Last Time

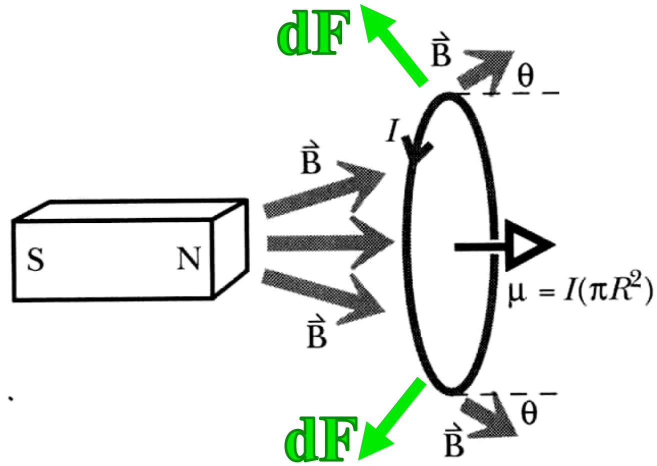
Magnetic Torque

Magnetic Dipole in a B-Field:

- Potential Energy
- (Force)

(Motors and Generators)

Force on a Magnetic Dipole



$$d\vec{F} = I d\vec{l} \times \vec{B}$$

→ Net force to the **left**

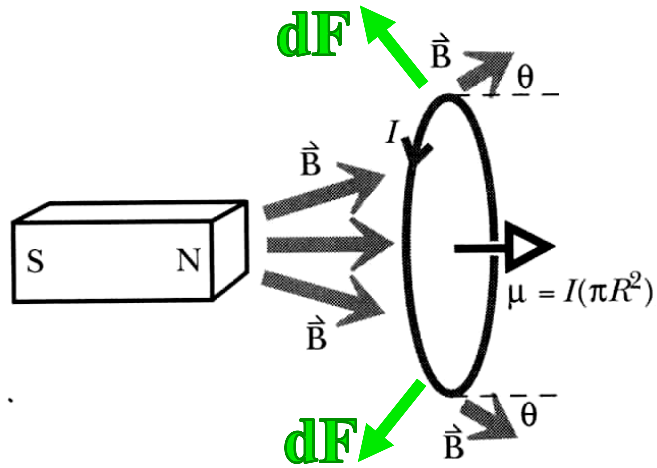
Our expression for B due to dipole only works on the axis, so we need a *different* way to calculate F.

Sneaky Way to find F_{magnet} : Use $W = \Delta U$ and $U = -\vec{\mu} \cdot \vec{B}$

Force on a Magnetic Dipole

Magnet pulls
coil to the **left**

F_{magnet} ←



We pull
coil to the **right**

F_{hand} →

Sneaky Way to find F_{magnet} : Use $W = \Delta U$ and $U = -\vec{\mu}\cdot\vec{B}$

$$\Delta U = W_{\text{by hand}} = F_{\text{hand}}\Delta x = -F_{\text{magnet}}\Delta x$$

$$F_{\text{magnet}} = -\frac{\Delta U}{\Delta x} \rightarrow \boxed{F_{\text{magnet}} = -\frac{dU}{dx}}$$

ALWAYS

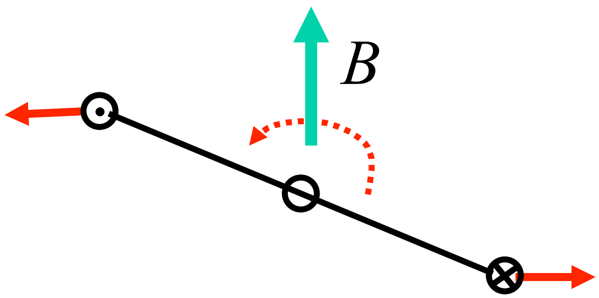
$$\boxed{F_{\text{magnet}} = -\mu\frac{dB}{dx}}$$

Only if μ is constant
(typical case)

If B is constant, net force is zero.

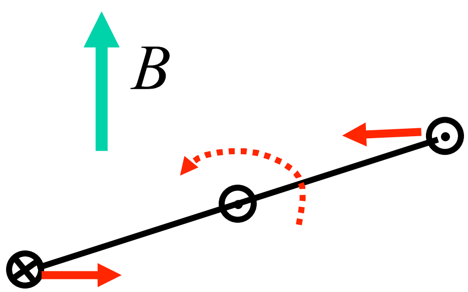
How to Make an Electric Motor

$$\vec{F}_m = I\Delta\vec{l} \times \vec{B}$$

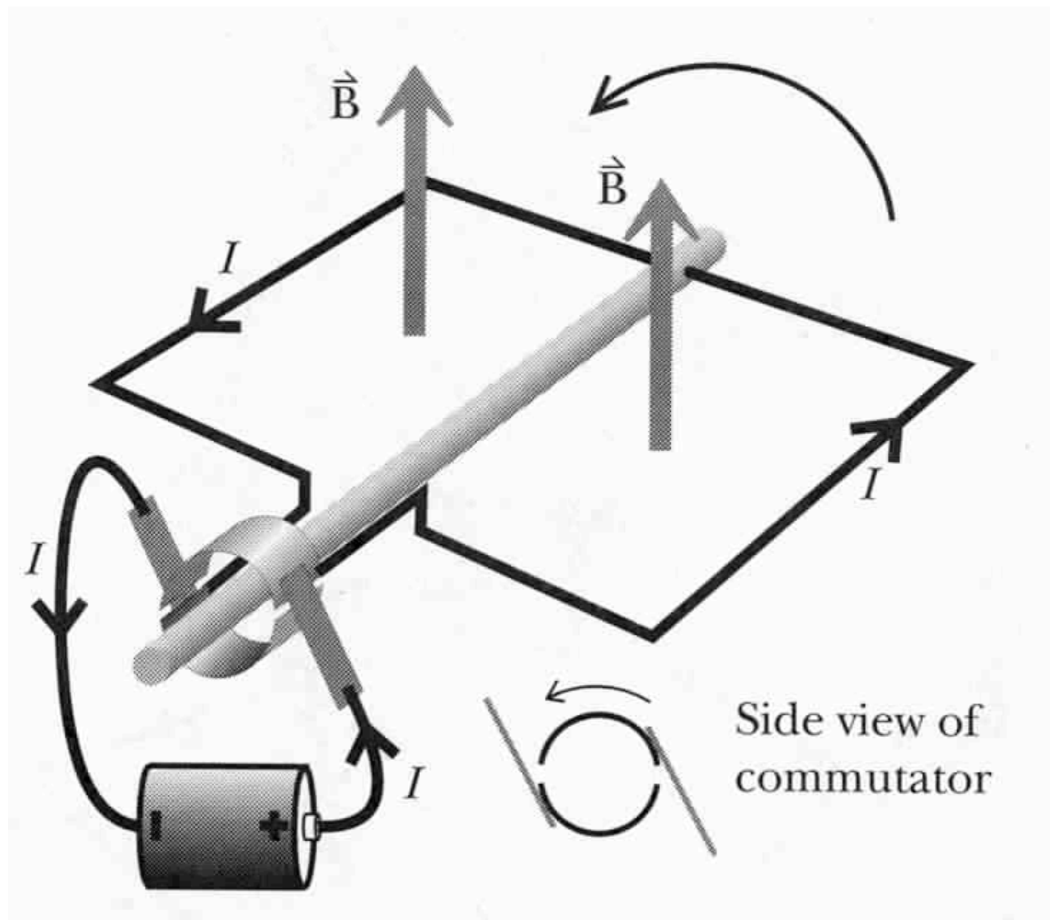


Run the current one way

Then reverse the current



Torque is in same direction



Today

Gauss' Law

Where's the Source? Follow the Flow!

We only see water flowing out.

We see the water flow in.
We see the same amount flow out.

The source of this fountain **must** be in the bowl.

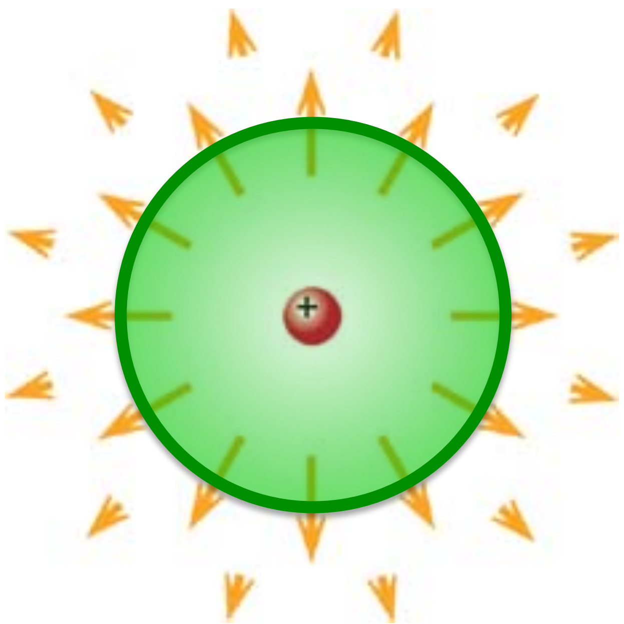
The source of this fountain is not in the picture.

What is the Source?

The **Source** of **Water**
is a **Water Faucet**

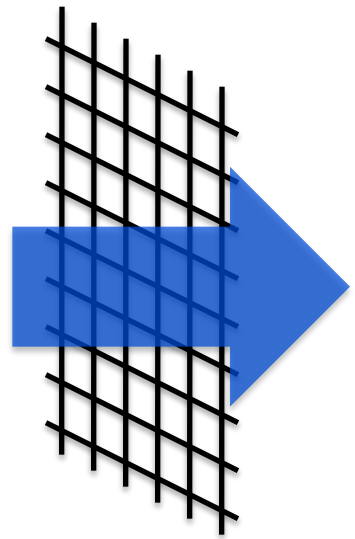
Water flowing OUT of bowl
→ there is a **Source** inside

The **Source** of **E-Field**
is **Charge**



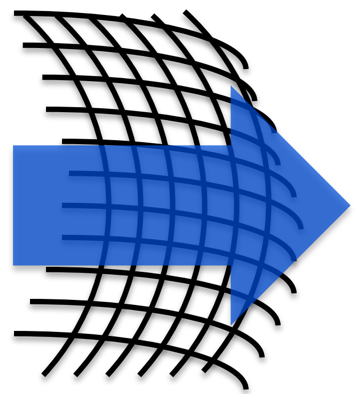
E-field "flowing" out of sphere
→ there is a **Source** inside

Electric Flux is like Water Flow



Water flows through a net
at a certain rate.

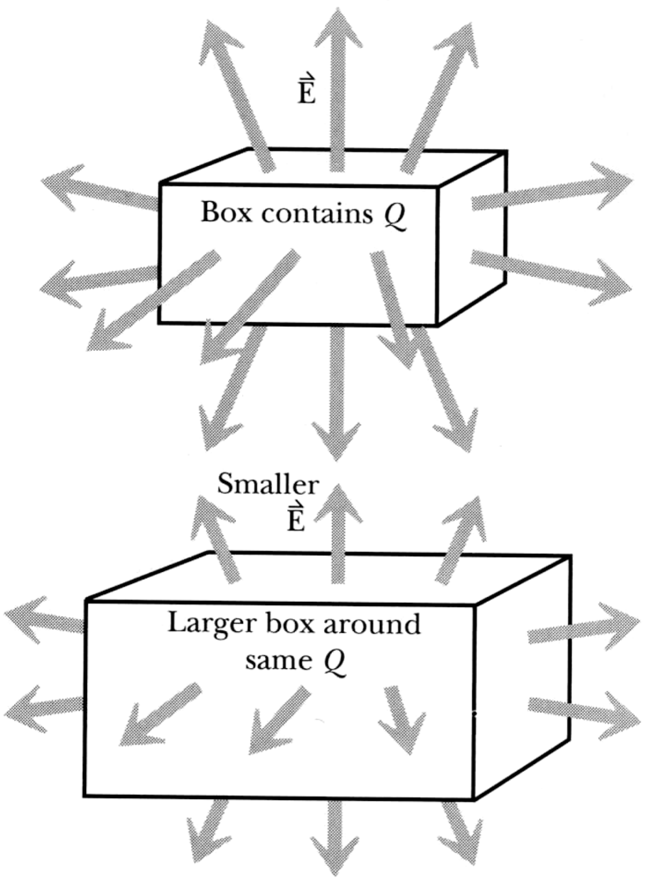
Molecules/second
through each square



The net can deform,
but the flow rate is the same.

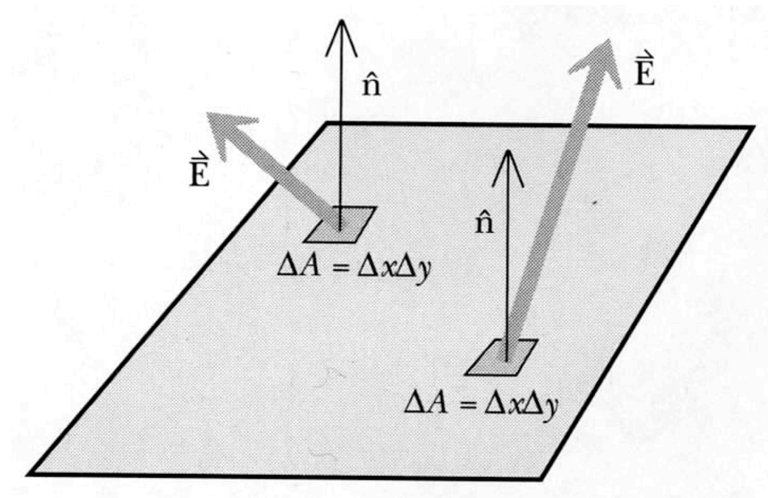
Same number of
molecules/second
through each square

Electric Flux: Surface Area



Flux through small area:

$$flux \sim \vec{E} \cdot \hat{n} \Delta A$$



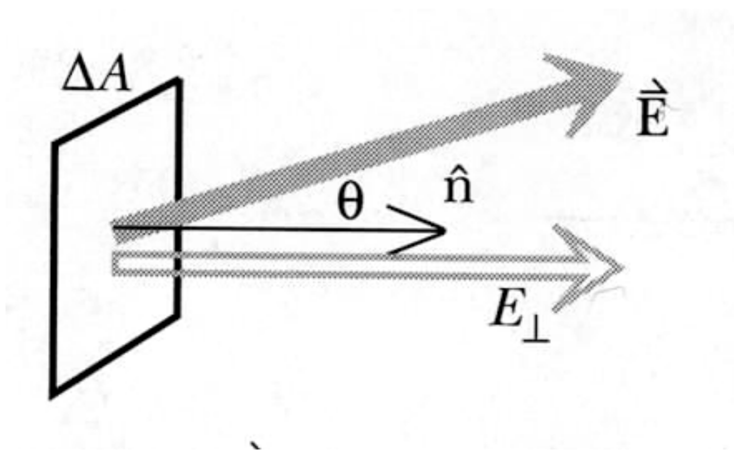
Definition of electric flux on a surface:

$$\sum_{surface} \vec{E} \cdot \hat{n} \Delta A$$

Electric Flux: Perpendicular Field or Area

Perpendicular field

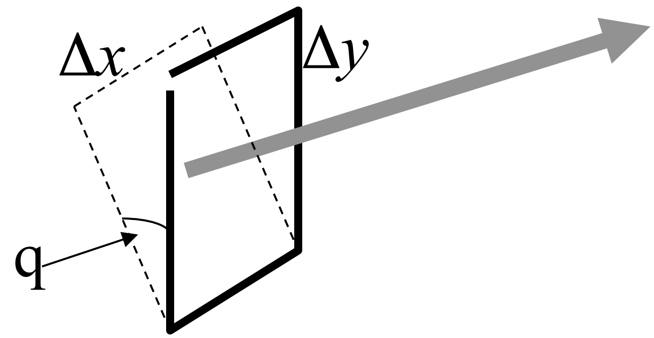
$$\vec{E} \cdot \hat{n}\Delta A = \Delta A E \cos\theta$$



$$\vec{E} \cdot \hat{n}\Delta A = \Delta A E_{\perp}$$

Perpendicular area

$$\vec{E} \cdot \hat{n}\Delta A = E\Delta A \cos\theta = E\Delta x \Delta y \cos\theta$$



$$\vec{E} \cdot \hat{n}\Delta A = E\Delta A_{\perp}$$

Adding up the Flux

$$\boxed{\sum_{surface} \vec{E} \cdot \hat{n} \Delta A} \longrightarrow \int \vec{E} \cdot \hat{n} dA$$

$$\hat{n} dA \rightarrow d\vec{A}$$

$$\int \vec{E} \cdot d\vec{A}$$

electric flux on a closed surface = $\oint \vec{E} \cdot d\vec{A}$

Gauss's Law

$$\sum_{surface} \vec{E} \cdot \hat{n} \Delta A = \frac{\sum q_{inside}}{\varepsilon_0}$$

$$\oint \vec{E} \cdot \hat{n} dA = \frac{\sum q_{inside}}{\varepsilon_0}$$

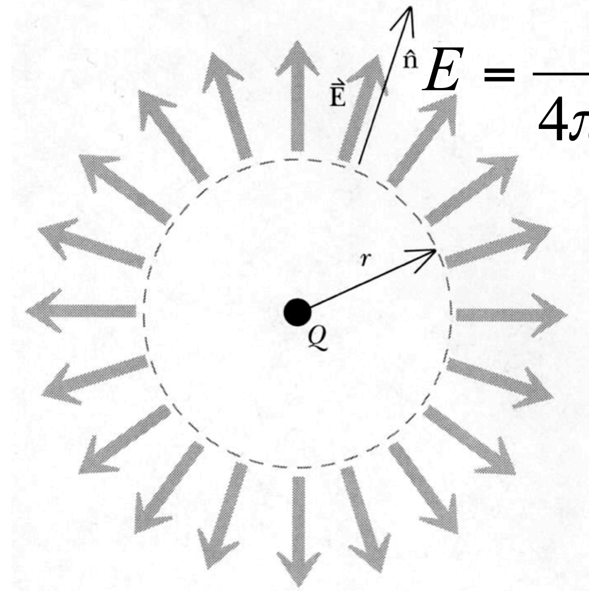
Features:

1. Proportionality constant
2. Size and shape independence
3. Independence on number of charges inside
4. Charges outside contribute zero

1. Gauss's Law: Proportionality Constant

$$\boxed{\sum_{surface} \vec{E} \cdot \hat{n} \Delta A = \frac{\sum q_{inside}}{\varepsilon_0}}$$

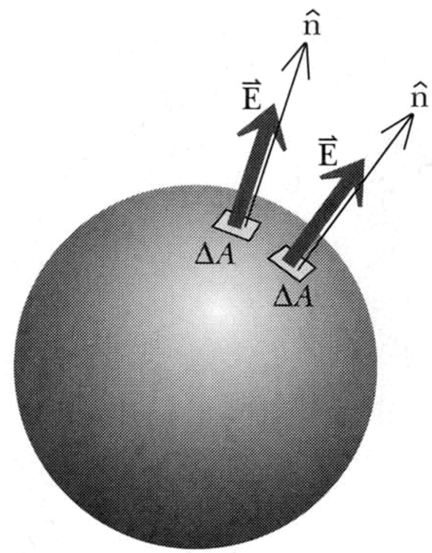
$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$



$$\sum_{surface} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r} \cdot \hat{n} \Delta A$$

$$\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \sum_{surface} \Delta A$$

$$\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} 4\pi r^2 = \frac{Q}{\varepsilon_0}$$



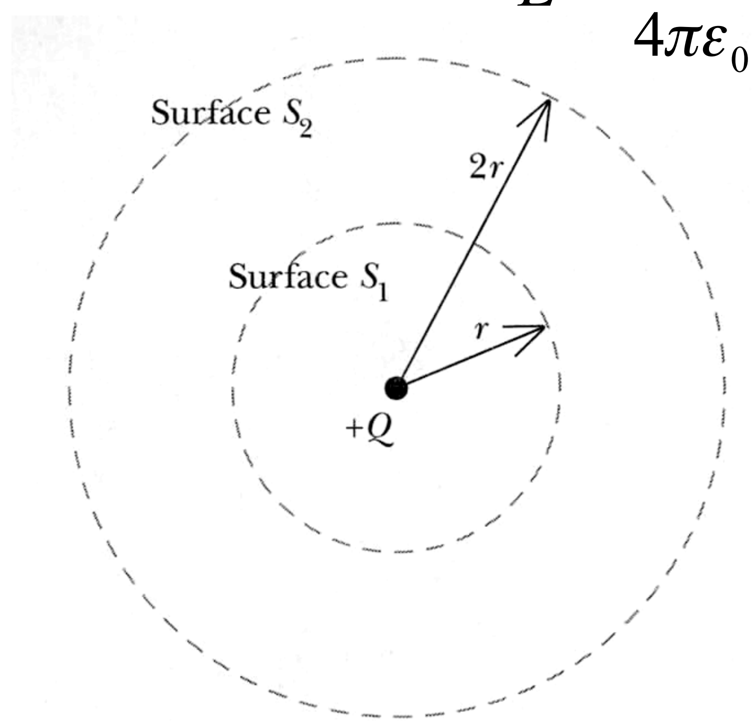
What if charge is negative?

Works at least for one charge and spherical surface

2. Gauss's Law: The Size of the Surface

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$\sum_{surface} \vec{E}\cdot\hat{n}\Delta A = \frac{\sum q_{inside}}{\varepsilon_0}$$



$$E \sim \frac{1}{r^2}$$

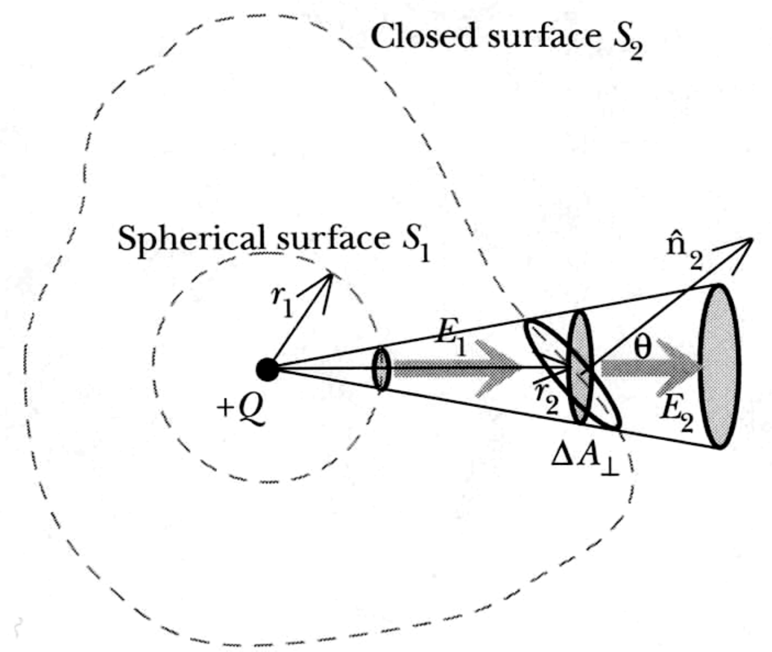
$$A \sim r^2$$

$$\boxed{E \sim \frac{1}{r^2}} \longleftarrow$$

universe would be much different if exponent was not exactly 2!

3. Gauss's Law: The Shape of the Surface

$$\sum_{surface} \vec{E}\cdot\hat{n}\Delta A = \frac{\sum q_{inside}}{\varepsilon_0}$$

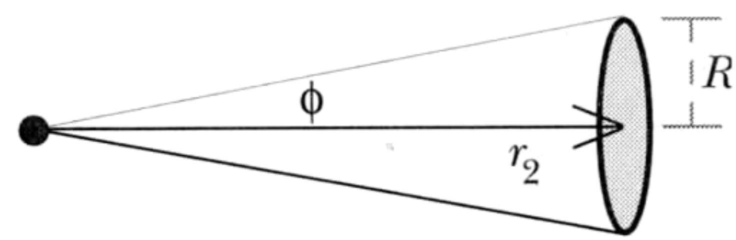


$$\sum_{surface} \vec{E}\cdot\hat{n}\Delta A = \sum_{surface} E\Delta A_\perp$$

$$\Delta A_{2\perp} = \pi R^2 = \pi(r_2 \tan\phi)^2 \propto r_2^2$$

$\downarrow$

All elements of the outer surface can be projected onto corresponding areas on the inner sphere with the same flux

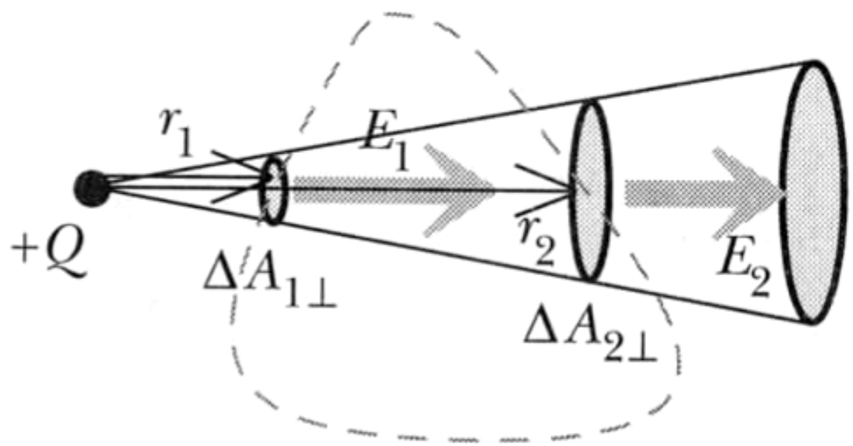


$$\Delta A_{2\perp} / \Delta A_{1\perp} = r_2^2 / r_1^2$$

$$E_2\Delta A_{2\perp} / E_1\Delta A_{1\perp} = 1$$

4. Gauss's Law: Outside Charges

$$\sum_{surface} \vec{E} \cdot \hat{n} \Delta A = \frac{\sum q_{inside}}{\varepsilon_0}$$



$$\sum_{surface} \vec{E} \cdot \hat{n} \Delta A = \sum_{surface} E \Delta A_{\perp}$$

$$\Delta A_{\perp} \sim r^2$$

$$E \sim \frac{1}{r^2}$$

$$\Delta A_{1\perp} E_1 = -\Delta A_{2\perp} E_2$$

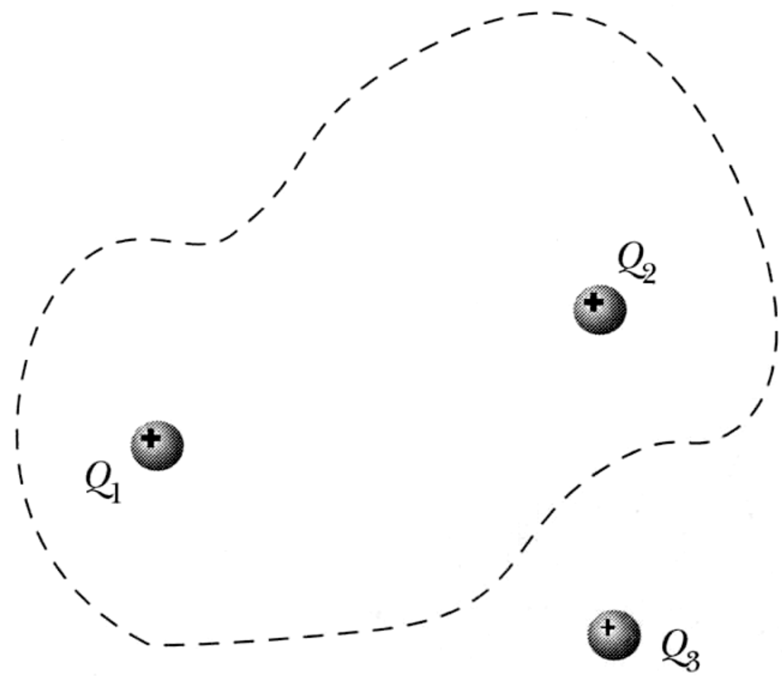
Outside charges contribute 0 to total flux

5. Gauss's Law: Superposition

$$\sum_{surface} \vec{E}_1 \cdot \hat{n} \Delta A = \frac{Q_1}{\varepsilon_0}$$

$$\sum_{surface} \vec{E}_2 \cdot \hat{n} \Delta A = \frac{Q_2}{\varepsilon_0}$$

$$\sum_{surface} \vec{E}_3 \cdot \hat{n} \Delta A = 0$$



$$\sum_{surface} \vec{E} \cdot \hat{n} \Delta A = \frac{\sum q_{inside}}{\varepsilon_0}$$

Today

Gauss' Law