Aung Kyi San
Summer 2012

LECTURE 11

- Switched Capacitors Circuits

- Reference: Decarlo/Lin PP 645-652

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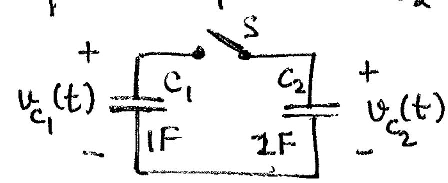
Switched Capacitor Networks

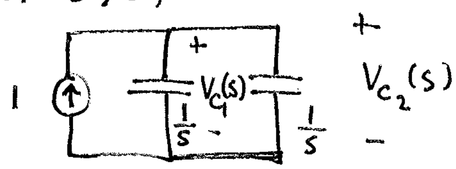
- Resistorless, inductorless
- only sources, switches, capacitors and op-amps

Example 1: Consider the circuit as shown below. The switch 'S' is closed at $t=0$. $v_{c_1}(0^-) = 1V$, $v_{c_2}(0^-) = 0$. Compute $v_{c_1}(t)$ and $v_{c_2}(t)$ for $t > 0$.



Method 1

For $t > 0$,



$$V_{c_1}(s) = V_{c_2}(s) = \frac{\frac{1}{s}\cdot\frac{1}{s}}{\frac{1}{s}+\frac{1}{s}} \cdot 1 = \frac{\frac{1}{s^2}}{\frac{2}{s}} = \frac{0.5}{s}$$

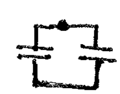
$$v_{c_1}(t) = v_{c_2}(t) = 0.5\,u(t)$$

Method 2 - Conservation-of-charge approach

For $t < 0$, $q_{total} = C_1 v_{c_1}(0^-) + C_2 v_{c_2}(0^-)$

$$= (1)(1) + (1)(0) = 1$$

For $t > 0$, $q_{total} = C_1 v_{c_1}(0^+) + C_2 v_{c_2}(0^+)$



$$= (1+1)\,v_{c_1}(0^+) \qquad (\because v_{c_1}(0^+) = v_{c_2}(0^+))$$

By conservation-of-charge

$$1 = 2\,v_{c_1}(0^+) \Rightarrow v_{c_1}(0^+) = \frac{1}{2} = v_{c_2}(0^+)$$

For $t > 0$, $v_{c_1}(t) = v_{c_2}(t) = 0.5\,V$ (or) $v_{c_1}(t) = v_{c_2}(t) = 0.5\,u(t)$

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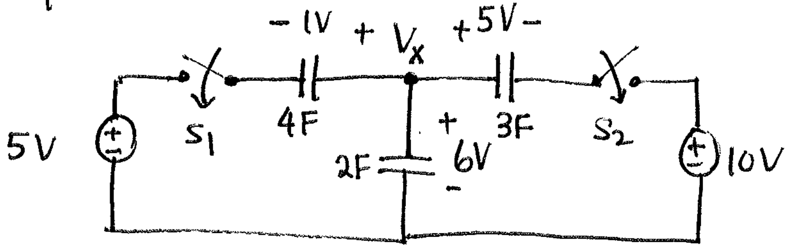
Example 2:

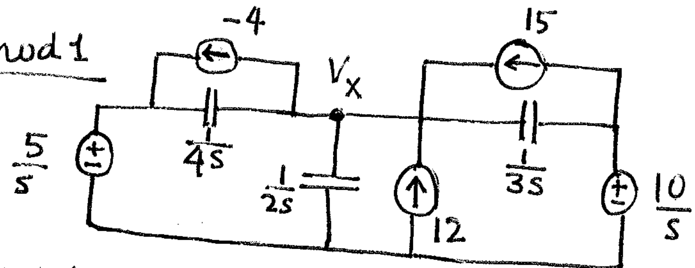


Initial conditions shown in figure at $t=0^-$.

Switches – closed at $t=0$

Find the node voltage V_x for $t>0$

Method 1



Nodal Eqn. at node V_x

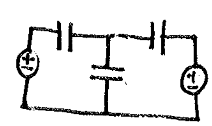
$$\left(V_x - \frac{5}{s}\right)4s + (-4) + V_x(2s) - 12 - 15 + \left(V_x - \frac{10}{s}\right)3s = 0$$

$$V_x = \frac{9}{s}$$

$$v_x(t) = 9\,u(t) \text{ V} \leftarrow$$

Method 2 – conservation-of-charge approach

For $t<0$, $q_{total} = (4)(1) + (3)(5) + (2)(6) = 31$



For $t>0$, $q_{total} = (4)(V_x - 5) + 2V_x + 3(V_x - 10)$

$$= 9V_x - 50$$

$\therefore$ By conservation-of-charge

$$31 = 9v_x - 50$$

$$v_x = \frac{81}{9} = 9 \text{ V}$$

$\therefore v_x(t) = 9\text{V}$ for $t>0$ (or) $v_x(t) = 9\,u(t)$ V.

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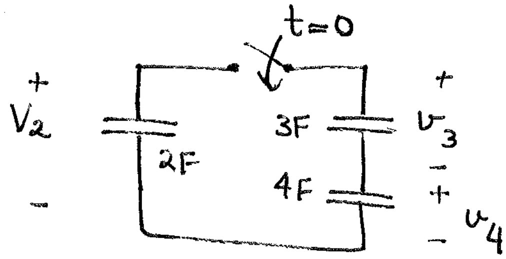
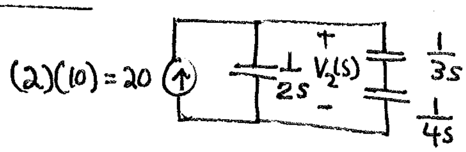
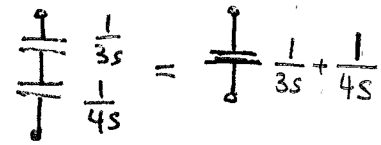
Example 3. Given $v_2(t) = 10V$, $v_3(t) = v_4(t) = 0V$ for $t < 0$.

Find $v_2(t)$, $v_3(t)$ and $v_4(t)$ for $t > 0$

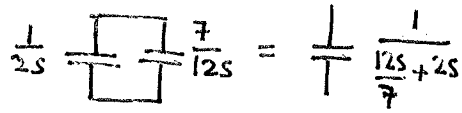


__Method 1.__





$$= \frac{7}{12s}$$



$$= \frac{7}{26s}$$

$$V_2(s) = \frac{7}{26s} \cdot 20 = \frac{70}{13s}$$

$$v_2(t) = \frac{70}{13} u(t) \leftarrow$$

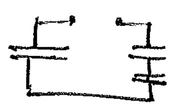
$$V_3(s) = V_2(s) \cdot \frac{\frac{1}{3s}}{\frac{7}{12s}} = \frac{70}{13s} \cdot \frac{4}{7} = \frac{40}{13s}$$

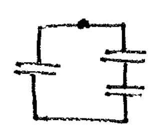
$$v_3(t) = \frac{40}{13} u(t) \leftarrow$$

$$V_4(s) = V_2(s) \cdot \frac{\frac{1}{4s}}{\frac{7}{12s}} = \frac{70}{13s} \cdot \frac{3}{7} = \frac{30}{13s}$$

$$v_4(t) = \frac{30}{13} u(t) \leftarrow$$

__Method 2__



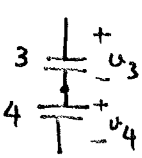


$t < 0 \quad q_{total} = 2(10) = 20$

$t > 0 \quad q_{total} = 2v_2 + 3v_3$

$$\therefore 20 = 2v_2 + 3v_3$$

But



$3v_3 = 4v_4$ and $v_2 = v_3 + v_4$

$v_3 = \frac{4}{3} v_4$

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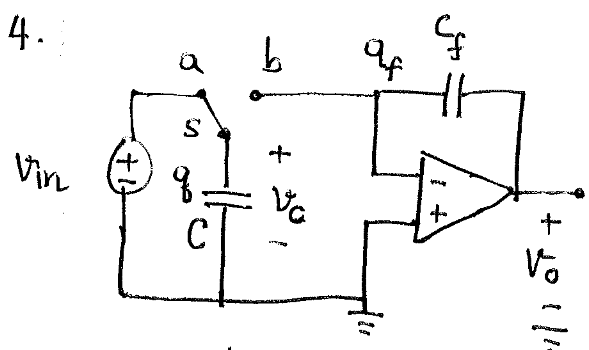
$$\therefore\ 20 = 2v_2 + 3v_3$$
$$= 2(v_3 + v_4) + 3v_3$$
$$= 5v_3 + 2v_4$$
$$= 5\,\frac{4}{3}v_4 + 2v_4$$
$$20 = \frac{26}{3}v_4$$
$$v_4 = \frac{60}{26} = \frac{30}{13}\ \text{V} \leftarrow$$
$$v_3 = \frac{4}{3}v_4 = \frac{4}{3}\cdot\frac{30}{13} = \frac{40}{13}\ \text{V} \leftarrow$$
$$v_2 = v_3 + v_4 = \frac{70}{13}\ \text{V} \leftarrow$$

Example 4.



S is operated in the following manner.

1. At $t=0$, S is at position a, has been there for a long time.
2. At $t=T$, S is moved to position b.
3. At $t=2T$, S is moved to position a.
4. At $t=3T$, S is moved to position b.
5. ⋮

Determine V_o and sketch the output waveform.

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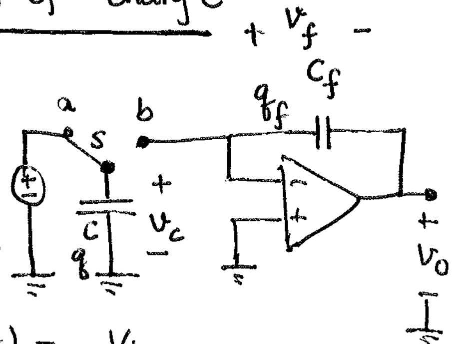
ECE 20200 : Linear Circuit Analysis II
School of ECE, Purdue University

Conservation of charge

$t < T$

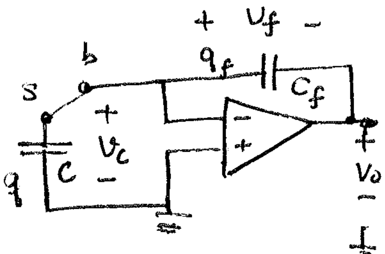


$$V_C(t) = V_{in}$$

$$v_f(t) = 0 \Rightarrow V_o = -V_f = 0 \leftarrow$$

$$q = C\,V_{in}\ ,\quad q_f = 0 \qquad \therefore q_{total} = C\,V_{in}$$

$T < t < 2T$

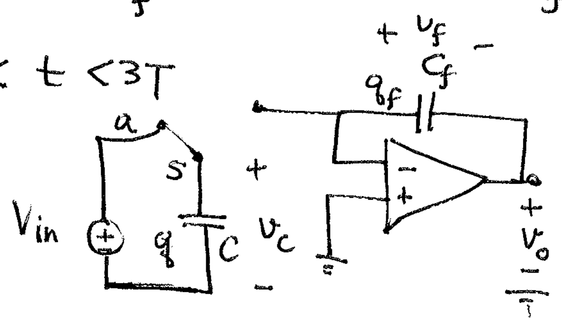


$$v_c = V_- = V_+ = 0$$

$$q = 0\ ,\quad q_f = q_{total} = C\,V_{in}$$

$$v_f = \frac{C}{C_f} V_{in} \Rightarrow v_o = -\frac{C}{C_f} V_{in} \leftarrow$$

$2T < t < 3T$



$$v_c(t) = V_{in}$$

$$q = C\,v_{in}\ ,\quad q_f = C\,V_{in} \qquad \therefore q_{total} = 2C\,v_{in}$$

$$v_o = -\frac{C}{C_f} v_{in} \leftarrow$$

$3T < t < 4T$

$$v_C = v_- = v_+ = 0$$

$$q = 0, \quad q_f = q_{total} = 2Cv_{in}$$

$$v_f = \frac{2Cv_{in}}{C_f} \Rightarrow v_o = -\frac{2Cv_{in}}{C_f}$$

$4T < t < 5T$

$$v_c(t) = v_{in}, \quad v_f(t) = \frac{2Cv_{in}}{C_f}$$

$$q = Cv_{in}, \quad q_f = 2Cv_{in}$$

$$q_{total} = 3Cv_{in}$$

$$v_o = -\frac{2Cv_{in}}{C_f}$$

$5T < t < 6T$

$$v_C = V_- = V_+ = 0$$

$$q = 0, \quad q_f = q_{total} = 3Cv_{in}$$

$$v_f = \frac{3Cv_{in}}{C_f} \Rightarrow v_o = -\frac{3Cv_{in}}{C_f}$$

⋮

$$v_o(t) = \begin{cases} 0 & t < T \\ -Cv_{in}/C_f & T < t < 3T \\ -2Cv_{in}/C_f & 3T < t < 5T \\ -3Cv_{in}/C_f & 5T < t < 7T \end{cases}$$

Let all the parameters have units of 1.