x floop x fdon &

20. For the purposes of Simplicity - I will leave out the global environment & in my informal whicher proof:

Baze Cases:
1. Consider the rule for LITERAL:

(LITERALLY), \$\P\) \(\v,\P,P\)

Examing this role reveals that it can be implemented as follows: pop P, then put P back and the environment strell - after which the only copy is at the top of the stack.

2. Consider the role for FORMAL VAR: X E dom P

(VAR (x), D, P) (P(x), D, P)

We see that this rule can be implemented as pop P, test $x \in dom P$, and compute P(x) — then push P brick ento the environment stack — after which the enty copy is at the top of the shell.

This, our induction hypothesis is that every preof rule ending in the filgrand form Le, ϕ , P > V $\langle V, \phi, P' \rangle$ can be changed to foll P of P Restand and P onto the strik.

Inductive Steps:

Consilor to role for formal assign: X E dom P (e, p, P) 11/(V, p, P') (SET(X, e), p, P) 11/(V, p, P') XI->V3)

By the industrial hospethess, we can evaluate Le & PTULV, P, P)

Using a stack and the evaluation will pop P and policy P' without

making a copy of P. Became P does not appear any here clee

in the rule, it is safe to pop it and discard it. P' is only used all

as a part of the regulate the rule! This, we can suffer a soldy

Conclude that when xis in the domain P, we can soldy evaluate

SET(X, e) and the entiring officially pops P juntoch is

Never used again, and postes P!

Consider the My for IFTRUE: Le, Ø, P> 1/2 LV, Ø, P'> V, ≠0 Le, Ø, P'>1/2 LV, Ø, P"> LIFLE, E, E, E3>, Ø, P> 1/2 LV3, Ø, P">

By the whiten hyphess, we can evaluate (e, 0, P) 11(V, 0, P')

Using a stack and the evaluation will pop P' and Pith P' without

making a copy of P. Because P does not appear injure of the

in the rule, it is safe to pop it and lixed it. We used to

the induction theylothers again to show that the evaluation of ex
can pop p' and puth pl' and p' is not copied. Also, P' is

not used again in the rule appearant for evaluation of P', so

P' may be Is carded firstly or Jee that P' is used only

as as fast of the routh of the rule, They we

Can conclude that when effectively pops P' and reports

Value, we can solely earling If Leigez, e3) on a stack,

and the evaluation effectively pops P' and reports

Never 1979

To Prove for IFFARSE,

(e, φ, P) U(V, φ, P'). V, =0 (e, φ, P') U(V, φ, P') [[(e, φ, P), U(V, φ, P'), U(V, φ, P')]

ine relighber that by IH, we can eminte Le, b, P) ULV, b, P)

Using a stock and the puntinher will peop P and path P' willout "

copying P. Once again, becarse P does not show up ony-love else
in the Nb, it is safe to pay is and prove it away. The IH can be

used again to she that the entrake of e3 can pop P' and pish P".

and P' is not copied. — P' is also not said again in the rule after

the entraken of e3.

Also, P" is only used on fait of the result of the rule.

This, when e, evaluables to D we can safely evaluate IF(0, e2, e3) on
a shock — the evaluables pays P and Poles P".

Now, consider the role WHILEITERATE

Lei, Ø, P) 11, LVI, Ø, P'> VI +0

Loz, Ø, P'> 11/(V2, Ø, P') LUMILE (e., P2), 4, P') 11/(V3, Ø, P'')

LWHILE Lei, P2), Ø, P) 11/(V3, Ø, P'')

As before, we can use the IH to see that it is pegybly to endose Lei, O, P) U, Lvi, D, P') and Lei, O, P'> U, Lvi, O, P'') ving a speck and discarding P and P', respectively. We can also apply the IH to zee that LWHILE (eigel), P, P'') U, Lvi, D, P''') can be contacted on a stack because P'' is rover used again, and can thus to observed when PIII is pull anto the stack. Finally, PIII is only used as for the conclusion that I the route of the route. This leads us to the conclusion that LWHILE (eigel), P, P) can be evaluated on a shock, at the annual after effectively gops P, which is rover used again. The property per P'', then continues this process.

For the Rule WHILEEND

(e1, 0, P) 11, (V1, 0, P') V.=0 (WHILE(01, 01), 0, P) 11, (0,0,P')

By the IH, we know that we can evaluate Le, \$, \$7 \ LVI, 0, \$, \$7 using a special and the evaluation will fish \$ and \$pop \$? without copying \$.

Because \$\beta \text{ low rot affect any-for the in the rule, it is never used again, and can be folked and thrown away.

We also rothe that \$1 is only prount in the result to fine.

The rule, Thosp we can conclude that if experiences to a non-zero subset then it is sufe to sendate WHILE(e, e)

on astrole and the end-other offerthely \$pops \$P\$ and \$porters

I', here using \$P\$ again.

Congiler to rules for BEGIN: The Me for EMPTY BEGIN:

does not require the Timer of to their that Pean be Papped, then produced and LM remains on the log of the stacks

The Rule for BEGIN: Lei, Ø, Po) U (Vi, Ø, Pr) Lea, Ø, Pr) U (Vi, Ø, Pr) Len, Ø, Pr) U (Vin, Ø, Pn)

LBEGIN (eijez,..., en), PPOT U, Vn, &, Pn)
We say by the IH, that each expression of the form Lex, Ø, Px) U, LVx, Ø, Px
can be evaluable usly a speck - discarding Px and pushing Px', The
Who is evaluable - Pn Only affers in the result of the rule.
Thus, we can conclude that bottom five evaluate BEGIN (ei...en) for
enth expression en, it is sole to log Pn and I need it , then A push
Pn',

Centiler the Ne APPLYUSER: P(F)= USER (Lx, ..., xn), e) Xii. , Xn all datend (e, d, PU) 1/ (v, d, P) Len Ø, Pn. > U (Vn, Ø, Pn) Le, Ø, ξX, → Vi, ... Xn → Vn3) 11, ∠V, Ø, P) ∠APPLY (F,e, ... en), Ø, Po) 11, ∠V, Ø, Pn) From the IH, we can evaluate any exprossion of the form (end, Pa-1) U (Vn, Q, Pn) using a stack - Popping Pn, diseady is, and the power for the last line of the promote Le, 0, {x, tov, ... xntovn})

U(1, 0, p) is also of the form - a new p is ported onto the

stack and the older P is popped and discorded. Thus, we can gifely conclude that we can gutely evaluate APPLY(F, el., en) on a 14 tack and the evaluable effectively pops each Pr and pries Pr - never wy Pr again Lustly, consily the who APPLY ADD (While LM 265194 for all formings) O(F) = PRIMITIVE (+) Le, 0, Po> y Lv, 0, Pi) Lez, 0, P.) U (V1, 16, P2) LAPPLY(Fie, ex), Q, Po) 1 LV, +V1, Q, P27

By the IH, both of the expressions in the Premie can be evaluated using a strick - Lei, P, Po II (Vi, P, P) I pape Po and pulse Pi, directly Po in the Process, hile Lea, P, Pi V (Vi, P, P) pape Pi and publis Pd Jirearly Pa in the Process, Finally, he see that Pd is only used in the regulat of the rule. We can this conclude that in the case of Primitive APPLYS, he can solely qualitate APPLY(F, e, e) using a struck - and the arrange of the process P and popes P and popes P2 - directory P,

From this femon, we can sufely say that the post of post of of former can be replaced by the example, "mutake p in place to become P!"

13a. AWK X & dom P X & dom Z VAR(x), Z, P, P) U, LO, Z, Z, D, P) X & dom S Cer, E, B, P) U LV, E, D, P') L SET (x,c), Z, D, P) U LV, E' EXHV3, P, P') L SET (x,c), Z, D, P) U LV, E' EXHV3, P, P')

LSET(x,e), \(\frac{2}{2}, \phi, \beta\) \(\frac{1}{2}\) \(\frac{1}\) \(\frac{1}{2}\) \(\frac{1}{2}\) \(\frac{1}{2}\) \(\frac{1

C. I prefer the charge to I(ON-like Sementics because it seems mere logical to want to both any newly introduced variables as focal translates within as given procedure. In the AMK-like conseque any created variable can be changed by any procedure or function; which seems produced and unsafe. The ICON-like nules make it very easy to use temporary variables within functions—which to very useful.

LSETC VARLX), LITERAL (3)), &, Ø, P) U, L3, É, Ø, P'EX+33) X & don P P'(x)=3 FORMALWAR LVAR 1 LBEGIN (SET LVARW), LITC3)), VARW), £, Ø, 87 U, L3, £', Ø, 8') 11. (VARLY), 2, 087 11, (84), 2, 0, 8)

10. (VARLY), 2, 0, P) 11 (VO2, 0, 8') V +0 (VARLY), 2, 0, 8') (VI, 2", 0, 9")

[IF (NARLY), VARLX), LIT(0)), 2, 0, P) 11 (VI, 2", 0, 8") 2. (VARCE), & , P, 874 (V), 2", P, 8") X E JAM P (VAR(x), E, Ø, 8) y (84), E, Ø, 8) 16. LVMU) 2, 0,8> U (Vo, 8,0,8) V=0 (LITCO), 8,0, 8) U (V, 8",0,8) IE(VARCA), VARCA), LITCO)), 2, 0, P>11, LV1, 2", 0, P") If N. 70, Hon la = 2 V1 = V2

If 1 = 0 / then 16 = 2