ECE201 HW1 Solution

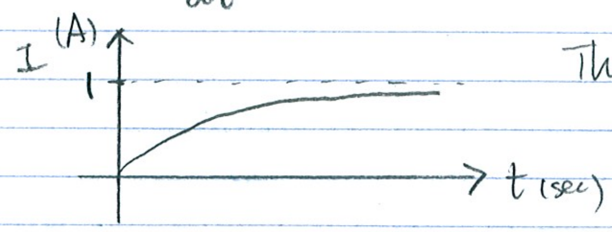
1.(a) $Q = 7.573 \times 10^{17} \times (-1.602 \times 10^{-19}) = -0.1213$ C

(b) $I = \frac{Q}{t} = \frac{0.1213}{10^{-3}} = 121.3$ A

(The problem does not specify the reference direction for current, which is why there is no +/− sign here.)

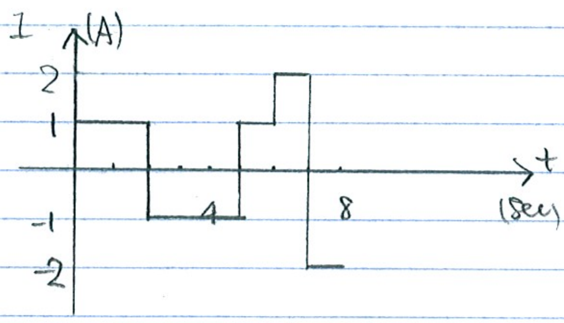
(c) $n \times 1.602 \times 10^{-19} / 60 = 10 \Rightarrow n = 3.75 \times 10^{21}$

(d) $I(t) = \frac{dq(t)}{dt} = 1 - e^{-5t}$ A



The current flows from left to right

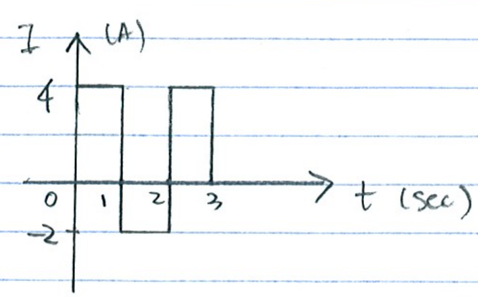
(e)



7(a) $q(t) = \int_0^t i(t)\,d\tau + q(0) = \int_0^t (2 - e^{-2\tau} - \cos(2\tau))\,d\tau + 0$

$= 2t + \frac{1}{2}e^{-2t} - \frac{1}{2}\cancel{\cos}\,\sin 2t \qquad (t \geq 0)$

8. $I(t) = \frac{dq(t)}{dt}$



ECE201

HW2 Solution

$\boxed{1}$ (a) (ii) & (iii)

(b) (i) $20V(-2A) = -40\ W$

(ii) $10V(2A) = 20\ W$

(iii) $20V(3A) = 60\ W$

$\boxed{2}$ (a) element 1 $\Rightarrow$ $10V(-1A) = -10W$

element 2 $\Rightarrow$ $6V(2A) = 12W$

(b) $-30 - 10 - 24 + 12 + 20 + 32 = 0$

$\boxed{3}$ (a) (.1) $\frac{10¢}{kWh}\left(8\text{-hours}\right) = 8¢$ a day

$\frac{100\ watt}{1000\ watts} = .1$

$\frac{8¢}{day}(30\ day) = \boxed{\$2.40}$

(b) $\frac{\$9}{30 days} = .30¢$ a day

$\frac{.30¢}{6 hour} = .05 = b\ \frac{.01}{kWh}$

$\boxed{\text{bulbs} = 5}$

Homework #3

Prob. 1-19

(a) $2.575\left(\frac{\Omega}{1000ft}\right) \times 800\,feet = 2.575\ \times 0.8\ (\Omega) =\ 2.06(\Omega)$

(b) From table 1.2, 200 feet of 14-gauge nickel wire has resistance:

$$5.1\ \times\ 2.575\left(\frac{\Omega}{1000ft}\right) \times 200\,feet = 5.1\ \times 2.575 \times 0.001\ \times 200 = 2.6265(\Omega)$$

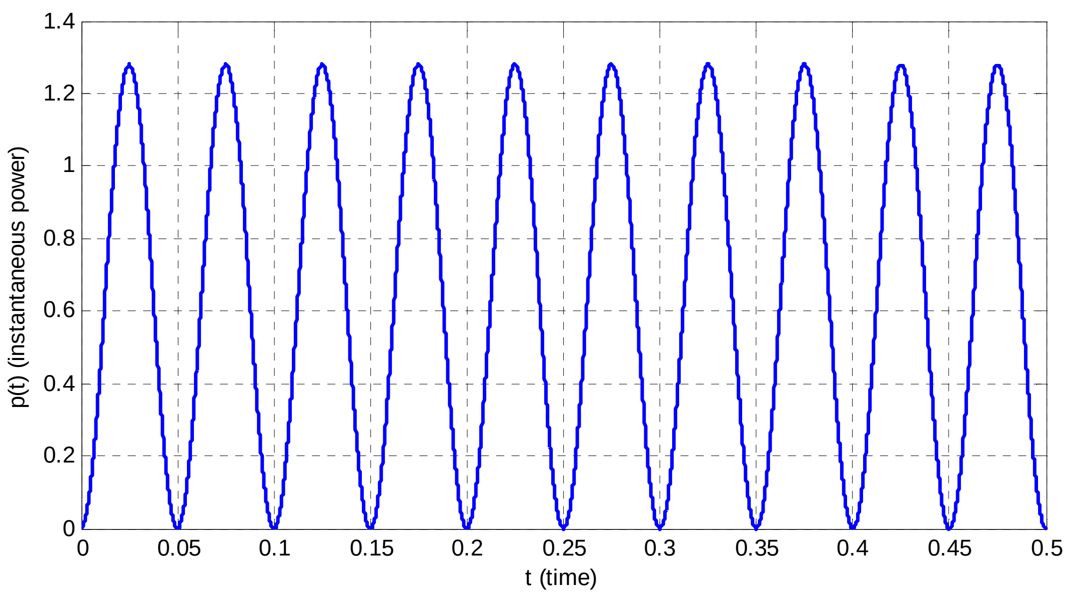
(c) $V = IR_{total} = I\left(R_{copper} + R_{nickel}\right)$

$$R_{total} = \left(R_{copper} + R_{nickel}\right) = 2.06 + 2.6265 = 4.6865(\Omega)$$

Prob. 1-22

(a) $P(t) =\ i^2(t)R = (400\sin(20\pi t) \times 10^{-3})^2 \times (R + 2R + 5R) = 0.16 \times (\sin(20\pi t))^2 \times 8 =$

$$= 1.28 \times sin^2(20\pi t)(\mathrm{W})$$




```
>> t=[0:0.0001:0.5];
>> p=1.28*(sin(20*pi*t)).^2;
>> plot(t,p);
>> grid;
>> xlabel('t (time)');
>> ylabel('p(t) (instantaneous power)');
```

Prob 1-27

(a) $P = \frac{{V_0}^2}{R}$

$\therefore R = \dfrac{{V_0}^2}{P} = \dfrac{120 \times 120}{60} = 240(\Omega)$

(b) $P = \frac{{V_0}^2}{R} \times 2$

$\therefore R = \dfrac{2 \times {V_0}^2}{P} = \dfrac{2 \times 120 \times 120}{150} = 192(\Omega)$

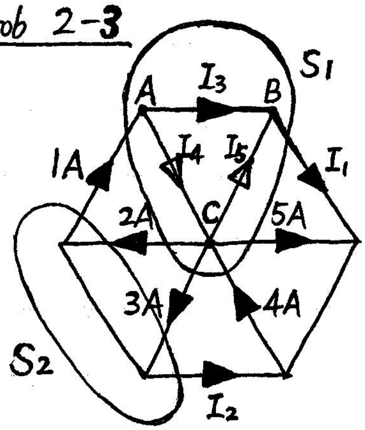
Prob 1-37

$V_{in} = I_{in} \times R_1$

$I_{out} = \dfrac{\mu V_{in}}{R_2} = \dfrac{\mu I_{in} R_1}{R_2}$

$P_{R2} = I_{out}^2 \times R_2 = \dfrac{\mu^2 I_{in}^2 R_1^2}{R_2^2} \times R_2 = \dfrac{\mu^2 I_{in}^2 R_1^2}{R_2}$

Prob 2-3



(a) Use Gaussian Surface S_1:

$$I_1 + 5 + 3 + 2 = 1 + 4$$

$$\therefore I_1 = \boxed{-5A}$$

(b) Use Gaussian Surface S_2:

$$I_2 + 1 = 2 + 3$$

$$\therefore I_2 = \boxed{4\ A}$$

(c) $\boxed{\text{False}}$

Proof (c): Use KCL: (Define new variables I_4 and I_5)

Node A: $I_3 + I_4 = 1$ ①

Node B: $I_3 + I_5 = I_1 = -5$ ②

Node C: $I_4 + 4 = I_5 + 2 + 3 + 5$ ③

$\Rightarrow \begin{cases} I_3 + I_4 = 1 \\ I_3 + I_5 = -5 \end{cases}$ It's underdetermined, no unique soln.

Prob 2-6

(a) For closed path A-B-D-A:

$$V_{AB} + V_{BD} + V_{DA} = 0$$

$$-80 + 70 + V_4 = 0$$

$$\therefore V_4 = \boxed{10\ V} \quad P_4 = \frac{V_4^2}{10K} = \boxed{0.01W}$$

For closed path C-D-G-C:

$$V_{CD} + V_{DG} + V_{GC} = 0$$

$$V_3 + 60 + 50 = 0$$

$$\therefore V_3 = \boxed{-110\ V} \quad P_3 = \frac{V_3^2}{10K} = \boxed{1.21W}$$

For closed path A-G-D-A:

$$V_{AG} + V_{GD} + V_{DA} = 0$$

$$V_2 - 60 + V_4 = V_2 - 60 + 10 = 0$$

$$\therefore V_2 = \boxed{50\ V} \quad P_2 = \frac{V_2^2}{10K} = \boxed{0.25\ W}$$

For closed path B-C-D-B:

$$V_{BC} + V_{CD} + V_{DB} = 0$$

$$V_1 + V_3 - 70 = V_1 - 110 - 70 = 0$$

$$\therefore V_1 = \boxed{180\ V} \quad P_1 = \frac{V_1^2}{10K} = \boxed{3.24W}$$

(b) $V_A = V_{AG} = V_2 = \boxed{50\ V}$

$V_B = V_{BG} = V_{BD} + V_{DG} = 70 + 60 = \boxed{130\ V}$

$V_C = V_{CG} = \boxed{-50\ V}$

$V_D = V_{DG} = \boxed{60\ V}$

(c) $V_{CA} = V_{CD} + V_{DA} = V_3 + V_4 = -110 + 10 = \boxed{-100\ V}$

Prob 2-14

(a) Let $R_1 = 20\,\Omega$, $R_2 = 40\,\Omega$, R_L unknown.

From current division formula (Eqn. 2.6):

$$\begin{cases} I_1 = \dfrac{1/R_1}{1/R_1 + 1/R_2 + 1/R_L}(I_{in} - aI_1) & ① \\ I_2 = \dfrac{1/R_2}{1/R_1 + 1/R_2 + 1/R_L}(I_{in} - aI_1) & ② \\ I_3 = \dfrac{1/R_L}{1/R_1 + 1/R_2 + 1/R_L}(I_{in} - aI_1) & ③ \end{cases}$$

$$I_{in} = aI_1 + I_1 + \underbrace{I_2 + I_3} = aI_1 + I_1 + \frac{1/R_2 + 1/R_L}{1/R_1 + 1/R_2 + 1/R_L}(I_{in} - aI_1)$$

$$\Rightarrow \left(1 - \frac{1/R_2 + 1/R_L}{1/R_1 + 1/R_2 + 1/R_L}\right) I_{in} = \left(1 - \frac{1/R_2 + 1/R_L}{1/R_1 + 1/R_2 + 1/R_L}\right) aI_1 + I_1$$

$$\Rightarrow I_{in} = aI_1 + \frac{1/R_1 + 1/R_2 + 1/R_L}{1/R_1} I_1 = aI_1 + R_1(1/R_1 + 1/R_2 + 1/R_L)I_1$$

$$= \left(a + 1 + \frac{R_1}{R_2} + \frac{R_1}{R_L}\right) I_1 = \boxed{\left(a + 1.5 + \frac{20}{R_L}\right) I_1}$$

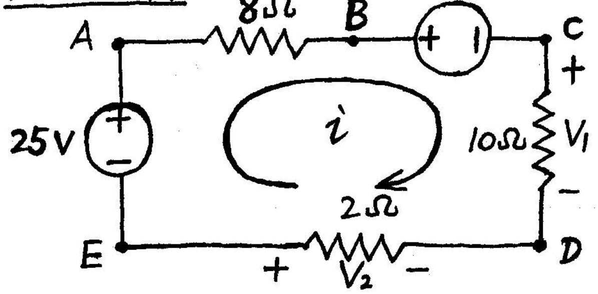
(b) $I_{in} = 4$ A, $I_1 = 2$A, $a = 0.25$:

$$4 = \left(0.25 + 1.5 + \frac{20}{R_L}\right) 2$$

$$\Rightarrow \frac{40}{R_L} = 0.5 \quad \Rightarrow R_L = \boxed{80\,\Omega} \qquad = \boxed{20\text{ W}}$$

$$P_L = (I_3)^2 R_L = \left(\frac{1/80}{1/20 + 1/40 + 1/80}\right)^2 (4 - 0.25\times 2)^2 80 = \left(\tfrac{1}{7}\right)^2 \left(\tfrac{7}{2}\right)^2 80$$

Prob 2-17



(a) For closed path A-B-C-D-E-A:

$$V_{AB} + V_{BC} + V_{CD} + V_{DE} + V_{EA} = 0$$

$$8i + 15 + 10i + 2i - 25 = 0$$

$$\Rightarrow i = \boxed{0.5\text{ A}}$$

(a) $V_1 = V_{CD} = 10i = 10 \times 0.5 = \boxed{5\text{ V}}$

$P_1 = i^2 R_1 = (0.5)^2\, 10 = \boxed{2.5\text{ W}}$

(b) $V_2 = -V_{DE} = -0.5 \times 2 = \boxed{-1\text{ V}}$

$P_2 = i^2 R_2 = (0.5)^2\, 2 = \boxed{0.5\text{ W}}$

(c) $P_{S1} = 25i = \boxed{12.5\text{ W}}$

$P_{S2} = -15i = \boxed{-7.5\text{ W}}$

ECE 201 Spring 2010

Homework 5 Solutions

Problem 26

Since $V_2 = 60\ V$, the current through 60 Ω branch is 1 A. The resistors 90 Ω and 180 Ω are in parallel. Their equivalent resistance is

$$\begin{aligned} R_{eq} &= \frac{90 \times 180}{90 + 180} \\ &= 60 \end{aligned}$$

Now 60 Ω and 60 Ω are in series. Thus the voltage drop across the parallel combination of 40 Ω and 120 Ω is 120 V. Thus current through 40 Ω resistor is 120/40=3 A. Hence,

$$\begin{aligned} I_s &= 3 + 1 \\ &= 4\ A \end{aligned}$$

Applying KVL around the loop containing the source and 40 Ω resistor,

$$\begin{aligned} V_s - 180 \times 4 - 120 &= 0 \\ \Rightarrow V_s &= 840\ V \end{aligned}$$

Power delivered by the source is given by,

$$\begin{aligned} P_s &= V_s \times I_s \\ &= 840 \times 4 \\ &= 3360\ W \end{aligned}$$

Problem 32(a)

Using voltage division, the following equations can be written,

$$R_1 + R_2 + R_s = 2400 \tag{1}$$

$$\frac{R_1 + R_2}{R_1 + R_2 + R_s} = 0.75 \tag{2}$$

$$\frac{R_2}{R_1 + R_2 + R_s} = 0.25 \tag{3}$$

Using (1) and (3), $R_2 = 600\ \Omega$. Using (2), $R_1 = 1200\ \Omega$. Using (1), $R_s = 600\ \Omega$.

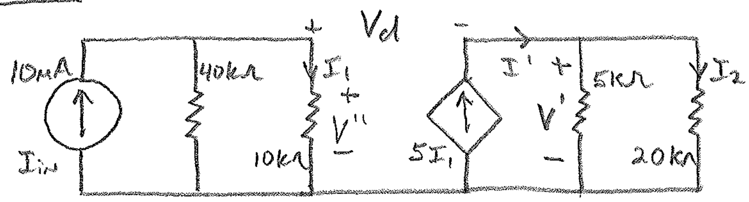
Problem 40

The resistors 9 $k\Omega$ and 18 $k\Omega$ are in parallel. Thus their equivalent resistance of 6 $k\Omega$ is in series with 6 $k\Omega$. Now the combination 4 $k\Omega$ and 12 $k\Omega$ are in parallel. The current I_{in} divides into I_1 and I_2 through the parallel resistors in inverse ratio of their resistances. Similarly, the current I_2 divides in inverse ratio of resistances 9 $k\Omega$ and 18 $k\Omega$ in parallel. Thus we get,

$$\begin{aligned}
I_1 &= 120 \times \frac{12}{12+4} \\
&= 90\ mA \\
I_2 = 120 - 90 &= 30\ mA \\
I_3 &= 30 \times \frac{9}{9+18} \\
&= 10\ mA \\
V_{in} &= 0.09 \times 4 \times 10^3 \\
&= 360\ V \\
P_{source} &= V_{in} I_{in} \\
&= 43.2\ W
\end{aligned}$$

ECE 201 HW # 6 | Solution

2-39



Find I_1, I_2, V_d, $P_{dep,del}$

$I_1 = 10mA \cdot \frac{40k\Omega}{10k\Omega + 40k\Omega}$ (current division) $= \boxed{8mA}$

$I' = 5I_1 = 5 \cdot 8mA = 40mA$ $\qquad I_2 = 40mA \cdot \frac{5k\Omega}{5k\Omega + 20k\Omega} = \boxed{8mA}$

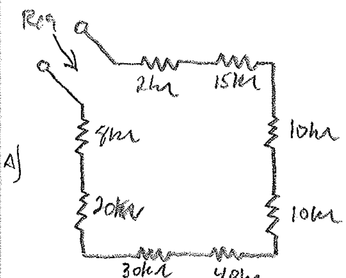
$V' = 20k\Omega\, I_2 = 20k\Omega\ 8mA = 160V$ $\qquad V'' = 8mA \cdot 10k\Omega = 80V$

$V_d = V'' - V' = 80V - 160V = \boxed{-80V}$

$P_{dep,del} = I' \cdot V' = 40mA \cdot 160V = \boxed{6.4W}$

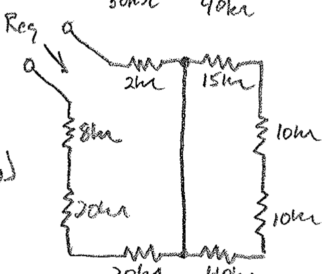
2-46

A)



A) $R_{eq} = \sum R_i$
$= (2+15+10+10+40+30+20+8)k\Omega$
$= \boxed{135k\Omega}$

b)

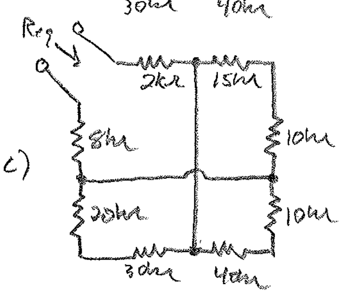


b) $R_{eq} = (2+30+20+8)k = \boxed{60k\Omega}$

The 15, 10, 10, 40 kΩ R's are in parallel with a short, which has $R = 0\Omega$

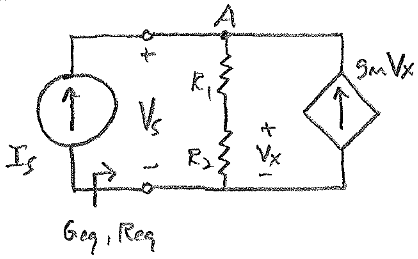
$R' = \frac{0 \cdot (15+10+10+40)k\Omega}{0 + (15+10+10+40)k\Omega} = 0$

c)



c) $R_{eq} = 2k\Omega + 8k\Omega + [25k\Omega \,//\, 50k\Omega \,//\, 50k\Omega]$
$= 10k\Omega + [25k\Omega \,//\, 25k\Omega]$
$= 10k\Omega + 12.5k\Omega = \boxed{22.5k\Omega}$

2-63



KCL @ A:

$$V_x = \frac{R_2}{R_1+R_2} \cdot V_S$$

$$I_S + g_m V_x = \frac{V_S}{R_1+R_2}$$

$$\Rightarrow (R_1+R_2) I_S + (R_1+R_2) g_m V_x = V_S$$

$$\Rightarrow (R_1+R_2) I_S = V_S - (R_1+R_2) g_m \cdot \frac{R_2}{R_1+R_2} V_S$$

$$\Rightarrow (R_1+R_2) I_S = V_S [1 - R_2 g_m]$$

$$\Rightarrow \boxed{R_{eq} = \frac{V_S}{I_S} = \frac{R_1+R_2}{1-R_2 g_m}}$$

$$\boxed{G_{eq} = R_{eq}^{-1} = \frac{1-R_2 g_m}{R_1+R_2}}$$

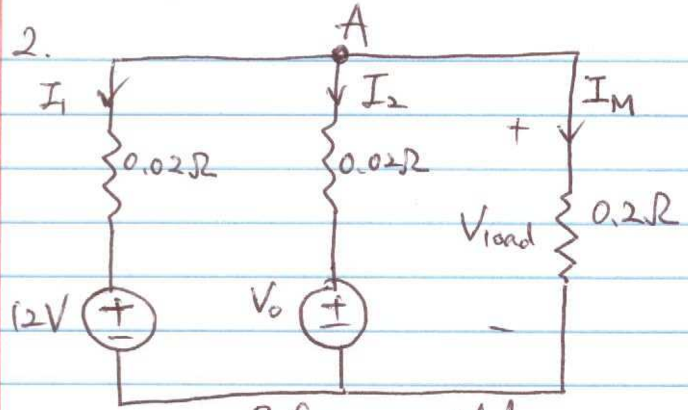
When $R_1 = 1k\Omega$, $R_2 = 3k\Omega$, $g_m = 0.2mS$:

$$R_{eq} = \frac{1k\Omega + 3k\Omega}{1 - 3k\Omega \cdot 0.0002 S} = \boxed{10k\Omega}$$

$$G_{eq} = R_{eq}^{-1} = \boxed{0.1 mS}$$

HW #7.

2.



At node A, KCL equation is

$$I_1 + I_2 + I_M = 0$$

$$\therefore \frac{V_A - 12}{0.02} + \frac{V_A - V_0}{0.02} + \frac{V_A}{0.2} = 0$$

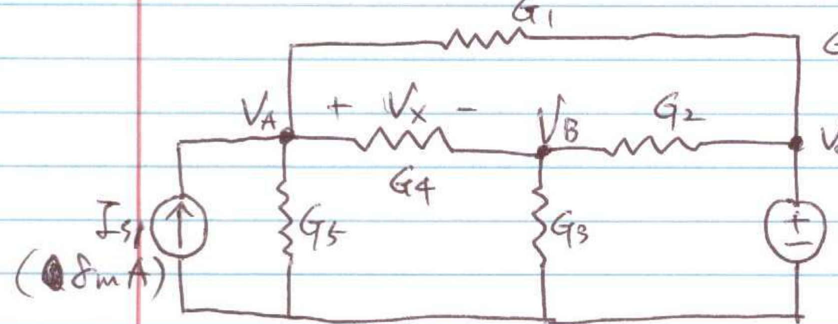
$$V_0 = 2.1 V_A - 12$$

$$I_M \geq 50A \Rightarrow V_A = I_M \times 0.2 \geq 10V$$

$$\therefore V_0 \geq 2.1 \times 10 - 12 = 9(V)$$

$\therefore$ The minimum value of voltage V_0 is $\underline{9V}$.

6. (a)



$G_1 = 1/5k\Omega$

$G_2 = G_3 = G_4 = 1/20k\Omega$

$G_5 = 1/10K$

At node A $\Rightarrow$ $I_{S1} = (V_A \times G_5) + (V_A - V_B) G_4 + (V_A - V_C) G_1$ --- ①

At node B $\Rightarrow$ $0 = (V_B \times G_3) + (V_B - V_A) G_4 + (V_B - V_C) \cdot G_2$ --- ②

At node C $\Rightarrow$ KCL is not useful.

$V_C = V_{S2} = 40 (V)$

$\therefore$ From ① and ②

$$(G_1 + G_4 + G_5) V_A + (-G_4) V_B = I_{S1} + V_C \cdot G_1$$

$$(-G_4) V_A + (G_2 + G_4 + G_3) V_B = V_C \cdot G_2$$

In matrix form

$$\begin{bmatrix} G_1 + G_4 + G_5 & -G_4 \\ -G_4 & G_2 + G_3 + G_4 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} I_{S1} + V_C \cdot G_1 \\ V_C \cdot G_2 \end{bmatrix}$$

$$\begin{bmatrix} 0.35 \times 10^{-3} & -0.05 \times 10^{-3} \\ -0.05 \times 10^{-3} & 0.15 \times 10^{-3} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 16 \times 10^{-3} \\ 2 \times 10^{-3} \end{bmatrix}$$

$\therefore V_A = 50V$, $V_B = 30V$,

$V_X = V_A - V_B = 20V$

the power absorbed by $R_4 = \dfrac{V_X^2}{R_4} = \dfrac{20^2}{(20K)} = 0.02(W)$

the power delivered by $I_{S1} = I_{S1} \times V_A = 8 \times 10^{-3} \times 50$

$= 0.4(W)$

the power delivered by $V_{S2} =$

$$V_{S2} \times \left(\frac{V_C - V_B}{R_2} + \frac{V_C - V_A}{R_1} \right)$$

$$= 40 \times \left(\frac{40-30}{20K} + \frac{40-50}{5K} \right) = -0.06(W)$$

ECE 201 HW8 Solutions

3-13 1) KCL on node A: $G_1 V_A + G_2(V_A - V_B) = G_S(V_S - V_A)$

$\Rightarrow (G_1 + G_2 + G_S)V_A - G_2 V_B = G_S V_S \Rightarrow (1.3)V_A + (-0.1)V_B = 150$

2) KCL on node B: $G_2(V_A - V_B) + g_m(V_A - V_B) = G_3 V_B$

$\Rightarrow (G_2 + g_m)V_A - (G_2 + G_3 + g_m)V_B = 0 \Rightarrow (7.6)V_A + (-7.7)V_B = 0$

(a) $$\begin{bmatrix} (G_1 + G_2 + G_3) & -G_2 \\ (G_2 + g_m) & -(G_2 + G_3 + g_m) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} G_S V_S \\ 0 \end{bmatrix}$$

(b) $$\begin{bmatrix} 1.3 & -0.1 \\ 7.6 & -7.7 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 150 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 124.8649\,V \\ 123.2432\,V \end{bmatrix}$$

$P_{V_S} = V_S I_S = V_S \cdot G_S(V_S - V_A) = 150 \cdot \frac{1}{1k}(150 - 124.8649) = \boxed{3.7703\,W}$

$P_{R_3} = \frac{(V_B)^2}{R_3} = \frac{(123.2432)^2}{10k} = \boxed{1.5189\,W}$

(c) $I_2 = G_2(V_A - V_B) = \frac{1}{10k}(124.8649 - 123.2432) = \boxed{0.16217 \times 10^{-3}\,A}$

3-18 Define current I_{AC} from node A to node C.

1) KCL on node A: $G_1(V_{AC} - V_{BC}) + I_{AC} = I_S$

$\Rightarrow G_1 V_{AC} - G_1 V_{BC} + I_{AC} = I_S \Rightarrow \frac{1}{9}V_{AC} - \frac{1}{9}V_{BC} + 1000 I_{AC} = 20$

2) KCL on node B: $G_2(V_{BC} - V_{DC}) + G_3 V_{BC} = G_1(V_{AC} - V_{BC})$

$\Rightarrow G_1 V_{AC} - (G_1 + G_2 + G_3)V_{BC} + G_2 V_{DC} = 0 \Rightarrow \frac{1}{9}V_{AC} - \frac{1}{3}V_{BC} + \frac{1}{18}V_{DC} = 0$

3) KCL on node D: $G_2(V_{BC} - V_{DC}) = G_4 V_{DC} + I_S$

$\Rightarrow G_2 V_{BC} - (G_2 + G_4)V_{DC} = I_S \Rightarrow \frac{1}{18}V_{BC} - \frac{1}{6}V_{DC} = 20$

4) For CCVS: $V_{AC} = r_m i_x = r_m \cdot G_1(V_{AC} - V_{BC})$

$\Rightarrow (r_m G_1 - 1)V_{AC} - r_m G_1 V_{BC} = 0 \Rightarrow -\frac{2}{3}V_{AC} - \frac{1}{3}V_{BC} = 0$

$$\begin{bmatrix} \frac{1}{9} & -\frac{1}{9} & 0 & 1000 \\ \frac{1}{9} & -\frac{1}{3} & \frac{1}{18} & 0 \\ 0 & \frac{1}{18} & -\frac{1}{6} & 0 \\ -\frac{2}{3} & -\frac{1}{3} & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{AC} \\ V_{BC} \\ V_{DC} \\ I_{AC} \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 20 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} V_{AC} \\ V_{BC} \\ V_{DC} \\ I_{AC} \end{bmatrix} = \begin{bmatrix} 9\,V \\ -18\,V \\ -126\,V \\ 0.017\,A \end{bmatrix}$$

$P_{I_S} = (V_{AC} - V_{DC})I_S = (9 + 126)\,20 \times 10^{-3} = \boxed{2.7\,W}$

$P_{CCVS} = r_m G_1(V_{AC} - V_{BC})(-I_{AC}) = -3k\,\frac{1}{9k}(9 + 18)(0.017) = \boxed{-0.153\,W}$

(3-18) $R_{eq} = \dfrac{V_{AC} - V_{DC}}{I_S} = \dfrac{9 + 126}{20 \times 10^{-3}} = \boxed{6.75\ k\Omega}$

3-26 (a) Define currents I_{AB} from node A to node B, and I_{BC} from node B to node C.

1) KCL on node A : $G_2(V_A - V_{S1}) + G_4 V_A + I_{AB} = I_{S4}$

$\Rightarrow (G_2 + G_4) V_A + I_{AB} = I_{S4} + G_2 V_{S1} \Rightarrow 0.03\, V_A + I_{AB} = 15$

2) KCL on node B : $G_3 V_B + I_{BC} = I_{AB}$

$\Rightarrow G_3 V_B - I_{AB} + I_{BC} = 0 \Rightarrow 0.01\, V_B - I_{AB} + I_{BC} = 0$

3) KCL on node C : $G_1 (V_C - V_{S1}) = I_{BC}$

$\Rightarrow G_1 V_C - I_{BC} = G_1 V_{S1} \Rightarrow 0.1\, V_C - I_{BC} = 10$

4) For V_{S2} : $V_B - V_C = V_{S2} = 60$

5) For V_{S3} : $V_A - V_B = V_{S3} = 100$

(b)
$$\begin{bmatrix} 0.03 & 0 & 0 & 1 & 0 \\ 0 & 0.01 & 0 & -1 & 1 \\ 0 & 0 & 0.1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ I_{AB} \\ I_{BC} \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 10 \\ 60 \\ 100 \end{bmatrix} \Rightarrow \begin{bmatrix} V_A \\ V_B \\ V_C \\ I_{AB} \\ I_{BC} \end{bmatrix} = \begin{bmatrix} 300\text{ V} \\ 200\text{ V} \\ 140\text{ V} \\ 6\text{ A} \\ 4\text{ A} \end{bmatrix}$$

(c) $P_{S1} = V_{S1} I_{S1} = V_{S1}\left[-G_1(V_C - V_{S1}) - G_2(V_A - V_{S1})\right] = -100(4+2) = \boxed{-600\text{ W}}$

$P_{S2} = V_{S2}(-I_{BC}) = 60 \times 4 = \boxed{-240\text{ W}}$

$P_{S3} = V_{S3}(-I_{AB}) = 100 \times 6 = \boxed{-600\text{ W}}$

$P_{S4} = V_A I_{S4} = 300 \times 14 = \boxed{4200\text{ W}}$

ECE 201 Spring 2010

Homework 9 Solutions

Problem 37

(a)

The following loop equation can be written,

$$
\begin{aligned}
-(I_1 - 0.75)200 - 300I_1 - (I_1 + 0.1)500 &= 0 \\
\Rightarrow I_1 &= 0.1\ A \\
P &= VI \\
P_{s1} &= 200(0.75 - 0.1)0.75 \\
&= 97.5\ W \\
P_{s2} &= 500(0.1 + 0.1)0.1 \\
&= 10\ W
\end{aligned}
$$

(b)

Again, writing the loop equation and comparing the equation obtained with that given in the problem,

$$
\begin{aligned}
-(I_1 - 0.4)R_1 - 600I_1 - (I_1 + 0.1)R_2 &= 0 \\
(R_1 + R_2 + 600)I_1 &= 0.4R_1 - 0.1R_2 \\
\Rightarrow R_1 + R_2 &= 1400 \\
4R_1 - R_2 &= 600 \\
\Rightarrow R_1 &= 400\ \Omega \\
R_2 &= 1000\ \Omega
\end{aligned}
$$

Problem 42

(a)

The following loop equations can be written,

$$\begin{aligned}
21 - 20I_1 - 80(I_1 - I_2) &= 0 \\
24 + 80I_2 - 80(I_1 - I_2) &= 0 \\
\Rightarrow I_1 &= 0.15\ A \\
I_2 &= -0.075\ A \\
V_{R_3} &= (I_1 - I_2)R_3 \\
&= 0.225 \times 80 = 18\ V \\
P_{R_3} &= 18(0.225) = 4.05\ W
\end{aligned}$$

(b)

Again, writing the loop equations,

$$\begin{aligned}
21 - (I_1 + I_2)20 - 80I_2 - 24 &= 0 \\
21 - (I_1 + I_2)20 - 80I_1 &= 0 \\
\Rightarrow I_1 &= 0.225\ A \\
I_2 &= -0.075\ A \\
V_{R_3} &= I_1 R_3 \\
&= 0.225 \times 80 = 18\ V \\
P_{R_3} &= 18(0.225) = 4.05\ W
\end{aligned}$$

Problem 52

(a)

Assume clockwise loop currents I_1 and I_2 in the two rightmost loops respectively. The following equations can then be written for the two loops,

$$\begin{aligned}
-(I_1 - I_{s1})R_1 - r_m I_x - R_2(I_1 - I_2) &= 0 \\
I_x &= I_{s1} - I_1 \\
-R_2(I_2 - I_1) - R_3 I_2 - V_{s2} &= 0
\end{aligned}$$

Putting in matrix form,

$$\begin{bmatrix} r_m - R_1 - R_2 & R_2 \\ R_2 & -(R_2 + R_3) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} (r_m - R_1)I_{s1} \\ V_{s2} \end{bmatrix}$$

(b)

Putting the respective literal values,

$$\begin{aligned} \begin{bmatrix} -80 & 40 \\ 40 & -120 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} &= \begin{bmatrix} -40 \\ 40 \end{bmatrix} \\ \Rightarrow I_1 &= 0.4\ A \\ I_2 &= -0.2\ A \end{aligned}$$

(c)

$$\begin{aligned} V_A = (1 - 0.4)100 &= 60\ V \\ V_B = 0.6(40) &= 24\ V \end{aligned}$$

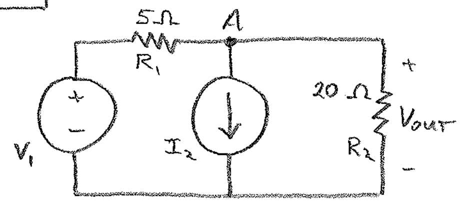
(d)

$$\begin{aligned} P_{s1} = 60 \times 1 &= 60\ W \\ P_{s2} = 40 \times 0.2 &= 8\ W \end{aligned}$$

(e)

$$P_{dep} = 60 \times 0.6 \times -0.4 = -14.4\ W$$

5-1



A) Node equation @ A: $\dfrac{V_{OUT}-V_1}{5} + I_2 + \dfrac{V_{OUT}}{20} = 0$

$\Rightarrow 4V_{OUT} - 4V_1 + 20I_2 + V_{OUT} = 0$

$\Rightarrow 5V_{OUT} = 4V_1 - 20I_2 \Rightarrow V_{OUT} = \frac{4}{5}V_1 - 4I_2 \Rightarrow \boxed{\alpha_1 = 4/5,\ \beta_2 = -4}$

b) $V_1(t) = 10\cos(10t)\,V \quad I_2 = 2A$

$\Rightarrow V_{OUT}(t) = \frac{4}{5}[10\cos(10t)] - 4(2) = \boxed{8\cos(10t) - 8\ V}$

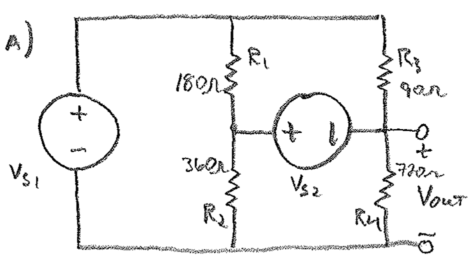
c) $G_1(V_{OUT} - V_1) + I_2 + G_2(V_{OUT}) = 0$

$\Rightarrow V_{OUT}(G_1 + G_2) = G_1V_1 - I_2 \Rightarrow \boxed{V_{OUT} = \dfrac{G_1}{G_1+G_2}V_1 - \dfrac{1}{G_1+G_2}I_2}$

$\alpha_1 = \dfrac{G_1}{G_1+G_2} \qquad \beta = -\dfrac{1}{G_1+G_2}$

5-21

A)

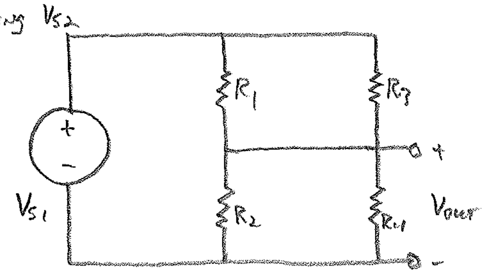


Zeroing V_{S2}: $R_1 // R_3$, $R_2 // R_4$

$V_{OUT} = V_{S1} \cdot \dfrac{R_2 // R_4}{R_1 // R_3 + R_2 // R_4}$

$= V_{S1} \cdot \dfrac{240}{60 + 240} = \dfrac{4}{5}V_{S1}$

Zeroing V_{S2}



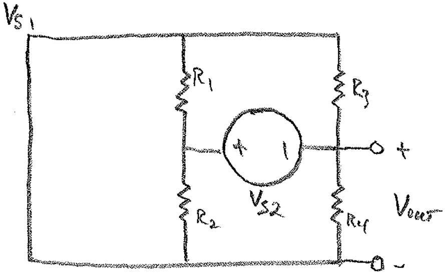
Zeroing V_{S1}: $R_1 // R_2$, $R_3 // R_4$

$-V_{OUT} = V_{S2} \cdot \dfrac{R_4 // R_3}{R_4 // R_3 + R_1 // R_2}$

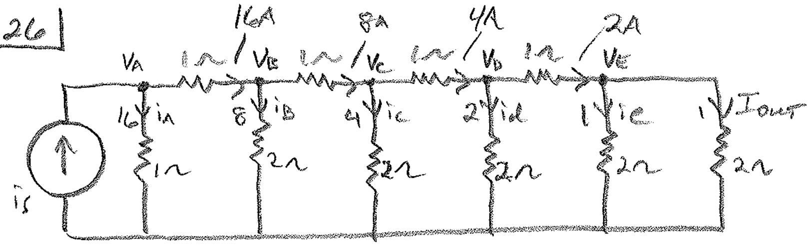
$= V_{S2} \cdot \dfrac{80}{80 + 120} = \dfrac{2}{5}V_{S2}$

$\Rightarrow \boxed{V_{OUT} = \dfrac{4}{5}V_{S1} - \dfrac{2}{5}V_{S2}}$

Zeroing V_{S1}



5-26



$i_s = 64mA$

$I_{out}' = 1A \Rightarrow V_E = 2V \Rightarrow i_e = 1A$

So $i_{DE} = 2A \Rightarrow V_D = V_E + 2V = 4V$ So $i_d = 4V/2\Omega = 2A$

$V_C = 4V + V_D = 8V$ So $i_C = 4A$

$V_B = 8V + V_C = 16V$ $i_B = 16V/2\Omega = 8A$

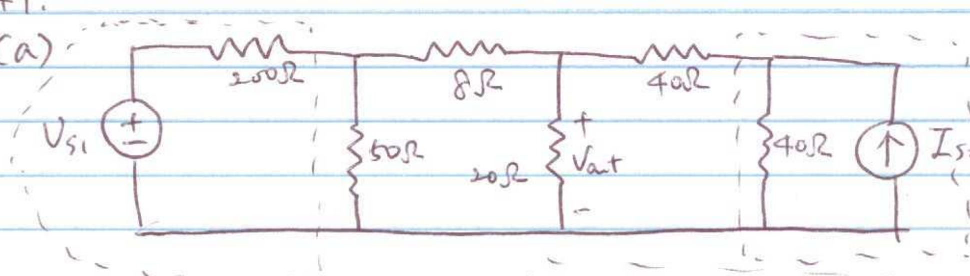
$V_A = 16V + V_B = 32V$ $i_A = 32A$ $\Rightarrow i_s' = 48A$

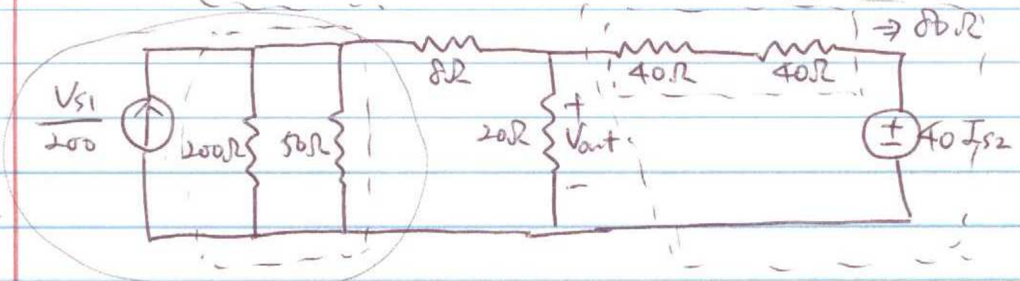
$$\frac{i_s'}{i_s} = \frac{I_{out}'}{I_{out}} \Rightarrow I_{out} = I_{out}' \cdot \frac{i_s}{i_s'} = 1A \cdot \frac{64mA}{48A} = \boxed{4/3\ mA}$$

HW #11

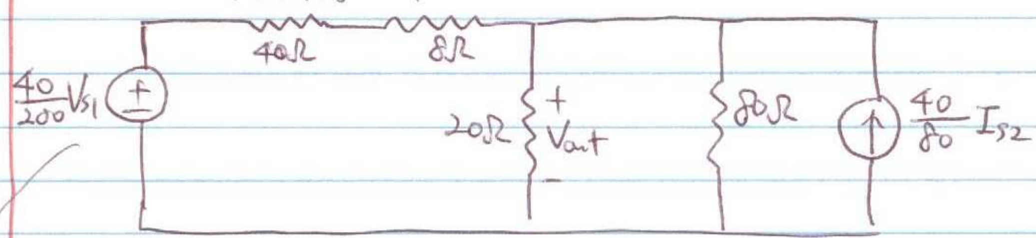
41.

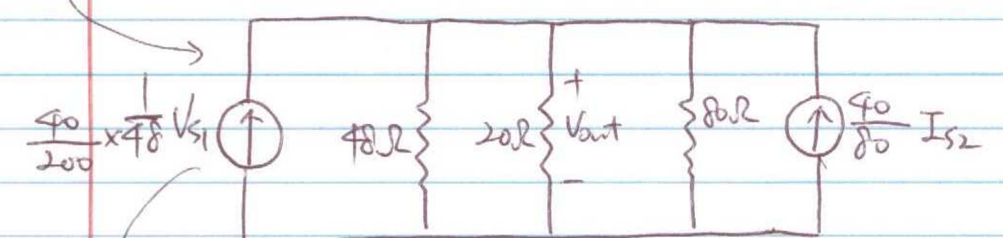
(a)

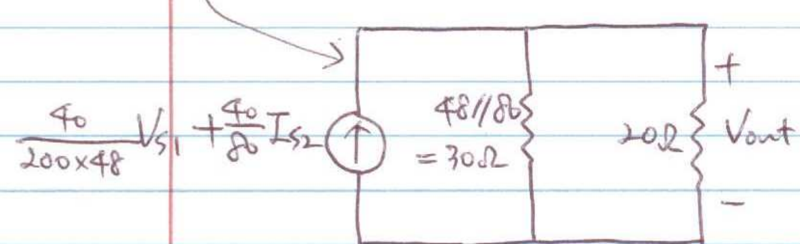




⇒ 200 // 50 = 40







$$V_{out} = 20 \times \frac{30}{30+20} \times \left(\frac{40}{200 \times 48} \times 240 + \frac{40}{80} \times 0.25 \right)$$

$$= 13.5\ (V)$$

(b) power dissipated

$$= \frac{V_{out}^2}{R} = \frac{13.5^2}{20} = 9.1125 (W)$$

(c) $V_{S1} = 480V$, $I_{S2} = 0.5A$

$$V_{out} = 20 \times \frac{30}{30+20} \times \left(\frac{40}{200 \times 48} \times 480 + \frac{40}{80} \times 0.5 \right)$$

$$= \underline{27 (V)}$$

By inspection,

From "linearity" property,

$$V_{out} = a \cdot V_{S1} + b \cdot I_{S2} \quad \left(\text{a and b are constants} \right)$$

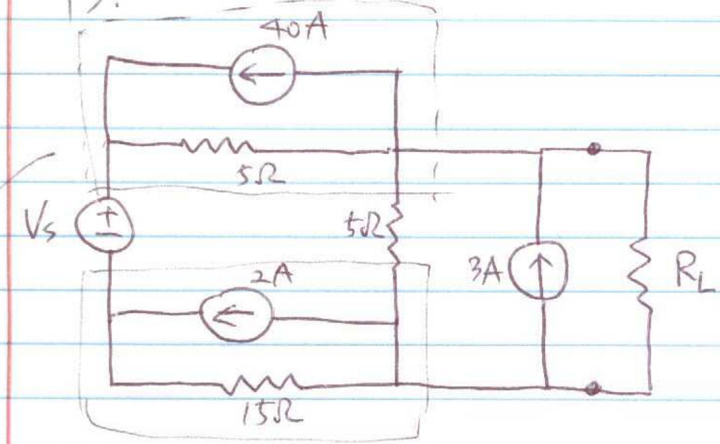
$$\therefore V_{out}' = a \cdot (2V_{S1}) + b \cdot (2I_{S2})$$

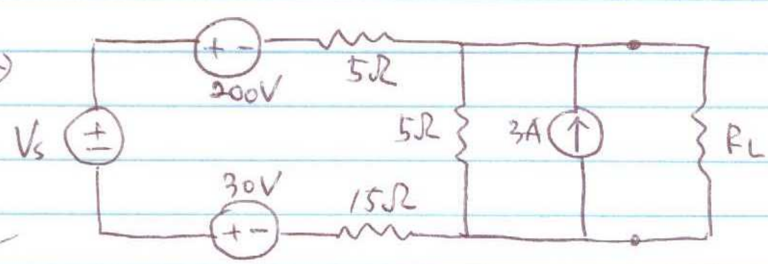
$$= 2 \left(a \cdot V_{S1} + b \cdot I_{S2} \right)$$

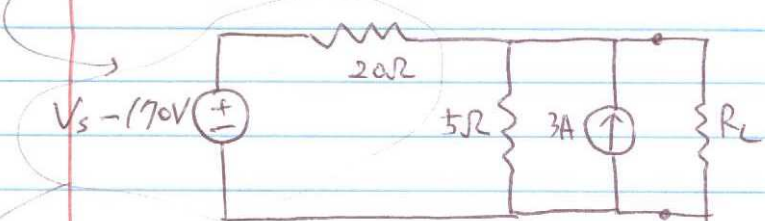
$$= 2 \cdot V_{out}$$

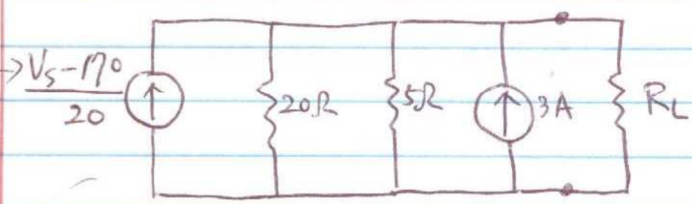
$\therefore V_{out}$ is also increased by a factor of two

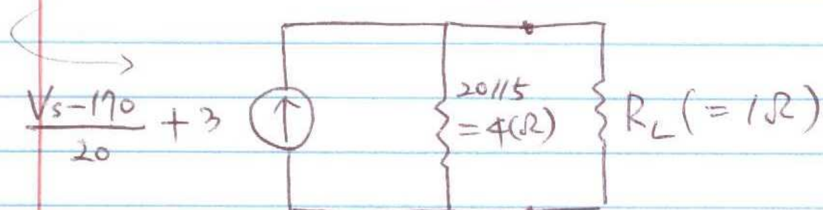
43.







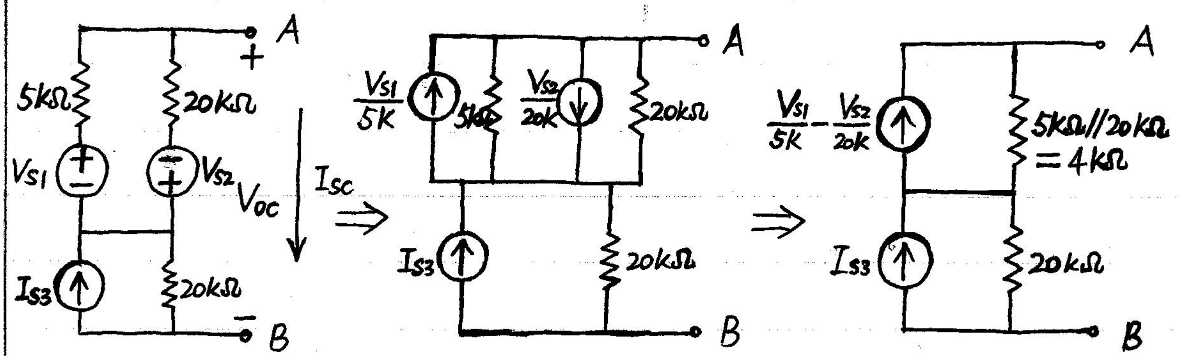


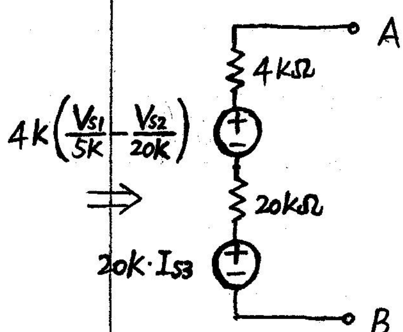


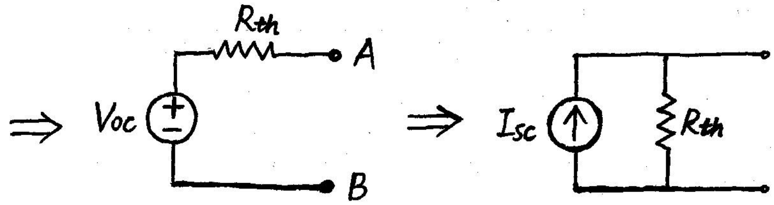
$$P_{R_L} = 1 \times \left\{ \frac{4}{4+1} \times \left(\frac{V_s - 170}{20} + 3 \right) \right\}^2 = 16$$

$$\therefore \quad \underline{V_s = 210(V) \text{ or } 10(V)}$$

6-2



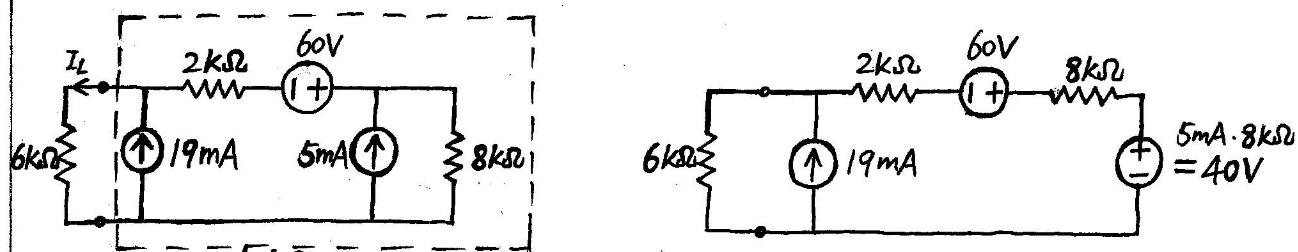


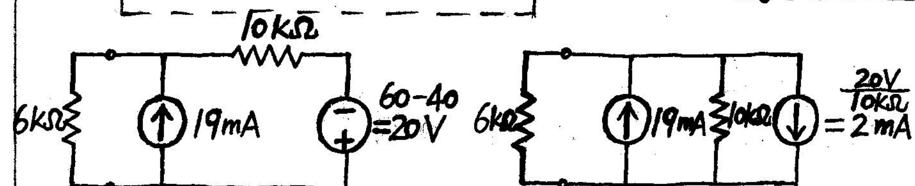


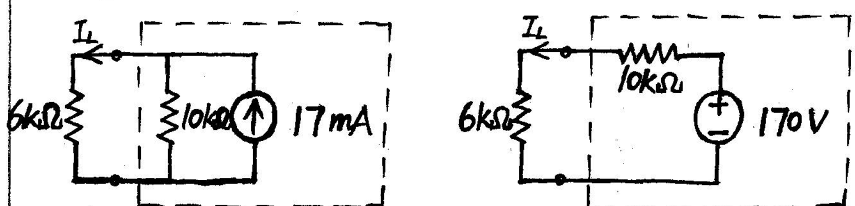
$$V_{oc} = \boxed{\frac{4}{5}V_{s1} - \frac{1}{5}V_{s2} + 20000 I_{s3}\ \text{V}} \qquad I_{sc} = \frac{V_{oc}}{R_{th}} = \boxed{\frac{1}{30000}V_{s1} - \frac{1}{120000}V_{s2} + \frac{5}{6}I_{s3}\ \text{A}}$$

$$R_{th} = 4\,k\Omega + 20\,k\Omega = \boxed{24000\,\Omega}$$

6-5







$$V_{oc} = \boxed{170\ \text{V}}$$
$$I_{sc} = \boxed{17\ \text{mA}}$$
$$R_{th} = \boxed{10\ k\Omega}$$

$$I_L = \frac{\frac{1}{6K}}{\frac{1}{6K}+\frac{1}{10K}}\times 17 = \frac{5}{8}\times 17 = \boxed{10.625\ \text{mA}} \qquad I_L = \frac{170}{6K+10K} = \frac{170}{16K} = \boxed{10.625\ \text{mA}}$$

$$P_L = (I_L)^2 R_L = \left(\frac{10.625}{1000}\right)^2 \times 6000 \approx \boxed{0.6773\,\text{W}}$$

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Homework 13 Solutions

Problem 9

(a)

2R and 6R in series gives 8R. 8R and 8R in parallel gives 4R. Thus 12R is in series with V_s in the simplified circuit. Thus the Thevenin voltage is given by

$$\begin{aligned} V_{oc} &= \frac{V_s}{12R} \times \frac{1}{2} \times 6R \\ &= \frac{V_s}{4} \\ &= 30\ V \end{aligned}$$

To find the Thevenin resistance, we short circuit V_s. Thus 8R and 8R are in parallel, which gives 4R, which in turn is in series with 2R. This gives 6R and 6R in parallel. Thus $R_{th} = 3R = 900\ \Omega$.

(b)

Using the Thevenin equivalent circuit to simplify our calculations,

$$\begin{aligned} P_{R_L} &= \left(\frac{30}{900 + R_L}\right)^2 R_L \\ &= 0.1875\ W,\ 0.24\ W,\ 0.244898\ W\ (R_L = 300\ \Omega,\ 600\ \Omega,\ 1200\ \Omega) \end{aligned}$$

The Thevenin equivalent circuit analysis allows us to analyze the circuit without the load once and then plug in the various load resistor values to compute the relevant quantities. However, using earlier techniques would require 3 separate analyses to compute these values. Hence the use of a Thevenin equivalent reduces the effort needed to obtain the answers.

(c)

Using **Maximum Power Transfer Theorem**, for maximum power transfer, $R_L = 900\ \Omega$ and resultant power delivered to the load is given by $P_{max} = (30)^2/(4 \times 900) = 0.25\ W$.

(d)

If V_s is doubled, power becomes four times, as $P_{max} \propto V_s^2$. Thus power delivered to the load becomes $P_{load} = 0.25 \times 4 = 1\ W$.

Problem 17

We proceed to find the Thevenin voltage V_{oc} and Thevenin resistance R_{th}. Norton current is then given by $I_{sc} = V_{oc}/R_{th}$ and Norton resistance is same as Thevenin resistance. Since this problem involves a dependent source, we cannot apply the conventional method. We write KVL equations to find the required values.

$$\begin{aligned}
v_{AB} - 300 i_A - v_x &= 0 \\
v_x - 200(i_A - v_x/18000) - \frac{4}{3}v_x &= 0 \\
\Rightarrow v_x &= -\frac{18000}{29} i_A \\
v_{AB} &= -320.689655 i_A \\
v_{AB} &= R_{th} i_A + V_{oc} \\
\Rightarrow V_{oc} &= 0 \\
R_{th} &= -320.69\ \Omega \\
\Rightarrow I_{sc} &= 0 \\
R_{norton} &= -320.69\ \Omega
\end{aligned}$$

Problem 21

(a)

Again, we write KVL equations to find the Thevenin and Norton equivalent circuits.

$$\begin{aligned}
v_{AB} - 40 i_A - 200(i_A + k v_x + v_x/50) &= 0 \\
V_s - v_x + 40 i_A - v_{AB} &= 0
\end{aligned}$$

$$
\begin{aligned}
\Rightarrow v_{AB}\left[1+200(k+0.02)\right] &= i_A\left[240+8000(k+0.02)\right]+200V_s(k+0.02)\\
v_{AB} &= 60i_A+18\\
v_{AB} &= i_AR_{th}+V_{oc}\\
\Rightarrow V_{oc} &= 18\ V\\
R_{th} &= 60\ \Omega\\
I_{sc} &= 0.3\ A
\end{aligned}
$$

(b)

Clearly, using the previous set of equations, V_{oc} is zero for $k=-0.02$ and for this value of k $R_{th}=240\ \Omega$.

Problem 37

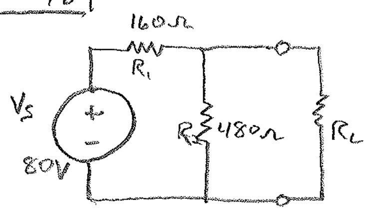
(a)

$$
\begin{aligned}
v_{AB} &= i_AR_{th}+V_{oc}\\
54 &= 0.01R_{th}+V_{oc}\\
66 &= 0.04R_{th}+V_{oc}\\
\Rightarrow R_{th} &= 400\ \Omega\\
V_{oc} &= 50\ V\\
I_{sc} &= 0.125\ A
\end{aligned}
$$

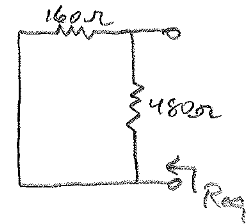
(b)

$R_L=400\ \Omega,\ P_{max}=V_{oc}^2/4R_L=2500/(4\times 400)=1.5625\ W.$

6-48



For R_{TH}

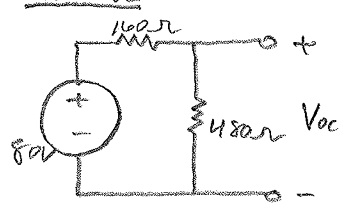


$$160\Omega \,//\, 480\Omega = 120\Omega = R_{TH}$$

For max. power,

$$\boxed{R_L = R_{TH} = 120\Omega}$$

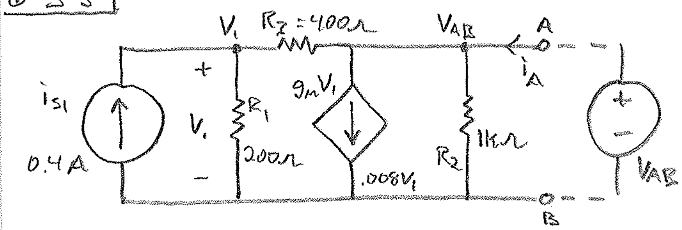
For V_{oc}



$$V_{oc} = 80V \cdot \frac{480\Omega}{(160+480)\Omega} = 60V$$

$$P_{MAX} = \frac{V_{oc}^2}{4R_{TH}} = \frac{(60V)^2}{480\Omega} = \boxed{7.5W}$$

6-53



$$V_{AB} = i_A \cdot R_{TH} + V_{oc}$$

Node eq. for V_1: $0.4A = \frac{V_1}{200\Omega} + \frac{V_1 - V_{AB}}{400\Omega} \Rightarrow \frac{160 + V_{AB}}{3} = V_1$

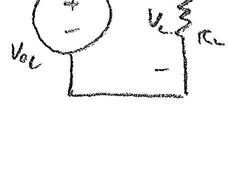
Node eq. for V_{AB}: $i_A = \frac{V_{AB} - V_1}{400\Omega} + 0.008V_1 + \frac{V_{AB}}{1k\Omega}$

$$2000 i_A = 5V_{AB} - 5V_1 + 16V_1 + 2V_{AB} = 7V_{AB} + 11V_1 = 7V_{AB} + 11\left(\frac{160 + V_{AB}}{3}\right)$$

$$6000 i_A = 21V_{AB} + 11V_{AB} + 1760 = 32V_{AB} + 1760$$

$$\Rightarrow V_{AB} = 187.5 i_A - 55 \Rightarrow \boxed{R_{TH} = 187.5\Omega = R_L \text{ for max power}}$$

$$V_{oc} = -55V$$



$$V_L = -55V \cdot \frac{187.5}{375} = \boxed{-27.5V}$$

$$P_L = \frac{V_{oc}^2}{4R_{TH}} = \frac{(-55V)^2}{4 \cdot 187.5\Omega} = \boxed{4.0\bar{3}\,W}$$

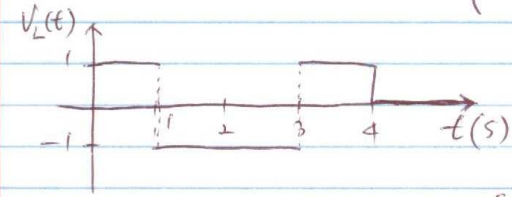
HW #15.

2.

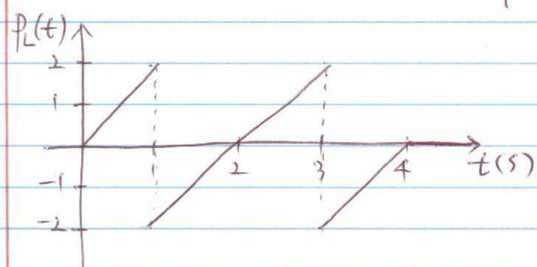
(a) $i_{in}(t) = \begin{cases} 2t & (0 \leq t \leq 1) \\ -2t+4 & (1 \leq t < 3) \\ 2t-8 & (3 \leq t < 4) \\ 0 & (t \geq 4) \end{cases}$

$V_L(t) = L \cdot \frac{d\,i_{in}(t)}{dt}$

$= 0.5 \times \frac{d\,i_{in}(t)}{dt} = \begin{cases} 1 & (0 \leq t < 1) \\ -1 & (1 \leq t < 3) \\ 1 & (3 \leq t < 4) \\ 0 & (t \geq 4) \end{cases}$



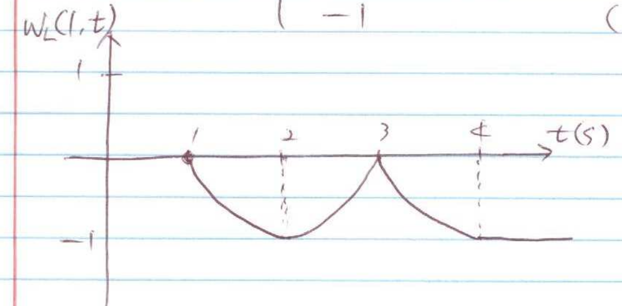
(b) $p_L(t) = V_L(t) \cdot i_{in}(t) = \begin{cases} 2t & (0 \leq t < 1) \\ 2t-4 & (1 \leq t < 3) \\ 2t-8 & (3 \leq t < 4) \\ 0 & (t \geq 4) \end{cases}$



(c) $W_L(t_0, t_1) = \int_{t_0}^{t_1} p_L(\tau)\, d\tau = \frac{1}{2} L \left[i_m^2(t_1) - i_m^2(t_0) \right]$

$\therefore W_L(1, t) = \frac{1}{2} \times 0.5 \times \left[i_m^2(t) - i_m^2(1) \right]$, $(t \geq 1)$

$$\therefore W_L(1,t) = \begin{cases} \text{not defined} & (t < 1) \\ t^2 - 4t + 3 & (1 \leq t < 3) \\ t^2 - 8t + 15 & (3 \leq t < 4) \\ -1 & (t \geq 4) \end{cases}$$



4. $i_s(t) = 10\sin(2000t) \times 10^{-3}$ (A)

(a) $V_{in}(t) = 0.2 \times 10^{-3} \times \frac{d\, i_s(t)}{dt}$

$= 0.2 \times 10^{-3} \times 10 \times 10^{-3} \times 2000 \times \cos(2000t)$

$= 4 \times 10^{-3} \times \cos(2000t)$ (V)

$i_{out}(t) = \frac{1}{2 \times 10^{-3}} \times \int_0^t 10 \cdot V_{in}(\tau)\, d\tau$

$= 10^{-2} \sin(2000t)$ (A) $= 10\sin(2000t)$ (mA)

(b) $P(t) = V \cdot I$

$= 10 V_{in}(t) \times i_{out}(t)$

$= 10 \times 4 \times 10^{-3} \cos(2000t) \times 10^{-2} \times \sin(2000t)$

$= 4 \times 10^{-4} \sin(2000t)\cos(2000t)$ (W)

$= 2 \times 10^{-4} \sin(4000t)$ (W)

$= 0.2 \times \sin(4000t)$ (mW)

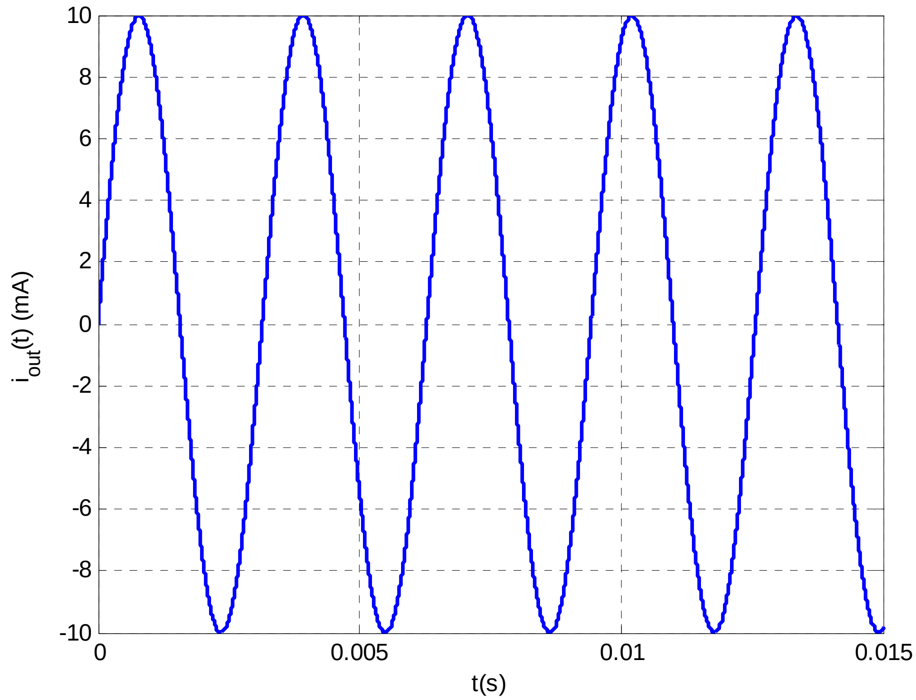
(c) $W_L(t) = \frac{1}{2} \times 2 \times 10^{-3} \times i_{out}^2(t)$

$= 10^{-3} \times 10^{-4} \sin^2(2000t)$ (J)

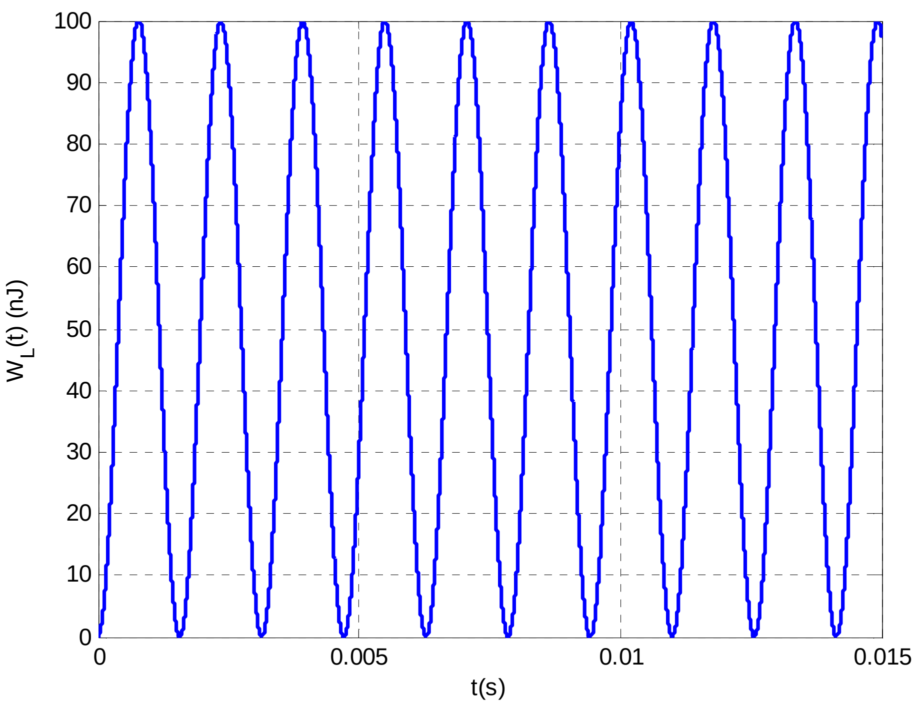
$= 10^{-7} \times \sin^2(2000t)$ (J)

$= 100 \sin^2(2000t)$ (nJ)

P 4 (a)


10
8
6
4
2
0
-2
-4
-6
-8
-10
$i_{out}(t)$ (mA)
0
0.005
0.01
0.015
t(s)

P 4 (c)


100
90
80
70
60
50
40
30
20
10
0
$W_L(t)$ (nJ)
0
0.005
0.01
0.015
t(s)

7-12 (a) $i_c(t) = C\frac{dV_c(t)}{dt} = 2\times10^{-6}\cdot\frac{d}{dt}\left[100\left(1+\cos(1000\pi t)\right)\right]$

$$= 2\times10^{-6}\cdot 100\times 1000\pi\times\left(-\sin(1000\pi t)\right) = \underline{-0.2\pi\sin(1000\pi t)\ A}$$

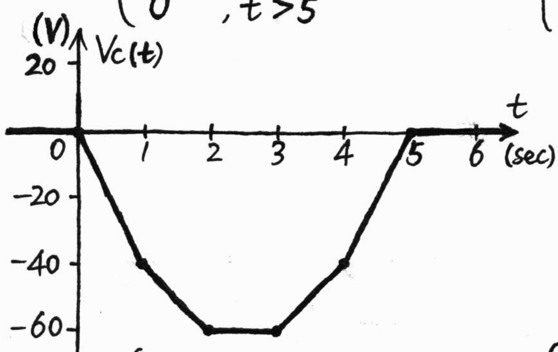
(b) $C = \dfrac{i_c(t)}{\frac{dV_c(t)}{dt}} = \dfrac{10^{-3}\times10\cos(2000t)}{\frac{d}{dt}[10\sin(2000t)]} = \dfrac{10^{-2}\cos(2000t)}{2\times10^{4}\cos(2000t)} = \underline{0.5\times10^{-6}\ F}$

7-16 $V_c(t) = \frac{1}{C}\int_{t_0}^{t} i_c(\tau)\,d\tau + V_c(t_0) = \overset{(\frac{1}{mF})}{4}\int_{t_0}^{t} \overset{(mA)}{i_c(\tau)}\,d\tau + V_c(t_0)$ (V)

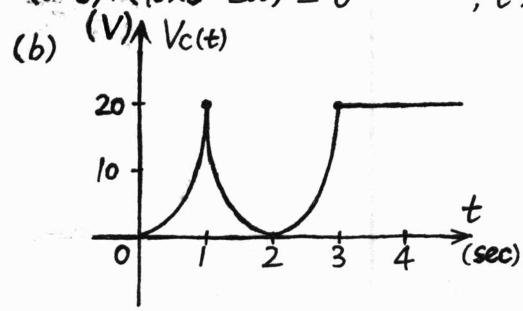
(a)

$$\underset{(mA)}{i_{in}(t)} = \begin{cases} 0 & ,t\le0 \\ -10 & ,0<t\le1 \\ -5 & ,1<t\le2 \\ 0 & ,2<t\le3 \\ 5 & ,3<t\le4 \\ 10 & ,4<t\le5 \\ 0 & ,t>5 \end{cases} \qquad \underset{(V)}{V_c(t)} = \begin{cases} 4\times0 = 0 & ,t\le0 \\ 4\times(-10)(t-0)+(0) = -40t & ,0<t\le1 \\ 4\times(-5)(t-1)+(-40\times1) = -20t-20 & ,1<t\le2 \\ 4\times0\times(t-2)+(-20\times2-20) = -60 & ,2<t\le3 \\ 4\times5(t-3)+(-60) = 20t-120 & ,3<t\le4 \\ 4\times10(t-4)+(20\times4-120) = 40t-200 & ,4<t\le5 \\ 4\times0\times(t-5)+(40\times5-200) = 0 & ,t>5 \end{cases}$$

(a)



(b)



(b)

$$\underset{(mA)}{i_{in}(t)} = \begin{cases} 0 & ,t\le0 \\ 10t & ,0<t\le1 \\ 10t-20 & ,1<t\le3 \\ 0 & ,t>3 \end{cases} \qquad \underset{(V)}{V_c(t)} = \begin{cases} 4\times0 = 0 & ,t\le0 \\ 4\times[5\tau^2]_0^t+(0) = 20t^2 & ,0<t\le1 \\ 4\times[5\tau^2-20\tau]_1^t+(20\times1^2) = 20(t-2)^2 & ,1<t\le3 \\ 4\times0+(20\times(3-2)^2) = 20 & ,t>3 \end{cases}$$

7-17 (a) $V_{C1}(t) = V_{C1}(0) + \frac{1}{C_1}\int_0^t i_{in}(t)\,dt = V_s(t) \Rightarrow \frac{dV_s(t)}{dt} = \frac{1}{C_1}i_{in}(t) \Rightarrow i_{in}(t) = C_1\frac{dV_s(t)}{dt}$

$V_{C2}(t) = V_{out}(t) = V_{out}(0) + \frac{1}{C_2}\int_0^t 5i_{in}(t)\,dt = V_{out}(0) + \frac{5C_1}{C_2}V_s(t) \leftarrow$ (Yes) Independent of $V_{C1}(0)$.

(b) $i_{in}(t) = C_1\frac{dV_s(t)}{dt} = 20\times10^{-6}\cdot5\times2000\times\cos(2000t) = \underline{0.2\cos(2000t)\ A}$

$V_{out}(t) = V_{out}(0) + \frac{1}{C_2}\int_0^t 5i_{in}(t)\,dt = V_{out}(0) + \frac{5C_1}{C_2}V_s(t)$

$$= 10 + \frac{5\times20\times10^{-6}}{0.1\times10^{-3}}\times5\sin(2000t) = \underline{10+5\sin(2000t)\ V}$$

$P(t) = 5i_{in}(t)V_{out}(t) = 5\times0.2\cos(2000t)\times\left(10+5\sin(2000t)\right)$

$$= 10\cos(2000t) + 5\sin(2000t)\cos(2000t) = \underline{10\cos(2000t)+2.5\sin(4000t)\ W}$$

(c) $W(0,t) = \frac{1}{2}C_2V_{out}^2(t) - \frac{1}{2}C_2V_{out}^2(0) = \frac{1}{2}\times0.1\times10^{-3}\left[\left(10+5\sin(2000t)\right)^2-10^2\right]$

$$= 0.05\times10^{-3}\times\left[100\sin(2000t)+25\sin^2(2000t)\right] = \underline{5\sin(2000t)+1.25\sin^2(2000t)\ mJ}$$

ECE 201 Spring 2010

Homework 17 Solutions

Problem 27

(a)

Let V be the common voltage across C_1 and C_2. Thus

$$\begin{aligned}(C_1 + C_2)\frac{dV}{dt} &= i_s(t) \\ C_2\frac{dV}{dt} &= i_{C2}(t) \\ \Rightarrow i_{C2}(t) &= \frac{C_2}{C_1 + C_2} i_s(t)\end{aligned}$$

(b)

Using KVL across the second loop and relation from part (a), we get

$$\begin{aligned}r_m i_{C2}(t) - (L_1 + L_2)\frac{di}{dt} &= 0 \\ v_{out}(t) &= L_2\frac{di}{dt} \\ &= r_m \frac{L_2 C_2}{(L_1 + L_2)(C_1 + C_2)} i_s(t)\end{aligned}$$

Problem 38

(a)

Starting from the right end and combining the inductors in series and parallel consecutively, we get L_{eq} as

$$\begin{aligned}L_{eq} &= 4 + ((36)||(10 + ((1 + 5)||(3)))) \\ &= 4 + ((36)||(10 + ((6)||(3))))\end{aligned}$$

$$
\begin{aligned}
&= 4 + ((36)||(10 + (2))) \\
&= 4 + ((36)||(12)) \\
&= 4 + 9 \\
&= 13\ mH
\end{aligned}
$$

(b)

Clearly, the given circuit has 1.2 mH and 0.6 mH in parallel, and this combination is in series with 2.4 mH, the equivalent of which is in parallel with 7 mH. Thus

$$
\begin{aligned}
L_{eq} &= 7||(2.4 + (1.2||0.6)) \\
&= 7||(2.4 + 0.4) \\
&= 7||2.8 \\
&= 2\ mH
\end{aligned}
$$

Problem 41

(a)

Remember that the equivalent capacitance expressions for series and parallel connections are opposite of that used for resistances. The equivalent capacitance can be written as

$$
\begin{aligned}
C_{eq} &= C_1||(C_2 + (C_3||C_4)) \\
&= 4 + \frac{6 \times 3}{9} \\
&= 6\ \mu F \\
v_s(t) &= \frac{1}{C_{eq}} \int_{-\infty}^{t} i_s(\tau)\, d\tau \\
&= \frac{1}{6} \sin(10^4 t)\ V
\end{aligned}
$$

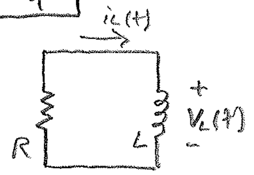
(b)

$$
C_{eq} = C_1||(C_2 + (C_3||C_4) + C_5)
$$

$$
\begin{aligned}
(C_2 + (C_3||C_4) + C_5) &= \left(\frac{1}{18} + \frac{1}{54} + \frac{1}{10.8}\right)^{-1} \\
&= 6\ \mu F \\
\Rightarrow C_{eq} &= 6 + 60 \\
&= 66\ \mu F \\
v_s(t) &= \frac{1}{C_{eq}} \int_{-\infty}^{t} i_s(\tau)\, d\tau \\
&= \frac{1}{660}(1 - \cos(10^5 t))\ V
\end{aligned}
$$

ECE 201 HW #18 Solution

8-4



$R = 2.5k\Omega \qquad i_L(0) = 20mA$

$i_L(1ms) = 2.7067\ mA = 20mA\ e^{-\frac{2.5k\Omega}{L}\cdot 1ms}$

$$\frac{2.7067}{20} = e^{-\frac{2.5}{L}}$$

A) $$-\ln\left(\frac{2.7067}{20}\right) = \frac{2.5}{L}$$

$$L = \frac{2.5}{-\ln\frac{2.7067}{20}} \approx \boxed{1.25\ H}$$

b) $i_L(t) = .02\ e^{-\frac{2500}{1.25}t} = \boxed{20\ e^{-2000t}\ mA}$

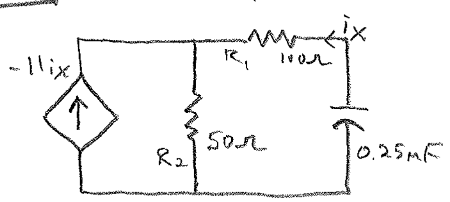
8-5

$V_c(t) = V_c(t_0)\ e^{-\frac{(t-t_0)}{\tau}} \Rightarrow \frac{V_c(t)}{V_c(t_0)} = e^{-(t-t_0)/\tau} \Rightarrow -\ln\left(\frac{V_c(t)}{V_c(t_0)}\right) = \frac{t-t_0}{\tau}$

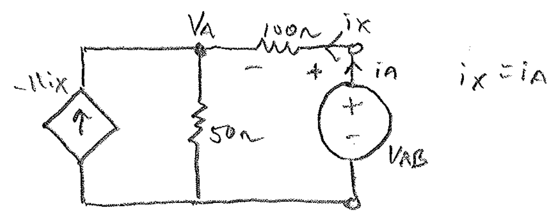
$$\Rightarrow \tau = \frac{t-t_0}{-\ln\left(\frac{V_c(t)}{V_c(t_0)}\right)} = \frac{2s - 0s}{-\ln\left(\frac{14.715\ V}{40\ V}\right)} \approx \boxed{2s}$$

$$\tau = RC \Rightarrow R = \frac{\tau}{C} = \frac{2s}{0.25mF} = \boxed{8k\Omega}$$

8-9 b) $\alpha = -11$



Th $\Rightarrow$



$i_x = i_A$

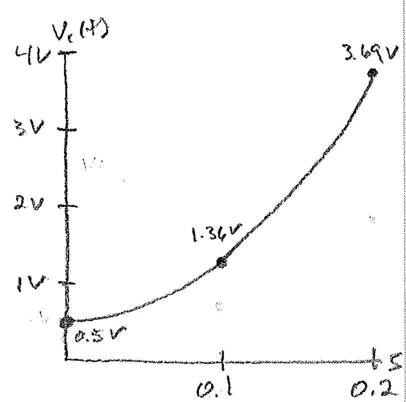
$$\frac{V_{AB} - V_A}{100\Omega} = i_A$$

$-11i_A + i_A = \frac{V_A}{50\Omega} = \frac{V_{AB} - 100i_A}{50} = -10i_A \Rightarrow V_{AB} = -400i_A$

So $R_{TH} = \boxed{-400\Omega}$

$$\boxed{V_c(t) = \frac{1}{2}e^{10t}}$$

$\tau = -400\Omega \cdot 0.25mF = -0.1s$



HW # 19.

8-18 (a)

$$V_c(t) = [\text{Final value}] + ([\text{Initial value}] - [\text{Final value}]) e^{-\frac{\text{elapsed time}}{\tau}}$$

Initial value : 0 V ($V_c(0) = 0V$)

Final value : 20V ($V_c(\infty) = 20V$)

$$\tau = RC = 10\times10^3 \times 0.4\times10^{-3} = 4 (s)$$

$$\therefore V_c(t) = 20 - 20e^{-\frac{t}{4}} = \underline{20(1-e^{-\frac{t}{4}})} \ (V)$$

(b)

Initial value : 10V ($V_c(0) = 10V$)

Final value : 0V ($V_c(\infty) = 0$)

$$\therefore V_c(t) = \underline{10e^{-\frac{t}{4}} \ (V)}$$

8-19. (a)

$$i_L(t) = [\text{Final value}] + ([\text{Initial value}] - [\text{Final value}]) e^{-\frac{\text{elapsed time}}{\tau}}$$

$$\tau = \frac{L}{R} = \frac{0.2}{100} = 2\times10^{-3}(s) = 2 (ms)$$

Initial value : 0 A ($i_L(0) = 0$)

Final value : $\frac{20V}{100\Omega} = 0.2(A)$ ($i_L(\infty) = \frac{20V}{R}$)

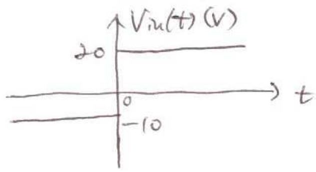
$$\therefore i_L(t) = 0.2 - 0.2e^{-\frac{t}{2\times10^{-3}}} = \underline{0.2(1-e^{-500t}) \ (A)}$$

(b) Initial Value : $-50mA = -0.05A$ $\left(i_L(0) = -50mA\right)$

Final Value : $0A$ $\left(i_L(\infty) = \frac{0V}{R} = 0\right)$

$\therefore \underline{i_L(t) = -0.05 \times e^{-500t} \ (A)}$ //

8-20. (a) $V_{in}(t) = -10 \cdot u(-t) + 20 \cdot u(t) \ (V)$



At $t = 0^-$, there is no current through C.

$\therefore V_c(0^-) = V_{in}(0^-) \times \frac{R_2}{R_1 + R_2} = -10 \times \frac{200}{200+50} = -8(V)$

By continuity property, $\underline{V_c(0^+) = V_c(0^-) = \boxed{-8V}}$ //

At $t = +\infty$, there is no current through C.

$\therefore V_C(\infty) = V_{in}(\infty) \times \frac{R_2}{R_1 + R_2} = 20 \times \frac{200}{200+50} = 16(V)$

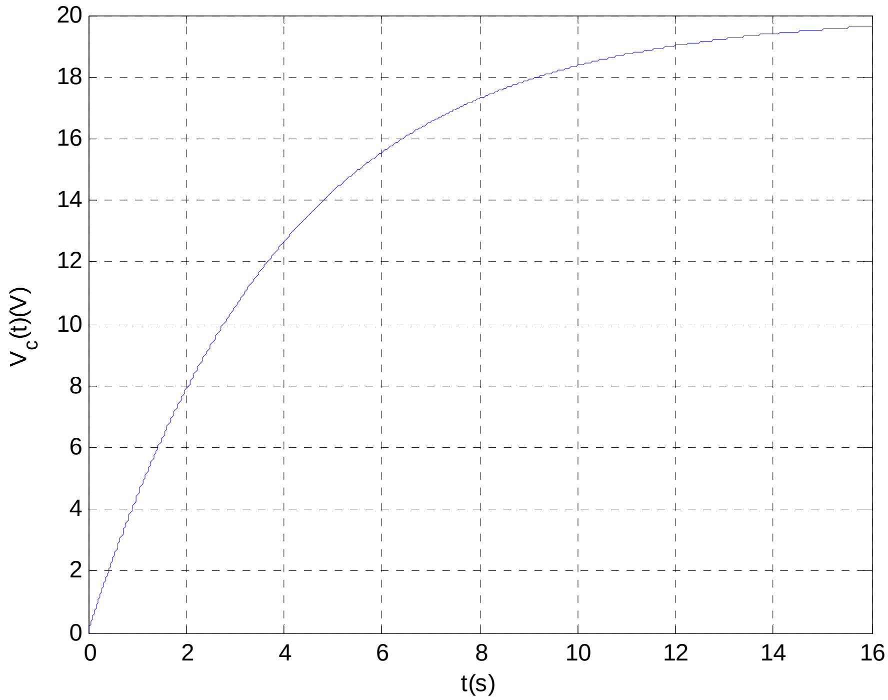
$\underline{V_c(t) = V_c(\infty) + \left[V_c(0^+) - V_c(\infty)\right] e^{-\frac{t}{\tau}}}$ $(t > 0)$

$\tau = (R_1 // R_2) \times C = (50 // 200) \times 2.5 \times 10^{-3} = 0.1(s)$

$\therefore V_c(t) = 16 + (-8 - 16) e^{-10t}$

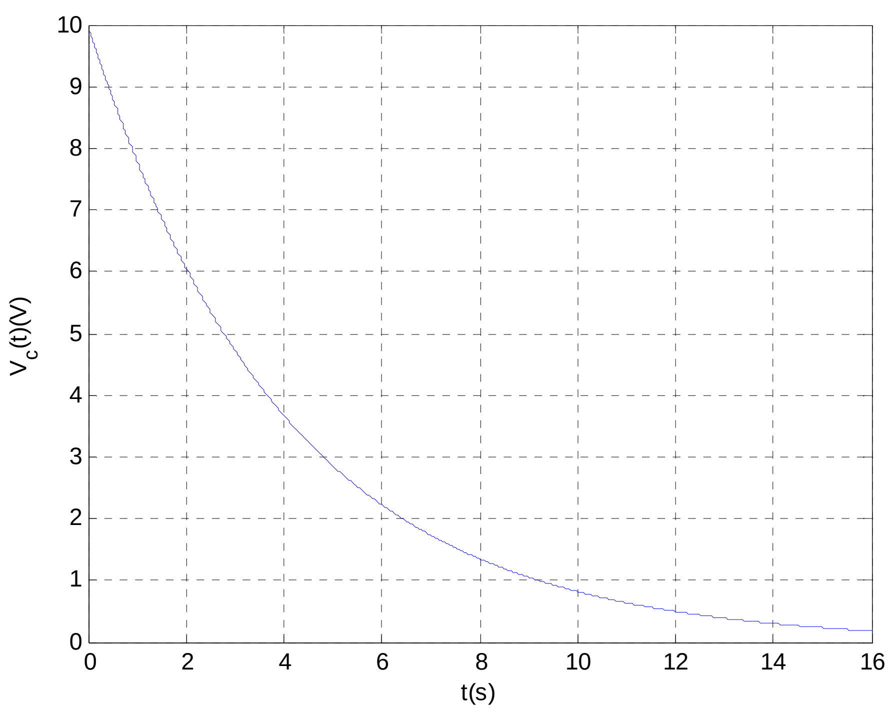
$\underline{= 16 - 24e^{-10t} \ (t > 0)}$ //

8-18(a)



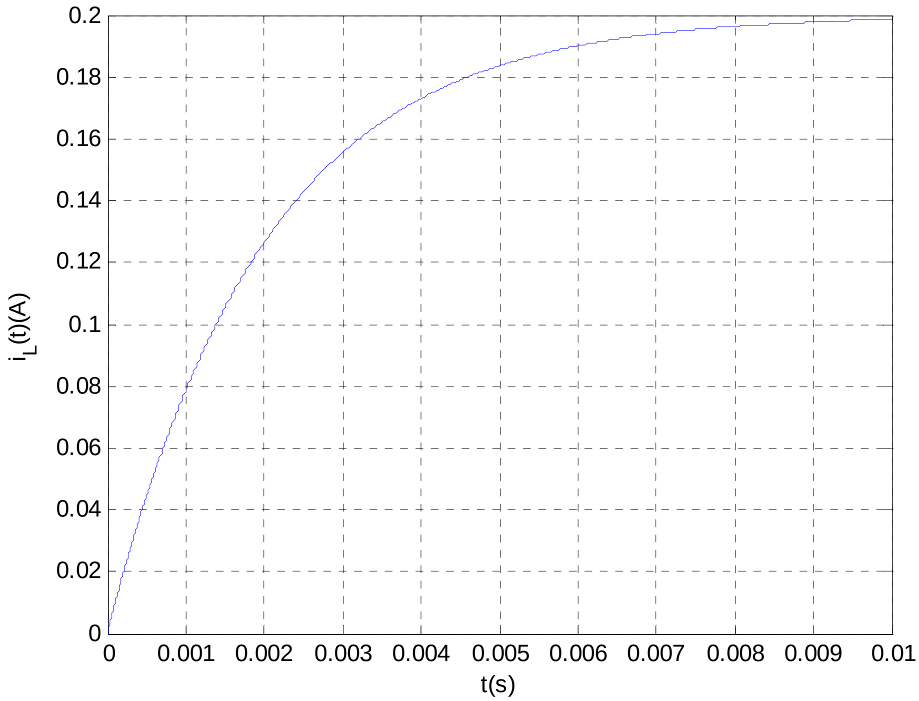
20
18
16
14
12
10
8
6
4
2
0
0
2
4
6
8
10
12
14
16
t(s)
Vc(t)(V)

8-18(b)

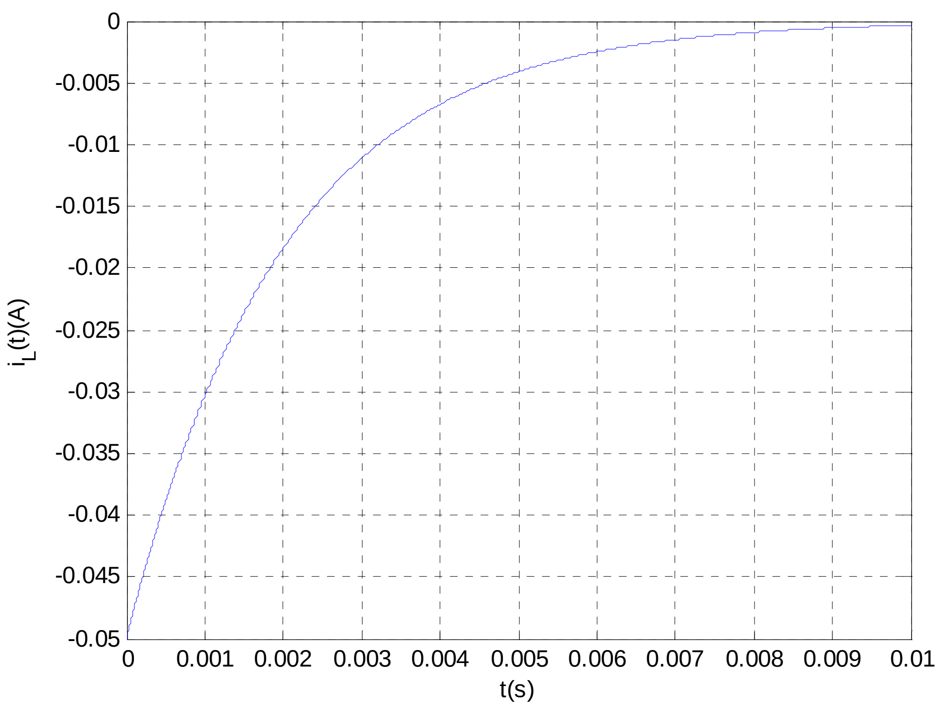


10
9
8
7
6
5
4
3
2
1
0
0
2
4
6
8
10
12
14
16
t(s)
Vc(t)(V)

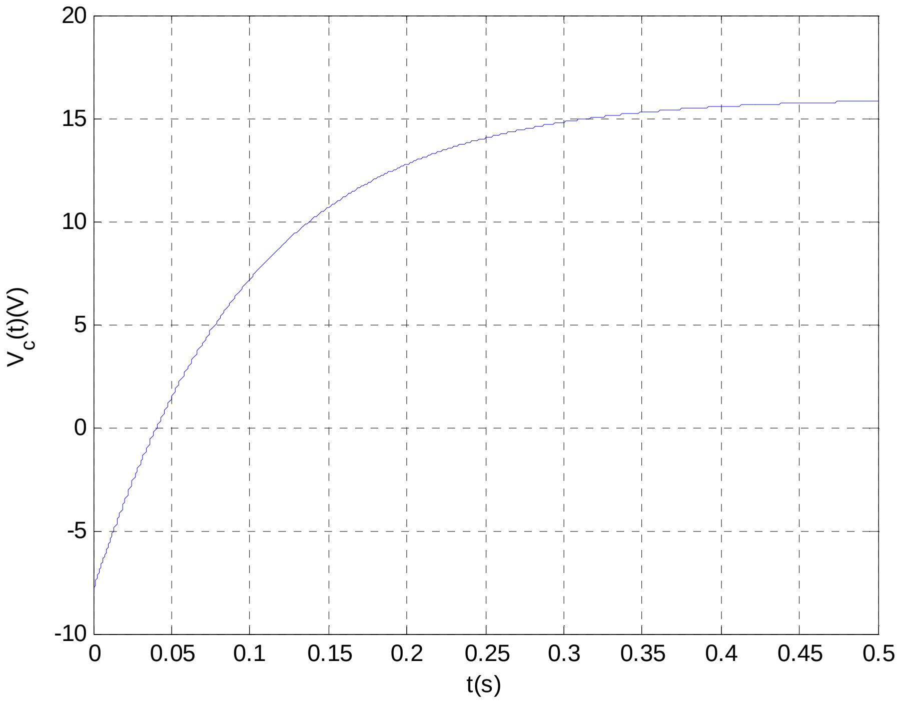
8-19(a)


0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
$i_L(t)$(A)
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01
t(s)

8-19(b)


0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
-0.035
-0.04
-0.045
-0.05
$i_L(t)$(A)
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01
t(s)

8-20(b)



8–18 $V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]\, e^{-\frac{t}{\tau}}$, $t \geq 0$. $V_C(\infty)$ open voltage

(c) $V_C(0^+) = 10V$, $V_C(\infty) = V_{in}(t) = 20V$, $\tau = R_{th}C = RC = 4\,s$

$V_C(t) = 20 + [10 - 20]\, e^{-\frac{t}{4}} = \underline{20 - 10 e^{-\frac{t}{4}}\ (V),\ t \geq 0}$

(d) $V_C(0) = -20V$, $V_C(\infty) = V_{in}(t) = -10V$, $\tau = R_{th}C = RC = 4\,s$

$V_C(t) = -10 + [-20 - (-10)]\, e^{-\frac{t}{4}} = \underline{-10 - 10 e^{-\frac{t}{4}} (V),\ t \geq 0}$

(e) Ohm's Law: $V_R(t) = I_C(t) R$, KVL: $V_{in}(t) = V_R(t) + \underbrace{V_C(t)}_{(c)}$

$I_C(t) = \frac{V_R(t)}{R} = \frac{V_{in}(t) - V_C(t)}{R} = \frac{1}{10k}\left[20 - (20 - 10 e^{-\frac{t}{4}})\right]$

$= \underline{1\times 10^{-3} e^{-\frac{t}{4}}\ (A),\ t \geq 0}$

8–19 $I_L(t) = I_L(\infty) + [I_L(0^+) - I_L(\infty)]\, e^{-\frac{t}{\tau}}$, $t \geq 0$. $I_L(\infty)$ short current

(c) $I_L(0^+) = -0.05\,A$, $I_L(\infty) = \frac{V_{in}(\infty) - \cancel{V_L(\infty)}^{\,0}}{R} = 0.2\,A$, $\tau = \frac{L}{R_{th}} = \frac{L}{R} = 2\times 10^{-3}\,s$

$I_L(t) = 0.2 + [-0.05 - 0.2]\, e^{-\frac{t}{2\times 10^{-3}}} = \underline{0.2 - 0.25 e^{-500t} (A),\ t \geq 0}$

(d) $I_L(0) = 0.025\,A$, $I_L(\infty) = \frac{V_{in}(\infty) - \cancel{V_L(\infty)}^{\,0}}{R} = -0.1\,A$, $\tau = \frac{L}{R_{th}} = \frac{L}{R} = 2\times 10^{-3}\,s$

$I_L(t) = -0.1 + [0.025 - (-0.1)]\, e^{-\frac{t}{2\times 10^{-3}}} = \underline{-0.1 + 0.125 e^{-500t} (A),\ t \geq 0}$

(e) KVL: $V_{in}(t) = V_R(t) + V_L(t)$, Ohm's Law: $V_R(t) = \underbrace{I_L(t)}_{(c)} R$

$V_L(t) = V_{in}(t) - I_L(t) R = 20 - (0.2 - 0.25 e^{-500t})\, 100$

$= \underline{25 e^{-500t}\ (V),\ t \geq 0}$

$\tau = R_{th}C = (R_1 \| R_2)C = 0.1\,s$

$V_C(\infty) = \frac{R_2}{R_1 + R_2} V_{in}(\infty) = 16V$

8–20 $V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]\, e^{-\frac{t}{\tau}}$, $t \geq 0$. $V_C(0^+) = V_C(0^-) = -8V$

(c) zero-input response: $V_C(\infty) = V_{in}(t) = 0$, no source input.

$V_{C1}(t) = V_C(t)\Big|_{V_C(\infty)=0} = V_C(0^+)\, e^{-\frac{t}{\tau}} = \underline{-8 e^{-10t}\ (V),\ t \geq 0}$

zero-state response: $V_C(0^+) = 0$, no initial condition.

$V_{C2}(t) = V_C(t)\Big|_{V_C(0^+)=0} = V_C(\infty) - V_C(\infty)\, e^{-\frac{t}{\tau}} = \underline{16 - 16 e^{-10t} (V),\ t \geq 0}$

Note: $V_C(t)$ is complete response: $V_C(t) = V_{C1}(t) + V_{C2}(t)$

(d) For $0 < t \leq 0.25$, switch is closed, diagram is the same as (c).

$V_C^I(t) = V_C^I(\infty) + [V_C^I(0^+) - V_C^I(\infty)]\, e^{-\frac{t}{\tau_I}} = 16 - 24 e^{-10t} (V)$, $0 \leq t \leq 0.25$

$$I_c^{I}(t) = C\frac{dV_c^{I}(t)}{dt} = 2.5\times10^{-3}\left(240e^{-10t}\right) = \underline{0.6e^{-10t}(A),\ 0<t\le 0.25}$$

For $0.25 < t \le 0.5$ (or ∞), switch is open, diagram is changed.

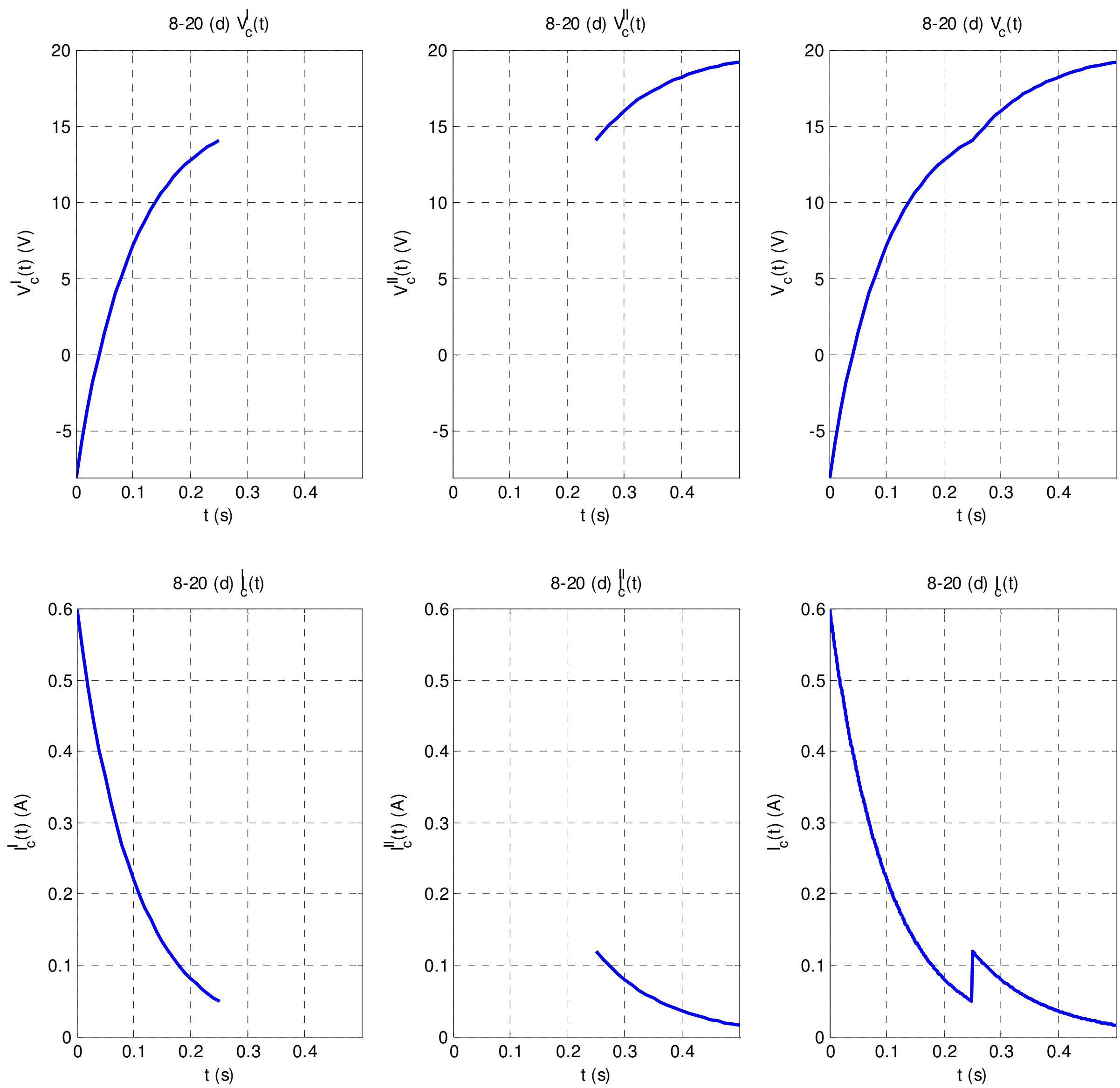
$$\tau_{II} = R_{th}^{II}C = R_1C = 50\left(2.5\times10^{-3}\right) = 0.125\ s,\quad V_c(\infty) = V_{in}(\infty) = 20V$$

Initial condition: $V_c^{II}(0.25^+) = V_c^{I}(0.25^-) = V_c^{I}(t)\big|_{t=0.25} = 16-24e^{-2.5} = 14.03\ V$

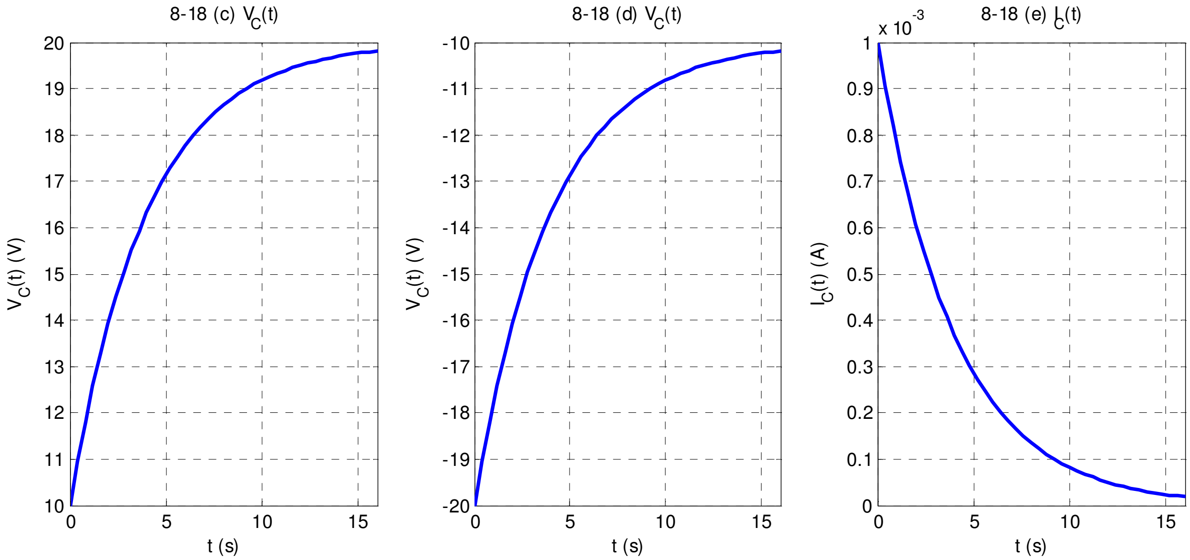
$$V_c^{II}(t) = V_c^{II}(\infty) + \left[V_c^{II}(0.25^+) - V_c^{II}(\infty)\right]e^{-\frac{t-0.25}{\tau_{II}}} = \underline{20-5.97e^{-8(t-0.25)}(V)}$$

$$I_c^{II}(t) = C\frac{dV_c^{II}(t)}{dt} = 2.5\times10^{-3}\left(47.76e^{-8(t-0.25)}\right) = \underline{0.1194e^{-8(t-0.25)}(A),\ 0.25<t\le 0.5\ (\text{or}\ \infty)}$$

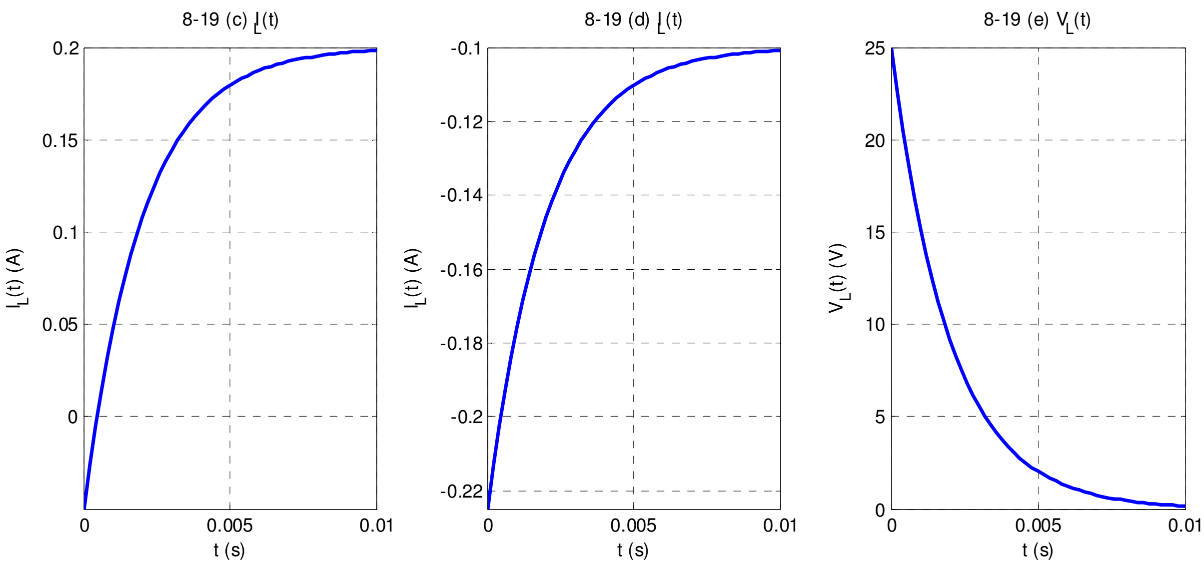
$$\Rightarrow V_c(t) = \begin{cases} 16-24e^{-10t}(V), & 0\le t\le 0.25 \\ 20-5.97e^{-8(t-0.25)}(V), & 0.25<t\le 0.5\ (\text{or}\ \infty)\end{cases} \qquad I_c(t) = \begin{cases} 0.6e^{-10t}(A), & 0\le t\le 0.25 \\ 0.1194e^{-8(t-0.25)}(A), & 0.25<t\le 0.5\ (\text{or}\ \infty)\end{cases}$$



8-18


8-18 (c) V_C(t)
V_C(t) (V)
t (s)
8-18 (d) V_C(t)
V_C(t) (V)
t (s)
8-18 (e) I_C(t)
x 10^-3
I_C(t) (A)
t (s)

8-19


8-19 (c) I_L(t)
I_L(t) (A)
t (s)
8-19 (d) I_L(t)
I_L(t) (A)
t (s)
8-19 (e) V_L(t)
V_L(t) (V)
t (s)

ECE 201 Spring 2010

Homework 21 Solutions

Problem 31

(a)

Using voltage division,

$$\begin{aligned} v_C(0-) &= \frac{20R}{50R}V_0 \\ &= 0.4V_0 \\ &= v_C(0+) \end{aligned}$$

(b)

The Thevenin equivalent is given by

$$\begin{aligned} V_{oc} &= V_0\frac{R_3}{R_3+(R_1||R_2)} \\ &= 0.8V_0 \\ R_{th} &= R_1||R_2||R_3 \\ &= 4R \end{aligned}$$

(c)

$$\begin{aligned} v_C(\infty) &= 0.8V_0 \\ v_C(0+) &= 0.4V_0 \\ \Rightarrow v_C(t) &= v_C(\infty)+[v_C(t_0+)-v_C(\infty)]e^{-\frac{t-t_0}{R_{th}C}} \\ &= 0.8V_0 - 0.4V_0e^{-\frac{t}{4RC}} \end{aligned}$$

(d)

$$v_C(T-) = v_C(T+) = 0.8V_0 - 0.4V_0 e^{-1.5}$$
$$= 0.71075V_0$$

(e)

$$\tau = R_{th}C$$
$$= 4RC$$

(f)

Using the same relation as in part (c), with $t_0 = T$,

$$v_C(t) = 0.71075V_0 e^{-\frac{t-T}{4RC}}$$

(g)

For simplicity, let all quantities be normalized to 1. Then the plot for $v_C(t)$ would look as shown on the next page.

Problem 33

(a)

$$i_L(0+) = i_L(0-) = \frac{-10}{80+20}$$
$$= -0.1\ A$$

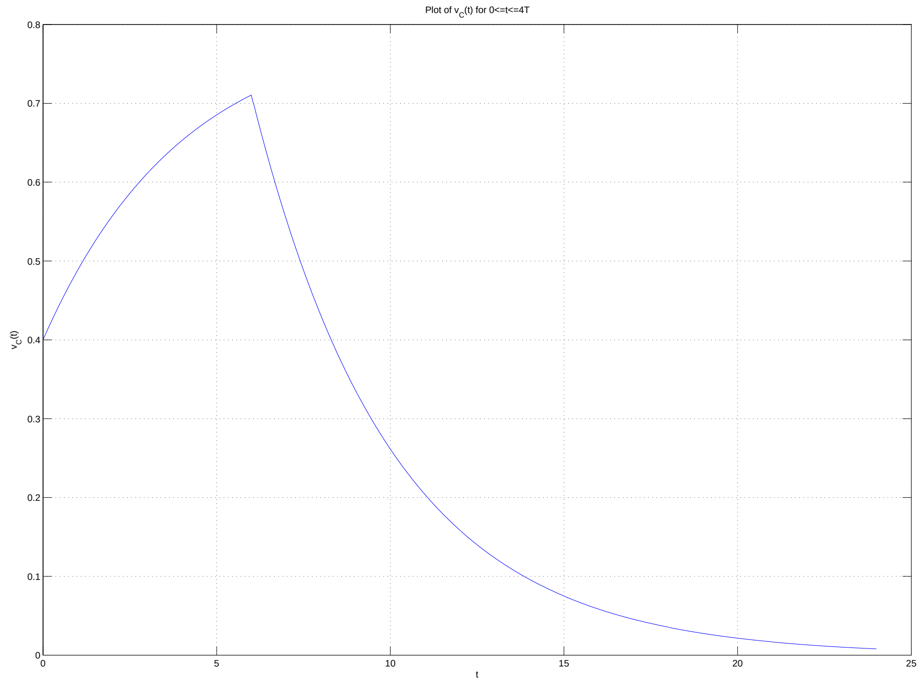


Figure 1: Plot of $v_C(t)$ for problem 31

(b)

Here we have to consider two intervals, $0 \leq t < 80\ ms$ and $80\ ms \leq t < 160\ ms$. For $0 \leq t < 80\ ms$,

$$\begin{aligned} i_L(t) &= i_L(\infty) + [i_L(0+) - i_L(\infty)]e^{-tR/L} \\ &= \frac{20}{100} + [-0.1 - 0.2]e^{-25t} \\ &= 0.2 - 0.3e^{-25t} \\ i_L(80-) = i_L(80+) &= 0.2 - 0.3e^{-2} \\ &= 0.1594\ A \end{aligned}$$

Again, for the interval $80\ ms \leq t < 160\ ms$,

$$\begin{aligned} i_L(t) &= i_L(\infty) + [i_L(80+) - i_L(\infty)]e^{-(t-80)R/L} \\ &= -0.1 + [0.1594 + 0.1]e^{-25(t-80)} \\ &= -0.1 + 0.2594e^{-25(t-80)} \end{aligned}$$

We can write $v_{out}(t)$ as the following,

$$v_{out}(t) = v_{in}(t) - R_1 i_L(t)$$

$$= \quad 4 + 24e^{-25t}, \; 0 \leq t < 80 \; ms$$
$$= \quad -2 - 20.752e^{-25(t-80ms)}, \; 80 \; ms \leq t < 160 \; ms$$

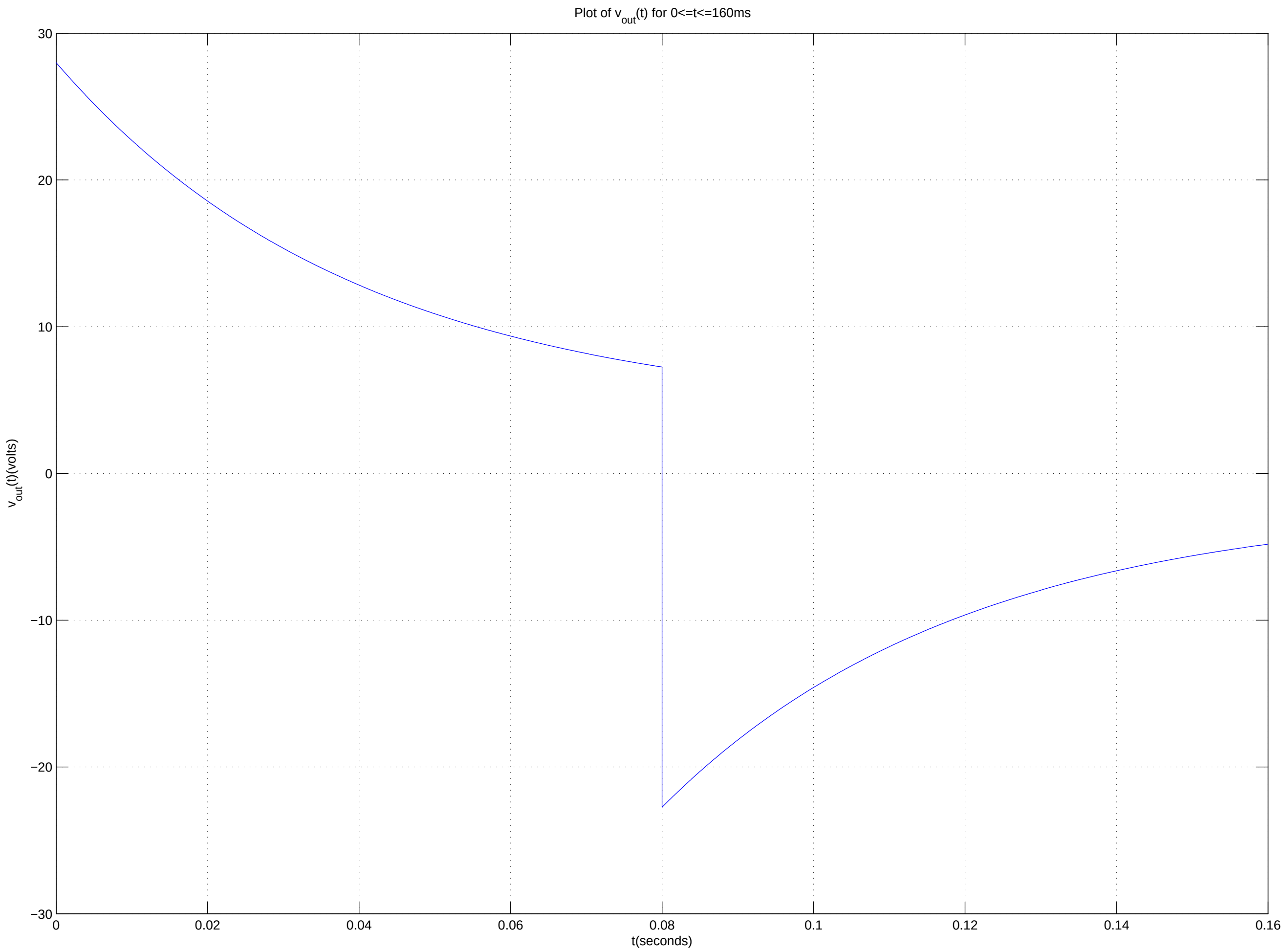


Figure 2: Plot of $v_{out}(t)$ for problem 33

ECE 201 HW 22 Solution

9.1

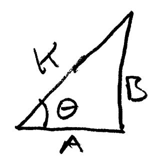
Let $A = K\cos\Theta$ and $B = K\sin\Theta$. Then

$A\cos(wt) + B\sin(wt) = K\cos\Theta\cos(wt) + K\sin\Theta\sin(wt)$

$= K\cos(wt)\cos\Theta + K\sin(wt)\sin\Theta$ by commutativity of multiplication

$= K\cos(wt-\Theta)$ by the cosine angle sum formula

$= K\cos(wt+\Theta)$ by a simple relabelling of Θ.



Letting $K = \sqrt{A^2+B^2}$ and $\Theta = \tan^{-1}(-B/A)$, we get

$K\cos(wt+\Theta) = \sqrt{A^2+B^2}\cos(wt+\tan^{-1}(-B/A))$

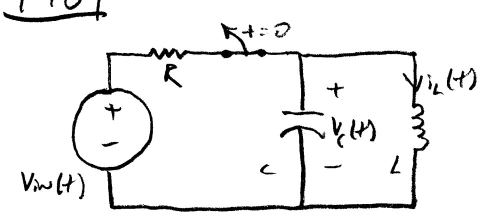
$= \sqrt{A^2+B^2}\left[\cos(wt)\cos(\tan^{-1}(B/A)) + \sin(wt)\sin(\tan^{-1}(B/A))\right]$

$= \sqrt{A^2+B^2}\left[\cos(wt)\cdot\frac{A}{\sqrt{A^2+B^2}} + \sin(wt)\cdot\frac{B}{\sqrt{A^2+B^2}}\right]$ (also: $\cos(-\Theta) = \cos(\Theta)$, $\sin(-\Theta) = -\sin(\Theta)$)

$= A\cos(wt) + B\sin(wt)$ □

9-10

$V_{in}(t) = 10\,V$, $R = 10\,\Omega$, $C = 0.4\,mF$, $L = \frac{1}{4}\,H$



A) $v_c(0^-) = v_c(0^+) = \boxed{0V}$ ∵ $V_L(0^-) = 0V$ since it goes short

$i_L(0^-) = i_L(0^+) = \frac{V_{in}(t)}{R} = \frac{10V}{10\Omega} = \boxed{1A}$

b) $W_c(0) = \frac{1}{2}CV_c^2(0) = \frac{1}{2}(0.4\,mF)(0V)^2 = \boxed{0\,J}$

$W_L(0) = \frac{1}{2}Li_L^2(0) = \frac{1}{2}(\frac{1}{4}H)(1A)^2 = \boxed{0.125\,J}$

c) $W_c = \frac{1}{2}CV_c^2 \Rightarrow V_c = \sqrt{2W_c/C} = \sqrt{2\cdot 0.125J/0.4\,mF} = \boxed{25V}$

d) $V_c(t) = K\cos(wt+\Theta)$ $w = 1/\sqrt{LC} = 100$

$V_c(0^+) = 0 = K\cos\Theta \Rightarrow \Theta = 90°$ ∴ $\boxed{V_c(t) = 25\cos(100t + \pi/2)\,V}$

$V_c'(0^+) = \frac{i_c(0^+)}{C} = \frac{-i_L(0^+)}{C} = \frac{-1A}{0.4mF} = -2500 = -100K\sin(90°) \Rightarrow K = 25$

HW #23.

9-16(b) $\dfrac{d^2 V_c}{dt^2} + \dfrac{1}{RC}\dfrac{dV_c}{dt} + \dfrac{1}{LC}V_c = 0$ (parallel RLC)

$b' = \dfrac{1}{RC} = \dfrac{1}{0.4 \times 0.5} = 5, \quad c' = \dfrac{1}{LC} = \dfrac{1}{0.5 \times 0.5} = 4$

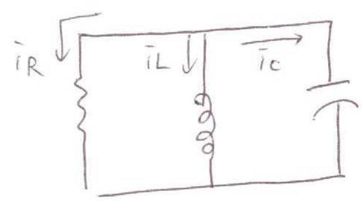
characteristic equation $\Rightarrow s^2 + 5s + 4 = (s+1)(s+4) = 0$

$\therefore s_1 = -1, \quad s_2 = -4$

$b'^2 - 4c' = 5^2 - 16 = 9 > 0$

$\therefore V_c(t) = K_1 e^{-t} + K_2 e^{-4t}$

$V_c(0) = K_1 + K_2 = 2(V) \quad \cdots ①$



$i_R + i_L + i_c = 0$

$\therefore i_c(0) = -i_R(0) - i_L(0)$

$= -\dfrac{V_c(0)}{R} - 2.5 = -\dfrac{2}{0.4} - 2.5$

$= -7.5(A)$

$i_c(t) = C \cdot \dfrac{dV_c(t)}{dt}$

$\therefore i_c(0) = 0.5 \times (-K_1 - 4K_2) = -7.5 \quad \cdots ②$

From ① and ②, $K_1 = -\dfrac{7}{3}, \quad K_2 = \dfrac{13}{3}$

$\therefore V_c(t) = -\dfrac{7}{3}e^{-t} + \dfrac{13}{3}e^{-4t}$

9-18. $i_{in}(t) = 0.5u(-t)\ (A)$

(a) $V_c(0^-) = 0.5 \times R = 0.5 \times 20 = 10(V)$

$V_c(0^+) = V_c(0^-)$ (by continuity property)

$i_L(0^-) = i_L(0^+) = 0\ (A)$

$$\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC} = V_c = 0 \quad (\text{when } t>0,\ \text{series } RLC)$$

$b' = \frac{R}{L} = 20,\qquad c' = \frac{1}{LC} = 100 \quad (C = 10mF)$

$\therefore$ characteristic equation : $s^2 + b's + c' = s^2 + 20s + 100$

$\therefore$ roots : $s_1 = -10$, $s_2 = -10$. $\underline{b'^2 - 4c' = 0}$

$\therefore V_c(t) = (k_1 + k_2 t)e^{-10t}$

$V_c(0) = K_1 = 10$

$i_L(0) = C \cdot \left.\frac{dV_c(t)}{dt}\right|_{t=0} = 0 \Rightarrow -10k_1 + k_2 = 0$

$\therefore k_1 = 10,\ k_2 = 100$

$\underline{V_c(t) = (10 + 100t)e^{-10t}\ (V)}$

$i_L(t) = C \cdot \frac{dV_c(t)}{dt} = 10 \times 10^{-3} \times \left\{ -100e^{-10t} + 100e^{-10t} - 1000te^{-10t} \right\}$

$= \underline{-10te^{-10t}\ (A)}$

(b) $b' = \frac{R}{L} = \frac{22.5}{1} = 22.5$ $\quad c' = \frac{1}{LC} = 100$

$\therefore$ characteristic equation : $s^2 + 22.5s + 100 = 0$

$\therefore$ roots $\Rightarrow s_1 = -6.1, \quad s_2 = -16.4$

$b'^2 - 4c' = 106.25 > 0$

$\therefore V_c(t) = K_1 e^{-6.1t} + K_2 e^{-16.4t}$

$V_c(0) = K_1 + K_2 = 0.5 \times 22.5 = 11.25 \cdots ①$

$i_L(0) = C \cdot \frac{dV_c(t)}{dt}\Big|_{t=0} = 0 \Rightarrow -6.1K_1 - 16.4K_2 = 0 \cdots ②$

From ① and ②, $K_1 = 17.91 \quad K_2 = -6.65$

$\therefore \underline{V_c(t) \cong 17.9e^{-6.1t} - 6.7e^{-16.4t} \text{ (V)}} \ (t > 0)$

$\therefore i_L(t) = C \cdot \frac{dV_c(t)}{dt}$

$= 10 \times 10^{-3} \times (-109.2e^{-6.1t} + 109.9e^{-16.4t})$

$= \underline{-1.09e^{-6.1t} + 1.10e^{-16.4t} \text{ (A)}} \ (t > 0)$

9-20. $R = 25k\Omega$.

(a) In parallel RLC,

$$\frac{d^2V_c}{dt^2} + \frac{1}{RC}\frac{dV_c}{dt} + \frac{1}{LC}V_c = 0$$

$b' = \frac{1}{RC}$, $c' = \frac{1}{LC}$.

The response of the graph is underdamped.

therefore. $V_c(t) = A \cdot e^{-\sigma t}\cos(w_d t + \theta)$

$s^2 + b's + c' = 0 \Rightarrow (s - s_1)(s - s_2) = 0$

$s_1 = -\sigma + jw_d$, $s_2 = -\sigma - jw_d$.

From the graph, $W_d = \frac{2\pi}{0.5\times10^{-3}} = 4000\pi$

When $t = 1msec$, we know that $e^{-\sigma\times1ms} = \frac{12}{20}$

$\therefore \sigma \cong 510.83$

$\therefore s_1 = -510.83 + j\cdot(4000\pi)$, $s_2 = -510.83 - j(4000\pi)$

$\therefore \frac{1}{RC} = b' \cong 1021.7$ $\quad\therefore C \cong 39.15nF$

$\frac{1}{LC} = \sigma^2 + W_d^2 = 510.83^2 + (4000\pi)^2$ $\quad\therefore L \cong 0.162H$

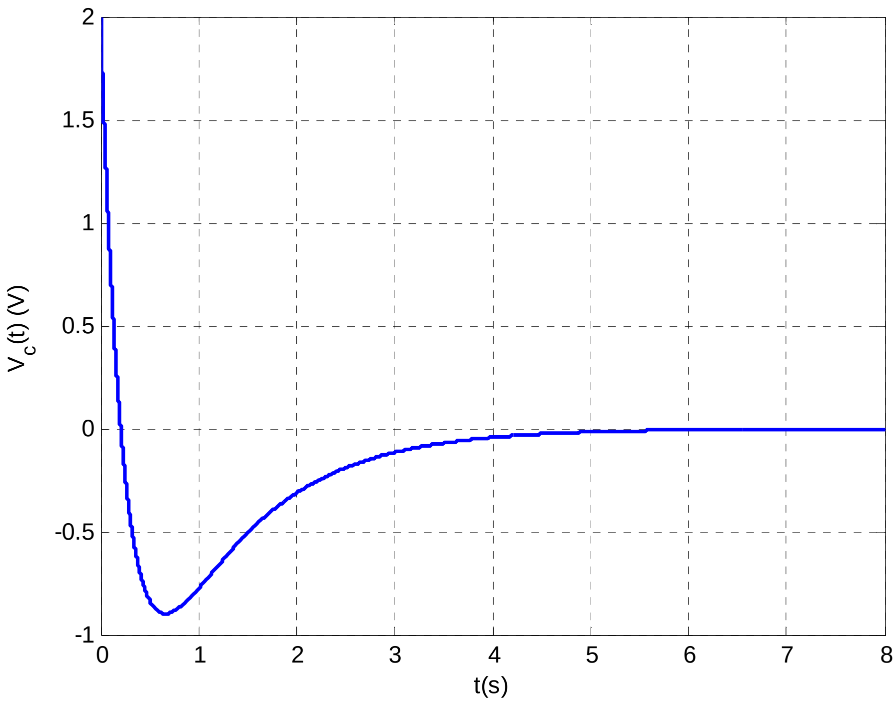
From the graph, $\theta = 0$, $A = 20$,

$\therefore V_c(t) = 20\cdot e^{-510.83t}\cos(4000\pi t)$

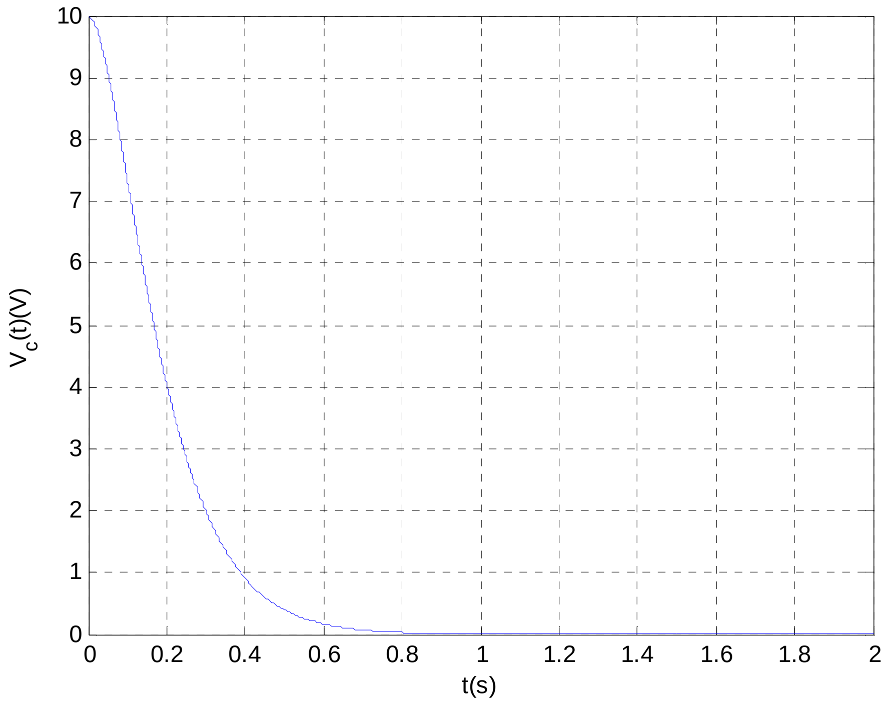
$\therefore V_c(0) = 20(V)$, $i_c(0) = C\cdot\frac{dV_c}{dt}\Big|_{t=0} \cong -0.4mA$

$i_L(0) = -\frac{V_c(0)}{R} - i_c(0) = -0.8mA + 0.4mA = -0.4mA$

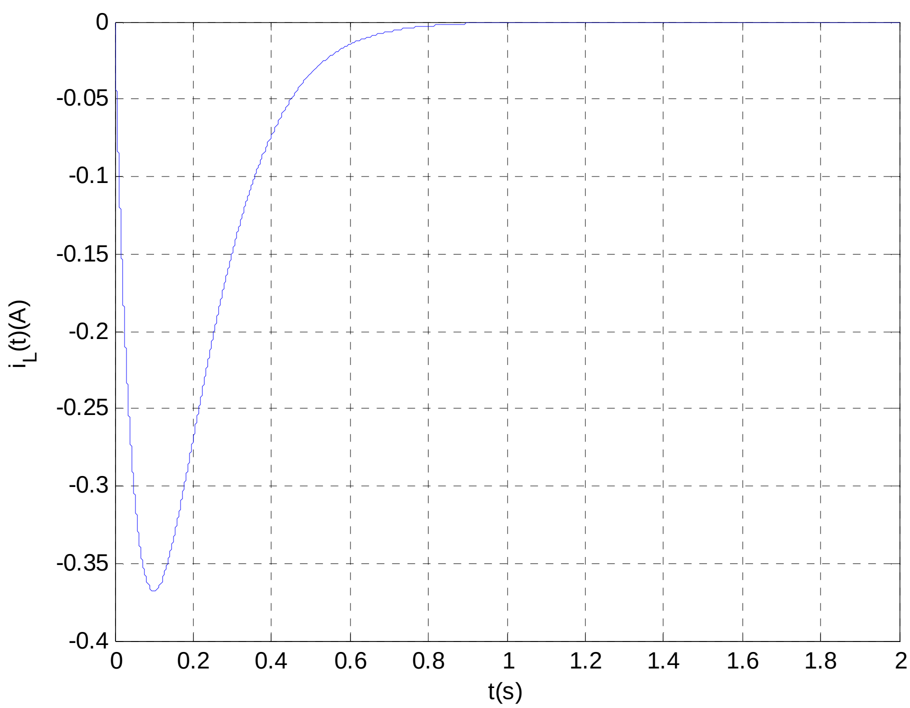
9-16(b)



9-18(a)

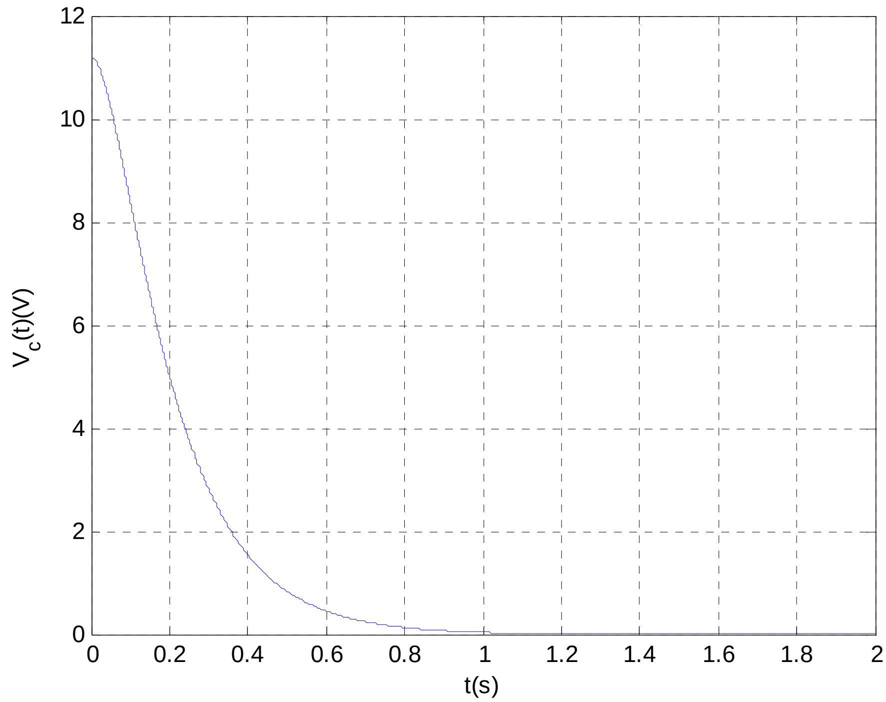


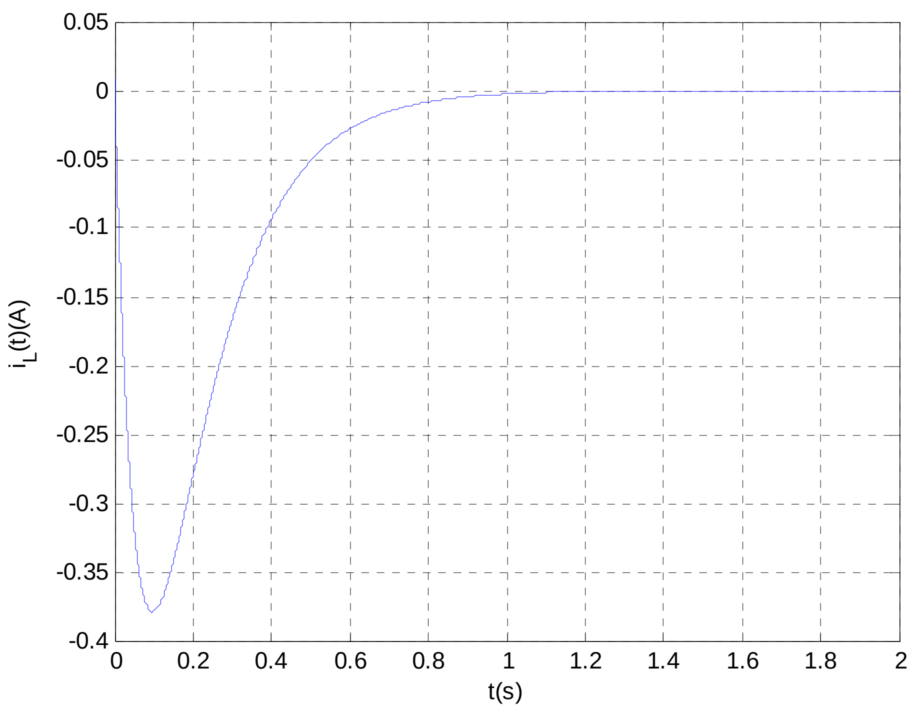
10
9
8
7
6
5
4
3
2
1
0
$V_c(t)(V)$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
t(s)



0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
$i_L(t)(A)$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
t(s)

9-18(b)

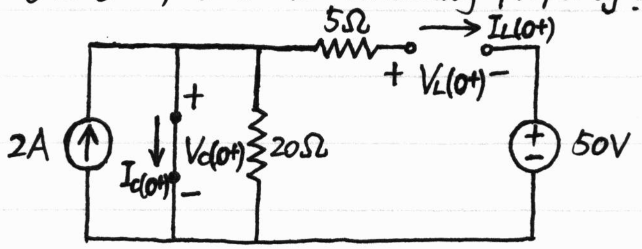

12
10
8
6
4
2
0
$V_c(t)(V)$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
t(s)


0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
$i_L(t)(A)$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
t(s)

ECE HW24 Solutions

9-28 $t = 0^-$, both sources are off: $V_C(0^-) = 0\,V$, $I_L(0^-) = 0\,A$.

$t = 0^+$, due to continuity property: $\begin{cases} V_C(0^+) = V_C(0^-) = \underline{0V} \text{ (short)} \\ I_L(0^+) = I_L(0^-) = \underline{0A} \text{ (open)} \end{cases}$



$I_C(0^+) = \underline{2A}$ (short current)

$KVL: 50 + V_L(0^+) + 0\times 5 + V_C(0^+) = 0$

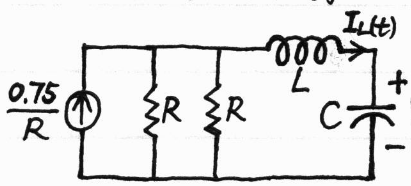
$V_L(0^+) = \underline{-50\,V}$ (open voltage)

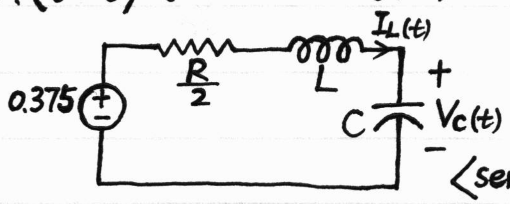
9-32 (a) $-\infty < t \le 0^-$, $V_{in}(t) = -0.25u(-t)\,V$ for a long time.

$t=0^-$, L is short, C is open: $I_L(0^-) = 0\,A$, $V_C(0^-) = \frac{R}{R+R}(-0.25) = -0.125\,V$

$t=0^+$, due to continuity property: $\begin{cases} I_L(0^+) = I_L(0^-) = 0\,A \\ V_C(0^+) = V_C(0^-) = -0.125V \end{cases}$

use source transformation: $(t>0)$





$X_F = V_C(\infty) = 0.375$

⟨series RLC circuits⟩

$$\frac{d^2V_C(t)}{dt^2} + \frac{R}{2L}\frac{dV_C(t)}{dt} + \frac{1}{LC}V_C(t) = F, \quad \tilde{b} = \frac{R}{2L} = 0.25, \quad \tilde{c} = \frac{1}{LC} = 100.$$

$$s^2 + \tilde{b}s + \tilde{c} = 0 \xrightarrow{\tilde{b}^2 - 4\tilde{c} < 0} s_1, s_2 = -\frac{\tilde{b}}{2} \pm j\frac{\sqrt{4\tilde{c} - \tilde{b}^2}}{2} = -\sigma \pm jW_d \approx -0.125 \pm j10$$

$$V_C(t) = e^{-\sigma t}[A\cos(W_d t) + B\sin(W_d t)] + X_F = Ke^{-\sigma t}\cos(W_d t + \theta) + X_F \ \langle\text{underdamped case}\rangle$$

$$= e^{-0.125t}[A\cos(10t) + B\sin(10t)] + X_F = Ke^{-0.125t}\cos(10t + \theta) + X_F, \ t > 0$$

① $V_C(0^+) = -0.125 = [A\cdot 1 + B\cdot 0] + 0.375 \Rightarrow A = \underline{-0.5}$

$$I_L(t) = I_C(t) = C\frac{dV_C(t)}{dt} = 0.01\left(e^{-0.125t}[-10A\sin(10t) + 10B\cos(10t)] - 0.125e^{-0.125t}[A\cos(10t) + B\sin(10t)]\right)$$

② $I_L(0^+) = 0 = C\left.\frac{dV_C(t)}{dt}\right|_{t=0}$

$$0 = 0.1B - 0.00125A \Rightarrow B = 0.0125A = \underline{-0.00625}$$

$$K = \sqrt{A^2 + B^2} \approx \underline{0.5}, \quad \theta = \tan^{-1}\left(\frac{-B}{A}\right) = \tan^{-1}(-0.0125) = \underline{\angle -0.716^\circ}$$

The complete response of $V_C(t)$:

$$V_C(t) = \underline{e^{-0.125t}[-0.5\cos(10t) - 0.00625\sin(10t)] + 0.375\,(V), \ t>0}$$

$$= \underline{0.5e^{-0.125t}\cos(10t + \angle -0.716^\circ) + 0.375\,(V), \ t > 0}$$

ECE 201 Spring 2010

Homework 25 Solutions

Problem 43

(a)

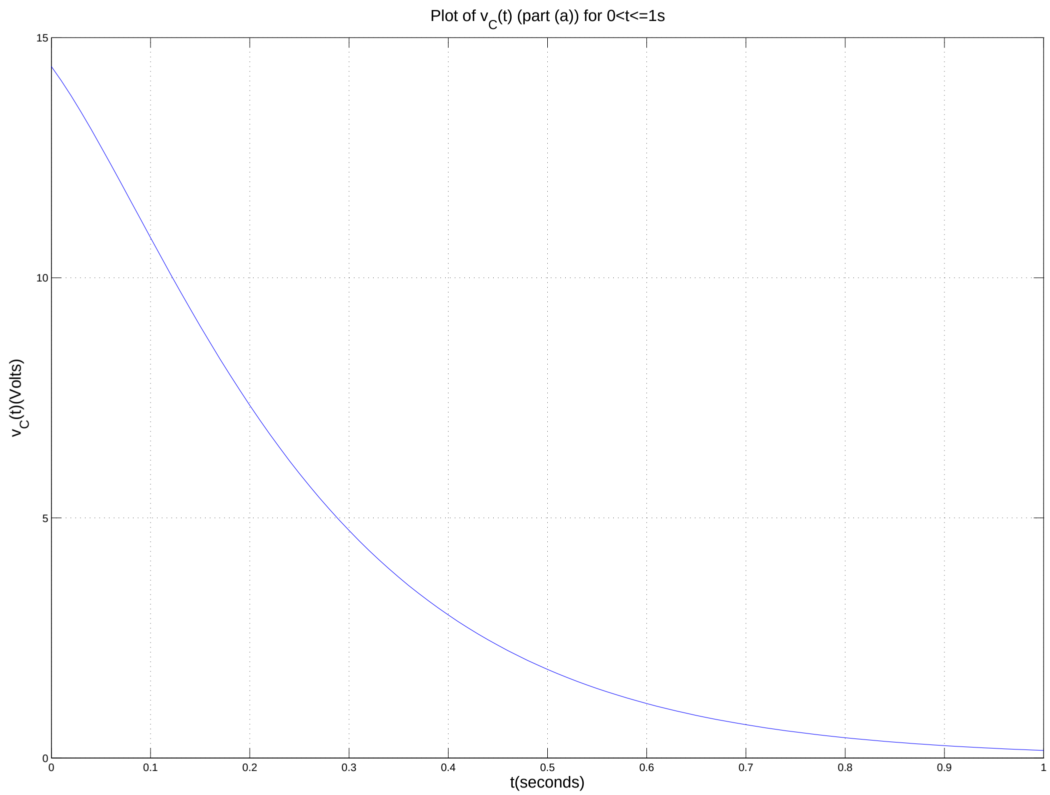
To find the initial conditions at t=0-, we can write the following KVL equations,

$$\begin{aligned}
20 + 40i_L(0-) - 60(0.1 - i_L(0-)) &= 0 \\
\Rightarrow i_L(0-) = i_L(0+) = i_C(0+) &= -0.14\ A \\
\Rightarrow v_C(0+) = v_C(0-) &= 20 - 0.14 \times 40 \\
&= 14.4\ V
\end{aligned}$$

After t=0, the circuit is a series RLC circuit with the following characteristic equation,

$$\begin{aligned}
s^2 + \frac{R}{L}s + \frac{1}{LC} &= 0 \\
s^2 + 15s + 50 &= 0 \\
\Rightarrow s_1 = -5,\ s_2 &= -10 \\
\Rightarrow v_C(t) &= c_1e^{-5t} + c_2e^{-10t} \\
\Rightarrow c_1 + c_2 &= 14.4 \\
(5 \times 10^{-3})(5c_1 + 10c_2) &= 0.14 \\
\Rightarrow c_1 = 23.2,\ c_2 &= -8.8 \\
\Rightarrow v_C(t) &= (23.2e^{-5t} - 8.8e^{-10t})\ V
\end{aligned}$$

The plot of $v_C(t)$ for $0 < t \leq 1$ is shown on the next page.

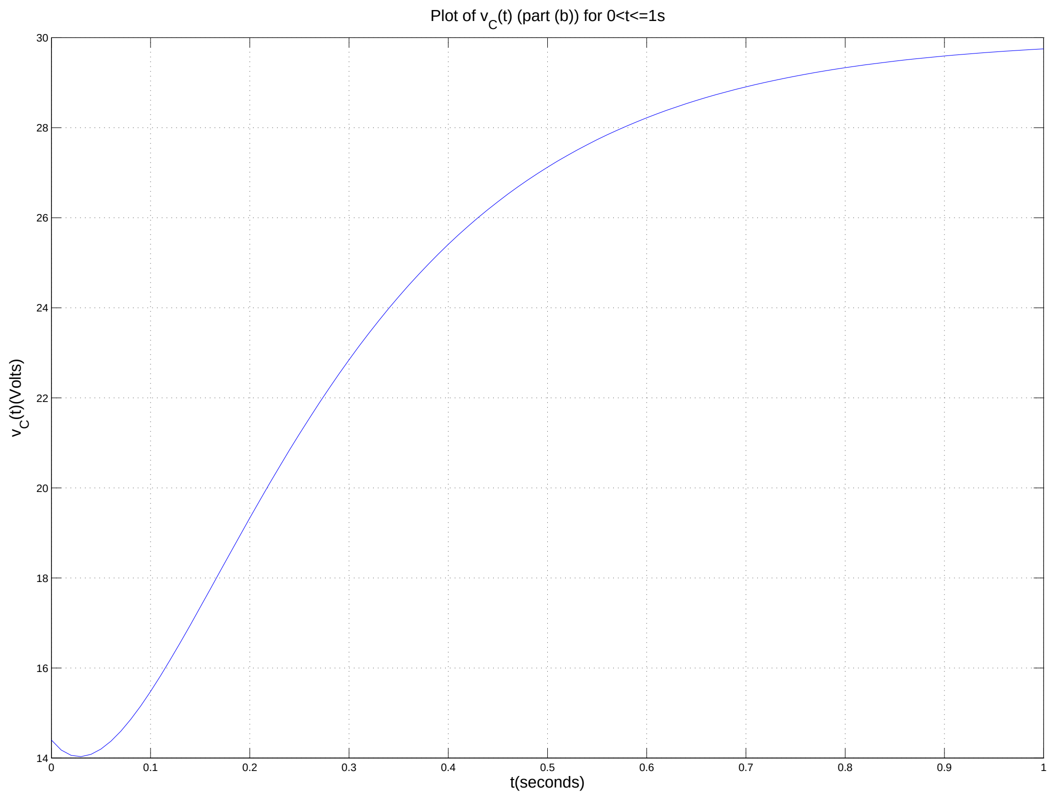


(b)

In this case, the situation after t=0 can be visualized as a series RLC circuit with a constant input voltage source (after source transformation) which will have the value $0.5 \times 60 = 30\ V$. The initial conditions and the characteristic equation do not change. However, the expression for $v_C(t)$ will now have a term due to the constant input also. Thus,

$$\begin{aligned} v_C(t) &= c_1 e^{-5t} + c_2 e^{-10t} + 30 \\ \Rightarrow c_1 + c_2 + 30 &= 14.4 \\ 5c_1 + 10c_2 &= 28 \\ \Rightarrow v_C(t) &= (-36.8e^{-5t} + 21.2e^{-10t} + 30)\ V \end{aligned}$$

The plot of $v_C(t)$ for $0 < t \leq 1$ follows.



Problem 49

(a)

At t=0-, the capacitor is open circuit and the inductor is short circuit. At t=0+, the current source is off and the voltage source is still operating. Thus we can write the following equations,

$$\begin{aligned}
v_C(0-) = v_C(0+) &= 10\ V \\
i_L(0-) + 0.005 &= \frac{10}{1000} \\
\Rightarrow i_L(0+) = i_L(0-) &= 0.005\ A \\
10 - v_L(0+) - v_C(0+) &= 0 \\
\Rightarrow v_L(0+) &= 0 \\
i_C(0+) + \frac{10}{1000} &= 0.005 \\
\Rightarrow i_C(0+) &= -0.005\ A
\end{aligned}$$

(b)

For $t > 0$, we can write the following KVL and KCL equations,

$$\begin{aligned} 10 - L\frac{di_L}{dt} - v_C &= 0 \\ i_L &= C\frac{dv_C}{dt} + \frac{v_C}{R} \\ \Rightarrow \frac{d^2v_C}{dt^2} + \frac{1}{RC}\frac{dv_C}{dt} + \frac{v_C}{LC} &= \frac{10}{LC} \end{aligned}$$

Thus the characteristic equation is

$$\begin{aligned} s^2 + 2000s + 1.087 \times 10^7 &= 0 \\ \Rightarrow s_{1,2} &= -1000 \pm j1000\pi \\ \Rightarrow v_C(t) &= e^{-1000t}[A\cos(1000\pi t) + B\sin(1000\pi t)] + 10 \\ A = 0,\ B &= -\frac{10}{\pi}\ (Using\ initial\ conditions) \\ \Rightarrow v_C(t) &= 10 - \frac{10}{\pi}e^{-1000t}\sin(1000\pi t)\ V \\ i_L(t) &= C\frac{dv_C}{dt} + \frac{v_C}{R} \\ &= 0.005\left[2 - e^{-1000t}\left\{\frac{1}{\pi}\sin(1000\pi t) + \cos(1000\pi t)\right\}\right]\ A \end{aligned}$$

(c)

$v_C(t)$ is computed above in part (b).

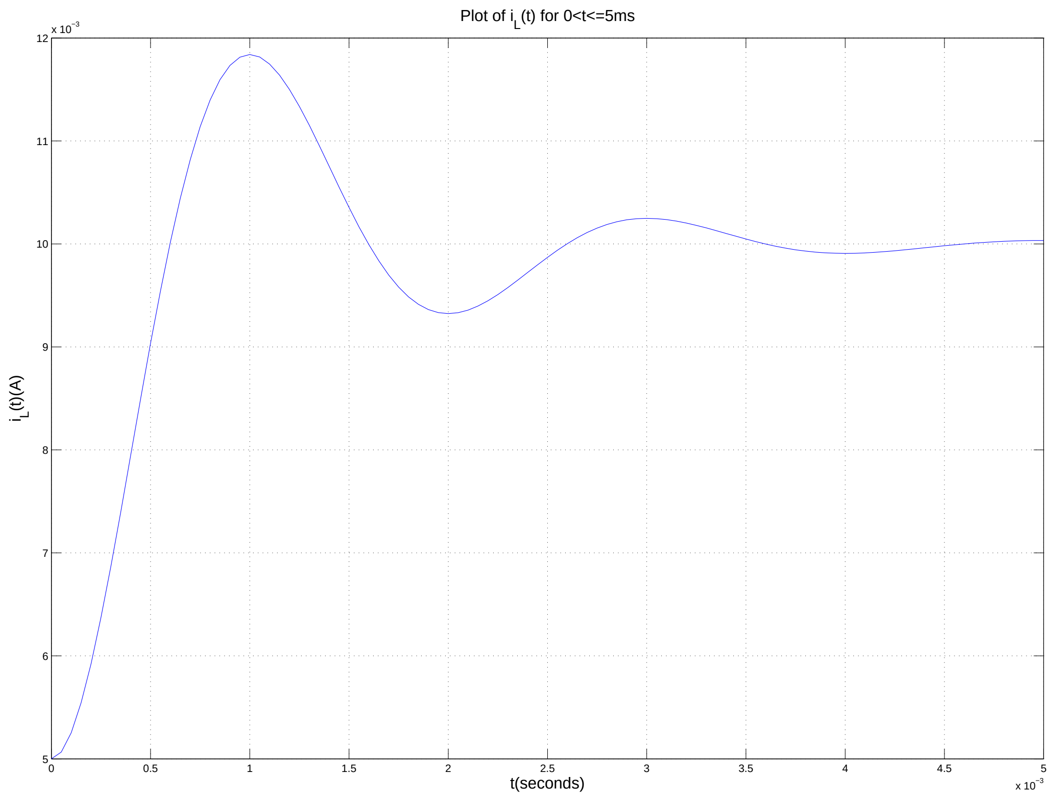
(d)

$$\begin{aligned} v_L(t) &= 10 - v_C(t) \\ &= \frac{10}{\pi}e^{-1000t}\sin(1000\pi t)\ V \end{aligned}$$

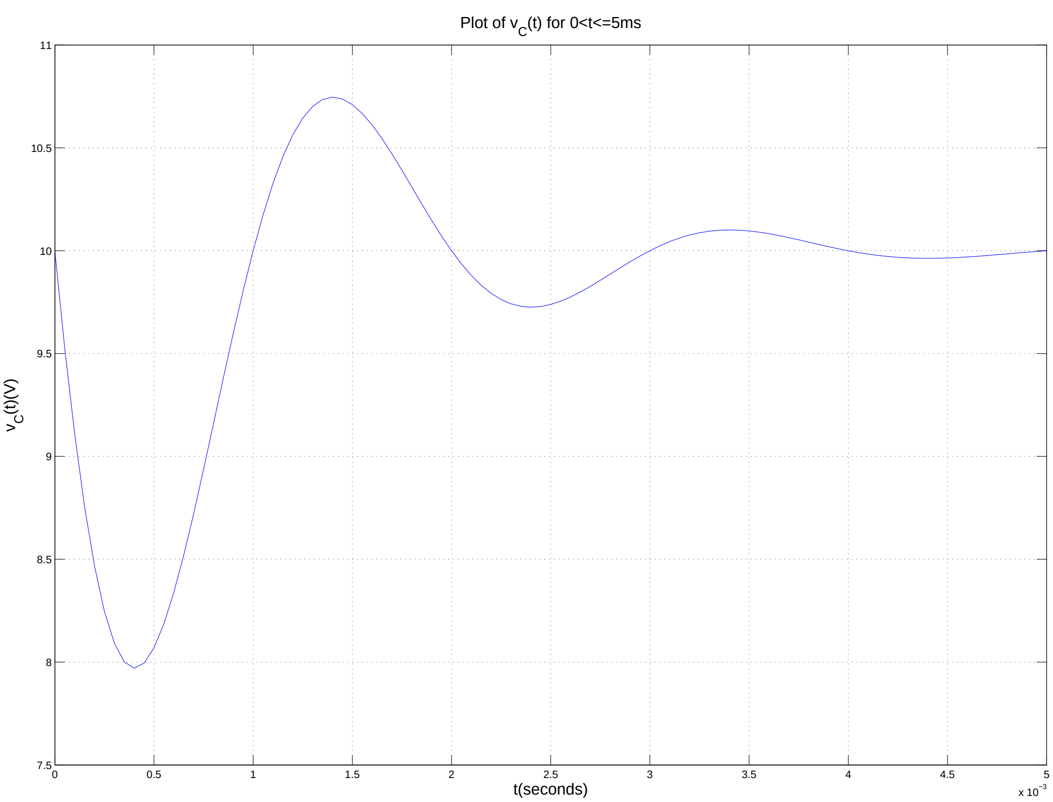
(e)

$$\begin{aligned} i_C(t) &= C\frac{dv_C}{dt} \\ &= \frac{0.005}{\pi}e^{-1000t}[\sin(1000\pi t) - \pi\cos(1000\pi t)]\ A \end{aligned}$$

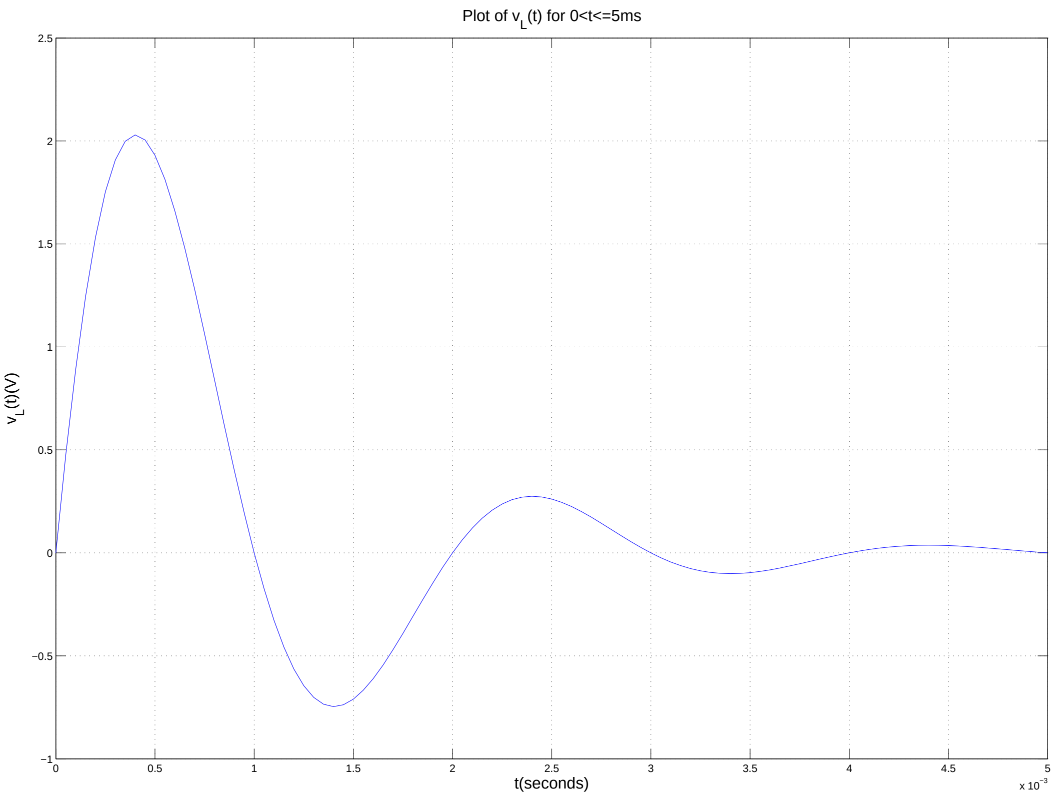
The plots for the respective quantities follow.



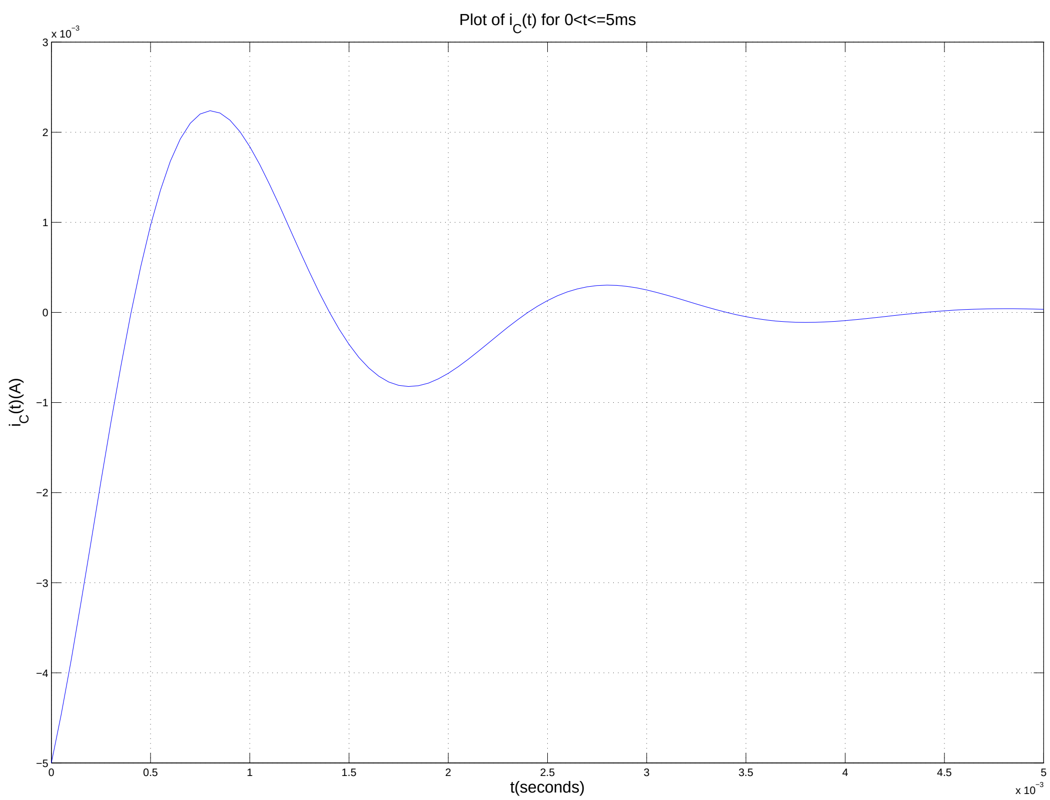
Plot of $i_L(t)$ for 0<t<=5ms
x 10^-3
12
11
10
9
8
7
6
5
$i_L(t)(A)$
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t(seconds)
x 10^-3



Plot of $v_C(t)$ for 0<t<=5ms
11
10.5
10
9.5
9
8.5
8
7.5
$v_C(t)(V)$
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t(seconds)
x 10^-3

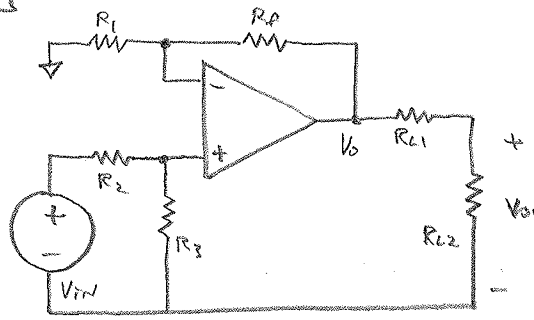


Plot of $v_L(t)$ for 0<t<=5ms
$v_L(t)$(V)
t(seconds)
x 10^{-3}
2.5
2
1.5
1
0.5
0
−0.5
−1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



Plot of $i_C(t)$ for 0<t<=5ms
x 10^{-3}
$i_C(t)$(A)
t(seconds)
x 10^{-3}
3
2
1
0
−1
−2
−3
−4
−5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

4-3



$G_+ = \frac{V_+}{V_{IN}}$ $\quad G_V = \frac{V_{OUT}}{V_{IN}}$

$i_+ = 0 \Rightarrow$

$V_+ = V_{IN} \cdot \frac{R_3}{R_2 + R_3}$

So $\boxed{G_+ = \frac{V_+}{V_{IN}} = \frac{R_3}{R_2 + R_3}}$

Node eq. @ V_-:

$$\frac{V_- - 0}{R_1} + \frac{V_- - V_0}{R_f} = 0 \qquad \text{and} \qquad V_{OUT} = V_0 \cdot \frac{R_{L2}}{R_{L1} + R_{L2}}$$

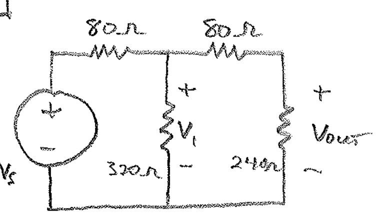
$$R_f V_- + R_1 V_- - R_1 V_0 = 0 \Rightarrow V_-(R_f + R_1) = R_1 V_0 \Rightarrow V_0 = \frac{V_-(R_f + R_1)}{R_1}$$

$$\text{So, } V_{out} = V_0 \cdot \frac{R_{L2}}{R_{L1} + R_{L2}} = \frac{V_-(R_f + R_1)}{R_1} \cdot \frac{R_{L2}}{R_{L1} + R_{L2}} = V_{IN} \cdot \frac{R_3}{R_2 + R_3} \cdot \frac{R_f + R_1}{R_1} \cdot \frac{R_{L2}}{R_{L1} + R_{L2}}$$

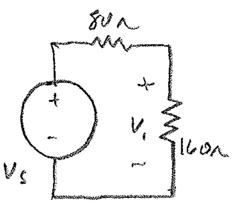
$$\Rightarrow \boxed{G_V = \frac{V_{OUT}}{V_{IN}} = \frac{R_3 R_{L2} (R_f + R_1)}{R_1 (R_2 + R_3)(R_{L1} + R_{L2})} = \frac{R_3 R_{L2}}{(R_2 + R_3)(R_{L1} + R_{L2})} \cdot \left(1 + \frac{R_f}{R_1}\right)}$$

4-5

a)



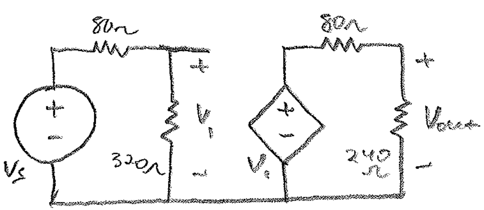
eq. ⇒



$V_1 = V_S \cdot \frac{160}{240} = \boxed{\frac{2}{3} V_S}$

$V_{out} = V_1 \cdot \frac{240}{320} = \frac{3}{4} \cdot \frac{2}{3} V_S = \boxed{\frac{V_S}{2}}$

b)



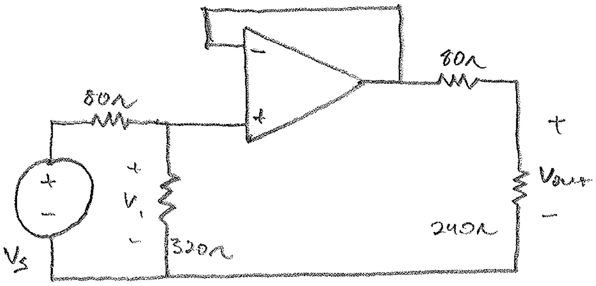
$V_1 = \frac{320}{400} V_S = \boxed{\frac{4}{5} V_S}$

$V_{1,b} > V_{1,A}$

$V_{out} = \frac{240}{320} V_1 = \frac{3}{4} \cdot \frac{4}{5} V_S = \boxed{\frac{3}{5} V_S}$

$V_{out,b} > V_{out,A}$

c)



$i_+ = 0 \Rightarrow V_+ = \boxed{\frac{4}{5} V_S}$

$V_{1,c} = V_{1,b}$

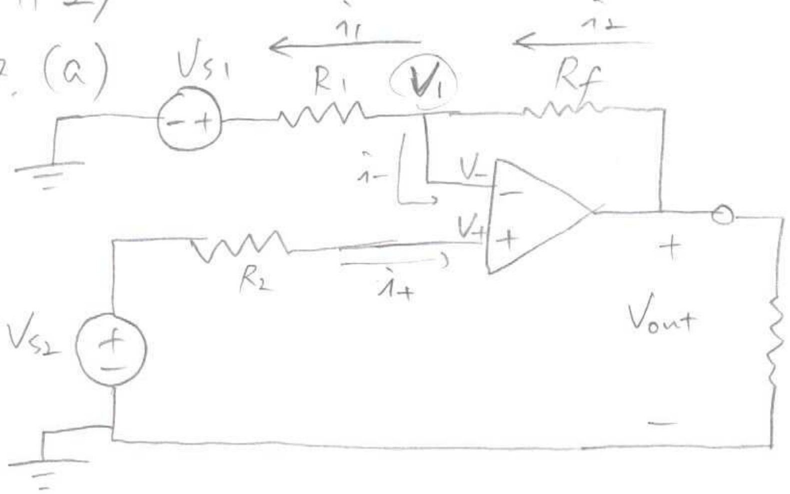
$V_- = V_+ \Rightarrow$

$V_{out} = \frac{240}{320} \cdot V_- = \frac{3}{4} \cdot \frac{4}{5} V_S = \boxed{\frac{3}{5} V_S}$

$V_{out,c} = V_{out,b}$

HW #27

4-7. (a)



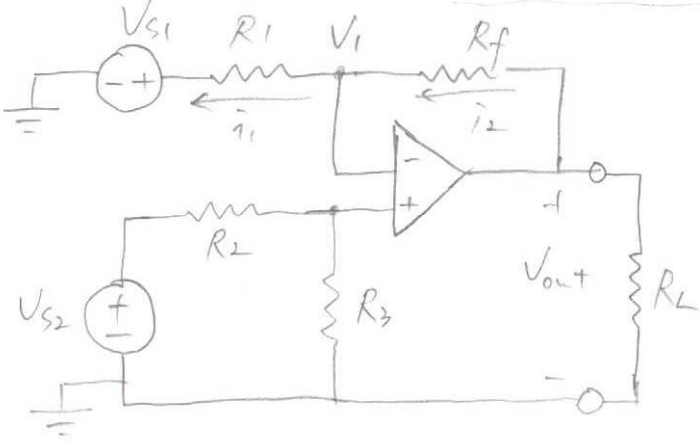
$$i_+ = 0 \Rightarrow V_+ = V_{S2} \Rightarrow V_1 = V_- = V_+ = V_{S2}$$

$$\therefore i_1 = \frac{V_1 - V_{S1}}{R_1} = \frac{V_{S2} - V_{S1}}{R_1}, \quad i_2 = i_1 \ (\because i_- = 0)$$

$$\therefore V_{out} = V_1 + i_2 \cdot R_f = V_{S2} + \frac{R_f}{R_1}(V_{S2} - V_{S1})$$

$$= \left(1 + \frac{R_f}{R_1}\right) V_{S2} - \frac{R_f}{R_1} V_{S1}$$

(b)



$\Leftarrow \quad i_2 = i_1 \ (\because i_- = 0)$

$$V_+ = \frac{R_3}{R_2 + R_3} \cdot V_{S2}, \quad \therefore i_1 = \frac{V_1 - V_{S1}}{R_1} = \left(\frac{R_3}{R_2 + R_3} V_{S2} - V_{S1}\right) \Big/ R_1$$

$$V_1 = V_- = V_+ \qquad = \frac{R_3 V_{S2} - R_2 V_{S1} - V_{S1} R_3}{R_1 (R_2 + R_3)}$$

$$\therefore V_{out} = V_1 + R_f \cdot i_2 = V_1 + R_f \cdot i_1$$

$$= \frac{R_3}{R_2 + R_3} V_{S2} + \frac{R_f}{R_1} \times \frac{(R_3 V_{S2} - R_2 V_{S1} - V_{S1} R_3)}{(R_2 + R_3)} = \frac{R_3 (R_1 + R_f)}{R_1 (R_2 + R_3)} V_{S2} - \frac{R_f}{R_1} V_{S1}$$

(c) $R_f = 12k\Omega$, $R_1 = 3k\Omega$, $R_2 = 1k\Omega$, $R_3 = 4k\Omega$.

$V_{S1} = 1.5V$, $V_{S2} = 2V$.

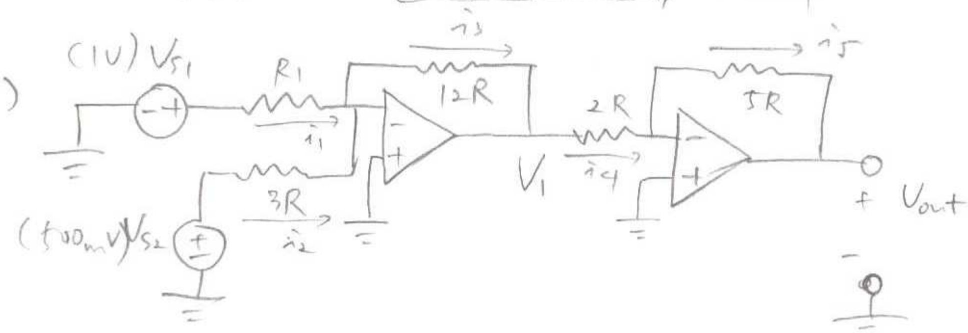
$$V_{out} = \frac{R_3(R_1+R_f)}{R_1(R_2+R_3)} V_{S2} - \frac{R_f}{R_1} V_{S1} = \frac{4\times(3+12)}{3\times(1+4)}\times 1.5 - \frac{12}{3}\times 2$$

$$= -2(V)$$

$\therefore$ power delivered to the load $R_L = 100\Omega$

$$\Rightarrow \frac{V_{out}^2}{100} = \underline{40\,mW} = \underline{0.04(w)}$$

13. (a)



$$i_1 = \frac{V_{S1}}{R_1},\quad i_2 = \frac{V_{S2}}{3R}$$

$$i_3 = i_1 + i_2 = \frac{V_{S1}}{R_1} + \frac{V_{S2}}{3R} \qquad \therefore V_1 = -12R\times i_3$$

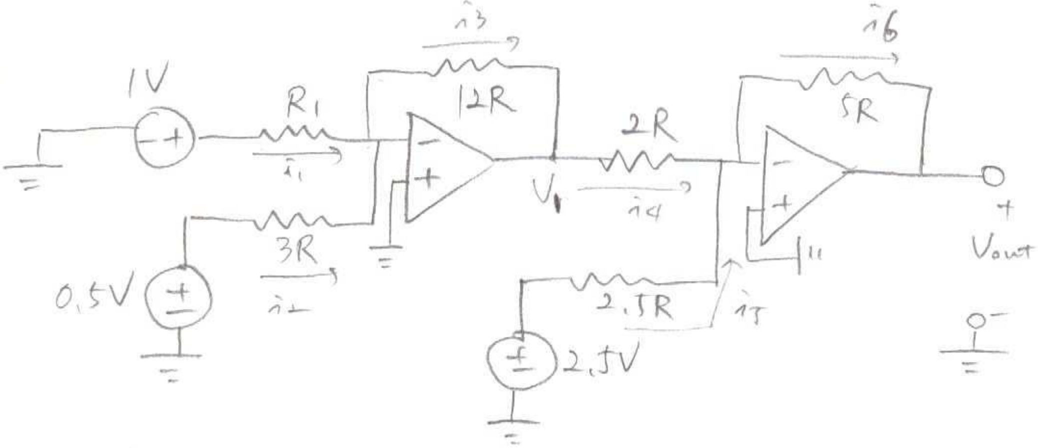
$$= \frac{1}{R_1} + \frac{0.5}{3R} \qquad = -\frac{12R}{R_1} - 2$$

$$i_4 = V_1/(2R) = -\frac{6}{R_1} - \frac{1}{R} = i_5$$

$$\therefore V_{out} = -i_5\times 5R = \underline{\frac{30R}{R_1} + 5 = 10V}$$

$$\therefore \underline{R_1 = 6R}$$

(b)



$$i_1 = \frac{1}{R_1}. \quad i_2 = \frac{0.5}{3R}. \quad i_3 = i_1 + i_2 = \frac{1}{R_1} + \frac{0.5}{3R}$$

$$V_1 = -12R \cdot i_3 = -\frac{12R}{R_1} - 2$$

$$i_4 = \frac{V_1}{2R} = -\frac{6}{R_1} - \frac{1}{R}$$

$$i_5 = \frac{2.5}{2.5R} = \frac{1}{R}$$

$$i_6 = i_4 + i_5 = -\frac{6}{R_1}$$

$$V_{out} = -i_6 \cdot 5R = \frac{30R}{R_1} = 10V$$

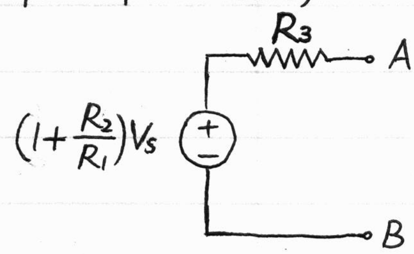
$$\therefore \underline{R_1 = 3R}$$

ECE 201 HW 28 Solutions

6-29 (a) $V_{CB} = \left(1+\frac{R_2}{R_1}\right)V_s$ (non-inverting op amp circuit)

$V_{AB} = R_3 I_A + V_{CB}$ (KVL)

$\therefore V_{AB} = R_3 I_A + \left(1+\frac{R_2}{R_1}\right)V_s$



Match coefficients with (6.5):

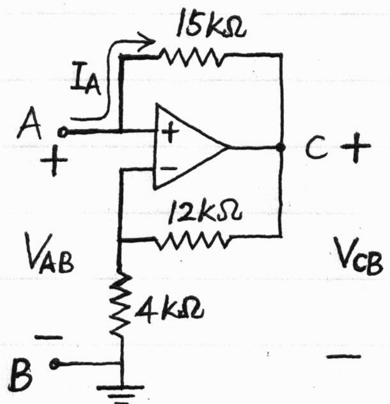
$R_{th} = R_3$, $V_{oc} = \left(1+\frac{R_2}{R_1}\right)V_s$.

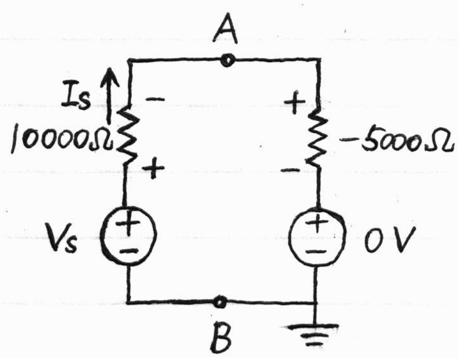
(b) $R_L = R_{th} = R_3$ (maximum power transfer)

$V_L = \frac{R_L}{R_3+R_L}V_{oc} = \frac{V_{oc}}{2}$

$P_L = \left(\frac{V_{oc}}{2}\right)^2 / R_L = \frac{1}{4R_L}\left(1+\frac{R_2}{R_1}\right)^2 V_s^2$

6-31





(a) $V_{CB} = \left(1+\frac{12k}{4k}\right)V_{AB} = 4\,V_{AB}$ (non-inverting op amp circuit)

$V_{AB} = (15k)\,I_A + V_{CB}$ (KVL) $\Longrightarrow -3\,V_{AB} = (15k)\,I_A$

$\therefore V_{AB} = (-5000)\,I_A$

Match coefficients with (6.5):

$R_{th} = -5000\ \Omega$, $V_{oc} = 0\ V$.

(b) KVL: $V_s = (10000)\,I_s + (-5000)\,I_s + 0 = (5000)\,I_s$

$\therefore I_s = \frac{V_s}{5000}$

ECE 201 Spring 2010

Homework 29 Solutions

Problem 40

(a)

The following differential equation can be written using KCL at the inverting terminal of the op amp and using the virtual ground concept,

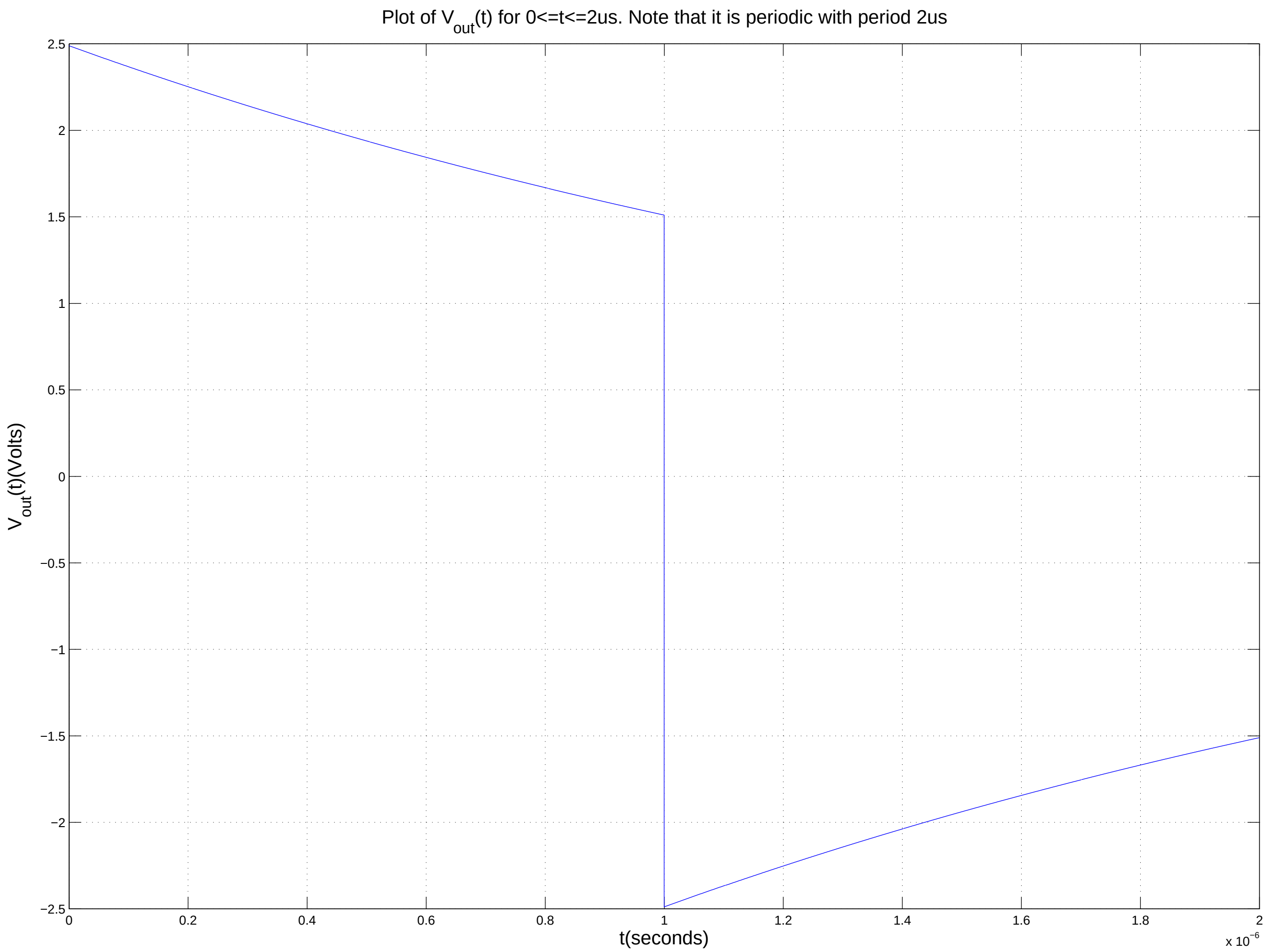
$$\begin{aligned}
\frac{v_s(t)}{R_1} + \frac{v_{out}(t)}{R_2} + C\frac{dv_{out}(t)}{dt} &= 0 \\
v_s(t) &= -100u(t) \\
&= -100,\ t \geq 0 \\
\Rightarrow v_{out}(t) &= \frac{100R_2}{R_1}(1 - e^{-t/R_2C}) \\
&= 400(1 - e^{-2t})\ mV,\ t \geq 0
\end{aligned}$$

Problem not from the book

If we denote the voltage at the non-inverting terminal of the op amp as $v_L(t)$, then using the virtual ground concept, the output voltage is given by $V_{out}(t) = 10v_L(t)$. Also, $v_L(t) = V_{IN}(t) - 1000i_L(t)$. The problem then reduces to finding $i_L(t)$. Due to the square wave shape of $V_{IN}(t)$, the graph for $i_L(t)$ will also be periodic with maximum and minimum values of x and $-x$ (say). The task is to find the value of x, which can be done using the following equation,

$$\begin{aligned}
i_L(t) &= i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau} \\
\tau &= \frac{L}{R} \\
&= 2\ \mu s \\
x &= 2 \times 10^{-4} + [-x - 2 \times 10^{-4}]e^{-0.5} \\
\Rightarrow x &= 0.489 \times 10^{-4}
\end{aligned}$$

The plot of $V_{out}(t)$ for $0 \leq t \leq 2\mu s$ is drawn below. Note that $V_{out}(t)$ will be periodic with period $2\mu s$.



ECE 201 HW #30 Solution

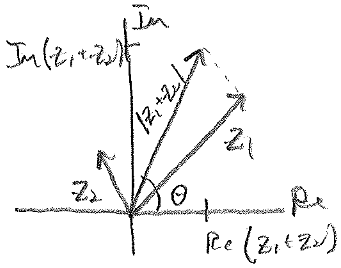
1. $z_1 = 10e^{j\pi/4} \quad z_2 = 5e^{j5\pi/8}$

$z_1 = 10\cos\pi/4 + j\,10\sin\pi/4 = 5\sqrt{2} + j5\sqrt{2}$

$z_2 = 5\cos 5\pi/8 + j\,5\sin 5\pi/8 = -\dfrac{5\sqrt{2-\sqrt{2}}}{2} + j\,\dfrac{5\sqrt{2+\sqrt{2}}}{2}$

$$z_1+z_2 = \underbrace{5\sqrt{2} - \frac{5\sqrt{2-\sqrt{2}}}{2}}_{c} + j\underbrace{\left(5\sqrt{2} + \frac{5\sqrt{2+\sqrt{2}}}{2}\right)}_{d} = \boxed{5.158 + j\,11.69} \quad \text{A)}$$

$$= \sqrt{c^2+d^2}\angle\tan^{-1}(d/c) = \boxed{12.778\angle 66.2^\circ} \quad \text{b)}$$

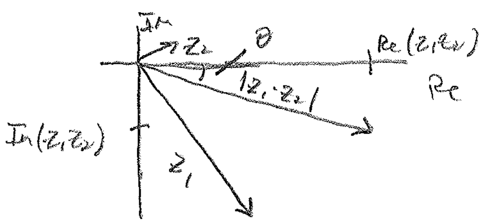


2. $z_1 = 12-16j \qquad z_2 = 1+3/4\,j$

$z_1 = \sqrt{12^2+16^2}\ \tan^{-1}(-16/12) = 20\angle -53.1^\circ$

$z_2 = \sqrt{1+9/16}\ \tan^{-1}(3/4) = 5/4\angle 36.9^\circ$

$$z_1 \cdot z_2 = 20\cdot 5/4\,\angle(-53.1^\circ + 36.9^\circ) = \overset{\text{b)}}{\boxed{25\angle -16.2^\circ}} = \overset{\text{A)}}{\boxed{24 - j7}}$$



c) $|z_1||z_2| = |z_1z_2|$

d) $\angle(z_1z_2) = \angle z_1 + \angle z_2$

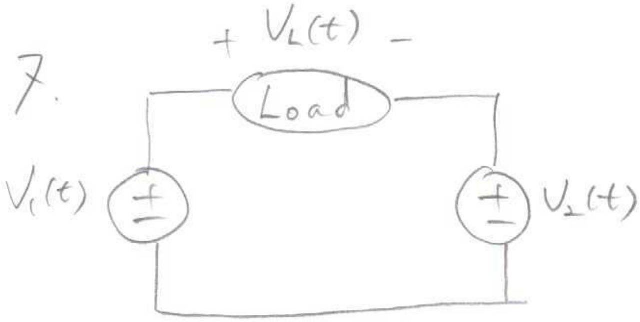
3. $$\frac{z_1}{z_2} = \frac{20}{5/4}\angle(-53.1^\circ - 36.9^\circ) = \overset{\text{b)}}{\boxed{16\angle -90^\circ}} = \overset{\text{A)}}{\boxed{-j16}}$$

c) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

d) $\angle\left(\dfrac{z_1}{z_2}\right) = \angle z_1 - \angle z_2$

HW # 31.

7.



from KVL, $V_1(t) - V_L(t) - V_2(t) = 0 \Rightarrow V_L(t) = V_1(t) - V_2(t)$

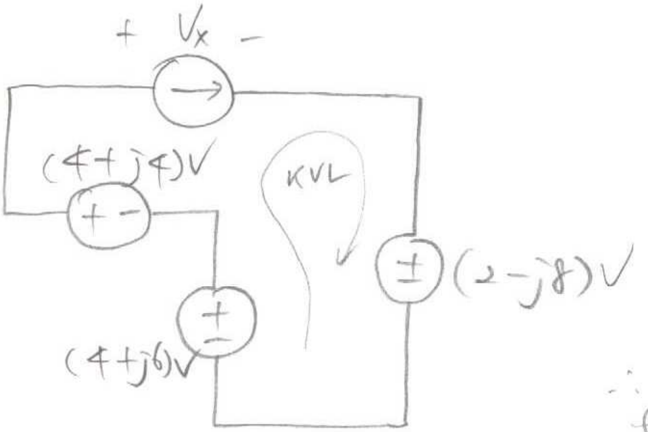
Voltage phasors must satisfy KVL,

$$V_L = V_1 - V_2 = 4\angle 0 - 4\sqrt{2}\angle -0.25\pi$$

$$= 4 - (4 - j4) = j4 = 4\angle \pi/2$$

$\therefore$ $\underline{V_L(t) = 4\cos(wt + \pi/2)\,(V)}$ $(K = 4,\ \phi = \pi/2)$

10.

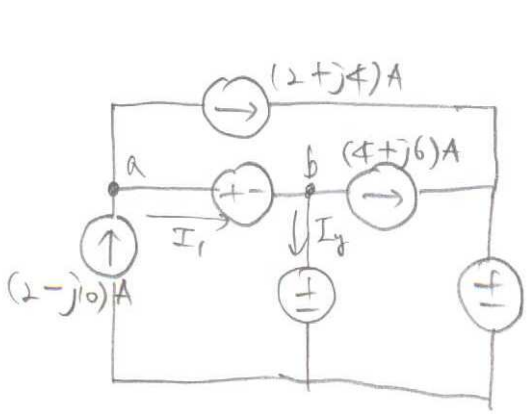


From KVL,

$$(4+j6) + (4+j4) - V_x - (2-j8) = 0$$

$\therefore$ $\underset{\text{(phasor)}}{V_x} = \underline{6 + j18} = \underline{18.97\angle 71.56°}$

$\therefore$ $\underline{V_x(t) = 18.97\cos(2000\pi t + 71.56°)\,(V)}$



At node a, from KCL,

$2 - j10 = (2+j4) + I_1 \Rightarrow I_1 = -j14$

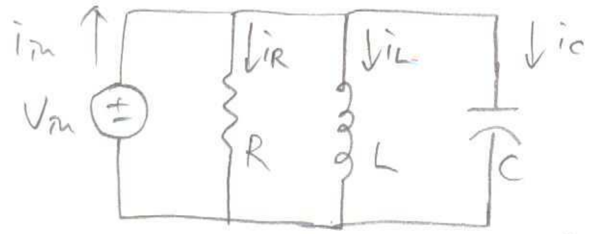
At node b, from KCL,

$I_1 = I_y + (4+j6)$

$\therefore$ $\underline{I_y = -4 - j20 = 20.4\angle -101.31°}$

$\therefore$ $i_y(t) = 20.4\cos(2000\pi t - 101.31°)\,(A)$

14.



$R = 1K\Omega$, $L = 0.5H$, $C = 1\mu F$.

$V_{in}(t) = 20\cos(1000t + 60^\circ)$ V.

$\omega = 1000$

phasor of $V_{in}(t) = 20\angle 60^\circ = V_{in}$

$$I_R = \frac{V_{in}}{Z_R(j\omega)} = \frac{20\angle 60^\circ}{1K} = \underline{0.02\angle 60^\circ}$$

$$I_L = \frac{V_{in}}{Z_L(j\omega)} = \frac{20\angle 60^\circ}{j\omega L} = \frac{20\angle 60^\circ}{500\angle 90^\circ} = \underline{0.04\angle -30^\circ}$$

$$I_c = \frac{V_{in}}{Z_c(j\omega)} = 20\angle 60^\circ \times 0.001\angle 90^\circ = \underline{0.02\angle 150^\circ}$$

From KCL, $I_{in} = I_R + I_L + I_c$

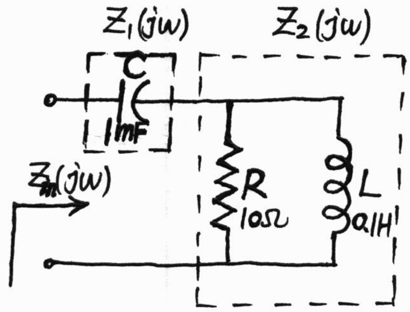
$$= 0.02\angle 60^\circ + 0.04\angle -30^\circ + 0.02\angle 150^\circ$$

$$= (0.01 + j0.017) + (0.035 - j0.02) + (-0.017 + j0.01)$$

$$= 0.028 + j0.007 = \underline{0.0289\angle 14.04^\circ}$$

$\therefore$ $\underline{i_{in}(t) = 0.0289\cos(1000t + 14.04^\circ) \text{ (A)}}$

ECE 201 HW 32 Solutions



10-21 $Z_1(jw) = \frac{1}{jwC} = -j\frac{1}{wC}$

$Z_2(jw) = \frac{1}{Y_2(jw)} = \frac{1}{\frac{1}{R}+\frac{1}{jwL}} = \frac{jwLR}{jwL+R}$

$= \frac{jwLR(R-jwL)}{R^2+w^2L^2} = \frac{w^2L^2R}{R^2+w^2L^2} + j\frac{wLR^2}{R^2+w^2L^2}$

$Z_{in}(jw) = Z_1(jw) + Z_2(jw)$ (in series)

$= \frac{w^2L^2R}{R^2+w^2L^2} + j\left(\frac{wLR^2}{R^2+w^2L^2} - \frac{1}{wC}\right)$

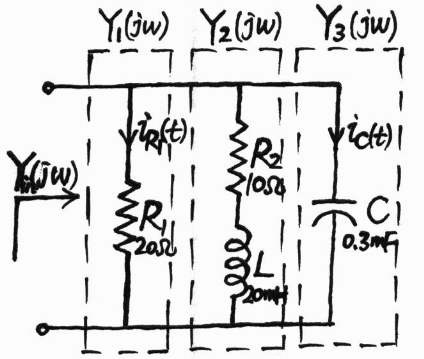
(a) $w = 100$ rad/sec $\quad Z_{in}(j100) = \frac{1000}{200} + j\left(\frac{1000}{200} - 10\right) = \underline{5 - j5 = 5\sqrt{2}\angle -45^\circ\,\Omega}$

(b) When $w \to \infty$: $\frac{w^2L^2R}{R^2+w^2L^2} \to R$, and $\left(\frac{wLR^2}{R^2+w^2L^2} - \frac{1}{wC}\right) \to 0$. So $\underline{Z_{in}(jw) \to R \text{ as } w \to \infty}$.

(c) $V_{in}(t) = 10\sqrt{2}\cos(100t)$ V, $\quad V_{in} = 10\sqrt{2}\angle 0^\circ = 10\sqrt{2} + j0$ V.

$I_{in} = \frac{V_{in}}{Z(j100)} = \frac{10\sqrt{2}\angle 0^\circ}{5\sqrt{2}\angle -45^\circ} = 2\angle 45^\circ = \sqrt{2} + j\sqrt{2}$ A.

$\therefore i_{in}(t) = \underline{2\cos(100t + 45^\circ)\ \text{A}}$.



10-23 $Y_1(jw) = \frac{1}{R_1}$

$Y_2(jw) = \frac{1}{Z_2(jw)} = \frac{1}{R_2+jwL} = \frac{R_2}{R_2^2+w^2L^2} - j\frac{wL}{R_2^2+w^2L^2}$

$Y_3(jw) = jwC$

$Y_{in}(jw) = Y_1(jw) + Y_2(jw) + Y_3(jw)$ (in parallel)

$= \left(\frac{1}{R_1} + \frac{R_2}{R_2^2+w^2L^2}\right) + j\left(wC - \frac{wL}{R_2^2+w^2L^2}\right)$

(a) $w = 500$ rad/sec, $\quad Y_{in}(j500) = \left(\frac{1}{20} + \frac{10}{200}\right) + j\left(\frac{3}{20} - \frac{10}{200}\right) = \underline{0.1 + j0.1 = 0.1414\angle 45^\circ\ \text{S}}$

(b) $wC - \frac{wL}{R_2^2+w^2L^2} = 0 \Rightarrow C = \frac{L}{R_2^2+w^2L^2} = \underline{1\times 10^{-4}\,\text{F} = 0.1\ \text{mF}}$

(c) $i_{in}(t) = 0.1\cos(500t)$ A. $\quad I_{in} = 0.1\angle 0^\circ = 0.1 + j0$ A.

$I_{R_1} = \frac{Y_1(j500)}{Y_{in}(j500)} I_{in} = \frac{0.05\angle 0^\circ}{0.1414\angle 45^\circ}(0.1\angle 0^\circ) = 0.0354\angle -45^\circ$ A. (current division)

$I_C = \frac{Y_3(j500)}{Y_{in}(j500)} I_{in} = \frac{0.15\angle 90^\circ}{0.1414\angle 45^\circ}(0.1\angle 0^\circ) = 0.1061\angle 45^\circ$ A.

$\therefore i_{R_1}(t) = \underline{0.0354\cos(500t - 45^\circ)\ \text{A}}$, $\quad i_C(t) = \underline{0.1061\cos(500t + 45^\circ)\ \text{A}}$.

10-25 (a) $Z_{in}(jw) = R + jwL + \frac{1}{jwC} = R + j\left(wL - \frac{1}{wC}\right)$. $\quad w = \frac{1}{\sqrt{LC}} = 2500$ rad/sec, $Z_{in}(jw)$ is real.

$|Z_{in}(jw)| = \sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2} \geq \sqrt{R^2 + 0} = R$. $\quad \therefore |Z_{in}(jw)|_{min} = R = \underline{5\Omega}$.

(b) $Y_{in}(jw) = \frac{1}{R} + \frac{1}{jwL} + jwC = \frac{1}{R} + j\left(wC - \frac{1}{wL}\right)$. $\quad w = \frac{1}{\sqrt{LC}} = 2500$ rad/sec, $Y_{in}(jw)$ is real.

$|Y_{in}(jw)| = \sqrt{\frac{1}{R^2} + \left(wC - \frac{1}{wL}\right)^2} \geq \sqrt{\frac{1}{R^2} + 0} = \frac{1}{R}$. $\quad \therefore |Y_{in}(jw)|_{min} = \frac{1}{R} = \underline{0.2\ \text{S}}$.

ECE 201 Spring 2010

Homework 33 Solutions

Problem 30

(a)

$$\begin{aligned}
Y_{in}(j\omega) &= \frac{1}{R} + \frac{1}{j\omega L} \\
&= 0.05 - \frac{j0.25}{\omega} \\
Z_{in}(j\omega) &= \frac{1}{Y_{in}(j\omega)} \\
&= \frac{j20\omega}{5 + j\omega}
\end{aligned}$$

(b)

$$\begin{aligned}
\mathbf{I_L} &= \frac{\mathbf{I_{in}} Z_{in}(j\omega)}{j\omega L} \\
&= 10 \times \frac{10(i + j)}{j20} \\
&= 5\sqrt{2}e^{-j\pi/4} \\
\Rightarrow i_L(t) &= 10\cos(5t - \pi/4)\ mA
\end{aligned}$$

Problem 40

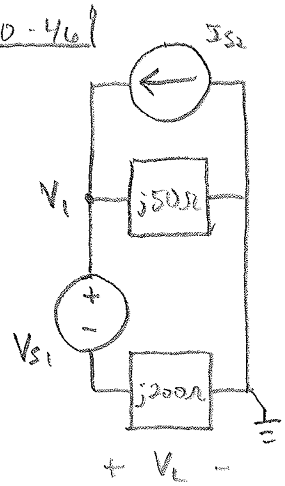
Let the impedances of $R_1,\ C$ and $R_2,\ L$ combinations be Z_1 and Z_2 respectively.

$$\begin{aligned}
Z_1 &= \left[\frac{1}{500} + j(400 \times 5 \times 10^{-6})\right]^{-1} \\
&= 250(1 - j) \\
Z_2 &= \left[\frac{1}{100} + \frac{1}{j(400 \times 0.125)}\right]^{-1} \\
&= 20(1 + 2j)
\end{aligned}$$

Let the phasor current through the source be $\mathbf{I}$.

$$\begin{aligned}
\mathbf{I} &= \frac{\mathbf{V}}{270 - j210} \\
\Rightarrow \mathbf{V_C} &= \mathbf{I} \times 250(1 - j) \\
&= \frac{120}{\sqrt{2}}(1.0256 - j0.1282) \\
\Rightarrow v_C(t) &= 124.032\cos(400t - 7.125^\circ)\ V \\
\mathbf{I_L} &= \left(\frac{\mathbf{V}}{270 - j210}\right)\left(\frac{R_2}{R_2 + j\omega L}\right) \\
&= \frac{\mathbf{V} \times 100}{(270 - j210)(100 + j50)} \\
&= \frac{240}{\sqrt{2}}(0.00128205 + j0.0002564) \\
\Rightarrow i_L(t) &= 0.3138\cos(400t + 11.3099^\circ)\ A
\end{aligned}$$

10-46



a) $V_L = A V_{S1} + b I_{S2}$

Node eq. @ V_1 : $I_{S2} = \dfrac{V_1}{j50} + \dfrac{V_L}{j200}$

and $V_1 = V_L + V_{S1}$

$$\Rightarrow \left[I_{S2} = \frac{V_L + V_{S1}}{j50} + \frac{V_L}{j200} \right] j200$$

$$\Rightarrow j200\, I_{S2} = 4V_L + 4V_{S1} + V_L$$

$$\Rightarrow 5V_L = -4V_{S1} + j200\, I_{S2}$$

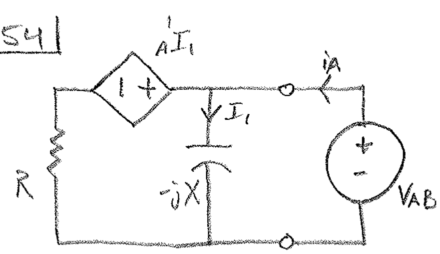
$$\Rightarrow \boxed{V_L = -\tfrac{4}{5} V_{S1} + j40\, I_{S2}} \quad A = -0.8\ \Omega,\ b = j40\ \Omega$$

b) $V_{S1} = 10\cos(100\pi t)\ V = 10\angle 0^\circ$

$I_{S2} = 200\sin(100\pi t)\ mA = 0.2\cos(100\pi t - 90^\circ) \Rightarrow 0.2\angle -90^\circ$

$$V_L = -\tfrac{4}{5}(10\angle 0^\circ) + (40\angle 90^\circ)(0.2\angle -90^\circ) = \boxed{0\ V}$$

10-54



a) $I_1 = \dfrac{V_{AB}}{-jX}$

$$i_A = I_1 + \frac{V_{AB} - A' I_1}{R}$$

$$= j\frac{V_{AB}}{X} + \frac{V_{AB}}{R} - \frac{A'}{R}\cdot j\frac{V_{AB}}{X}$$

$V_{AB} = i_A \cdot Z_{TH} + V_{OC}$

$$= V_{AB}\left(\frac{j}{X} + \frac{1}{R} - \frac{jA'}{RX}\right)$$

So $V_{AB} = \left[\dfrac{1}{R} + j\left(\dfrac{1}{X} - \dfrac{A'}{RX}\right)\right]^{-1} \cdot i_A \Rightarrow \boxed{Z_{TH} = \left[\dfrac{1}{R} + j\left(\dfrac{1}{X} - \dfrac{A'}{RX}\right)\right]^{-1}}$

b) $i_{IN} = I_M \angle 0^\circ$

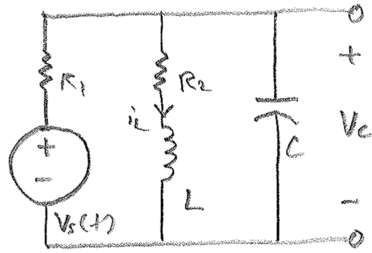
$$V_{AB} = i_{IN}\cdot Z_{TH} = (I_M\angle 0^\circ)\left[\frac{1}{10} + j\left(\frac{1}{20} - \frac{5}{200}\right)\right]^{-1}$$

$$= (I_M\angle 0^\circ)\left[\frac{1}{10} + j\frac{5}{200}\right]^{-1}\cdot\frac{\frac{1}{10} - j\frac{1}{40}}{\frac{1}{10} - j\frac{1}{40}} = (I_M\angle 0^\circ)\left[\frac{\frac{1}{10} - j\frac{1}{40}}{\frac{1}{100} + \frac{1}{1600}}\right]$$

$$= \frac{1600}{17}(I_M\angle 0^\circ)\left(\frac{\sqrt{17}}{40}\angle -14.04^\circ\right) = \boxed{\begin{aligned} &9.7\, I_M \angle -14.04^\circ \\ =\ &9.7\cdot I_M \cos(\omega_0 t - 14.04^\circ)\ V \end{aligned}}$$

10-60)

$\omega = 1000 \qquad V_s = 20\angle 0°$



A) $\dfrac{V_c - V_s}{R_1} + \dfrac{V_c}{R_2 + j\omega L} + \dfrac{V_c}{1/j\omega C} = 0$

$\Rightarrow \dfrac{V_c - 20}{40} + \dfrac{V_c}{20 + j20} + j V_c \cdot 0.075 = 0$

$\Rightarrow V_c\left(\dfrac{1}{40} + \dfrac{1}{20+j20} + j\,0.075\right) = \dfrac{1}{2}$

$\Rightarrow V_c\left(\dfrac{1}{40} + \dfrac{1-j}{20(1+j)(1-j)} + j\,0.075\right) = \dfrac{1}{2} = V_c\left(\dfrac{2-j}{40} + j\dfrac{75}{1000}\right)$

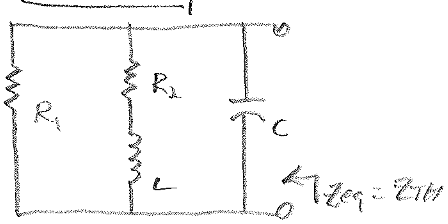
$\Rightarrow V_c\left(\dfrac{1}{20} + j\dfrac{1}{20}\right) = \dfrac{1}{2} \Rightarrow V_c = 0.5\cdot\left(\dfrac{1}{20} + j\dfrac{1}{20}\right)^{-1} = 5 - j5$

$\Rightarrow \boxed{V_c = 5\sqrt{2}\angle -45°\,V = 5\sqrt{2}\cos(1000t - 45°)\,V}$

b) $i_L = \dfrac{V_c}{R_2 + j\omega L} = \dfrac{5\sqrt{2}\angle -45°}{20 + j20} = \dfrac{5\sqrt{2}\angle -45°}{20\sqrt{2}\angle 45°}$

$\boxed{i_L = 0.25\angle -90°\,A = 0.25\cos(1000t - 90°)\,A}$

c) $\boxed{V_{oc} = V_c}$

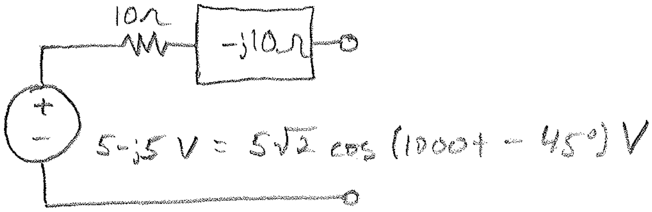


$Z_{eq} = \left(\dfrac{1}{R_1} + \dfrac{1}{R_2 + j\omega L} + j\omega C\right)^{-1}$

$= \left(\dfrac{1}{40} + \dfrac{1}{20+j20} + j\dfrac{75}{1000}\right)^{-1}$

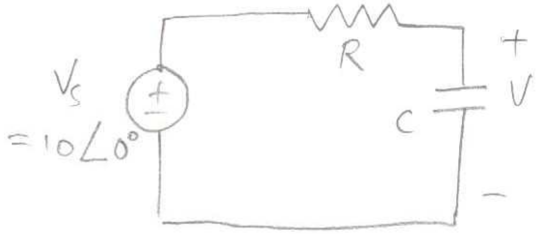
$= \left(\dfrac{1}{20} + j\dfrac{1}{20}\right)^{-1} = \boxed{10 - j10\,\Omega}$

Thevenin eq:



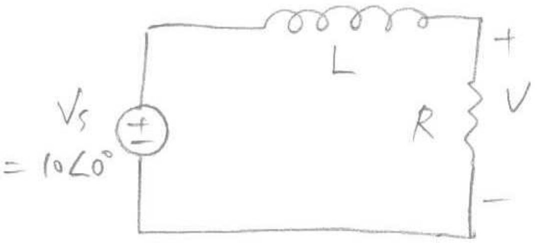
HW #35.

66. (a) Figure P 10.66 shows that we have a low-pass filter



$$\frac{V}{V_s} = \frac{\frac{1}{jwC}}{R + \frac{1}{jwC}} = \frac{1}{jwCR + 1}$$

$$\therefore \left|\frac{V}{V_s}\right| = \frac{1}{\sqrt{w^2C^2R^2+1}} \quad (\text{low-pass filter})$$



$$\frac{V}{V_s} = \frac{R}{R + jwL} = \frac{1}{1 + j\frac{wL}{R}}$$

$$\therefore \left|\frac{V}{V_s}\right| = \frac{1}{\sqrt{1 + \frac{w^2L^2}{R^2}}} \quad (\text{low-pass filter})$$

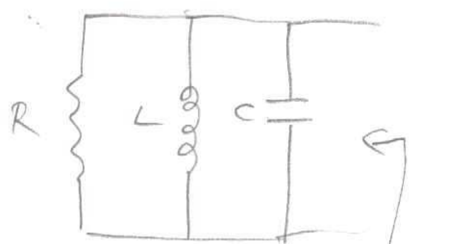
(b) when $w = 2000$ (rad/s), we have $V = |V_s| \times \frac{\sqrt{2}}{2}$ $\left(\frac{\sqrt{2}}{2} \cong 0.707\right)$

$$\therefore \frac{1}{\sqrt{w^2C^2R^2+1}} = \frac{1}{\sqrt{2}} \qquad \therefore C = \frac{1}{wR} = \frac{1}{2000 \times 10} = 50\,\mu F$$

$$\frac{1}{\sqrt{1 + \frac{w^2L^2}{R^2}}} = \frac{1}{\sqrt{2}} \qquad \therefore L = \frac{R}{w} = \frac{10}{2000} = 5\,mH$$

68. At $w = 500$ (rad/s), we have maximum $Y(jw)$

①

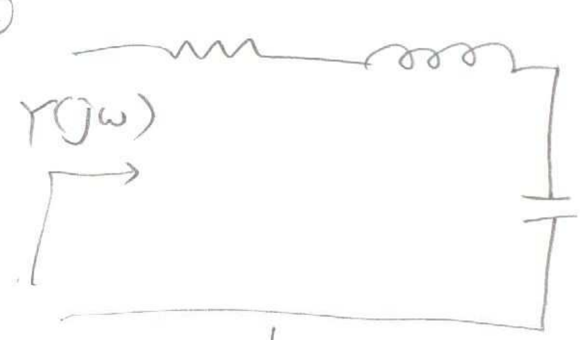


$$Y(jw) = 1/R + j\left(wC - \frac{1}{wL}\right)$$

$$|Y(jw)|^2 = (1/R)^2 + \left(wC - \frac{1}{wL}\right)^2$$

$\therefore$ $Y(jw)$ has the minimum at a particular w (because $\left(wC - \frac{1}{wL}\right)^2 \geq 0$)

②



$$Y(jw) = \frac{1}{R + j\left(wL - \frac{1}{wC}\right)}$$

$$|Y(jw)|^2 = \frac{1}{R^2 + \left(wL - \frac{1}{wC}\right)^2}$$

$\therefore$ $Y(jw)$ has the maximum at a particular w

$\therefore$ We select ②

$|Y(jw)|$ is maximized when $wL - \frac{1}{wC} = 0$

$\therefore$ $W = \frac{1}{\sqrt{LC}} = 500$ $\quad\therefore$ $C = \frac{1}{500^2 \times L} = 0.1\,mF$ $(L = 40mH)$

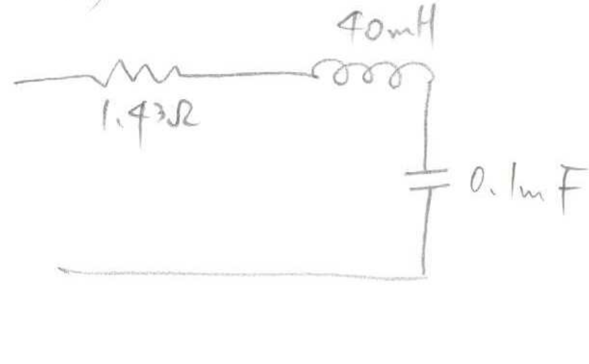
Suppose magnitude of V_s is V_s

then. $Y(j500) = \frac{1}{R} = \frac{1}{V_s}$ $\quad\therefore$ $R = V_s$ $\quad\cdots(1)$

when $w \cong 420$ (rad/s), $Y(jw)$ is $\frac{0.2}{V_s}$ as shown in Figure P10.68

$$\therefore \sqrt{\frac{1}{R^2 + \left(420L - \frac{1}{420C}\right)^2}} = 0.2 \times \frac{1}{R} \quad (\text{From } (1))$$

$\therefore$ $R \cong 1.43\,\Omega$



ECE 201 HW 36 Solutions

11-1 $P(t) = i^2(t) R = \begin{cases} 2(e^t-1)^2, & 0 \le t < 1 \\ 0, & 1 \le t < 2 \end{cases}$ *period* $T = 2$.

$$P_{ave} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{2}\int_0^1 2(e^t-1)^2\,dt = \int_0^1 e^{2t} - 2e^t + 1\,dt$$
$$= \left[\frac{1}{2}e^{2t} - 2e^t + t\right]_0^1 = \frac{1}{2}e^2 - 2e^1 + 1 - \frac{1}{2} + 2 - 0 = \underline{0.758 \text{ W}}$$

11-2 $w = 2\pi f = 2\pi\frac{1}{T}$, $R = 1000\,\Omega$, $\cos^2(\theta) = \frac{1}{2}(1+\cos(2\theta))$

(a) $i(t) = 0.01\cos(10t)$ A, $w = 10$, $T = \frac{2\pi}{10}$.

$$P(t) = i^2(t) R = 0.1\cos^2(10t) = 0.05(1+\cos(20t))$$
$$P_{ave} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{\frac{2\pi}{10}}\int_0^{\frac{2\pi}{10}} 0.05(1+\cos(20t))\,dt$$
$$= \frac{1}{4\pi}\left[t + \frac{1}{20}\sin(20t)\right]_0^{\frac{2\pi}{10}} = \frac{1}{4\pi}\left[\frac{2\pi}{10} + \frac{1}{20}\cancelto{0}{\sin(4\pi)} - 0 - \frac{1}{20}\cancelto{0}{\sin(0)}\right] = \underline{0.05\text{W}}$$

(b) $i(t) = 0.01|\cos(10t)|$ A, $w = 20$, $T = \frac{2\pi}{20}$.

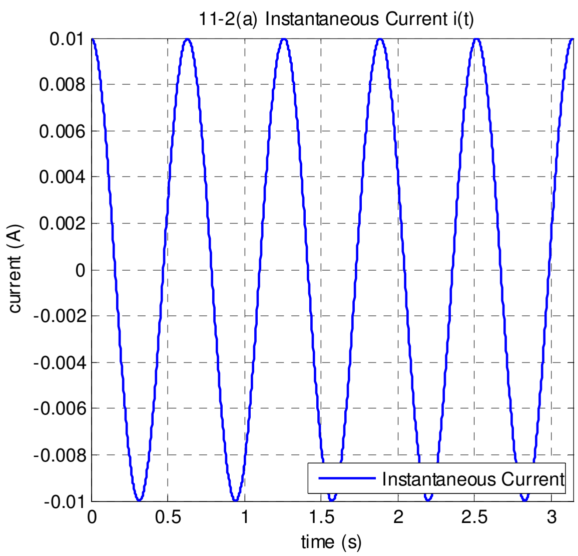
$$P(t) = i^2(t) R = 0.1\,|\cos(10t)|^2 = 0.1\cos^2(10t) = 0.05(1+\cos(20t))$$

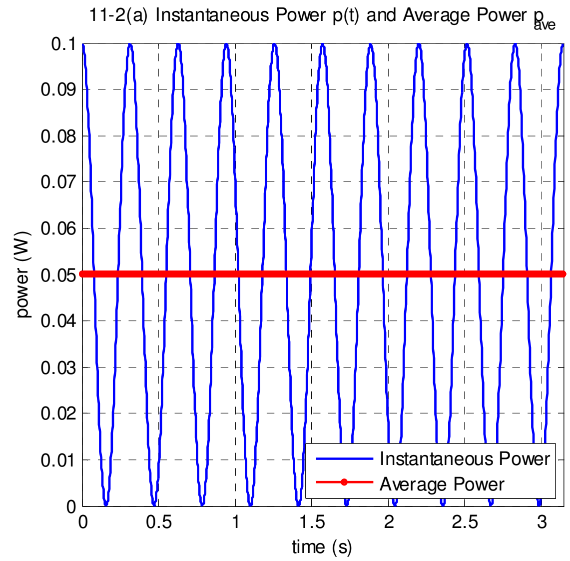
Similar to (a), $P_{ave} = \frac{1}{T}\int_0^T P(t)\,dt = \underline{0.05 \text{ W}}$

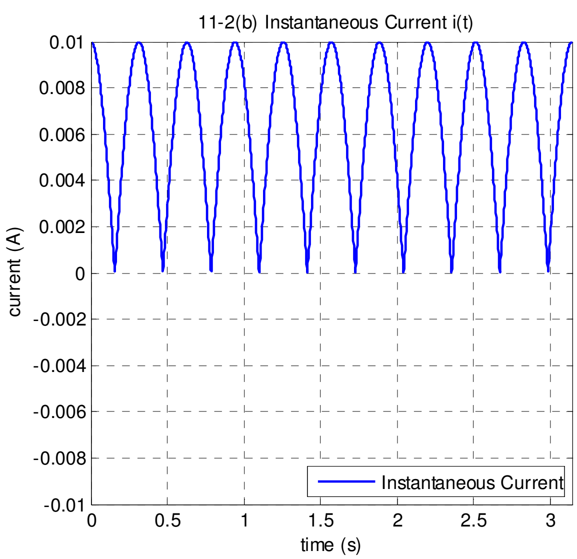
(c) $i(t) = 0.01\cos^2(10t)$ A $= 0.005(1+\cos(20t))$ A, $w = 20$, $T = \frac{2\pi}{20}$.

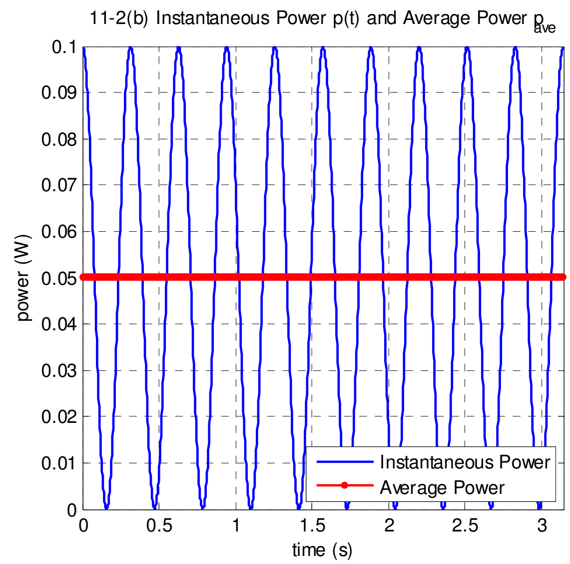
$$P(t) = i^2(t) R = 0.025(1+\cos(20t))^2 = 0.025[1 + 2\cos(20t) + \cos^2(20t)]$$
$$= 0.025[1.5 + 2\cos(20t) + 0.5\cos(40t)]$$
$$P_{ave} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{\frac{2\pi}{20}}\int_0^{\frac{2\pi}{20}} 0.025[1.5 + 2\cos(20t) + 0.5\cos(40t)]\,dt$$
$$= \frac{20}{2\pi}\left[0.0375t + 0.05\left(\frac{1}{20}\right)\sin(20t) + 0.0125\left(\frac{1}{40}\right)\sin(40t)\right]_0^{\frac{2\pi}{20}}$$
$$= \frac{20}{2\pi}\left[0.0375\left(\frac{2\pi}{20}\right) + 0.05\left(\frac{1}{20}\right)\cancelto{0}{\sin(2\pi)} + 0.0125\left(\frac{1}{40}\right)\cancelto{0}{\sin(4\pi)}\right.$$
$$\left. - 0.0375(0) - 0.05\left(\frac{1}{20}\right)\cancelto{0}{\sin(0)} + 0.0125\left(\frac{1}{40}\right)\cancelto{0}{\sin(0)}\right]$$
$$= \underline{0.0375 \text{ W}}$$

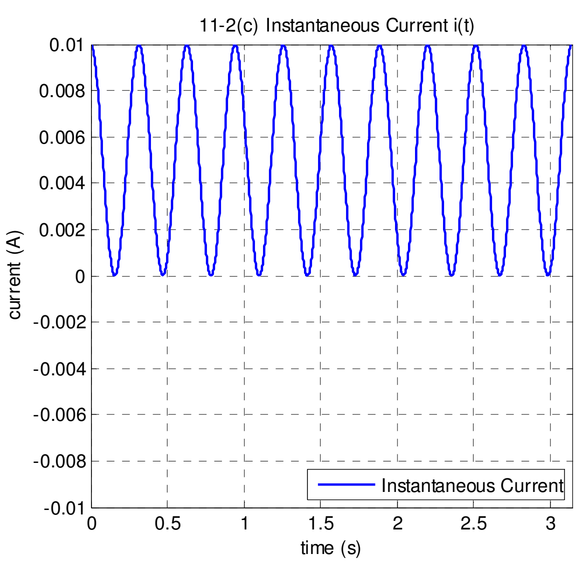
(d) see next page.

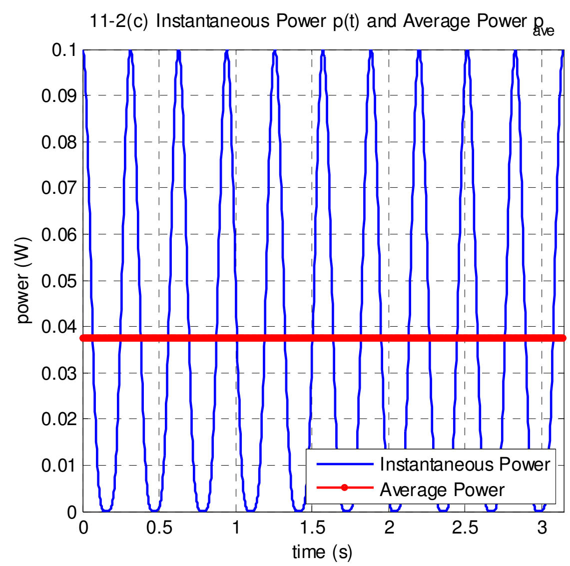

11-2(a) Instantaneous Current i(t)
current (A)
time (s)
Instantaneous Current


11-2(a) Instantaneous Power p(t) and Average Power p_{ave}
power (W)
time (s)
Instantaneous Power
Average Power


11-2(b) Instantaneous Current i(t)
current (A)
time (s)
Instantaneous Current


11-2(b) Instantaneous Power p(t) and Average Power p_{ave}
power (W)
time (s)
Instantaneous Power
Average Power


11-2(c) Instantaneous Current i(t)
current (A)
time (s)
Instantaneous Current


11-2(c) Instantaneous Power p(t) and Average Power p_{ave}
power (W)
time (s)
Instantaneous Power
Average Power

ECE 201 Spring 2010

Homework 37 Solutions

Problem 5

(a)

For Figure (a),

$$\begin{aligned}
I_{eff}^2 &= \frac{1}{T}\int_0^T i^2(t)\,dt \\
&= \frac{1}{9}\left[\int_0^3 9\,dt + \int_3^6 16\,dt\right] \\
&= \frac{25}{3} \\
\Rightarrow I_{eff} &= 5/\sqrt{3}
\end{aligned}$$

For Figure (b),

$$\begin{aligned}
I_{eff}^2 &= \frac{1}{4}\left[\int_0^1 9\,dt + \int_2^3 16\,dt\right] \\
&= 25/4 \\
\Rightarrow I_{eff} &= 2.5
\end{aligned}$$

(b)

Current through R_L is given by $i(t) \times 60/(30+60) = (2/3)i(t)$. Power absorbed by R_L is then given by the square of the effective current through it times the resistance. Thus

$$\begin{aligned}
P &= \frac{4}{9} \times \frac{25}{3} \times 30 \\
&= \frac{1000}{9}\,W
\end{aligned}$$

(c)

$$
\begin{aligned}
P &= \frac{4}{9} \times \frac{25}{4} \times 30 \\
&= \frac{250}{3}\, W
\end{aligned}
$$

Problem 7

In all parts of the problem, the following identities will be used,

$$
\int_0^{2\pi/m} \cos(mnx)\, dx = 0
$$
$$
\int_0^{2\pi/m} \sin(mnx)\, dx = 0
$$

Here m and n are integers.

(a)

$$
\begin{aligned}
V_{1eff}^2 &= \frac{1}{T}\int_0^T v_1^2(t)\, dt \\
&= \frac{1}{T}\int_0^T [102 + 40\cos(20t) + 2\cos(40t)]\, dt \\
&= 102 \\
\Rightarrow V_{1eff} &= 10.0995
\end{aligned}
$$

(b)

$$
\begin{aligned}
V_{2eff}^2 &= \frac{1}{T}\int_0^T v_2^2(t)\, dt \\
&= \frac{1}{T}\int_0^T [50\{1+\cos(4t)\} + 12.5\{1+\cos(8t)\} + 50\{\cos(6t)+\cos(2t)\}]\, dt \\
&= 62.5 \\
\Rightarrow V_{2eff} &= 7.9057
\end{aligned}
$$

(c)

$$
\begin{aligned}
V_{3eff}^2 &= \frac{1}{T}\int_0^T v_3^2(t)\, dt \\
&= \frac{1}{T}\int_0^T [50\{1+\cos(4t)\} + 36.4277\{1+\cos(8t)\} + 6.2499\{1-\cos(8t)\} + \ldots]\, dt \\
&= 92.6776 \\
\Rightarrow V_{3eff} &= 9.6269
\end{aligned}
$$

Problem 9

(a)

$$
\begin{aligned}
Z_L &= \left[\frac{1}{5} + j(30 \times 5 \times 10^{-3})\right]^{-1} \\
&= 0.8(4 - j3) \\
\Rightarrow \mathbf{V_L} &= \mathbf{I_{in}} \times Z_L \\
&= \frac{4}{\sqrt{2}}(4 - j3) \\
\Rightarrow v_L(t) &= 20\cos(30t - 36.87^\circ)\ V
\end{aligned}
$$

(b)

$$
\begin{aligned}
P_{inst}(t) &= i_{in}(t) v_L(t) \\
&= 100\cos(30t)\cos(30t - 36.87^\circ)\ W \\
P_{average} &= \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) \\
&= 50\cos 36.87^\circ \\
&= 40\ W
\end{aligned}
$$

Problem 10

(a)

$$
\begin{aligned}
\mathbf{I_s} &= \frac{50\angle -90^\circ}{6 + j12 - j4} \\
&= 5\angle -143.13^\circ\ (magnitude = 5)
\end{aligned}
$$

(b)

$$
\begin{aligned}
P_{average} &= V_{eff} I_{eff}\cos(\theta_v - \theta_i) \\
&= 5 \times 50\cos(-53.13^\circ) \\
&= 150\ W
\end{aligned}
$$

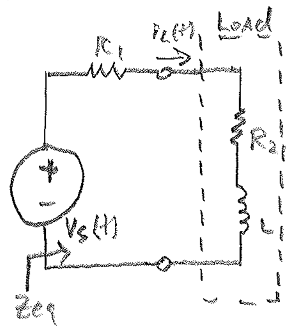
(c)

$$\begin{aligned} P_{average} &= R|\mathbf{I_s}|^2 \\ &= 6 \times 25 \\ &= 150\ W\ (same\ as\ part\ (b)) \end{aligned}$$

(d)

$$\begin{aligned} \mathbf{I_s} &= \frac{50\angle -90^\circ}{30 + j50 - j10} \\ &= 1\angle -143.13^\circ\ (magnitude = 1) \\ P_{average} &= V_{eff} I_{eff} \cos(\theta_v - \theta_i) \\ &= 1 \times 50 \cos(-53.13^\circ) \\ &= 30\ W \end{aligned}$$

11-17)



$V_s(t) = 100\sqrt{2}\cos(500t + 30°) \Rightarrow 100\angle 30°\ V_{rms}$

$R_1 = 100\,\Omega \quad R_2 = 700\,\Omega \quad L = 1.2\,H$

$Z_{eq} = R_1 + R_2 + j\omega L = 100 + 700 + j500 \cdot 1.2$
$= 800 + j600$

$i_L = \dfrac{V_s}{Z_{eq}} = \dfrac{100\angle 30°}{800+j600} = \dfrac{3+4\sqrt{3}}{100} + j\dfrac{4-3\sqrt{3}}{100}$

$= 0.1\angle -6.87°$

$\Rightarrow \boxed{i_L(t) = 0.1\sqrt{2}\cos(500t - 6.87°)\ A}$

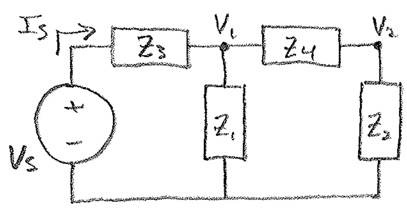
$V_L = V_s \cdot \dfrac{R_2 + j\omega L}{R_1 + R_2 + j\omega L} = (100\angle 30°)\left[\dfrac{700+j600}{800+j600}\right] = 46\sqrt{3} - 3 + j(46 + 3\sqrt{3})\ V_{rms}$

$= 92.195\angle 33.73°\ V_{rms}$

$S = V_{eff} I_{eff}^* = \left(46\sqrt{3} - 3 + j[46 + 3\sqrt{3}]\right)\left[\dfrac{3+4\sqrt{3}}{100} - j\dfrac{4-3\sqrt{3}}{100}\right]$

$\boxed{S = 7 + j6\ VA}$ $\quad \boxed{P_{Ave} = 7W} = \mathrm{Re}\{S\}$ $\quad |S| = \sqrt{49+36} = \boxed{\sqrt{85}\ VA}$

11-20)



$V_s = 2300\ V_{rms}$ @ 60 Hz

$S_1 = 20 + j8 \quad S_2 = 20 + j18 \quad S_3 = 5 + j6 \quad S_4 = 3 + j4$

A) $S_s = S_1 + S_2 + S_3 + S_4$
$= 20 + 20 + 5 + 3 + j(8 + 18 + 6 + 4) = \boxed{48 + j36\ kVA}$

$S_s = V_s I_s^* \Rightarrow I_s = \left(\dfrac{S_s}{V_s}\right)^*$

$I_s = \left(\dfrac{48 + j36\ kVA}{2300\ V_{rms}}\right)^* = \dfrac{480}{23} - j\dfrac{360}{23}\ A_{rms} = \boxed{20.8696 - j15.6522\ A_{rms}}$

b) $\boxed{I_s = 26.087\angle -36.87°\ A_{rms}}$

c) $S_s = S_3 + S_{1,2,4} \Rightarrow S_{1,2,4} = S_s - S_3 = 48 + j36 - (5 + j6) = \boxed{43 + j30\ kVA}$

d) $S_{1,2,4} = V_1 I_s^* \Rightarrow V_1 = \dfrac{S_{1,2,4}}{I_s^*} = \dfrac{43 + j30\ kVA}{\left(\frac{480}{23} - j\frac{360}{23}\ A\right)^*} = \dfrac{6026}{3} - j69\ V$

$V_1 = \boxed{2008.67 - j69\ V_{rms}} = \boxed{2009.85\angle -1.97°\ V_{rms}}$

c) Claim: $V_2 = V_1 \cdot \frac{S_2}{S_2+S_4}$

Proof: $V_1 \cdot \frac{S_2}{S_2+S_4} = V_1 \cdot \frac{V_2 \cdot I^*}{V_2 \cdot I^* + (V_1 - V_2) I^*} = V_1 \cdot \frac{I^*}{I^*} \cdot \frac{V_2}{V_2 + V_1 - V_2}$

$= V_1 \cdot \frac{V_2}{V_1} = V_2 \quad \square$

So $V_2 = V_1 \cdot \frac{S_2}{S_2+S_4} = \left(\frac{6026}{3} - j69\right) \cdot \frac{20+j18}{23+j22} = 1695.58 - j109.86$

$= \boxed{1695.58 - j109.86 \text{ } V_{rms} = 1699.14 \angle -3.71^\circ \text{ } V_{rms}}$

HW # 39.

22. complex power $S = P + jQ$

P: average power

Q: reactive power.

$Q = \pm P\sqrt{\frac{1}{pf^2} - 1}$ (pf: power factor)

$Q > 0$ if pf is lagging, $Q < 0$ if pf is leading.

(a) $P = 2kW$, $pf = 0.90$ lagging

$\therefore Q = 2 \times 10^3 \times \sqrt{\frac{1}{0.9^2} - 1} \cong 968.64$ (VAR)

$\therefore S = P + jQ = 2000 + j968.64$ (VA)

(b) $P = 4kW$, $pf = 0.90$ leading.

$\therefore Q = -4 \times 10^3 \times \sqrt{\frac{1}{0.9^2} - 1} = -1937.29$ (VAR)

$\therefore S = 4000 - j1937.29$ (VA)

23.

(a) absorbed power by Z_1

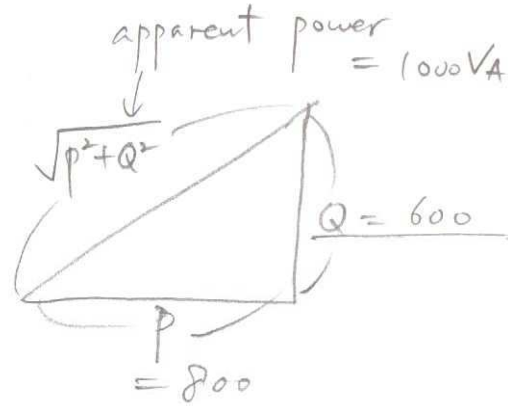
$\Rightarrow S_1 = 1600 \ (VA)$

absorbed power by Z_2

$\Rightarrow S_2 = P + jQ$

$= 1000 \times 0.8 + j\sqrt{1000^2 - 800^2}$

$= 800 + j600 \ (VA)$



$\therefore S_1 + S_2 = V_{in,eff} I_{s.eff}^*$

$2400 + j600 = 120 \times I_{s,eff}^*$

$\therefore I_{s,eff} = \left(\frac{2400+j600}{120}\right)^* = \underline{20 - j5 \ A_{rms}}$

(b) $S_1 = V_{1.eff} \cdot I_{s,eff}^*$

$\therefore 1600 = V_{1,eff} \cdot (20 + j5)$

$\therefore V_{1,eff} = \underline{75.29 - j18.82 \ V_{rms}}$

(c) $S_2 = V_{2,eff} \cdot I_{s.eff}^*$

$\therefore 800 + j600 = V_{2,eff} \cdot (20 + j5)$

$\therefore V_{2,eff} = \underline{44.71 + j18.82 \ V_{rms}}$

26. $\omega = 500$ rad/s. $V_s = 100\angle 0°$

(a) without capacitance.

$$S_L = V_{s,eff} \cdot I_{L,eff}^{*} = 86.6 + j50 \ (VA)$$

$$= V_{s,eff} \cdot \left(\frac{V_{s,eff}}{R + j\omega L} \right)^{*}$$

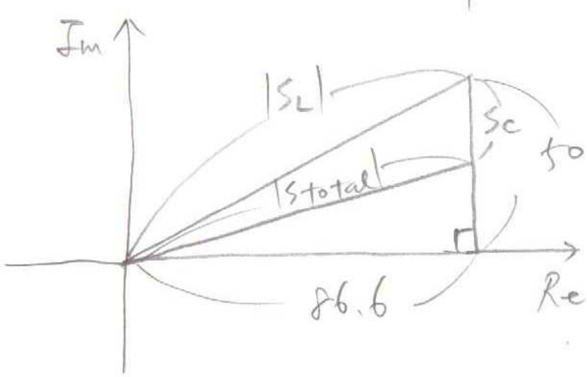
$$= 100 \cdot \left(\frac{100}{R + j500L} \right)^{*} = 10^4 \times \frac{R + j500L}{R^2 + 500^2 L^2} = 86.6 + j50$$

$$\therefore 86.6 = \frac{R \times 10^4}{R^2 + 500^2 L^2} \qquad 50 = \frac{500 \times L \times 10^4}{R^2 + 500^2 L^2}$$

$$R = 86.6\,\Omega, \qquad L = 0.1\,H$$

(b)

$$S_{total} = S_L + \underbrace{S_C}_{\text{power absorbed by capacitance}}$$



$$\frac{86.6}{|S_{total}|} = pf = 0.95$$

$$\therefore |S_{total}| \cong 91.158$$

$$\therefore S_{total} = 86.6 + j28.46$$

$$\therefore S_C = S_{total} - S_L = -j21.5356 \ (VA)$$

$$= V_s \cdot (V_s \times j\omega C)^{*} = -j(100 \times 100 \times 500 \times C) \ (VA)$$

$$\therefore \underline{C \cong 4.31\,\mu F}$$

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Homework 40 Solutions

Problem 30

(a)

$$
\begin{aligned}
Z_{th} &= (R_1||R_2) - j/\omega C \\
&= (20 - j10)\ \Omega \\
V_{oc} &= V_s \frac{R_2}{R_1 + R_2} \\
&= 20\ V_{rms}
\end{aligned}
$$

For maximum power transfer, $Z_L = Z_{th}^* = (20 + j10)\ \Omega$.

(b)

$$
\begin{aligned}
P_{avg} &= \frac{V_{oc}^2}{4R_L} \\
&= 5\ W
\end{aligned}
$$

Problem 37

(a)

Here we cannot apply the maximum power transfer theorem because R_L is fixed. The expression for average power across R_L is given by

$$
P_{avg} = \frac{V_s^2 R_L}{(R + R_L)^2 + (\omega L - 1/\omega C)^2}
$$

Clearly, the average power is maximum when the denominator is minimum, which happens when the following two conditions hold,

$$
R = 0,\ \omega L = \frac{1}{\omega C}
$$

$$\begin{aligned} C &= \frac{1}{\omega^2 L} \\ &= 2.5\ mF \\ P_{avg_{max}} &= \frac{V_s^2}{R_L} \\ &= \frac{50^2}{5} \\ &= 500\ W \end{aligned}$$

(b)

$$P_{avg} = \frac{V_s^2 R_L}{(R+R_L)^2 + (\omega L - 1/\omega C)^2}$$

For maximum P_{avg},

$$\begin{aligned} \frac{dP_{avg}}{dR_L} &= 0 \\ \Rightarrow R_L^2 &= R^2 + (\omega L - 1/\omega C)^2 \\ \Rightarrow R_L &= 4.011\ \Omega \end{aligned}$$