STAT 350 Review Set 3 Solution

1. True of False Questions:

T F a) If the population mean is known, there is no reason to run a hypothesis test on he population mean.

T F b) The P-value is usually chosen before an experiment is conducted.

T F c) In conducting a hypothesis test, it is impossible to simultaneously make both a Type I error and a Type II error.

T F d) The probability of a Type II error does not depend on the probability of a Type I error.

T F e) While the normal distribution is symmetric, the t-distributions are slightly skewed.

T F f) The smaller the df, the closer the t-distributions are to the normal distribution.

T F g) If there is sufficient evidence to reject a null hypothesis at the 5% level, then there is sufficient evidence to reject it at the 10% level.

T F h) In a study of high school students, a positive correlation (r = 0.95) was found between hours spent per week doing homework and scores on standardized achievement tests. We can conclude that doing homework helps prepare students for these tests from the data.

T F i) The sample correlation coefficient for the following 10 pairs of data $(1, 2), (2, 4), (3, 8), (4, 16), (5, 32), (6, 64), (7, 128), (8, 256), (9, 512), (10, 1024)$ is approximately 1 because $y = 2^x$.

T F j) When $r \approx 0$, there still can be strong relationship between the variables.

Answer: TF TF FF TF FT

2. A random sample of 26 offshore oil workers took part in a simulated escape exercise, and their times (sec) to complete the escape are recorded. The sample mean is 370.69 sec and the sample standard deviation is 24.36 sec.

a) Construct a 95% upper confidence bound on the true average escape time. Interpret your bound.

Solution: degree of freedom = 25, t* = 1.708

$$\bar{x} + t * \frac{s}{\sqrt{n}} = 370.69 + 1.708 * \frac{24.36}{\sqrt{n}} = 378.82$$

I am 95% confident that the mean escape time for the population is less than 378.82 seconds.

b) Construct a 95% lower confidence bound on the true average escape time. Interpret your bound.

Solution:

$$\bar{x} - t * \frac{s}{\sqrt{n}} = 370.69 - 1.708 * \frac{24.36}{\sqrt{n}} = 362.53$$

I am 95% confident that the mean escape time for the population is more than 362.53 seconds..

3. A study is to be conducted on the proportion of homeowners who own at least two TV sets. How large a sample is required if we wish to be 90% confident that the error in estimating this quantity is within $\pm 1.5\%$?

Solution:

$$z^* = 1.645 \quad \text{for } C = 90\%,$$

$$1.5\% = 1.645\sqrt{\frac{0.5 * 0.5}{n}}$$

$$n = 3006.69 \approx 3007$$

4. An investigator wishes to estimate the difference between population mean SAT-M scores of incoming freshmen in the College of Engineering and in the College of Science at Purdue University. The population standard deviations are both roughly 100 points and equal sample sizes are to be selected. What value of the common sample size n will be necessary to estimate the difference to within ± 10 points with 99% confidence?

Solution:

$$z^* = 2.575 \quad \text{for } C = 99\%,$$

$$\sigma_1 = \sigma_2 = 100, n_1 = n_2 = n$$

$$10 = 2.575 * \sqrt{\frac{100^2}{n} + \frac{100^2}{n}}$$

$$n = 1327$$

5. The life in hours of a battery is known to be approximately normally distributed. A random sample of 10 batteries has a mean life of $\bar{x} = 40.5$ hours and standard deviation $s = 1.25$ hours. Is there evidence to support the claim that the battery life exceeds 40 hours? Use $\alpha = 0.05$

Solution: Let μ be the mean battery life.

$$H_0 : \mu = 40$$

$$H_a : \mu > 40$$

We have a random sample of n=10 batteries, with $\bar{x} = 40.5$ and $s = 1.25$;

Use 1-sample T test since σ is unknown and population is approximately normal.

Test Statistic:

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{40.5 - 40}{1.25/\sqrt{10}} = 1.265$$

degree of freedom = 9, Use T-Table to get $0.10 < P - Value < 0.25$.

Since $\alpha = 0.05$ and $P - Value > \alpha$, we fail to reject the null hypothesis.

There is no evidence to support the claim that the battery life exceeds 40 hours.

6. A random sample of 500 registered voters in Indiana is asked if they favor the use of oxygenated fuels year-round to reduce air pollution. If more than 315 voters respond positively, we will conclude that more than 60% of the voters favor the use of these fuels.

a) Find the probability of type I error if exactly 60 % of the voters favor the use of these fuels.

b) What is the probability of type II error if 67% of the voters favor this action?

Solution: First, state the hypothesis being tested:

$$H_0 : p = 0.6 \text{ vs } H_1 : p > 0.6$$

n= 500, if $\hat{p} > 315/500$ then reject H_0 in favor of H_1.

a) P(Type I error) = P(Reject H_0| p = 0.6)

$$= P(\hat{p} > 315/500 | p = 0.6) = P(Z > \frac{0.63 - 0.6}{\sqrt{0.6 * 0.4/500}} = 1.37) = 0.0853$$

b) P (Type II error) = P(Fail to Reject H_0| p = 0.67)

$$= P(\hat{p} < 315/500 | p = 0.67) = P(Z < \frac{0.63 - 0.67}{\sqrt{0.67 * 0.33/500}} = -1.9) = 0.0287$$

7. The overall distance traveled by a golf ball is tested by hitting the ball with Iron Byron, a mechanical golfer with a swing that is said to emulate the legendary champion, Byron Nelson. Ten randomly selected balls of two different brands are tested and the overall distance measured. The data follow:
Brand 1: 275, 286, 287, 271, 283, 271, 279, 275, 263, 267
Brand 2: 258, 244, 260, 265, 273, 281, 271, 270, 263, 268

Using Minitab software, Harry and Hermione produced different results for a 95% confidence interval for the mean difference in overall distance between the two brands of golf balls. Answer the questions on the next page.

Harry's Output:

Two-Sample T-Test and CI: Ex10-29 Brand 1, Ex10-29 Brand 2

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Two-sample T for Ex10-29 Brand 1 vs Ex10-29 Brand 2

                   N    Mean  StDev  SE Mean
Ex10-29 Brand 1   10  275.70   8.03      2.5
Ex10-29 Brand 2   10   265.3   10.0      3.2


Difference = mu (Ex10-29 Brand 1) - mu (Ex10-29 Brand 2)
Estimate for difference:  10.40
95% CI for difference:  (1.82, 18.98)
T-Test of difference = 0 (vs not =): T-Value = 2.56  P-Value = 0.020  DF = 17
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Hermione's Output:

Paired T-Test and CI: Ex10-29 Brand 1, Ex10-29 Brand 2

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Paired T for Ex10-29 Brand 1 - Ex10-29 Brand 2

                    N    Mean  StDev  SE Mean
Ex10-29 Brand 1    10  275.70   8.03     2.54
Ex10-29 Brand 2    10  265.30  10.04     3.18
Difference         10   10.40  15.01     4.75


95% CI for mean difference: (-0.33, 21.13)
T-Test of mean difference = 0 (vs not = 0): T-Value = 2.19  P-Value = 0.056
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a) What is Harry's 95% confidence interval for the difference of the mean?

Answer: (1.82, 18.98)

b) What is Hermione's 95% confidence interval for the difference of the mean?

Answer: (-0.33, 21.13)

c) Whose result is correct? Why?

Answer:

These two sample are two **independent** samples, each from a different brand and they are in no way paired. Therefore, Harry's 2-sample T Test result is correct.

d) Using the correct Minitab output, carry out a hypothesis test to determine whether the mean overall distance for brand 1 and brand 2 are different? Use $\alpha = 0.01$.

Solution:

$H_0 : \mu_1 - \mu_2 = 0$ vs $H_1 : \mu_1 - \mu_2 \neq 0$

Test Statistic:

$$t_0 = \frac{10.4}{\sqrt{\frac{8.03^2}{10} + \frac{10^2}{10}}} = 2.56$$

P-Value = $0.02 > \alpha = 0.01$ (DF = 17)

We fail to reject the null hypothesis, and there is no strong evidence to suggest that the mean overall distance for brand 1and brand 2 are different.