Strings

- Strings Sorts
- Tries
- Substring Search
- Regular Expressions
- Data Compression

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- Tries
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String processing

String. Sequence of characters.

Important fundamental abstraction.

- Information processing.
- Genomic sequences.
- Communication systems (e.g., email).
- Programming systems (e.g., Java programs).
- …

> “ *The digital information that underlies biochemistry, cell biology, and development can be represented by a simple string of G’s, A’s, T’s and C’s. This string is the root data structure of an organism’s biology.* ” *— M. V. Olson*

The char data type

C char data type. Typically an 8-bit integer.

- Supports 7-bit ASCII.
- Need more bits to represent certain characters.

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SP	!	“	#	$	%	&	‘	(	)	*	+	,	–	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

Hexadecimal to ASCII conversion table

Java char data type. A 16-bit unsigned integer.

- Supports original 16-bit Unicode.
- Supports 21-bit Unicode 3.0 (awkwardly).

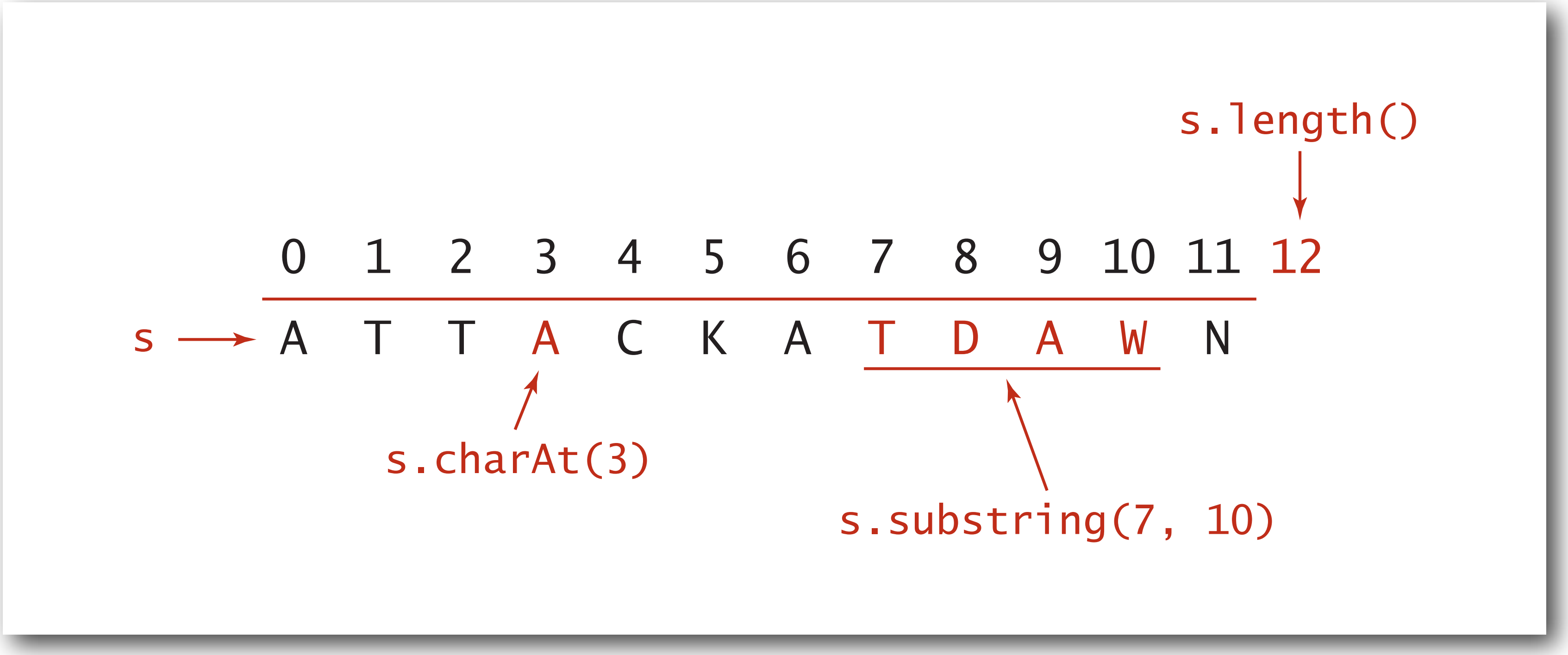
The String data type

String data type. Sequence of characters (immutable).

Indexing. Get the i^{th} character.

Substring extraction. Get a contiguous sequence of characters from a string.

String concatenation. Append one character to end of another string.



The String data type: Java implementation

```
public final class String implements Comparable<String>
{
   private char[] value;   // characters
   private int offset;     // index of first char in array
   private int count;      // length of string
   private int hash;       // cache of hashCode()

   private String(int offset, int count, char[] value)
   {
      this.offset = offset;
      this.count  = count;
      this.value  = value;
   }

   public String substring(int from, int to)
   {  return new String(offset + from, to - from, value);   }

   public char charAt(int index)
   {  return value[index + offset];   }

   public String concat(String that)
   {
      char[] val = new char[this.length() + that.length()];
      ...
      return new String(0, this.length() + that.length(), val);
   }
}
```

strings share underlying `char[]` array

The String data type: performance

String data type. Sequence of characters (immutable).

Underlying implementation. Immutable **`char[]`** array, offset, and length.

	`String`	
operation	guarantee	extra space
`charAt()`	1	1
`substring()`	1	1
`concat()`	N	N

Memory. $40 + 2N$ bytes for a virgin **`String`** of length N.

use **`byte[]`** or **`char[]`** instead of **`String`** to save space

Alphabets

Digital key. Sequence of digits over fixed alphabet.

Radix. Number of digits R in alphabet.

name	R()	lgR()	characters
BINARY	2	1	01
OCTAL	8	3	01234567
DECIMAL	10	4	0123456789
HEXADECIMAL	16	4	0123456789ABCDEF
DNA	4	2	ACTG
LOWERCASE	26	5	abcdefghijklmnopqrstuvwxyz
UPPERCASE	26	5	ABCDEFGHIJKLMNOPQRSTUVWXYZ
PROTEIN	20	5	ACDEFGHIKLMNPQRSTVWY
BASE64	64	6	ABCDEFGHIJKLMNOPQRSTUVWXYZabcdef ghijklmnopqrstuvwxyz0123456789+/
ASCII	128	7	*ASCII characters*
EXTENDED_ASCII	256	8	*extended ASCII characters*
UNICODE16	65536	16	*Unicode characters*

Standard alphabets

String Sorts

- key-indexed counting
- LSD string sort
- MSD string sort
- 3-way string quicksort
- suffix arrays

Review: summary of the performance of sorting algorithms

Frequency of operations = key compares.

algorithm	guarantee	random	extra space	stable?	operations on keys
insertion sort	$N^2/2$	$N^2/4$	no	yes	`compareTo()`
mergesort	N lg N	N lg N	N	yes	`compareTo()`
quicksort	1.39 N lg N *	1.39 N lg N	c lg N	no	`compareTo()`
heapsort	2 N lg N	2 N lg N	no	no	`compareTo()`

* probabilistic

Lower bound. $\sim N \lg N$ compares are required by any compare-based algorithm.

Q. Can we do better (despite the lower bound)?

A. Yes, if we don't depend on compares.

String Sorts

- key-indexed counting
- LSD string sort
- MSD string sort
- 3-way string quicksort
- suffix arrays

Key-indexed counting: assumptions about keys

Assumption. Keys are integers between 0 and $R - 1$.

Implication. Can use key as an array index.

Applications.

- Sort string by first letter.
- Sort class roster by section.
- Sort phone numbers by area code.
- Subroutine in a sorting algorithm.

Remark. Keys may have associated data $\Rightarrow$ can't just count up number of keys of each value.

input *name*	*section*	sorted result *(by section)*	
Anderson	2	Harris	1
Brown	3	Martin	1
Davis	3	Moore	1
Garcia	4	Anderson	2
Harris	1	Martinez	2
Jackson	3	Miller	2
Johnson	4	Robinson	2
Jones	3	White	2
Martin	1	Brown	3
Martinez	2	Davis	3
Miller	2	Jackson	3
Moore	1	Jones	3
Robinson	2	Taylor	3
Smith	4	Williams	3
Taylor	3	Garcia	4
Thomas	4	Johnson	4
Thompson	4	Smith	4
White	2	Thomas	4
Williams	3	Thompson	4
Wilson	4	Wilson	4

keys are small integers

Key-indexed counting

Goal. Sort an array **a[]** of N integers between 0 and $R-1$.

- Count frequencies of each letter using key as index.
-
-
-

count frequencies

```
int N = a.length;
int[] count = new int[R+1];

for (int i = 0; i < N; i++)
   count[a[i]+1]++;

for (int r = 0; r < R; r++)
   count[r+1] += count[r];

for (int i = 0; i < N; i++)
   aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
   a[i] = aux[i];
```

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

offset by 1

[stay tuned]

r	count[r]
a	0
b	2
c	3
d	1
e	2
f	1
-	3

Key-indexed counting

Goal. Sort an array `a[]` of N integers between 0 and $R-1$.

- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
-
-

compute cumulates

```
int N = a.length;
int[] count = new int[R+1];

for (int i = 0; i < N; i++)
   count[a[i]+1]++;

for (int r = 0; r < R; r++)
   count[r+1] += count[r];

for (int i = 0; i < N; i++)
   aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
   a[i] = aux[i];
```

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	0
b	2
c	5
d	6
e	8
f	9
-	12

6 keys < `d`, 8 keys < `e`
so `d`'s go in `a[6]` and `a[7]`

Key-indexed counting

Goal. Sort an array **a[]** of N integers between 0 and $R-1$.

- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
- Access cumulates using key as index to move records.
-

```
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int[] count = new int[R+1];

for (int i = 0; i < N; i++)
   count[a[i]+1]++;

for (int r = 0; r < R; r++)
   count[r+1] += count[r];

for (int i = 0; i < N; i++)
   aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
   a[i] = aux[i];
```

move records

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	0
b	2
c	5
d	6
e	8
f	9
-	12

i	aux[i]
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	

Key-indexed counting

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for (int r = 0; r < R; r++)
   count[r+1] += count[r];

for (int i = 0; i < N; i++)
   aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
   a[i] = aux[i];
```

move records

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	0
b	2
c	5
d	7
e	8
f	9
-	12

i	aux[i]
0	
1	
2	
3	
4	
5	
6	d
7	
8	
9	
10	
11	

Key-indexed counting

Goal. Sort an array **a[]** of N integers between 0 and $R - 1$.

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move records

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	1
b	2
c	5
d	7
e	8
f	9
-	12

i	aux[i]
0	a
1	
2	
3	
4	
5	
6	d
7	
8	
9	
10	
11	

Key-indexed counting

Goal. Sort an array **a[]** of N integers between 0 and $R - 1$.

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move records

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	1
b	2
c	6
d	7
e	8
f	9
-	12

i	aux[i]
0	a
1	
2	
3	
4	
5	c
6	d
7	
8	
9	
10	
11	

Key-indexed counting

Goal. Sort an array **a[]** of N integers between 0 and $R - 1$.

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move records

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	1
b	2
c	6
d	7
e	8
f	10
-	12

i	aux[i]
0	a
1	
2	
3	
4	
5	c
6	d
7	
8	
9	f
10	
11	

Key-indexed counting

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move records

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1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	1
b	2
c	6
d	7
e	8
f	11
-	12

i	aux[i]
0	a
1	
2	
3	
4	
5	c
6	d
7	
8	
9	f
10	f
11	

Key-indexed counting

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move records

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0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	1
b	3
c	6
d	7
e	8
f	11
-	12

i	aux[i]
0	a
1	
2	b
3	
4	
5	c
6	d
7	
8	
9	f
10	f
11	

Key-indexed counting

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move records

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0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	1
b	3
c	6
d	8
e	8
f	11
-	12

i	aux[i]
0	a
1	
2	b
3	
4	
5	c
6	d
7	d
8	
9	f
10	f
11	

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move records

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	1
b	4
c	6
d	8
e	8
f	11
-	12

i	aux[i]
0	a
1	
2	b
3	b
4	
5	c
6	d
7	d
8	
9	f
10	f
11	

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move records

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	1
b	4
c	6
d	8
e	8
f	12
-	12

i	aux[i]
0	a
1	
2	b
3	b
4	
5	c
6	d
7	d
8	
9	f
10	f
11	f

Key-indexed counting

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move records

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	1
b	5
c	6
d	8
e	8
f	12
-	12

i	aux[i]
0	a
1	
2	b
3	b
4	b
5	c
6	d
7	d
8	
9	f
10	f
11	f

Key-indexed counting

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0	d
1	a
2	c
3	f
4	f
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7	b
8	f
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a	1
b	5
c	6
d	8
e	9
f	12
-	12

i	aux[i]
0	a
1	
2	b
3	b
4	b
5	c
6	d
7	d
8	e
9	f
10	f
11	f

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move records

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0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	2
b	5
c	6
d	8
e	9
f	12
-	12

i	aux[i]
0	a
1	a
2	b
3	b
4	b
5	c
6	d
7	d
8	e
9	f
10	f
11	f

Key-indexed counting

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1	a
2	c
3	f
4	f
5	b
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7	b
8	f
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11	a

r	count[r]
a	2
b	5
c	6
d	8
e	9
f	12
-	12

i	aux[i]
0	a
1	a
2	b
3	b
4	b
5	c
6	d
7	d
8	e
9	f
10	f
11	f

Key-indexed counting

Goal. Sort an array **a[]** of N integers between 0 and $R - 1$.

- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
- Access cumulates using key as index to move records.
- Copy back into original array.

```
int N = a.length;
int[] count = new int[R+1];

for (int i = 0; i < N; i++)
   count[a[i]+1]++;

for (int r = 0; r < R; r++)
   count[r+1] += count[r];

for (int i = 0; i < N; i++)
   aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
   a[i] = aux[i];
```

copy back

i	a[i]
0	a
1	a
2	b
3	b
4	b
5	c
6	d
7	d
8	e
9	f
10	f
11	f

r	count[r]
a	2
b	5
c	6
d	8
e	9
f	12
-	12

i	aux[i]
0	a
1	a
2	b
3	b
4	b
5	c
6	d
7	d
8	e
9	f
10	f
11	f

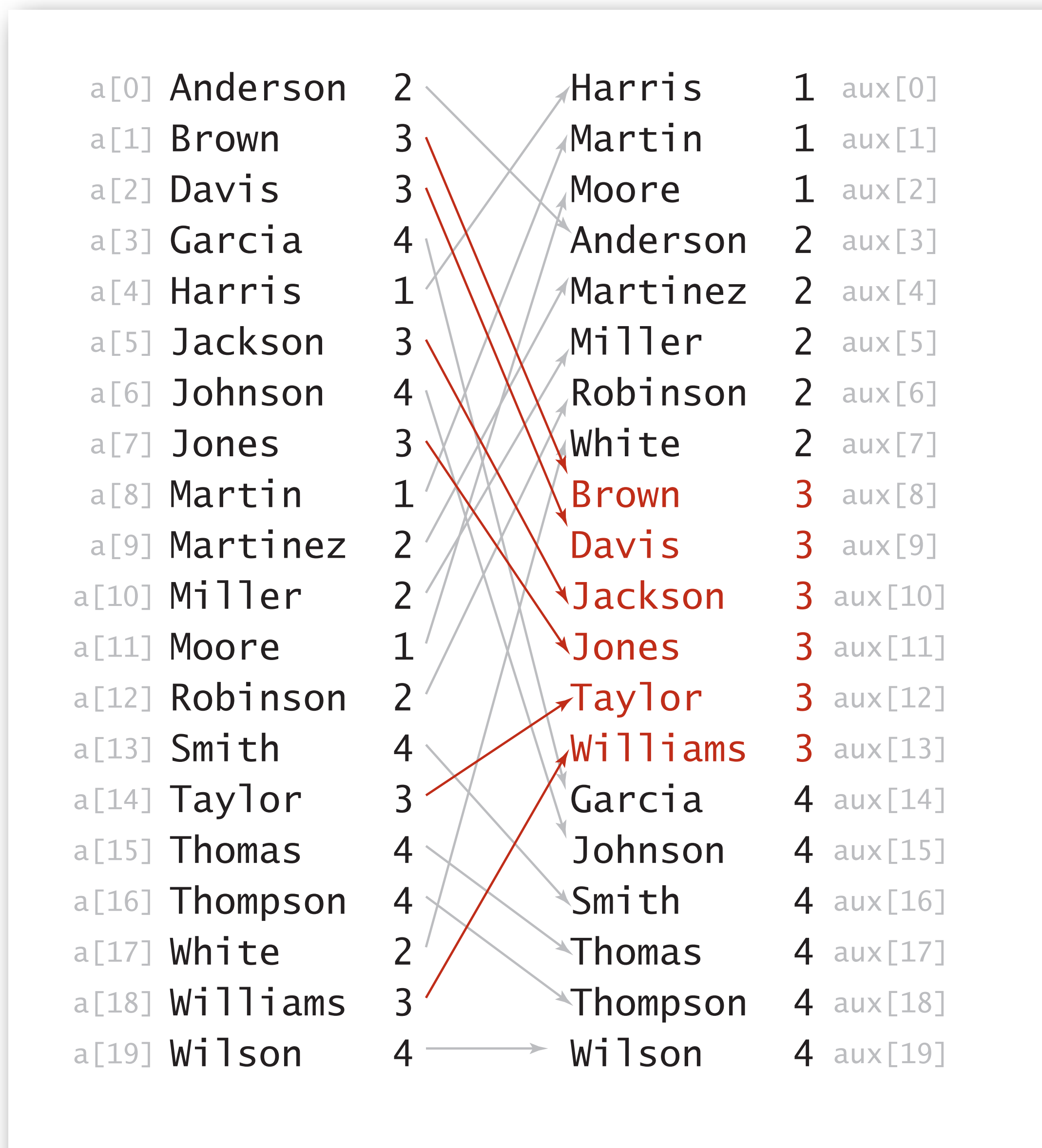
Key-indexed counting: analysis

Proposition. Key-indexed counting uses $8N + 3R$ array accesses to sort N records whose keys are integers between 0 and $R - 1$.

Proposition. Key-indexed counting uses extra space proportional to $N + R$.

Stable? Yes!

In-place? No.



String Sorts

- key-indexed counting
- LSD string sort
- MSD string sort
- 3-way string quicksort
- suffix arrays

Least-significant-digit-first string sort

LSD string sort.

- Consider characters from right to left.
- Stably sort using d^{th} character as the key (using key-indexed counting).

0	d	a	b
1	a	d	d
2	c	a	b
3	f	a	d
4	f	e	e
5	b	a	d
6	d	a	d
7	b	e	e
8	f	e	d
9	b	e	d
10	e	b	b
11	a	c	e

			sort key
0	d	a	b
1	c	a	b
2	e	b	b
3	a	d	d
4	f	a	d
5	b	a	d
6	d	a	d
7	f	e	d
8	b	e	d
9	f	e	e
10	b	e	e
11	a	c	e

		sort key	
0	d	a	b
1	c	a	b
2	f	a	d
3	b	a	d
4	d	a	d
5	e	b	b
6	a	c	e
7	a	d	d
8	f	e	d
9	b	e	d
10	f	e	e
11	b	e	e

	sort key		
0	a	c	e
1	a	d	d
2	b	a	d
3	b	e	d
4	b	e	e
5	c	a	b
6	d	a	b
7	d	a	d
8	e	b	b
9	f	a	d
10	f	e	d
11	f	e	e

sort must be stable
(arrows do not cross)

LSD string sort: correctness proof

Proposition. LSD sorts fixed-length strings in ascending order.

Pf. [thinking about the future]

- If the characters not yet examined differ, it doesn't matter what we do now.
- If the characters not yet examined agree, stability ensures later pass won't affect order.

0	d	a	b
1	c	a	b
2	f	a	d
3	b	a	d
4	d	a	d
5	e	b	b
6	a	c	e
7	a	d	d
8	f	e	d
9	b	e	d
10	f	e	e
11	b	e	e

in order

by previous passes

sort key

0	a	c	e
1	a	d	d
2	b	a	d
3	b	e	d
4	b	e	e
5	c	a	b
6	d	a	b
7	d	a	d
8	e	b	b
9	f	a	d
10	f	e	d
11	f	e	e

LSD string sort: Java implementation

```
public class LSD
{
   public static void sort(String[] a, int W)
   {
      int R = 256
      int N = a.length;
      String[] aux = new String[N];

      for (int d = W-1; d >= 0; d--)
      {
         int[] count = new int[R+1];
         for (int i = 0; i < N; i++)
            count[a[i].charAt(d) + 1]++;
         for (int r = 0; r < R; r++)
            count[r+1] += count[r];
         for (int i = 0; i < N; i++)
            aux[count[a[i].charAt(d)]++] = a[i];
         for (int i = 0; i < N; i++)
            a[i] = aux[i];
      }
   }
}
```

fixed-length W strings

radix R

do key-indexed counting
for each digit from right to left

key-indexed
counting

LSD string sort: example

Input	d = 6	d = 5	d = 4	d = 3	d= 2	d= 1	d = 0	Output
4PGC938	2IYE230	3CIO720	2IYE230	2RLA629	1ICK750	3ATW723	1ICK750	1ICK750
2IYE230	3CIO720	3CIO720	4JZY524	2RLA629	1ICK750	3CIO720	1ICK750	1ICK750
3CIO720	1ICK750	3ATW723	2RLA629	4PGC938	4PGC938	3CIO720	1OHV845	1OHV845
1ICK750	1ICK750	4JZY524	2RLA629	2IYE230	1OHV845	1ICK750	1OHV845	1OHV845
1OHV845	3CIO720	2RLA629	3CIO720	1ICK750	1OHV845	1ICK750	1OHV845	1OHV845
4JZY524	3ATW723	2RLA629	3CIO720	1ICK750	1OHV845	2IYE230	2IYE230	2IYE230
1ICK750	4JZY524	2IYE230	3ATW723	3CIO720	3CIO720	4JZY524	2RLA629	2RLA629
3CIO720	1OHV845	4PGC938	1ICK750	3CIO720	3CIO720	1OHV845	2RLA629	2RLA629
1OHV845	1OHV845	1OHV845	1ICK750	1OHV845	2RLA629	1OHV845	3ATW723	3ATW723
1OHV845	1OHV845	1OHV845	1OHV845	1OHV845	2RLA629	1OHV845	3CIO720	3CIO720
2RLA629	4PGC938	1OHV845	1OHV845	1OHV845	3ATW723	4PGC938	3CIO720	3CIO720
2RLA629	2RLA629	1ICK750	1OHV845	3ATW723	2IYE230	2RLA629	4JZY524	4JZY524
3ATW723	2RLA629	1ICK750	4PGC938	4JZY524	4JZY524	2RLA629	4PGC938	4PGC938

Summary of the performance of sorting algorithms

Frequency of operations.

algorithm	guarantee	random	extra space	stable?	operations on keys
insertion sort	$N^2/2$	$N^2/4$	1	yes	`compareTo()`
mergesort	N lg N	N lg N	N	yes	`compareTo()`
quicksort	1.39 N lg N *	1.39 N lg N	c lg N	no	`compareTo()`
heapsort	2 N lg N	2 N lg N	1	no	`compareTo()`
LSD †	2 W N	2 W N	N + R	yes	`charAt()`

* probabilistic

† fixed-length W keys

Q. What if strings do not have same length?

String sorting challenge 1

Problem. Sort a huge commercial database on a fixed-length key field.

Ex. Account number, date, SS number, ...

Which sorting method to use?

- Insertion sort.
- Mergesort.
- Quicksort.
- Heapsort.
- ✓ LSD string sort.

256 (or 65,536) counters;
Fixed-length strings sort in W passes.

B14-99-8765
756-12-AD46
CX6-92-0112
332-WX-9877
375-99-QWAX
CV2-59-0221
[illegible]87-SS-0321
KJ-0[illegible]12388
715-YT-013C
MJ0-PP-983F
908-KK-33TY
BBN-63-23RE
48G-BM-912D
982-ER-9P1B
WBL-37-PB81
810-F4-J87Q
LE9-N8-XX76
908-KK-33TY
B14-99-8765
CX6-92-0112
CV2-59-0221
332-WX-23SQ
332-6A-9877

String sorting challenge 2

Problem. Sort 1 million 32-bit integers.

Ex. Google interview (or presidential interview).

Which sorting method to use?

- Insertion sort.
- Mergesort.
- Quicksort.
- Heapsort.
- LSD string sort.

Google CEO Eric Schmidt interviews Barack Obama

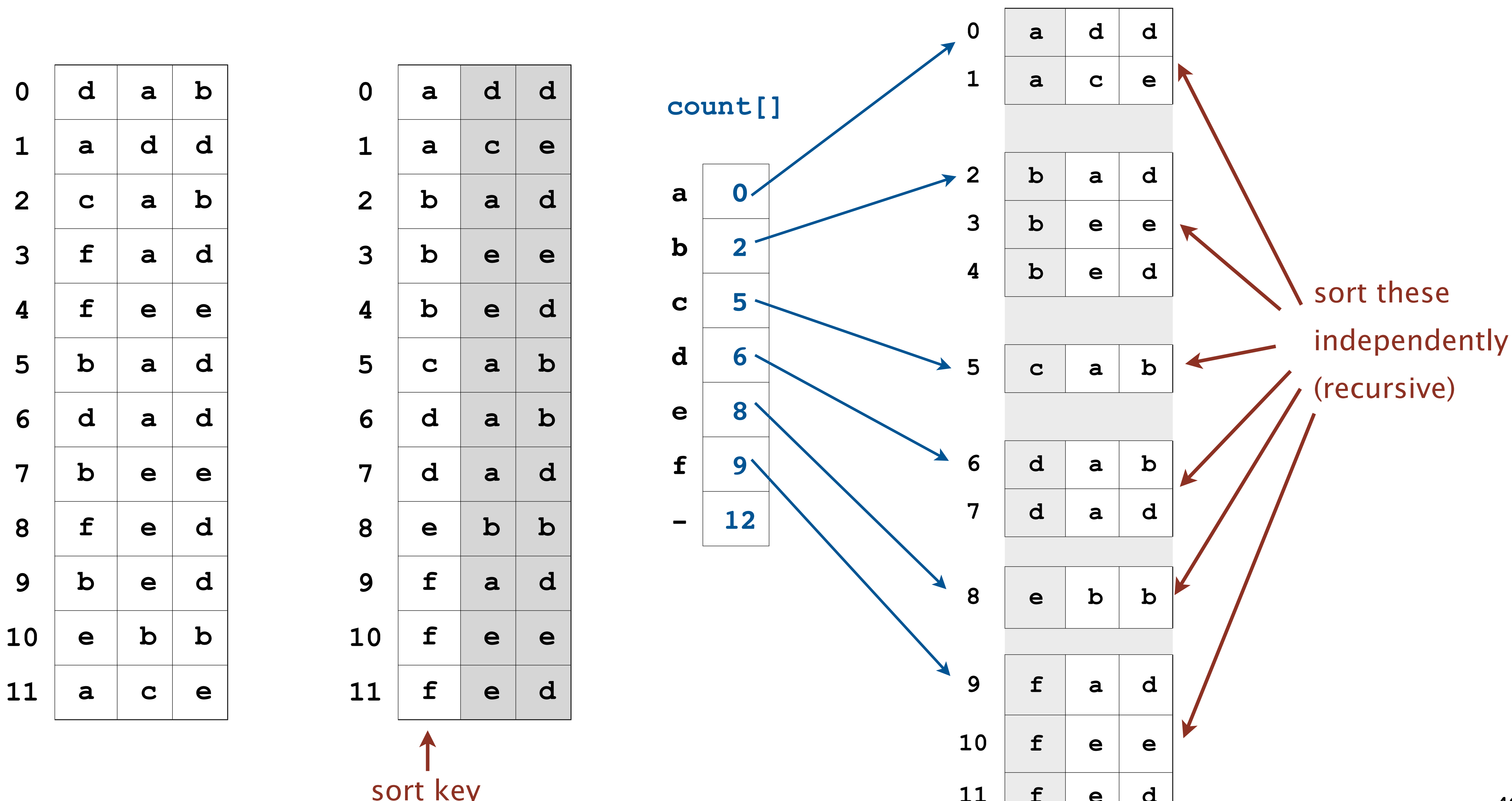
String Sorts

- key-indexed counting
- LSD string sort
- MSD string sort
- 3-way string quicksort
- suffix arrays

Most-significant-digit-first string sort

MSD string sort.

- Partition file into R pieces according to first character (use key-indexed counting).
- Recursively sort all strings that start with each character (key-indexed counts delineate subarrays to sort).



Variable-length strings

Treat strings as if they had an extra char at end (smaller than any char).

why smaller?

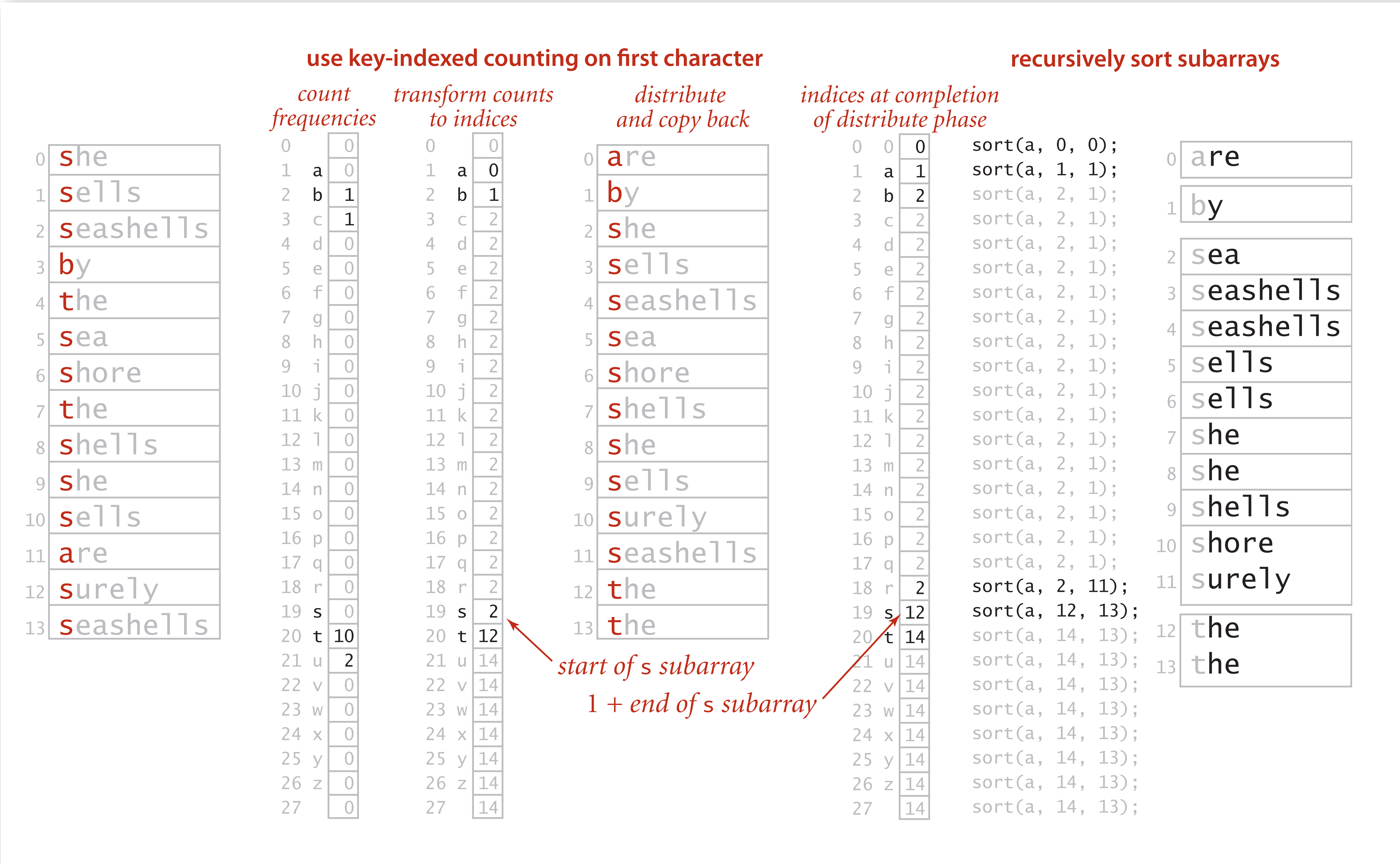
0	s	e	a	-1						
1	s	e	a	s	h	e	l	l	s	-1
2	s	e	l	l	s	-1				
3	s	h	e	-1						
4	s	h	e	-1						
5	s	h	e	l	l	s	-1			
6	s	h	o	r	e	-1				
7	s	u	r	e	l	y	-1			

she before **shells**

```
private static int charAt(String s, int d)
{
   if (d < s.length()) return s.charAt(d);
   else return -1;
}
```

C strings. Have extra char `'\0'` at end $\Rightarrow$ no extra work needed.

MSD string sort: top-level trace



MSD string sort: performance

Number of characters examined.

- MSD examines just enough characters to sort the keys.
- Number of characters examined depends on keys.
- Can be sublinear!

Random (sublinear)	Non-random with duplicates (nearly linear)	Worst case (linear)
1EIO402	are	1DNB377
1HYL490	by	1DNB377
1ROZ572	sea	1DNB377
2HXE734	seashells	1DNB377
2IYE230	seashells	1DNB377
2XOR846	sells	1DNB377
3CDB573	sells	1DNB377
3CVP720	she	1DNB377
3IGJ319	she	1DNB377
3KNA382	shells	1DNB377
3TAV879	shore	1DNB377
4CQP781	surely	1DNB377
4QGI284	the	1DNB377
4YHV229	the	1DNB377

Characters examined by MSD string sort

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Frequency of operations.

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mergesort	N lg N	N lg N	N	yes	`compareTo()`
quicksort	1.39 N lg N *	1.39 N lg N	c lg N	no	`compareTo()`
heapsort	2 N lg N	2 N lg N	1	no	`compareTo()`
LSD †	2 N W	2 N W	N + R	yes	`charAt()`
MSD ‡	2 N W	N $\log_R$ N	N + D R	yes	`charAt()`

stack depth D = length of longest prefix match

* probabilistic

† fixed-length W keys

‡ average-length W keys

MSD string sort vs. quicksort for strings

Disadvantages of MSD string sort.

- Accesses memory "randomly" (cache inefficient).
- Inner loop has a lot of instructions.
- Extra space for **`count[]`**.
- Extra space for **`aux[]`**.

Disadvantage of quicksort.

- Linearithmic number of string compares (not linear).
- Has to rescan long keys for compares.

Goal. Combine advantages of MSD and quicksort.