In all of the following questions, show the details of your work (it is not enough to just give the answer).

Question 1. (10 points) Let T be a tree whose 15 nodes are named $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o$. A preorder traversal of T lists its nodes in the following order:

$$g\ c\ e\ k\ h\ b\ o\ l\ i\ a\ f\ j\ d\ m\ n$$

A postorder traversal of T lists its nodes in the following order:

$$k\ e\ o\ l\ b\ h\ c\ j\ f\ m\ n\ d\ a\ i\ g$$

Draw T.

Question 2. (10 points) Let T be an undirected tree (i.e., a tree whose edges are undirected, hence it has no designated root node). In other words T is an undirected graph that is connected and contains no undirected cycle. A leaf in T is a node that is touched by only 1 undirected edge, and in what follows by "removing a leaf" we mean deleting both the leaf node and the edge that touches it. We use the notation $\ell(T)$ to denote the length of a longest path in T, where the length of a path is defined as the number of nodes on that path.

Let T_1 be the tree obtained from T by deleting all the leaves of T, T_2 be the tree obtained from T_1 by deleting all the leaves of T_1, ..., T_k be the tree obtained from T_{k-1} by deleting all the leaves of T_{k-1}.

1. Suppose that, for some $k \geq 1$, T_k consists of a single node (hence no edges). What is $\ell(T)$ as a function of k?
2. Suppose that, for some $k \geq 1$, T_k consists of two nodes joined by an edge (hence T_{k+1} is the empty tree). What is $\ell(T)$ as a function of k?

Question 3. (10 points) Convert the below infix arithmetic expression into Polish notation, and into reverse Polish notation.

$$(A + B * C) * (D * E - F * G)$$

Question 4. (10 points) This question is about Huffman's coding algorithm.

1. Suppose that the alphabet consists of 512 symbols each of which has a probability of 1/512. What does the Huffman tree look like? What is the average code length?
2. Suppose that the alphabet consists of 257 symbols, one of which has probability 0.5 and the remaining 256 have a probability of 1/512 each. What does the Huffman tree look like? What is the average code length?

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