Last Time

Gauss' Law: Examples
"Magnetic Gauss Law"
Ampere's Law

Today

Faraday's Law

Maxwell's Equations (so far)

For Steady State: Stationary net charge and constant current

Inward/Outward Pointing Fields

$$\oint \vec{E}\cdot\hat{n}\,dA = \frac{1}{\epsilon_o}\sum Q_{\text{enclosed}}$$

GAUSS' LAW

$$\oint \vec{B}\cdot\hat{n}\,dA = 0$$

GAUSS' LAW (Magnetism)

Curly Fields

$$\oint \vec{E}\cdot d\vec{l} = 0$$

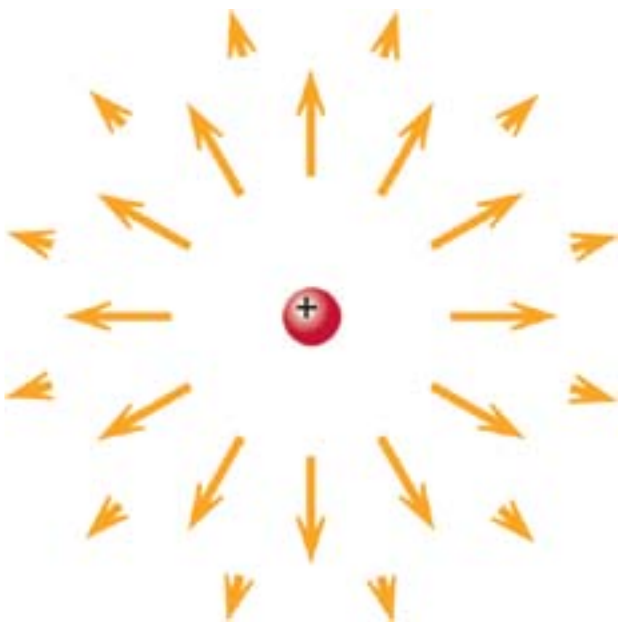
$$\oint \vec{B}\cdot d\vec{l} = \mu_o \sum I_{\text{enclosed}}$$

Inward/Outward and Curly Fields

Gauss' Law

$$\oint \vec{E}\cdot\hat{n}\,dA = \frac{1}{\epsilon_o}\sum Q_{\text{enclosed}}$$

"Flux" is about inward/outward pointing fields

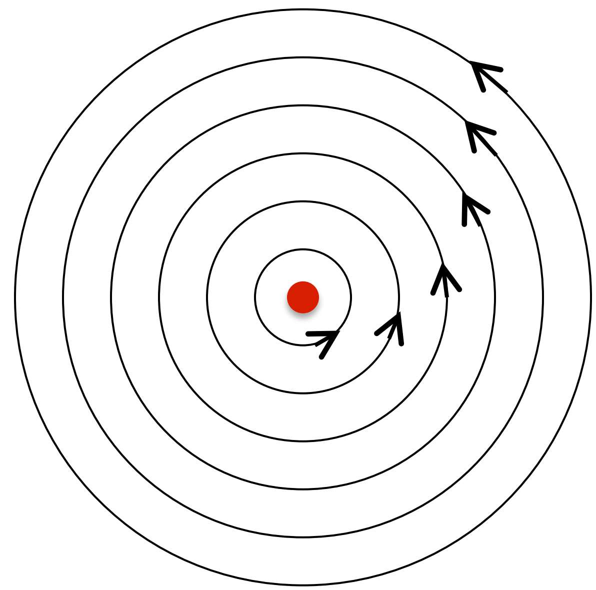


$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}$$ Point Charge

Biot-Savart law

$$\oint \vec{B}\cdot d\vec{l} = \mu_o \sum I_{\text{enclosed}}$$

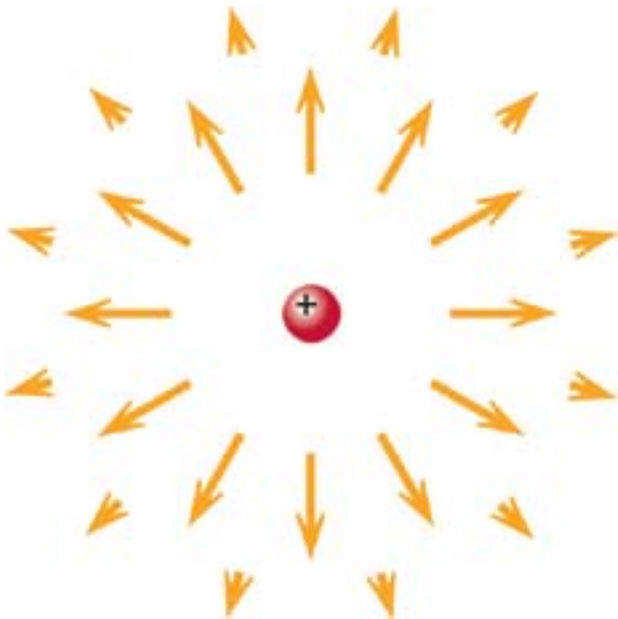
"Circulation" is about curly fields



$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{2I}{r}\hat{\theta}$$ Long Wire (I out of board)

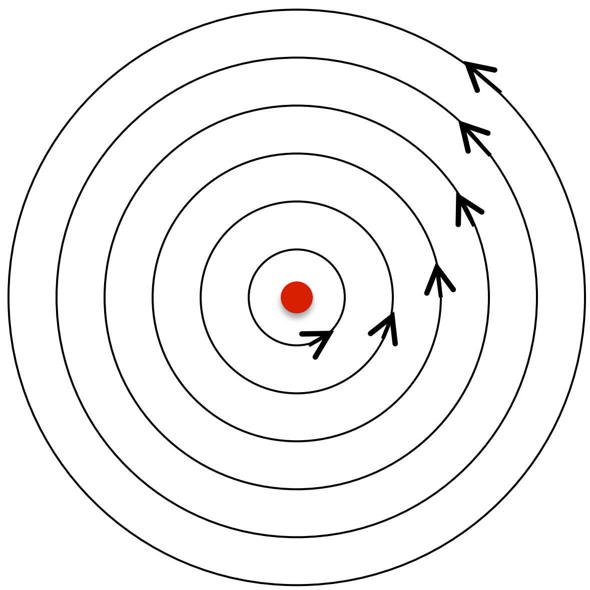
Inward/Outward and Curly Fields

Flux (inward/outward field)



$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}$$ Point Charge

Circulation (curly field)

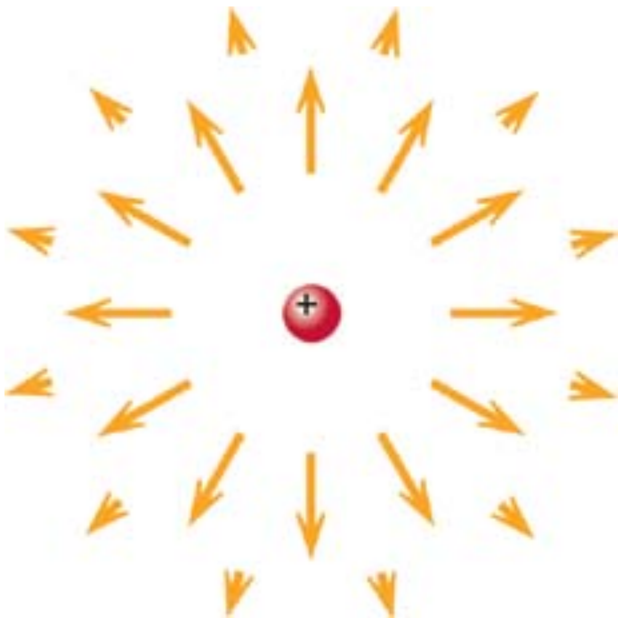


$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{2I}{r}\hat{\theta}$$ Long Wire

Compare this to different coordinates in a coordinate system:
Just like x,y are independent in Cartesian coordinates,
and r, theta are independent in Cylindrical coordinates,
"Flux" and "Circulation" are independent ***field configurations.***

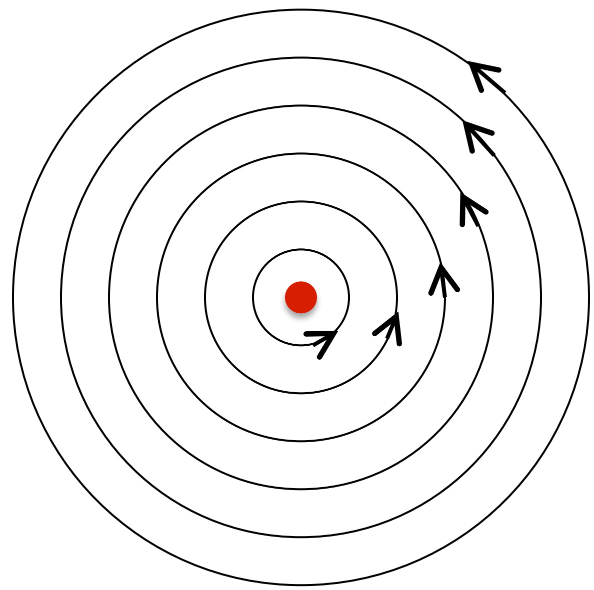
Inward/Outward and Curly Fields

Flux (inward/outward field)



$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}$$ Point Charge

Circulation (curly field)



$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{2I}{r}\hat{\theta}$$ Long Wire

Can we make a Curly E-field?

Demos

How to make a curly E-field

Evidently, we can make a curly E-field
by changing the "magnetic flux" through a loop of wire.

Maxwell's Equations (so far)

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_o} \sum Q_{\mathrm{enclosed}}$$

GAUSS' LAW

$$\oint \vec{B} \cdot \hat{n} dA = 0$$

GAUSS' LAW (Magnetism)

$$\oint \vec{E} \cdot d\vec{l} = \cancel{0}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \left[\sum I_{\mathrm{enclosed}} + \quad 0 \quad \right]$$

Maxwell's Equations (incomplete)

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_o} \sum Q_{\text{enclosed}}$$

GAUSS' LAW

$$\oint \vec{B} \cdot \hat{n} dA = 0$$

GAUSS' LAW (Magnetism)

Chapter 23 (This week)

$$\oint \vec{E} \cdot d\vec{l} = \boxed{-\frac{d}{dt} \int \vec{B} \cdot \hat{n} dA}$$

FARADAY'S LAW

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \left[\sum I_{\text{enclosed}} + \qquad 0 \qquad \right]$$

Maxwell's Equations (incomplete)

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_o} \sum Q_{\text{enclosed}}$$ **GAUSS' LAW**

$$\oint \vec{B} \cdot \hat{n} dA = 0$$ **GAUSS' LAW (Magnetism)**

Chapter 23 (This week)

$$\oint \vec{E} \cdot d\vec{l} = \boxed{-\frac{d}{dt} \int \vec{B} \cdot \hat{n} dA}$$ **FARADAY'S LAW**

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \left[\sum I_{\text{enclosed}} + \cancel{0} \right]$$

Maxwell's Equations – The Full Story

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_o} \sum Q_{\text{enclosed}}$$

GAUSS' LAW

$$\oint \vec{B} \cdot \hat{n} dA = 0$$

GAUSS' LAW (Magnetism)

Chapter 23 (This week)

$$\oint \vec{E} \cdot d\vec{l} = \boxed{-\frac{d}{dt} \int \vec{B} \cdot \hat{n} dA}$$

FARADAY'S LAW

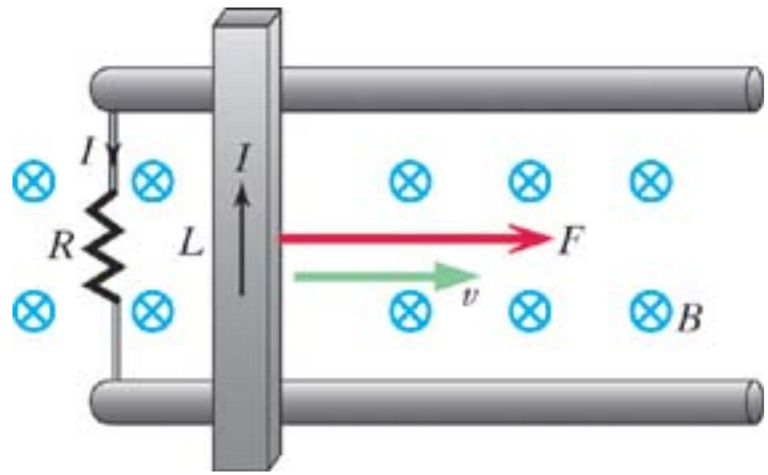
$$\oint \vec{B} \cdot d\vec{l} = \mu_o \left[\sum I_{\text{enclosed}} + \boxed{\epsilon_o \frac{d}{dt} \int \vec{E} \cdot \hat{n} dA} \right]$$

AMPERE-MAXWELL LAW

Chapter 24 (Next week)

Curly E from "stretching" a loop of wire

$$\vec{F} = q\vec{v} \times \vec{B}$$

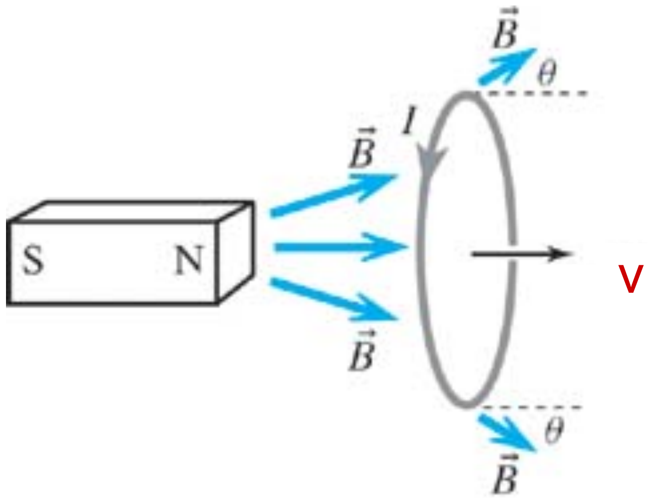


In the presence of a perpendicular field,
increasing the area of a loop of wire causes a "motional emf"

(Section 21.5 "Motional emf")

Curly E from changing B

$$\vec{F} = q\vec{v} \times \vec{B}$$



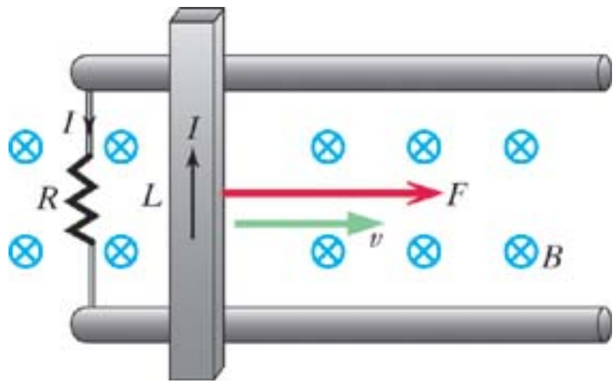
Move the loop away from the magnet.
A current will run.

-OR-

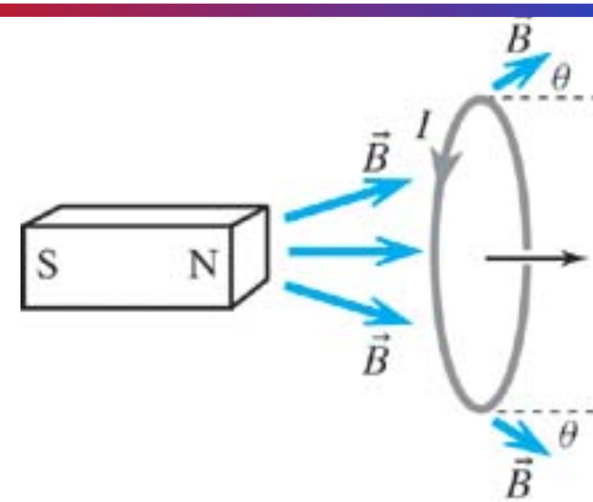
Move the magnet away from the loop.
A current will run.

Curly E from changing *Flux of B*

$$\vec{F} = q\vec{v} \times \vec{B}$$



Increase the area of the loop.
A current will run.



v

Move the magnet away from the loop.
A current will run.

Both of these effects can be summed up as:

$$\boxed{emf = -\frac{d\Phi_{\text{mag}}}{dt}}$$

FARADAY'S LAW

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$$emf = -\frac{d\Phi_{\mathrm{mag}}}{dt}$$

Faraday's Law

$$emf = -\frac{d\Phi_{\mathrm{mag}}}{dt}$$

$$emf = \oint \vec{E} \cdot d\vec{l}$$

Same thing!

$$\Phi \equiv \int \vec{B} \cdot \hat{n} dA$$

Magnetic Flux

Therefore another way to state Faraday's Law is this:

$$\boxed{\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot \hat{n} dA}$$

FARADAY'S LAW

Maxwell's Equations (incomplete)

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_o} \sum Q_{\text{enclosed}}$$

GAUSS' LAW

$$\oint \vec{B} \cdot \hat{n} dA = 0$$

GAUSS' LAW (Magnetism)

Chapter 23 (This week)

$$\oint \vec{E} \cdot d\vec{l} = \boxed{-\frac{d}{dt} \int \vec{B} \cdot \hat{n} dA}$$

FARADAY'S LAW

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \left[\sum I_{\text{enclosed}} + \qquad 0 \qquad \right]$$

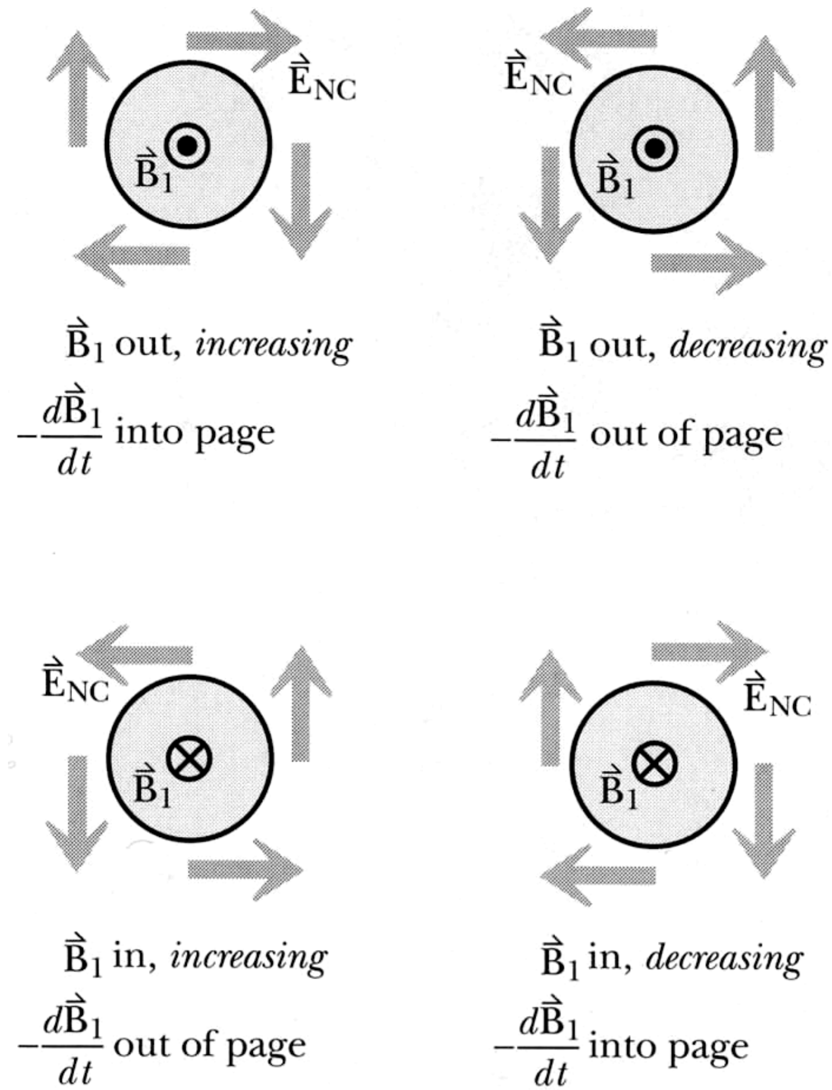
Direction of the Curly Electric Field

Right hand rule:

Thumb in direction of $-\dfrac{d\vec{B}_1}{dt}$
fingers: E_{NC}

Exercise:

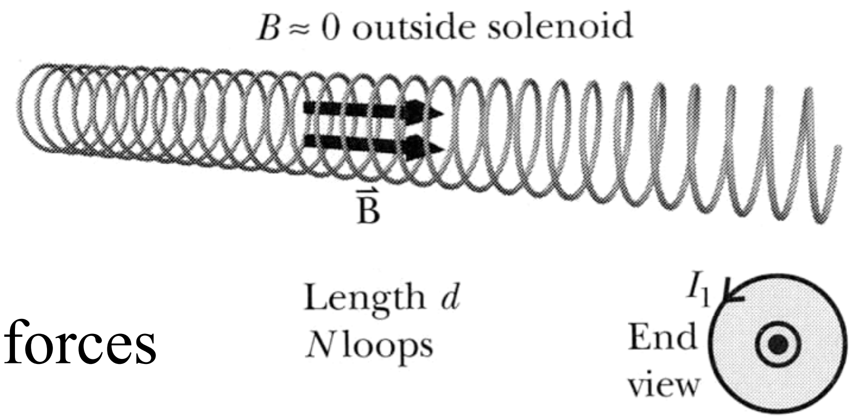
Magnetic field points down
from the ceiling and is increasing.
What is the direction of E?



Changing Magnetic Field

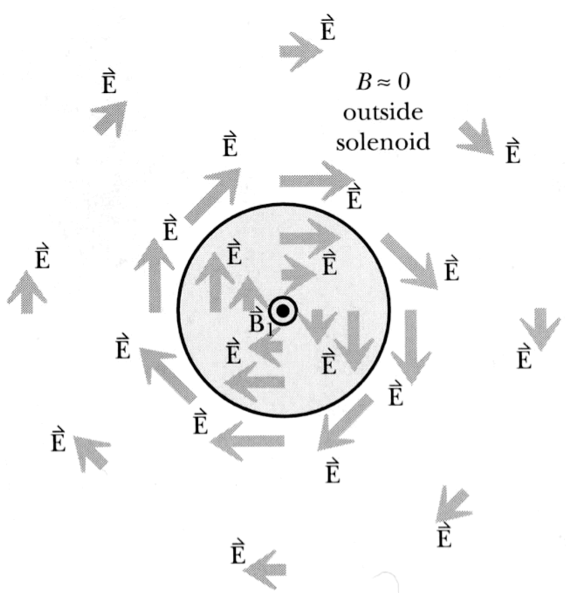
Solenoid: inside $B = \dfrac{\mu_0 NI}{d}$

outside $B \approx 0$

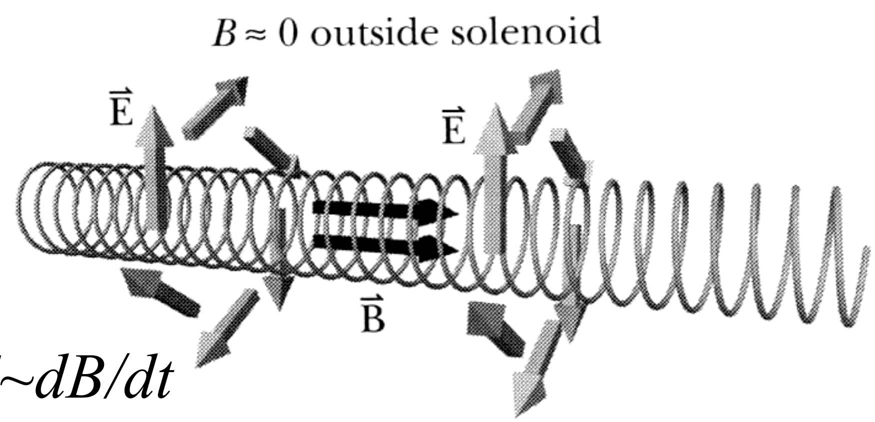


Constant current: there will be no forces on charges outside (B=0, E=0)

What if current is not constant in time? Let B increase in time



$E \sim 1/r$



$E \sim dB/dt$

Non-Coulomb E_{NC} !

Two Ways to Produce Electric Field

1. Coulomb electric field: produced by charges $\vec{E}_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1}{r^2}\hat{r}$

2. Non-Coulomb electric field:
using changing magnetic field

$$\vec{E}_1 \sim \frac{d\vec{B}_1}{dt}\frac{1}{r}$$

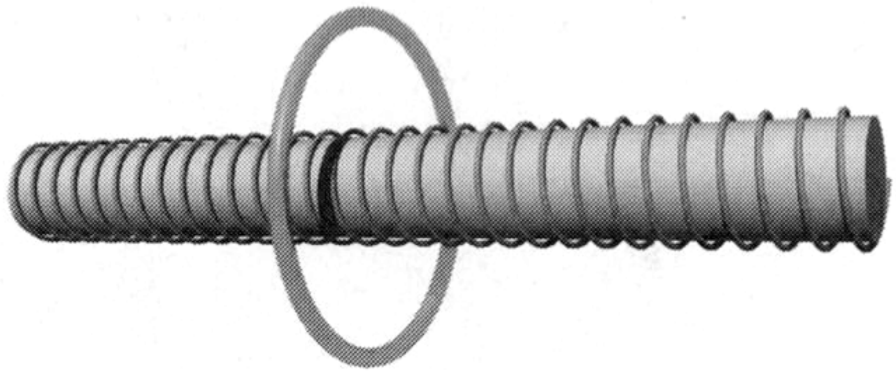
Field outside of solenoid

$$emf = -\frac{d\Phi_{\mathrm{mag}}}{dt}$$

Using changing magnetic flux, by changing area or magnetic field.

Driving Current by Changing *B*

E_{NC} causes current to run along the ring



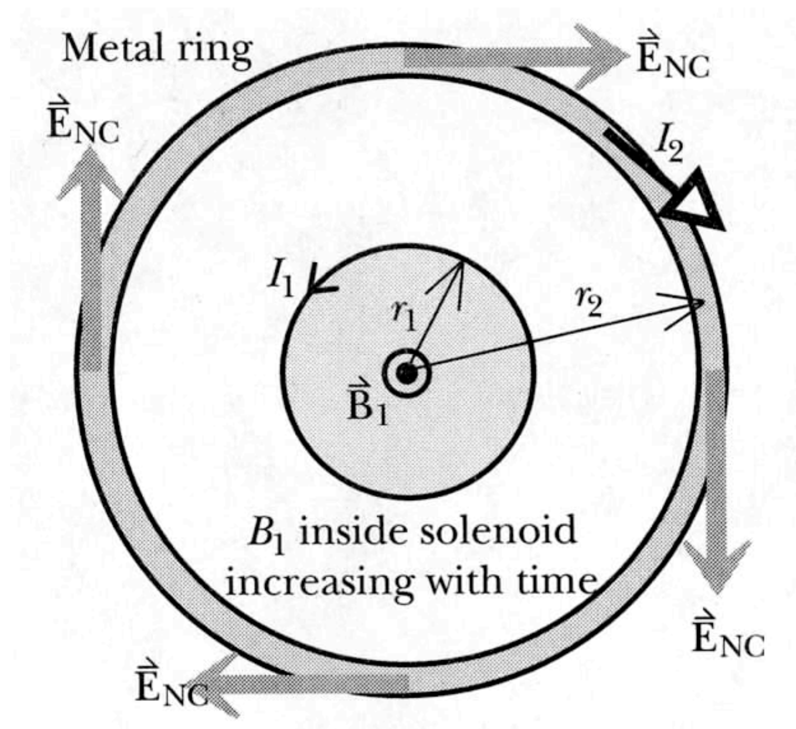
What is the surface charge distribution?

What is *emf* and *I*?

$$emf = \oint \vec{E}_{NC} \cdot d\vec{l} = E_{NC} 2\pi r_2$$

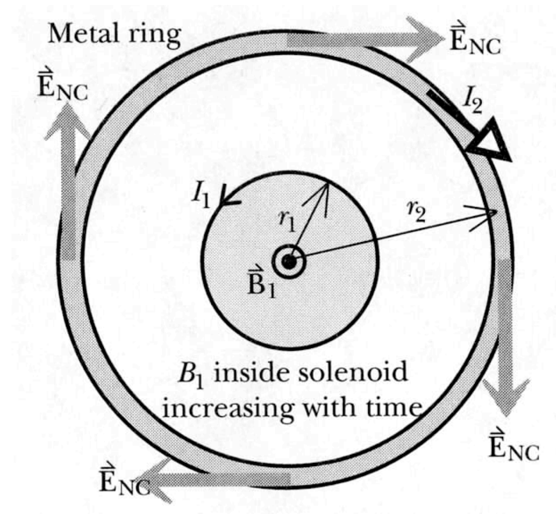
$$I = \frac{E_{NC} 2\pi r_2}{R}$$

Ring has resistance, R



Effect of the Ring Geometry

$$emf = \oint \vec{E}_{NC} \cdot d\vec{l} = E_{NC} 2\pi r_2$$



1. Change radius r_2 by a factor of 2.

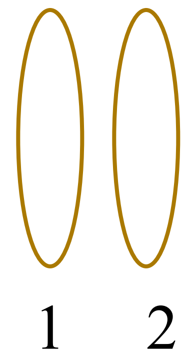
$$E_{NC} \sim 1/r_2 \qquad L = 2\pi r_2 \longrightarrow$$

emf does not depend on radius of the ring!

2. One can easily show that *emf* will be the same for any circuit surrounding the solenoid

Complication

Two loops: one produces changing B_1



$$emf_2 \sim \frac{dB_1}{dt} \sim \frac{dI_1}{dt}$$

$$\downarrow$$

$$I_2 = \frac{emf_2}{R_2} \sim \frac{dB_1}{dt}$$

If I_2 changes in time it creates additional *emf* in the first loop!

$$emf_1 \sim \frac{dB_2}{dt} \sim \frac{dI_2}{dt} \sim \frac{d^2 B_1}{dt^2}$$

Today

Faraday's Law