Chapter 10: Collisions

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Key Ideas of Chapter 10

- **Collisions are brief interactions involving large forces**

- **To analyze collisions in detail, we must apply both**
  The Energy Principle and The Momentum Principle

- **In analyzing collisions it is useful to choose all colliding objects as the system. With this choice of system:**
  - External forces are negligible during the collision
  - The total momentum of the system is constant
  - The total energy of the system is constant

- **Collisions in which there is no change in internal energy are called "elastic."**
  Other collisions are "inelastic."

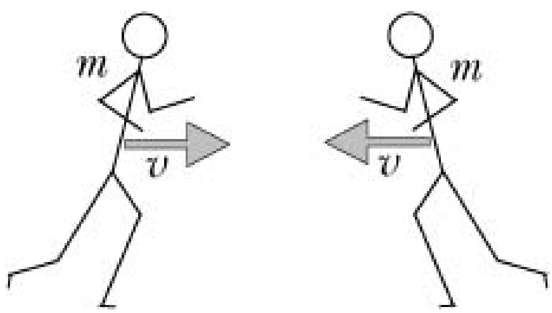
- **Changing to a reference frame moving with the center of mass of the system simplifies the analysis of a collision.**

What Qualifies as a Collision?

Key Idea: Collisions are brief interactions involving large forces

- Two objects interacting during a small time interval, with little interaction before or after interval
- During this time interval, the interaction between the two objects is much stronger than any other external interactions

Example: System = Two Colliding Students (p. 80)



Exert $|\vec{F}|$ = 21000 N on each other for 0.017 seconds.

External friction force is present, but negligible during interaction of students.

What Qualifies as a Collision?

Key Idea: Collisions are brief interactions involving large forces

Example: Hitting a Home Run

Bat and ball exert $|\vec{F}|$ = 10000 N on each other for 1 millisecond.

The gravitational force is present, but negligible during collision of ball and bat.

Note: Both before and after collision, gravity is non-negligible.

Examples: What *Wouldn't* Qualify as a Collision

- p^+ and e^- in a hydrogen atom: The p^+-e^- electric interaction may be much stronger than external interactions, but Δt isn't small!

Energy and Momentum Conservation

Key Idea: Apply both The Energy Principle and The Momentum Principle

$$\left.\begin{aligned}\Delta\vec{p}_{system} + \Delta\vec{p}_{surroundings} &= 0 \\ \Delta E_{system} + \Delta E_{surroundings} &= 0\end{aligned}\right\}$$ **Always** True

- Let system = objects participating in a collision.
- By our definition of a collision, system and surroundings don't interact significantly during the collision.
- Therefore, neither momentum nor energy may be transferred between system and surroundings:

$$\Delta E_{system} = 0 \qquad \Delta\vec{p}_{system} = 0$$

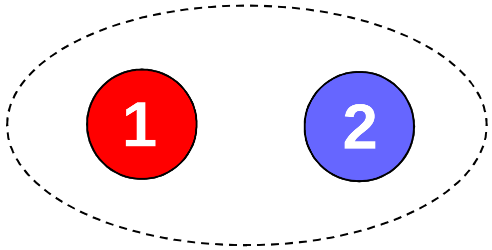
during a collision process (system defined as above)

Now let's look at each of these conservation equations more closely.

Momentum Conservation in a Collision

Key Idea: Total Momentum of The System (all colliding objects) is constant

The System = all colliding objects



$$\Delta\vec{p}_{system} = \cancelto{0}{\vec{F}_{net,ext}}\Delta t$$

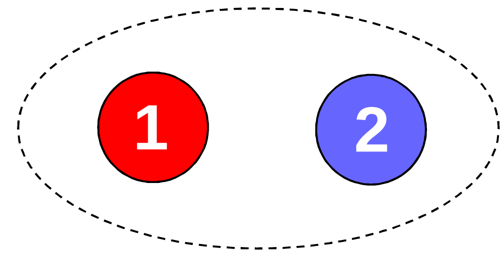
During the collision, external forces ≈ 0.

$$\boxed{\vec{p}_{f,system} = \vec{p}_{i,system}} \qquad \text{where} \quad \vec{p}_{system} = \vec{p}_1 + \vec{p}_2$$

Energy Conservation in a Collision

Key Idea: Total Energy of The System (all colliding objects) is constant

The System = all colliding objects



$$\Delta E_{system} = \cancelto{0}{W_{surr}} + \cancelto{0}{Q_{surr}}$$

During the collision, external interactions ≈ 0.

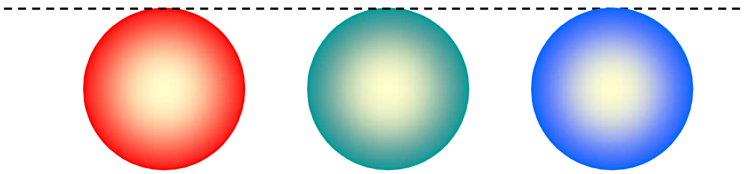
$$\Delta E_{system} = \Delta\left(K_{1,trans} + K_{2,trans} + \underbrace{E_{internal}}_{U + K_{rot} + K_{vib} + E_{therm} + E_{chem} + \ldots}\right) = 0$$

We usually write this as $\boxed{\Delta E_{system} = \Delta K_1 + \Delta K_2 + \Delta E_{int} = 0}$

with the understanding that ΔK_1 means $\Delta K_{1,trans}$, etc.

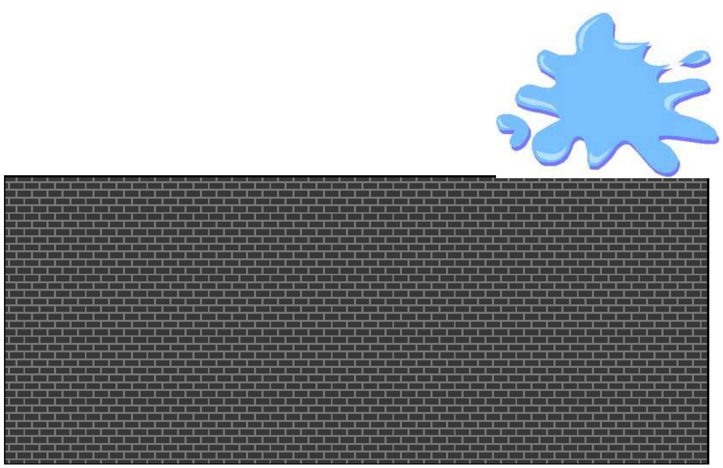
Elastic and inelastic collision

Key Idea: Collisions in which $\Delta E_{int} = 0$ are elastic. Other collisions are inelastic.



ELASTIC COLLISION:
the internal energy of the objects in the system does not change: $\Delta E_{int}=0$

INELASTIC COLLISION:
the internal energy of the objects in the system changes: $\Delta E_{int} \neq 0$



MAXIMALLY INELASTIC COLLISION:
Objects stick together – maximum energy dissipation

Elastic and Inelastic Collisions

Key Idea: Collisions in which ΔE_{int} = 0 are elastic. Other collisions are inelastic.

$$\boxed{\Delta E_{system} = \Delta K_1 + \Delta K_2 + \Delta E_{int} = 0}$$

If $\Delta E_{int} = 0$, collision is **elastic**.

- The total macroscopic kinetic energy is constant.
- A low-energy proton collision is perfectly elastic.
- A billiard ball collision is approximately elastic.

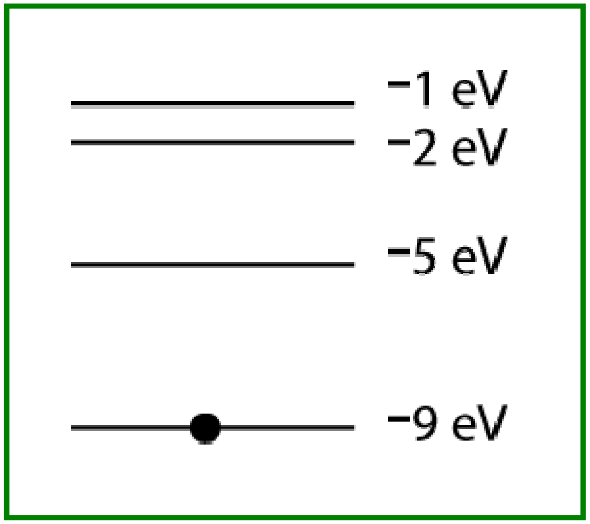
If $\Delta E_{int} \neq 0$, collision is **inelastic**.

- Some macroscopic kinetic energy got transformed into internal energy, or vice versa.
- A basketball bouncing off a wall is an inelastic collision.
- If two objects collide and stick together, collision is perfectly inelastic (head-on car-crash, or two lumps of clay colliding).

Quantum Collision

Key Idea: Collisions in which $\Delta E_{int} = 0$ are elastic. Other collisions are inelastic.

energy levels for atom A



An electron ($K_{electron}$ = 1eV) is heading straight toward atom A, which is at rest and in its ground state.

Can the collision be inelastic? ***No.***

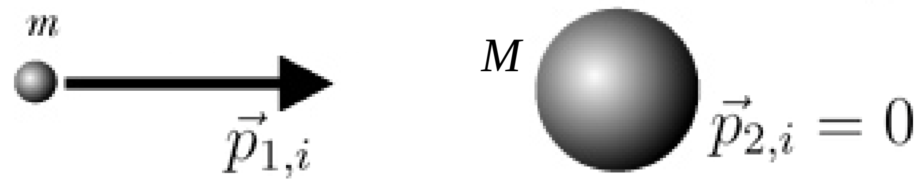
Inelastic means $\Delta E_{int} \neq 0$. Only possibility is for atom to get excited. But the necessary (threshold) energy to excite the atom is 4 eV.

The incident electron isn't energetic enough to excite the atom. Further, U = 0 before and after (since atom will be far away from electron). Hence, $\Delta E_{int} \neq 0$ isn't possible here.

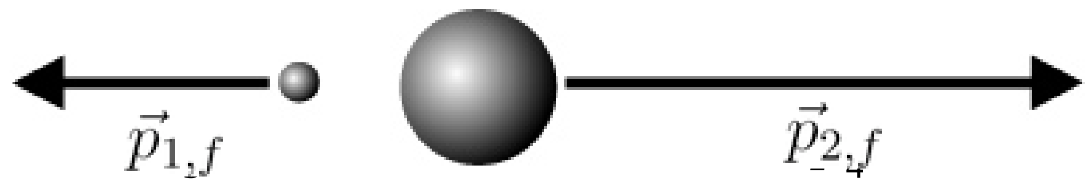
Elastic Head-On (1-D) Collisions

Key Idea: Apply both The Energy Principle and The Momentum Principle

Initial
(just before collision)



Final
(just after collision)



Momentum conservation:

$$\Delta \vec{P}_{tot} = \Delta \vec{p}_m + \Delta \vec{p}_M = 0$$

$$(\vec{p}_{1,f} - \vec{p}_{1,i}) + (\vec{p}_{2,f} - \vec{p}_{2,i}) = 0$$

$$(p^x_{1,f} - p^x_{1,i}) + (p^x_{2,f} - 0) = 0$$

two unknowns

Energy conservation:

elastic

$$\Delta E_{sys} = \Delta K_m + \Delta K_M + \cancel{\Delta E_{int}} = 0$$

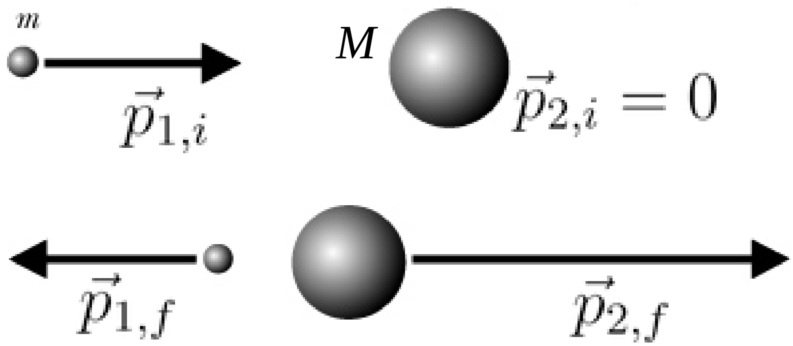
$$(K_{1,f} - K_{1,i}) + (K_{2,f} - K_{2,i}) = 0$$

$$\left(\frac{(p^x_{1,f})^2}{2m} - \frac{(p^x_{1,i})^2}{2m}\right) + \left(\frac{(p^x_{2,f})^2}{2M} - 0\right) = 0$$

same two unknowns

Elastic Head-On (1-D) Collisions

Key Idea: Apply both The Energy Principle and The Momentum Principle



$$p^x_{1,i} = p^x_{1,f} + p^x_{2,f}$$

$$\frac{(p^x_{1,i})^2}{2m} = \frac{(p^x_{1,f})^2}{2m} + \frac{(p^x_{2,f})^2}{2M}$$

2 unknowns, 2 equations: we can solve for $p^x_{1,f}$ and $p^x_{2,f}$.

Note: if collision had been <u>inelastic</u>, we'd have too many unknowns!

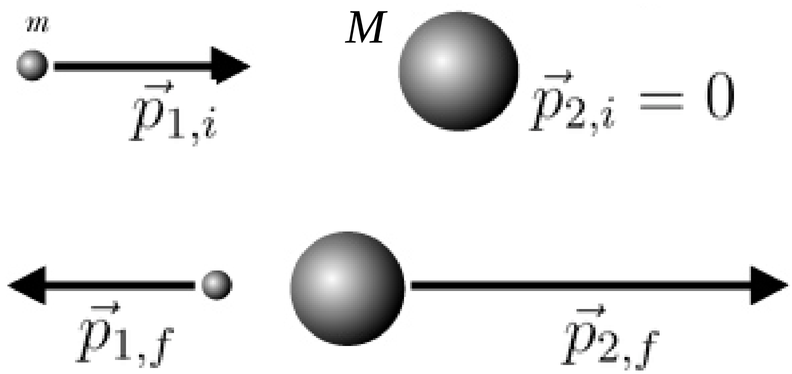
Do algebra. For **m ≤ M**, result is a quadratic equation with two solutions:

Solution #1: $p^x_{1,f} = p^x_{1,i}$ and $p^x_{2,f} = 0$ **(no collision! – objects not lined up)**

Solution #2: $p^x_{1,f} = \left(\frac{m - M}{m + M}\right) p^x_{1,i}$ and $v^x_{2,f} = \frac{2m}{m + M} v^x_{1,i}$ **m ≤ M here**

Two Special Cases

Key Idea: Apply both The Energy Principle and The Momentum Principle



$$p^x_{1,f} = \left(\frac{m - M}{m + M}\right) p^x_{1,i}$$

$$v^x_{2,f} = \frac{2m}{m + M} v^x_{1,i}$$

Case I: m ≈ M $\quad p^x_{1,f} \approx 0$ and $v^x_{2,f} \approx v^x_{1,i}$

First ball stops in place, second ball moves ahead with same speed.

Case II: m << M $\quad p^x_{1,f} \approx -p^x_{1,i}$ and $v^x_{2,f} \approx 0$

First ball stops bounces straight back, second ball doesn't move.

A Concluding Puzzle

James Clerk Maxwell once walked into a pool hall to observe a game of billiards. He remarked that he knew with certainty, without making any measurements, that the collisions between the billiard balls were inelastic. How did he know this?

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