WebAssign CH 4.9 - 2 (Homework) Yinglai Wang MA 265 Spring 2013, section 132, Spring 2013 Instructor: Alexandre Eremenko

**Current Score :** 20 / 20 **Due :** Thursday, March 21 2013 11:40 PM EDT

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1. 2/2 points | Previous Answers

KolmanLinAlg9 4.9.012.

Compute the row and column ranks of the following matrices.

(a) 
$$\begin{bmatrix} 9 & 6 & 9 \\ -3 & 2 & 1 \\ 9 & 1 & 2 \end{bmatrix}$$

row rank

column rank

(b) 
$$\begin{bmatrix} 2 & -4 & -4 \\ 2 & -1 & 6 \\ 7 & -8 & 6 \end{bmatrix}$$

row rank

column rank

(c) 
$$\begin{bmatrix} 1 & -2 & -2 \\ 4 & -2 & 12 \\ 7 & -8 & 6 \\ 5 & -7 & 0 \end{bmatrix}$$

row rank

column rank

#### 2. 2/2 points | Previous Answers

KolmanLinAlg9 4.9.014.

Compute the rank and nullity of each given matrix and verify the following theorem.

If A is an  $m \times n$  matrix, then rank A + nullity A = n.

(a) 
$$\begin{bmatrix} 1 & 3 & -2 & 4 \\ -1 & 4 & -5 & 10 \\ 3 & 2 & 1 & -2 \\ 3 & -5 & 8 & -16 \end{bmatrix}$$

(b) 
$$\begin{bmatrix} 1 & 1 & 4 & 1 \\ 2 & -1 & 0 & 0 \\ 0 & 4 & -16 & 8 \\ 1 & 1 & -4 & 2 \end{bmatrix}$$
 rank 
$$\begin{bmatrix} 4 & & & \checkmark \\ \text{nullity} & 0 & & \checkmark \end{bmatrix}$$

## 3. 2/2 points | Previous Answers

KolmanLinAlg9 4.9.016.

Determine which of the given linear systems are consistent by comparing the ranks of the coefficient and augmented matrices.

(a) 
$$\begin{bmatrix} 1 & 2 & 5 & -2 \\ 2 & 3 & -2 & 4 \\ 5 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- consistent
- inconsistent

(b) 
$$\begin{bmatrix} 1 & 2 & 5 & -2 \\ 2 & 3 & -2 & 4 \\ 5 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -5 \\ -12 \\ 4 \end{bmatrix}$$

- consistent
- inconsistent

#### 4. 2/2 points | Previous Answers

KolmanLinAlg9 4.9.020.

Use the following corollary to find out whether rank A = 3 for each given matrix.

If A is an  $n \times n$  matrix, then rank A = n if and only if  $det(A) \neq 0$ .

(a) 
$$A = \begin{bmatrix} 3 & 1 & 2 \\ 10 & 5 & 10 \\ 4 & 2 & 5 \end{bmatrix}$$
  

$$det(A) = \begin{bmatrix} 5 & & & \\ & &$$

## **5.** 2/2 points | Previous Answers

KolmanLinAlg9 4.9.022.

Use the following corollary to find which of the given homogeneous systems have a nontrivial solution.

The homogeneous system  $A\mathbf{x} = \mathbf{0}$ , where A is  $n \times n$ , has a nontrivial solution if and only if rank A < n.

(a) 
$$\begin{bmatrix} 1 & 5 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- trivial solution
- nontrivial solution

(b) 
$$\begin{bmatrix} 7 & 6 & 9 & 18 \\ -2 & -2 & -3 & -6 \\ 1 & 1 & 1 & 2 \\ -3 & -3 & -3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- trivial solution
- nontrivial solution

**6.** 2/2 points | Previous Answers

KolmanLinAlg9 4.9.029.

Solve using the concept of rank.

Is

$$S = \left\{ \begin{bmatrix} -1 & 4 & 8 \end{bmatrix}, \begin{bmatrix} 2 & -7 & -14 \end{bmatrix}, \begin{bmatrix} -2 & 6 & 12 \end{bmatrix} \right\}$$

a linearly independent set of vectors in  $R_3$ ?



# 7. 2/2 points | Previous Answers

KolmanLinAlg9 4.9.030.

Solve using the concept of rank.

Does the set

$$S = \left\{ \begin{bmatrix} 4\\3\\9 \end{bmatrix}, \begin{bmatrix} 1\\1\\3 \end{bmatrix}, \begin{bmatrix} -1\\-1\\-2 \end{bmatrix} \right\}$$

span  $R^3$ ?



## 8. 2/2 points | Previous Answers

KolmanLinAlg9 4.9.032.

Solve using the concept of rank.

Is

$$S = \left\{ \begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ 0 & 3 \end{bmatrix} \right\}$$

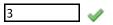
a basis for  $M_{22}$ ?



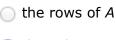
<b>9.</b> 2/2 points	Previous Answers

KolmanLinAlg9 4.9.034.

(a) If A is a  $3 \times 5$  matrix, what is the largest possible value for rank A?



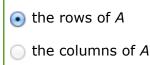
(b) If A is a  $3 \times 4$  matrix, which can be guaranteed to be linearly dependent of A?



the columns of A



(c) If A is a  $4 \times 3$  matrix, which can be guaranteed to be linearly dependent of A?



10.2/2 points | Previous Answers

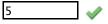
KolmanLinAlg9 4.9.036.

Let A be a  $5 \times 9$  matrix.

(a) Give all possible values for the rank of A. (Enter your answers as a comma-separated list.)



(b) If the rank of A is 5, what is the dimension of its column space?



(c) If the rank of A is 5, what is the dimension of the solution space of the homogeneous system Ax = 0?

