Purdue University
School of Electrical and Computer Engineering

ECE 20200 : Linear Circuit Analysis II

Summer 2012
Instructor: Aung Kyi San

Midterm Examination I
June 28, 2012

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet.
2. Enter your name, student ID number, e-mail address and your full signature in the space provided on this page.
3. You have one hour to complete all **5** questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
4. Read the questions carefully. Unless otherwise stated, you must fully justify your answers. You may use any method you want unless you are asked to use a specific method.
5. This booklet contains **14** pages including the Laplace Transform Tables. Since only this booklet will be graded, make sure you have all your answers written in this booklet.
6. Notes, books, calculators, cell phones, pagers and any other electronic communication device are strictly forbidden.

Name:____

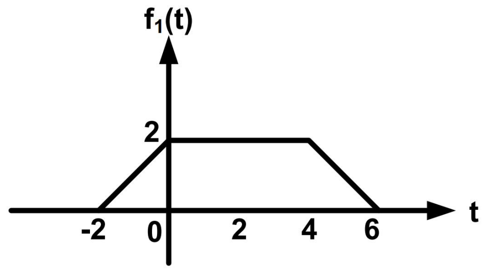
Student ID:____________________________________

Email:____

Signature:_____________________________________

(Total 20 pts) **1.**

(a) (8 pts)



(i) (4 pts) Find the Laplace transform of $f_1(t)$.

(ii) (4 pts) Find the Laplace transform of $f_2(t) = \dfrac{d}{dt}\Big[f_1(t)\Big]$.

(b) (6 pts) Find the Laplace transform of $f_3(t) = te^{-t}u(t-2)$.

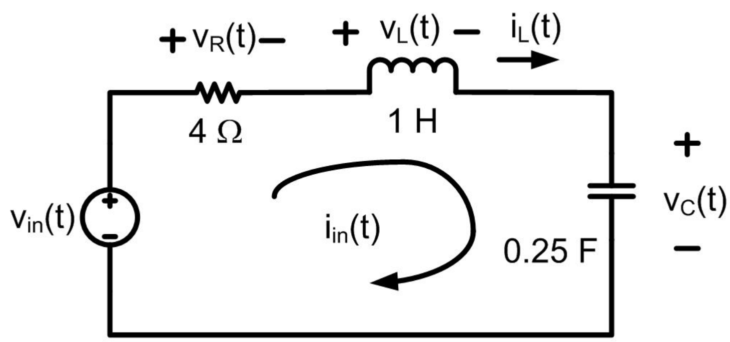
(c) (6 pts) Find the inverse Laplace transform of the function

$$F_4(s) = \frac{24s - 72}{s^2 + 4s + 40}.$$

(Total 30 pts) **2.**

(a) (15 pts)

Consider the series RLC circuit given below. Suppose the initial conditions are $i_L(0^-) = 1\text{A}$ and $v_C(0^-) = -2\text{V}$.



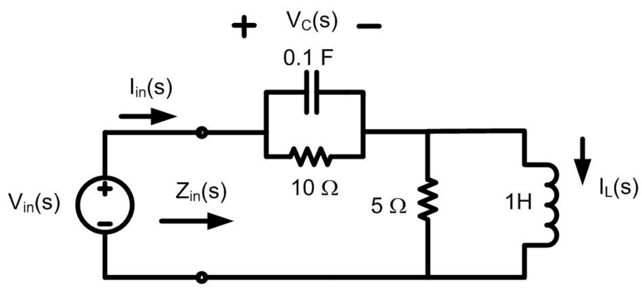
(i) (2 pts) Show that the integro-differential equation of the circuit is given by

$$4i_{in}(t) + \frac{di_{in}(t)}{dt} + 4\int_{-\infty}^{t} i_{in}(q)dq = v_{in}(t)$$

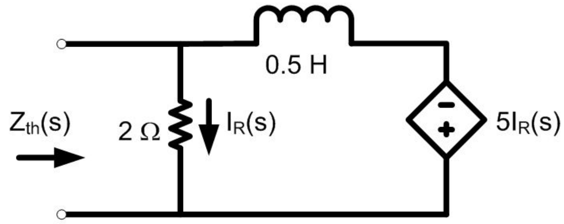
(ii) (7 pts) Using the result from part (i), find $I_{in}(s)$ if $v_{in}(t) = \delta(t)$ V.

(iii) (6 pts) Find $i_{in}(t)$.

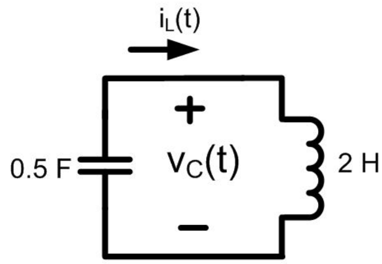
(b) (7 pts) Compute the input impedance $Z_{in}(s)$ of a series connection of two pairs of parallel elements, as shown in the figure below. Suppose the initial conditions are $i_L(0^-) = 2\text{A}$ and $v_C(0^-) = -1\text{V}$.



(c) (8 pts) Compute the Thevenin equivalent impedance of the following circuit.

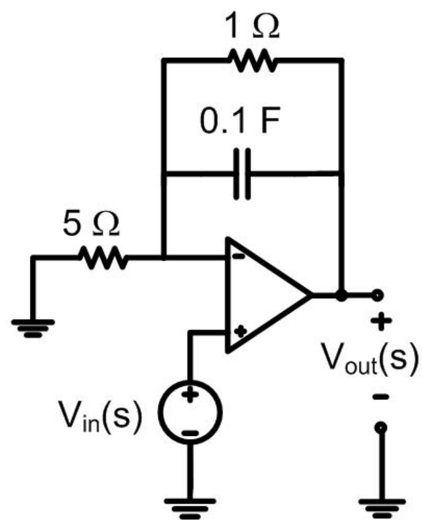


(10 pts) **3.** Consider the circuit given below. Suppose the initial conditions are $i_L(0^-) = -1\text{A}$ and $v_C(0^-) = 2\text{V}$. Find $v_c(t)$ for $t \geq 0$.



(Total 15 pts) **4.**

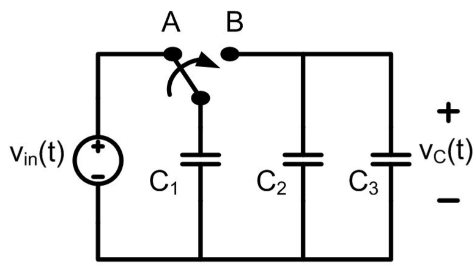
(a) (6 pts) Find the transfer function of the ideal op amp circuit shown below.



(b) (4 pts) Find the impulse response of the circuit.

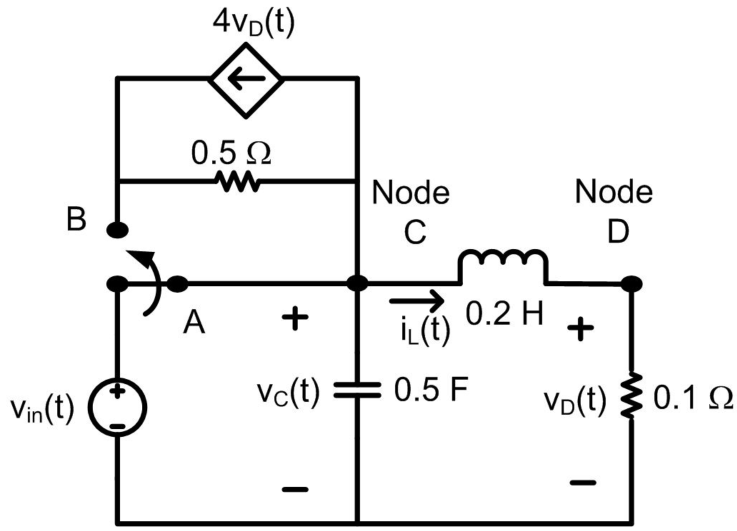
(c) (5 pts) Find the zero-state response to $v_{in}(t) = u(t)$ V.

(Total 25 pts) **5.**
(a) (5 pts) Consider the circuit given below. The switch is moved from position A to position B at t = 2 s.



If $v_{in}(t) = 25$V, $C_1 = 150$ mF, $C_2 = C_3 = 50$ mF and $v_C(2^-) = 10$ V, find $v_C(t)$ for $t \geq 2$ s.

(b) (20 pts) In the circuit below, the switch has been in position A for a long time and then moves to position B at t = 0. It is given that $v_{in}(t) = 4u(-t) + 80u(t)$ V.



(i) (4 pts) Compute $i_L(0^-)$ and $v_C(0^-)$

(ii) (5 pts) Draw an equivalent s-domain circuit valid for $t \geq 0$ using current-source equivalent models of charged capacitor and charged inductor.

(iii) (6 pts) Using the circuit obtained in part (ii), write down the nodal equations at node C and node D.

(iv) (2 pts) Write the equations obtained in part (iii) in matrix form.

(v) (3 pts) Write down the formula you would use to solve $V_C(s)$ using Cramer's Rule.

Table 12.1 LAPLACE TRANSFORM PAIRS

Item Number	$f(t)$	$\mathcal{L}[f(t)] = F(s)$
1	$K\delta(t)$	K
2	$Ku(t)$ or K	K/s
3	$r(t)$	$1/s^2$
4	$t^n u(t)$	$\dfrac{n!}{s^{n+1}}$
5	$e^{-at}u(t)$	$1/(s+a)$
6	$te^{-at}u(t)$	$1/(s+a)^2$
7	$t^n e^{-at}u(t)$	$\dfrac{n!}{(s+a)^{n+1}}$
8	$\sin(\omega t)u(t)$	$\dfrac{\omega}{s^2+\omega^2}$
9	$\cos(\omega t)u(t)$	$\dfrac{s}{s^2+\omega^2}$
10	$e^{-at}\sin(\omega t)u(t)$	$\dfrac{\omega}{(s+a)^2+\omega^2}$
11	$e^{-at}\cos(\omega t)u(t)$	$\dfrac{(s+a)}{(s+a)^2+\omega^2}$
12	$t\sin(\omega t)u(t)$	$\dfrac{2\omega s}{(s^2+\omega^2)^2}$
13	$t\cos(\omega t)u(t)$	$\dfrac{s^2-\omega^2}{(s^2+\omega^2)^2}$
14	$\sin(\omega t+\phi)u(t)$	$\dfrac{s\sin(\phi)+\omega\cos(\phi)}{s^2+\omega^2}$
15	$\cos(\omega t+\phi)u(t)$	$\dfrac{s\cos(\phi)-\omega\sin(\phi)}{s^2+\omega^2}$
16	$e^{-at}[\sin(\omega t)-\omega t\cos(\omega t)]u(t)$	$\dfrac{2\omega^3}{[(s+a)^2+\omega^2]^2}$

17	$te^{-at}\sin(\omega t)u(t)$	$2\omega\dfrac{s+a}{[(s+a)^2+\omega^2]^2}$
18	$e^{-at}\left[C_1\cos(\omega t)+\left(\dfrac{C_2-C_1a}{\omega}\right)\sin(\omega t)\right]u(t)$	$\dfrac{C_1s+C_2}{(s+a)^2+\omega^2}$

Table 12.2 LAPLACE TRANSFORM PROPERTIES

Property	Transform Pair
Linearity	$\mathcal{L}[a_1f_1(t) + a_2f_2(t)] = a_1F_1(s) + a_2F_2(s)$
Time Shift	$\mathcal{L}[f(t-T)u(t-T)] = e^{-sT}F(s),\ T > 0$
Multiplication by t	$\mathcal{L}[tf(t)u(t)] = -\dfrac{d}{ds}F(s)$
Multiplication by t^n	$\mathrm{L}[t^n f(t)] = (-1)^n \dfrac{d^nF(s)}{ds^n}$
Frequency Shift	$\mathcal{L}[e^{-at}f(t)] = F(s+a)$
Time Differentiation	$\mathcal{L}\left[\dfrac{d}{dt}f(t)\right] = sF(s) - f(0^-)$
Second-Order Differentiation	$\mathrm{L}\left[\dfrac{d^2f(t)}{dt^2}\right] = s^2F(s) - sf(0^-) - f^{(1)}(0^-)$
nth-Order Differentiation	$\mathrm{L}\left[\dfrac{d^nf(t)}{dt^n}\right] = s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f^{(1)}(0^-)$ $- \ldots - f^{(n-1)}(0^-)$
Time Integration	(i) $\mathrm{L}\left[\int_{-\infty}^{t} f(q)dq\right] = \dfrac{F(s)}{s} + \dfrac{\int_{-\infty}^{0^-} f(q)dq}{s}$ (ii) $\mathrm{L}\left[\int_{0^-}^{t} f(q)dq\right] = \dfrac{F(s)}{s}$
Time/Frequency Scaling	$\mathcal{L}[f(at)] = \dfrac{1}{a}F\left(\dfrac{s}{a}\right)$