

(A) What is the total linear momentum  $\vec{p}_{total}$  of the system?

Sol'n:

System: The two small objects

Surroundings: Nothing significant. (In outer space)

Assumptions: Assuming  $|v| \ll c$

Definition:  $\vec{p} = m\vec{v}$  and  $\vec{p}_{total} = \sum_i \vec{p}_i \rightarrow \vec{p}_{total} = m\vec{v}_1 + m\vec{v}_2 = \langle mv_1 + mv_2, 0, 0 \rangle$

(B) What is the velocity of the center of mass,  $\vec{v}_{CM}$ ?

Sol'n:

System, surroundings and assumptions: See previous part.

Definition:  $\vec{v}_{CM} = \frac{\vec{p}_{CM}}{M_{total}}$

We know that  $\vec{p}_{CM} = \vec{p}_{total}$

Thus:  $\vec{v}_{CM} = \frac{m\vec{v}_1 + m\vec{v}_2}{2m} = \frac{\vec{v}_1 + \vec{v}_2}{2} = \langle \frac{v_1 + v_2}{2}, 0, 0 \rangle$

(C) What is the total angular momentum  $\vec{L}_{total,C}$  of the system relative to point C?

Sol'n:

System, surroundings and assumptions: See previous part.

Definition:  $\vec{L}_A = \vec{r}_A \times \vec{p}$  and  $\vec{L}_{total} = \sum_i \vec{L}_i$

Thus:  $\vec{L}_{total,C} = \langle -d, h + L, 0 \rangle \times \langle mv_1, 0, 0 \rangle + \langle -d, h, 0 \rangle \times \langle mv_2, 0, 0 \rangle$

So,  $\vec{L}_{total,C} = \langle 0, 0, -mv_1(h + L) - mv_2h \rangle$

(D) What is the translational angular momentum  $\vec{L}_{trans,C}$  of the system relative to point C?

Sol'n:

System, surroundings and assumptions: See previous part.

Definition:  $\vec{L}_{trans} = \vec{r}_{CM} \times \vec{p}_{total}$

Thus:  $\vec{L}_{trans,C} = \langle -d, h + L/2, 0 \rangle \times \langle mv_1 + mv_2, 0, 0 \rangle = \langle 0, 0, -(mv_1 + mv_2)(h + L/2) \rangle$

(E) What is the rotational angular momentum  $\vec{L}_{rot}$  of the system?

Sol'n:

System, surroundings and assumptions: See previous part.

Definition:  $\vec{L}_{Tot} = \vec{L}_{rot} + \vec{L}_{trans}$

Thus:  $\vec{L}_{rot} = \vec{L}_{tot} - \vec{L}_{trans} = \langle 0, 0, -mv_1(h + L) - mv_2h \rangle - \langle 0, 0, -(mv_1 + mv_2)(h + L/2) \rangle$

Which is:  $\vec{L}_{rot} = \langle 0, 0, -mv_1(L/2) - mv_2(L/2) \rangle = \langle 0, 0, -\frac{mv_1 L}{2} - \frac{mv_2 L}{2} \rangle$

**After a short amount of time  $\Delta t$ ,**

(F) What is the total (linear) momentum  $\vec{p}_{total}$  of the system?

Sol'n:

System: Same as above

Surroundings: Whatever is exerting the forces on the masses

Assumptions:  $|v| \ll c$

Momentum Principle:  $\Delta \vec{p} = \vec{F}_{net} \Delta t$

Thus:  $\vec{p}_{system,f} = \vec{p}_{system,i} + \vec{F}_{net} \Delta t$

Giving us:  $\vec{p}_{system,f} = \langle mv_1 + F_1 \Delta t + mv_2 - F_2 \Delta t, 0, 0 \rangle$

(G) What is the total angular momentum  $\vec{L}_{total,C}$  of the system?

Sol'n:

System, surroundings and assumptions: see previous part.

Angular Momentum Principle:  $\Delta \vec{L} = \vec{\tau}_{net} \Delta t$  where  $\vec{\tau} = \vec{r} \times \vec{F}$

For this system:

$\vec{\tau}_{net} = \langle -d, h + L, 0 \rangle \times \langle F_1, 0, 0 \rangle + \langle -d, h, 0 \rangle \times \langle -F_2, 0, 0 \rangle$

$\vec{\tau}_{net} = \langle 0, 0, -F_1(h + L) + F_2 h \rangle$

Since  $\vec{L}_{total,f} = \vec{L}_{total,i} + \vec{\tau}_{net} \Delta t$

$\vec{L}_{total,f} = \langle 0, 0, -mv_1(h + L) - mv_2 h \rangle + \langle 0, 0, -F_1(h + L)\Delta t + F_2 h \Delta t \rangle$

$\vec{L}_{total,f} = \langle 0, 0, -(mv_1 + F_1 \Delta t)(h + L) - (mv_2 - F_2 \Delta t)h \rangle$

or: compute new momenta & plug into definition of  $\vec{L}$