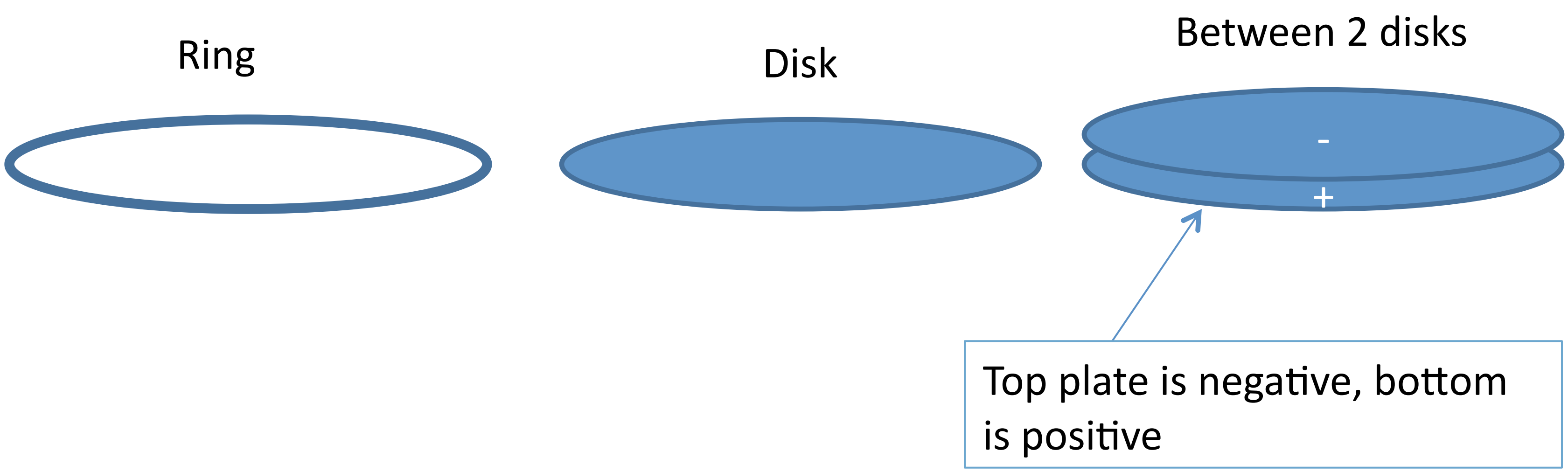
iClicker Poll (not graded)

A. I want to keep HW due Tuesdays and Fridays.

B. I want to ***change*** HW to be due Tuesdays and Thursdays, starting next week.

Last Time

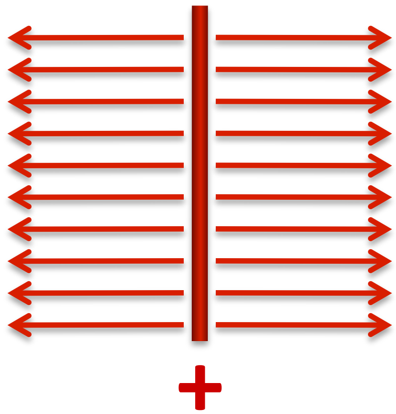
- We found the fields of:



Two Infinite Planes

$$E = \frac{\sigma}{2\epsilon_o}$$

$\sigma =$ Charge per Area



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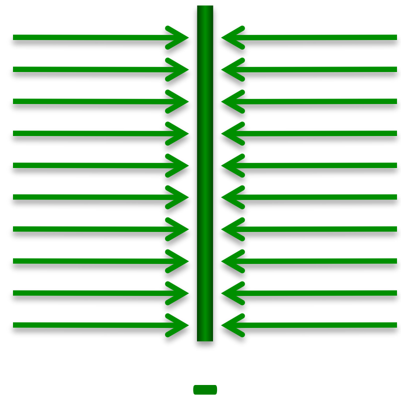
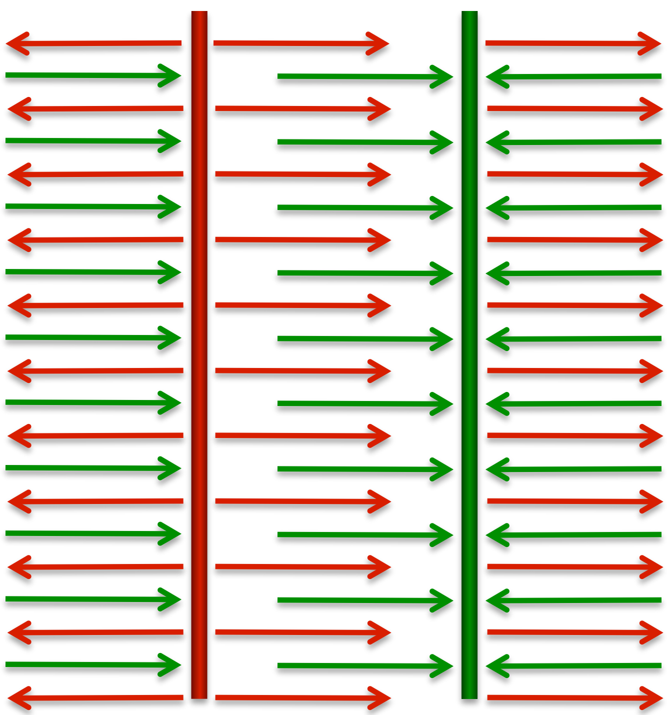


Same but Opposite Charge

SUPERPOSITION:

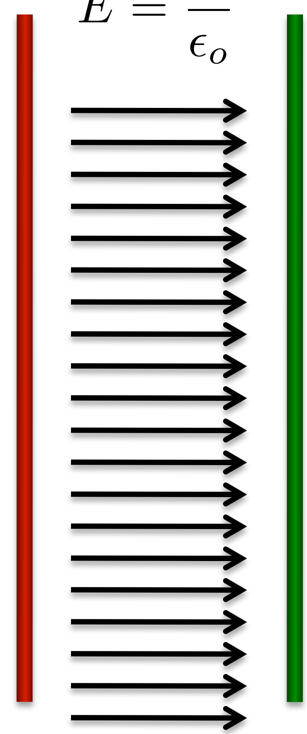


=

$E = 0$

$E = \frac{\sigma}{\epsilon_o}$

$E = 0$



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Today

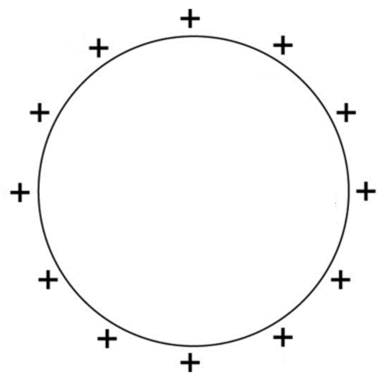
- Electric field of a hollow sphere
  - Inside the sphere
  - Outside the sphere
- Electric field of a solid sphere
  - Inside the sphere
  - Outside the sphere

Finding E from a Hollow Sphere*

- Like before:
  - Break up the sphere into point charges
  - Calculate the electric field due to the point charges
  - Sum to get the total field
  - Sanity Check!
- Use symmetry of the sphere

** **Insulating** Hollow Sphere: Charges uniformly distributed, but cannot move.*

Hollow Sphere



Total Charge = Q

Area = $4\pi R^2$

Charge Density = $Q/(4\pi R^2)$

(How did I know to use area, and not volume?)

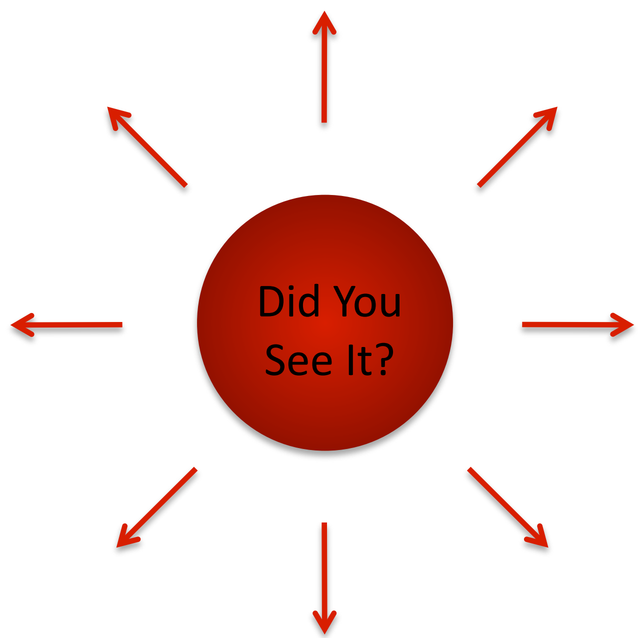
FINE PRINT: *(Void in New Hampshire)*

- Up close ($< 10^{-7}$ m) we can see discrete charges on the surface
- Approximate: uniform, continuous charge density

Symmetries of a sphere

- What direction do you expect for E?

  → Watch this animated rotation closely:



E field has the same symmetries as the sphere

In physics, if you can't tell the difference, *there is no difference*!

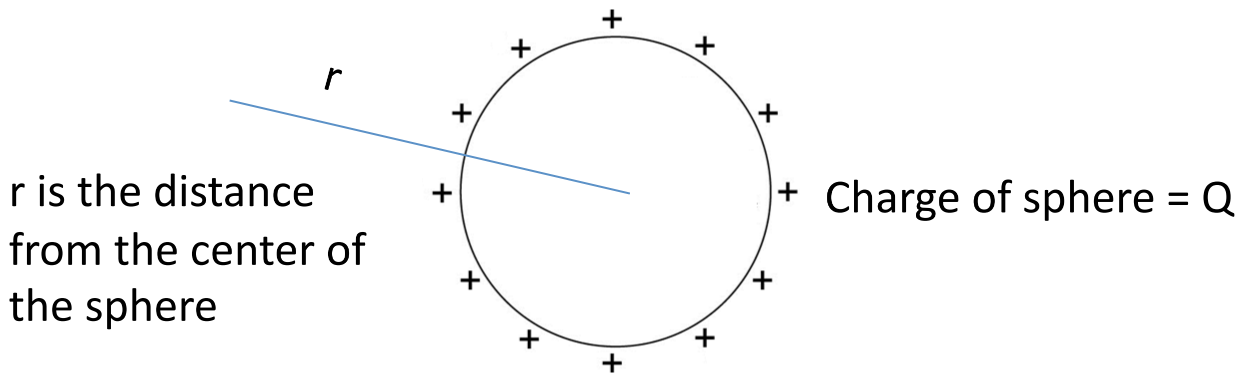
Electric field from far away

- From far away, every charged object looks like a point charge

Hollow Sphere: Looks *EXACTLY* like a point charge

Field outside

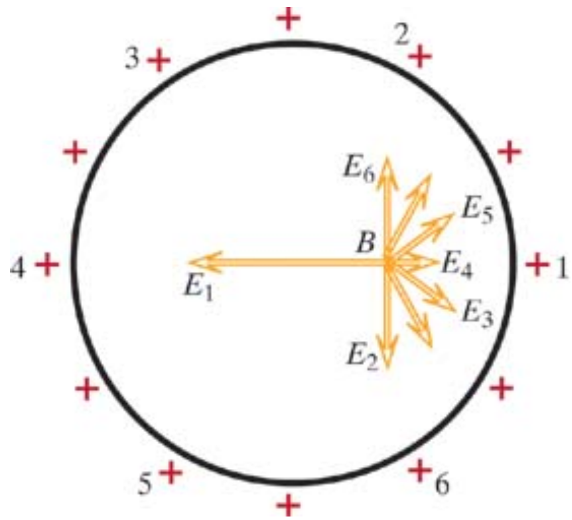
- We could do integration, but it's complicated
- We would find that $E_{outside}$ is the same as for a point charge located at the center, with charge Q



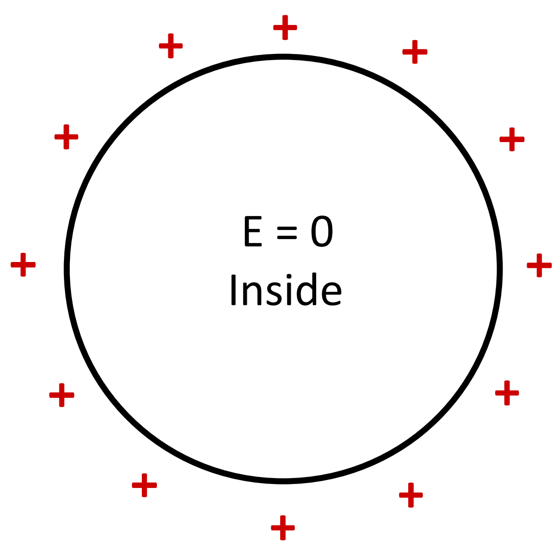
- $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$

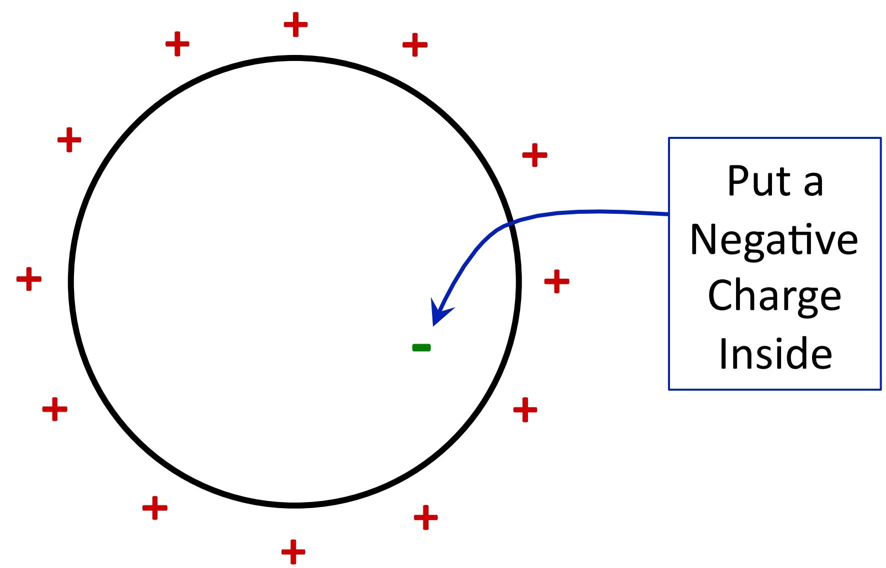
iClicker question

E inside the Hollow Sphere



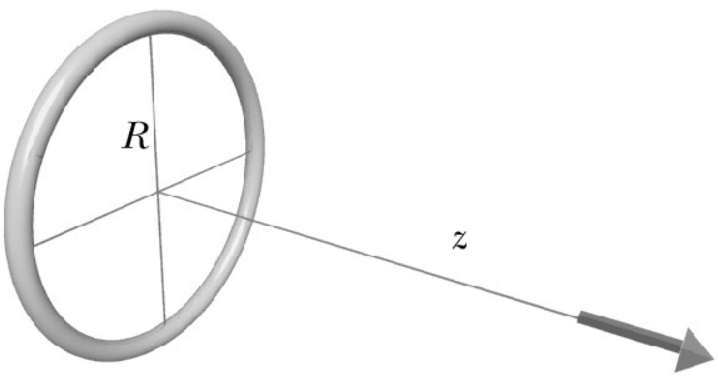
Complete Cancellation Everywhere Inside!

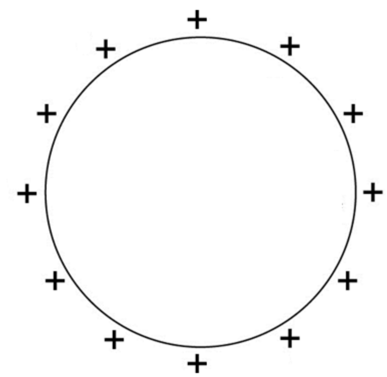


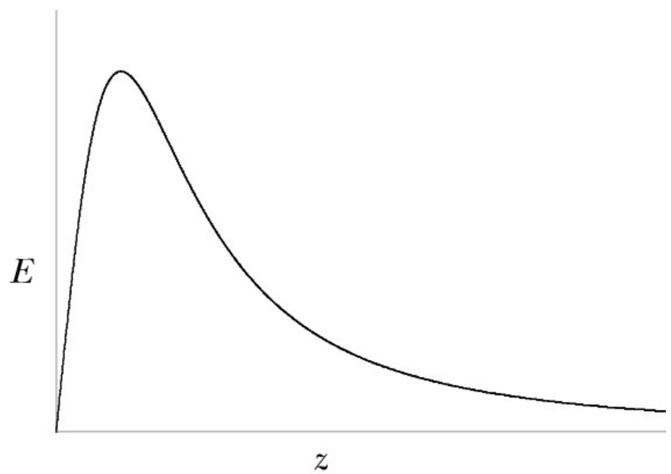


- What is the field inside now?
- What force is on the negative charge?
- What if the sphere were conducting?

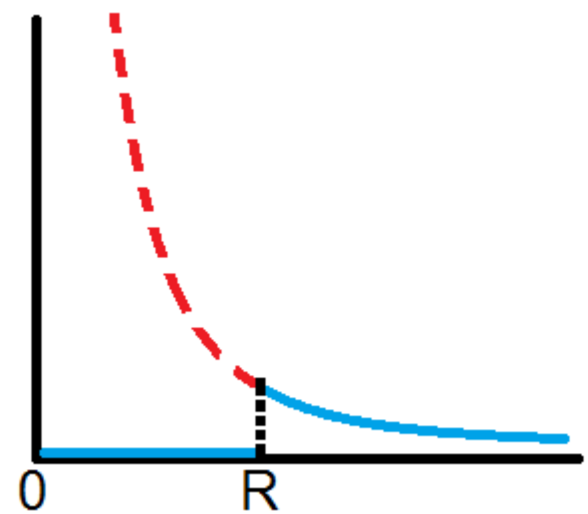
Two different situations







Electric field changes continuously

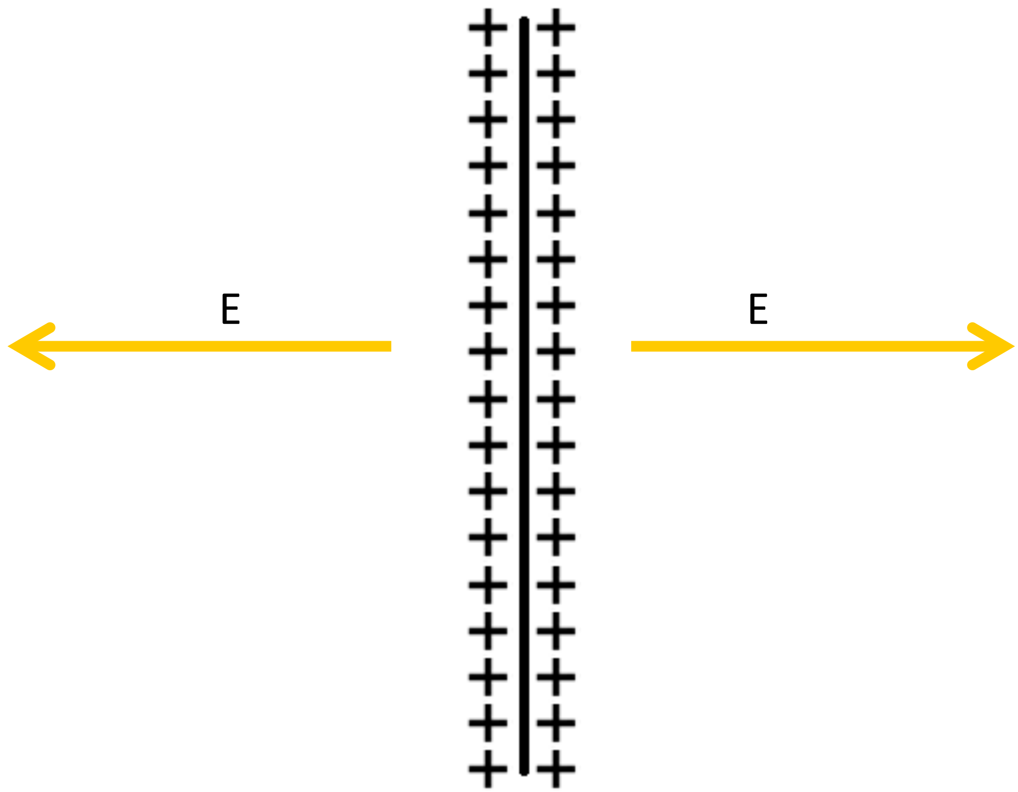


Electric field jumps from 0 to $1/R^2$

Why are these so different?

E at a Charged Surface

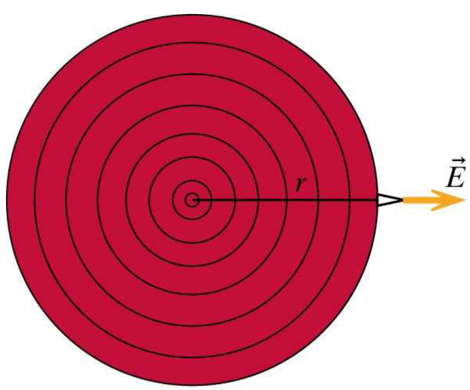
- Electric field can change suddenly when you pass through a charged surface



Electric field of a solid sphere

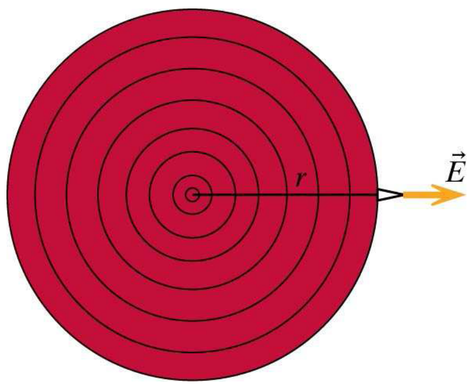
- How can we build a solid sphere out of hollow spheres?

→ Build it out of several concentric hollow spheres
→ Add up their fields



Charge Density of Solid Sphere

- Total charge on entire sphere: Q
- Volume: $V = \frac{4}{3}\pi R^3$



- Charge density of a SOLID sphere of radius R

$$\frac{Q}{(4/3)\pi R^3}$$

Integrating the Spheres

- Charge on an object = Charge density * Volume

  (For a uniform charge density, otherwise, you must integrate)
- Volume of a spherical shell is approximately $4\pi r_s^2 \Delta r_s$ – Surface area of the shell * thickness
- This approximation becomes better as Δr_s -> 0
- When integrating, Δr -> dr_s

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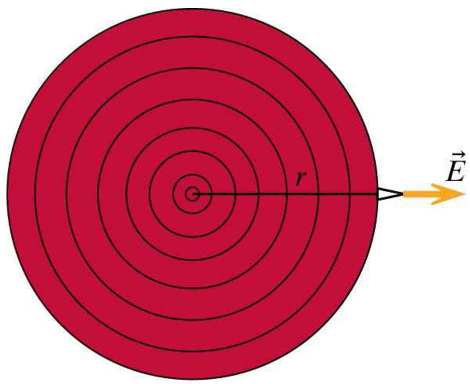
E from each Spherical Shell

E of a shell $$\vec{E} = \frac{q}{4\pi\epsilon_o |r|^2}\hat{r}$$

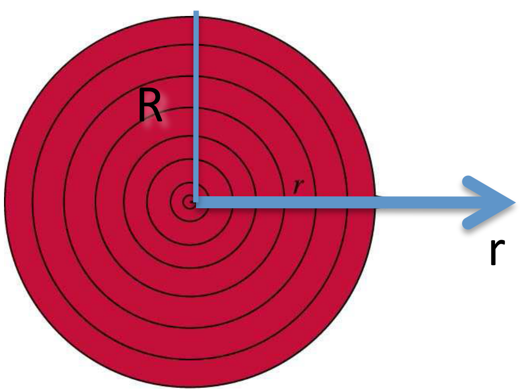
Charge on each shell $$q_{\text{shell}} = \frac{3Qr_s^2}{R^3} dr_s$$

$$\Delta E_{\text{shell}} = \frac{3}{4\pi\epsilon_o} \frac{Qr_s^2}{R^3 |r|^2} \hat{r} dr_s$$



E Outside the Solid Sphere

$$\Delta E_{\text{shell}} = \frac{3}{4\pi\epsilon_o}\frac{Qr_s^2}{R^3|r|^2}\hat{r}dr_s$$



Add up (Integrate) all spheres

$$\vec{E}_{\text{total}} = \sum_{\text{shells}} \Delta E_{\text{shell}}$$

$$\rightarrow \frac{3}{4\pi\epsilon_o}\frac{Q}{R^3|r|^2}\hat{r}\int_0^R r_s^2 dr_s$$

$$= \frac{3}{4\pi\epsilon_o}\frac{Q}{R^3|r|^2}\hat{r}\frac{1}{3}R^3$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_o|r|^2}\hat{r}}$$

OUTSIDE THE SPHERE

E Inside the Solid Sphere

$$\Delta E_{\text{shell}} = \frac{3}{4\pi\epsilon_o}\frac{Qr_s^2}{R^3|r|^2}\hat{r}dr_s$$

What is the field at **r** due to the grey shells?

→ Zero

Now, integrate from 0 to r.

Add up (Integrate) the inner spheres

$$\vec{E}_{\text{total}} = \sum_{\text{shells}} \Delta E_{\text{shell}} \rightarrow \frac{3}{4\pi\epsilon_o}\frac{Q}{R^3|r|^2}\hat{r}\int_0^r r_s^2 dr_s$$

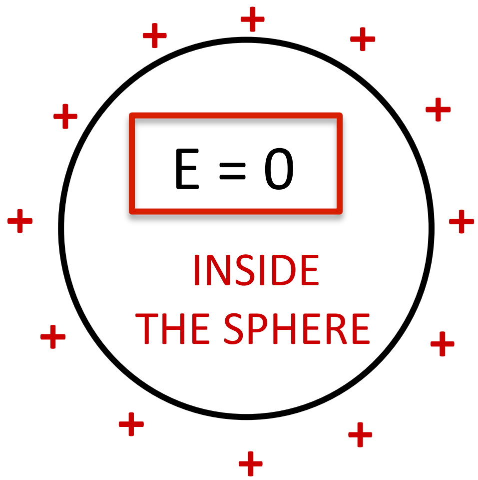
$$= \frac{3}{4\pi\epsilon_o}\frac{Q}{R^3|r|^2}\hat{r}\frac{1}{3}|r|^3$$

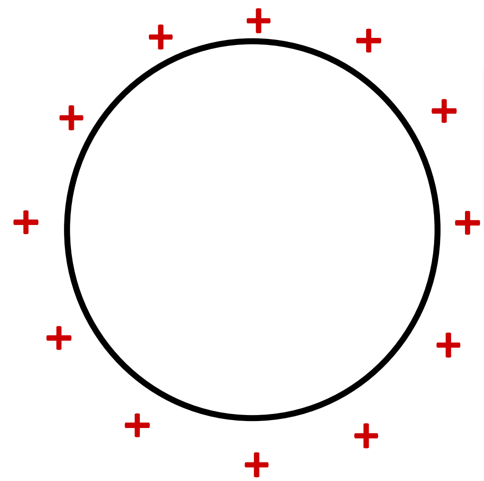
$$\boxed{\vec{E} = \frac{Q|r|}{4\pi\epsilon_o R^3}\hat{r}}$$

INSIDE THE SPHERE

What We Did Today

- Electric field of a hollow sphere





$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_o |r|^2}\hat{r}}$$

OUTSIDE
THE SPHERE

- Electric field of a solid sphere

$$\boxed{\vec{E} = \frac{Q|r|}{4\pi\epsilon_o R^3}\hat{r}}$$

INSIDE
THE SPHERE

$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_o |r|^2}\hat{r}}$$

OUTSIDE
THE SPHERE