

**Table 12.1 LAPLACE TRANSFORM PAIRS**

<i>Item Number</i>	<i>f(t)</i>	$\mathcal{L}[f(t)] = F(s)$
1	$K\delta(t)$	$K$
2	$Ku(t)$ or $K$	$K/s$
3	$r(t)$	$1/s^2$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{-at}u(t)$	$1/(s+a)$
6	$te^{-at}u(t)$	$1/(s+a)^2$
7	$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin(\omega t)u(t)$	$\frac{\omega}{s^2 + \omega^2}$
9	$\cos(\omega t)u(t)$	$\frac{s}{s^2 + \omega^2}$
10	$e^{-at}\sin(\omega t)u(t)$	$\frac{\omega}{(s+a)^2 + \omega^2}$
11	$e^{-at}\cos(\omega t)u(t)$	$\frac{(s+a)}{(s+a)^2 + \omega^2}$
12	$t\sin(\omega t)u(t)$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
13	$t\cos(\omega t)u(t)$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
14	$\sin(\omega t + \phi)u(t)$	$\frac{s \sin(\phi) + \omega \cos(\phi)}{s^2 + \omega^2}$
15	$\cos(\omega t + \phi)u(t)$	$\frac{s \cos(\phi) - \omega \sin(\phi)}{s^2 + \omega^2}$
16	$e^{-at}[\sin(\omega t) - \omega t \cos(\omega t)]u(t)$	$\frac{2\omega^3}{[(s+a)^2 + \omega^2]^2}$

17	$te^{-at}\sin(\omega t)u(t)$	$2\omega \frac{s+a}{[(s+a)^2 + \omega^2]^2}$
18	$e^{-at} \left[ C_1 \cos(\omega t) + \left( \frac{C_2 - C_1 a}{\omega} \right) \sin(\omega t) \right] u(t)$	$\frac{C_1 s + C_2}{(s+a)^2 + \omega^2}$

**Table 12.2 LAPLACE TRANSFORM PROPERTIES**

Property	Transform Pair
Linearity	$\mathcal{L}[a_1 f_1(t) + a_2 f_2(t)] = a_1 F_1(s) + a_2 F_2(s)$
Time Shift	$\mathcal{L}[f(t-T)u(t-T)] = e^{-sT}F(s), T > 0$
Multiplication by $t$	$\mathcal{L}[tf(t)u(t)] = -\frac{d}{ds}F(s)$
Multiplication by $t^n$	$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n F(s)}{ds^n}$
Frequency Shift	$\mathcal{L}[e^{-at}f(t)] = F(s+a)$
Time Differentiation	$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0^-)$
Second-Order Differentiation	$\mathcal{L}\left[\frac{d^2 f(t)}{dt^2}\right] = s^2 F(s) - sf(0^-) - f^{(1)}(0^-)$
$n$ th-Order Differentiation	$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1}f(0^-) - s^{n-2}f^{(1)}(0^-) - \dots - f^{(n-1)}(0^-)$
Time Integration	<p>(i) <math>\mathcal{L}\left[\int_{-\infty}^t f(q)dq\right] = \frac{F(s)}{s} + \frac{\int_{-\infty}^{0^-} f(q)dq}{s}</math></p> <p>(ii) <math>\mathcal{L}\left[\int_{0^-}^t f(q)dq\right] = \frac{F(s)}{s}</math></p>
Time/Frequency Scaling	$\mathcal{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$