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Summer 2012

ECE 20200 : Linear Circuit Analysis II
School of ECE, Purdue University

LECTURE 10

- Switching in linear circuits

Reference: Decarlo/ Lin PP 640-645

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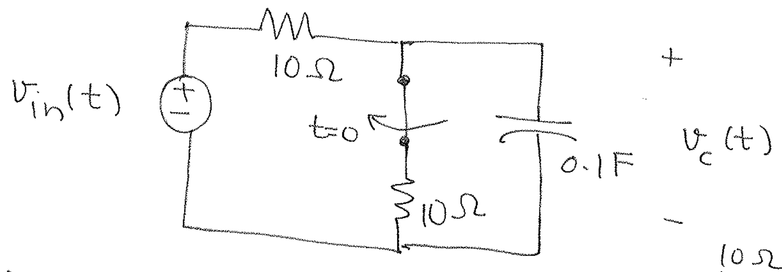
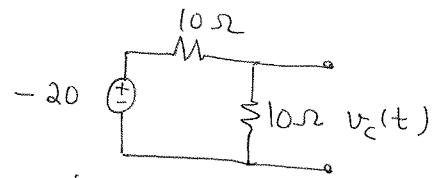
Switching in RLC circuits

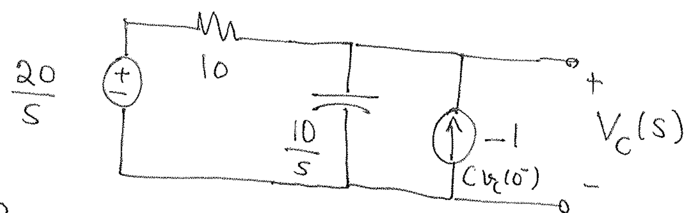
Example 1. Find $v_c(t)$ for $t \geq 0$ when $v_{in}(t) = -20\,u(-t) + 20u(t)$ V



1) $v_c(0^-) = -10V$



2) Draw equivalent s-domain circuit for $t \geq 0$



3) By superposition,

$$V_{c_V} = \frac{10/s}{10/s + 10} \cdot \frac{20}{s} = \frac{200/s}{10 + 10s} = \frac{20}{s(s+1)}$$

$$V_{c_I} = \frac{10 \cdot \frac{10}{s}}{\frac{10}{s} + 10} \cdot (-1) = \frac{-100/s}{\frac{10}{s} + 10} = \frac{-10}{s+1}$$

$$\therefore V_c(s) = V_{c_V} + V_{c_I} = \frac{20}{s(s+1)} - \frac{10}{s+1} = \frac{20}{s} - \frac{20}{s+1} - \frac{10}{s+1}$$

$$V_c(s) = \frac{20}{s} - \frac{30}{s+1}$$

$$\therefore v_c(t) = 20u(t) - 30e^{-t}u(t) \text{ V}$$

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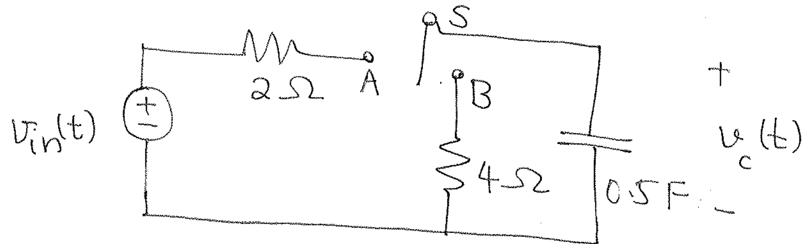
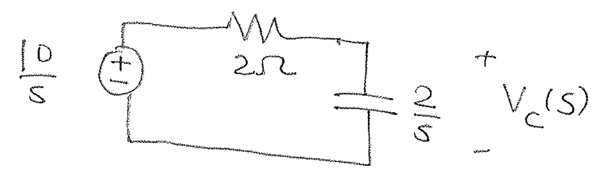
Example 2. $v_{in}(t) = 10u(t) + 10u(t-3)$ V



Switch 'S' is initially in position A. Switch moves to position B at $t = 2s$ and back to A at $t = 4s$. Find $v_c(t)$ for all t.

$t < 0 \Rightarrow v_c(t) = 0 \qquad v_c(0^-) = 0$

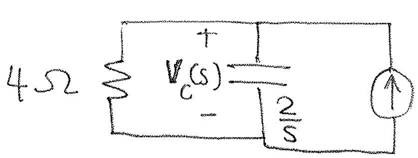
$0 \le t < 2s \Rightarrow$ Draw equivalent s-domain circuit. Find $V_c(s)$ and $v_c(t)$



$$V_c(s) = \frac{\frac{2}{s}}{2 + \frac{2}{s}} \cdot \frac{10}{s} = \frac{20/s}{(2s+2)} = \frac{10}{s(s+1)} = \frac{10}{s} - \frac{10}{s+1}$$

$$v_c(t) = 10u(t) - 10e^{-t}u(t) \qquad 0 \le t < 2$$

$2 \le t < 4s \Rightarrow$ Draw equivalent s-domain circuit. Find $V_c(s)$ and $v_c(t)$. $\quad v_c(2^-) = 10 - 10e^{-2} = 8.65$ V



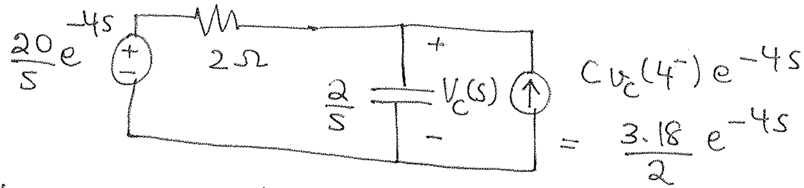
$$Cv_c(2^-)e^{-2s} = \frac{(8.65)e^{-2s}}{2}$$

$$V_c(s) = \frac{\frac{2}{s} \cdot 4}{\frac{2}{s} + 4} \cdot \left(\frac{8.65}{2}\right)e^{-2s} = \frac{\cancel{2}}{s+0.5}\left(\frac{8.65}{\cancel{2}}\right)e^{-2s}$$

$$= \frac{8.65}{s+0.5}e^{-2s}$$

$$v_c(t) = 8.65 e^{-0.5(t-2)} u(t-2) \quad , \quad 2 \leq t < 4$$

$t \geq 4 \Rightarrow$ Draw equivalent s-domain circuit. Find $V_c(s)$ and $v_c(t)$. $\quad v_c(4^-) = 3.18\,V$



Use superposition,

$$V_c(s) = \frac{20}{s} e^{-4s} \cdot \frac{2/s}{2/s + 2} + \frac{2(\frac{2}{s})}{2 + 2/s} \cdot \frac{3.18}{2} e^{-4s}$$

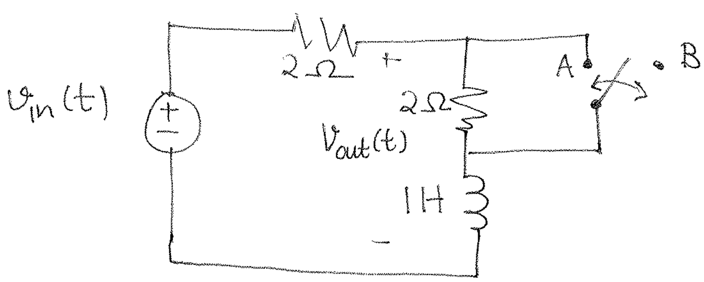
$$= \frac{20\, e^{-4s}}{s(s+1)} + \frac{\cancel{2}}{s+1} \cdot \frac{3.18}{\cancel{2}} e^{-4s}$$

$$v_c(t) = 20\left(1 - e^{-(t-4)}\right) u(t-4) + 3.18\, e^{-(t-4)} u(t-4)$$

$$t \geq 4$$

Remark: The voltage across capacitor is continuous, so each segment matches at the switching instant.

Example 3. RL circuit. Let $v_{in}(t) = 4u(-t) + 8u(t) + 8u(t-2)\,V$. Initially switch is in position B; moves to A at $t = 0\,s$ and back to B at $t = 4\,s$. Find $v_{out}(t)$ for $t \geq 0$



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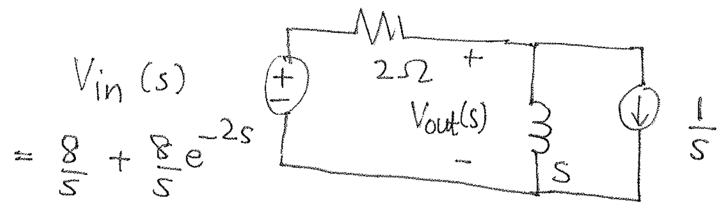
$t<0$ Inductor short

$$\therefore\ i_L(0^-) = \frac{4}{2+2} = 1\ A$$

$0 \leq t < 4s$



By superposition,

$$V_{out}(s) = \frac{s}{s+2}V_{in}(s) - \frac{2\cancel{s}}{2+s}\frac{1}{\cancel{s}}$$

$$= \frac{s}{s+2}\left(\frac{8}{s} + \frac{8}{s}e^{-2s}\right) - \frac{2}{s+2}$$

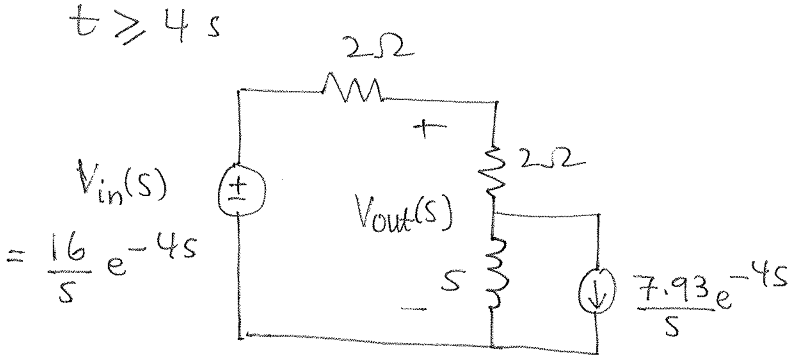
$$= \frac{8}{s+2} + \frac{8}{s+2}e^{-2s} - \frac{2}{s+2}$$

$$= \frac{6}{s+2} + \frac{8}{s+2}e^{-2s}$$

$$v_{out}(t) = 6e^{-2t}u(t) + 8e^{-2(t-2)}u(t-2)$$

$0 \leq t < 4s$

$t \geq 4s$



$i_L(4^-) = ?$

$$i_L(4^-) = \frac{v_{in}(4^-) - v_{out}(4^-)}{2}$$

$$= \frac{15.85}{2}$$

$$= 7.93\ A$$

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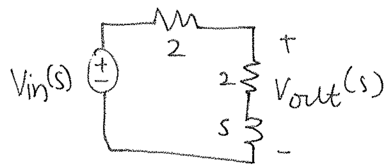
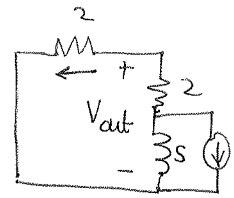
By superposition,

$$V_{out}(s) = \frac{s+2}{s+2+2} V_{in}(s) + \left(\frac{s}{s+4}\right)\left(-\frac{7.93}{s} e^{-4s}\right)$$



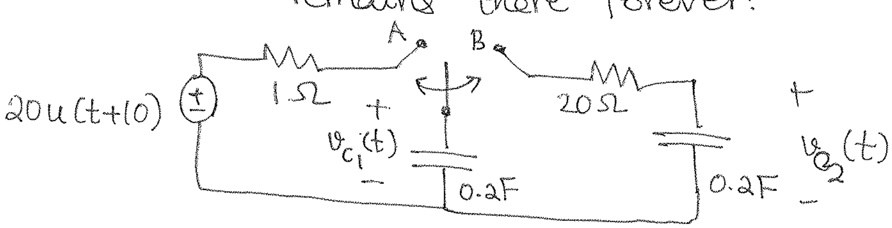


$$= \frac{s+2}{s+4} \frac{16}{s} e^{-4s} + \frac{-\cancel{7}.93}{s+4} e^{-4s}$$

$$= \left(\frac{8}{s} + \frac{8}{s+4}\right) e^{-4s} - \frac{7.93}{s+4} e^{-4s}$$

$$v_{out}(t) = 8\left(1 + e^{-4(t-4)}\right) u(t-4) - 7.93 e^{-(t-4)} u(t-4) \text{ V.}$$

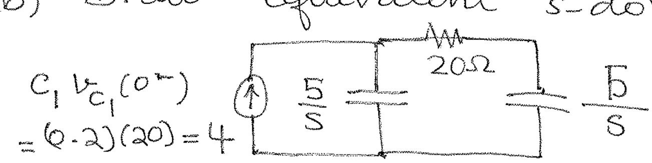
Example 4. The switch is in position A for a long time. At $t=0$, the switch moves to position B and at $t=2s$, the switch moves back to position A and remains there forever.



(a) $v_{c_1}(0) = ?$ $v_{c_2}(0) = ?$

Since $R_1 C_1 = 0.2$, the charging time constant for C_1, is much less than 10 s, the capacitor C_1 is fully charged to 20V. $\therefore$ $v_{c_1}(0) = 20\text{ V}$, $v_{c_2}(0) = 0\text{ V}$.

(b) Draw equivalent s-domain circuit for $0 \le t < 2s$



(c) Compute $v_{c_1}(t)$ for $0 \leqslant t \leqslant 2\,s$.

$$V_{C_1}(s) = 4 \cdot \frac{1}{\frac{s}{5} + \frac{1}{20 + \frac{5}{s}}} = \frac{4}{\frac{s}{5} + \frac{s}{20s+5}}$$

$$= \frac{4}{\frac{s}{5} + \frac{s}{20(s+0.25)}}$$

$$= \frac{4}{\frac{4s(s+0.25)+s}{20(s+0.25)}}$$

$$= \frac{80(s+0.25)}{s(4s+1+1)}$$

$$= \frac{\cancel{80}^{\,20}(s+0.25)}{\cancel{4}s(s+0.5)}$$

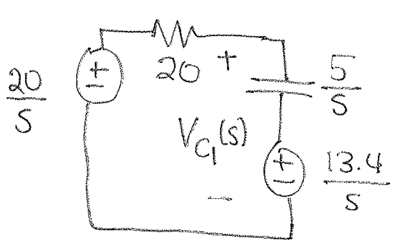
$$= \frac{20(s+0.25)}{s(s+0.5)}$$

$$= \frac{10}{s} + \frac{10}{s+0.5}$$

$\therefore v_{c_1}(t) = 10(1+e^{-0.5t})u(t)$ $\quad 0 \leqslant t < 2s$

(d) $v_{c_1}(2) \approx 10(1+e^{-0.5(2)}) = 13.4\,V$

(e) Compute $v_1(t)$ for $t \geqslant 2 \rightarrow t' = (t-2)$



Superposition:

$$V_{C_1}(s) = \frac{\frac{5}{s}}{20+\frac{5}{s}} \cdot \frac{20}{s} + \frac{20}{20+\frac{5}{s}} \cdot \frac{13.4}{s}$$

$$= \frac{20}{s} - \frac{20}{s+5} + \frac{13.4}{s+5}$$

$\therefore v_{c_1}(t') = 20 - 6.6e^{-5t'}u(t')\ V, \quad t' \geqslant 0$

(or) $v_c(t) = 20 - 6.6e^{-5(t-2)}u(t-2)\ V \qquad t \geqslant 2$