

Question 1. (10 points) See Figure on next page.

Question 2. (10 points) Every time all leaves are removed, the length of a longest path is decreased by 2. Therefore after k iterations it has been decreased by $2k$. If what remains after those k iterations is a single node, then $\ell(k) = 2k + 1$, but if what remains is an edge then $\ell(k) = 2k + 2$.

Question 3. (10 points)

Polish:

$$* + A * BC - * DE * FG$$

Reverse Polish:

$$ABC * + DE * FG * - *$$

Question 4. (10 points)

1. Complete binary tree of height 9. Average code length is 9.
2. Binary tree whose root has one child that is a leaf for the symbol of probability 0.5, and the other child is the root of a complete binary tree of height 8. Average code length is $1 * 0.5 + 256 * 9 * (1/512) = 5$.

