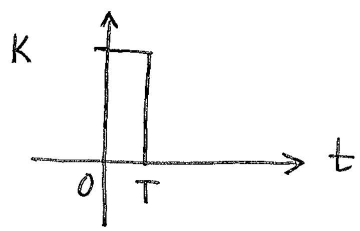
3) delta function (unit impulse function)

- Defined implicitly as

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

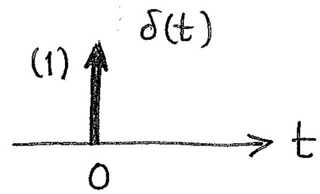
We loosely interpret $\delta(t)$ as

$$\delta(t) = \frac{d}{dt} u(t)$$



$KT = 1$, $K \to \infty$, $T \to 0$

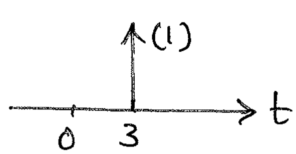
$\Rightarrow$



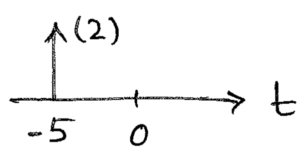
$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases} \quad \text{and} \quad \int_{0^-}^{0^+} \delta(t)\, dt = 1$$

- Shifted delta functions

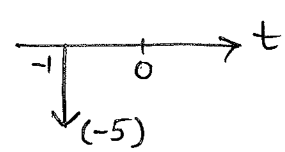
$\delta(t-3)$



$2\delta(t+5)$



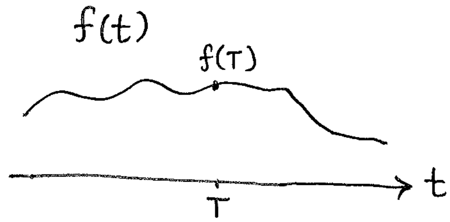
$-5\delta(t+1)$

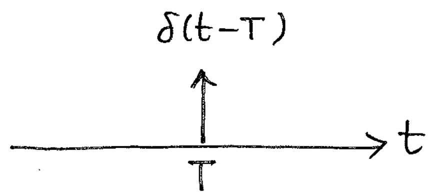


Properties of delta function

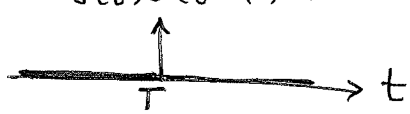
★ Sifting Property

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-T)\,dt = f(T)$$





$$f(t)\delta(t-T) = f(T)\delta(t-T)$$



Thus
$$\int_{-\infty}^{\infty} f(t)\,\delta(t-T)\,dt = \int_{-\infty}^{\infty} f(T)\,\delta(t-T)\,dt$$
$$= f(T)\int_{-\infty}^{\infty} \delta(t-T)\,dt$$
$$= f(T)$$

★ $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

★ $$\frac{d}{dt}\Big(f(t)u(t)\Big) = f'(t)u(t) + f(t)\frac{d}{dt}u(t)$$
$$= f'(t)u(t) + f(t)\,\delta(t)$$
$$= f'(t)u(t) + f(0)\,\delta(t)$$

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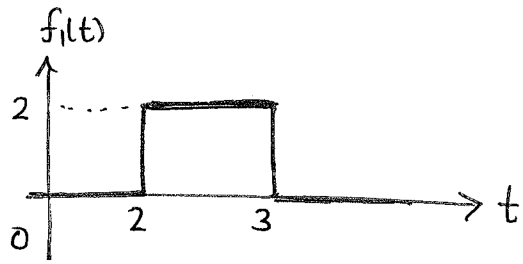
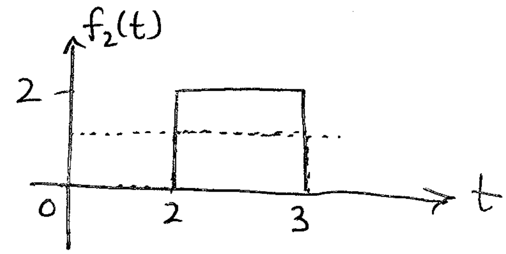
Signal representation

1) Sketch the following signals

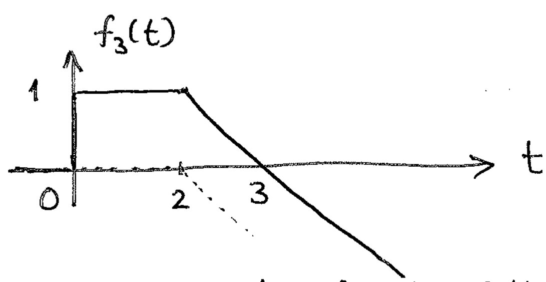
(i) $f_1(t) = 2u(t-2) - 2u(t-3)$



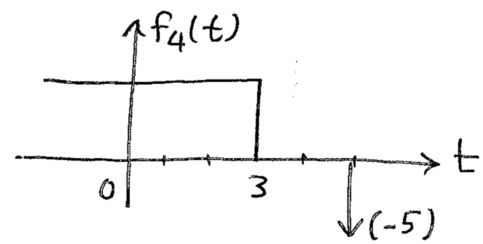
(ii) $f_2(t) = 2u(t-2)u(3-t)$



(iii) $f_3(t) = u(t) - r(t-2)$

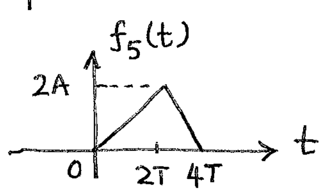


(iv) $f_4(t) = u(3-t) - 5\delta(t-5)$



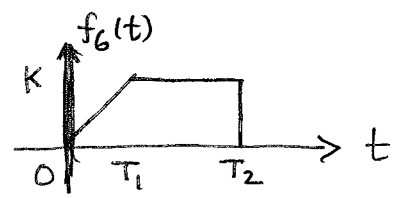
2) Represent each of the following signals using basic signals.

(i)



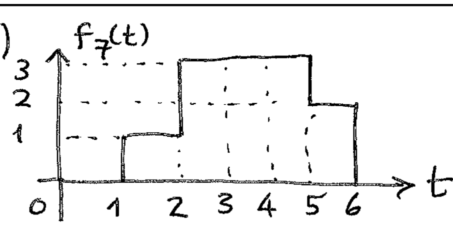
$$f_5(t) = \frac{2A}{2T}r(t) - 2\frac{2A}{2T}r(t-2T) + \frac{2A}{2T}r(t-4T)$$
$$= \frac{A}{T}r(t) - 2\frac{A}{T}r(t-2T) + \frac{A}{T}r(t-4T)$$

(ii)



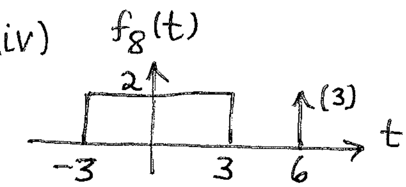
$$f_6(t) = \frac{K}{T_1}r(t) - \frac{K}{T_1}r(t-T_1) - Ku(t-T_2)$$

(iii)



$$f_7(t) = u(t-1) + 2u(t-2) - u(t-5) - 2u(t-6)$$

(iv)



$$f_8(t) = 2u(t+3) - 2u(t-3) + 3\delta(t-6)$$