PHYSICS 272
Electric & Magnetic Interactions

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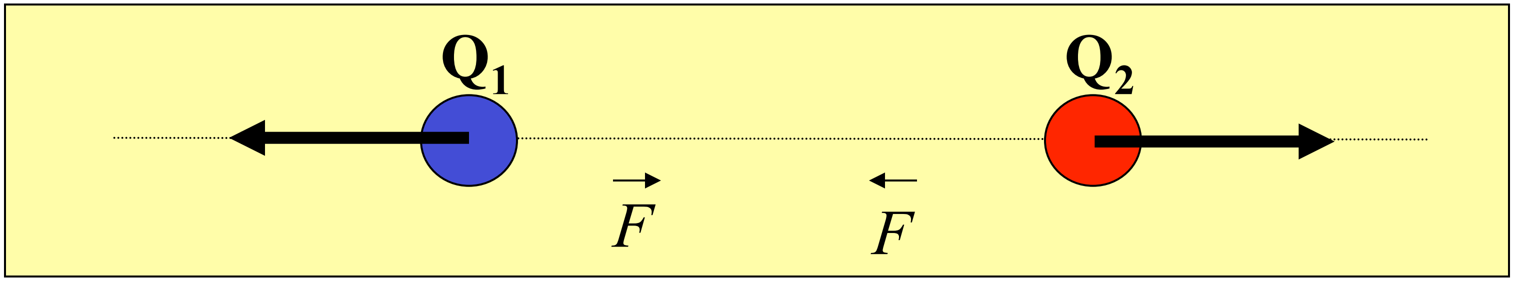
Question 1 (Chap. 14)

Key Ideas in Chapter 14: Electric Field

- **A charged particle makes an electric field at every location in space (except its own location).**
- **The electric field due to one particle affects other charged particles.**
- **The electric force on a charged particle is proportional to the net electric field at the location of that particle.**
- **The Superposition Principle:**
  - The net electric field at any location is the vector sum of the individual electric fields of all charge particles at other locations.
  - The field due to one charged particle is not changed by the presence of other charged particles.
- **An electric dipole consists of two particles with charges equal in magnitude and opposite in sign, separated by a short distance.**
- **Changes in electric fields travel at the speed of light ("retardation").**

The Coulomb Force Law

Key Idea: Charges exert forces on each other



$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}\hat{r}$$

Direction of Force is:

ATTRACTIVE if charges have OPPOSITE sign

REPULSIVE if charges have SAME sign

Always acts along a line connecting the charges

Question 2 (Chap. 14)

How Strong is the Coulomb Force?

$$F_{\text{elec}} = \frac{1}{4\pi\epsilon_o}\frac{Q_1 Q_2}{r^2}\hat{r} \qquad F_{\text{grav}} = G\frac{m_1 m_2}{r^2}\hat{r}$$

Example: Hydrogen Atom = proton and electron

$$|Q1| = |Q2| = e = 1.6 \times 10^{-19} C$$

$$m_{\text{proton}} = 1.7 \times 10^{-27} kg \qquad m_{\text{electron}} = 9 \times 10^{-31} kg$$

$$F_{\text{elec}} = \frac{(8.99 \times 10^9 Nm^2/C^2)(1.60 \times 10^{-19} C)^2}{(5.3 \times 10^{-11} m)^2} \approx O(10^{-7})N$$

$$F_{\text{grav}} = \frac{(6.7 \times 10^{-11} Nm^2/kg^2)(1.7 \times 10^{-27} kg)(9 \times 10^{-31} kg)}{(5.3 \times 10^{-11} m)^2} \approx O(10^{-46})N$$

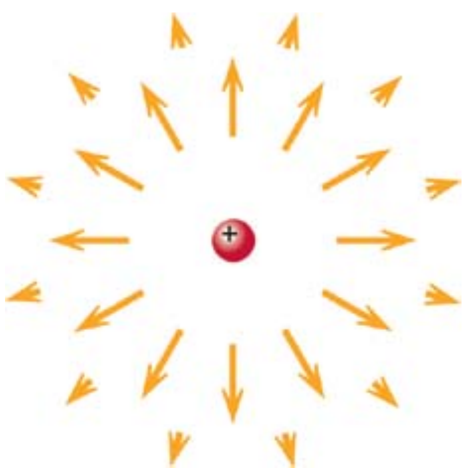
$$\boxed{\frac{F_{\text{elec}}}{F_{\text{grav}}} \approx 2.27 \times 10^{39}}$$

Electricity is ***much stronger*** than Gravity

Electric Field of Point Charges

Key Idea: The electric force on a charged particle is proportional to the net electric field at the location of that particle.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}\hat{r} \qquad \vec{F}_2 = q_2\vec{E}_1 \qquad \vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r}$$



What happens as $\mathbf{r} \to \mathbf{0}$?

Can the particle exert a force on itself?

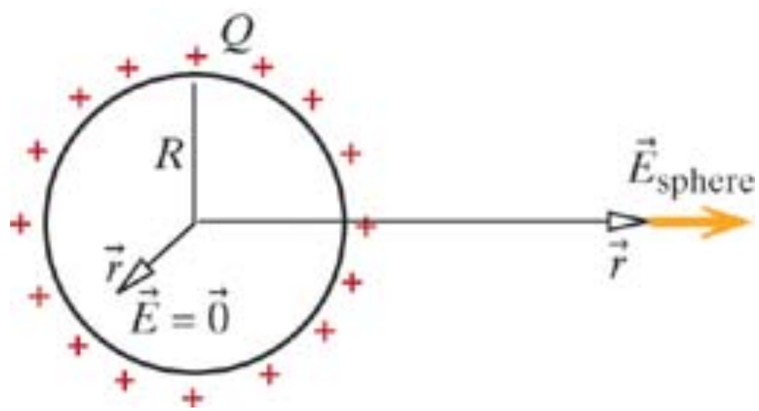
No.

E is undefined at the origin, since $\hat{\mathbf{r}}$ is self-contradictory.

If there were a force at the origin, which way would it point?

Electric Field of a Uniformly Charged Spherical Shell

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r}$$ ← for a point particle



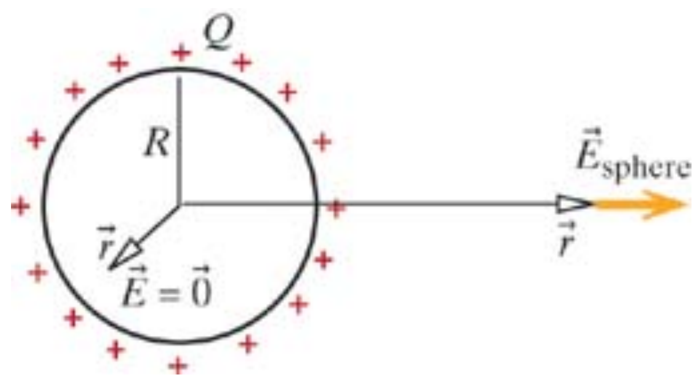
What should the electric field look like from far away?

$$\vec{E}_{sphere} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \qquad \text{for } r >> R$$

Which direction does the field point close to the sphere?

Spherical symmetry $\Longrightarrow$ The field is in the $\hat{\mathbf{r}}$ direction even close up to the sphere

Electric Field of a Uniformly Charged Spherical Shell

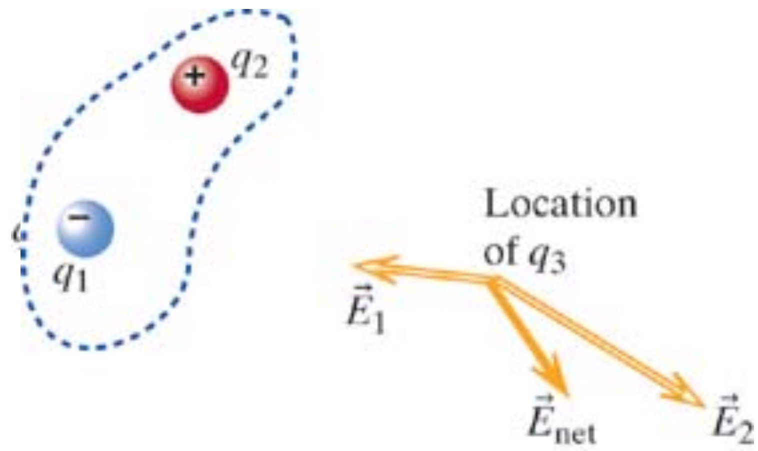


We'll calculate the whole thing in Ch. 16 using the principle of superposition:

$$\vec{E}_{sphere} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \qquad \text{for } r > R \text{ (outside)}$$

$$\vec{E}_{sphere} = 0 \qquad \text{for } r < R \text{ (inside)}$$

The Superposition Principle

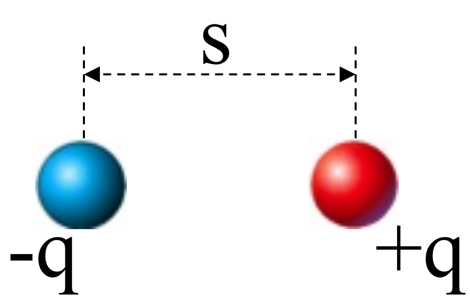


Key Idea: The net electric field at a location in space is a vector sum of the individual electric fields contributed by all charged particles located elsewhere.

Key Idea: The electric field contributed by a charged particle is unaffected by the presence of other charged particles.

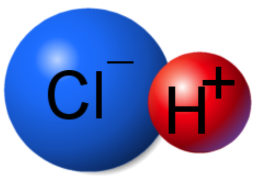
The Superposition Principle

The electric field of a dipole:



Electric dipole:
Two equally but oppositely charged point-like objects

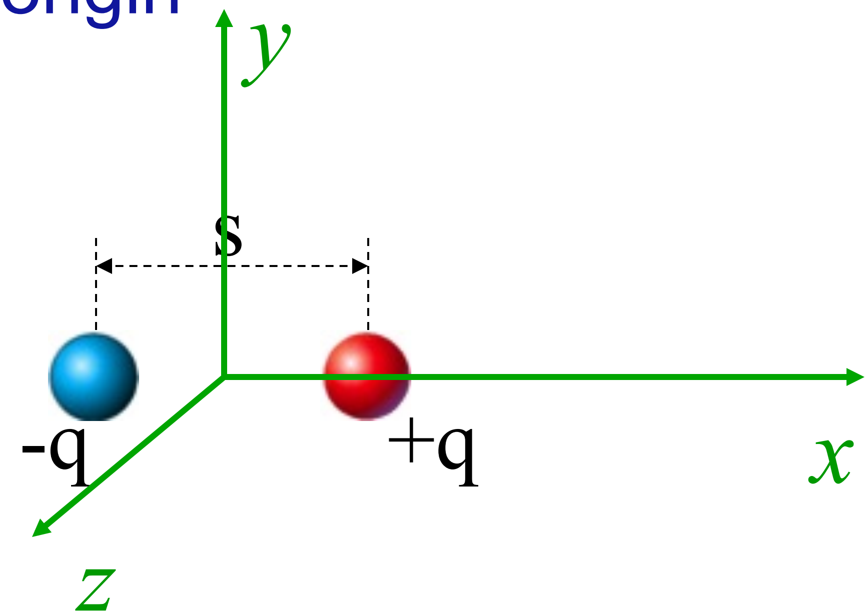
Example of electric dipole: HCl molecule



What is the E field **far** from the dipole ($r >> s$)?

Calculating Electric Field

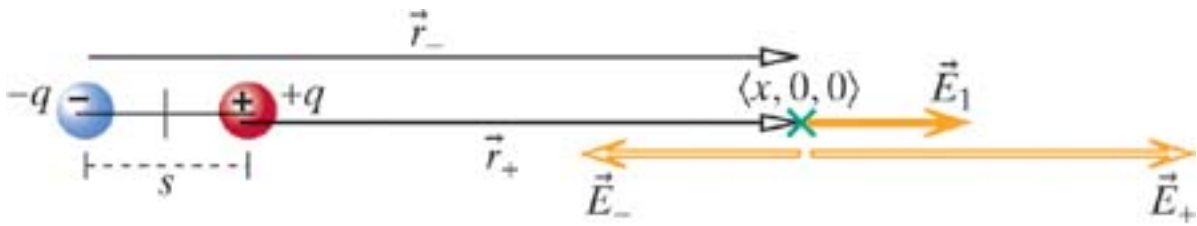
Choice of the origin



Choice of origin: use symmetry

1. *E* along the *x*-axis

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r}$$ for a point particle



$$E_{1,x} = \boxed{E_{+,x}} + \boxed{E_{-,x}} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-s/2)^2}} + \boxed{\frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+s/2)^2}}$$

$$E_{1,x} = \frac{1}{4\pi\varepsilon_0}\frac{qr^2 + qrs + qs^2/4 - qr^2 + qrs - qs^2/4}{(r-s/2)^2(r+s/2)^2}$$

$$E_{1,x} = \frac{1}{4\pi\varepsilon_0}\frac{2qrs}{(r-s/2)^2(r+s/2)^2}$$

Far from the Dipole on the x-axis

$$E_{1,x} = \frac{1}{4\pi\varepsilon_0} \frac{2srq}{\left(r - \frac{s}{2}\right)^2 \left(r + \frac{s}{2}\right)^2}$$

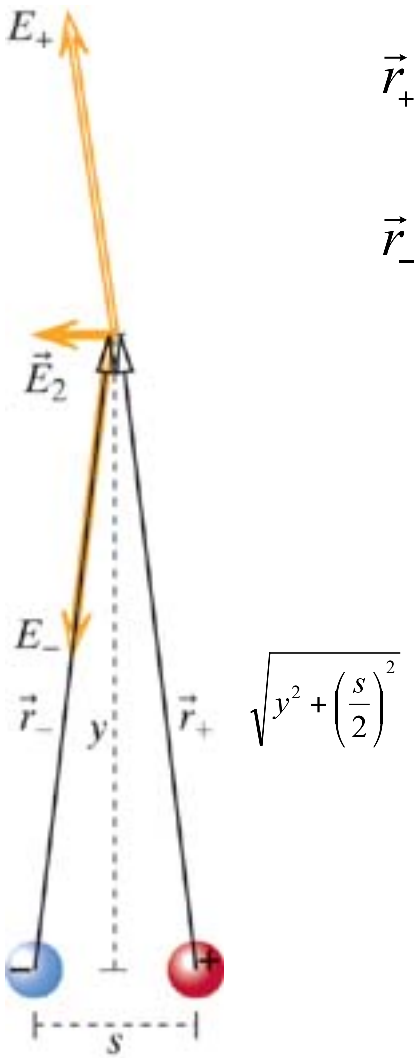
if r>>s, then $\left(r - \frac{s}{2}\right)^2 \approx \left(r + \frac{s}{2}\right)^2 \approx r^2$

$$E_{1,x} = \frac{1}{4\pi\varepsilon_0} \frac{2sq}{r^3} \qquad \vec{E}_1 = \left\langle \frac{1}{4\pi\varepsilon_0} \frac{2sq}{r^3}, 0, 0 \right\rangle$$

While the electric field of a point charge is proportional to $1/r^2$, the electric field created by several charges may have a different distance dependence.

2. *E* along the *y*-axis

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r} \longleftarrow$$ for a point particle



$$\left.\begin{aligned}\vec{r}_+ &= \langle 0,y,0\rangle - \left\langle \frac{s}{2},0,0\right\rangle = \left\langle -\frac{s}{2},y,0\right\rangle \\ \vec{r}_- &= \langle 0,y,0\rangle - \left\langle -\frac{s}{2},0,0\right\rangle = \left\langle \frac{s}{2},y,0\right\rangle\end{aligned}\right\} \Rightarrow |r_+| = |r_-| = \sqrt{y^2+\left(\frac{s}{2}\right)^2}$$

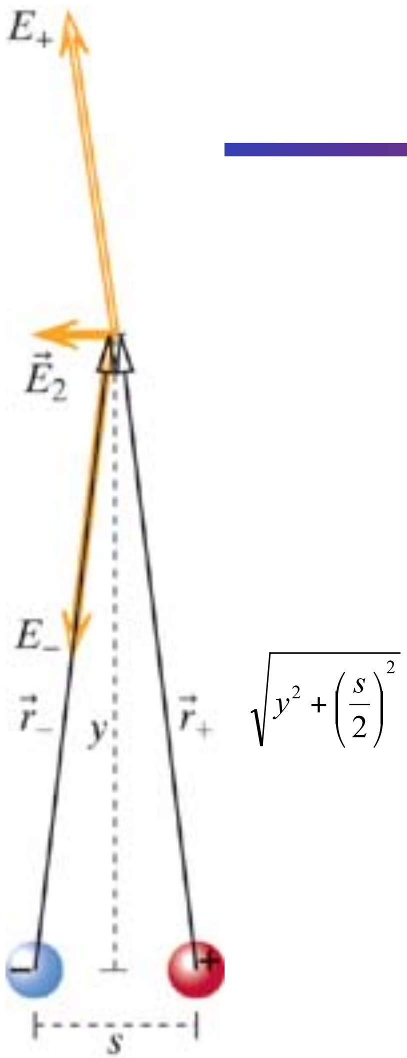
$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{y^2+\left(\frac{s}{2}\right)^2}\frac{\left\langle -\frac{s}{2},y,0\right\rangle}{\sqrt{y^2+\left(\frac{s}{2}\right)^2}}$$

$$\vec{E}_- = \frac{1}{4\pi\varepsilon_0}\frac{-q}{y^2+\left(\frac{s}{2}\right)^2}\frac{\left\langle \frac{s}{2},y,0\right\rangle}{\sqrt{y^2+\left(\frac{s}{2}\right)^2}}$$

2. *E* along the *y*-axis

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{y^2+\left(\frac{s}{2}\right)^2}\frac{\left\langle -\frac{s}{2}, y, 0\right\rangle}{\sqrt{y^2+\left(\frac{s}{2}\right)^2}}$$

$$\vec{E}_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{y^2+\left(\frac{s}{2}\right)^2}\frac{\left\langle -\frac{s}{2}, -y, 0\right\rangle}{\sqrt{y^2+\left(\frac{s}{2}\right)^2}}$$



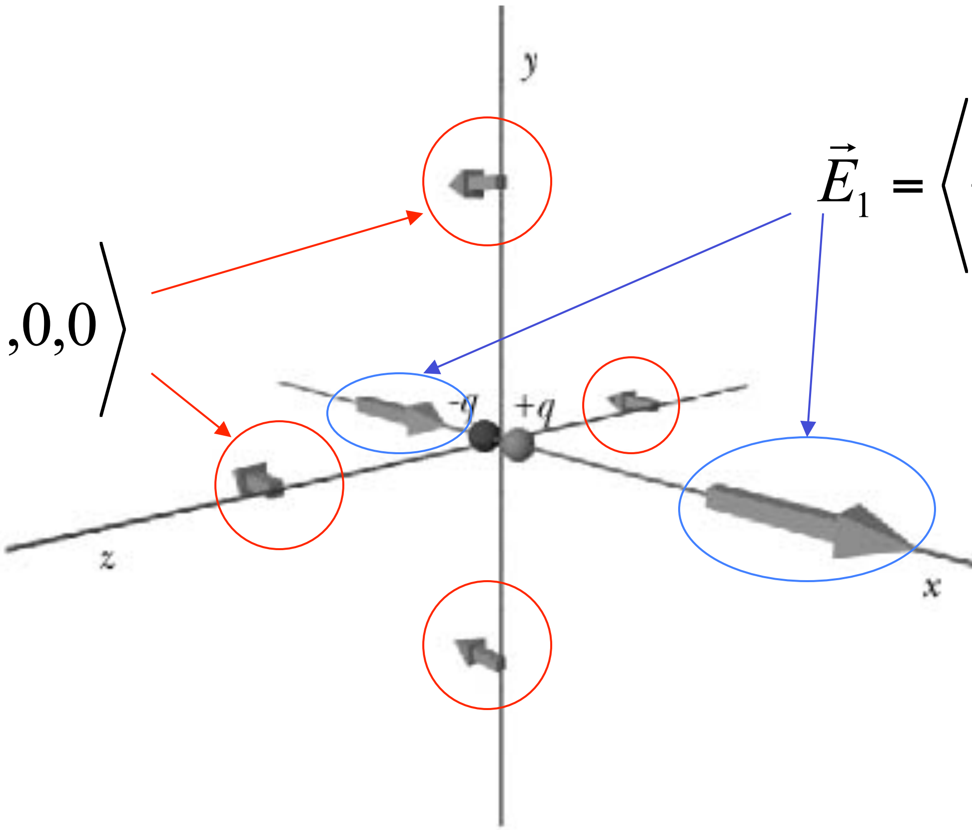
$$\vec{E}_2 = \vec{E}_+ + \vec{E}_- = \frac{1}{4\pi\varepsilon_0}\frac{qs}{\left[y^2+\left(\frac{s}{2}\right)^2\right]^{\frac{3}{2}}}\langle -1,0,0\rangle$$

if r>>s, then $\vec{E}_2 \approx \left\langle \frac{-1}{4\pi\varepsilon_0}\frac{qs}{r^3}, 0, 0\right\rangle$ at <0,r,0>

3. *E* along the *z*-axis

$$\vec{E}_2 = \left\langle \frac{-1}{4\pi\varepsilon_0}\frac{qs}{r^3},0,0 \right\rangle$$

at <0,r,0>
or <0,0,r>

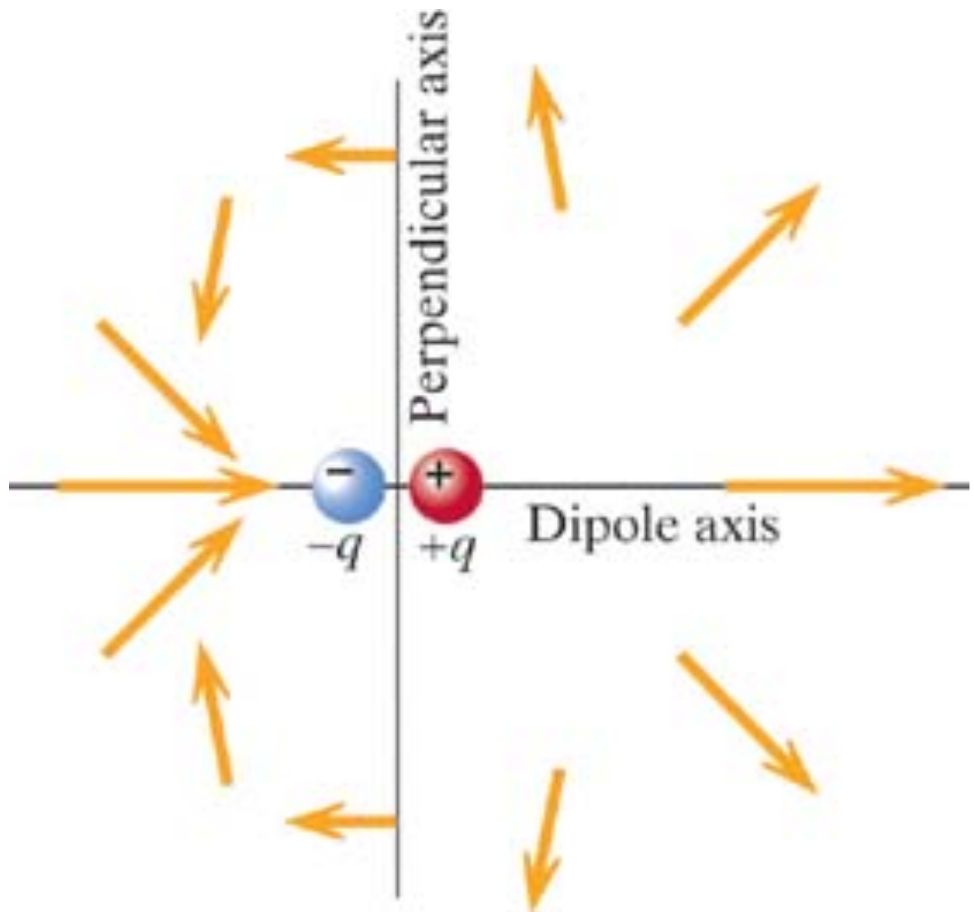


$$\vec{E}_1 = \left\langle \frac{1}{4\pi\varepsilon_0}\frac{2sq}{r^3},0,0 \right\rangle$$

at <r,0,0>

By symmetry,
E along the *z*-axis must be the same as
E along the *y*-axis!

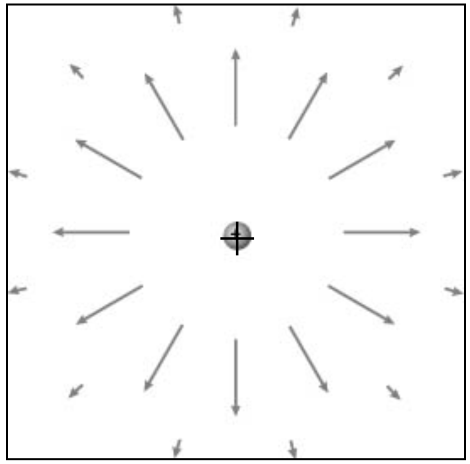
Other Locations

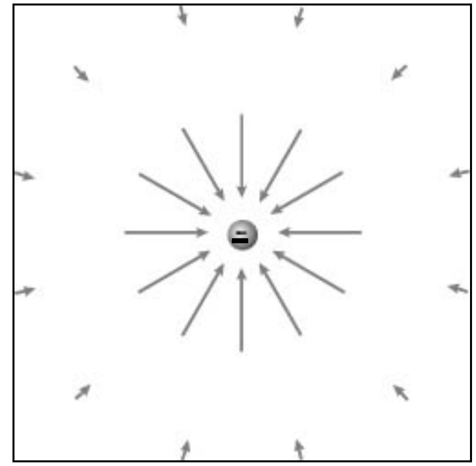


Summary: Point Charge and Dipole

Point Charge:

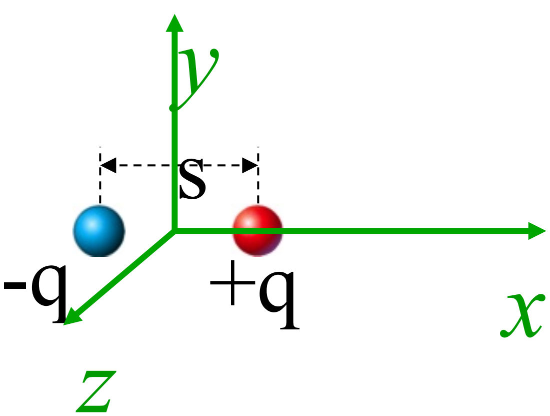
$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r}$$





Dipole:

for r>>s :

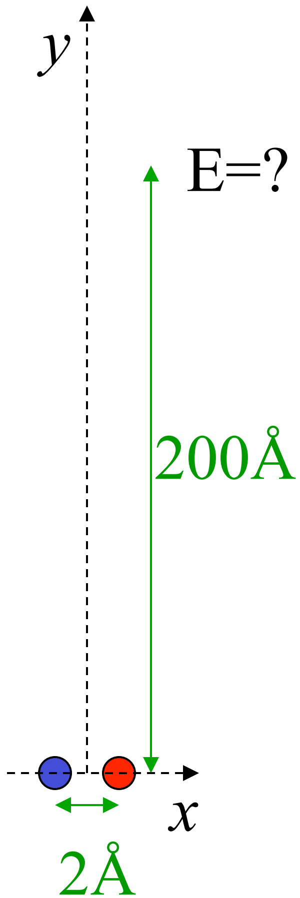


$$\vec{E} = \left\langle \frac{1}{4\pi\varepsilon_0}\frac{2qs}{r^3},0,0 \right\rangle$$ at <r,0,0>

$$\vec{E} = \left\langle \frac{-1}{4\pi\varepsilon_0}\frac{qs}{r^3},0,0 \right\rangle$$ at <0,r,0>

$$\vec{E} = \left\langle \frac{-1}{4\pi\varepsilon_0}\frac{qs}{r^3},0,0 \right\rangle$$ at <0,0,r>

Example Problem



A dipole is located at the origin, and is composed of particles with charges e and $-e$, separated by a distance 2×10^{-10} m along the x-axis. Calculate the magnitude of the E field at <0,2×10^{-8},0> m.

Since $r \gg s$: $E_{1,x} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{sq}{r^3}$

$$E_{1,x} = \left(9\times10^9\,\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\right)\left(\frac{2\times10^{-10}\,\mathrm{m}\times1.6\times10^{-19}\,\mathrm{C}}{\left(2\times10^{-8}\,\mathrm{m}\right)^3}\right)$$

$$E_{1,x} = 7.2\times10^4\,\frac{\mathrm{N}}{\mathrm{C}}$$

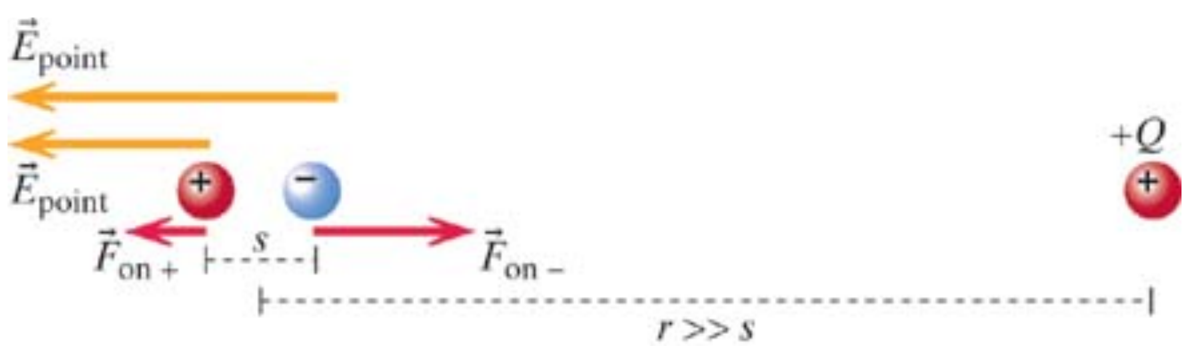
Using exact solution:

$$E_{1,x} = 7.1999973\times10^4\,\frac{\mathrm{N}}{\mathrm{C}}$$

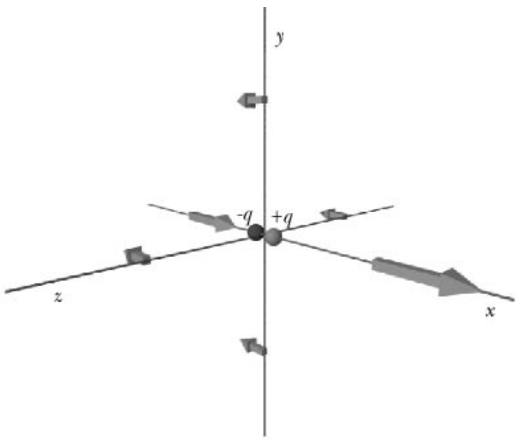
Interaction of a Point Charge and a Dipole

$$\vec{F} = Q\vec{E}_{dipole} = Q\left\langle \frac{-1}{4\pi\varepsilon_0}\frac{2qs}{d^3},0,0 \right\rangle$$

- Direction makes sense?
  - negative end of dipole is closer, so its net contribution is larger
- What is the force exerted on the dipole by the point charge?
  - Newton's third law: equal but opposite sign



Definition of "Dipole Moment"



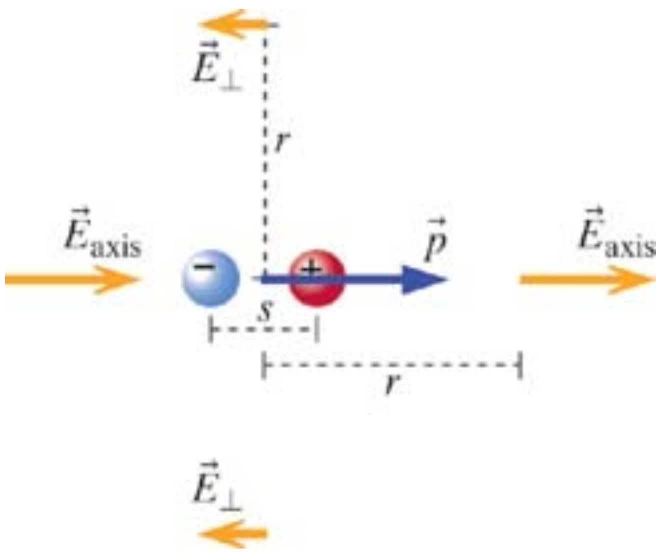
x: $\vec{E}_1 = \left\langle \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}, 0, 0 \right\rangle$

y, z: $\vec{E}_2 = \left\langle \frac{-1}{4\pi\varepsilon_0}\frac{p}{r^3}, 0, 0 \right\rangle$

$r >> s$

The electric field of a dipole is proportional to the

Dipole moment: $p = qs$



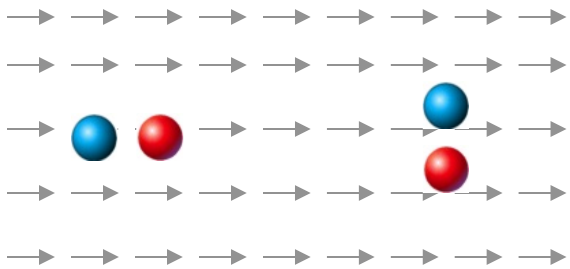
$\left|\vec{p}\right| = qs$, direction from $-q$ to $+q$

Dipole moment is a vector pointing from negative to positive charge

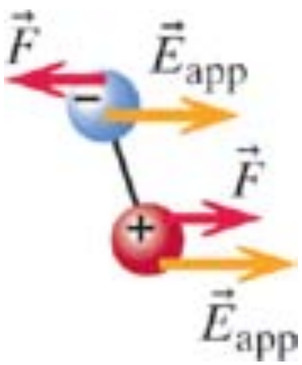
Dipole in a Uniform Field

$$\vec{F} = q\vec{E}$$

Forces on $+q$ and $-q$ have the same magnitude but opposite direction



$$\vec{F}_{net} = +q\vec{E} - q\vec{E} = 0$$

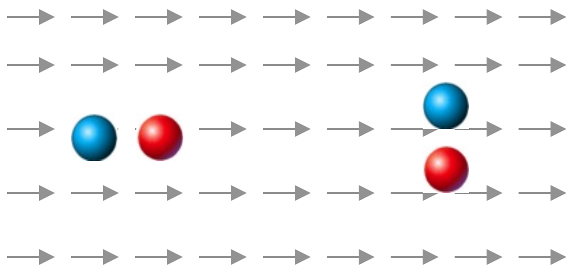


What happens in this case?

Dipole in a Uniform Field

$$\vec{F} = q\vec{E} \longrightarrow$$

Forces on $+q$ and $-q$ have the same magnitude but opposite direction



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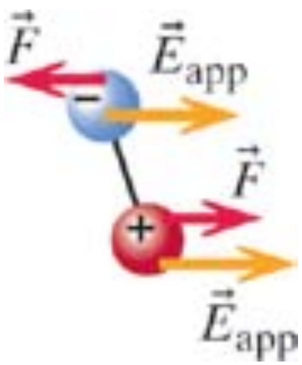


Dipole experiences a torque about its center of mass.

What is the equilibrium position?

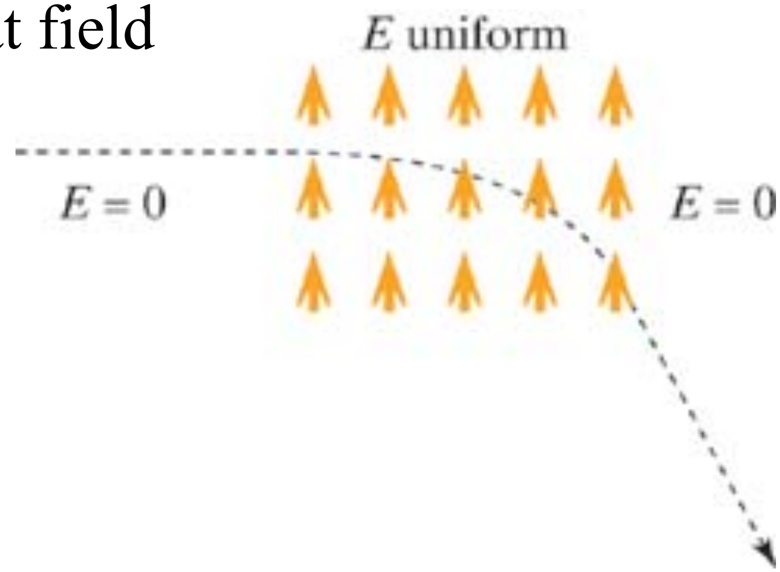
Electric dipole can be used to measure the direction of electric field.

Choice of System

Multiparticle systems: Split into objects to include into system and objects to be considered as external.

To use field concept instead of Coulomb's law we split the Universe into two parts:

- the charges that are the sources of the field
- the charge that is affected by that field



Example: Oscilloscope

Charges on metal plates are the sources of an uniform E field

A Fundamental Rationale

- If know E at some location, then we know the electric force on any charge: $\vec{F} = q\vec{E}$
- Can describe the electric properties of matter in terms of electric field – independent of how this field was produced.

Example: if $E>3\times10^{6}$ N/C air becomes conductor

- **Retardation**

Nothing can move faster than light c

c = 300,000 km/s = 30 cm/ns

Coulomb's law is not completely correct – it does not contain time t nor speed of light c.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \qquad v << c \text{ !!!}$$

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  - The field due to one charged particle is not changed by the presence of other charged particles.
- **An electric dipole consists of two particles with charges equal in magnitude and opposite in sign, separated by a short distance.**
- **Changes in electric fields travel at the speed of light ("retardation").**