ECE 202: Linear Circuit Analysis II – Fall2013

HOMEWORK SET 2: DUE THURSDAY, AUGUST 29, 5 PM IN MSEE 180

ALWAYS CHECK THE ERRATA on the web.

5. Find (i) the SIMPLIFIED partial fraction expansion and (ii) the inverse Laplace transforms of each of the following functions by hand. Assume $a, b > 0$. Show all work:

(a) $F_1(s) = \frac{1}{s^2+2s-8}$

(b) $F_2(s) = \frac{7s^2-19s-2}{s^3-4s^2+s+6}$

(c) $F_3(s) = \frac{s^4-3s^3-6s^2+20s-12}{s^4-3s^3+2s^2}$

(d) $F_4(s) = \frac{2s^2+(a-6b)s+a^2-4ab}{(s^2-a^2)(s-2b)}$

(e) The Laplace transform of $f_5(t) = \left[K_1 e^{-at}\cos(\omega t) + K_2 e^{-at}\sin(\omega t) + K_3 e^{-bt}\right]u(t)$ is$F_5(s) = \frac{2s^2+4s+12}{s^3+5s^2+17s+13}$. Find a, b, K_1, K_2, K_3, and ω. **Hint 1**: roots(coef) where coef = [coefficients of the polynomial] produces the roots or zeros of the polynomial in MATLAB.

Hint 2: check your answersusing MATLAB's residue command. Type: help residue….

6. Find the partial fraction expansion and the inverse Laplace transforms of each of the indicated output voltages or currents. All answers must be in terms of real functions with real coefficients or symbols. Show ALL work.

(a) For the circuit of figure 6a, find the partial fraction expansion of $V_c(s)$ and then find $v_c(t)$when $V_{s1}(s) = \frac{3s+4}{s^2-16}$ and $V_{s2}(s) = 12 \cdot \frac{s+1}{s^2+6s+8}$ withR_1=1 kΩ,R_2=6 kΩ, andR_3=3 kΩ.

Hint: use superposition, or a single node equation for $V_1(s)$.

(b) For the circuit of figure 6b, find the partial fraction expansion of $I_c(s)$ and then find $i_c(t)$for the input current $I_{s1}(s) = \frac{20e^{-s}}{s^2+1}$ and $I_{s2}(s) = 90\frac{s^3-2s^2+16s-2}{(s^2+1)(s^2+16)}$ when R_1=12Ω, R_2=24Ω, and R_3=48Ω.

Hint: usesuperposition or loop analysis in the time domain to find $i_1(t)$ in terms of the symbols (not the actual waveform) $i_{s1}(t)$ and $i_{s2}(t)$ before taking Laplace transforms.

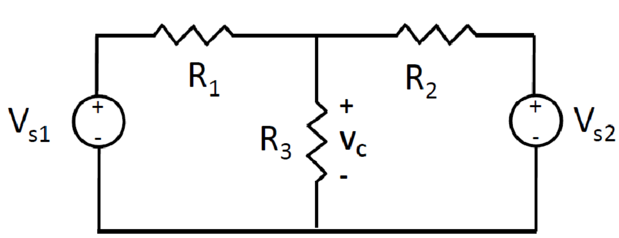


Figure 6.a

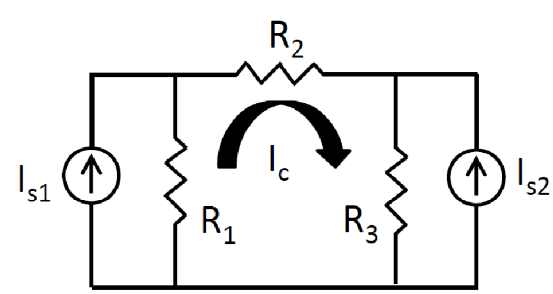


Figure 6.b

7. Use Laplace transform properties to find the indicated transform for each question below. For each part indicate the specific property or properties you used for each step.

(a) The Laplace transform of a circuit is $H(s)=\dfrac{24s}{s^2+64}$. Define $G(s)=L\left[e^{-4t}h(t)\right]$, where $h(t)=L^{-1}\left[H(s)\right]$. Then compute $G(s)$.

(b) Reconsider problem (a) except suppose now that $G(s)=L\left[t\,h(t)\right]$. Then compute $G(s)$.

(c) Reconsider problem (a) except suppose now that $G(s)=L\left[te^{-4t}h(t)\right]$. Then compute $G(s)$.

(d) Reconsider problem (a). Find $G(s)=L\left[\dfrac{d}{dt}\left(te^{-2t}h(t)\right)\right]$ by inspection.

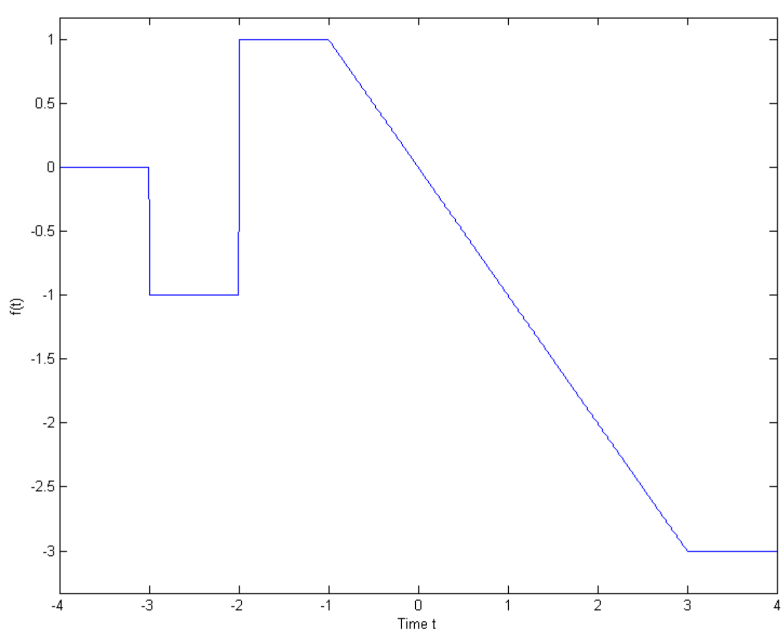
(e) Suppose $F(s)=\dfrac{s+4a}{(s+2a)^2}$, $a>0$. If $g(t)=e^{2at}f(t-2T)u(t-2T)$, $T>0$, then compute $G(s)$.

8. (a) Consider $f(t)$ in the figure below.

(i) Express $f(t)$ as a sum of appropriate (shifted) step functions and ramp functions. Compute $F(s)$.

(ii) Compute $\mathcal{L}\left[g_1(t)\right]=\mathcal{L}\left[\dfrac{d}{dt}f(t)\right]$ using the derivative property.

(iii) Compute $\mathcal{L}\left[g_2(t)\right]=\mathcal{L}\left[\displaystyle\int_{-\infty}^{t} f(\tau)\,d\tau\right]$ using the integral property.



(b) Repeat part (a) for $f_2(t)=f(t+3)$.