CS182 **Homework 8 Sketch of Solution** **Spring 2012**

Question 1. (10 points)

$$\begin{aligned} f(n) &= 3f(n/3)+1 = 3(3f(n/3^2)+1)+1 = 3^2 f(n/3^2)+3+1 \\ &= 3^2(3f(n/3^3)+1)+3+1 = 3^3 f(n/3^3)+3^2+3+1 = \cdots \\ &= 3^i f(n/3^i)+3^{i-1}+3^{i-2}+\cdots+3^1+3^0 = 3^i f(n/3^i)+(3^i-1)/(3-1) \end{aligned}$$

For $3^i = n$, this becomes

$$f(n) = nf(1)+(n-1)/2 = (3n-1)/2$$

Question 2. (10 points)

$$\begin{aligned} f(n) &= 3f(n/3)+n = 3(3f(n/3^2)+n/3)+n = 3^2 f(n/3^2)+n+n \\ &= 3^2(3f(n/3^3)+n/3^2)+n+n = 3^3 f(n/3^3)+n+n+n = \cdots = 3^i f(n/3^i)+in \end{aligned}$$

For $3^i = n$, this becomes $f(n) = nf(1)+n\log_3 n = n+n\log_3 n$

Question 3. (10 points) The number of favorable outcomes is $N*(N-1)*\cdots*(N-n+1)$ because, if they are distinct, the number of choices for the first is N, for the second it is $(N-1)$, for the third it is $(N-2)$, etc. The total number of possible outcomes (whether favorable or not) is N^n. Therefore the probability that the a_i's are all distinct is

$$N*(N-1)*\cdots*(N-n+1)/N^n = N!/((N-n)!N^n) = n!C(N,n)/N^n$$

Question 4. (10 points)

1. The net gain is 100 times the following:

   $$2^k-(2^{k-1}+2^{k-2}+\cdots+2^1+2^0) = 2^k-(2^{k-1+1}-1)/(2-1) = 1$$

   which is \$100.

2. The probability that the game ends at round k is 0.5^k, in which case the gain is \$100. Hence the expected gain is:

   $$\sum_{k=1}^{\infty} 100*0.5^k = -100+\sum_{k=0}^{\infty} 100*0.5^k = -100+100*(0-1)/(0.5-1) = 100$$

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