Lec2

Monday, January 09, 2012
2:21 PM

Homework set 1 is due today. (last problem should be prob.8)

Homework set 2 is assigned. Due on Friday.

TA office hours and location are available at the course homepage.

Review and objective

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Review:

- Electrical Charge Q
  - Positive and negative charge
- Electrical current I
  - Flow rate of charge
  - Flow of negative charge viewed as flow of positive charge in the reverse direction
  - Positive and negative current

$$I = \frac{\Delta Q}{\Delta t}$$

$$i = \frac{dq}{dt}$$

Objective:

- Introduce the notion of voltage
- Introduce voltage source/battery
- Introduce the notion of 2-terminal circuit element
- Study absorbed and delivered power

Charge revisited-10min

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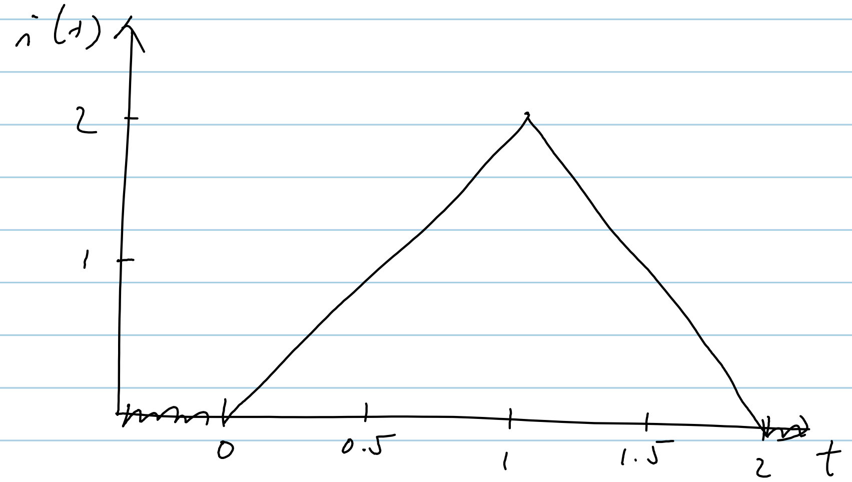
- Conversely, the "sum" of the instantaneous currents through a cross-sectional area from $-\infty$ to t equals the net charge transported through the boundary

$$q(t) = \int_{-\infty}^{t} i(\tau)\, d\tau = \text{"Area below } i(t)\text{"}$$

Ex 1.4. Suppose that i(t) is given as follows



Find q(t)

$$q(t) = \int_{-\infty}^{t} i(\tau)\, d\tau = \cancel{\int_{-\infty}^{0} i(\tau)\, d\tau} + \int_{0}^{t} i(\tau)\, d\tau$$

$$= \int_{0}^{t} i(\tau)\, d\tau$$

Case 1: $0 \le t < 1$. Here $i(t) = 2t$

$$q(t) = \int_0^t 2\tau\, d\tau = \tau^2 \Big|_0^t = t^2 \; (C)$$

Case 2: $1 \le t < 2$. Here $i(t) = -2t + 4$

$$q(t) = q(1) + \int_1^t i(\tau)\, d\tau$$

$$= 1 + \int_1^t (-2\tau + 4)\, d\tau$$

$$= 1 + \left[-\tau^2 + 4\tau \right]_1^t$$

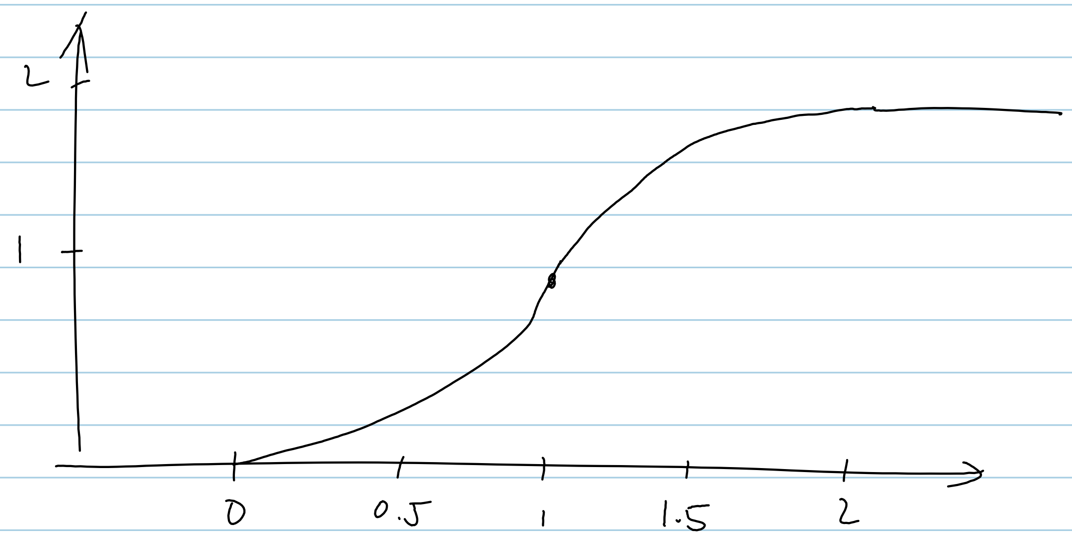
$$= 1 + \left[-t^2 + 4t - (-1 + 4) \right]$$

$$= -2 + 4t - t^2 \quad (C)$$

Case 3: $t \ge 2$

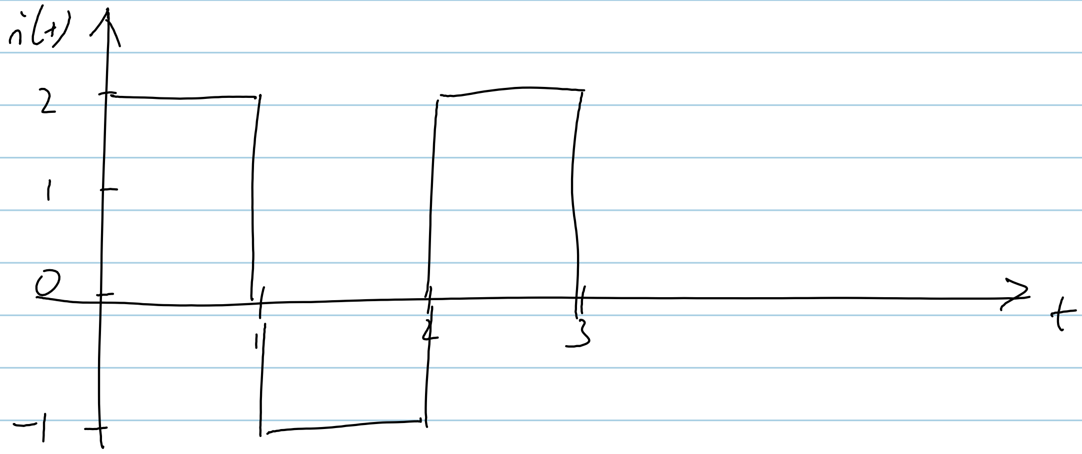
$$q(t) = q(2) + \int_2^t \underbrace{i(\tau)}_{=0}\, d\tau$$

$$= q(2) = 2 \quad (C)$$



Ex 1.5 (P7)

May skip



Find the total charge Q transported.

The total charge equals the total area beneath the curve.

$= 2 - 1 + 2 = 3$ (C)

Voltage

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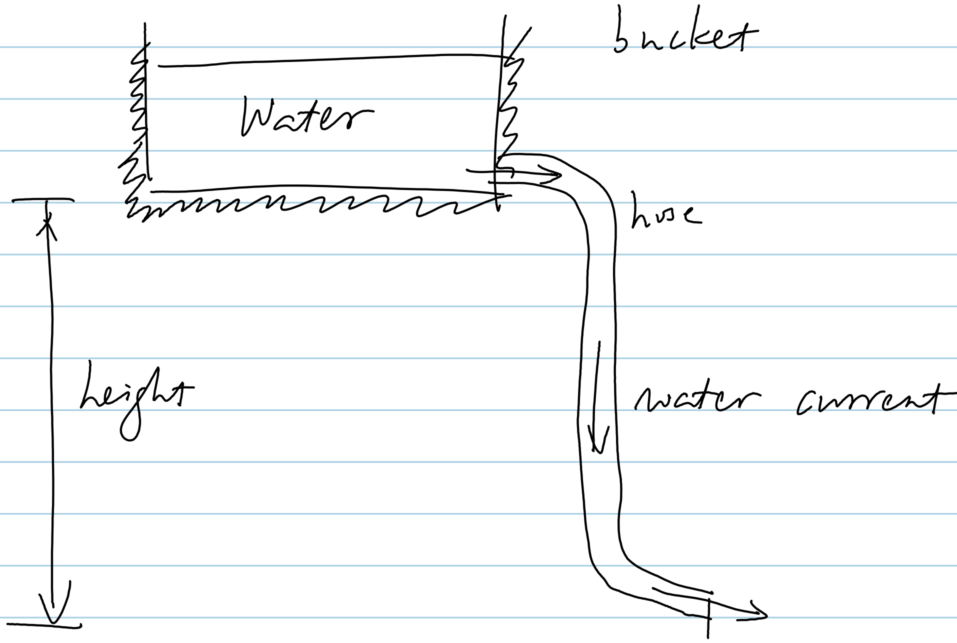
Recall the water analogy



high potential energy

$\propto h, m$

Water bucket/hose	Electrical Circuit
Water Mass	Electrical charge
Water current	Electrical current
Height	Voltage

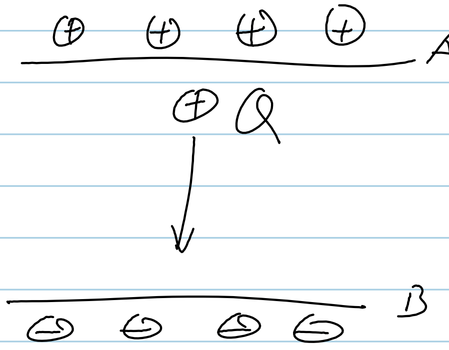
What causes current to flow?

- Gravity forces water to flow from a higher elevation to a lower elevation.

- Water mass at higher altitude contains more potential energy $\propto h$ $(= mgh)$

- As water flows, potential energy is converted to kinetic energy

- Similarly, due to electrical forces, positive charge tends to flow from a positively charged plate to a negatively charged plate



high potential energy $\propto Q, V$.

- Thus, positive charge at point A has higher potential energy.

- As it moves from A to B, its potential energy is converted to other form of energy, e.g. heat.

- Similar to height, the potential energy at A is proportional to the "voltage" "potential difference" drop between A & B.

Voltage (potential difference $A \to B$) /ju:l/

$$= \frac{\text{Energy converted (Joules)}}{\text{Quantity of Charge Moved (C)}}$$

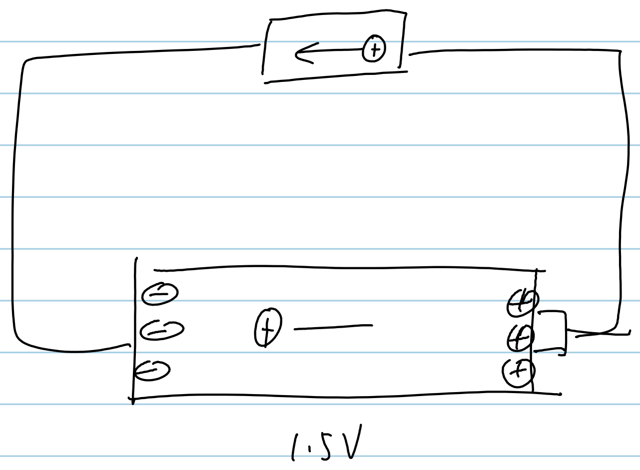
- Unit : Volt $1 \text{ Volt} = 1 \text{ J/C}$

— V :

$v(t)$:

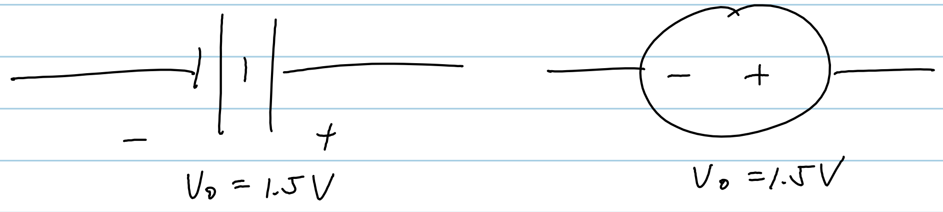
Battery

- In battery, such a voltage is produced by accumulation of charged particles at different terminals due to the chemistry reaction.



energy released $= 1.5V \times Q$

- A battery produces a nearly-constant voltage, which is determined by the chemestry. e.g. 1.5V.



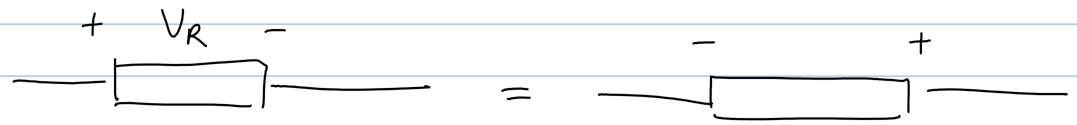
- Then, when the two sides are connected,

e.g. by a coil, current flows,
which converts potential energy to heat

Chemical energy → electrical energy
→ heat

Positive and negative voltage (drop)

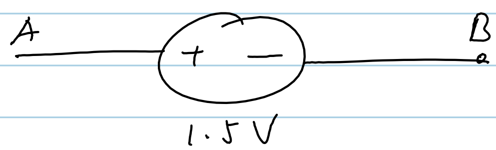
- The plus and negative sign represent the polarity
- Like for current, in circuit analysis we often need to guess the polarity in advance. What if the actual polarity is the reverse?



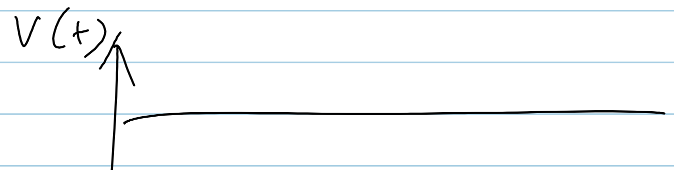
$V_R = -1.5V$ 1.5 V

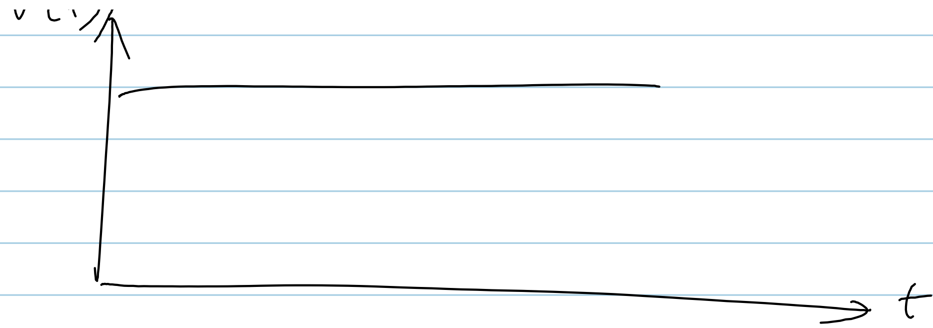
- useful if we do not know the voltage drop direction in advance.

- Or, we can use V_{AB} to denote the voltage drop form A to B

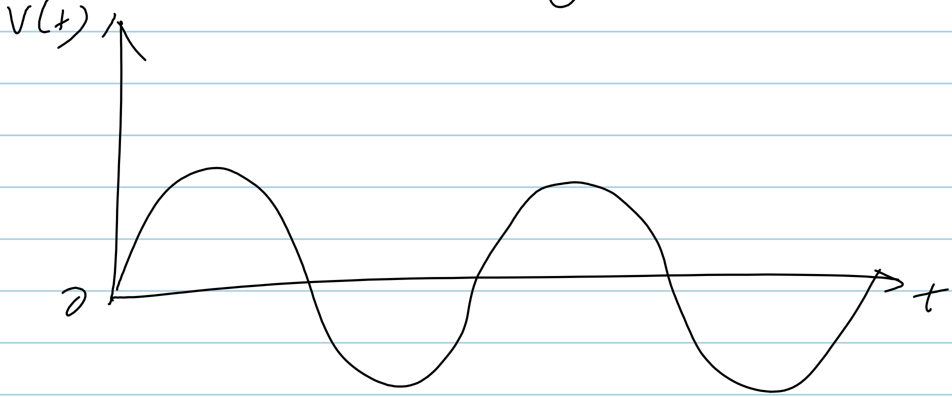


$V_{AB} = 1.5V$
$V_{BA} = -1.5V$





A DC battery (direct current)

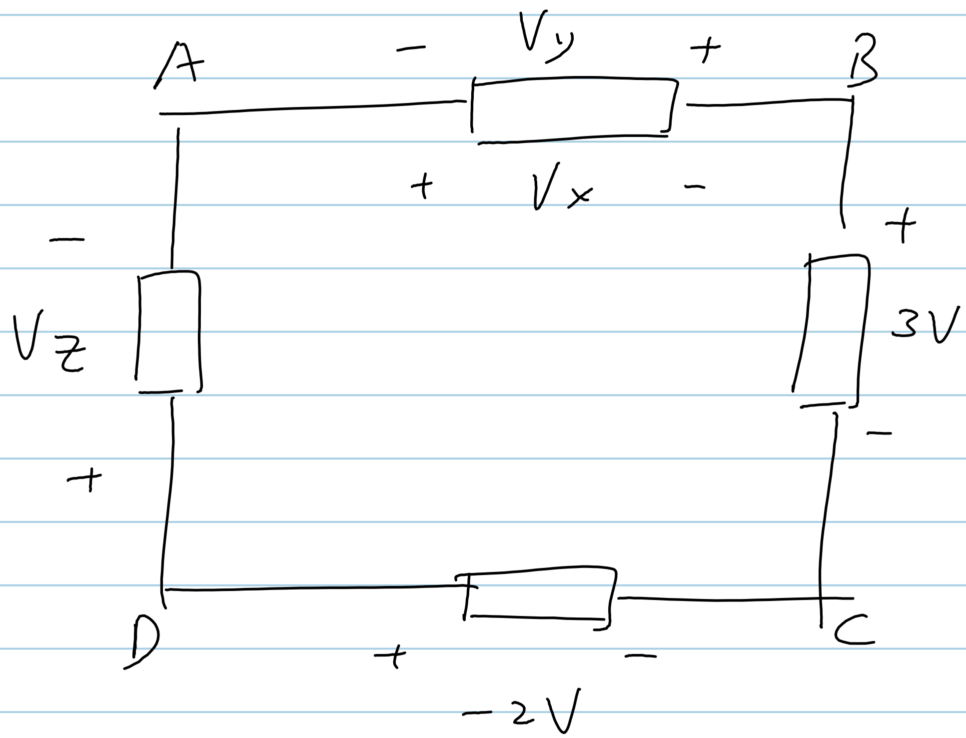


AC voltage source (alternate current)

Voltage example-10min

Tuesday, January 03, 2012
5:08 PM

Ex 2.1 (P13)



Suppose V_{AB}=4V, V_{AD}=9V
Find Vx, Vy, Vz, V_{BC}, V_{CD}

$V_x = V_{AB} = 4V$

$V_y = V_{BA} = -V_{AB} = -4V$

$V_z = V_{DA} = -V_{AD} = -9V$

$V_{BC} = 3V$

$V_{CD} = -V_{DC} = -(-2V) = 2V$

Resistance and ohm's law-10min

Thursday, January 05, 2012
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- Ohm found that for many materials (metal, etc.), the current through a device made of it is proportional to the voltage

$$I \propto V$$

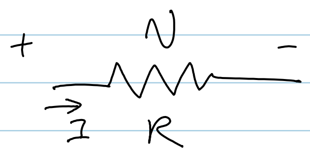
Write $\quad I = \frac{V}{R} = GV \quad , \quad V = IR$

The proportional constant R is called resistance, the inverse G is called the conductance of the device.

- Such a device is called a Resistor

- This relationship between V & I is called Ohm's Law

Ohm's Law



$V = IR, \quad I = \frac{V}{R}$

$V(t) = i(t)\,R, \quad i(t) = \frac{V(t)}{R}$

- Note the signs:
  - Water flows from high altitude to low altitude
  - Electric current flows from high voltage to low voltage.

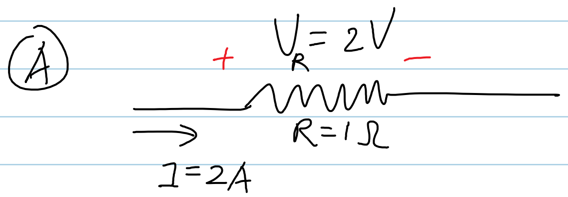
- Basic unit:

  - Resistance R: Ohm (Ω)

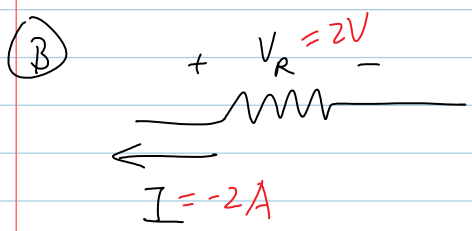
$$1\,\Omega = \frac{1\,V}{1\,A}$$

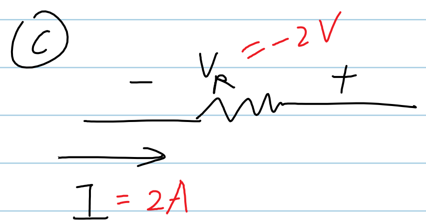
– Conductance G: Siemens (S or $\mho$)

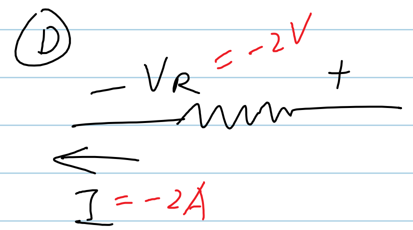
Here is a problem:



– What if I put the reference directions differently?







$V_R = -IR$ $\quad\quad$ $V_R = -IR$ $\quad\quad$ $V_R = IR$

– Problem: When do I need to/not to add the minus sign??

- For both cases A and D, the reference directions of voltage & current are "naturally" compatible"
- Current flows from high voltage to low voltage
- Not for cases B and C

Passive sign convention-5min

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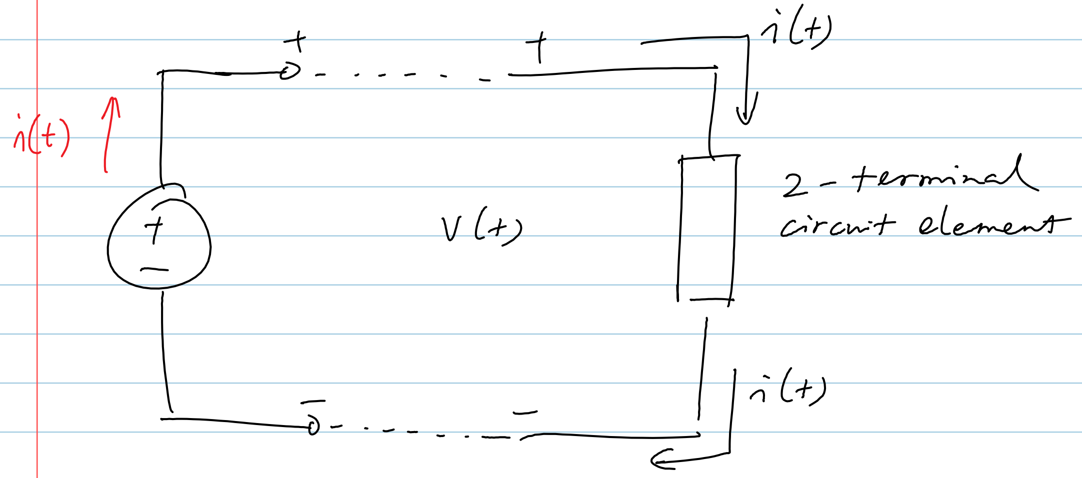
Passive sign convention

- For a passive component (resistor, inductor, capacity, i.e. RHS)



- The passive-sign convention is said to be followed when we take the reference direction of current from ⊕ to ⊖ of the reference direction of voltage drop.

- For a resistor:
  - positive voltage drop will then produce positive current
  - negative voltage drop will produce negative current

  } Ohm's Law without additional minus sign!

  - The current into one terminal equals the exit current

- The situation is however different at the battery (or voltage source)

- If voltage is positive, usually current flow ⊖ → ⊕ inside the battery

↑ an active component

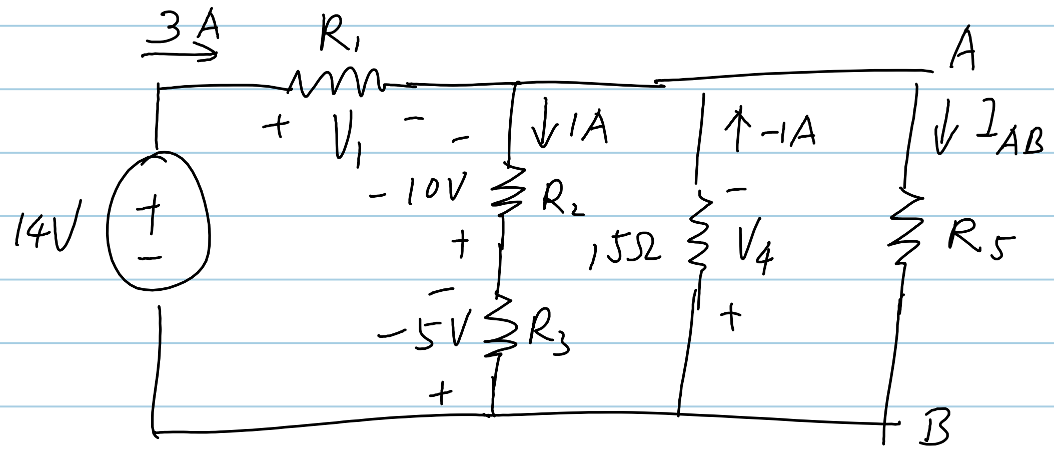
This convention is very important:

- Critical for writing down Ohm's Law correctly
- Most common mistakes for ECE201 students (Imaging omitting a minus sign in equations)
- Also important for power calculation

Example - 5min

Tuesday, January 08, 2013
11:50 AM

Ex 2.2



Which of the following relationship does not follow the passive-sign convention? Which of them is incorrect?

(1) For R_1 : $V_1 = 3R_1$

X (2) For R_2 : $-10 = 1R_3$

(3) For R_3 : $-5 = -(1R_3)$

(4) For 15Ω : $V_4 = (-1) \times 15$

X (5) For R_5 : $V_{BA} = I_{AB} R_5$

Power-10min

Wednesday, January 04, 2012
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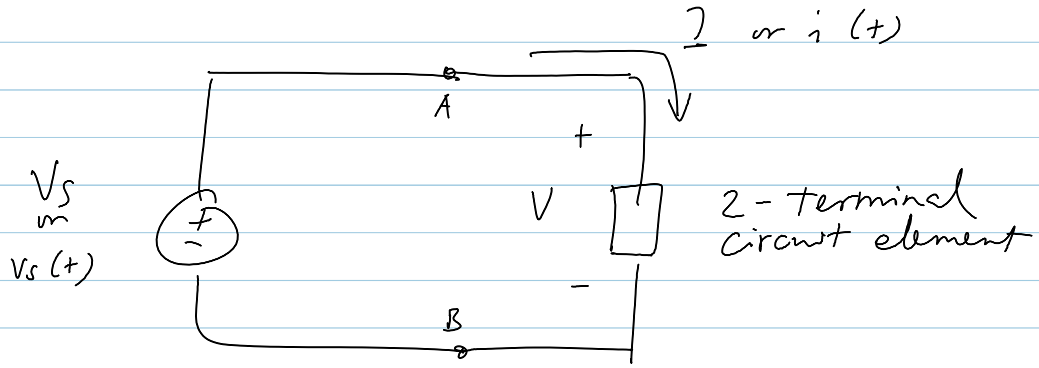


Assume the passive-sign convention on any circuit element.

The power absorbed by the circuit element in Watts is given by

- DC case:

$$P_{CE} = V \cdot I = \frac{V \cdot \Delta Q}{\Delta t}$$

energy released

$$\boxed{P_{CE} = V \cdot I = V_{AB} \cdot I = V_S \cdot I}$$

due to KVL, which we will study soon.

(Note the signs!)

- Instantaneous absorbed power (depends on t)

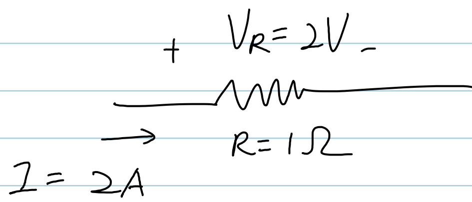
$$p_{CE}(t) = v(t) \cdot i(t) = v_{AB}(t) \cdot i(t) = v_s(t) \cdot i(t)$$

- Integrating power over time leads to the total energy consumed

$$w(t) = \int_{-\infty}^{t} p(\tau)\, d\tau = \int_{-\infty}^{t} v(\tau)\, i(\tau)\, d\tau$$

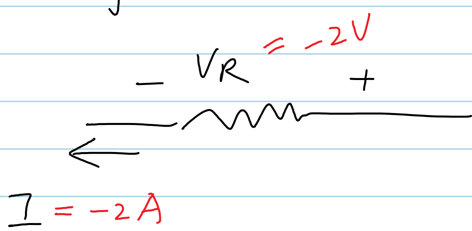
Over finite intervals $\int_{t_0}^{t_1} p(\tau)\,d\tau$

For a resistor (e.g. a heating coil on oven)



$P_{abs}^{R} = V_R I = 4W$

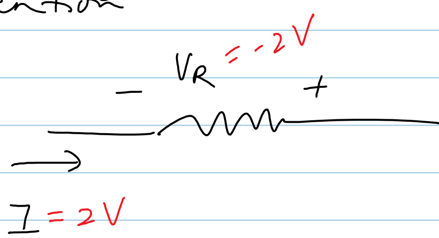
- If the reference directions are both reversed



$P_{abs}^{R} = V_R I = 4W$

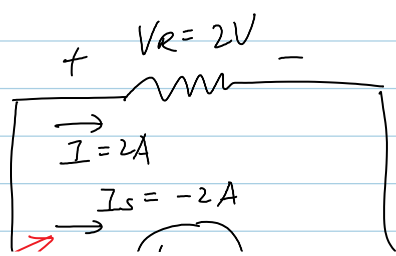
- As long as the passive-sign convention is followed, the absorbed power of a resistor is always positive (good!)

- The following represents a case not following the passive-sign convention



$V_R \cdot I = -4W$ ✗ Avoid!!!

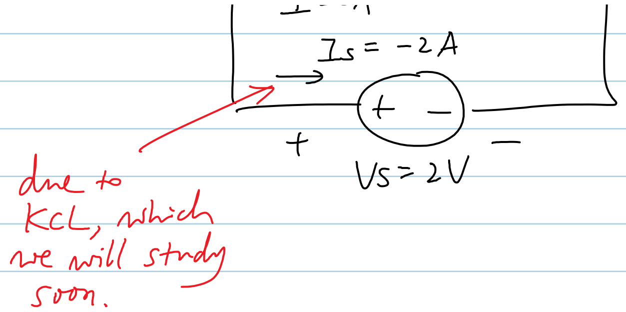
What about the source?



passive-sign for the source

$P_{abs}^{S} = V_s I_s = -4W$

$P_{del}^{S} = V_s I = 4W$



P_{abs} …

$P^S_{del} = V_s I = 4W$

passive-sign for other parts of the circuit

- If we had applied the passive-sign convention to the source, the absorbed power would have been negative!
- This is because the source is delivering power, not absorbing power.

If you want to compute the delivered power from a source, apply the passive sign convention to the opposite components

- Note ± label on voltage source
- current direction leaving the source ⊕
- Delivered power

$$\left.\boxed{\begin{aligned} P_{del} &= V_s I \\ P_{del} &= V_s(t)\, i(t) \end{aligned}}\right\}$$

positive value indicating that power is delivered from the source

- This is the same expression as P^{CE}_{abs}

$$\boxed{P^S_{del} = P^{CE}_{abs}}$$

What is delivered from the battery is consumed by the other circuit elements!

— Conservation of power.

- $P^S_{abs} = -P^S_{del}$

Summary:

- Positive P_{abs} means consuming power
  negative P_{abs} means delivering power

- Positive P_{del} means delivering power
  negative P_{del} means absorbing power

Example

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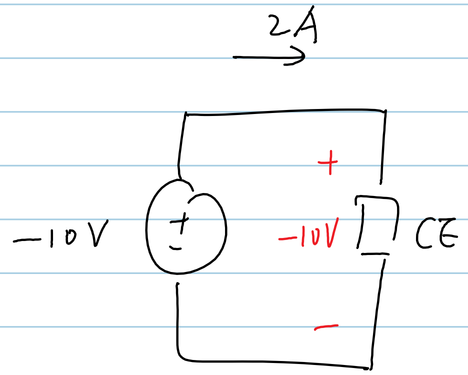
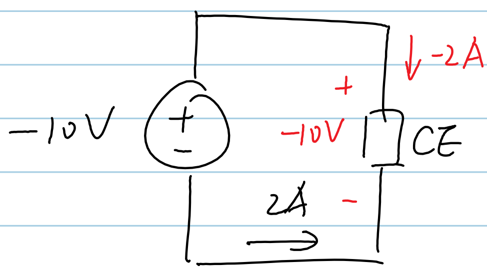
Ex 2.3. Goal: Practice the use of passive sign convention!!!

Find P_{del}^{S} and P_{abs}^{CE}



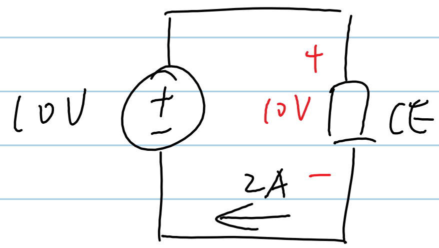
$P_{del}^{S} = -10 \times 2 = -20W$ absorbing

$P_{abs}^{CE} = (-10) \times 2 = -20W$ delivering



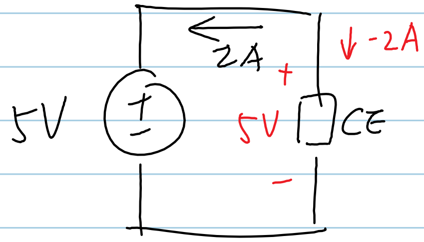
$P_{del}^{S} = (-10) \times (-2) = 20W$

$P_{abs}^{CE} = (-10) \times (-2) = 20W$



$P_{del}^{S} = 10 \times 2 = 20W$

$P_{abs}^{CE} = 10 \times 2 = 20W$



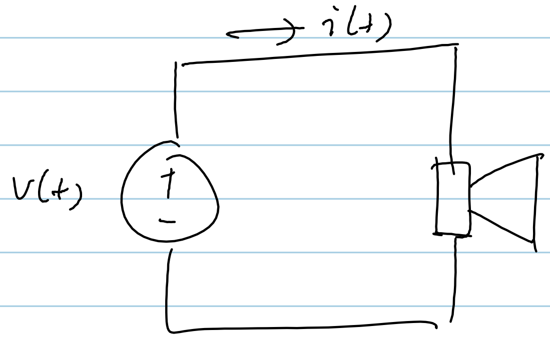
$P_{del}^{S} = 5 \times (-2) = -10W$

$P_{abs}^{CE} = 5 \times (-2) = -10W$

- P_{del}^{S} & P_{abs}^{CE} are essentially the same expression

- $P_{del}^{S} = P_{abs}^{CE}$ — Conservation of Power.

May skip



$v(t) = 40 \sin(200\pi t)$ $\quad$ $i(t) = 5 \sin(200\pi t)$

$p(t) = v(t) \cdot i(t) = 200 \sin^2(200\pi t)$