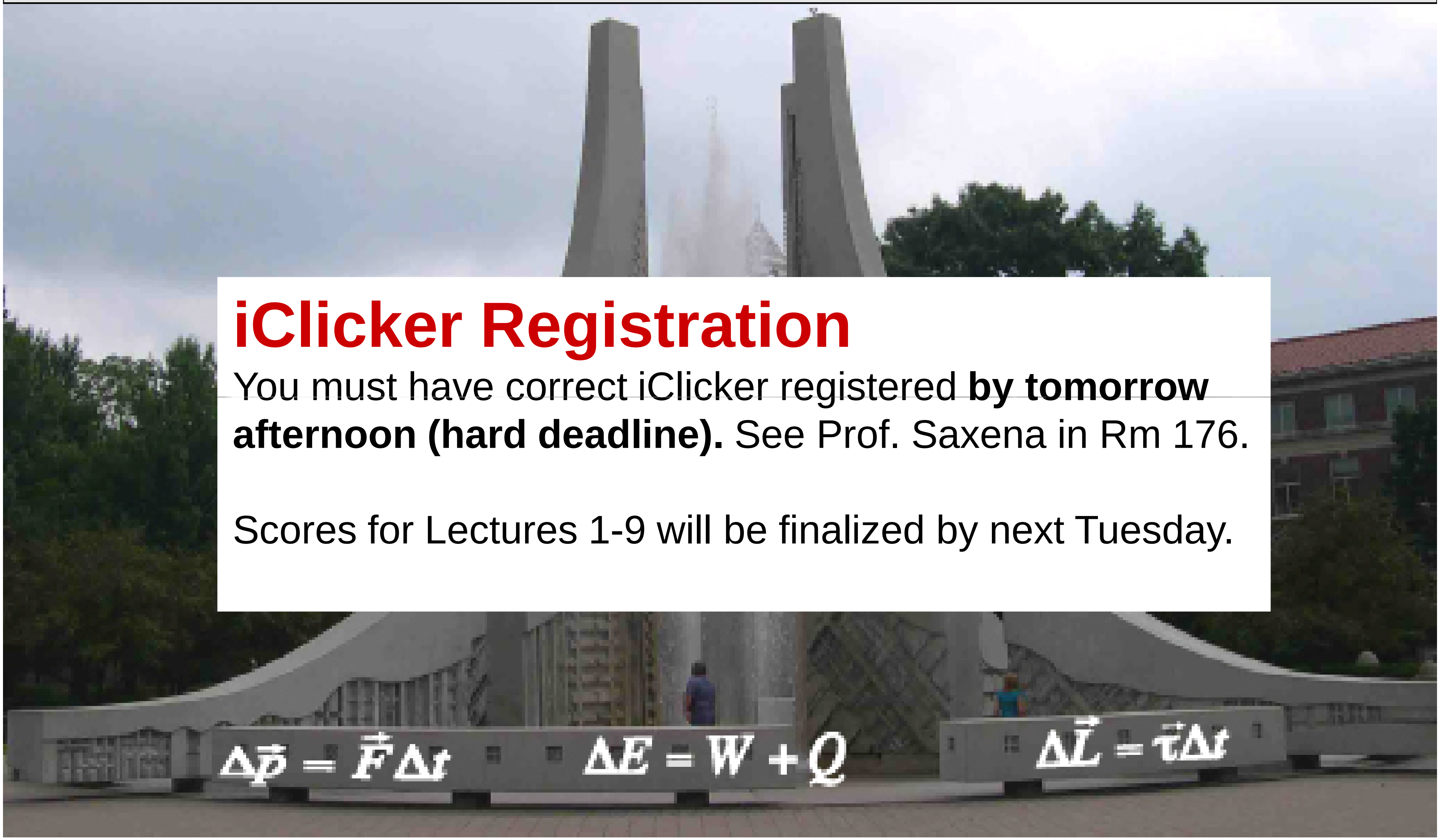


iClicker Registration

You must have correct iClicker registered **by tomorrow afternoon (hard deadline).** See Prof. Saxena in Rm 176.

Scores for Lectures 1-9 will be finalized by next Tuesday.

TODAY

- Multiparticle Systems and Potential Energy
- Relationship of Force and Potential Energy
- Energy Graphs

Last Time: Single Particle System

Energy principle (single particle system):

$$\Delta E_{\text{single particle system}} = W + \cancel{Q}$$

= 0 for now

where energy is

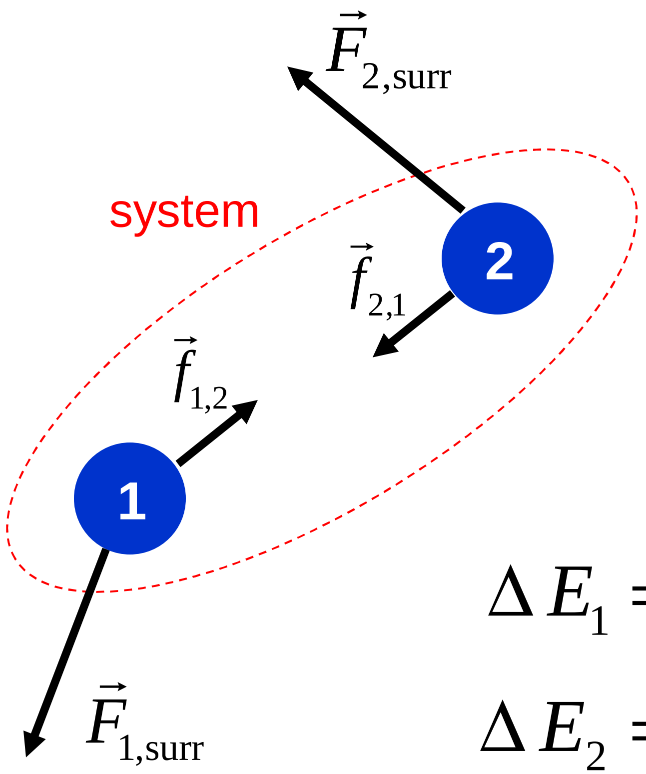
$$E_{\text{single particle system}} = \gamma mc^2 = \frac{mc^2}{\sqrt{1-v^2/c^2}}$$

and work is

$$W_{\text{on particle}} = \vec{F}_{\text{net on particle}} \bullet \Delta\vec{r}$$

How do we generalize these results to multiparticle systems?

Example: Energy in 2-Particle System



$$\Delta E_{\text{system}} = W + \underbrace{\cancel{Q}}_{= 0}$$

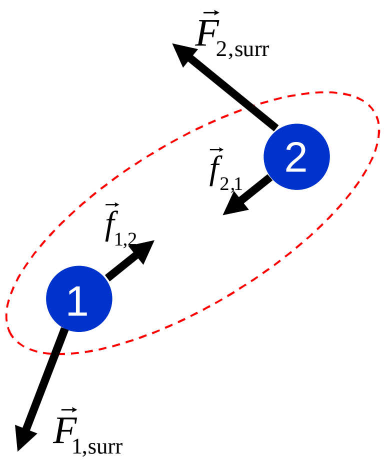
Let's find the energy change of each particle:

$$\Delta E_1 = \underline{\vec{f}_{1,2} \bullet \Delta \vec{r}_1} + \underline{\vec{F}_{1,\text{surr}} \bullet \Delta \vec{r}_1} = \underline{W_{1,\text{internal}}} + \underline{W_{1,\text{surr}}}$$

$$\Delta E_2 = \vec{f}_{2,1} \bullet \Delta \vec{r}_2 + \vec{F}_{2,\text{surr}} \bullet \Delta \vec{r}_2 = W_{2,\text{internal}} + W_{2,\text{surr}}$$

We're counting the work done by internal forces.

Example: Energy in 2-Particle System



$$\text{Thus } \Delta\left(\mathrm{E}_1 + \mathrm{E}_2\right) = W_{\mathrm{int}} + W_{surr} + \cancel{Q}$$

$$\text{where} \quad \mathrm{W}_{\mathrm{int}} = W_{1,\mathrm{internal}} + W_{2,\mathrm{internal}}$$

$$\text{and} \quad \mathrm{W}_{\mathrm{surr}} = W_{1,\mathrm{surr}} + W_{2,\mathrm{surr}}$$

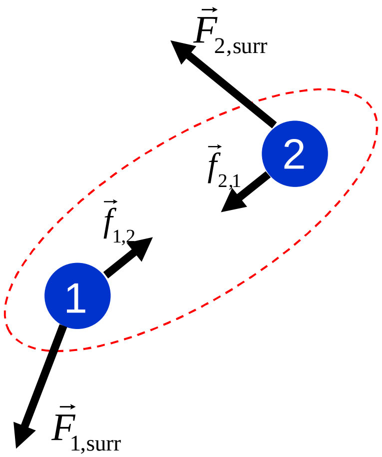
Put system on left side, surroundings on right side:

$$\Delta\left(\mathrm{E}_1 + \mathrm{E}_2\right) - W_{\mathrm{int}} = W_{surr} + Q$$

Now <u>define</u> the change in potential energy as $\Delta U \equiv -W_{\mathrm{int}}$:

$$\Delta\left(\mathrm{E}_1 + \mathrm{E}_2\right) + \Delta U = W_{surr} + Q$$

Potential Energy (in 2-Particle System)



$$\Delta U = -W_{\mathrm{int}} = -\vec{f}_{1,2} \bullet \Delta \vec{r}_1 - \vec{f}_{2,1} \bullet \Delta \vec{r}_2$$

The potential energy U represents a sum of interaction energies between all pairs of particles inside the system.

NOTE: i) U is defined to take into account both terms above.

ii) we choose U=0 for infinite separation

Question: If $\vec{f}_{1,2} + \vec{f}_{2,1} = 0$, why isn't ΔU = 0 here?

Answer: Usually, $\Delta r_1 \neq \Delta \vec{r}_2$

ΔU is related to a system changing shape.

Energy of a Multiparticle System

Energy of system = sum of single particle energies + sum of interaction energies of all pairs

$$\boxed{E_{sys} = \underbrace{\left(m_1c^2 + m_2c^2 + \ldots\right) + \left(K_1 + K_2 + \ldots\right)}_{\text{sum of single particle energies}} + \underbrace{\left(U_{12} + \ldots\right)}_{\text{sum of interaction energies of all pairs}}}$$

We now write $\boxed{\Delta E_{sys} = W_{surr} + Q}$

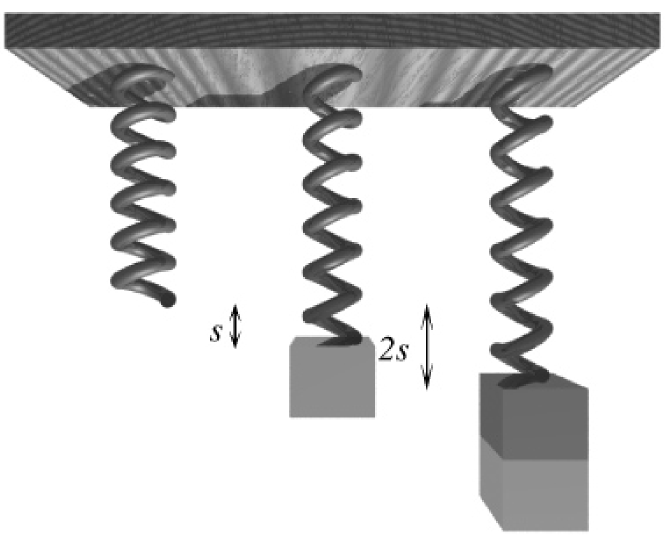
Now W is about external forces only

(internal forces show up in U).

Energy of a Multiparticle System

$$\boxed{\Delta E_{sys} = W_{surr} + Q}$$

W_{surr} is about external forces only

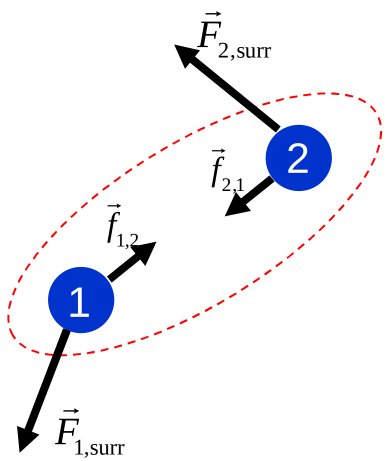


Add a mass to the spring.
How much work does the gravitational force do on the mass?

What if the mass oscillates before coming to equilibrium?

Connection: Force and Potential Energy

$$\Delta U = -W_{\text{int}} = -\vec{f}_{2,1} \bullet \Delta\vec{r}_2 - \vec{f}_{1,2} \bullet \Delta\vec{r}_1 \cdot$$



$$= -\vec{f}_{2,1} \bullet \left(\Delta\vec{r}_2 - \Delta\vec{r}_1\right)$$

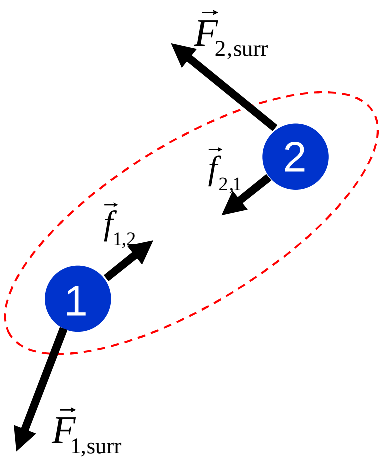
$$= -\underbrace{\vec{f}_{2,1} \bullet \Delta\vec{r}}$$ where $\vec{r} = \vec{r}_2 - \vec{r}_1$

$\vec{f}_{1,2} = -\vec{f}_{2,1}$
Equal and opposite

The combination is independent
of coordinate system.

Thus $\vec{f}_{2,1} = -\dfrac{\Delta U}{\Delta\vec{r}}$ (gradient) $\rightarrow$ $\boxed{f_r = -\dfrac{dU}{dr}}$

Connection: Force and Potential Energy



$$\boxed{f_r = -\frac{dU}{dr}}$$

For Gravity:

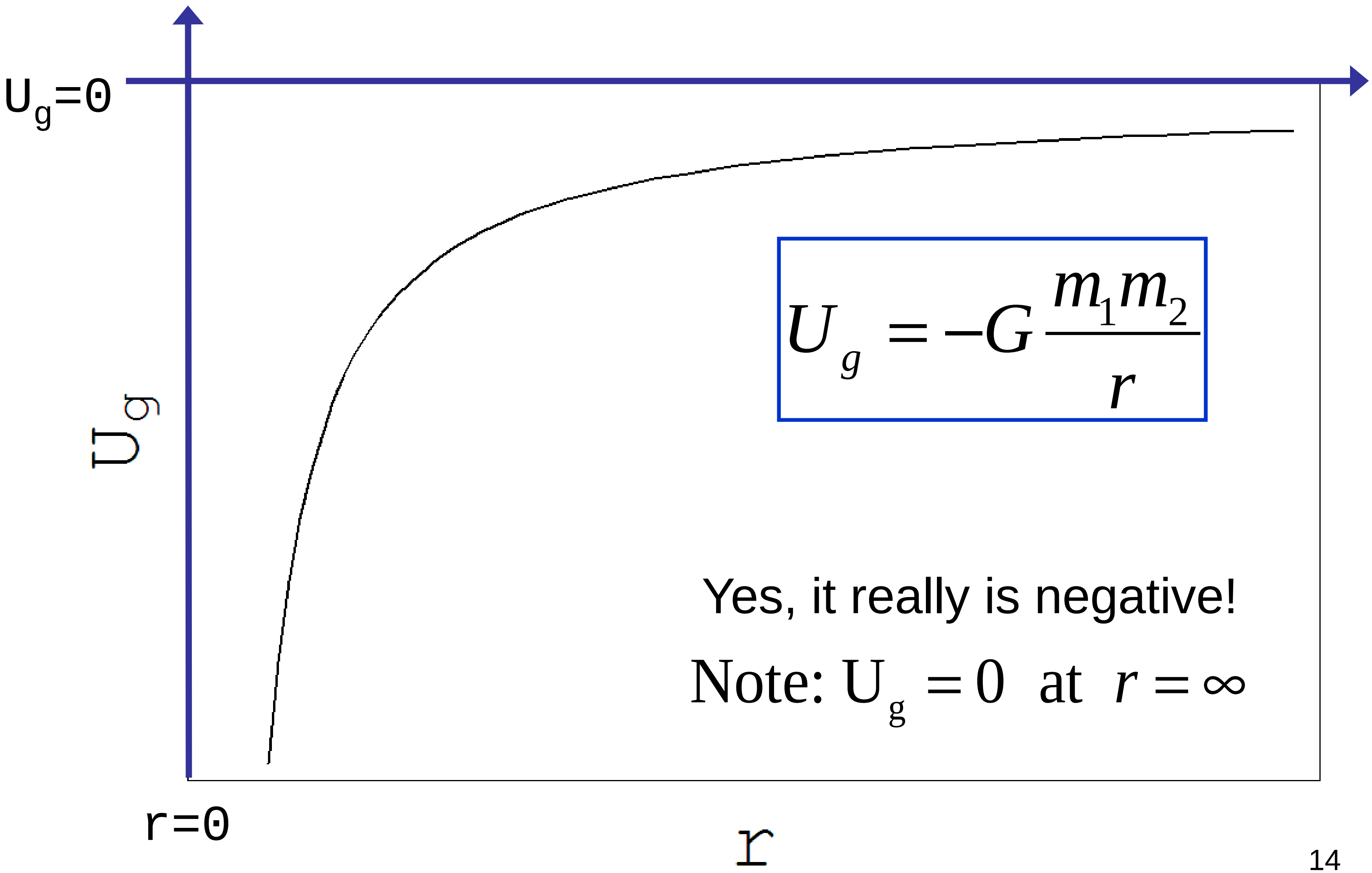
$$f_r = -G\frac{m_1 m_2}{r^2} \quad \Leftrightarrow \quad U_g = -G\frac{m_1 m_2}{r}$$

To see this:

$$f_r = -\frac{dU}{dr} = -\frac{d}{dr}\left(-G\frac{m_1 m_2}{r}\right)$$

$$= G m_1 m_2 \frac{d}{dr}\left(\frac{1}{r}\right) = G m_1 m_2\left(\frac{-1}{r^2}\right) = -G\frac{m_1 m_2}{r^2}$$

Gravitational Potential Energy



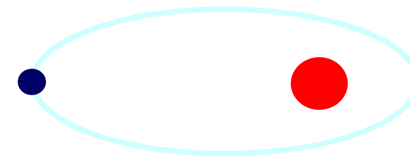
$$U_g = -G\frac{m_1 m_2}{r}$$

Yes, it really is negative!

Note: $U_g = 0$ at $r = \infty$

Example: Planet and Star

System: planet+star



$$\Delta \mathrm{E}_{\mathrm{sys}} = W_{surr} + Q = 0$$

$$\Delta\left[E_{particles} + U_{system} \right] = 0 \quad \rightarrow$$

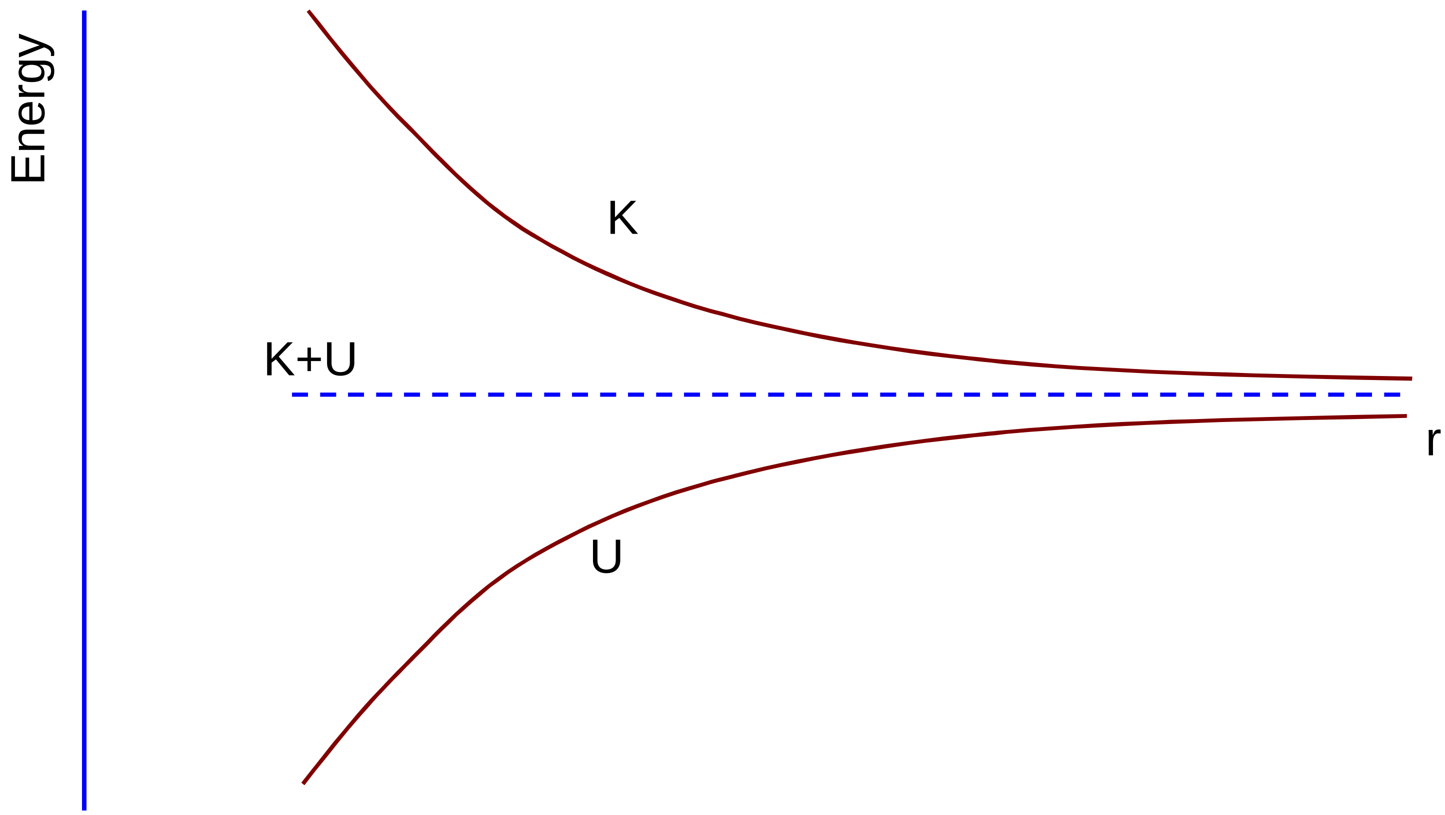
$$\Delta\left[m_{star}c^2 + K_{star} + m_{planet}c^2 + \underbrace{K_{planet} + U_{system}}_{\text{So these together must be constant}} \right] = 0$$

Each of these is constant

$$\boxed{\mathrm{K}_{\mathrm{planet}} + U_{system} = const}$$

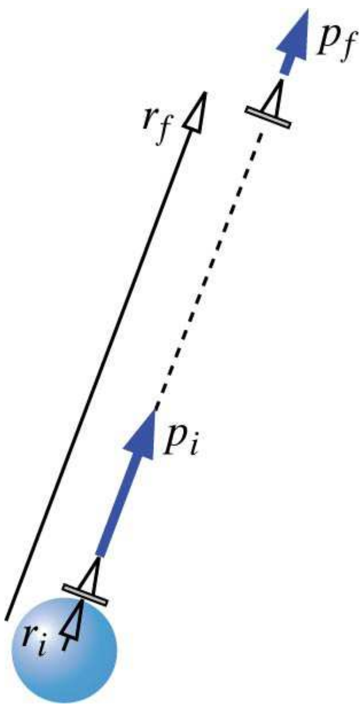
Example: Planet and Star

$$\boxed{K_{\text{planet}} + U_{system} = const}$$



Application: Escape Speed

What does it take to launch a rocket so it leaves the Earth's gravitational well?



Minimal condition for escape: $K + U = 0$

Assume: kinetic energy of a planet is negligible, *v*<<*c*

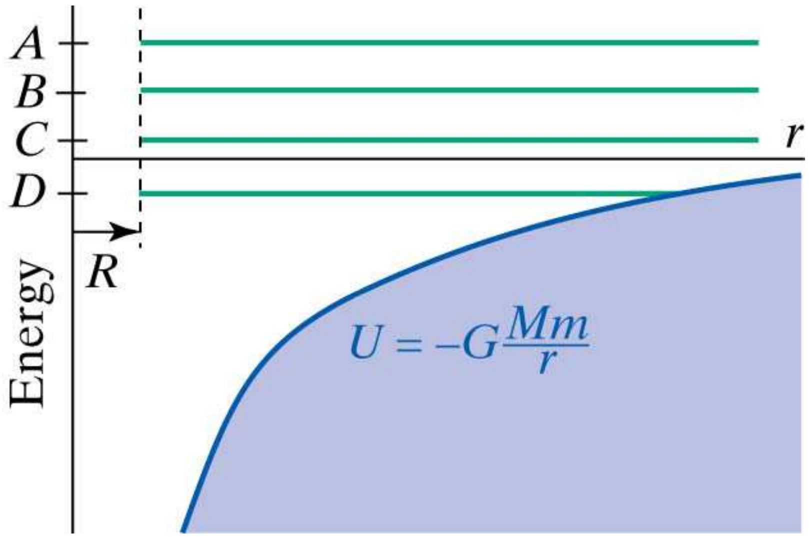
$$K_i + U_i = \frac{mv_{esc}^2}{2} + \left[-G\frac{Mm}{R} \right] = 0$$

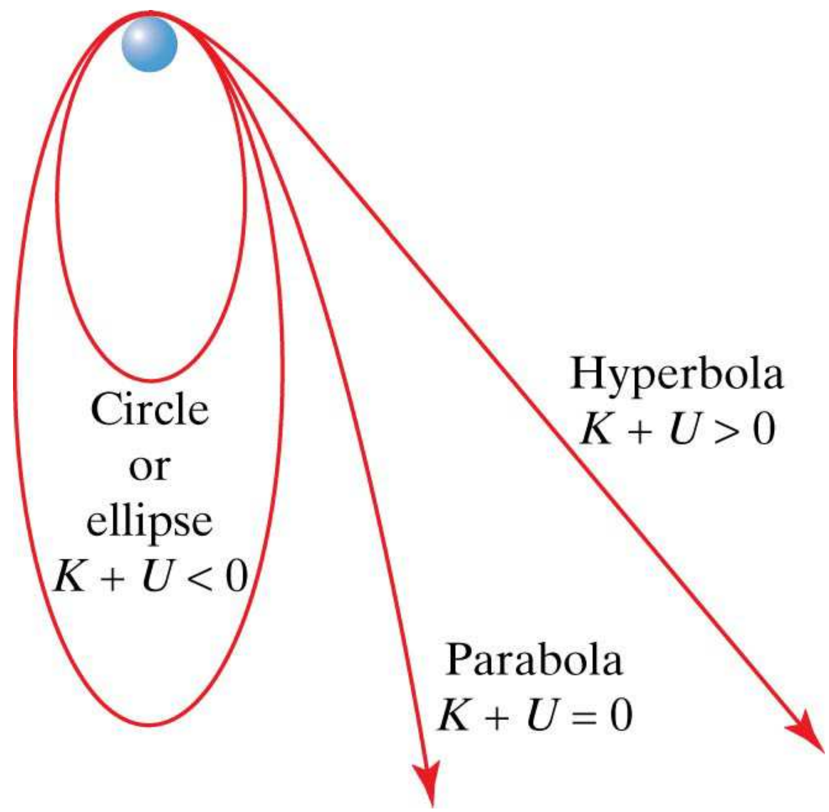
$$\frac{mv_{esc}^2}{2} = G\frac{Mm}{R}$$

$$\boxed{v_{esc} = \sqrt{\frac{2GM}{R}}}$$

Application: Escape Speed

What does it take to launch a rocket so it leaves the Earth's gravitational well?





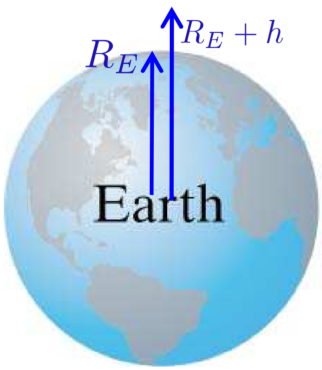
Bound state: $K + U < 0$

Unbound state: $K + U \geq 0$

Gravitational U Near Earth's Surface

$$U = -G\frac{Mm}{r}$$ ← **Are these the same?** → $$U = mgh$$

They are the same near the Earth's Surface.



$$\frac{1}{1+x} \approx 1 - x + \ldots$$

Taylor Expansion

$$\Delta U = -G\frac{Mm}{R_E + h} - \left(-G\frac{Mm}{R_E}\right)$$

$$= -G\frac{Mm}{R_E}\left(\frac{1}{1+h/R_E} - 1\right)$$

$$\approx -G\frac{Mm}{R_E}\left(1 - h/R_E - 1\right)$$

$$= m\left(\frac{GM}{R_E^2}\right)h \quad \equiv \boxed{mgh = \Delta U}$$

$$g = GM/R_E^2$$

WHAT WE DID TODAY

- Multiparticle Systems and Potential Energy
- Relationship of Force and Potential Energy
- Energy Graphs