PHYS 172 Problem of the Week #4 Solution – Spring 2012

A spring has a relaxed length of 12 cm. You suspend it vertically and hang a 35 gram mass from it, and you observe that with the mass hanging motionless the spring is 19 cm long. You then pull down on the mass until the spring is 22 cm long and you release the mass without giving it any initial speed.

a. How long does it take the mass to make a round trip, up and back down?
System: mass + spring, Surroundings: Earth

When the mass is hanging motionless with the spring length 19 cm, the system is in equilibrium. Therefore the momentum of the mass-spring system is zero, and the rate of change of the mass-spring system is zero as well.

With the mass-spring system motionless, the Momentum Principle gives:

$$\Delta\vec{p} = \vec{F}_{net}\Delta t = 0$$

Therefore the net force must equal zero. This is essentially a one-dimensional problem, so we need only consider the y-direction.

$$\vec{F}_{net} = \langle 0, (k\Delta y - Mg), 0 \rangle = \langle 0, 0, 0 \rangle$$

where k is the spring constant and Δy is the displacement of the spring from its equilibrium (relaxed length). Therefore

$$\frac{k}{M} = \omega^2 = \frac{g}{\Delta y}$$

The period is related to the natural frequency by

$$T = \frac{2\pi}{\omega}$$

For simple harmonic motion, the period does not depend on the amplitude of the motion!

Putting in numbers, we find that $T = 0.53\ s$ and $k = 4.9\ N/m$.

b. You stop the oscillation, you pull down on the mass until the spring is 25 cm long, and you release the mass without giving it any initial speed. Now how long does it take the mass to make a round trip, up and back down?

Because the period of oscillation does not depend on the amplitude of the motion, a round trip still will take 0.53 s.

[Note: a simple pendulum has a period of oscillation that, for small angular displacements from the vertical, does not depend on the amplitude of the motion. However, the period for amplitudes that approach 45 degrees will depend on the initial angular displacement. This system is no longer in the linear (small angle) regime.

c. When the mass returns to the bottom (so again the spring is 25 cm long), the mass is momentarily at rest before continuing its oscillation. What is the magnitude of the net force at this instant?

In contrast to the situation in which the mass-spring system was in equilibrium, now it is not. At the bottom of the motion, the system's momentum is zero, **but its rate of change of momentum is not zero!**

Applying the momentum principle:

$$\frac{\Delta\vec{p}}{\Delta t} = \vec{F}_{net} = \langle 0, (k\Delta y - Mg), 0 \rangle$$
$$\Delta y = .25 - .12 = 0.13\ m$$
$$\vec{F}_{net} = \langle 0, 0.29, 0 \rangle\ N$$

d. You cut the spring into two equal lengths, each with relaxed length 6 cm. You take one of these half-length springs and hang the 35 gram mass from it. You then pull down on the mass until the spring is 9 cm long and you release the mass without giving it any initial speed. How long does it take the mass to make a round trip, up and back down?

When we halve the length of the original spring, we reduce the number of atomic bonds by a factor of two. This **increases** the macroscopic spring stiffness by a factor of two. Thus, the new natural frequency is given by:

$$\omega_{new}^2 = \frac{k_{new}}{M} = \frac{2k_{old}}{M} = 2\omega_{old}^2$$

Therefore the new period is: $T_{new} = \frac{T_{old}}{\sqrt{2}} = 0.375\ s$