Purdue University
School of Electrical and Computer Engineering

ECE 20200 : Linear Circuit Analysis II

Summer 2013
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Midterm Examination I
June 26, 2013

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet.
2. Enter your name, student ID number, e-mail address and your full signature in the space provided on this page.
3. You have 90 minutes to complete all **5** questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
4. Read the questions carefully. Unless otherwise stated, you must fully justify your answers. You may use any method you want unless you are asked to use a specific method.
5. This booklet contains **14** pages including the Laplace Transform Tables. Since only this booklet will be graded, make sure you have all your answers written in this booklet.
6. Notes, books, calculators, cell phones, pagers and any other electronic communication device are strictly forbidden.

Name:____

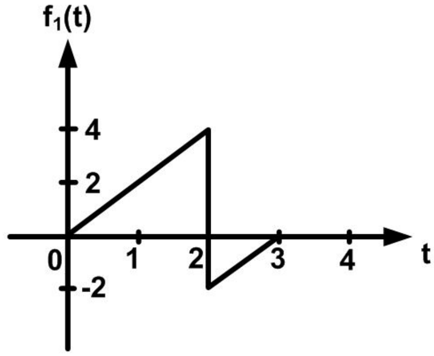
Student ID:____

Email:____

Signature:____

(Total 20 pts) **1.**

(a)



(i) (3 pts) Represent $f_1(t)$ using sums of steps, ramps and shifts of basic signals.

(ii) (2 pts) Find the Laplace transform of $f_1(t)$.

(b) (3 pts) Find the Laplace transform of $f_2(t) = [(t^2+1) + (t^2-1)]\,[\delta(t+1) + \delta(t-1)]$.

(c) (6 pts) Find the inverse Laplace transform of the function

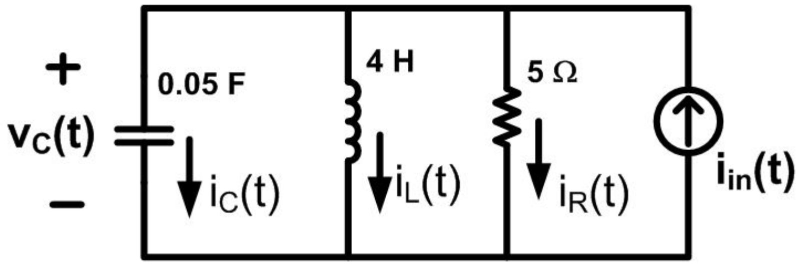
$$F_3(s) = \frac{10s + 40 - (20s + 40)e^{-s}}{s^3 + 6s^2 + 8s}.$$

(d) (6 pts) Find the inverse Laplace transform of the function

$$F_4(s) = \frac{3s}{(s+3)^2} + \frac{2s-3}{s^2+4}.$$

(Total 20 pts) **2.**

(a) Consider the parallel RLC circuit given below. Suppose the initial conditions are $i_L(0^-) = 1\text{A}$ and $v_C(0^-) = 1\text{V}$.



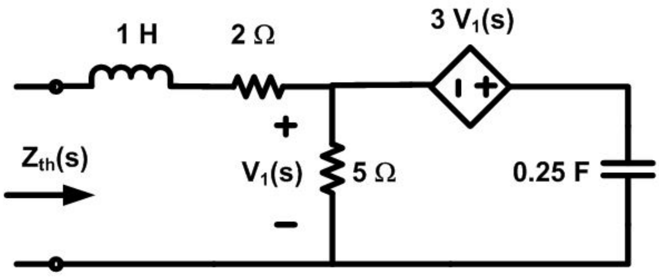
(i) (2 pts) Show that the integro-differential equation of the circuit is given by

$$\frac{1}{5}v_c(t) + \frac{1}{20}\frac{dv_c(t)}{dt} + \frac{1}{4}\int_{-\infty}^{t} v_c(q)dq = i_{in}(t)$$

(ii) (7 pts) By taking Laplace transform of the equation obtained in part (i), find $V_c(s)$ if $i_{in}(t) = u(t)$ A.

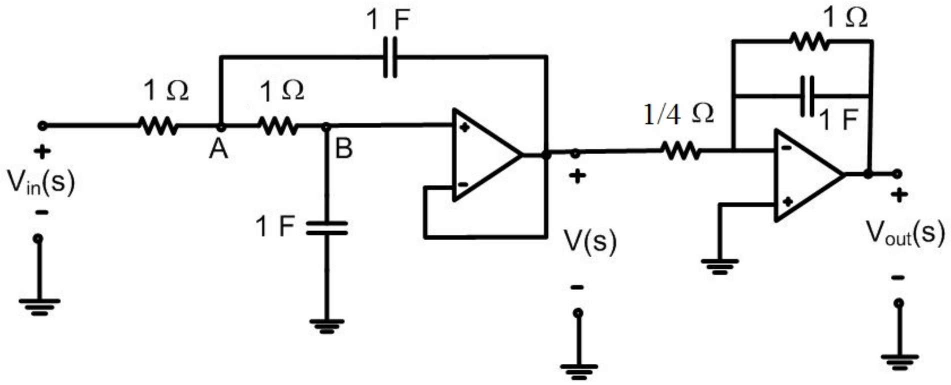
(iii) (4 pts) Find $v_c(t)$.

(b) (7 pts) Compute the Thevenin equivalent impedance of the following circuit.



(Total 15 pts) **3.**

(a) (12 pts) Compute $H_1(s) = \dfrac{V(s)}{V_{in}(s)}$ and $H_2(s) = \dfrac{V_{out}(s)}{V(s)}$ using nodal analysis, basis op amp properties and/or the transfer function formula of an inverting op amp amplifer. Then find $H(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$.

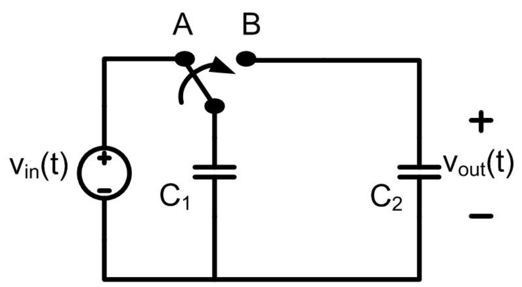


(b) (3 pts) Find the impulse response.

(Total 20 pts) **4.**

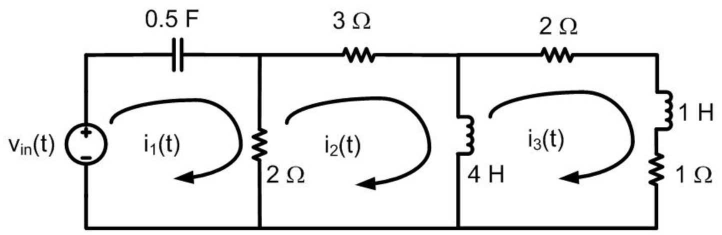
(a) (4 pts) The Laplace transform of a capacitor voltage is given by $V_C(s) = \frac{2}{s} - \frac{1}{2s+5}$. Find the initial capacitor voltage $v_C(0^+)$ using the Initial Value Theorem.

(b) (6 pts) Consider the circuit given below. The switch moves from position A to position B at $t = 2$ s, back to position A at $t = 4$ s, and then back to position B at $t = 6$ s, where it remains forever.



If $C_1 = 1$ F, $C_2 = 4$ F, $v_{out}(0^-) = 0$, and $v_{in}(t) = 25u(t)$ V, find $v_{out}(t)$ at $t = 7$ s.

(c) Consider the circuit below. $v_{in}(t)$ is given as $u(t)$. Assume all the initial conditions to be zero.

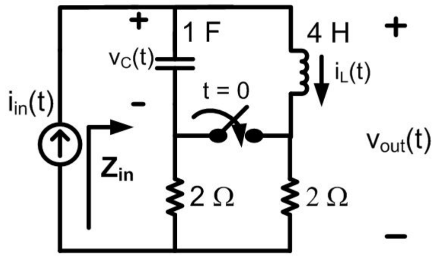


(i) (6 pts) Write down the loop equations in s-domain.

(ii) (2 pts) Write the equations obtained in part (i) in matrix form.

(iii) (2 pts) Write down the formula you would use to solve $I_2(s)$ using Cramer's Rule.

(Total 25 pts) **5.** Consider the circuit given below. Suppose $i_{in}(t) = 10u(-t) + 10u(t)$.



(a) Compute $v_C(0^-)$ and $i_L(0^-)$. (2 pts)

(b) Compute $Z_{in}(s)$ for the case of $t < 0$. (3 pts).

(c) Compute $Z_{in}(s)$ for the case of $t \geq 0$. (3 pts).

For parts (d), (e) and (f), if you do not have answers for part(a), then assume $v_C(0^-) = \mathbf{2}$ V and $i_L(0^-) = \mathbf{1}$ A and proceed to obtain partial credits.

(d) Draw the equivalent s-domain circuit for $t \geq 0$. Use your results from part (a) and use only **CURRENT SOURCE MODELS** of charged capacitor/inductor. (3 pts)

(e) Compute the zero-input response for $t \geq 0$ assuming the input current is zero. (7 pts)

(f) Compute the zero-state response for $t \geq 0$ assuming all initial conditions are zero. (7 pts)

Table 12.1 LAPLACE TRANSFORM PAIRS

Item Number	$f(t)$	$\mathcal{L}[f(t)] = F(s)$
1	$K\delta(t)$	K
2	$Ku(t)$ or K	K/s
3	$r(t)$	$1/s^2$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{-at}u(t)$	$1/(s+a)$
6	$te^{-at}u(t)$	$1/(s+a)^2$
7	$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin(\omega t)u(t)$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos(\omega t)u(t)$	$\frac{s}{s^2+\omega^2}$
10	$e^{-at}\sin(\omega t)u(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
11	$e^{-at}\cos(\omega t)u(t)$	$\frac{(s+a)}{(s+a)^2+\omega^2}$
12	$t\sin(\omega t)u(t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
13	$t\cos(\omega t)u(t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
14	$\sin(\omega t+\phi)u(t)$	$\frac{s\sin(\phi)+\omega\cos(\phi)}{s^2+\omega^2}$
15	$\cos(\omega t+\phi)u(t)$	$\frac{s\cos(\phi)-\omega\sin(\phi)}{s^2+\omega^2}$
16	$e^{-at}[\sin(\omega t)-\omega t\cos(\omega t)]u(t)$	$\frac{2\omega^3}{[(s+a)^2+\omega^2]^2}$

17	$te^{-at}\sin(\omega t)u(t)$	$2\omega\dfrac{s+a}{[(s+a)^2+\omega^2]^2}$
18	$e^{-at}\left[C_1\cos(\omega t)+\left(\dfrac{C_2-C_1a}{\omega}\right)\sin(\omega t)\right]u(t)$	$\dfrac{C_1s+C_2}{(s+a)^2+\omega^2}$

Table 12.2 LAPLACE TRANSFORM PROPERTIES

Property	**Transform Pair**
Linearity	$\mathcal{L}[a_1f_1(t)+a_2f_2(t)] = a_1F_1(s)+a_2F_2(s)$
Time Shift	$\mathcal{L}[f(t-T)u(t-T)] = e^{-sT}F(s),\ T>0$
Multiplication by t	$\mathcal{L}[tf(t)u(t)] = -\dfrac{d}{ds}F(s)$
Multiplication by t^n	$\mathrm{L}[t^n f(t)]=(-1)^n\dfrac{d^nF(s)}{ds^n}$
Frequency Shift	$\mathcal{L}[e^{-at}f(t)] = F(s+a)$
Time Differentiation	$\mathcal{L}\left[\dfrac{d}{dt}f(t)\right] = sF(s)-f(0^-)$
Second-Order Differentiation	$\mathrm{L}\left[\dfrac{d^2f(t)}{dt^2}\right]=s^2F(s)-sf(0^-)-f^{(1)}(0^-)$
nth-Order Differentiation	$\mathrm{L}\left[\dfrac{d^nf(t)}{dt^n}\right]=s^nF(s)-s^{n-1}f(0^-)-s^{n-2}f^{(1)}(0^-)$ $-\ldots-f^{(n-1)}(0^-)$
Time Integration	(i) $\mathrm{L}\left[\int_{-\infty}^{t}f(q)dq\right]=\dfrac{F(s)}{s}+\dfrac{\int_{-\infty}^{0^-}f(q)dq}{s}$ (ii) $\mathrm{L}\left[\int_{0^-}^{t}f(q)dq\right]=\dfrac{F(s)}{s}$
Time/Frequency Scaling	$\mathcal{L}[f(at)] = \dfrac{1}{a}F\left(\dfrac{s}{a}\right)$