CS381 **Homework 2** **Fall 2013**

Question 1. (25 points) Recall that we covered in class the "simultaneous minimum and maximum" problem described in Section 9.1 of the textbook. This question explores a different algorithm for that problem.

1. (10 points) Assuming n is a power of 2, write down a recursive divide-and-conquer algorithm for solving the "simultaneous minimum and maximum" problem described in the second half of Section 9.1. Make sure you give enough details that, based on your description, any competent programmer could easily implement it.
2. (5 points) If we let $T(n)$ denote the number of comparisons done by your algorithm, write down the recurrence relation satisfied by $T(n)$.
3. (10 points) Solve exactly (without using the "big O" notation) the recurrence relation for $T(n)$, by
   (a) using the algebraic method described in class to "guess" what the solution is (repeatedly using the recurrence until a pattern appears), and then
   (b) proving the correctness of your answer by induction.

   How does your answer compare to the number of comparisons taken by the non-recursive algorithm covered in class?

Question 2. (25 points) Given a set S of n distinct integers, we want to find the k smallest in sorted order. Give an $O(n + k \log k)$ time algorithm for doing this.

Question 3. (25 points) Given n distinct integers $y_1, \ldots, y_n$ (not in sorted order), we want to find an integer y that minimizes the following quantity:

$$\sum_{i=1}^{n} |y - y_i|$$

where $|y - y_i|$ denotes the absolute value of $y - y_i$. Give an $O(n)$ time algorithm for finding such a y, and prove the correctness of your algorithm.

Question 4. (25 points) Let A be an $n \times n$ matrix of n^2 arbitrary numbers (positive or negative), and let $\mathcal{R} = \{R_1, \ldots, R_m\}$ be a set of $m = n^2$ rectangular regions in A where each R_i is specified by 2 row indices i_1, i_2 ($i_1 \leq i_2$) and two column indices j_1, j_2 ($j_1 \leq j_2$); therefore the 4 corners of rectangle R_i are the matries entries $A[i_1, j_1]$, $A[i_1, j_2]$, $A[i_2, j_1]$, $A[i_2, j_2]$. Let r_i denote the sum of the entries of A that are covered by rectangle R_i:

$$r_i = \sum_{i_1 \leq k \leq i_2, j_1 \leq \ell \leq j_2} A[k, \ell]$$

Give an $O(n^2)$ time algorithm for computing all of $r_1, \ldots, r_m$.

Date due: Thursday September 12, 2013