Announcements

Machine-graded scores are on Blackboard Learn.

Hand-graded scores will appear this week.

Correction to Last Lecture!

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{q\vec{v} \times \vec{r}}{|r|^2}$$

BIOT-SAVART LAW
point charge

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{q\vec{v} \times \hat{r}}{|r|^2}$$

BIOT-SAVART LAW
point charge

... Corrected slides will be posted on BBL.

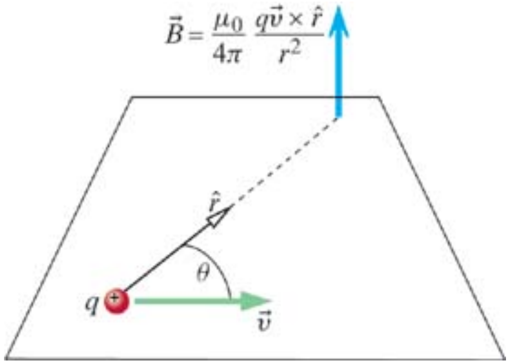
Key Ideas in Chapter 18: Magnetic Field

- **Moving charged particles make a magnetic field, which is different from an electric field.**
- **The needle of a magnetic compass aligns with the direction of the net magnetic field at its location.**

- **A current is a continuous flow of charge.**
  - Electron current is a number of electrons per second entering a section of a conductor.
  - Conventional current (Coulombs/second) is opposite in direction to the electron current, and is assumed to be due to positively charged particles.
- **The superposition principle can be applied to calculate the expected magnetic field from current-carrying wires in various configurations.**
  - A current-carrying loop is a magnetic dipole.
  - A bar magnet is also a magnetic dipole.
  - Even a single atom can be a magnetic dipole!

Last Time

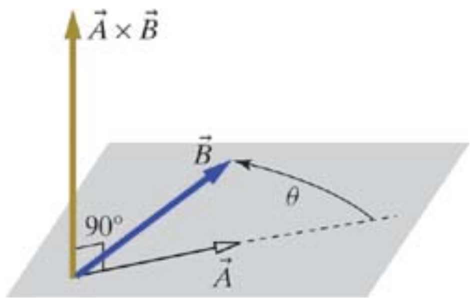
- Energy stored in a field
- Sources of Magnetic Field
- Magnetic Field due to Moving Charges
- Cross Products: Right-hand Rule
- (Cross Products: Mathematically)



Right-Hand Rule

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{q\vec{v}\times\hat{r}}{|r|^2}$$

BIOT-SAVART LAW
point charge

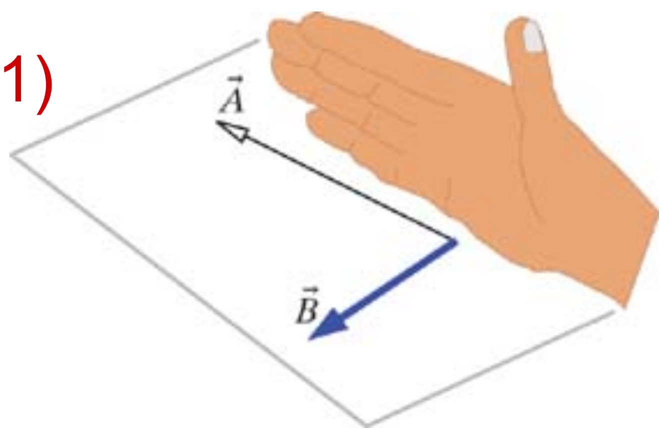


Result of Cross Product $\vec{v}\times\vec{r}$

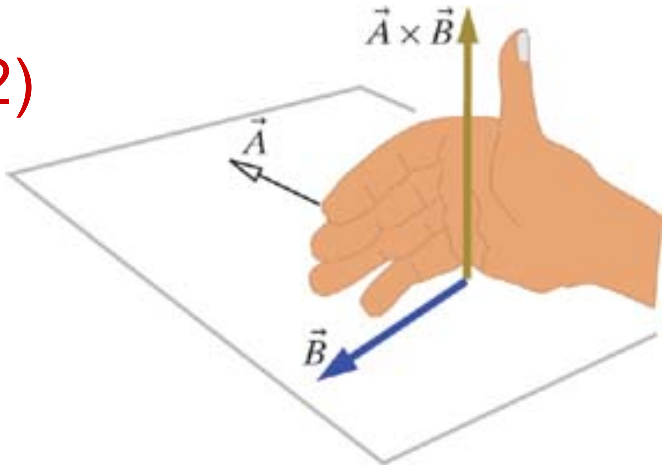
is Perpendicular to both $\vec{v}$ and $\vec{r}$

Right-Hand Rule:

1)



2)



iClicker

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{q\vec{v}\times\hat{r}}{|r|^2}$$

BIOT-SAVART LAW
point charge

Today

- (Cross Products: Mathematically)
- Electron Current and Conventional Current
- Calculating the Electron Current
- True *vs*. Useful
- Biot-Savart Law in a Wire
- Relativity??

Cross Product: Here's the Math

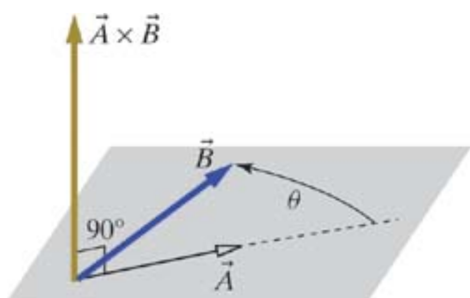
$$\vec{A} \times \vec{B} = \left| \begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{array} \right| \begin{array}{cc} \hat{x} & \hat{y} \\ A_x & A_y \\ \underbrace{B_x \quad B_y}_{\boxed{\text{copy 1st two colums}}} \end{array} = \underbrace{\begin{array}{l} +(\quad\quad - \quad\quad)\,\hat{x} \\ +(\quad\quad - \quad\quad)\,\hat{y} \\ +(\quad\quad - \quad\quad)\,\hat{z} \end{array}}_{\boxed{\text{set up the answer}}}$$

Cross Product: Here's the Math

$$\vec{A} \times \vec{B} = \left| \begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{array} \right| \begin{array}{cc} \hat{x} & \hat{y} \\ A_x & A_y \\ B_x & B_y \end{array} = \begin{array}{l} +\,(A_y B_z \;-\; A_z B_y)\,\hat{x} \\ +\,(A_z B_x \;-\; A_x B_z)\,\hat{y} \\ +\,(A_x B_y \;-\; A_y B_x)\,\hat{z} \end{array}$$

Cross Product: Here's the Math

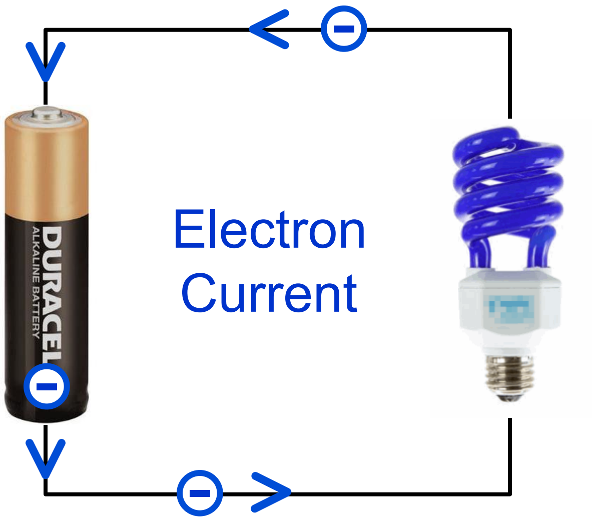
$$\vec{A} \times \vec{B} = \left| \begin{array}{ccc|cc} \hat{x} & \hat{y} & \hat{z} & \hat{x} & \hat{y} \\ A_x & A_y & A_z & A_x & A_y \\ B_x & B_y & B_z & B_x & B_y \end{array} \right| = \Big\langle (A_yB_z - A_zB_y), \; (A_zB_x - A_xB_z), \; (A_xB_y - A_yB_x) \Big\rangle$$



The resulting vector has magnitude:

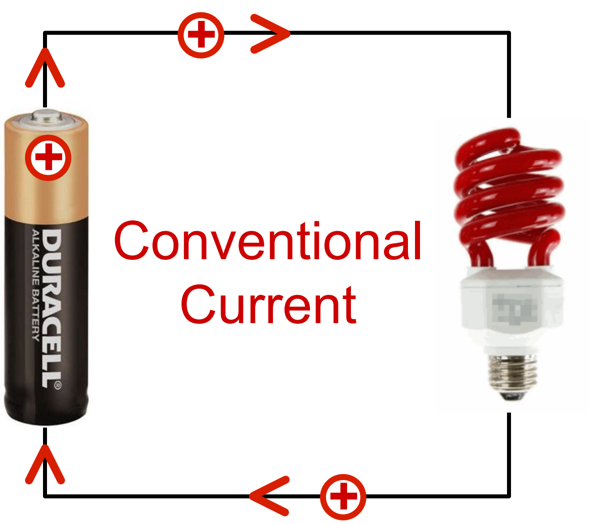
$$|\vec{A} \times \vec{B}| = |A||B|sin\theta$$

Conventional Current and Electron Current



Electron Current:

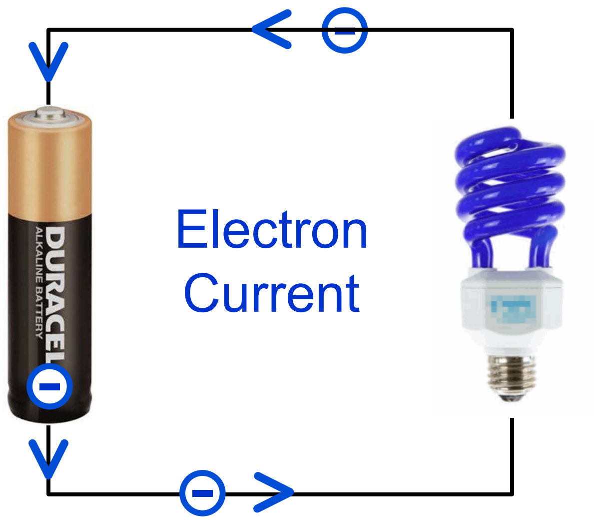
Electrons exit battery at (**-**) terminal,
and enter battery at (**+**) terminal

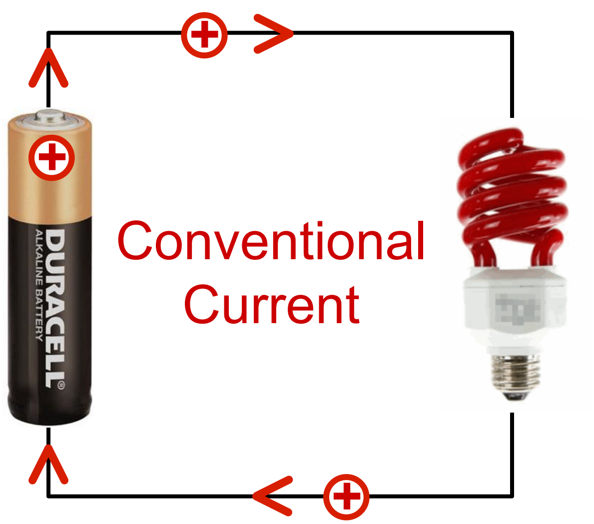


Conventional Current:

Positive charges exit battery at (**+**) terminal,
and enter battery at (**-**) terminal

Conventional Current and Electron Current

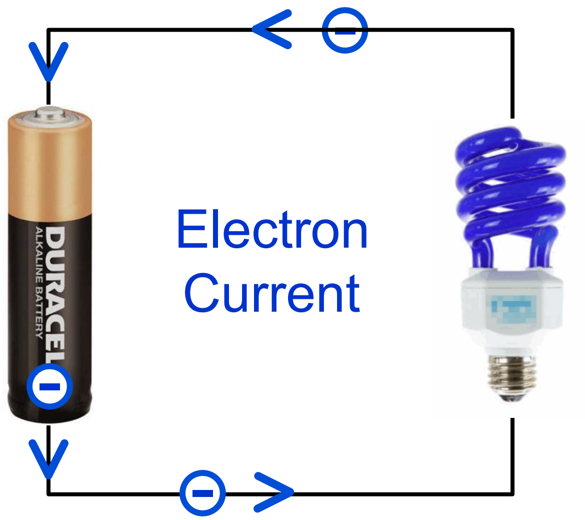


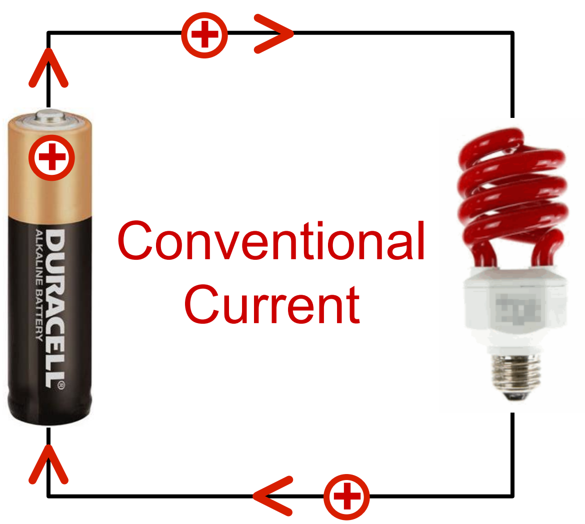


Why the difference? Benjamin Franklin guessed that current is carried by positive charges.

http://blog.marshotelonline.com/2012/01/08/if-grounded/

Conventional Current and Electron Current





Electron Current:

$$i = nA\bar{v} \quad = \left[\frac{\text{electrons}}{\text{second}}\right]$$

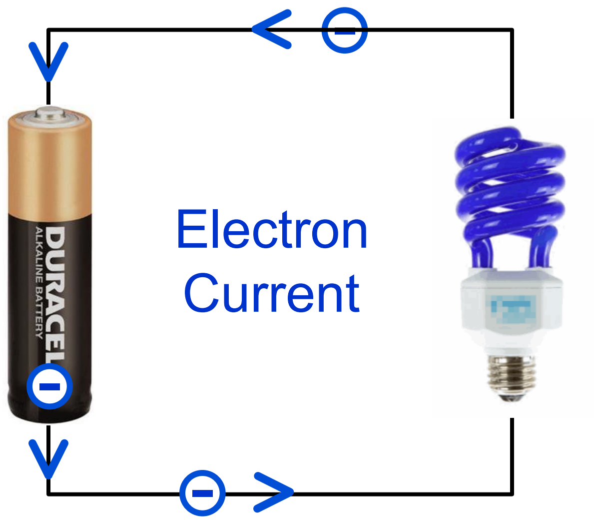
Square Brackets [ ]
Mean "Units"

Conventional Current:

$$I = |q|nA\bar{v} = \left[\frac{\text{Coulombs}}{\text{second}}\right]$$

$$= \left[\text{Amperes}\right]$$

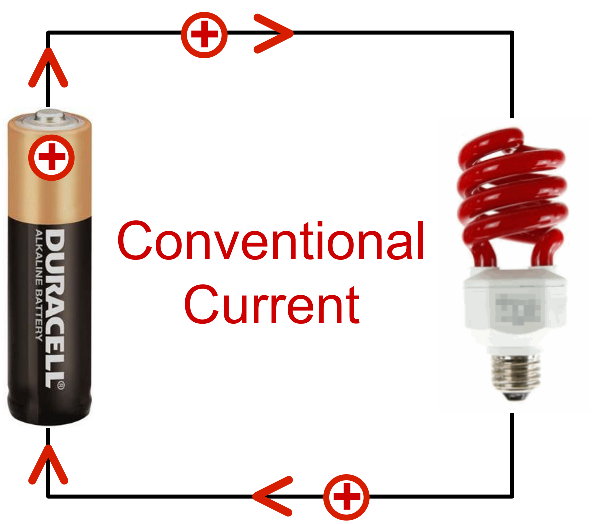
Conventional Current and Electron Current



Electron Current:

$$i = nA\bar{v} = \left[\frac{\text{electrons}}{\text{second}}\right]$$

We will show this one



Conventional Current:

$$I = |q|nA\bar{v} = \left[\frac{\text{Coulombs}}{\text{second}}\right]$$

$$= \left[\text{Amperes}\right]$$

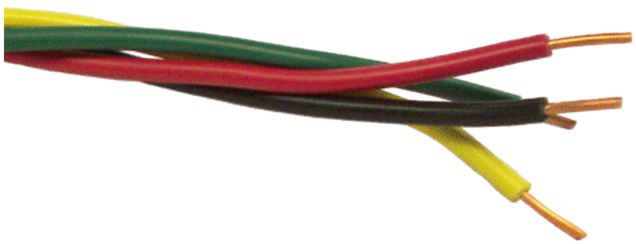
Calculating Electron Current

Find electron current in terms of:

$$n = \left[\frac{\text{electrons}}{\text{m}^3}\right]$$ Density of Electrons in the Wire

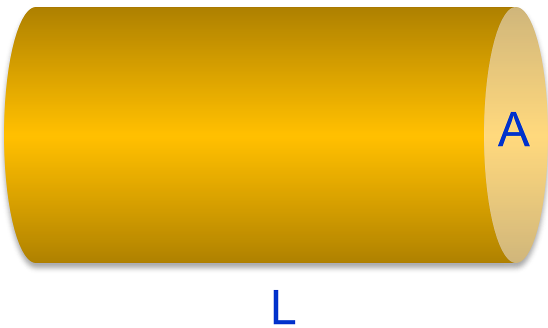
$$A = \left[\text{m}^2\right]$$ Cross Sectional Area of the Wire

$$\bar{v} = \left[\frac{m}{s}\right]$$ Average Velocity of Electrons in the Wire



Calculating Electron Current

$n = \left[\dfrac{\text{electrons}}{\text{m}^3}\right]$ $A = [\text{m}^2]$ Cross Sectional Area of Wire $\bar{v} = \left[\dfrac{m}{s}\right]$ Average Velocity of Electrons



1. How many Electrons are in here?

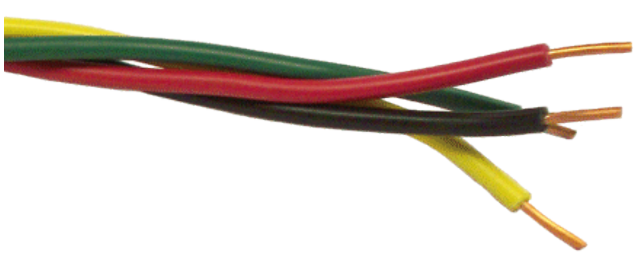
$$N = nV \quad = \left[\frac{\#}{\text{m}^3}\right][m^3] \quad = [\#]$$ ✔ UNITS

$$\boxed{N = nAL}$$

2. Electrons are moving at velocity $\bar{v}$.
How long does this take to pass through L?

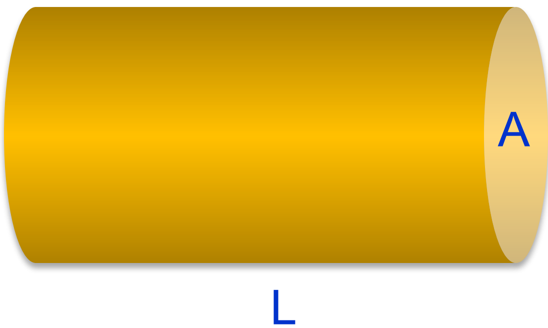
$$L = \bar{v}\Delta t$$

$$\Rightarrow \boxed{\Delta t = \frac{L}{\bar{v}}} = \frac{[m]}{[m/s]} = [s]$$ ✔ UNITS



Calculating Electron Current

$$N = nAL \qquad \Delta t = \frac{L}{\bar{v}}$$



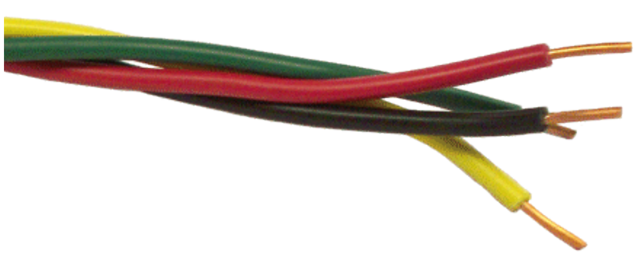
3. How many Electrons pass by per second?

$$i = \frac{N}{\Delta t} = \left[\frac{\#}{s}\right]$$

$$= \frac{nA\cancel{L}}{\cancel{L}/\bar{v}} = nA\bar{v}$$

$$= \left[\frac{\#}{\cancel{m}^3}\right]\left[\cancel{m}^2\right]\left[\frac{\cancel{m}}{s}\right] = \left[\frac{\#}{s}\right]$$

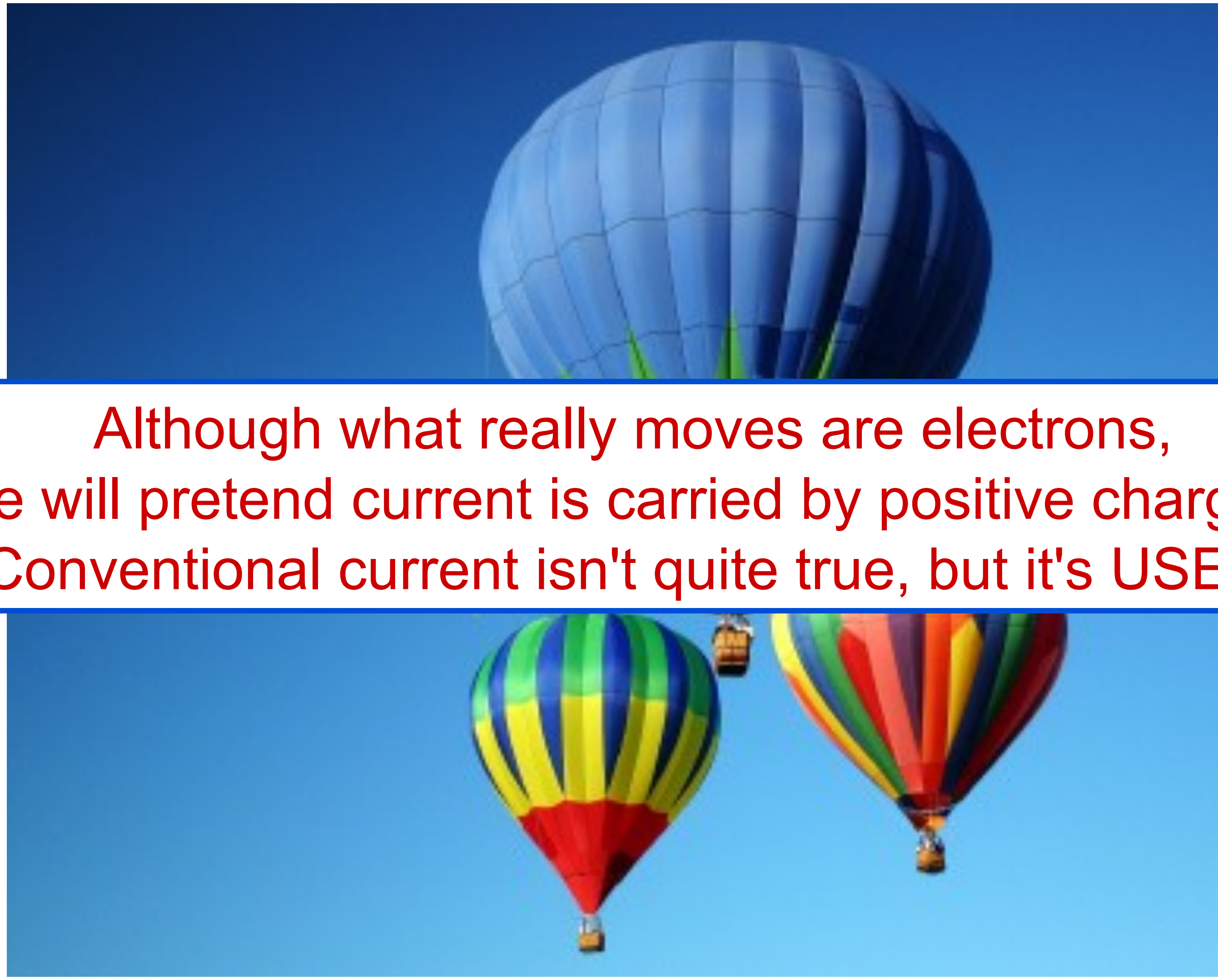
✔ UNITS



$$\boxed{i = nA\bar{v}}$$

ELECTRON CURRENT

True *vs.* Useful



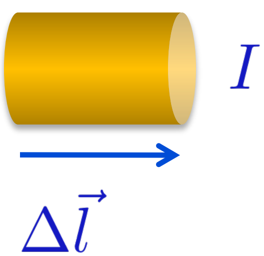
Although what really moves are electrons,
we will pretend current is carried by positive charges.
→ Conventional current isn't quite true, but it's USEFUL!

Biot-Savart Law

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{q\vec{v} \times \hat{r}}{|r|^2}$$

BIOT-SAVART LAW
point charge

We need to understand how these are related



$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{I\Delta\vec{l} \times \hat{r}}{|r|^2}$$

BIOT-SAVART LAW
current in a wire

$\Delta\vec{l}$ = length of this chunk of wire

GOAL: Show $q\vec{v} \longrightarrow I\Delta\vec{l}$

(point charge) (wire)

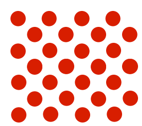
(POSITIVE) POINT CHARGE

q $\vec{v}$

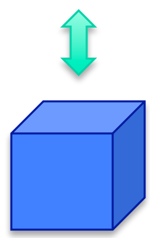
$q\vec{v}$

$q\vec{v}$

MANY POINT CHARGES



N particles



$n = \left[\frac{\#}{m^3}\right]$ particles per volume ΔV

ΔV

$Nq\vec{v}$

$\updownarrow$

$(n\Delta V)q\vec{v}$

GOAL: Show $q\vec{v} \longrightarrow I\Delta\vec{l}$

(point charge) (wire)

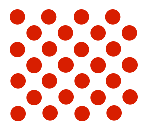
(POSITIVE) POINT CHARGE

q $\vec{v}$

$q\vec{v}$

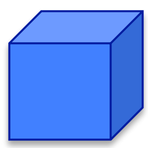
$q\vec{v}$

MANY POINT CHARGES



N particles

$N q\vec{v}$

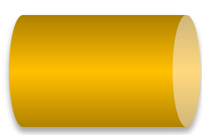


$n = \left[\frac{\#}{m^3}\right]$ particles per volume ΔV

ΔV

$(n\Delta V)q\vec{v}$

CHUNK OF WIRE



I

$\Delta\vec{l}$ = length of this chunk of wire

Move the vector symbol

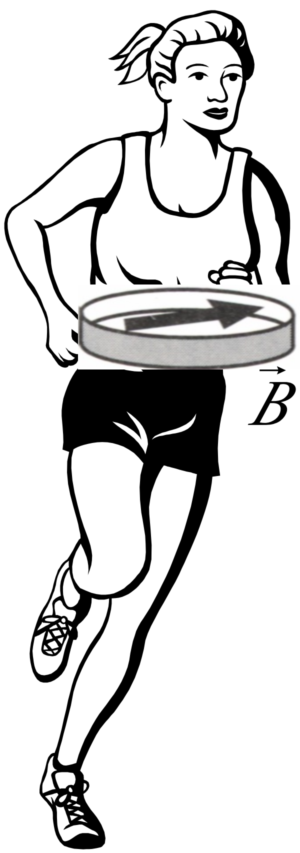
$$q\vec{v}n\Delta V = q\vec{v}nA\Delta l$$

$$= \underbrace{q|v|nA}_{\equiv I}\Delta\vec{l} \qquad I\Delta\vec{l}$$

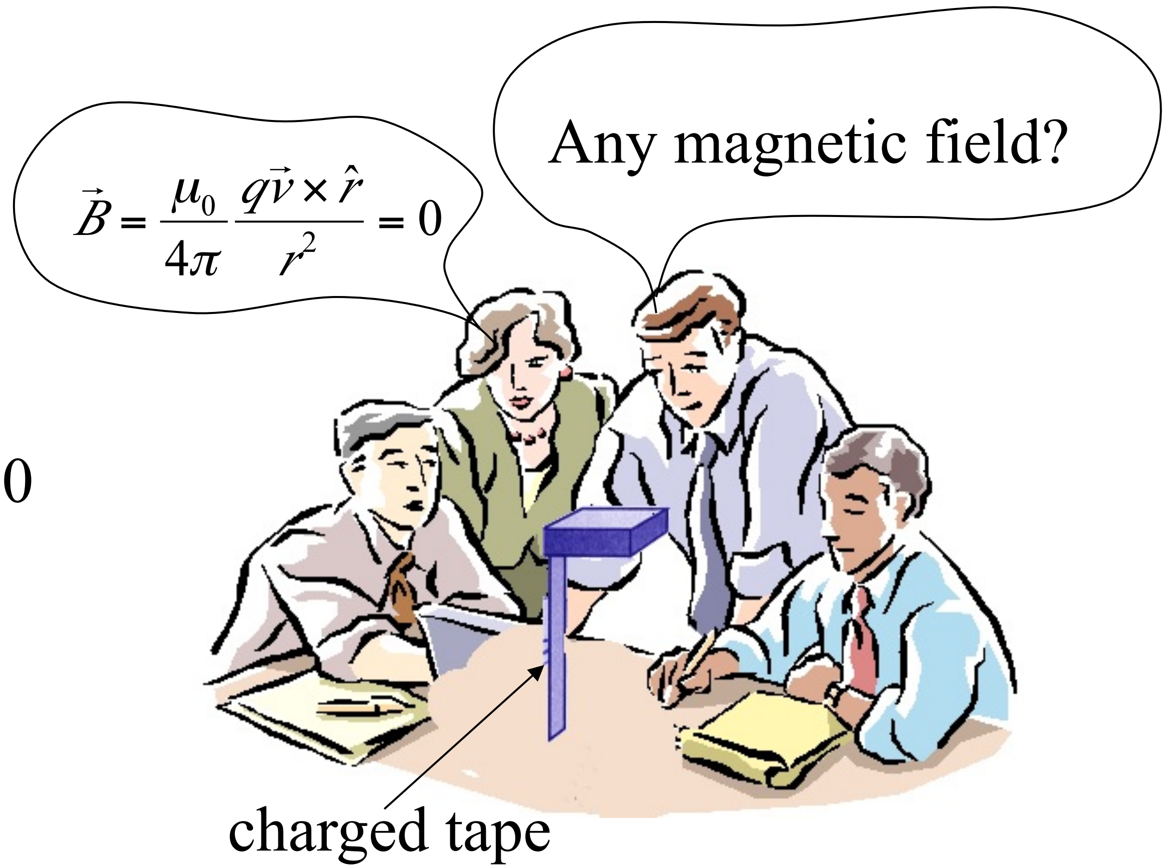
Frame of Reference

Electric fields: produced by charges

Magnetic fields: produced by *moving* charges



$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \neq 0$$



Frame of Reference

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

Must use the velocities of the charges as you observe them in *your reference frame*!

There is a deep connection between electric field and magnetic fields (Einstein's special theory of relativity)

Retardation

If we suddenly change the current in a wire:
Magnetic field will not change instantaneously.

Electron and positron collide:
Produce both electric and magnetic field, these fields exist even after annihilation.

Changes propagate at speed of light

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

Why is there no time in Biot-Savart law?
→ We assume speed of charges is small

Today

- (Cross Products: Mathematically)
- Electron Current and Conventional Current
- Calculating the Electron Current
- True *vs*. Useful
- Biot-Savart Law in a Wire
- Relativity!!

Today

Conventional Current and Electron Current

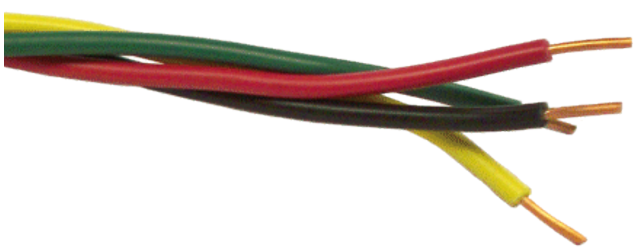
Calculating ConventionalCurrent

Find conventional current in terms of:

$$n = \left[\frac{\text{particles}}{\text{m}^3}\right]$$ Density of Particles in the Wire

$$A = \left[\text{m}^2\right]$$ Cross Sectional Area of the Wire

$$\bar{v} = \left[\frac{m}{s}\right]$$ Average Velocity of Particles in the Wire

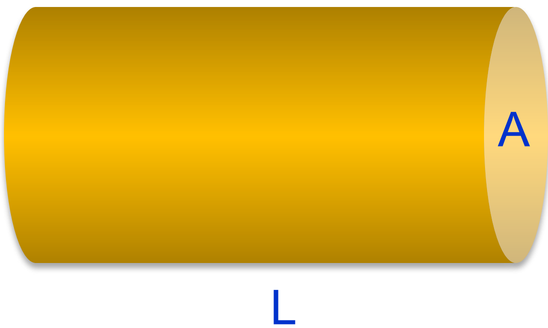


Calculating Conventional Current

$$n = \left[\frac{\text{particles}}{\text{m}^3}\right]$$

$$A = [\text{m}^2]$$ Cross Sectional Area of Wire

$$\bar{v} = \left[\frac{m}{s}\right]$$ Average Velocity of Particles



1. How many Particles are in here?

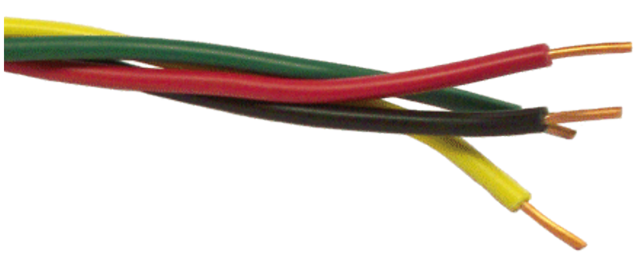
$$N = nV \quad = \left[\frac{\#}{\text{m}^3}\right][m^3] \quad = [\#]$$ ✔ UNITS

$$\boxed{N = nAL}$$

2. Particles are moving at velocity $\bar{v}$
   How long does this take to pass through L?

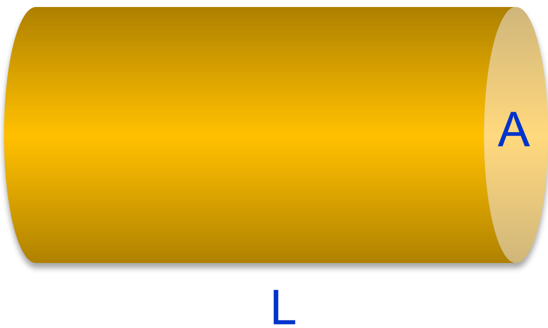
$$L = \bar{v}\Delta t$$

$$\Rightarrow \boxed{\Delta t = \frac{L}{\bar{v}}} = \frac{[m]}{[m/s]} = [s]$$ ✔ UNITS



Calculating Conventional Current

$$N = nAL \qquad \Delta t = \frac{L}{\bar{v}}$$



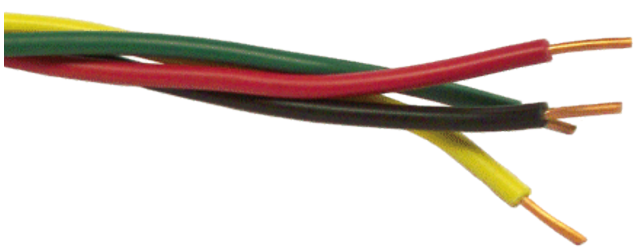
3. How much Charge passes by per second?

$$i = \frac{N}{\Delta t} = \left[\frac{\#}{s}\right]$$

$$= \frac{nA\cancel{L}}{\cancel{L}/\bar{v}} = nA\bar{v}$$

$$= \left[\frac{\#}{\cancel{m}^3}\right]\left[\cancel{m}^2\right]\left[\frac{\cancel{m}}{s}\right] = \left[\frac{\#}{s}\right]$$

✔ UNITS



$$\boxed{i = nA\bar{v}}$$