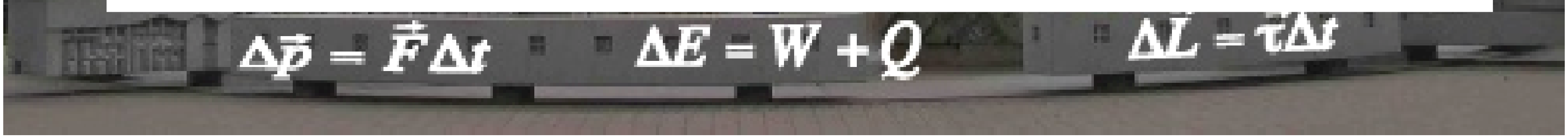
EXAM 1 is next week

Time: 8:00-9:30 pm Wed Feb 8
Place: Elliott Hall
Material: lectures 1-8, HW 1-8, Recitations 1-4, Labs 1-4
Problems: multiple choice, 10 questions (70 points)
write-up part, hand graded (30 points)
Equation sheet: provided with exam
Practice exam + equation sheet: will be posted at the end of this week

Note: no lecture on Thursday Feb 9!



TODAY

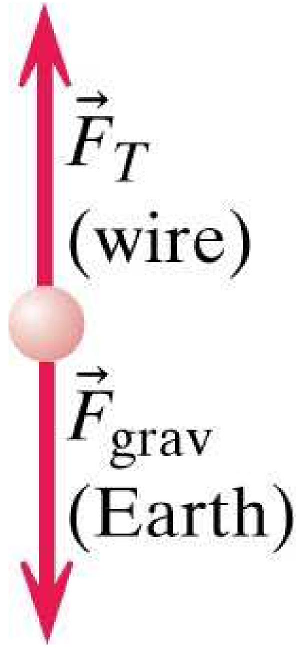
- Forces on a System
- Changing Momentum
- Circular Motion

Rules for Identifying Forces on a System

- Choose "The System"
- Draw arrows for all forces due to objects in the surroundings that interact with The System.
  - Objects that touch The System
  - Objects that interact from a distance
- Label each force with the name of the object in the surroundings

Example – Hanging Ball

System: Ball
Surroundings: Earth, Wire



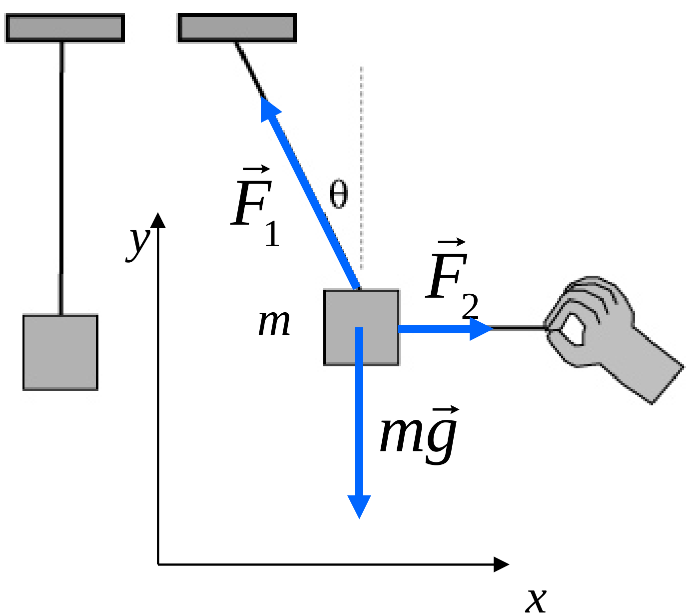
Free-body diagram

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

Equilibrium with two strings

Know mass.
Need to know the force F_2 required to achieve θ

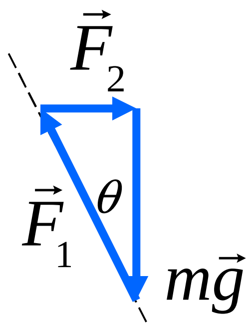
Static equilibrium



$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}_{net}} \qquad \frac{d\vec{p}}{dt} = \vec{0}$$

$$\vec{F}_1 + \vec{F}_2 + m\vec{g} = \vec{0}$$

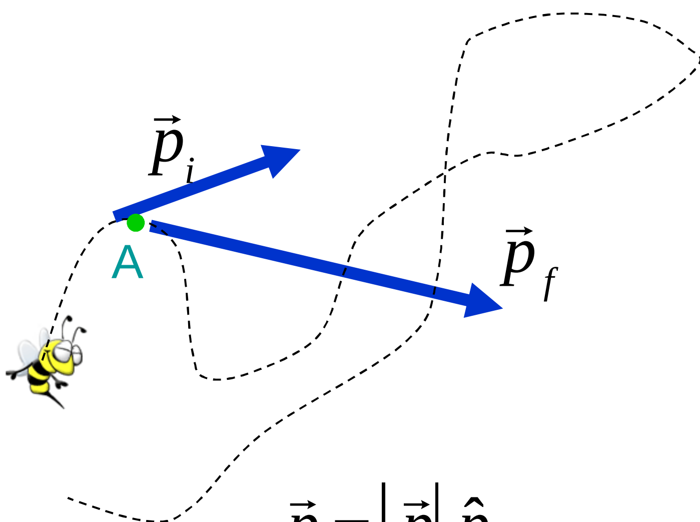
Geometrical:

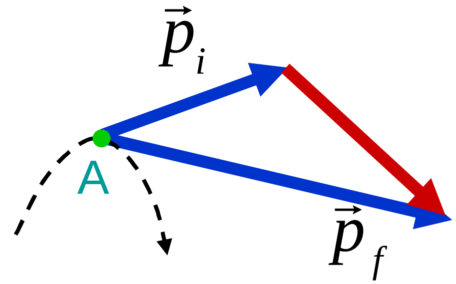


$$\left|\vec{F}_2\right| = mg\tan\theta$$

$$\vec{F}_2 = \langle mg\tan\theta, 0, 0\rangle$$

Changing momentum





$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

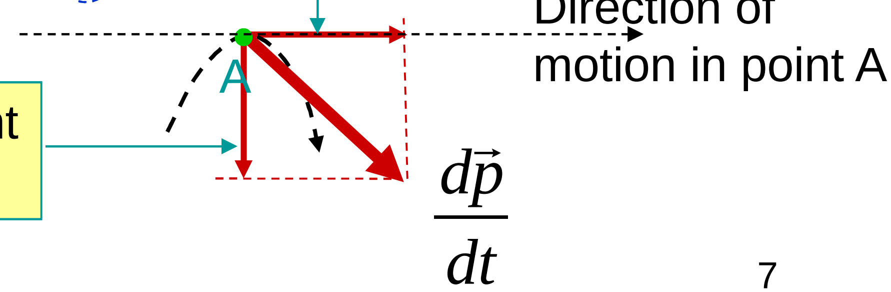
$$\vec{p} = |\vec{p}|\,\hat{p}$$

$$\frac{d\vec{p}}{dt} = \frac{d\left(|\vec{p}|\,\hat{p}\right)}{dt} = \boxed{\frac{d|\vec{p}|}{dt}\hat{p}} + \boxed{|\vec{p}|\frac{d\hat{p}}{dt}}$$

Parallel component
(Along trajectory)

Rate of change
of direction

Perpendicular component
(to trajectory)



Rate of change of direction

$$\Delta\hat{p} = \hat{p}_f - \hat{p}_i$$

Length of path A = $v\Delta t$

Angle = $v\Delta t / R$

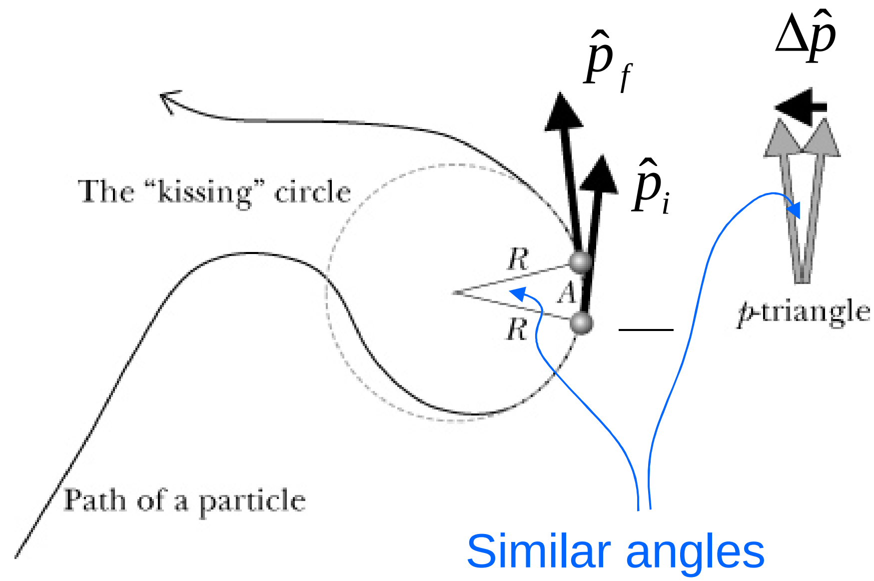
For small angle:

$$\frac{|\Delta\hat{p}|}{|\hat{p}|} = \frac{|\vec{v}|\Delta t}{R}$$

$$\frac{|\Delta\hat{p}|}{\Delta t} = \frac{|\vec{v}|}{R}|\hat{p}|$$

$$\left|\frac{\Delta\hat{p}}{\Delta t}\right| = \frac{|\vec{v}|}{R}$$

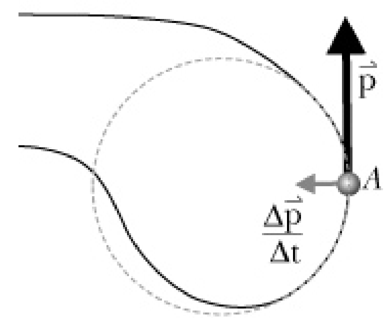
$$\left|\frac{d\hat{p}}{dt}\right| = \frac{|\vec{v}|}{R}$$



Direction:
For small angles $\Delta\hat{p}$ is perpendicular to $\vec{p}$

$d\hat{p}$ is directed toward the center of the kissing circle

$$\boxed{\frac{d\hat{p}}{dt} = \frac{|\vec{v}|}{R}\hat{n}}$$



$\hat{n}$ - Normal to the path
toward center of the "kissing" circle

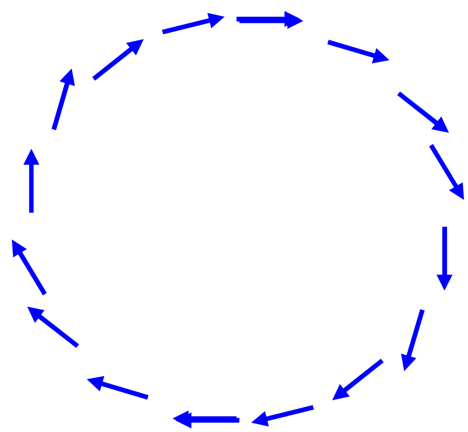
Components of Momentum Change

Perpendicular to Momentum

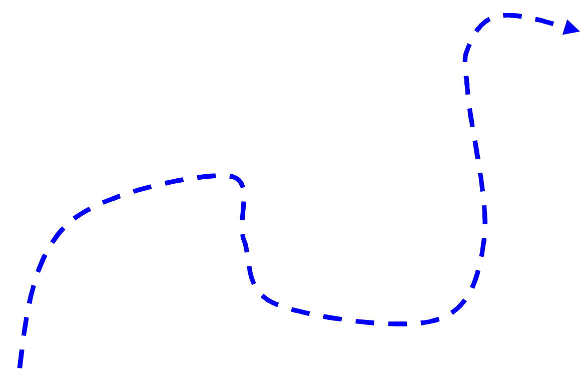
Parallel to Momentum

$$\frac{d\vec{p}}{dt} = \underbrace{|p|\frac{d\hat{p}}{dt}}_{= p\frac{\nu}{R}} + \hat{p}\frac{d|p|}{dt} \quad = \vec{F}_{\perp} + \vec{F}_{\parallel}$$

Change in Magnitude



Change in Direction



Change in Both

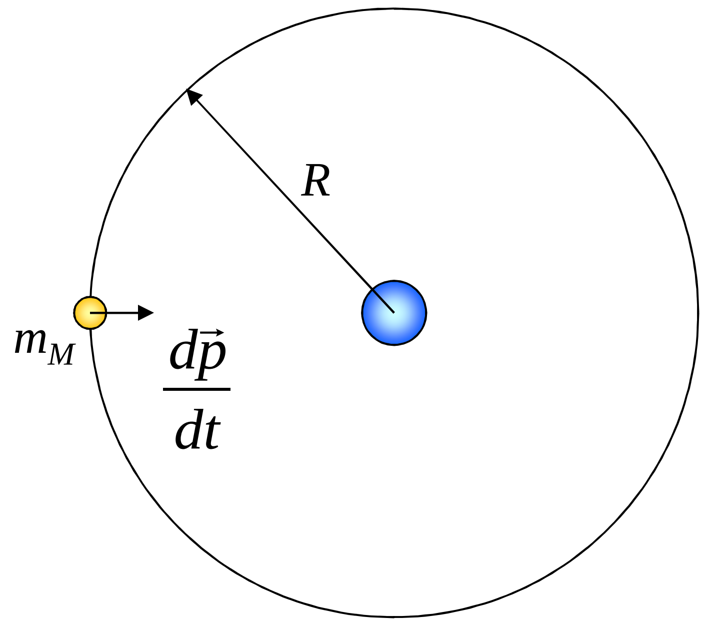
Example: the Moon and the Earth

Mass of the Moon: m_M = 7×10²² kg

Distance from the Earth: R = 4×10⁸ m

Period: T = 28 days

Question: $d\vec{p}/dt$ = ?



Solution:

Parallel: $\left(d\vec{p}/dt\right)_{\parallel} = dp/dt = 0$

Perpendicular: $$\left(\frac{d\vec{p}}{dt}\right)_{\perp} = p\frac{v}{R}$$

$$v = \frac{2\pi R}{T} = \frac{2\pi \cdot 4\times 10^{8}\,\mathrm{m}}{(28\text{ day})(24\text{h/day})(3600\text{s/h})} = 1\times 10^{3}\text{ m/s}$$

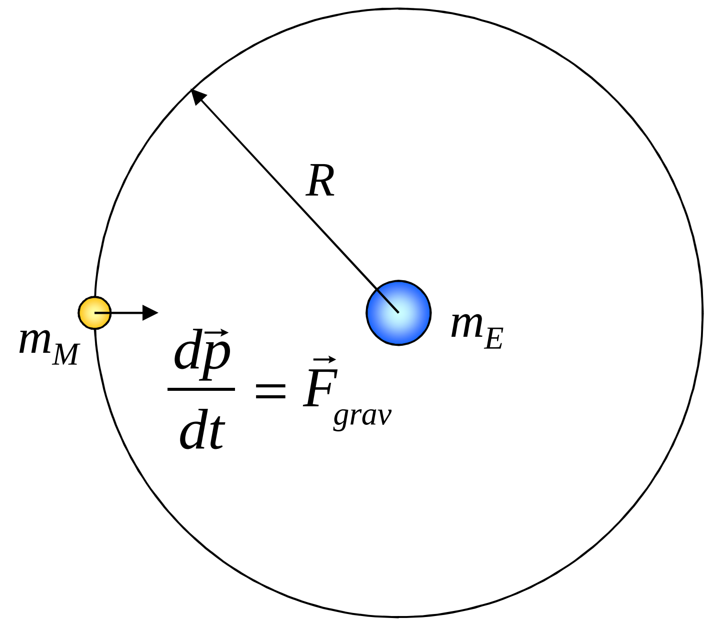
$$p \approx m_M v = 7.3\times 10^{25}\text{ kg}\cdot\text{m/s}$$

$$\left(\frac{d\vec{p}}{dt}\right)_{\perp} = p\frac{v}{R} = 7.3\times 10^{25}\text{ kg}\cdot\text{m/s}\,\frac{1\times 10^{3}\text{ m/s}}{4\times 10^{8}\text{ m}} = 1.8\times 10^{20}\,\text{kg}\cdot\text{m/s}^{2}$$

What is the direction of $\frac{d\vec{p}}{dt}$?

Example: the Moon and the Earth

Mass of the Moon:	$m_m = 7\times10^{22}$ kg
Distance from the Earth:	$R = 4\times10^{8}$ m
Period:	$T = 28$ days
Mass of the Earth:	$m_E = 6\times10^{24}$ kg
Question:	$F_{\text{Earth on }M}=?$



Solution:

$$\vec{F}_{Earth} = -G\frac{m_E m_M}{R^2}\hat{r}_{E-M}$$

$$F_{Earth} = 1.8\times10^{20}\ \text{N}$$

From motion path:

$$\left|\frac{d\vec{p}}{dt}\right| = 1.8\times10^{20}\,\text{kg}\cdot\text{m/s}^2$$

$$\boxed{\vec{F}_{grav} = \frac{d\vec{p}}{dt}}$$

any force

motion of any object

these are not the same things!

This is precise – works at any speed!

WHAT WE DID TODAY

- Forces on a System
- Changing Momentum
- Circular Motion – To be continued…