

What is the goal?

- achieve  $Q(t) = Q^*$
- or close to it:  $|Q(t) - Q^*| < \varepsilon$

How to: basic idea

- if  $Q(t) = Q^*$  do nothing
  - if  $Q(t) < Q^*$  increase  $\lambda(t)$ 
    - too little in the buffer
  - if  $Q(t) > Q^*$  decrease  $\lambda(t)$ 
    - too much in the buffer
- a rule of thumb: called control law

Since state of receiver buffer must be conveyed to sender who adjusts  $\lambda(t)$ :

- called feedback control
- also closed-loop control

Network protocol implementation:

- some design options available
- control action undertaken at sender
  - smart sender/dumb receiver
  - preferred mode of many Internet protocols
  - when might the opposite be better?
- receiver informs sender of  $Q^*$  and  $Q(t)$ 
  - feedback packet (control signaling/messaging)
  - feedback could just be gap  $Q^* - Q(t)$
  - or simply up/down binary indication

Key question in feedback congestion control:

- **how much** to increase/decrease  $\lambda(t)$
- already know in which direction

Desired state of the system:

$$Q(t) = Q^* \text{ and } \lambda(t) = \gamma$$

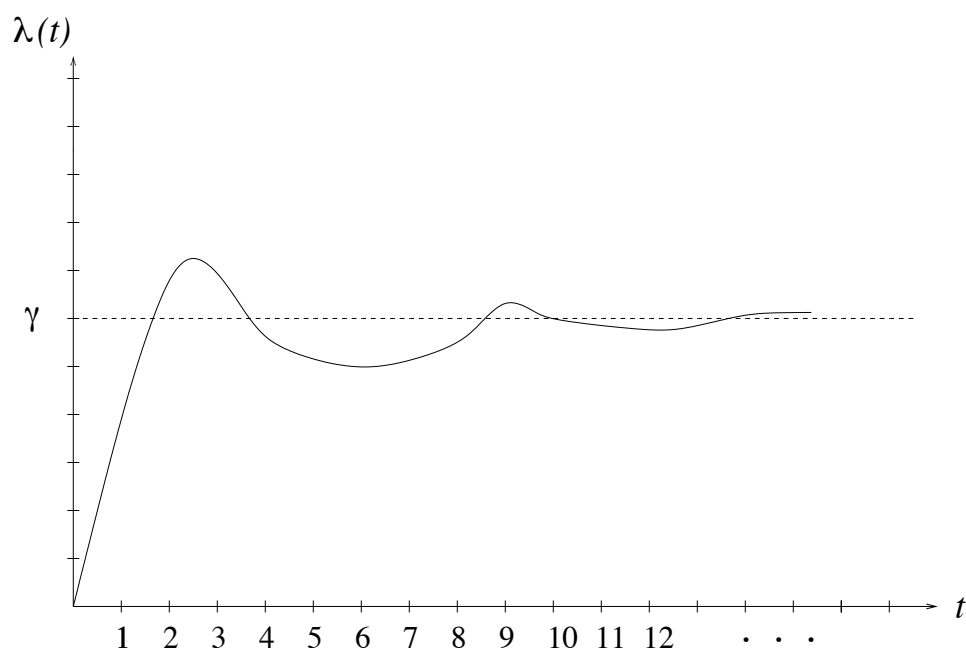
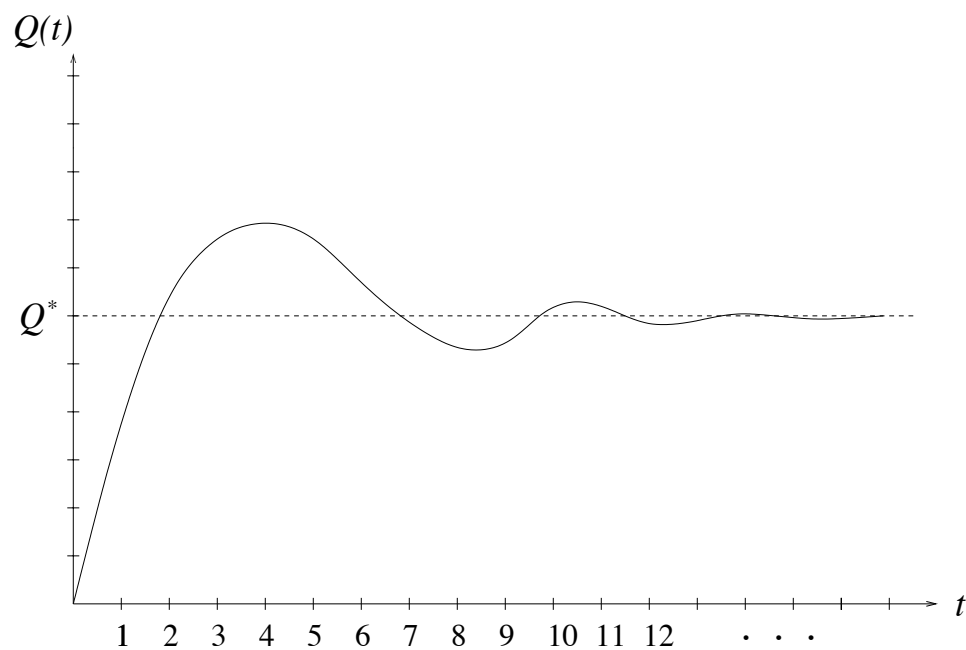
- why is “ $\lambda(t) = \gamma$ ” needed?

Starting state:

- empty buffer and nothing is being sent
- think of iTunes, Rhapsody, etc.

$$\text{i.e., } Q(t) = 0 \text{ and } \lambda(t) = 0$$

Time evolution (or dynamics): track  $Q(t)$  and  $\lambda(t)$



Congestion control methods: A, B, C and D

**Method A:**

- if  $Q(t) = Q^*$  then  $\lambda(t+1) \leftarrow \lambda(t)$
- if  $Q(t) < Q^*$  then  $\lambda(t+1) \leftarrow \lambda(t) + a$
- if  $Q(t) > Q^*$  then  $\lambda(t+1) \leftarrow \lambda(t) - a$

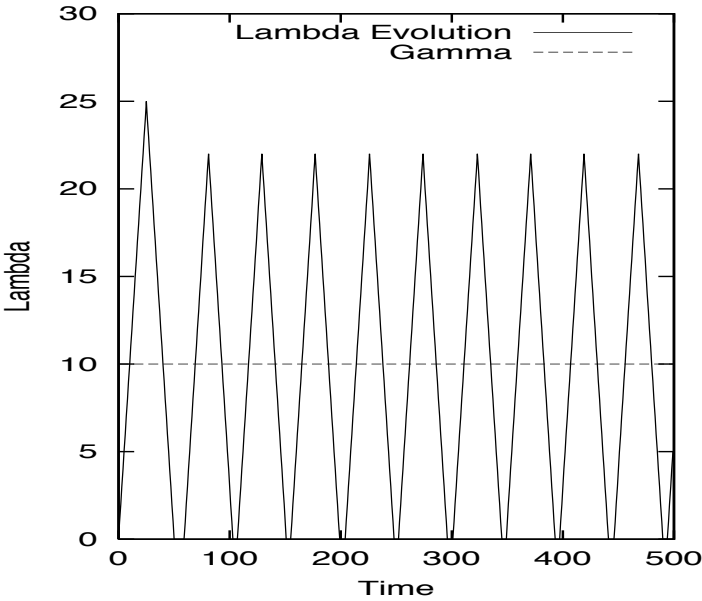
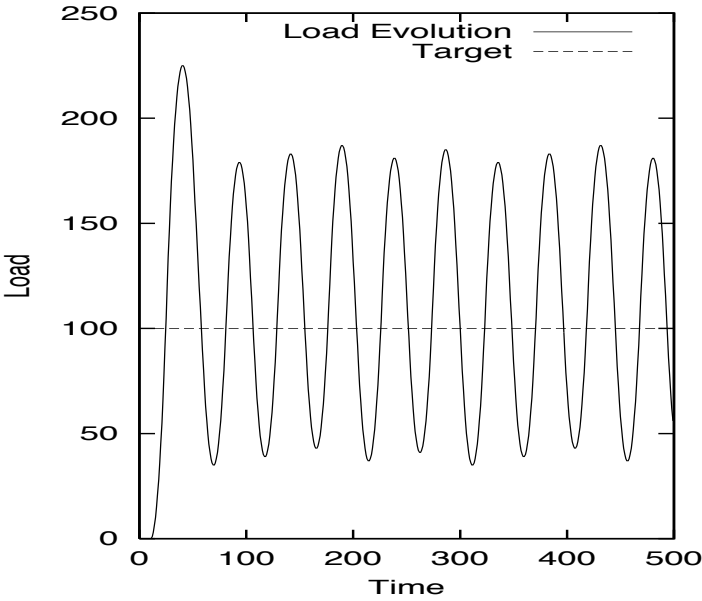
where  $a > 0$  is a fixed parameter

→ called linear increase/linear decrease

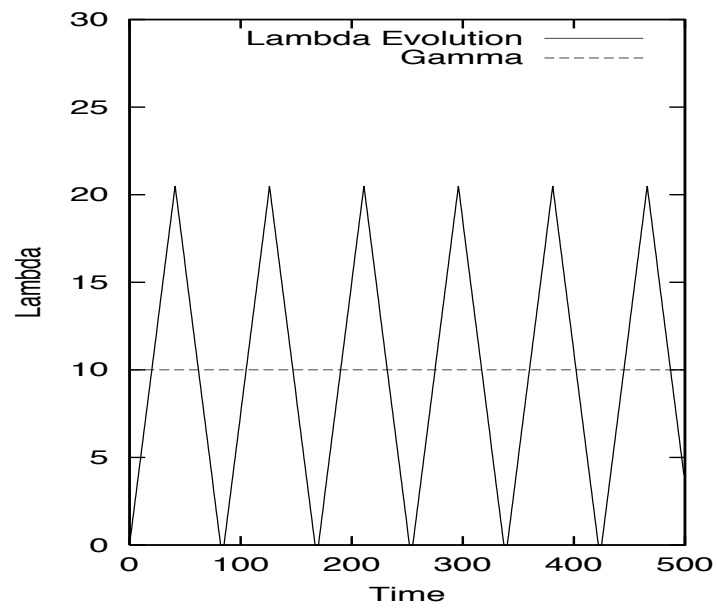
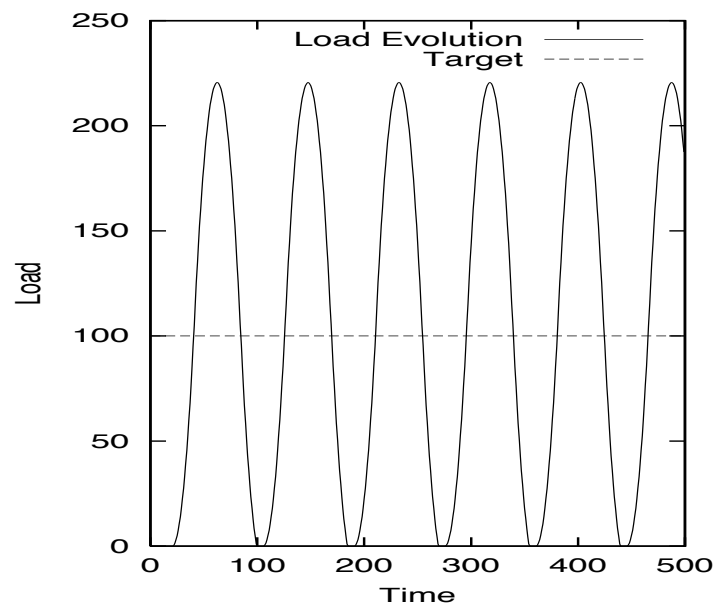
Question: how well does it work?

Example:

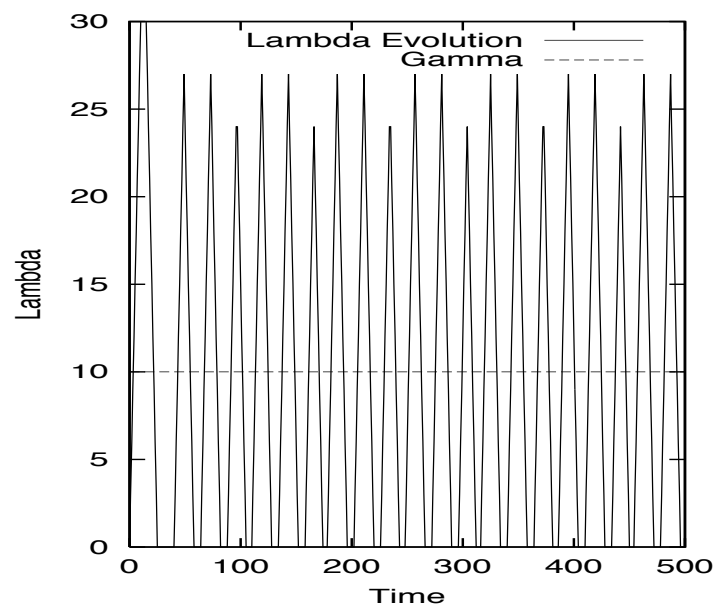
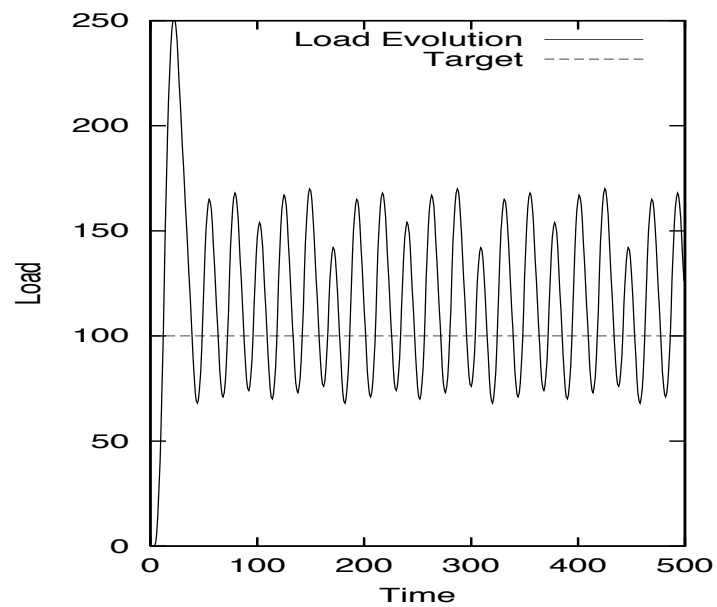
- $Q(0) = 0$
- $\lambda(0) = 0$
- $Q^* = 100$
- $\gamma = 10$
- $a = 1$



With  $a = 0.5$ :

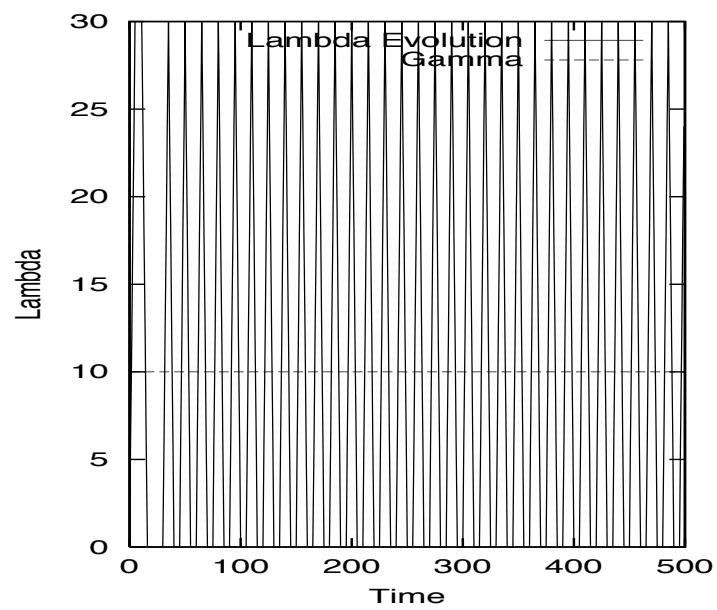
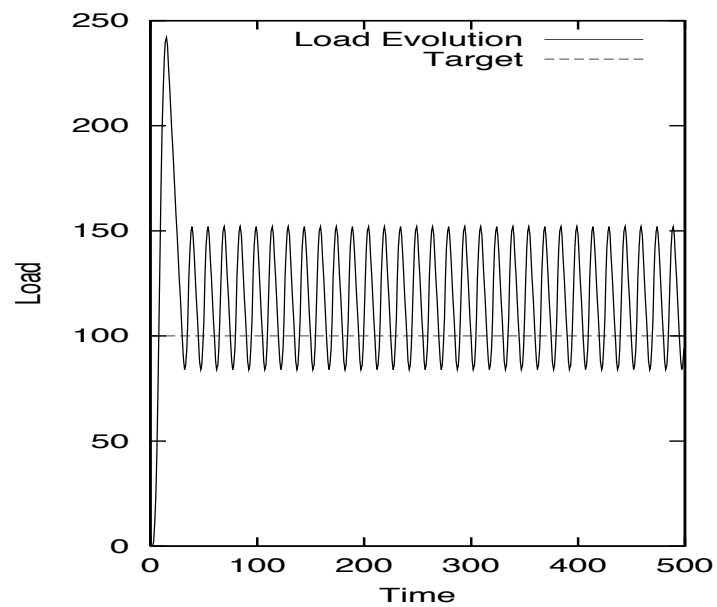


With  $a = 3$ :





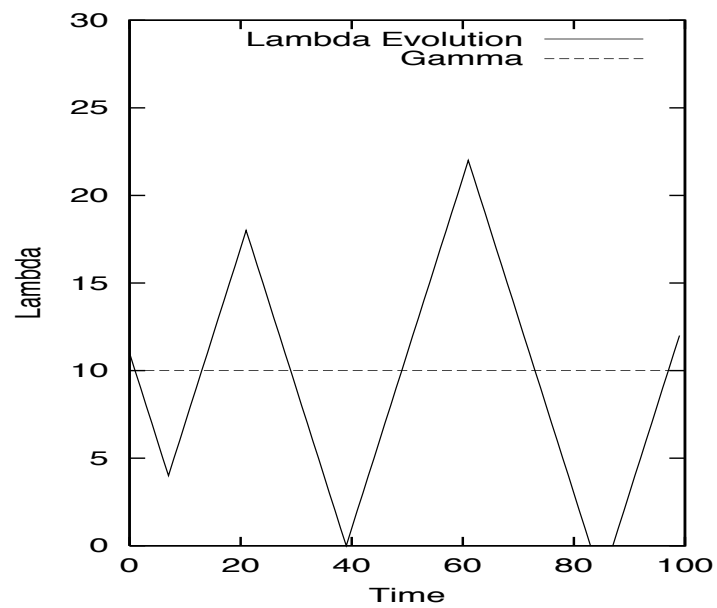
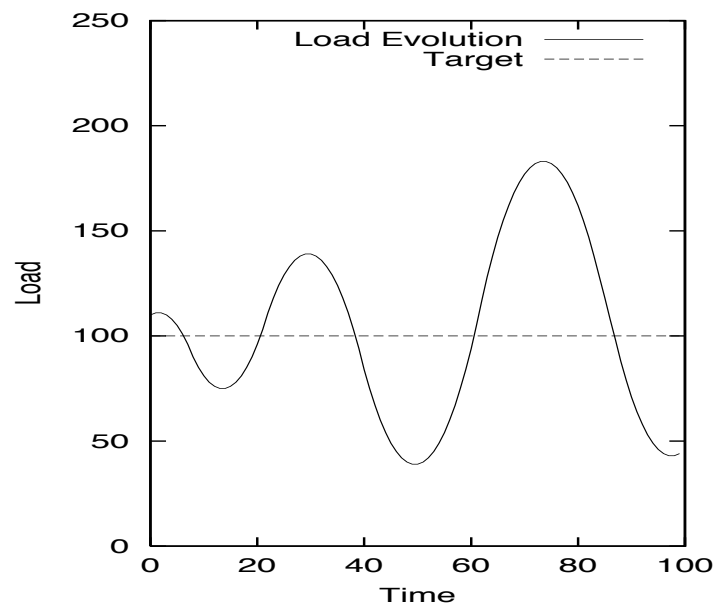
With  $a = 6$ :



Remarks:

- Method A isn't that great no matter what  $a$  value is used
  - keeps oscillating
- Actually: would lead to unbounded oscillation if not for physical restriction  $\lambda(t) \geq 0$  and  $Q(t) \geq 0$ 
  - i.e., bottoms out
  - easily seen: start from non-zero buffer
  - e.g.,  $Q(0) = 110$

With  $a = 1$ ,  $Q(0) = 110$ ,  $\lambda(0) = 11$ :



Method B:

- if  $Q(t) = Q^*$  then  $\lambda(t+1) \leftarrow \lambda(t)$
- if  $Q(t) < Q^*$  then  $\lambda(t+1) \leftarrow \lambda(t) + a$
- if  $Q(t) > Q^*$  then  $\lambda(t+1) \leftarrow \delta \cdot \lambda(t)$

where  $a > 0$  and  $0 < \delta < 1$  are fixed parameters

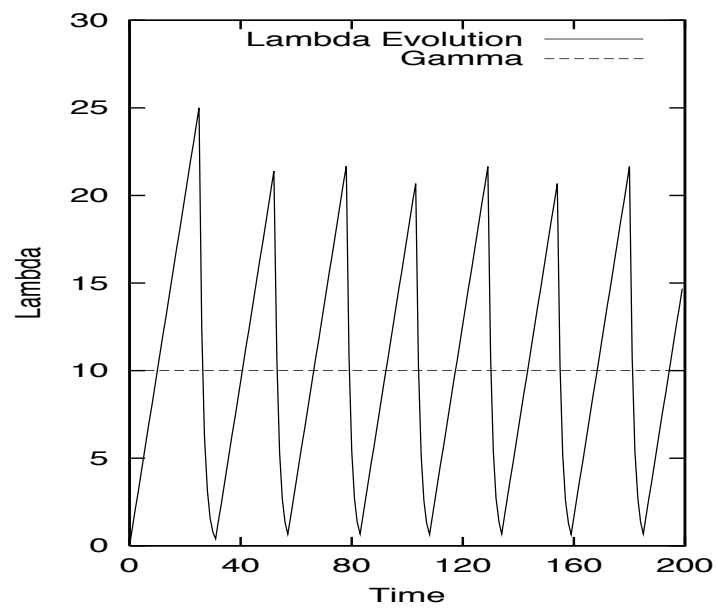
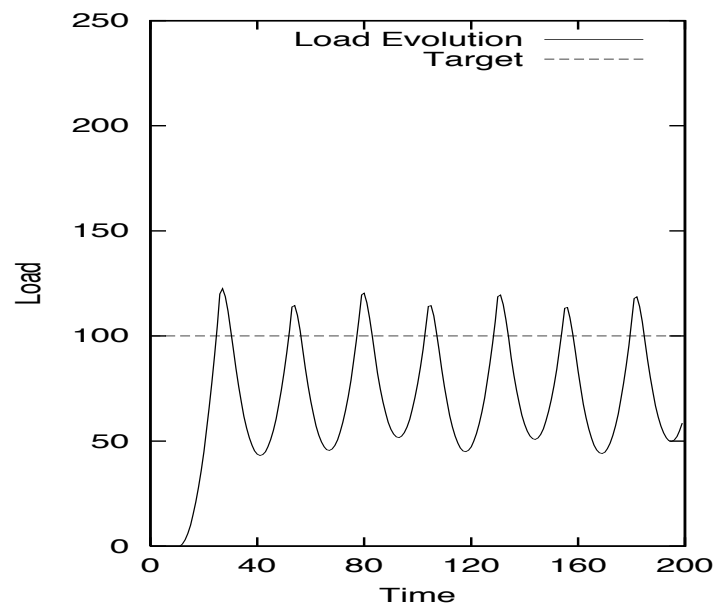
Note: only decrease part differs from **Method A**.

- linear increase with slope  $a$
- exponential decrease with backoff factor  $\delta$
- e.g., binary backoff in case  $\delta = 1/2$

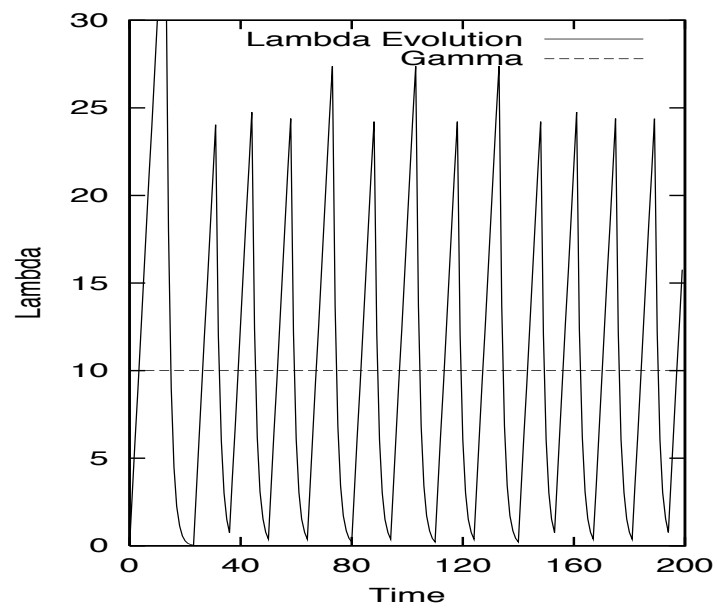
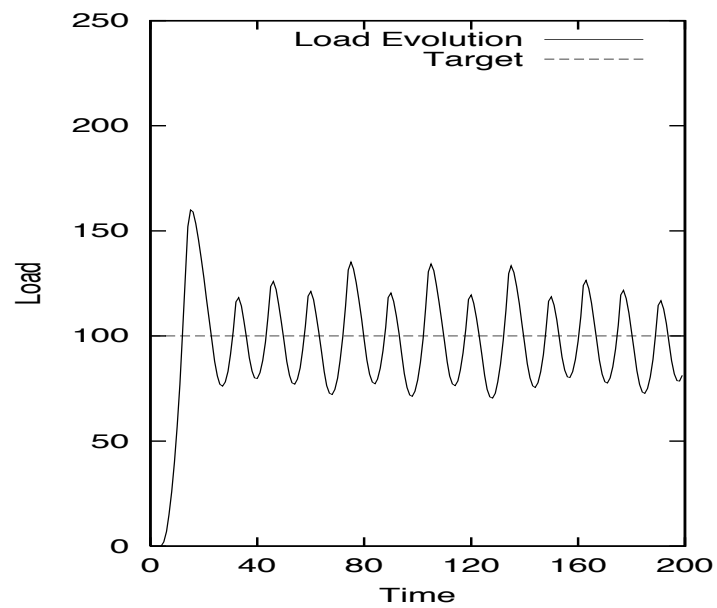
Similar to Ethernet and WLAN backoff

- question: does it work?

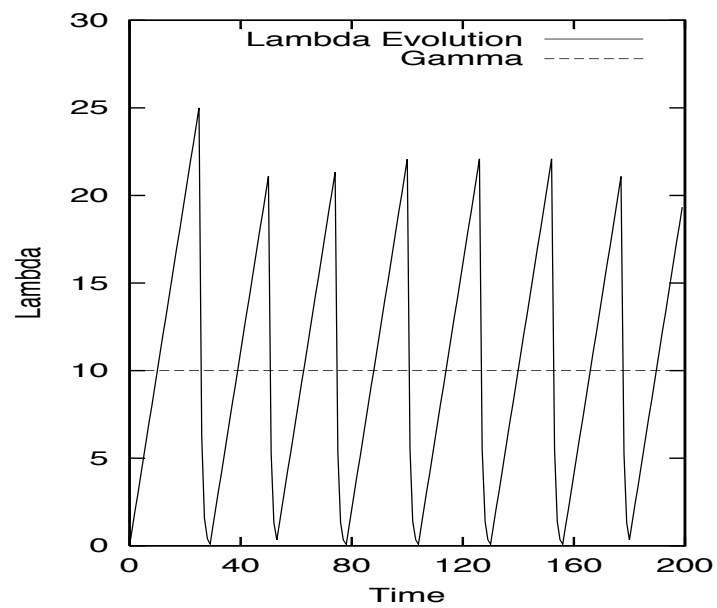
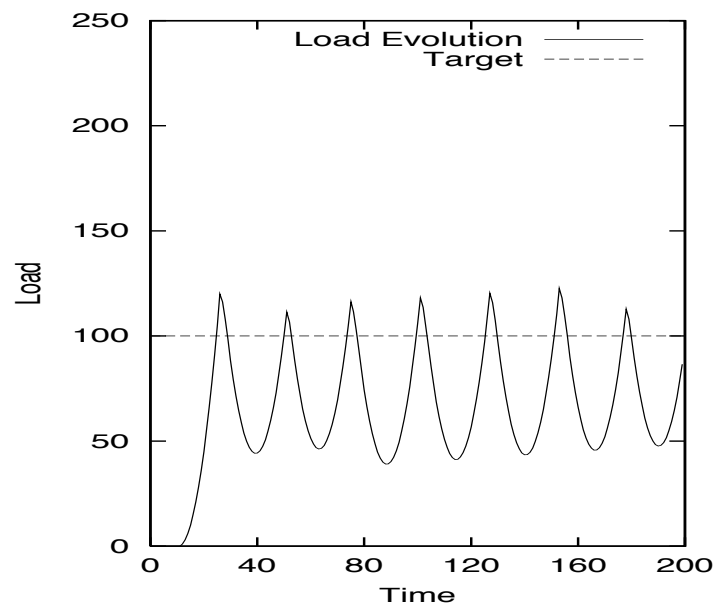
With  $a = 1$ ,  $\delta = 1/2$ :



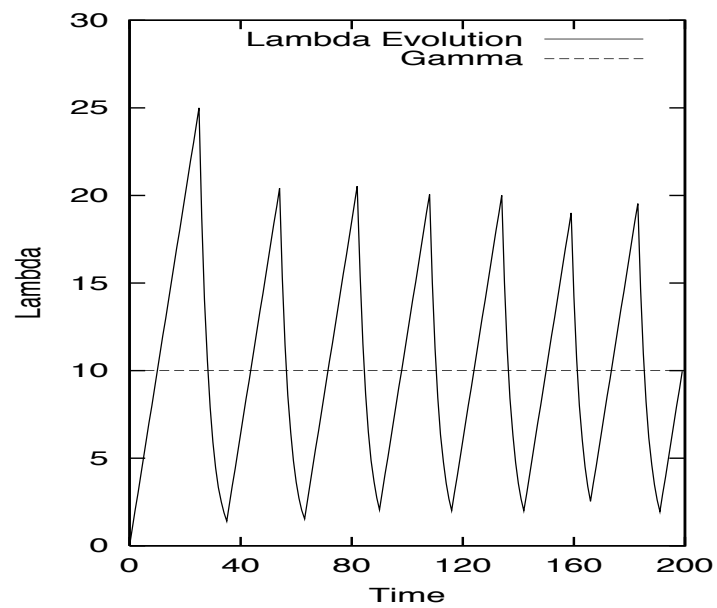
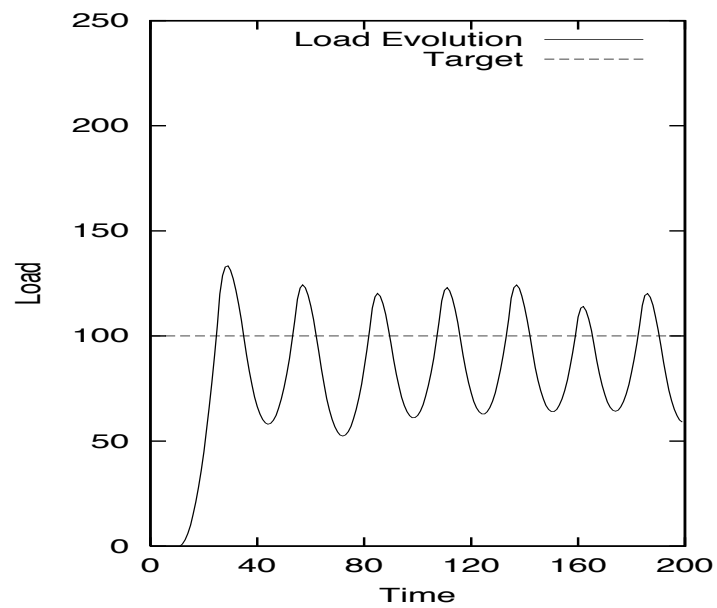
With  $a = 3$ ,  $\delta = 1/2$ :



With  $a = 1$ ,  $\delta = 1/4$ :



With  $a = 1$ ,  $\delta = 3/4$ :





Note:

- Method B isn't that great either
- One advantage over Method A: doesn't lead to unbounded oscillation
  - note: doesn't hit "rock bottom"
  - due to asymmetry in increase vs. decrease policy
  - we observe "sawtooth" pattern
- Method B is used by TCP
  - linear increase/exponential decrease
  - additive increase/multiplicative decrease (AIMD)

Question: can we do better?

→ what "freebie" have we not made use of?

Method C:

$$\lambda(t+1) \leftarrow \lambda(t) + \varepsilon(Q^* - Q(t))$$

where  $\varepsilon > 0$  is a fixed parameter

Tries to adjust magnitude of change as a function of the gap  $Q^* - Q(t)$

→ incorporate distance from target  $Q^*$

→ before: just the sign (above/below)

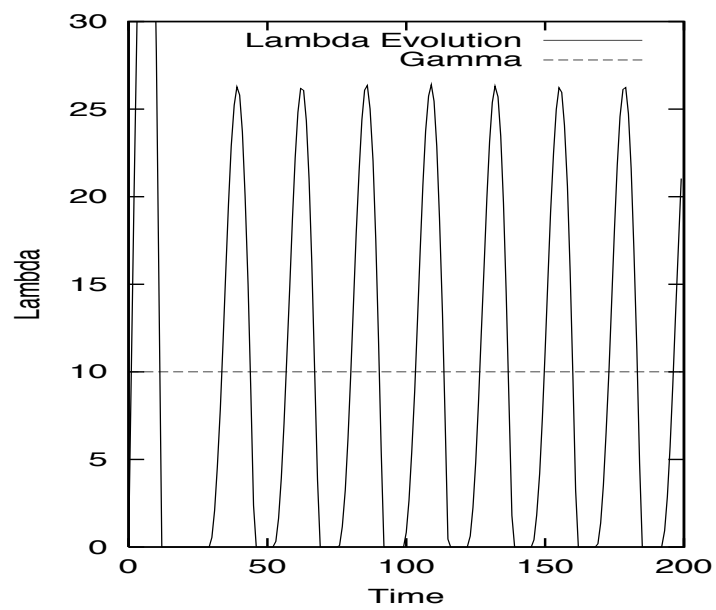
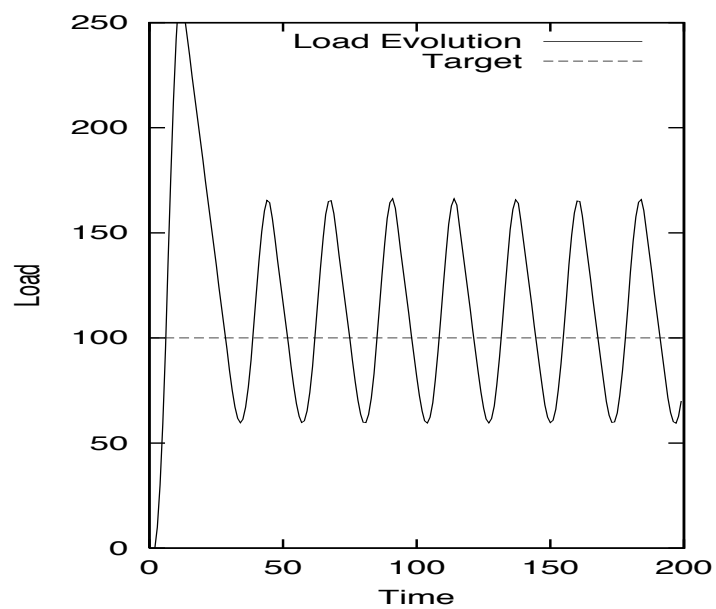
Thus:

- if  $Q^* - Q(t) > 0$ , increase  $\lambda(t)$  proportional to gap
- if  $Q^* - Q(t) < 0$ , decrease  $\lambda(t)$  proportional to gap

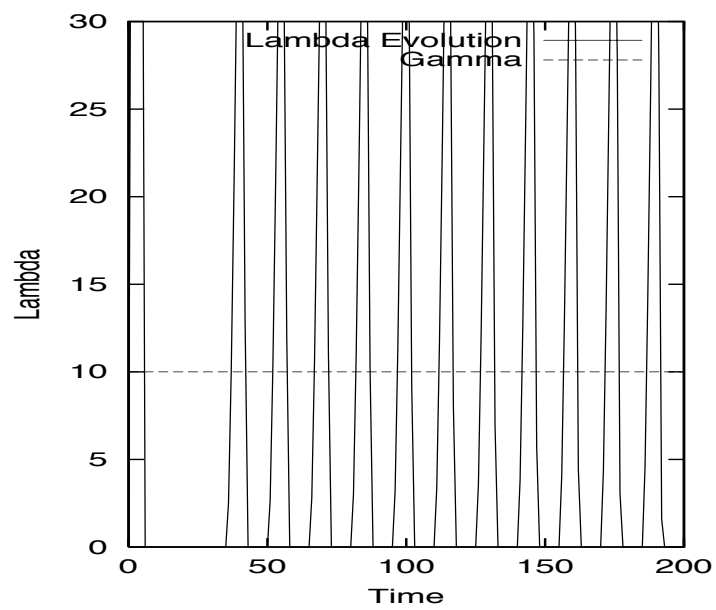
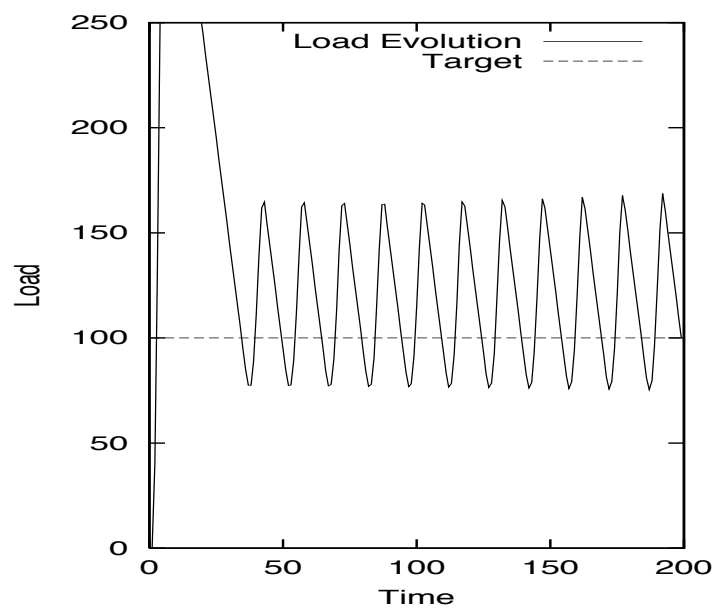
Trying to be more clever...

→ bottom line: is it any good?

With  $\varepsilon = 0.1$ :



With  $\varepsilon = 0.5$ :



Answer: no

→ control law looks good on the surface

→ but looks can be deceiving

Time to try something strange

→ any (crazy) ideas?

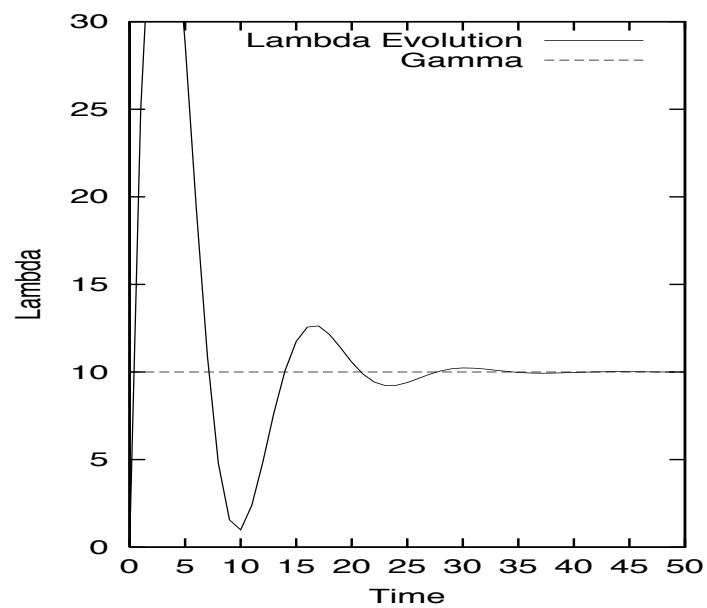
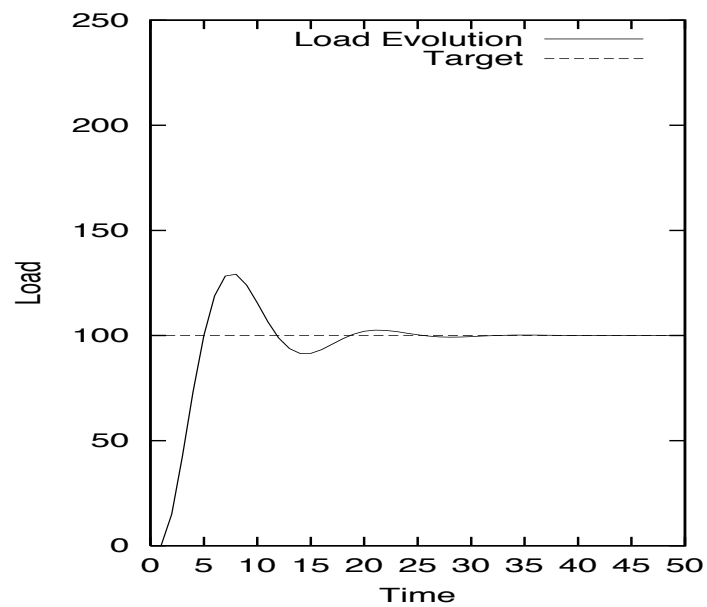
**Method D:**

$$\lambda(t+1) \leftarrow \lambda(t) + \varepsilon(Q^* - Q(t)) - \beta(\lambda(t) - \gamma)$$

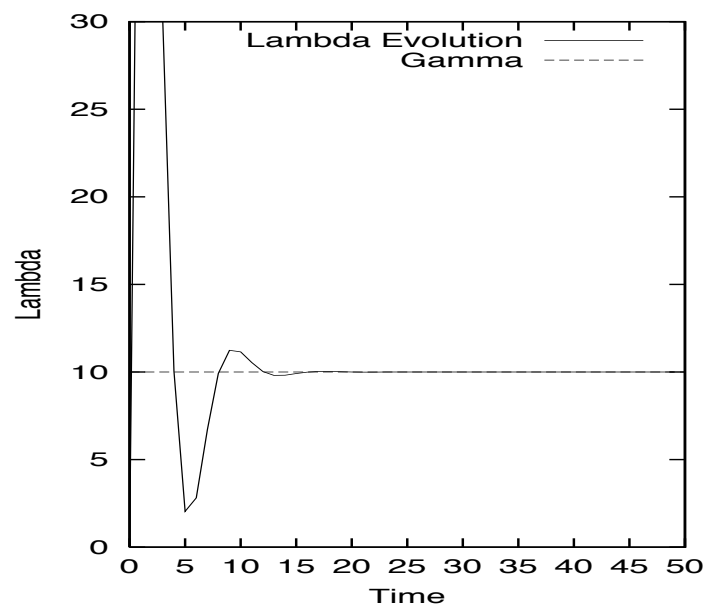
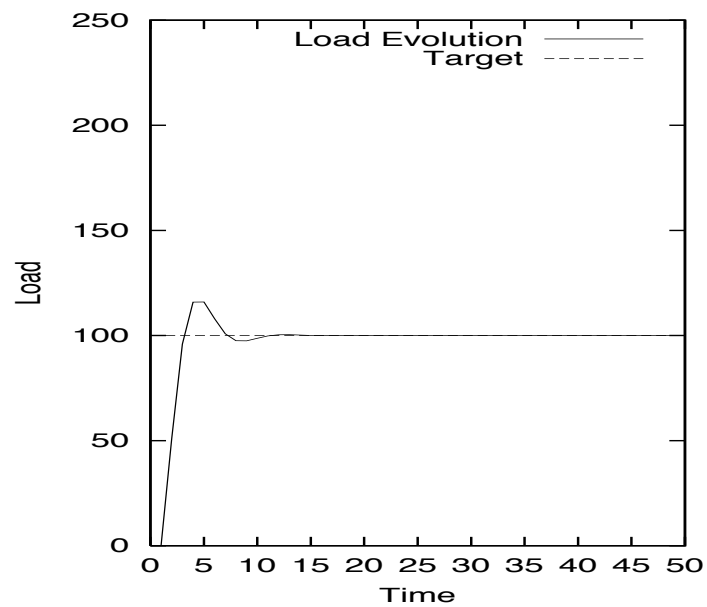
where  $\varepsilon > 0$  and  $\beta > 0$  are fixed parameters

- odd looking modification to **Method C**
- additional term  $-\beta(\lambda(t) - \gamma)$
- what's going on?
- does it work?

With  $\varepsilon = 0.2$  and  $\beta = 0.5$ :



With  $\varepsilon = 0.5$  and  $\beta = 1.1$ :





With  $\varepsilon = 0.1$  and  $\beta = 1.0$ :

