WebAssign
CH 4.9 - 2 (Homework)

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MA 265 Spring 2013, section 132, Spring 2013
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Current Score : 20 / 20 **Due :** Thursday, March 21 2013 11:40 PM EDT

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1. 2/2 points | Previous Answers KolmanLinAlg9 4.9.012.

Compute the row and column ranks of the following matrices.

(a) $\begin{bmatrix} 9 & 6 & 9 \\ -3 & 2 & 1 \\ 9 & 1 & 2 \end{bmatrix}$

row rank 3

column rank 3

(b) $\begin{bmatrix} 2 & -4 & -4 \\ 2 & -1 & 6 \\ 7 & -8 & 6 \end{bmatrix}$

row rank 2

column rank 2

(c) $\begin{bmatrix} 1 & -2 & -2 \\ 4 & -2 & 12 \\ 7 & -8 & 6 \\ 5 & -7 & 0 \end{bmatrix}$

row rank 2

column rank 2

2. 2/2 points | Previous Answers KolmanLinAlg9 4.9.014.

Compute the rank and nullity of each given matrix and verify the following theorem.

If A is an $m \times n$ matrix, then rank A + nullity $A = n$.

(a) $\begin{bmatrix} 1 & 3 & -2 & 4 \\ -1 & 4 & -5 & 10 \\ 3 & 2 & 1 & -2 \\ 3 & -5 & 8 & -16 \end{bmatrix}$

rank 2

nullity 2

(b) $\begin{bmatrix} 1 & 1 & 4 & 1 \\ 2 & -1 & 0 & 0 \\ 0 & 4 & -16 & 8 \\ 1 & 1 & -4 & 2 \end{bmatrix}$

rank 4

nullity 0

3. 2/2 points | Previous Answers KolmanLinAlg9 4.9.016.

Determine which of the given linear systems are consistent by comparing the ranks of the coefficient and augmented matrices.

(a) $\begin{bmatrix} 1 & 2 & 5 & -2 \\ 2 & 3 & -2 & 4 \\ 5 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

- (•) consistent
- ( ) inconsistent

(b) $\begin{bmatrix} 1 & 2 & 5 & -2 \\ 2 & 3 & -2 & 4 \\ 5 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -5 \\ -12 \\ 4 \end{bmatrix}$

- (•) consistent
- ( ) inconsistent

4. 2/2 points | Previous Answers KolmanLinAlg9 4.9.020.

Use the following corollary to find out whether rank A = 3 for each given matrix.

If A is an $n \times n$ matrix, then rank $A = n$ if and only if $\det(A) \neq 0$.

(a) $A = \begin{bmatrix} 3 & 1 & 2 \\ 10 & 5 & 10 \\ 4 & 2 & 5 \end{bmatrix}$

$\det(A)$ = 5 $\Rightarrow$ rank A = 3

(b) $A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 1 & -2 \\ -4 & -4 & 10 \end{bmatrix}$

$\det(A)$ = 2 $\Rightarrow$ rank A = 3

5. 2/2 points | Previous Answers KolmanLinAlg9 4.9.022.

Use the following corollary to find which of the given homogeneous systems have a nontrivial solution.

The homogeneous system $A\mathbf{x} = \mathbf{0}$, where A is $n \times n$, has a nontrivial solution if and only if rank $A < n$.

(a) $\begin{bmatrix} 1 & 5 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

- ○ trivial solution
- ● nontrivial solution

(b) $\begin{bmatrix} 7 & 6 & 9 & 18 \\ -2 & -2 & -3 & -6 \\ 1 & 1 & 1 & 2 \\ -3 & -3 & -3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

- ● trivial solution
- ○ nontrivial solution

6. 2/2 points | Previous Answers KolmanLinAlg9 4.9.029.

Solve using the concept of rank.

Is

$$S = \left\{ \begin{bmatrix} -1 & 4 & 8 \end{bmatrix}, \begin{bmatrix} 2 & -7 & -14 \end{bmatrix}, \begin{bmatrix} -2 & 6 & 12 \end{bmatrix} \right\}$$

a linearly independent set of vectors in R_3?

- ( ) Yes
- (•) No

✓

7. 2/2 points | Previous Answers KolmanLinAlg9 4.9.030.

Solve using the concept of rank.

Does the set

$$S = \left\{ \begin{bmatrix} 4 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} \right\}$$

span R^3?

- (•) Yes
- ( ) No

✓

8. 2/2 points | Previous Answers KolmanLinAlg9 4.9.032.

Solve using the concept of rank.

Is

$$S = \left\{ \begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ 0 & 3 \end{bmatrix} \right\}$$

a basis for M_{22}?

- (•) Yes
- ( ) No

✓

9. 2/2 points | Previous Answers KolmanLinAlg9 4.9.034.

(a) If A is a 3 × 5 matrix, what is the largest possible value for rank A?

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(b) If A is a 3 × 4 matrix, which can be guaranteed to be linearly dependent of A?

- ○ the rows of A
- ◉ the columns of A

(c) If A is a 4 × 3 matrix, which can be guaranteed to be linearly dependent of A?

- ◉ the rows of A
- ○ the columns of A

10. 2/2 points | Previous Answers KolmanLinAlg9 4.9.036.

Let A be a 5 × 9 matrix.

(a) Give *all* possible values for the rank of A. (Enter your answers as a comma-separated list.)

(b) If the rank of A is 5, what is the dimension of its column space?

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(c) If the rank of A is 5, what is the dimension of the solution space of the homogeneous system $A\mathbf{x} = \mathbf{0}$?

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