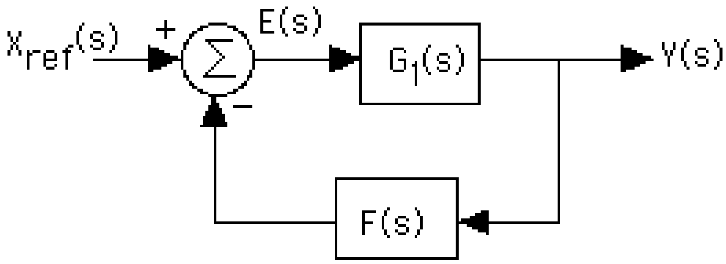
HW5 Solutions

ECE 202 Fall 2013

MK
MN

17.



$$G_1(s) = \frac{n_g(s)}{d_g(s)} \quad F(s) = \frac{n_f(s)}{d_f(s)}$$

a)

$$E(s) = X_{ref}(s) - Y(s)F(s)$$

b)

$$E(s) = X_{ref}(s) - Y(s)F(s)$$

And we know that $Y(s) = G_1(s)E(s)$ *plugging* $Y(s)$ *into the above equation*:

$$E(s) = X_{ref}(s) - G_1(s)E(s)F(s) \Rightarrow X_{ref}(s) = E(s) + G_1(s)E(s)F(s) \;\Rightarrow$$

$$X_{ref}(s) = E(s)\big(1 + G_1(s)F(s)\big) \;\Rightarrow H(s) = \frac{E(s)}{X_{ref}(s)} = \frac{1}{1 + G_1(s)F(s)}$$

We also know that $G_1(s) = \frac{n_g(s)}{d_g(s)} \quad F(s) = \frac{n_f(s)}{d_f(s)}$ *plugging* $F(s)$ *and* $G(s)$ *into* $H(s)$:

$$H(s) = \frac{1}{1 + \frac{n_g(s)}{d_g(s)}\frac{n_f(s)}{d_f(s)}} = \frac{d_g(s)d_f(s)}{d_g(s)d_f(s) + n_g(s)n_f(s)} = \frac{d(s)}{d(s) + n(s)}$$

$$\textit{where } F(s)G_1(s) = \frac{n_g(s)}{d_g(s)}\frac{n_f(s)}{d_f(s)} = \frac{n(s)}{d(s)}$$

c)

$$E(s) = H(s)X_{ref}(s) = X_{ref}(s)\frac{d_g(s)d_f(s)}{d_g(s)d_f(s) + n_g(s)n_f(s)} \Rightarrow$$

$$E(s) = \frac{K_0}{s}\frac{d_g(s)d_f(s)}{d_g(s)d_f(s) + n_g(s)n_f(s)}$$

Now $for\ lim_{t\to\infty} e(t) = 0.$

First of all, the poles of E(s) must be on the left-hand side of the complex plane. Then we can use final value theorem which says:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\left(K_0\frac{d_g(s)d_f(s)}{d_g(s)d_f(s) + n_g(s)n_f(s)}\right)$$

For this equation to be zero, the numerator $\left(d_g(s)d_f(s)\right)$ must have at least one zero at the origin which is not common with the denominator.

d)

$$\lim_{t\to\infty} e(t) = K_e \neq 0$$

The above statement is true if first of all, all the poles are on the left-hand side of the complex plane and secondly $d_g(s)d_f(s)$ has no zero at the origin.

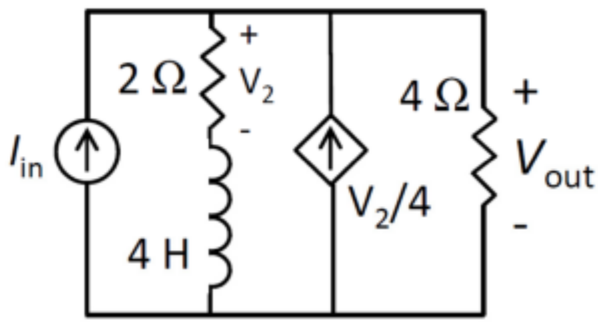
e)

We can determine $e(0^+)$ using the initial value theorem if E(s)= $H(s).X_{ref}(s)$ is a strictly proper rational function i.e. the numerator and the denominator of E(s) are both polynomials in s-domain and the order of numerator's polynomial is less than that of the denominator.

f)

F(s) must have zeroes at the same location as the poles of $G_1(s)$ on the imaginary axis.

18.



$R_1 = 2\Omega, L = 4H, g_m = 0.24S, and\ R_2 = 4\Omega$

$i_{in}(t) = 3e^{-t}\cos(2\mathrm{t})\, u(t)A$

KCL at $V_{out}(s)$:

$$I_{in}(s) = \frac{V_{out}(s)}{R_2} - g_m V_2(s) + \frac{V_{out}(s)}{R_1 + Ls}$$

$$V_2(s) = \frac{R_1}{R_1 + Ls} V_{out}(s)$$

$$I_{in}(s) = \frac{V_{out}(s)}{R_2} - \frac{g_m R_1}{R_1 + Ls} V_{out}(s) + \frac{V_{out}(s)}{R_1 + Ls}$$

Using MATLAB

```
clear all
close all
clc
syms s t Iin Vout vout R1 R2 gm L
R1=2;R2=4;gm=1/4;L=4;
Iin=Vout/R2-gm*R1*Vout/(R1+L*s)+Vout/(R1+L*s);
Iin=collect(Iin,Vout);
H=Vout/Iin;
H=collect(eval(H));
pretty(H)
```

$$H(s) = Z_{in}(s) = \frac{4s + 2}{s + 1}$$

```
iin=3*exp(-t)*cos(2*t);
Iin=laplace(iin);
Vout=H*Iin;
Vout=collect(Vout);
```

```
pretty(Vout)
```

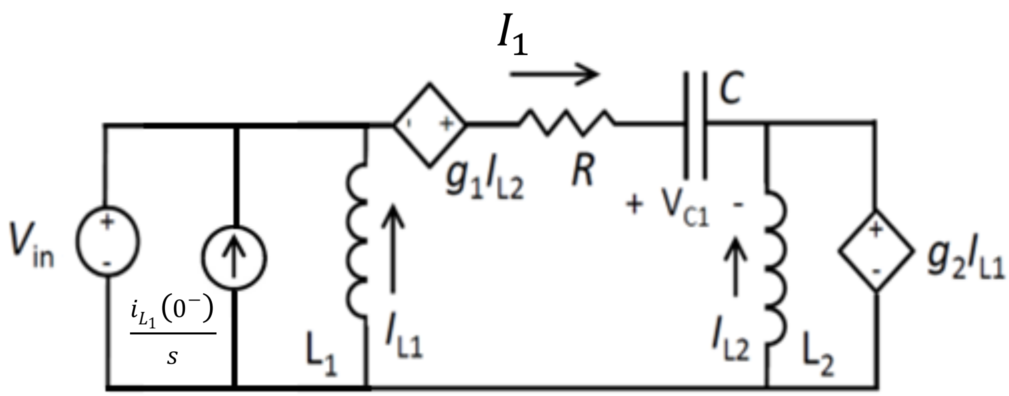
$$V_{out}(s) = \frac{12s + 6}{s^2 + 2s + 5}$$

```
vout=ilaplace(Vout);
pretty(vout)
```

$$v_{out}(t) = 12e^{-t}\left(\cos(2t) - \frac{\sin(2t)}{4}\right)u(t)$$

19.

a)



b)

$$V_1(s) = V_{in}(s)$$

$$V_2(s) = g_2 I_{L_1}(s)$$

$$I_{L_1}(s) = \frac{-V_1(s)}{L_1 s} + \frac{i_{L_1}(0^-)}{s} = \frac{-V_{in}(s)}{L_1 s} + \frac{i_{L_1}(0^-)}{s}$$

$$V_2(s) = g_2\left(\frac{-V_{in}(s)}{L_1 s} + \frac{i_{L_1}(0^-)}{s}\right)$$

$$-V_1(s) - g_1 I_{L_2}(s) + RI_1(s) + \frac{1}{Cs}I_1(s) + V_2(s) = 0$$

$$I_{L_2}(s) = \frac{-V_2(s)}{L_2 s}$$

$$-V_1(s) + \left(1 + \frac{g_1}{L_2 s}\right) V_2(s) + \left(R + \frac{1}{Cs}\right) I_1(s) = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 + \dfrac{g_1}{L_2 s} & R + \dfrac{1}{Cs} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} 1 \\ -\dfrac{g_2}{L_1 s} \\ 0 \end{bmatrix} V_{in}(s) + \begin{bmatrix} 0 \\ \dfrac{g_2}{s} \\ 0 \end{bmatrix} i_{L_1}(0^-)$$

c)

In MATLAB:

```
clear all
close all
clc
syms s t R C L1 L2 g1 g2 Vin vin iL1 IL1 M b1 b2 iL0
R=1e3;C=12e-3;L1=0.1;L2=0.2; g1=8;g2=0.5;vin=8*exp(-
2*t)*heaviside(t);
iL0=4;
Vin=laplace(vin);
M=[1 0 0;0 1 0;-1 1+g1/(L2*s) R+1/(C*s)];
b1=[1; -g2/(L1*s); 0];
b2=[0; g2/s; 0];
M=collect(eval(M))
b=collect(eval(b1)*Vin+eval(b2)*iL0)
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \dfrac{s+40}{s} & \dfrac{300s+250}{3s} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} \dfrac{8}{s+2} \\ \dfrac{2s-36}{s^2+2s} \\ 0 \end{bmatrix}$$

d)

$$I_{L_1}(s) = \frac{-V_1(s)}{L_1 s} + \frac{i_{L_1}(0^-)}{s} = \frac{-V_{in}(s)}{L_1 s} + \frac{i_{L_1}(0^-)}{s}$$

```
IL1=-Vin/(L1*s)+iL0/s;
IL1=collect(eval(IL1));
pretty(IL1)
iL1=ilaplace(IL1);
pretty(iL1)
```

$$I_{L_1}(s) = \frac{4s-72}{s^2+2s}$$

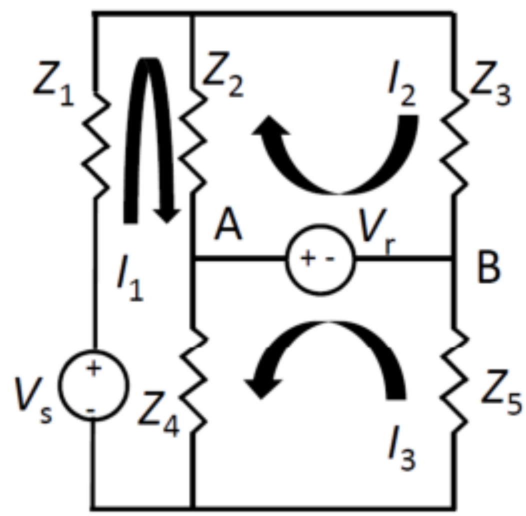
Zero-State: $i_{L_1}(t) = (40e^{-2t} - 40)u(t)$

Zero Input: $i_{L_1}(t) = 4u(t)$

$$i_{L_1}(t) = (40e^{-2t} - 36)u(t)$$

20.

a)



Apply KVL loop equations:

Loop 1:

$$Z_1I_1 + Z_2(I_1 - I_2) + Z_4(I_1 - I_3) = V_S \Rightarrow (Z_1 + Z_2 + Z_4)I_1 - Z_2I_2 - Z_4I_3 = V_s$$

Loop 2:

$$Z_2(I_2 - I_1) + Z_3I_2 = V_r \Rightarrow -Z_2I_1 + (Z_2 + Z_3)I_2 = V_r$$

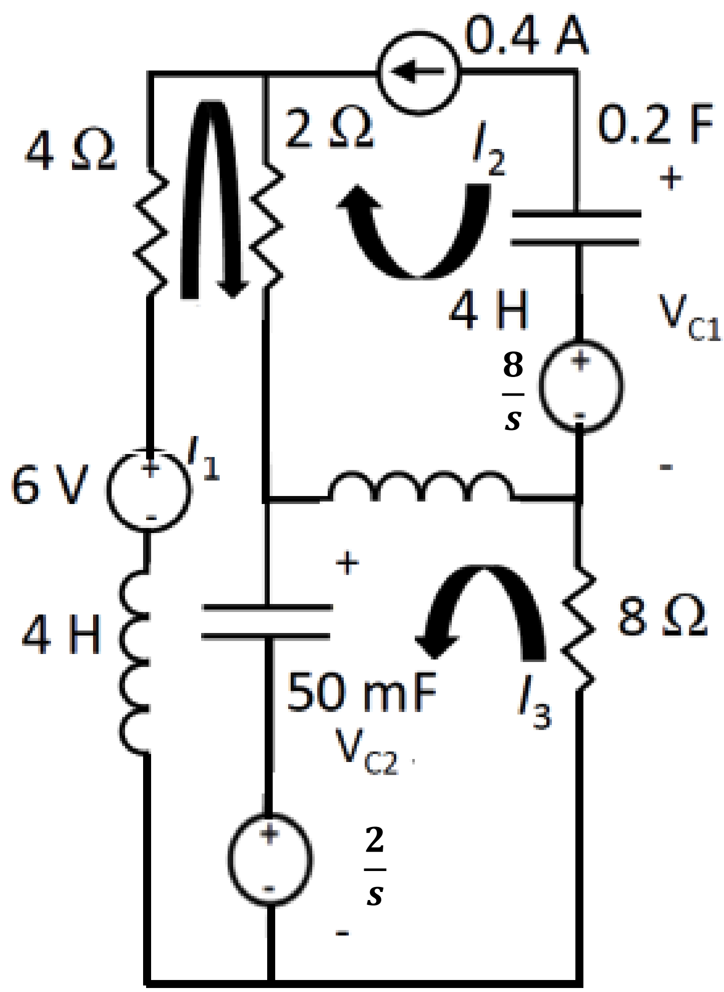
Loop 3:

$$Z_5I_3 + Z_4(I_3 + I_1) = V_r \Rightarrow Z_4I_1 + (Z_4 + Z_5)I_3 = V_r$$

Putting above equations in matrix form:

$$\begin{bmatrix} Z_1 + Z_2 + Z_4 & -Z_2 & -Z_4 \\ -Z_2 & Z_2 + Z_3 & 0 \\ Z_4 & 0 & Z_4 + Z_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_s \\ V_r \\ V_r \end{bmatrix}$$

b) We need to use voltage sources as equivalent frequency domain circuits for the inductor and the capacitor. The reason is that when we write KVL (loop equations), we consider the voltage across each component. So it's easier to have voltage sources. Of course with voltage sources, unlike current sources, we don't need to use extra constraint equations either.



Apply KVL loop equations:

Loop 1:

$$4sI_1(s) - \frac{6}{s} + 4I_1(s) + 2\big(I_1(s) - I_2(s)\big) + \frac{20}{s}\big(I_1(s) + I_3(s)\big) + \frac{2}{s} = 0$$

$$\left(4s + 6 + \frac{20}{s}\right) I_1(s) - 2I_2(s) + \frac{20}{s} I_3(s) = \frac{4}{s}$$

Loop 2:

$$I_2(s) = -\frac{0.4}{s}$$

Loop 3:

$$8I_3(s) + 4s\big(I_3(s) + I_2(s)\big) + \frac{20}{s}\big(I_3(s) + I_1(s)\big) + \frac{2}{s} = 0$$

$$\frac{20}{s}I_1(s) + 4sI_2(s) + \left(4s + 8 + \frac{20}{s}\right)I_3(s) = -\frac{2}{s}$$

Putting in MATRIX form:

$$\begin{bmatrix} 4s + 6 + \frac{20}{s} & -2 & \frac{20}{s} \\ 0 & 0 & 1 \\ \frac{20}{s} & 4s & 4s + 8 + \frac{20}{s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} \frac{4}{s} \\ -\frac{0.4}{s} \\ -\frac{2}{s} \end{bmatrix}$$