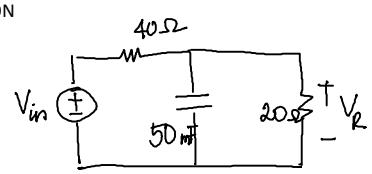
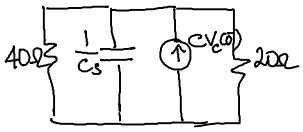
(13) a)



(i) Zero-input response

$$Z(s) = 40 \,//\, \frac{1}{cs} \,//\, 20$$

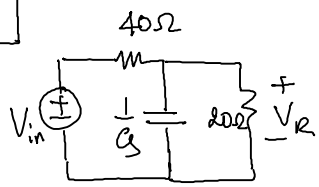
$$= \frac{1}{\frac{1}{40} + Cs + \frac{1}{20}} = \frac{40}{2s+3}$$



$$V_R(s) = Z(s) \cdot CV_c(0^-) = \frac{-24}{2s+3}$$

$$\Rightarrow \boxed{v_R^{(1)}(t) = -12e^{-\frac{3}{2}t}u(t)}$$

(ii) Zero-state response



$$Z_1(s) = \frac{1}{cs + \frac{1}{20}} = \frac{20}{s+1}$$

$$V_{in}(s) = \frac{3}{(s+2)^2}$$

$$V_R(s) = \frac{Z_1(s)}{Z_1(s)+40} \cdot V_{in}(s) = \frac{3}{2s^3 + 11s^2 + 20s + 12}$$

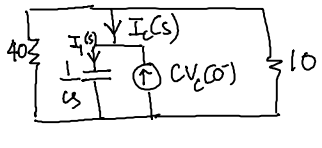
$$\Rightarrow \boxed{v_R^{(2)}(t) = \left(6e^{-\frac{3}{2}t} - 6e^{-2t} - 3te^{-2t}\right)u(t)}$$

(iii) complete response

$$v_R(t) = v_R^{(1)}(t) + v_R^{(2)}(t) = \left(-6e^{-\frac{3}{2}t} - 6e^{-2t} - 3te^{-2t}\right)u(t)$$

b) (i) Zero-input response

$$I_1(s) = CV_c(0^-) \cdot \frac{Cs}{Cs + \frac{1}{10} + \frac{1}{40}}$$



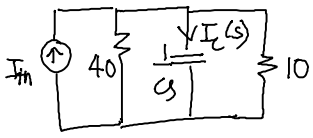
$$= \frac{9s}{20s + 100}$$

$$I_c(s) = I_1(s) - CV_c(0^-) = \frac{-9}{4s+20}$$

$$\Rightarrow \boxed{i_C^{(1)}(t) = \frac{-9}{4}e^{-5t}u(t)}$$

(ii) Zero-state response

$$I_c(s) = I_{in}(s) \cdot \frac{Cs}{Cs + \frac{1}{10} + \frac{1}{40}}$$



$$= \frac{6}{s^2+7s+10}$$

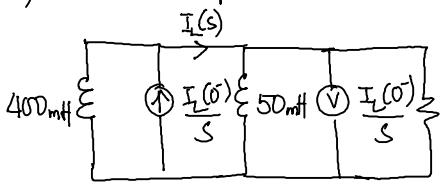
$$\Rightarrow \boxed{i_C^{(2)}(t) = \left[2e^{-2t} - 2e^{-5t}\right] u(t)}$$

(iii) Complete response

$$\boxed{i_C(t) = i_C^{(1)}(t) + i_C^{(2)}(t) = \left[2e^{-2t} - \frac{17}{4}e^{-5t}\right] u(t)}$$

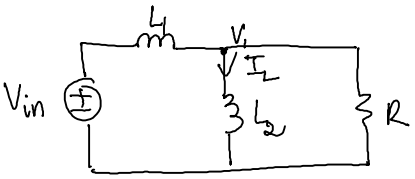
14

a) i. Zero-input



$$\boxed{I_L(s) = \frac{I_L(0^-)}{s} = \frac{-4}{s}} \qquad \boxed{i_L^{(1)}(t) = -4u(t)}$$

(ii) Zero-state



$L_1 = 400\,mH$
$L_2 = 50\,mH$
$R = 200\,m\Omega$

$$V_1(s) = V_{in}(s) \cdot \frac{L_2 s \,//\, R}{(L_2 s \,//\, R) + 4s} = \frac{2s}{2s^3 + 21s^2 + 70s + 72}$$

$$\boxed{I_L(s) = \frac{V_1(s)}{L_2 s} = \frac{40}{2s^3 + 21s^2 + 70s + 72}}$$

$$\boxed{i_L^{(2)}(t) = \left[4e^{-2t} - 20e^{-4t} + 16e^{-\frac{9t}{2}}\right] u(t)}$$

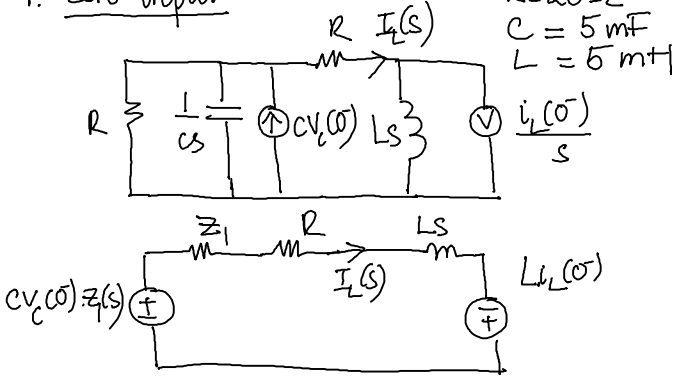
(iii) complete response

$$I_L(s) = \frac{-8s^3 - 84s^2 - 240s - 288}{2s^4 + 21s^3 + 70s^2 + 72s}$$

$$i_L(t) = i_L^{(1)}(t) + i_L^{(2)}(t)$$
$$= \left[-4 + 4e^{-2t} - 20e^{-4t} + 16e^{-\frac{9}{2}t}\right] u(t)$$

(b) 1. Zero-input

$R = 20\,\Omega$
$C = 5\,mF$
$L = 5\,mH$



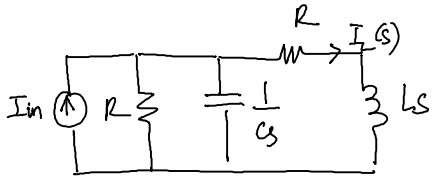
$$Z_1(s) = \frac{1}{Cs + \frac{1}{R}}$$

$$I_L(s) = \frac{\left[CV_c(0^-)Z_1(s) + Li_L(0^-)\right]}{Z_1(s) + R + Ls}$$

$$I_L(s) = \frac{2s + 220}{s^2 + 4010s + 80000}$$

$$i_L^{(1)}(t) = 2e^{-2005t}\left[\cosh(1985t) - .95\sinh(1985t)\right]$$

(ii) zero-state



$$Z_1(s) = \frac{1}{Cs + \frac{1}{R}}$$

$$I_L(s) = I_{in}(s) \cdot \frac{Z_1(s)}{Z_1(s) + R + Ls}$$

$$I_L(s) = \frac{280000s + 120000}{s^3 + 4014s^2 + 96040s + 320000}$$

$$\Rightarrow \quad i_L^{(2)}(t) = 15.6\, e^{-2005t}\left[\cosh(1985t) + 10\sinh(1985t)\right] - 15.6\, e^{-4t}$$

(iii) complete $\quad I_L(s) = \dfrac{2s^2 + 280288s + 120880}{s^3 + 4014s^2 + 96040s + 320000}$

$$i_L(t) = i_L^{(1)}(t) + i_L^{(2)}(t)$$
$$= e^{-2005t}\left[17.6\cosh(1985t) + 154\sinh(1985t)\right] - 15.6\, e^{-4t}$$

(15) KCL at V_{in} : $\quad V_{in}(s)\left[\frac{1}{R_1} + G_1 s\right] + V_{out}(s)\left[\frac{1}{R_2} + G_2 s\right] = 0$

$$\Rightarrow \quad H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{\frac{1}{R_1} + G_1 s}{\frac{1}{R_2} + C_2 s} = -\frac{G_1 + G_1 s}{G_2 + C_2 s}$$

If $C_2 = 0.2F \Rightarrow H(s) = -\dfrac{G_1 + G_1 s}{G_2 + \frac{s}{5}} = -\dfrac{5G_1 s + 5G_1}{s + 5G_2}$

$$= -\frac{2s + 2}{s + 5}$$

$$\Rightarrow \begin{cases} 5G_1 = 2 \\ 5G_1 = 2 \\ G_2 = 1 \end{cases} \Rightarrow \begin{array}{l} C_1 = .4F \\ R_1 = 2.5\,\Omega \\ R_2 = 1\,\Omega \end{array}$$

Step response :

$$V_{in}(t) = u(t) \Rightarrow V_{in}(s) = \frac{1}{s}$$
$$V_{out}(s) = H(s)\, V_{in}(s) = -\frac{1}{5}\,\frac{(2s+2)}{s+5}$$
$$\Rightarrow \quad V_{out}(t) = \left[-\frac{2}{5} - \frac{8}{5}\, e^{-5t}\right] u(t)$$

(15) KCL at −

$$\frac{V_{in}(s)}{R_1} + \left[V_{in}(s) - V_{out}(s)\right]\left(\frac{1}{R_2} + Cs\right) = 0$$

$$\Rightarrow V_{in}(s)\left[\frac{1}{R_1} + \frac{1}{R_2} + Cs\right] = V_{out}(s)\left[\frac{1}{R_2} + Cs\right]$$

$$\Rightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\left(\frac{1}{R_1} + \frac{1}{R_2} + Cs\right)}{\frac{1}{R_2} + Cs}$$

$$\boxed{H(s) = \frac{s+3}{s+1}}$$

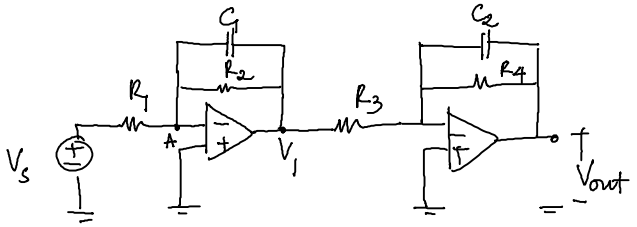
$$V_{in}(t) = 6te^{-3t}u(t) + 3e^{-t}u(t)$$

$$\Rightarrow V_{in}(s) = \frac{6}{(s+3)^2} + \frac{3}{s+1}$$

$$V_{out}(s) = H(s)\,V_{in}(s) = \frac{3s^2 + 24s + 33}{s^3 + 5s^2 + 7s + 3}$$

$$\boxed{v_{out}(t) = \left[6e^{-t} - 3e^{-3t} + 6te^{-t}\right]u(t)}$$

(16)



KCL at node A :

$$\frac{V_s(s)}{R_1} + V_1(s)\left[\frac{1}{R_2} + C_1 s\right] = 0$$

$$\Rightarrow H_1(s) = \frac{V_1(s)}{V_s(s)} = -\frac{1/R_1}{1/R_2 + C_1 s} = -\frac{10^{-3}}{1/R_2 + 10^{-3}s}$$

$$H_1(s) = -\frac{1}{s + \frac{10^3}{R_2}} = -\frac{1}{s+4}$$

$$\Rightarrow \boxed{R_2 = \frac{10^3}{4} = 250\ \Omega}$$

Similarly

$$H_2(s) = \frac{V_{out}(s)}{V_1(s)} = \frac{-1/R_3}{1/R_4 + C_2 s} = \frac{-10^{-3}}{10^{-3}s + \frac{1}{R_4}} = -\frac{1}{s + \frac{10^3}{R_4}}$$

$$= \frac{-1}{s+2}$$

$$\Rightarrow \boxed{R_4 = \frac{1000}{2} = 500\,\Omega}$$

$$\therefore H(s) = H_1(s) H_2(s) = \frac{V_{out}(s)}{V_s(s)} = \frac{-1}{s+4} \cdot \frac{-1}{s+2}$$

$$\Rightarrow V_s(s) = V_s(s) \cdot \frac{1}{(s+4)(s+2)}$$

- impulse response : $V_s(s) = 1 \Rightarrow V_{out}(s) = \frac{1}{(s+4)(s+2)}$

$$\Rightarrow \boxed{v_{out}(t) = \left(\frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t}\right) u(t)}$$

- step response : $V_s(s) = \frac{1}{s} \Rightarrow V_{out}(s) = \frac{1}{s(s+4)(s+2)}$

$$\Rightarrow \boxed{v_{out}(t) = \left(\frac{1}{8} - \frac{1}{4}e^{-2t} + \frac{1}{8}e^{-4t}\right) u(t)}$$

(b) KCL: $\cdot\ V_{in}(s)\left[\frac{1}{R_1} + C_1 s\right] = -V_1(s)\left[\frac{1}{R_2} + C_2 s\right]$

$$H_1(s) = \frac{V_1(s)}{V_{in}(s)} = -\frac{1/R_1 + C_1 s}{1/R_2 + C_2 s} = -\frac{20s - 25}{10s + 1}$$

$\cdot\ V_1(s)\left[\frac{1}{R_3 + \frac{1}{C_3 s}}\right] = -V_{out}(s)\left[\frac{1}{R_4} + C_4 s\right]$

$$H_2(s) = \frac{V_{out}(s)}{V_1(s)} = -\frac{\frac{1}{R_3 + \frac{1}{C_3 s}}}{1/R_4 + C_4 s} = \frac{-20s}{20s^2 + 13s + 2}$$

$$H(s) = H_1(s) H_2(s) = \frac{400s^2 + 500s}{200s^3 + 150s^2 + 33s + 2}$$

- impulse response : $V_{out}(s) = 1 \cdot H(s)$

$$v_{out}(t) = \left[\frac{200}{9}e^{-\frac{t}{4}} - \frac{136}{9}e^{-\frac{2t}{5}} - \frac{46}{9}e^{-\frac{t}{10}}\right]u(t)$$

- step response : $V_{out}(s) = \frac{1}{s}H(s)$

$$\Rightarrow v_{out}(t) = \left[-\frac{800}{9}e^{-\frac{t}{4}} + \frac{340}{9}e^{-\frac{2t}{5}} + \frac{460}{9}e^{-\frac{t}{10}}\right]u(t)$$

```
%% hw4.
clc, clear
% 13.
%% a
syms s t;
C = .050;
R1 = 20;
Rin = 40;
Z1 = 1/(C*s + 1 /R1);
f1 = 1/(s^2 +2*s -8) ;
vin = 3*t*exp(-2*t)* heaviside(t);
Vin = laplace(vin);
VR = Vin*(Z1/(Z1+Rin));
pretty(collect(Z1));
pretty((Vin));
pretty(collect(VR));
pretty(ilaplace(VR))

%ii.
Vc0 = -12;
Z = 1/(C*s + 1 /R1 + 1/Rin);
VR = Vc0*C*Z;
pretty(collect(Z));
pretty(collect(VR));
pretty(ilaplace(VR));

% b
syms s t
vc0 = 18;
C = .025;
Iin = 6/(s^2 + 2*s);
I1 = C*vc0*C*s/(C*s + 1/10 + 1/40);
Ic = I1 - C*vc0;
pretty(collect(Ic));
pretty(ilaplace(Ic))
Ic = Iin * C*s/(C*s + 1/10 + 1/40);
pretty(collect(Ic));
pretty(ilaplace(Ic))



disp('================================================================');
disp('14.  ');
%%14
%a.
L1 = .4;
L2 = .05;
R =.2;
Vin = 4/(s+4) - 2/(s+2);
```

```
Z1 = 1/(1/(L2*s) + 1/R);
V1 = Vin*Z1/(Z1 + L1*s);
IL = V1/(L2*s);
pretty(collect(V1));
pretty(collect(IL));
pretty(ilaplace(IL))
disp('14.b');
%b.
R = 20;
C = .005;
L = .005;
IL0 = 2; VC0 = 1;
Z1 = 1/(C*s + 1/R);
IL = (C*VC0*Z1 + L*IL0)/(Z1 + R + L*s);
pretty(collect(Z1));
pretty(collect(IL));
pretty((ilaplace(IL)))
Iin = (7*s + 3)/(s+4);
IL2 = Iin*Z1/(Z1 + R + L*s);
pretty(collect(Z1));
pretty(collect(IL2));
pretty((ilaplace(IL2)))
IL = IL + IL2;
pretty(collect(IL));
pretty((ilaplace(IL)))



disp('================================================================');
disp('15.  ');
%% 15
% a
pretty((ilaplace(-1/s/(s+5)*(2*s+2))))
% b
C = .1;
R1 = 5;
R2 = 10;
H = (1/R1 + 1/R2 + C*s)/(1/R2 + C*s);
vin = 6*t*exp(-3*t) + 3*exp(-t);
pretty(collect(H));
V = laplace(vin);
pretty((V));
Vout = H*V;
pretty(collect(Vout));
pretty((ilaplace(Vout)))



disp('================================================================');
disp('16 a');
%%16
%a.
Vout = 1/(s+4)/(s+2);
pretty((ilaplace(Vout)))
pretty((ilaplace(Vout/s)))
disp('16 b');
%b
R1 = 40; R2 = 1000; R3 = 500; R4 = 2000;
C1 = .02; C2 = .01; C3 = .005; C4 = .002;
H1 = -(1/R1 + C1*s)/(1/R2 + C2*s);
H2 = -1/(R3 + 1/C3/s)/(1/R4 + C4*s);
pretty(collect(H1));
pretty(collect(H2));
H = H1*H2;
pretty(collect(H));
pretty((ilaplace(H)))
pretty((ilaplace(H/s)))
```