Key Ideas in Chapter 18: Magnetic Field

- **Moving charged particles make a magnetic field, which is different from an electric field.**
- **The needle of a magnetic compass aligns with the direction of the net magnetic field at its location.**

- **A current is a continuous flow of charge.**
  - Electron current is a number of electrons per second entering a section of a conductor.
  - Conventional current (Coulombs/second) is opposite in direction to the electron current, and is assumed to be due to positively charged particles.
- **The superposition principle can be applied to calculate the expected magnetic field from current-carrying wires in various configurations.**
  - A current-carrying loop is a magnetic dipole.
  - A bar magnet is also a magnetic dipole.
  - Even a single atom can be a magnetic dipole!

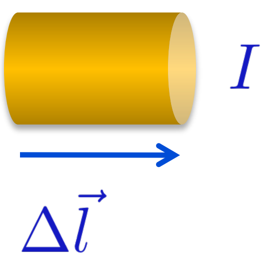
Last Time

- (Cross Products: Mathematically)
- Electron Current and Conventional Current
- Calculating the Electron Current
- True *vs.* Useful
- Biot-Savart Law in a Wire
- Relativity??

Biot-Savart Law

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{q\vec{v} \times \hat{r}}{|r|^2}$$

BIOT-SAVART LAW
point charge



$$\Delta\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{I\Delta\vec{l} \times \hat{r}}{|r|^2}$$

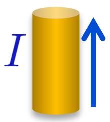
BIOT-SAVART LAW
current in a wire

$\Delta\vec{l}$ = length of this chunk of wire

Today

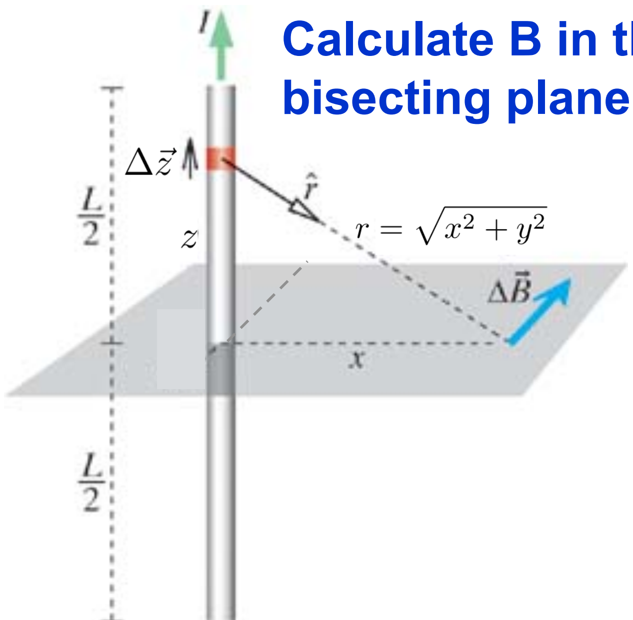
- Magnetic Field of a Straight Wire
- (Magnetic Field of a Current Loop)
- Magnetic Dipole Moment
- Bar Magnet
- Atomic Dipoles

Magnetic Field of a Straight Wire



$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{I\Delta\vec{l}\times\hat{r}}{|r|^2}$$

BIOT-SAVART LAW
current in a wire



Calculate B in the bisecting plane

$$\vec{r} = \langle x, 0, z\rangle \Rightarrow \left[\begin{array}{l} |r|^2 = (x^2 + z^2) \\ \hat{r} = \dfrac{\vec{r}}{|r|} = \dfrac{\langle x, 0, z\rangle}{\sqrt{x^2 + z^2}} \end{array}\right.$$

$$\Delta\vec{l} = \Delta\vec{z} = \langle 0, 0, \Delta z\rangle$$

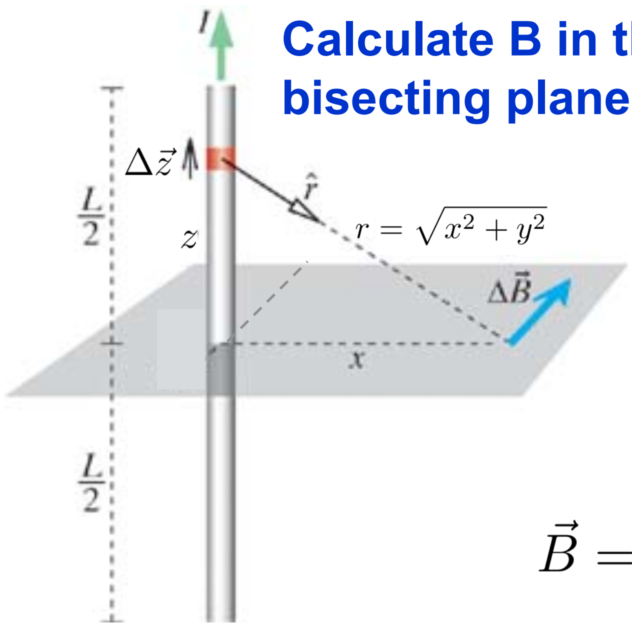
$$\Rightarrow \quad \Delta\vec{z}\times\hat{r} = \frac{1}{|r|}(\Delta\vec{z}\times\vec{r})$$

$$\Delta\vec{z}\times\hat{r} = \frac{1}{|r|}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \Delta z \\ x & 0 & z \end{vmatrix} = \frac{0\hat{x} + x\Delta z\hat{y} + 0\hat{z}}{|r|}$$

$$\boxed{\Delta\vec{z}\times\hat{r} = \frac{x\Delta z}{\sqrt{x^2 + z^2}}\hat{y}}$$

Magnetic Field of a Straight Wire

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{I\Delta\vec{l}\times\hat{r}}{|r|^2} \qquad |r|^2 = (x^2+z^2) \qquad \Delta\vec{l} = \Delta\vec{z} \qquad \Delta\vec{z}\times\hat{r} = \frac{x\Delta z}{\sqrt{x^2+z^2}}\hat{y}$$



Calculate B in the bisecting plane

$$\Delta\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{I\Delta\vec{l}\times\hat{r}}{|r|^2} = \left(\frac{\mu_o}{4\pi}\right)\frac{I\Delta\vec{z}\times\hat{r}}{(x^2+z^2)}$$

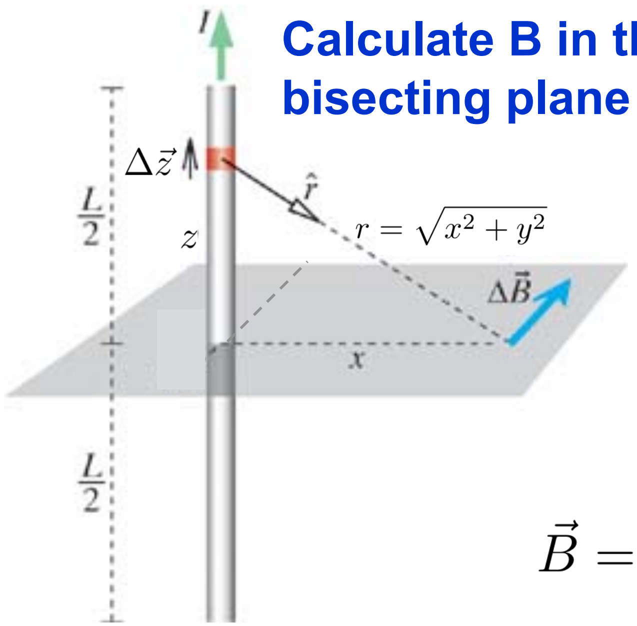
$$= \frac{\mu_o I}{4\pi}\hat{y}\frac{x\Delta z}{(x^2+z^2)^{3/2}}$$

Does x change during the integration?

$$\vec{B} = \int\Delta B = \frac{\mu_o I}{4\pi}\hat{y}\int_{-L/2}^{L/2}\frac{x}{(x^2+z^2)^{3/2}}dz$$

Magnetic Field of a Straight Wire

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{I\Delta\vec{l}\times\hat{r}}{|r|^2} \qquad |r|^2 = (x^2 + z^2) \qquad \Delta\vec{l} = \Delta\vec{z} \qquad \Delta\vec{z}\times\hat{r} = \frac{x\Delta z}{\sqrt{x^2+z^2}}\hat{y}$$



Calculate B in the bisecting plane

$$\Delta\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{I\Delta\vec{l}\times\hat{r}}{|r|^2} = \left(\frac{\mu_o}{4\pi}\right)\frac{I\Delta\vec{z}\times\hat{r}}{(x^2+z^2)}$$

$$= \frac{\mu_o I}{4\pi}\hat{y}\frac{x\Delta z}{(x^2+z^2)^{3/2}}$$

Observation point x does not change during the integration

$$\vec{B} = \int \Delta B = \frac{\mu_o I}{4\pi}\hat{y}\int_{-L/2}^{L/2}\frac{x}{(x^2+z^2)^{3/2}}dz$$

Look up the integral

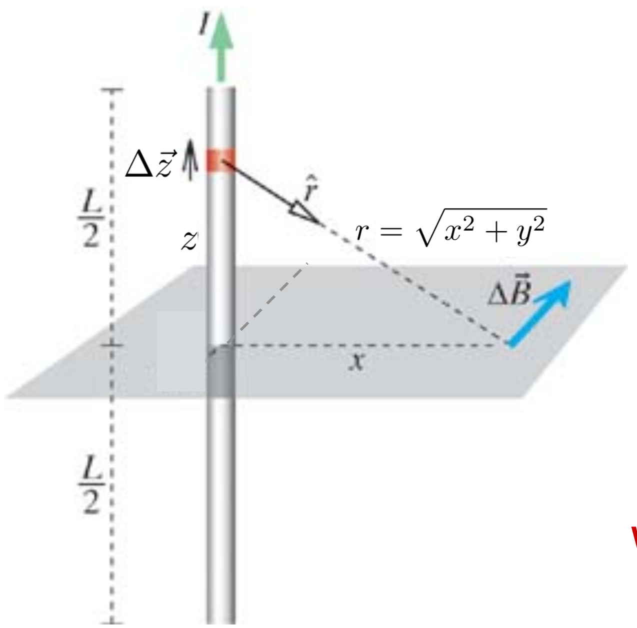
$$\boxed{|B_z| = \left(\frac{\mu_o}{4\pi}\right)\frac{IL}{x\sqrt{x^2+(L/2)^2}}}$$

B of a Long Straight Wire

Magnetic Field of a Straight Wire

$$|B_z| = \left(\frac{\mu_o}{4\pi}\right)\frac{IL}{x\sqrt{x^2+(L/2)^2}}$$

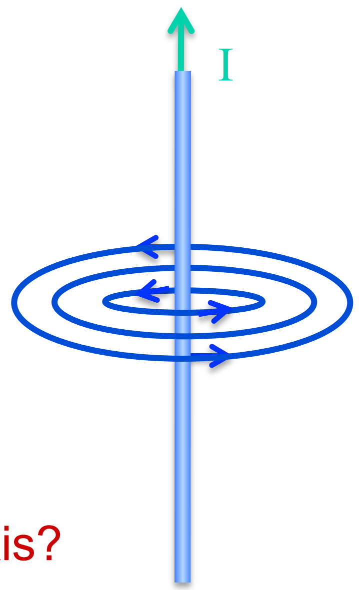
B in the bisecting plane



Which direction does B point?

→ Always along concentric circles

In cylindrical coordinates,
it points in the " $\hat{\theta}$ " direction



Will the y axis look different from the x axis?

No, so we can trade x → r

$$\boxed{\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{IL}{r\sqrt{r^2+(L/2)^2}}\hat{\theta}}$$

B of a Long Straight Wire
(cylindrical coord.)

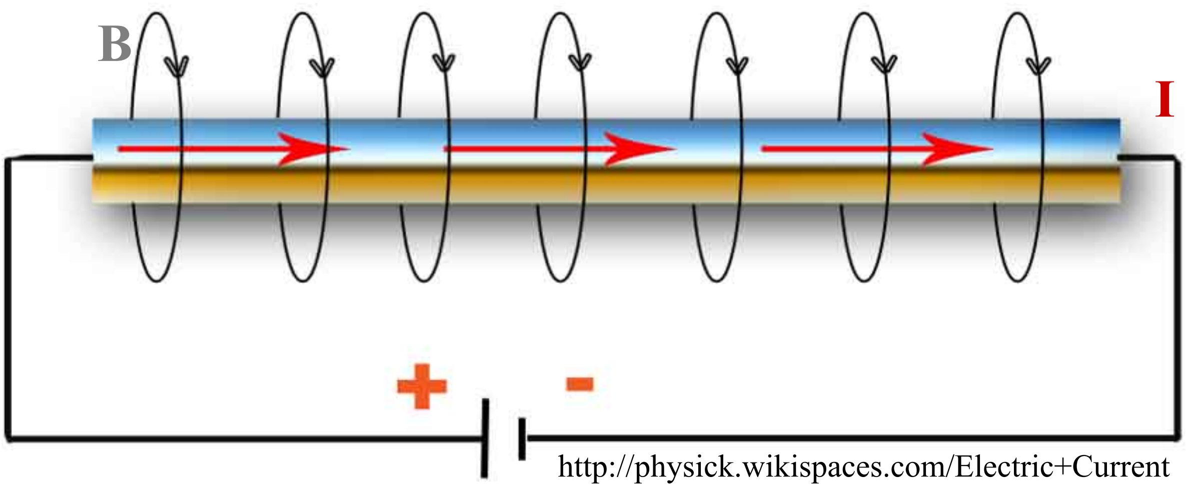
Very Close to the Wire

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{IL}{r\sqrt{r^2+(L/2)^2}}\hat{\theta}$$

Very close to the wire: r << L $\sqrt{r^2+(L/2)^2} \approx L/2$

$$\Rightarrow \vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{IL}{r(L/2)}\hat{\theta} = \boxed{\left(\frac{\mu_o}{4\pi}\right)\frac{2I}{r}\hat{\theta} = \vec{B}}$$

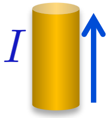
CLOSE TO THE WIRE



http://physick.wikispaces.com/Electric+Current

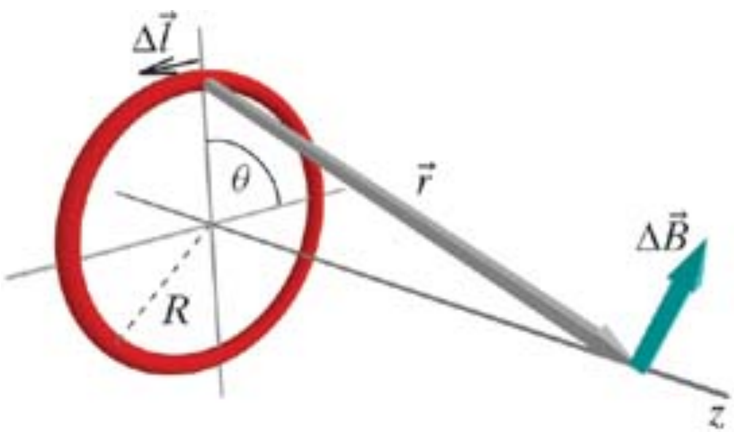
iClicker Question

B along Axis of Circular Loop of Wire

I  $\Delta\vec{l}$

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{I\Delta\vec{l}\times\hat{r}}{|r|^2}$$

BIOT-SAVART LAW
current in a wire



1) Draw $\Delta\vec{B}$ for one piece

→ Notice only z component survives!

2) Write $\Delta\vec{B}$ due to one piece

$$\Delta B_z = \frac{\mu_o}{4\pi}\frac{IR^2\Delta\theta}{(R^2+z^2)^{3/2}}$$ (lots of math!)

3) Integrate $\Delta\vec{B}$ to find total B

$$B_z = \int \Delta B_z = \int_0^{2\pi}\frac{\mu_o}{4\pi}\frac{IR^2 d\theta}{(R^2+z^2)^{3/2}} = \boxed{\frac{\mu_o}{4\pi}\frac{IR^2 2\pi}{(R^2+z^2)^{3/2}} = B_z}$$

4) Doulbecheck

$$B_z = \left[\left(\frac{T\cdot m}{A}\right)\frac{(m^2\cdot A)}{(m^2)^{3/2}}\right]$$

✔ UNITS   Right-hand Rule ✔

Magnetic Field of a Wire Loop

Special case: far from the loop

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(R^2 + z^2\right)^{3/2}}$$

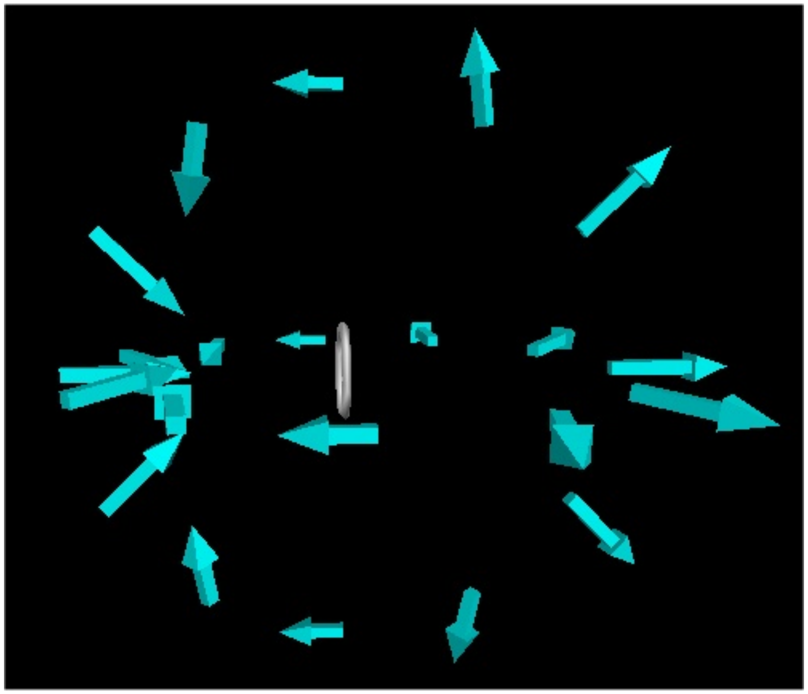
for $z \gg R$: $\left(z^2 + R^2\right)^{3/2} \approx (z^2)^{3/2} = z^3$

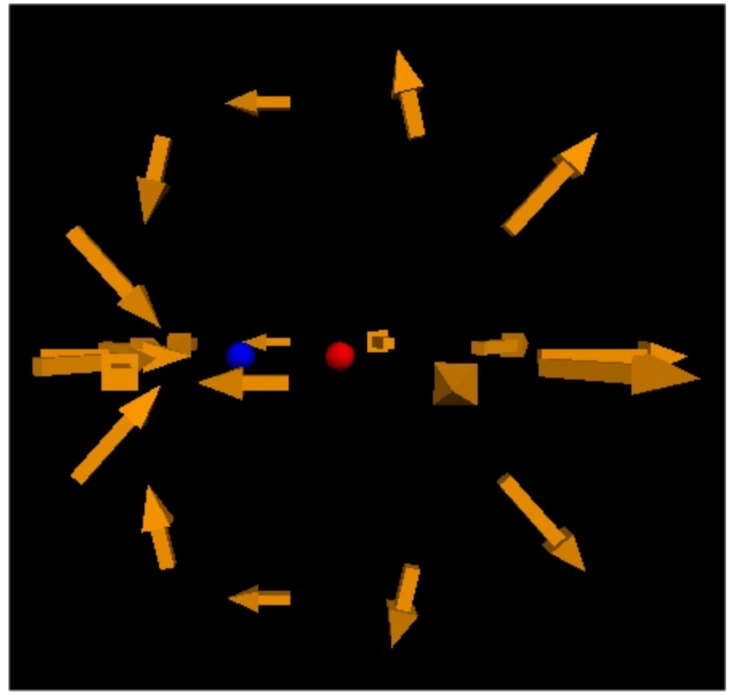
$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(z^2\right)^{3/2}}$$

$$\boxed{B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{z^3}}$$

The magnetic field of a circular loop falls off like $1/z^3$

Magnetic Dipole Moment





far from coil: $B_z = \dfrac{\mu_0}{4\pi}\dfrac{2\pi R^2 I}{z^3}$

far from dipole: $E_z = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2p}{z^3}$

$$B_z = \frac{\mu_0}{4\pi}\frac{2\mu}{z^3}$$

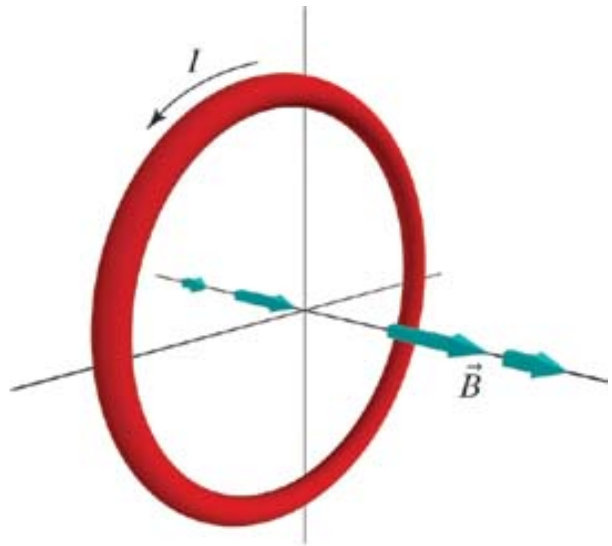
$$p = sq$$

magnetic dipole moment: $\mu = \pi R^2 I = AI$

$\vec{\mu}$ - vector in the direction of $\vec{B}$

Magnetic Dipole in a B Field

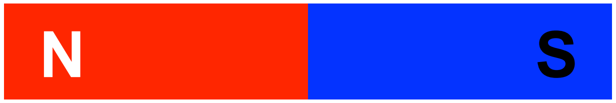
The magnetic dipole moment μ acts like a compass needle!

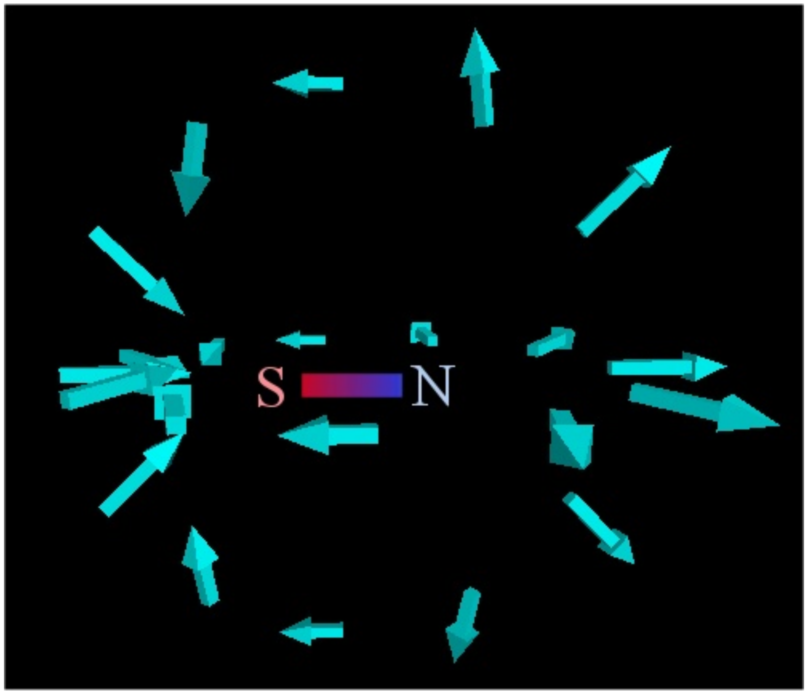


In the presence of external magnetic field a current-carrying loop rotates to align the magnetic dipole moment $\vec{\mu}$ along the field $\vec{B}$.

The Magnetic Field of a Bar Magnet

How does the magnetic field around a bar magnet look?





Atomic Structure of Magnets

Assume: Electrons make circular current loops

$$B_{atom} = \frac{\mu_0}{4\pi}\frac{2\mu}{z^3}, \qquad \mu = \pi R^2 I$$

What is the direction?

One loop:

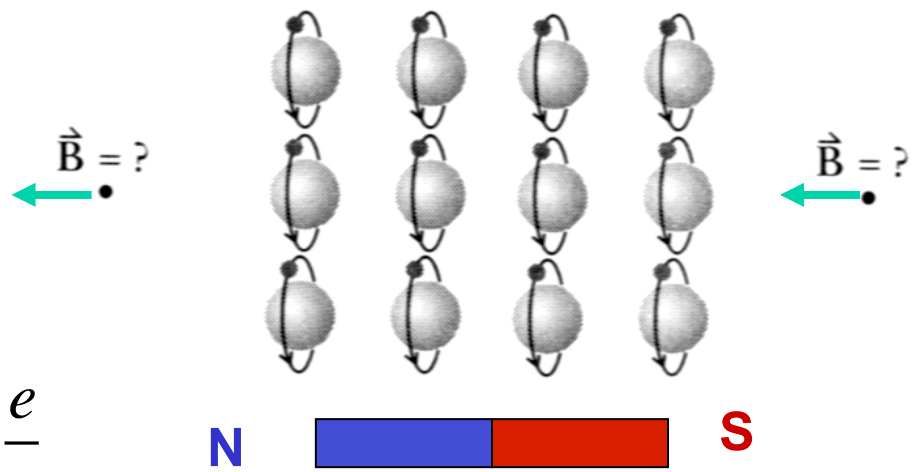
What is the average current I?

current=charge/second: $I = \frac{e}{t}$

$$T = \frac{2\pi R}{v} \longrightarrow I = \frac{ev}{2\pi R}$$

$$\mu = \pi R^2 \frac{ev}{2\pi R} = \frac{1}{2}eRv$$

Electrons

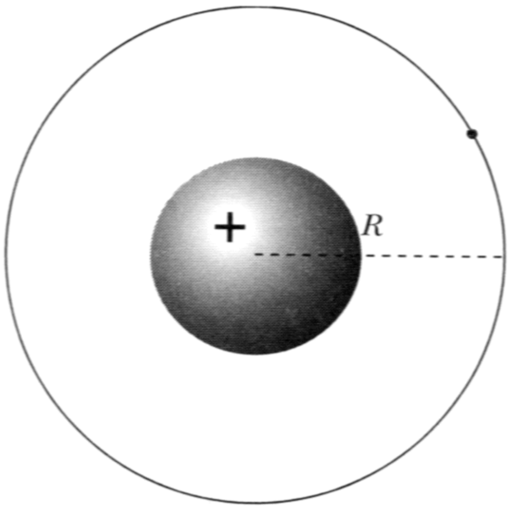


Atomic Structure of Magnets

Assume: Electrons make circular current loops

$$\mu = \frac{1}{2} eRv$$ **Dipole Moment of 1 Atom**

Angular Momentum is Quantized:



Orbital angular momentum: $L = Rmv$

$$\mu = \frac{1}{2} eRv = \frac{1}{2}\frac{e}{m} Rmv = \frac{1}{2}\frac{e}{m} L$$

Quantum mechanics: L is quantized:

$$L = n\hbar, \quad \hbar = 1.05 \times 10^{-34} \text{ J}\cdot\text{s}$$

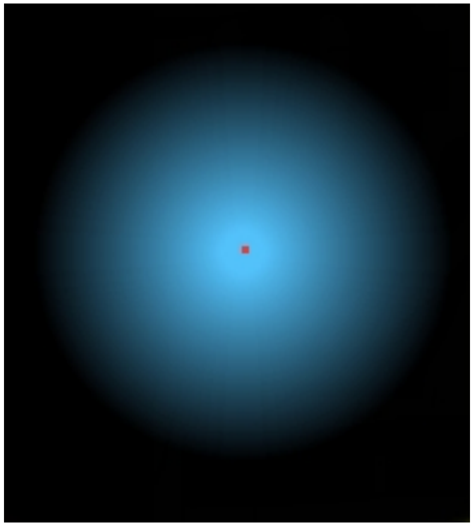
If n=1: $\mu = \frac{1}{2}\frac{e}{m} L = 0.9 \times 10^{-23} \text{ A}\cdot\text{m}^2$ per atom

Quantum Magnetism

Magnetic Moment = Orbital Motion + "Spin"

ORBITAL MOTION:

Fuzzy electron cloud.
Shapes are set by "spherical harmonics"



Spherically symmetric cloud (s-orbital) has no μ

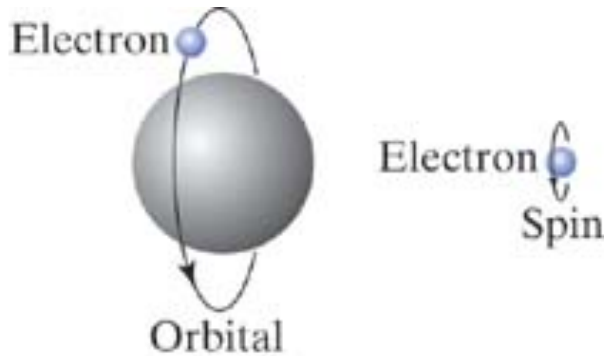
Only non spherically symmetric orbitals (p, d, f) contribute to μ

There is more than 1 electron in an atom

Quantum Magnetism

Magnetic Moment = Orbital Motion + "Spin"

SPIN:



Electron acts like spinning charge
- contributes to μ

Electron spin contribution to μ is of the same order as one due to orbital momentum

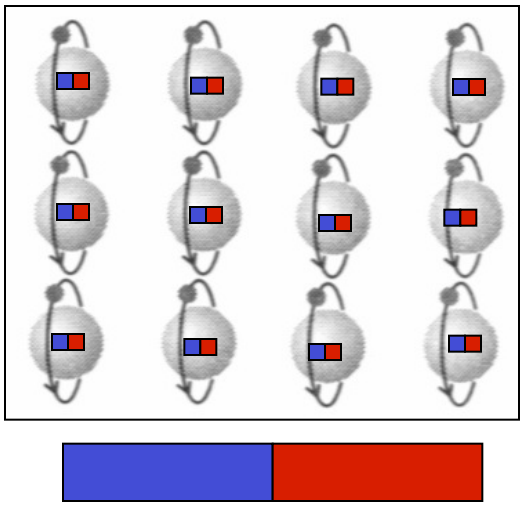
Neutrons and proton in nucleus also have spin but their μ's are much smaller than for electron

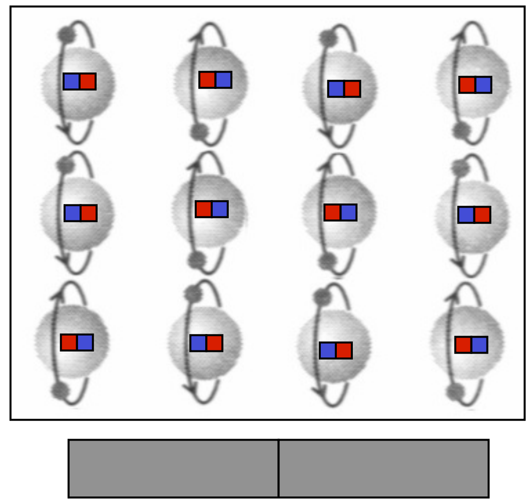
same angular momentum: $\mu \approx \frac{1}{2}\frac{e}{m}\hbar$

NMR, MRI – use nuclear μ

Refrigerator Magnets

Alignment of atomic dipole moments:



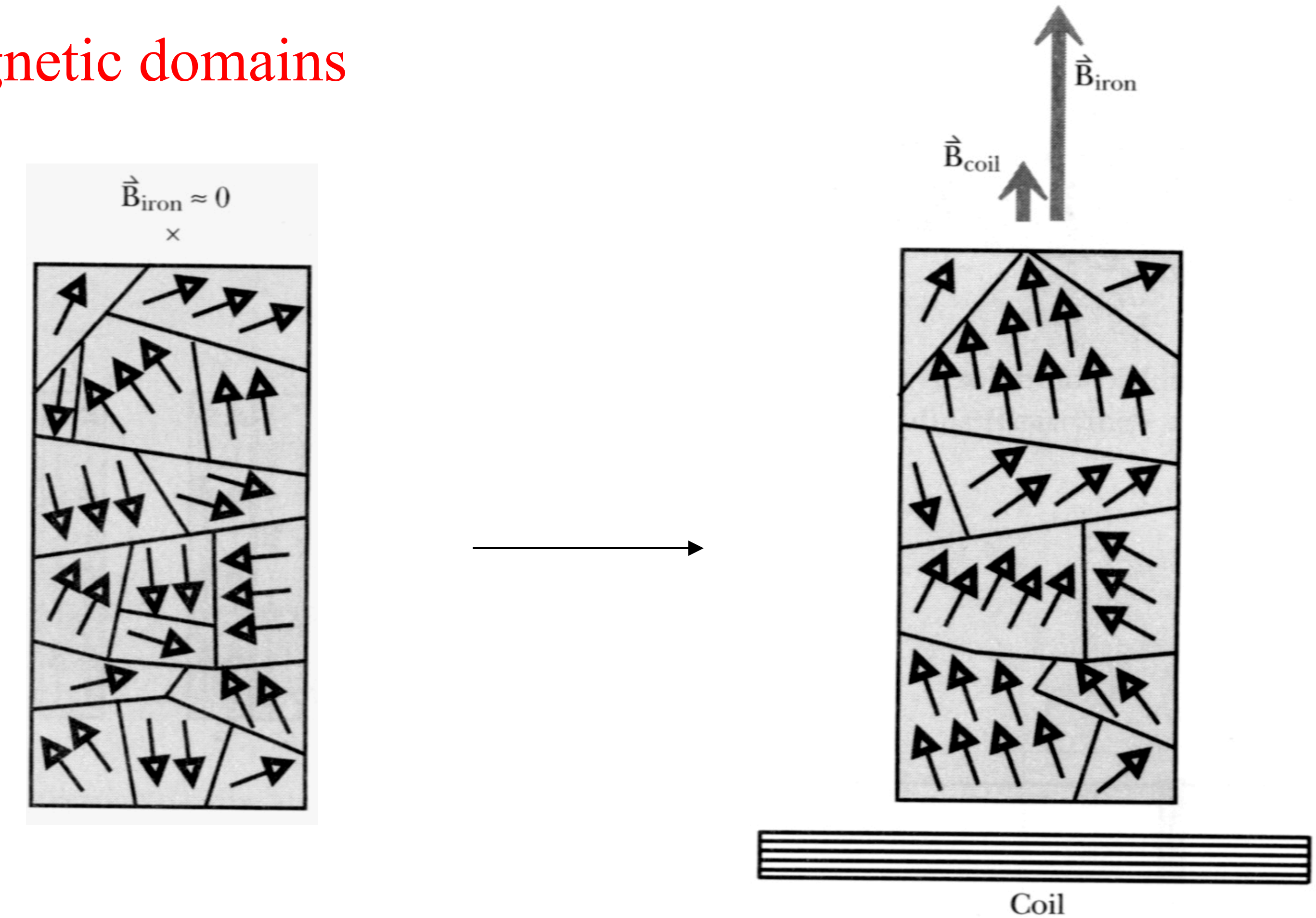


ferromagnetic materials:
iron, cobalt, nickel

most materials

Reality Physics - Domains

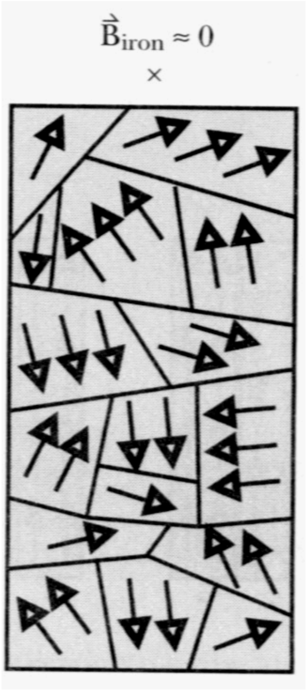
Magnetic domains

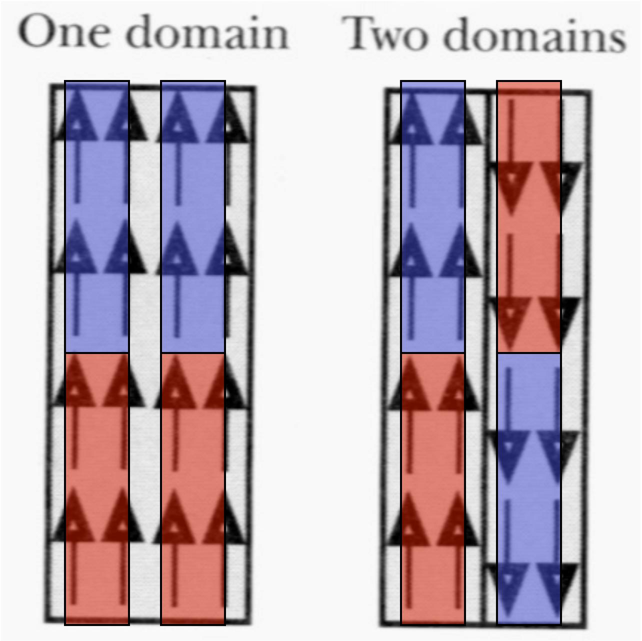


Hitting or heating can also demagnetize

Why are there Multiple Domains?

Magnetic domains





Today

- Magnetic Field of a Straight Wire
- (Magnetic Field of a Current Loop)
- Magnetic Dipole Moment
- Bar Magnet
- Atomic Dipoles