# Lec1-10min

Sunday, January 08, 2012
11:11 AM

- HW set 1 is assigned. Due next session.

## Circuits-5min

Wednesday, December 28, 2011
5:25 PM

What is a circuit?

- A circuit is an energy, signal, information processor

Examples of "processors"

- Stereo amplifier
    - amplifies a small electrical signal to a large signal that drives the speaker to reproduce music
- Radio/TV
    - converts electrical magnetic waves to sound and or picture
- Electrical grid
    - bring electrical power from one place to another
    - convert high-voltage power to low-voltage power.

Many other examples!

- computers
- toasters

- car ignition
- electrical motors
- computer chips
- microwave ovens

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- Depending on the processing that needs to be completed, circuits can be very complicated.
- In this class, we will study simpler <u>Linear</u> circuits
    - composed of resistors (R), inductors (L), capacitors (C).
    - voltage/current sources
    - linear op-amps
- Nonetheless, they are the basic components of other more complicated circuits.

# Charge current voltage-5min

Tuesday, January 03, 2012
12:19 PM

To begin with, we need to understand three basic quantities in any electrical circuits

- Charge, current and voltage

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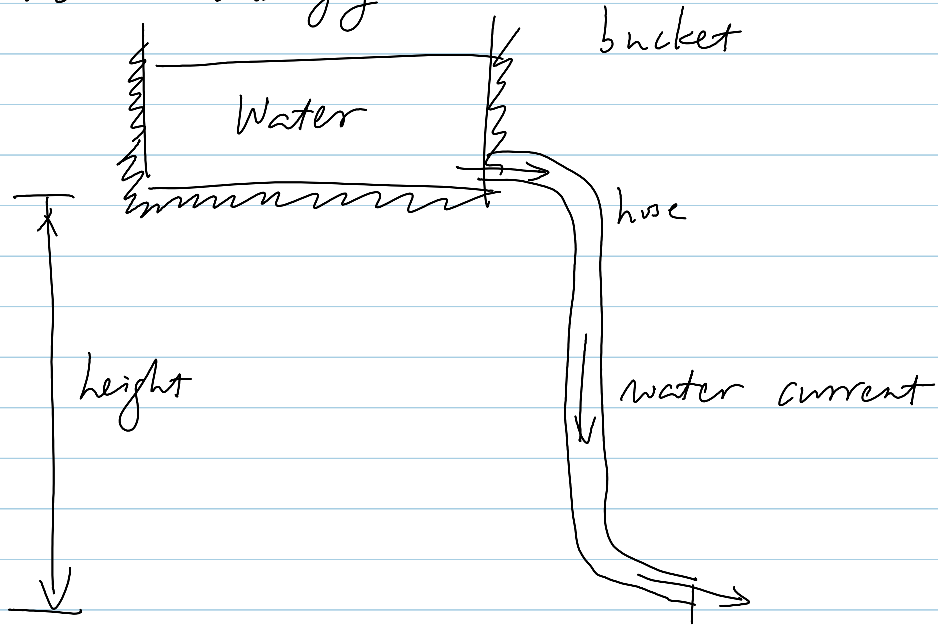
Use an analogy




Why does the water flow?

- gravity
- potential energy → kinetic energy

- A bucket holds water. There is a hole at the bottom, connected to a hose.
- If the bucket is held at a high place, water will flow through the hose.
- A water current is formed as water moves from one place to another.
- What is driving the water current?
  - The altitude, or the potential

energy of the water in the bucket due to gravitational fields.

- The higher the bucket is and the wider the hose, the larger the water current.

- As water flows down, potential energy is converted into kinetic energy, which can be used to drive a hydro-eletric generator

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Compared to electrical circuit

| Water bucket / hose | Electrical Circuit |
| --- | --- |
| Water Mass | Electrical charge |
| Water current | Electrical current |
| Height | Voltage |
| Potential / kinetic energy | Electrical energy |

Let us now go over these in closer details.

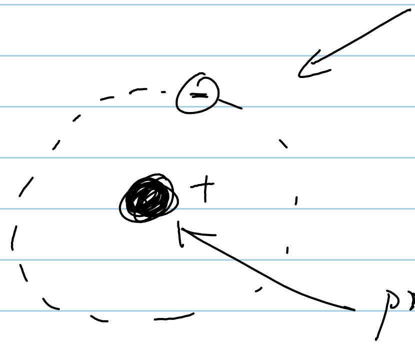
# Charge-10min

Tuesday, January 03, 2012
12:37 PM

- Charge is an electrical property of matter.
- Basic unit of charge: Coulomb (C)

  /ku:lʌm/

- Unlike water mass, however, there are two types of charges +, −
- Matter consists of atoms



electrons: negatively charged

protons: positively charged

neutrons: no charge

- In "normal" state, there are an equal # of electrons & protons. The matter is not charged.
- When there are more protons than eletrons, the matter is positively charged
- When there are less protons than electrons, the matter is negatively charged

– (–1) C = accumulated charge on $6.241 \times 10^{18}$ electrons

1 electron has charge

$$\frac{-1}{6.241 \times 10^{18}} = -1.6019 \times 10^{-19}\,C$$

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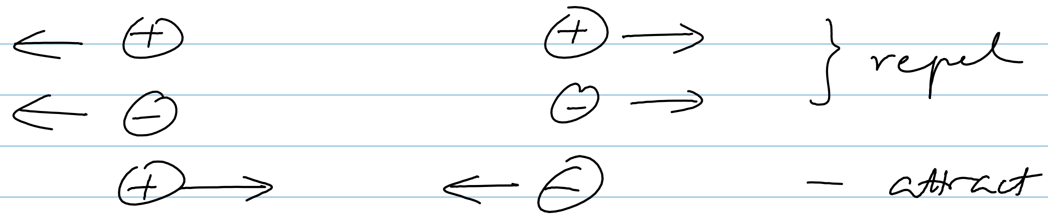
– Recall that due to gravity, masses are attracted to the earth. Similarly, there are forces between charged particles

– Particles with opposite charges attract each other.

Particles with similar charges repel.




– Coulomb's Law:

– The force of attraction or repulsion between two charged bodies is inversely proportional to the square of the distance between them.

– Similar to gravitational force, Electrically charged particles contain energy!

- Two equally charged particles with 1 C each at 1 meter apart repel each other with a force of

$$F = 10^{-7} c^2 \text{ Newton}$$

$c$ speed of light

- Notation:

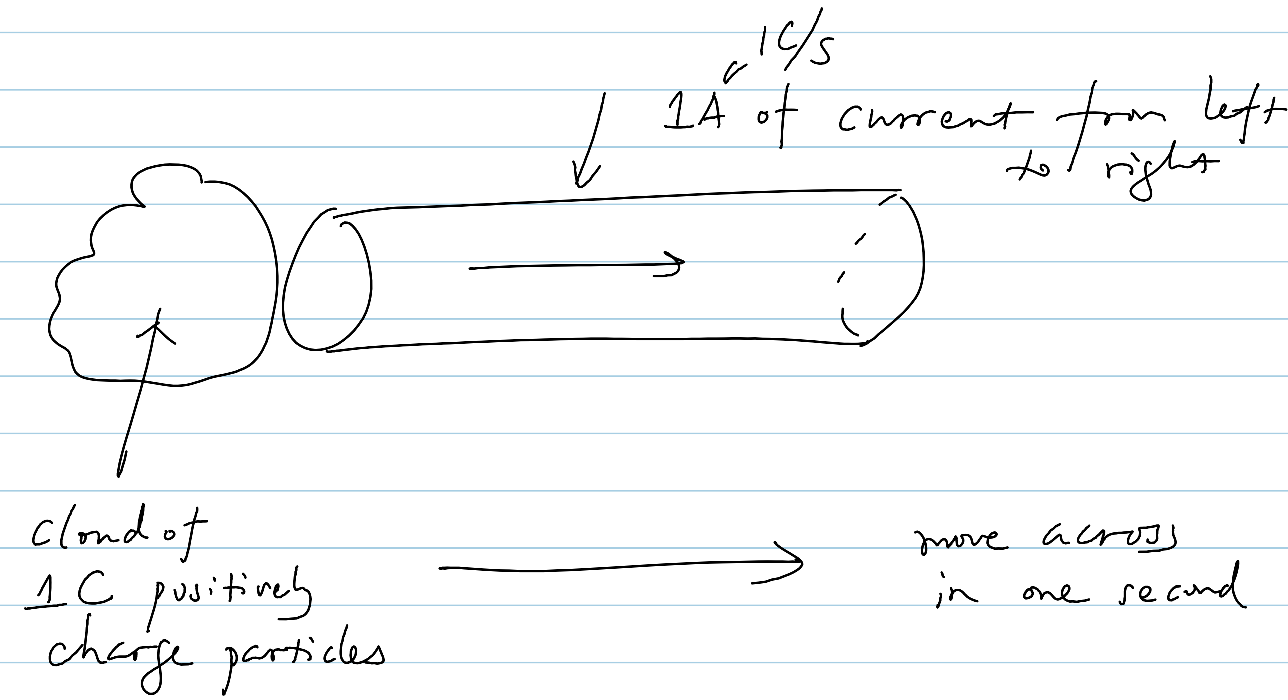
  $Q$ : fixed quantity of charge

  $q$ or $q(t)$ : time dependent charge

# Current

Tuesday, January 03, 2012
12:55 PM

Current is the net flow of charge through a cross-sectional area of a conductor.




- Basic unit of current: Ampere (A)

$$1A = 1C/s$$

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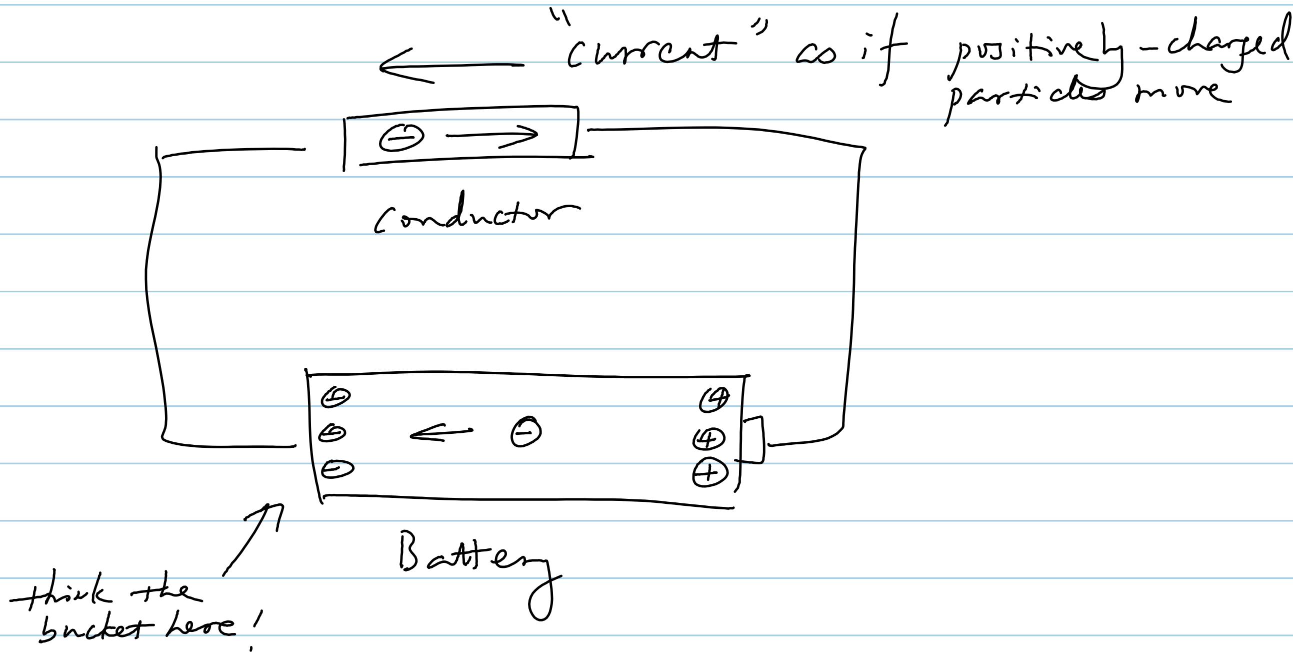
## What moves?

- Early in the history of electricity, it was erroneously believed that positively charged particles move. (As in the figure above)

- In reality, for metal conductors, only negatively charged electrons move.




- Chemistry inside the battery causes electrons to accumulate in the negative plate

- When a conductor is connected to the battery, the electron move from the negative plate to the positive plate

    - opposite charges attract

- This is as if positively charged particles move in the opposite direction from the positive plate to the negative plate.

    - In the earlier example, $-1C$ of electrons move from right to left in 1 second.

- For analysis of linear circuits, there will be no practical difference. Hence, we will mostly take the convention that the positive direction of current represents the movement of positively charged particles

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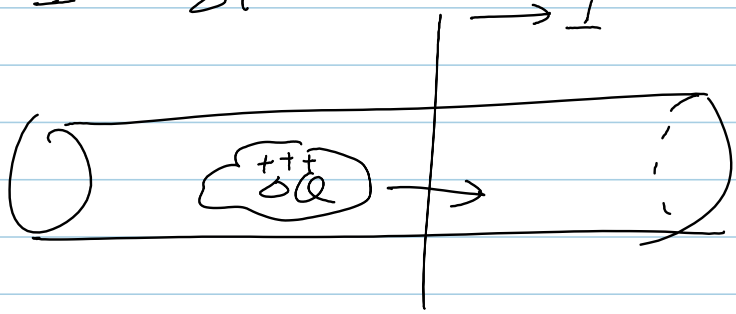
Formula for current flow

- $I = \frac{\Delta Q}{\Delta t}$




- Limiting case

$$i(t) = \frac{dq(t)}{dt}$$

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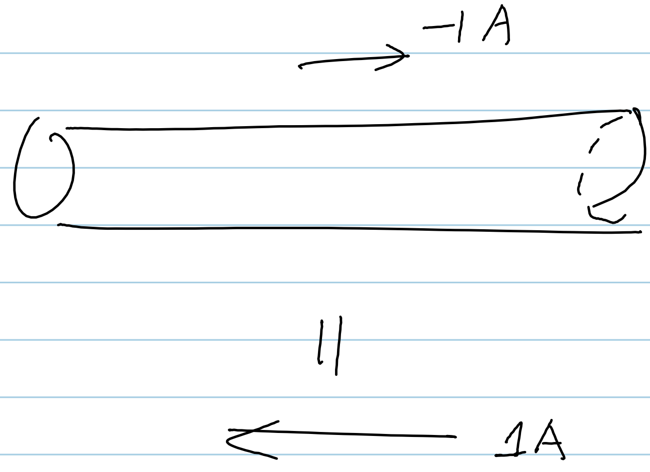
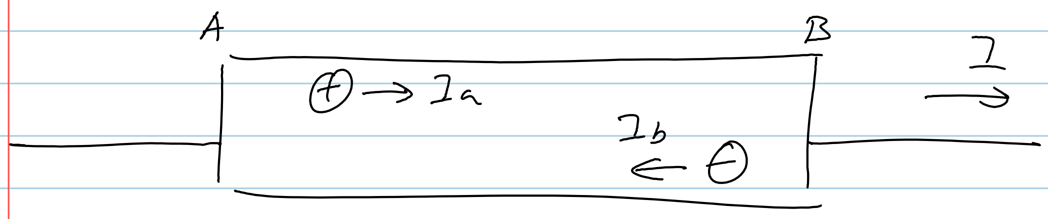
What is



# Current: examples-15min

Tuesday, January 03, 2012
3:52 PM

Ex 1.1. (p5)




- positive charged particles move from left to right at the rate of 0.2 C/s
- negatively charged particles move from right to left at the rate of 0.48 C/s
- Find $I_a$ & $I_b$ due to positive/negative particles, respectively (Note the reference directions!)
- Find the total current

Solution:

$I_a = 0.2A$

$I_b = -0.48A$ (Note the positive direction is from right to left)

Total current (⟶)

$I = I_a - I_b = 0.68A$

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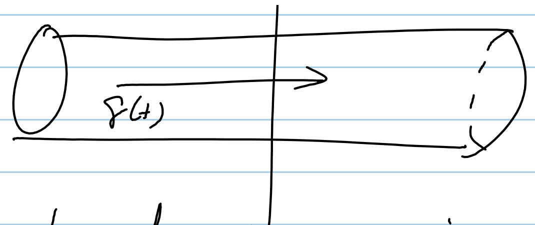
Ex 2

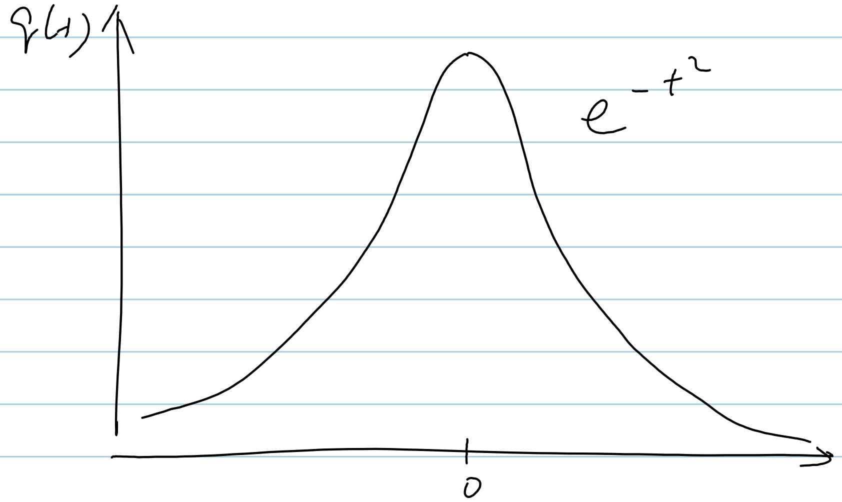



$q(t)$: the charge crossing the boundary from left to right

— between $-\infty$ to $t$

Suppose $q(t) = e^{-t^2}$

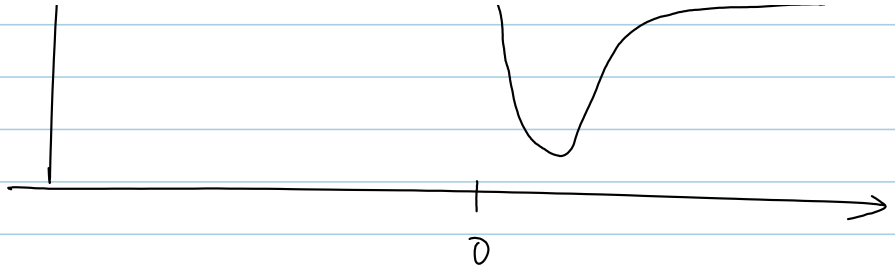
May skip to the next one



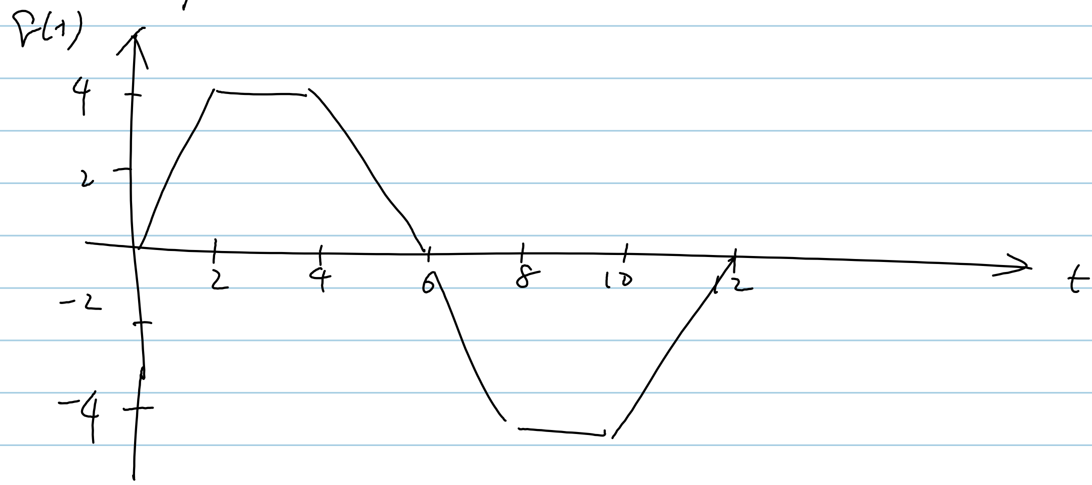

Find $i(t)$.

- Again, according to our convention, the arrow represents direction of positive charge flow

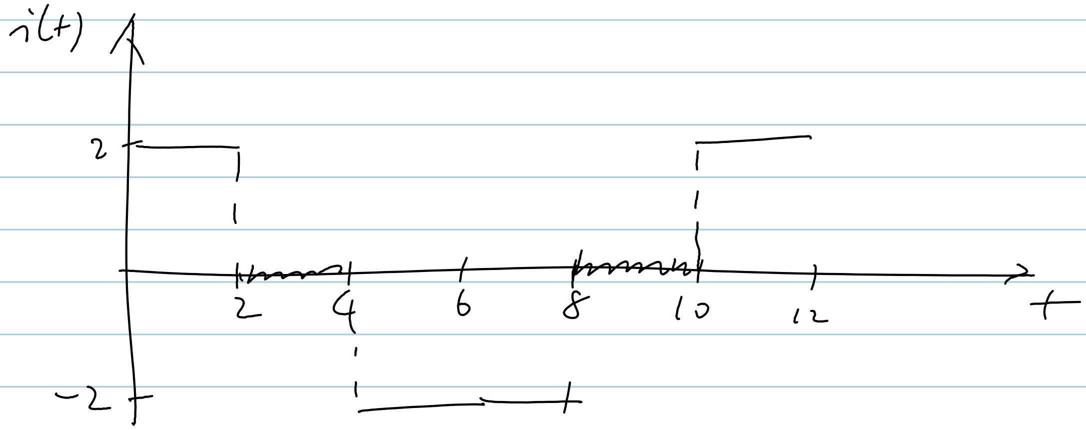
$$i(t) = \frac{d}{dt} q(t) = \frac{d}{dt} e^{-t^2} = e^{-t^2} \cdot (-2t)$$




Ex 1.2 (p6)




Find i(t)



# Charge revisited-5min

Tuesday, January 03, 2012
4:07 PM

- Conversely, the "sum" of the instantaneous currents through a cross-sectional area from $-\infty$ to $t$ equals the net charge transported through the boundary

$$q(t) = \int_{-\infty}^{t} i(\tau)\, d\tau$$

Ex 4 Suppose that $i(t)$ is given as follows


Find $q(t)$

$$q(t) = \int_{-\infty}^{t} i(\tau)\, d\tau = \cancel{\int_{-\infty}^{0} i(\tau)\, d\tau} + \int_{0}^{t} i(\tau)\, d\tau$$

$$= \int_0^t i(\tau)\, d\tau$$

Case 1: $0 \le t < 1$. Here $i(t) = 2t$

$$q(t) = \int_0^t 2\tau\, d\tau = \tau^2 \Big|_0^t = t^2 \ (C)$$

Case 2: $1 \le t < 2$. Here $i(t) = -2t + 4$

$$q(t) = q(1) + \int_1^t i(\tau)\,d\tau$$

$$= 1 + \int_1^t (-2\tau + 4)\,d\tau$$

$$= 1 + \left[-\tau^2 + 4\tau\right]_1^t$$

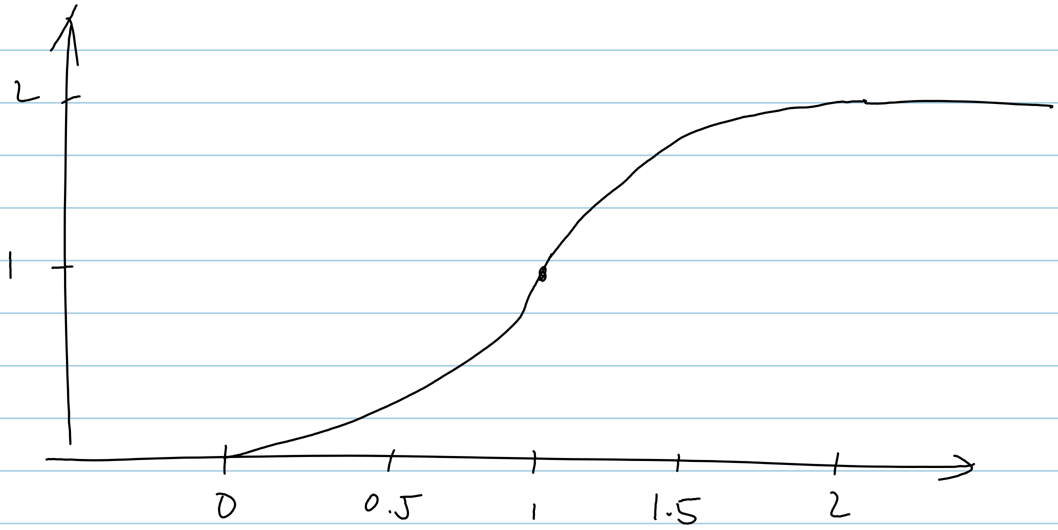
$$= 1 + \left[-t^2 + 4t - (-1 + 4)\right]$$

$$= -2 + 4t - t^2 \quad (C)$$

Case 3: $t \ge 2$

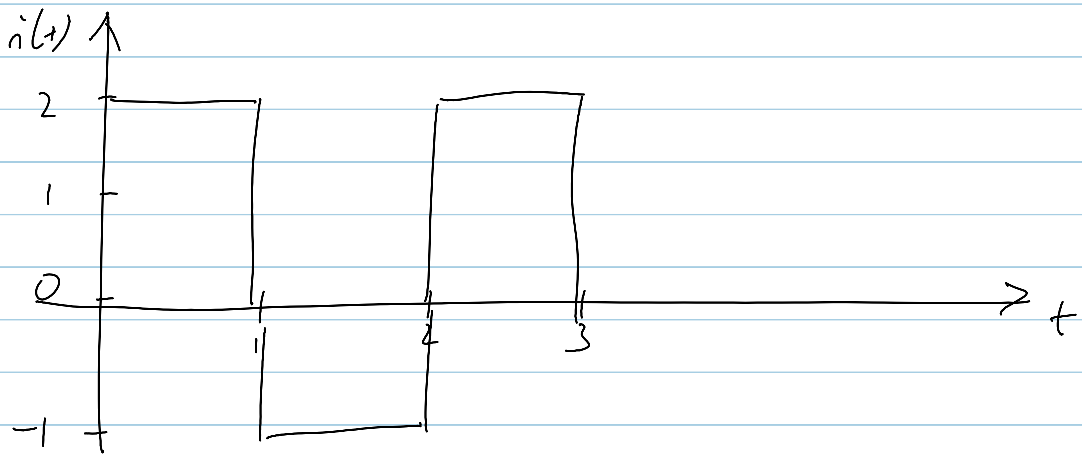
$$q(t) = q(2) + \int_2^t \underbrace{i(\tau)}_{=0}\,d\tau$$

$$= q(2) = 2 \ (C)$$




Ex 1.3 (p 7)

May skip




Find the total charge Q transported.

The total charge equals the total area beneath the curve.

$$= 2 - 1 + 2 = 3 \ (C)$$

# Summary

Tuesday, January 03, 2012
4:20 PM

| Water bucket / hose | Electrical Circuit |
|---|---|
| Water Mass | Electrical charge |
| Water current | Electrical current |

- $I = \frac{\Delta Q}{\Delta t}$ $\qquad i(t) = \frac{d q(t)}{dt}$

- $q(t) = \int_{-\infty}^{t} i(\tau)\, d\tau.$