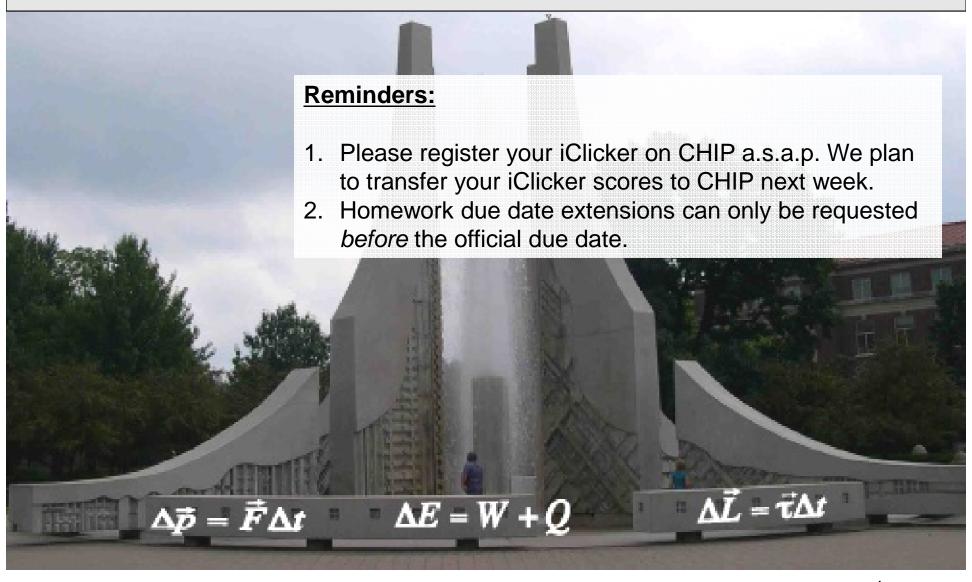
PHYS 172: Modern Mechanics

Spring 2012



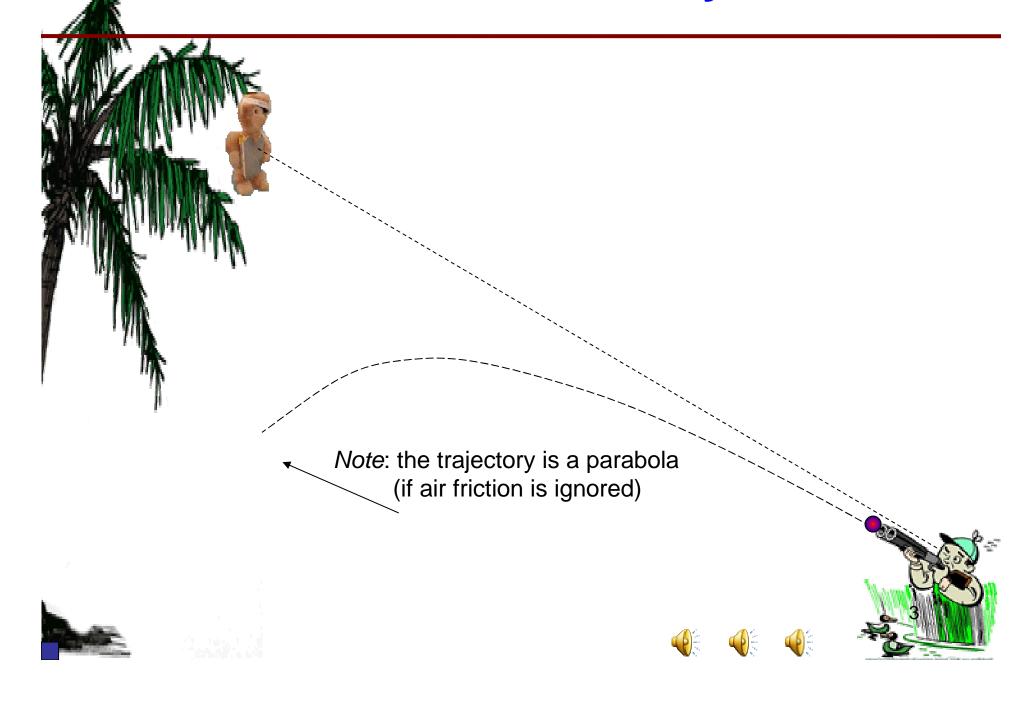
Lecture 4 – Physical Models, Fundamental Interactions

Read 2.7–2.8, 3.1-3.4

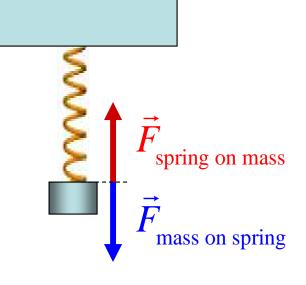
Today

- Poor Monkey
- Reciprocity (Equal and Opposite Forces)
- Example: Colliding Students
- Four Fundamental Forces
- Gravity... Lots of Gravity

Shoot the monkey



Reciprocity: Newton's 3d law



Force magnitudes are the same Directions are opposite They act on *different* objects

$$\Delta \vec{p} = \vec{F}_{net} \Delta t$$

$$\vec{F}_{
m spring \ on \ mass} = -\vec{F}_{
m mass \ on \ spring}$$

Reciprocity (Newton's 3rd law):

The forces of two objects on each other are always equal and are directed in opposite directions

NOTE: Velocity-dependent forces (e.g., magnetic forces) do not obey Newton's 3rd law!

Physical models



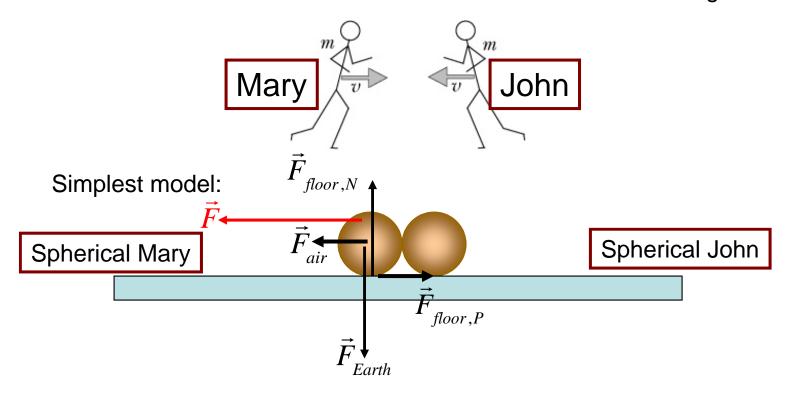
"Spherical cow"

Ideal model: ignore factors that have no significant effect on the outcome

Example: colliding students

Mary and John are late for class and run into each other head-on.

Q: Estimate the force that one student exerts on the other during collision



System:

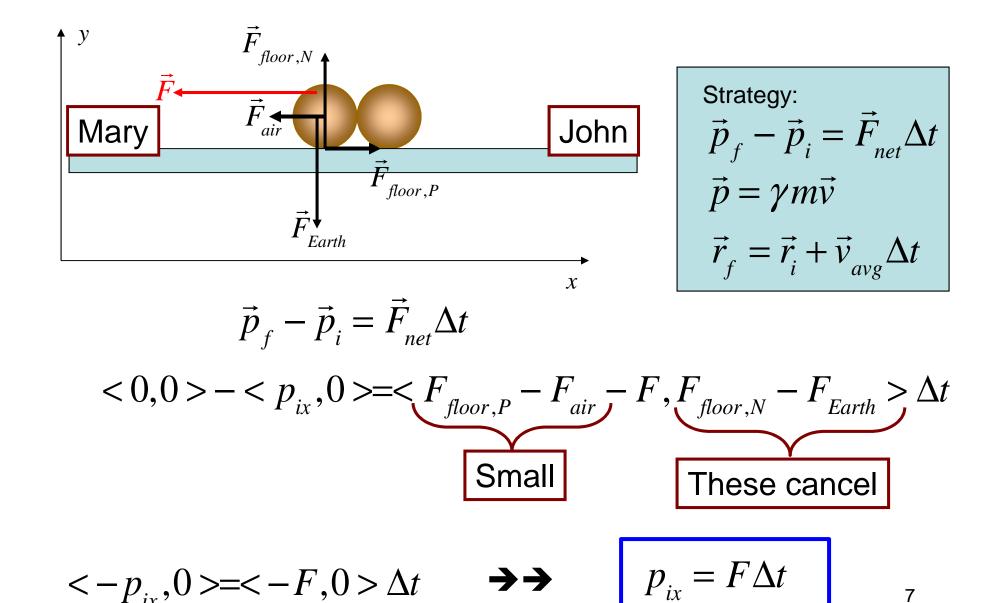
Spherical Mary

Surroundings: Earth, floor, air, Spherical John

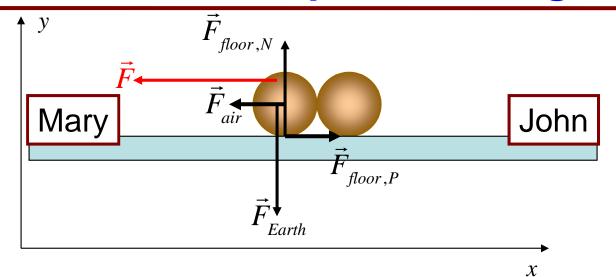
Forces acting on Mary: Earth, floor, air, John



Example: colliding students



Example: colliding students



Strategy:

$$\vec{p}_{f} - \vec{p}_{i} = \vec{F}_{net} \Delta t$$

$$\vec{p} = \gamma m \vec{v}$$

$$\vec{r}_{f} = \vec{r}_{i} + \vec{v}_{avg} \Delta t$$

$$p_{ix} = F \Delta t$$

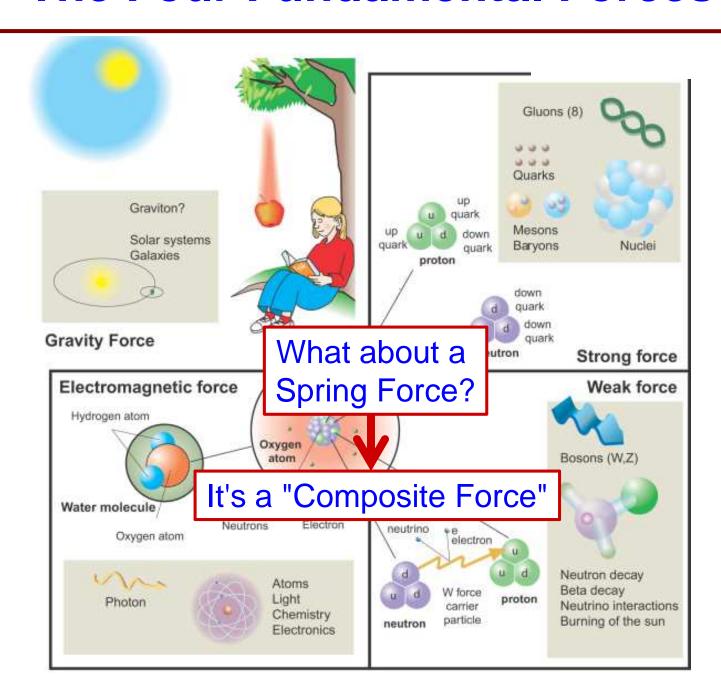
What is collision time? \longrightarrow Assume: $v_i = 5$ m/s, $\Delta x = 0.05$ m

$$v_{avg} = \frac{\Delta x}{\Delta t} \rightarrow \Delta t = \frac{\Delta x}{v_{avg}} \rightarrow \Delta t = \frac{\Delta x}{\left(v_i + v_f\right)/2}$$
 $\Delta t = 0.02 \text{ s}$

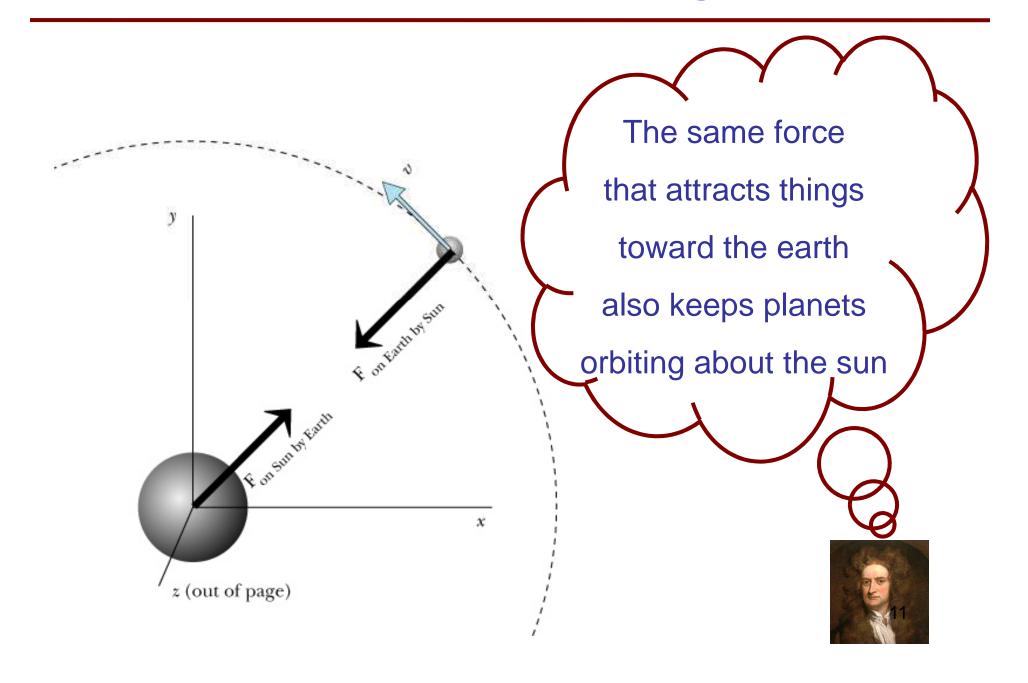
What is initial momentum? \rightarrow Assume: m=60 kg \rightarrow $p_{ix} = mv_{ix} = 300 \text{ kg} \cdot \text{m/s}$

Find *F*:
$$F = \frac{p_{ix}}{\Delta t} = \frac{300 \text{ kg} \cdot \text{m/s}}{0.02 \text{ s}} = 15000 \text{ N}$$
 John hits Mary with 15,000 N!

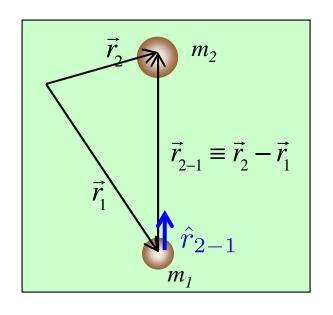
The Four Fundamental Forces



Newton's Great Insight:



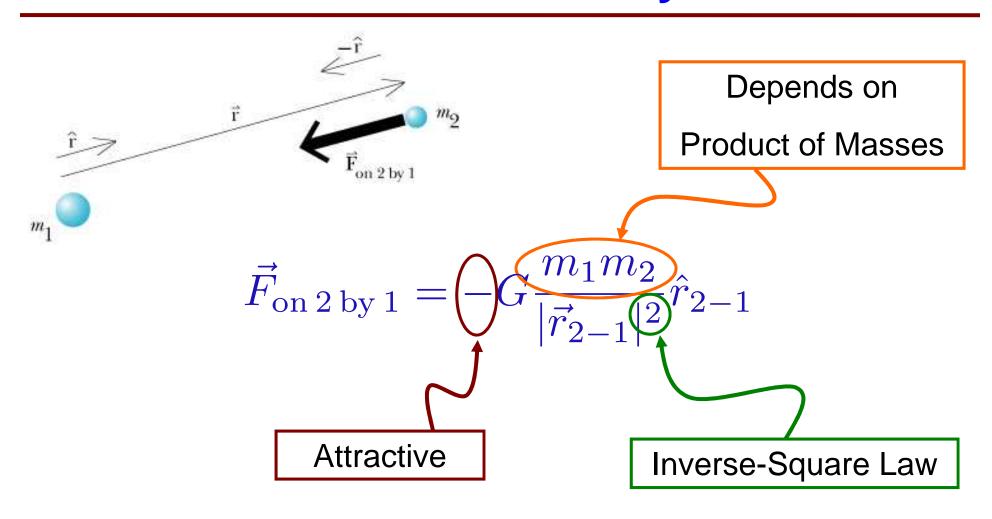
Force of Gravity



$$\vec{F}_{\text{on 2 by 1}} = -G \frac{m_1 m_2}{|\vec{r}_{2-1}|^2} \hat{r}_{2-1}$$

$$G = 6.7 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$$

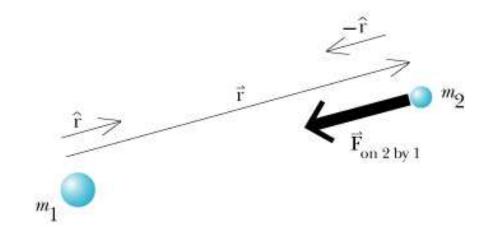
Force of Gravity



Features of gravitational force

$$\vec{F}_{grav \text{ on 2by1}} = G \frac{m_2 m_1}{|\vec{r}_{2-1}|^2} \hat{r}_{2-1}$$

gravity is always attractive



$$\vec{F}_{grav \text{ on 2 by 1}} = -G \frac{m_2 m_1}{|\vec{r}_{2-1}|^2} \hat{r}_{2-1}$$

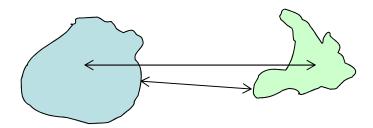
gravity is an inverse square law

$$\vec{F}_{grav \text{ on 2by1}} = -G \frac{m_2 m_1}{|\vec{r}_{2-1}|^2} \hat{r}_{2-1} \qquad \vec{F}_{grav \text{ on 2by1}} = -G \frac{m_2 m_1}{|\vec{r}_{2-1}|^2} \hat{r}_{2-1}$$

the force depends upon the product of the masses

Distance between two objects

Real objects have size & shape



Point object: idealized object which has no size, all mass is in one point

If distance between the two objects is >> than their size, can model the objects as point-masses

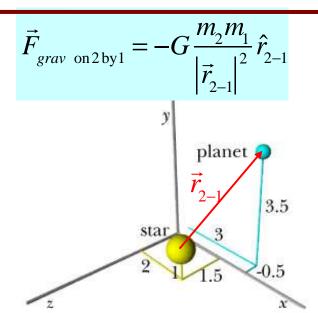
Special case: spherical objects (spherical symmetry)



Uniform-density spheres interact gravitationally in exactly the same way as if all their mass were concentrated at the center of the sphere.

Spheres act like a point mass!

Gravitational force on a planet



$$G = 6.7 \times 10^{-11} \text{ N} \cdot \text{m/kg}^2$$

STAR PLANET
$$m_1 = 4 \times 10^{30} \text{ kg}$$
 $m_2 = 3 \times 10^{24} \text{ kg}$ planet $\vec{r}_1 = \left\langle 2, 1, 1.5 \right\rangle \times 10^{11} \text{ m}$ $\vec{r}_2 = \left\langle 3, 3.5, -0.5 \right\rangle \times 10^{11} \text{ m}$

1. Calculate
$$\vec{r}_{2-1} \equiv \vec{r}_2 - \vec{r}_1$$

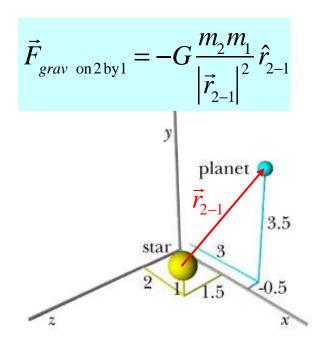
$$\vec{r}_{2-1} = \langle 1, 2.5, -2 \rangle \times 10^{11} \text{ m}$$

2. Distance
$$|\vec{r}_{2-1}| = \sqrt{(1 \times 10^{11} \text{ m})^2 + (2.5 \times 10^{11} \text{ m})^2 + (-2 \times 10^{11} \text{ m})^2} = 3.35 \times 10^{11} \text{ m}$$
3. Unit vector: $\hat{r}_{2-1} = \frac{\vec{r}_{2-1}}{|\vec{r}_{2-1}|} = \frac{\langle 1, 2.5, -2 \rangle \times 10^{11} \text{ m}}{3.35 \times 10^{11} \text{ m}} = \langle 0.299, 0.746, -0.597 \rangle$
3. Force: $\vec{F}_{grav \text{ on 2 by 1}} = -G \frac{m_2 m_1}{|\vec{r}_{2-1}|^2} \hat{r}_{2-1} = -7.16 \times 10^{21} \langle 0.299, 0.746, -0.597 \rangle \text{ N}$

$$\vec{F}_{grav \text{ on planet by star}} = 7.16 \times 10^{21} \langle -0.299, -0.746, 0.597 \rangle \text{ N}$$

$$magnitude \qquad direction$$

Gravitational force on a planet



star planet

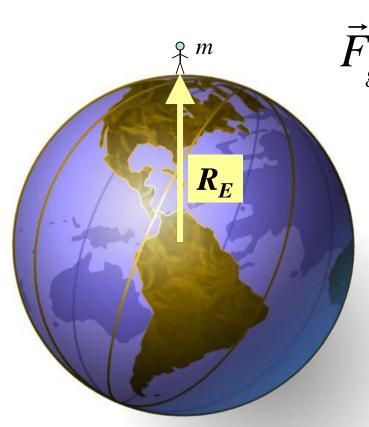
$$m_1 = 4 \times 10^{30} \text{ kg}$$
 $m_2 = 3 \times 10^{24} \text{ kg}$
 $\vec{r}_1 = \left\langle 2, 1, 1.5 \right\rangle \times 10^{11} \text{ m}$ $\vec{r}_2 = \left\langle 3, 3.5, -0.5 \right\rangle \times 10^{11} \text{ m}$

$$\vec{F}_{grav \text{ on planet by star}} = 7.16 \times 10^{21} \langle -0.299, -0.746, 0.597 \rangle \text{ N}$$

Checking results:

- 1. Diagram
- 2. Order of magnitude
- 3. Units
- 4. Unit vector

Gravitational force near the Earth's surface



 $ec{F}_{grav \text{ on } m \text{ by } M_E}$

~ The same for all objects on surface

$$\vec{F}_{grav \text{ on } mby M_E} = \vec{g}m$$

Gravitational field
$$\vec{g} = -G \frac{M_E}{\left|R_E\right|^2} \hat{r}$$

 $M_E = 5.976 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$

The magnitude:
$$g = 9.8 \text{ N/kg} = 9.8 \text{ m/s}^2$$

$$F_{g} = mg$$

What We Did Today

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