CS182 **Homework 8** **Spring 2012**

In all of the following questions, show the details of your work (it is not enough to just give the answer). The following identity, for $a \neq 1$, is useful for parts of this homework:

$$a^k + a^{k-1} + \cdots + a^1 + a^0 = (a^{k+1} - 1)/(a - 1)$$

Question 1. (10 points) Assuming n is a power of 3, solve the recurrence $f(n) = 3f(n/3) + 1$, with boundary condition $f(1) = 1$.

Question 2. (10 points) Assuming n is a power of 3, solve the recurrence $f(n) = 3f(n/3) + n$, with boundary condition $f(1) = 1$.

Question 3. (10 points) Let $a_1, \ldots, a_n$ be random integers that are selected uniformly and independently from the set $\{1, 2, \ldots, N\}$ where $n < N$. What is the probabiity that the n numbers are distinct (i.e., that no two of them are equal)?

Question 4. (10 points) Bob is playing at a casino the following game: Bob can bet any amount he wishes, then a fair coin is tossed, and if the outcome of the toss is "heads" he wins an amount equal to his bet, otherwise he loses the amount he bet. Bob can play as many rounds of this game as he wishes. Bob decides to follow the following strategy: He bets \$100 in round 1, and if he loses round 1 then he doubles the bet for the next round, and he keeps doubling the bet from one round to the next, until he wins; he stops as soon as he wins a round, collects his money, and walks out of the casino.

1. If Bob has lost the first k rounds and won round $k + 1$, what is his net gain out of playing these $k + 1$ rounds?
2. What is the expected gain of Bob in this game?

Date due: April 12, 2012