Last Time

Gauss' Law

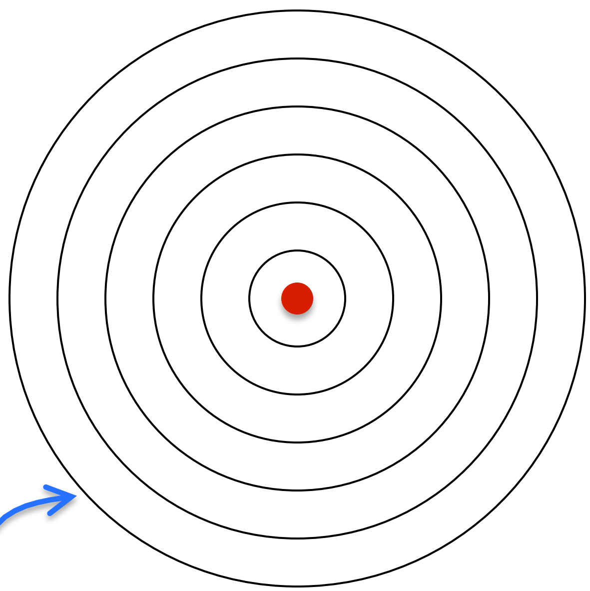
Today

Gauss' Law: Examples
"Magnetic Gauss Law"
Ampere's Law

Gauss' Law for Point Charge

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_o} \sum q_{\text{inside}}$$ Gauss' Law

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \frac{q}{r^2} \hat{r}$$ E-field Point Charge



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Works for any size sphere because r cancels

On each sphere:

Field $E \propto \frac{1}{r^2}$

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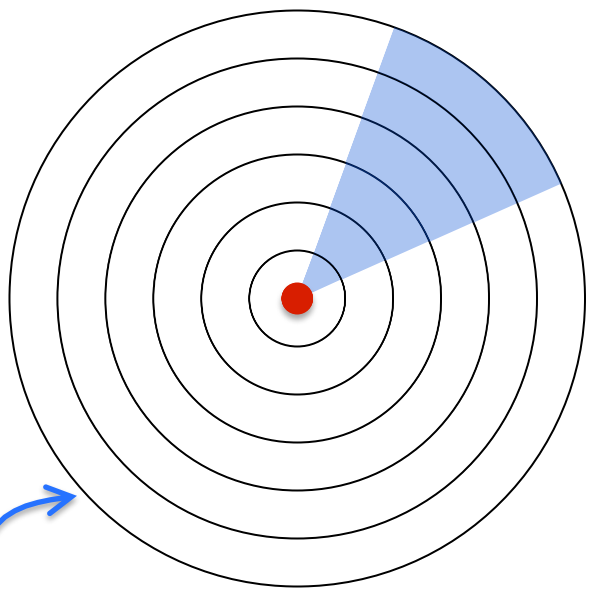

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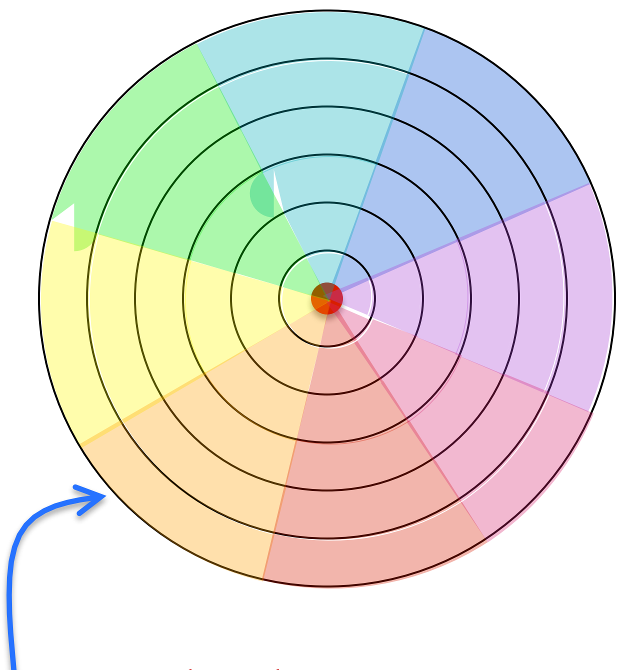

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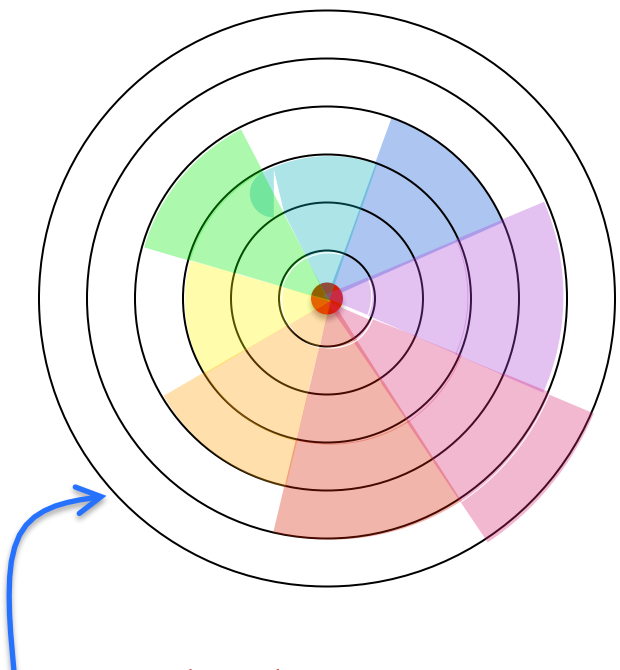

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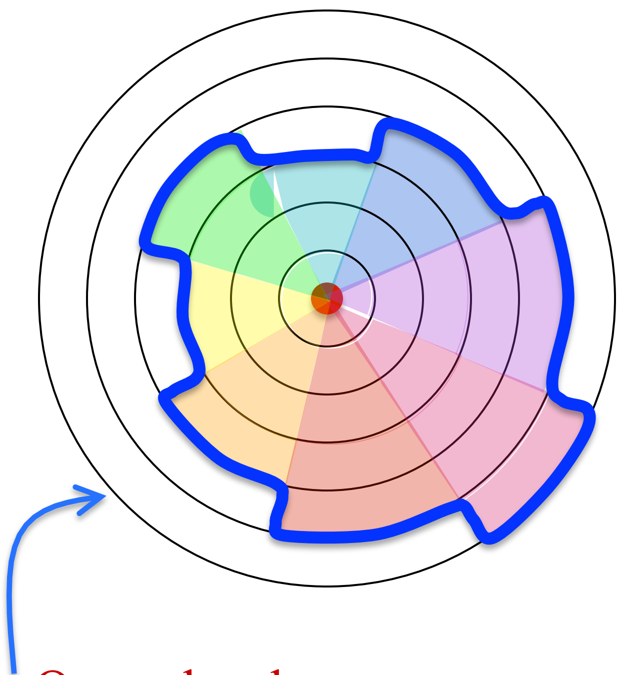

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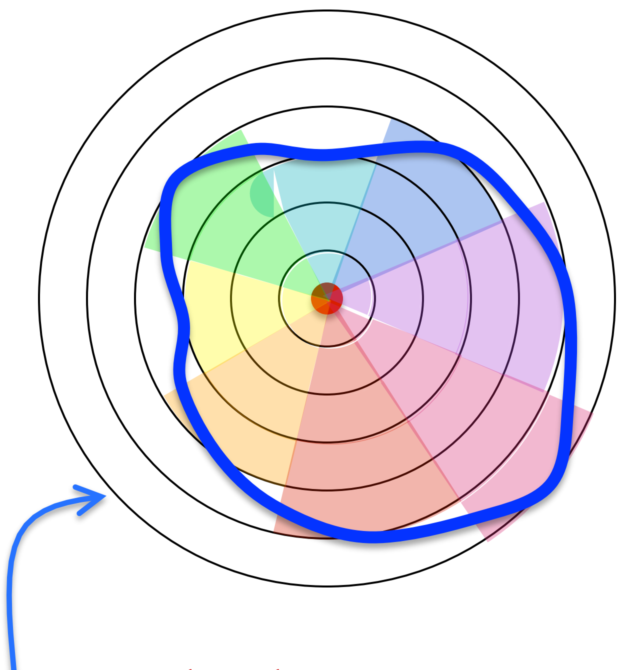

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→ works for any smooth shape.

Example: Gauss' Law for Plane

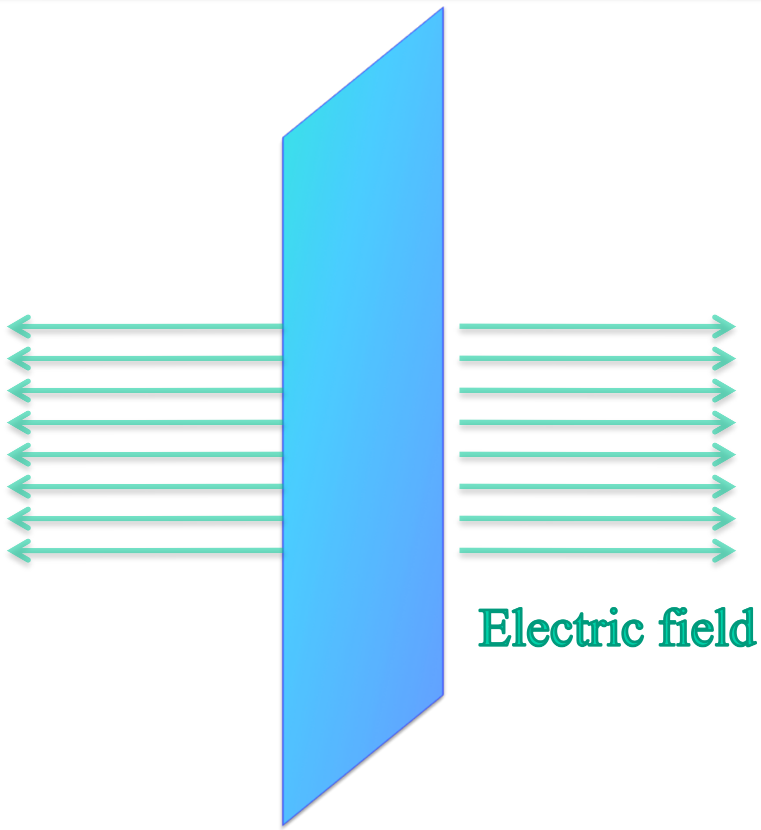
$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_o} \sum q_{\text{inside}}$$ GAUSS' LAW

GIVEN: Infinite plane.
Surface charge density σ = [Q/A]

FIND: E-field due to plane

Due to geometry of plane:
E-field is perpendicular to plane
E-field has same magnitude everywhere

→ USE Gauss' Law to find magnitude.



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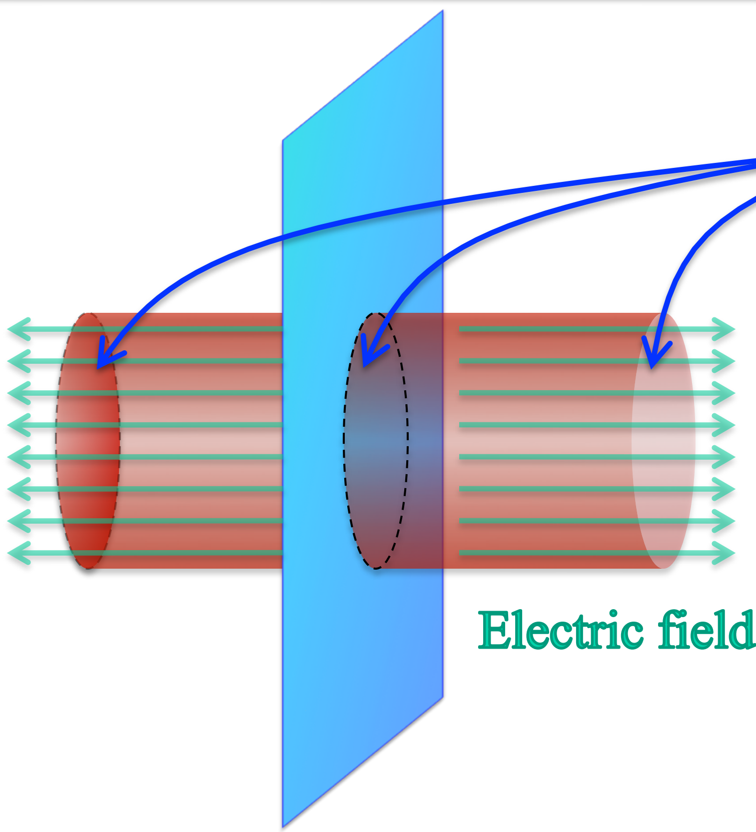


Choose "Gaussian box" wisely!
Same area A on ends of cylinders

E is constant and perpendicular
to the surface A of the "Gaussian box":

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_o} q_{\text{inside}}$$

q_{inside} = [Q/A]*A
=σA

$$2EA = \frac{1}{\epsilon_o} \sigma A$$

$$\boxed{E = \frac{1}{2\epsilon_o} \sigma}$$

ELECTRIC FIELD of a PLANE

Gauss' Law for Magnetism?

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_o} \sum q_{\text{inside}}$$ GAUSS' LAW for charge

So far, no experiment has found a "magnetic charge"
(*a.k.a. magnetic monopole)*

$$q_{\text{magnet}} = 0$$ Big fat ZERO!

→ Gauss' Law for Magnetism is simpler:

http://www.congresshotelstrike.info/

$$\boxed{\oint \vec{B} \cdot \hat{n} dA = 0}$$

GAUSS' LAW FOR MAGNETISM

Next Up: Ampere's Law

First review Biot-Savart Law

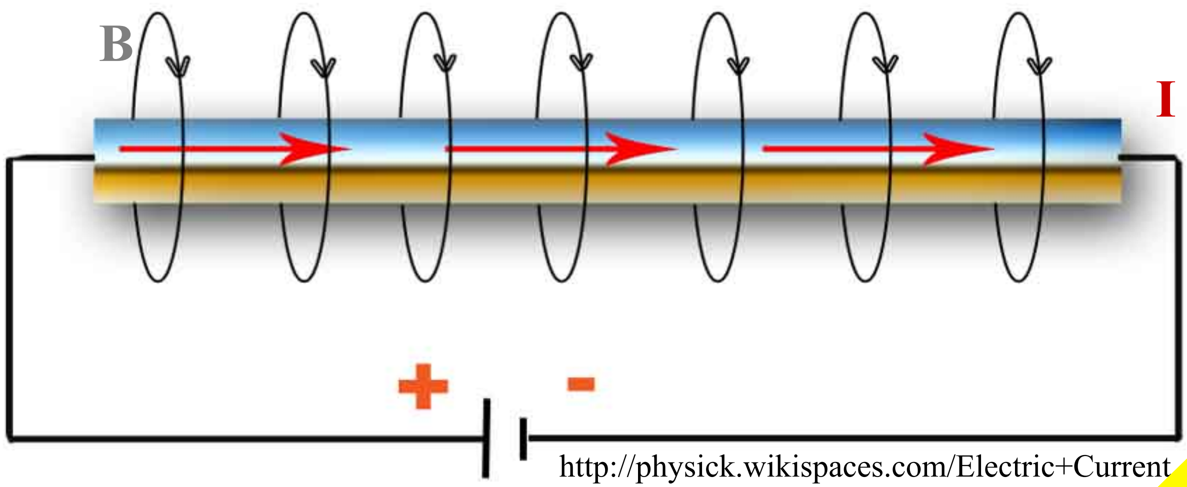
Very Close to the Wire

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{IL}{r\sqrt{r^2+(L/2)^2}}\hat{\theta}$$

Very close to the wire: r << L $\sqrt{r^2+(L/2)^2} \approx L/2$

$$\Rightarrow \vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{IL}{r(L/2)}\hat{\theta} = \boxed{\left(\frac{\mu_o}{4\pi}\right)\frac{2I}{r}\hat{\theta} = \vec{B}}$$

CLOSE TO THE WIRE



http://physick.wikispaces.com/Electric+Current

Blast from the Past
Lecture 13

Very *Long* Wire

$$\vec{B} = \left(\frac{\mu_o}{4\pi}\right)\frac{IL}{r\sqrt{r^2+(L/2)^2}}\hat{\theta}$$

Very *Long* wire: L >> r $\qquad \sqrt{r^2+(L/2)^2} \approx L/2$

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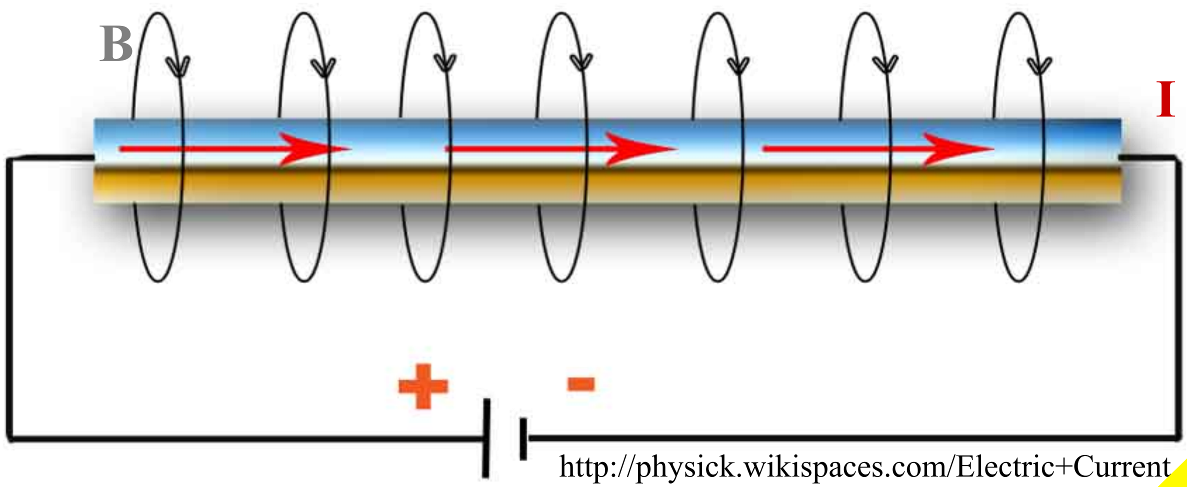

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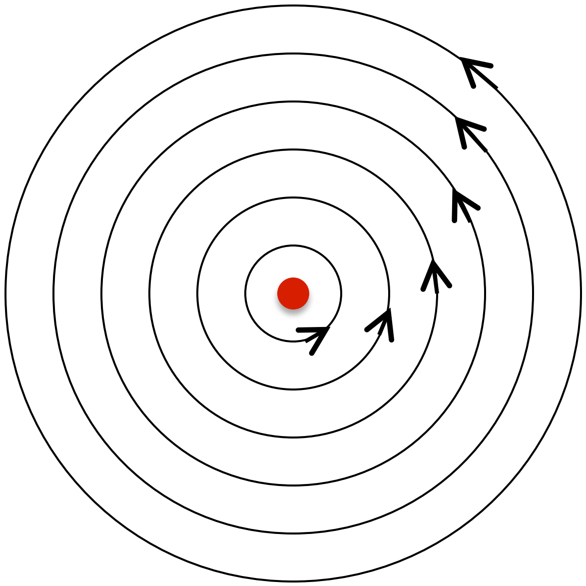

Viewed from the end
Current coming out of board

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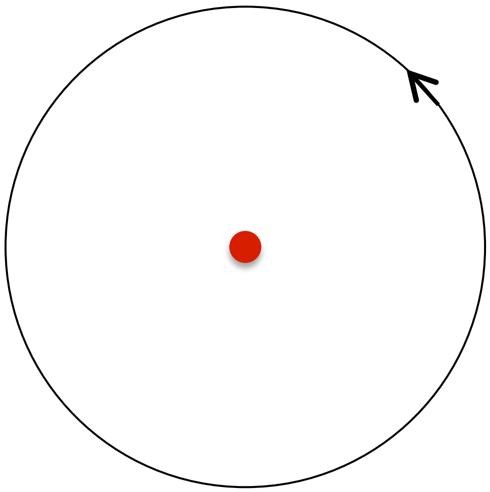

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Cylindrical pattern of B-field
→ Let's take a line integral along one circle

$$\oint \vec{B}\cdot d\vec{l} = \oint \left(\frac{\mu_o}{4\pi}\right)\frac{2I}{r}\hat{\theta}\cdot d\vec{l}$$

$$= \left(\frac{\mu_o}{4\pi}\right)\frac{2I}{r}\oint \hat{\theta}\cdot d\vec{l}$$

$$= \left(\frac{\mu_o}{4\pi}\right)\frac{2I}{r}(2\pi r) = \mu_o I$$

$d\vec{l}$ is along our circle

$$d\vec{l} = r d\hat{\theta}$$

$$\oint \hat{\theta}\cdot d\vec{l} = r\oint \hat{\theta} d\hat{\theta} = 2\pi r$$

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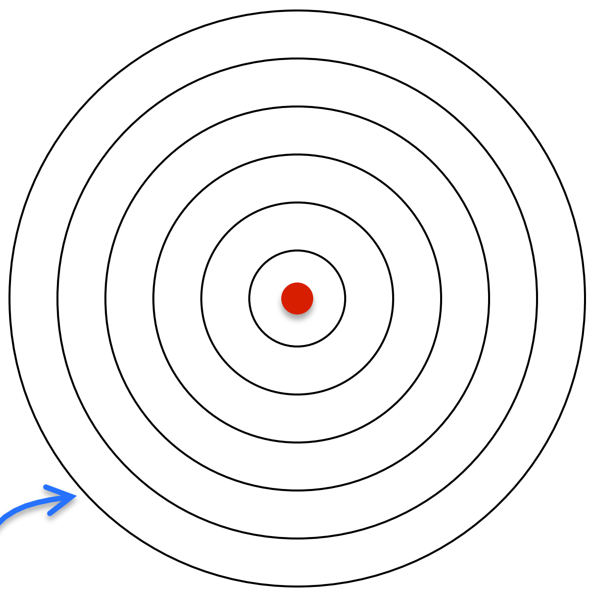

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Works for any size sphere because r cancels

Something similar is going to happen for B of a wire

Remember Me?
Slide 3

Ampere's Law for Long Wire

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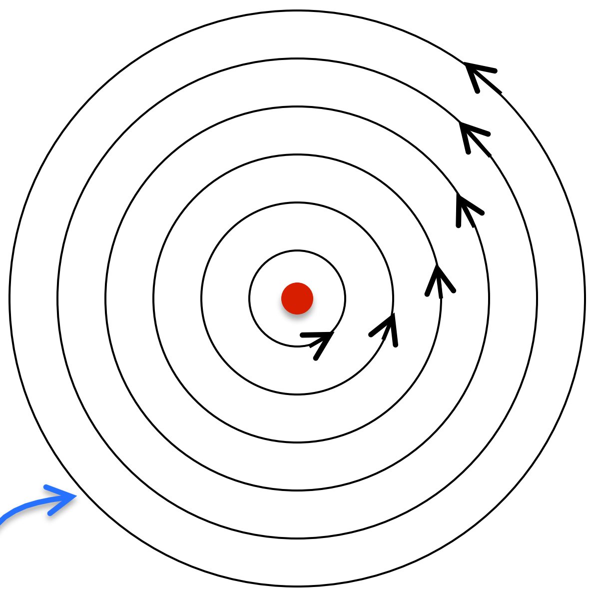

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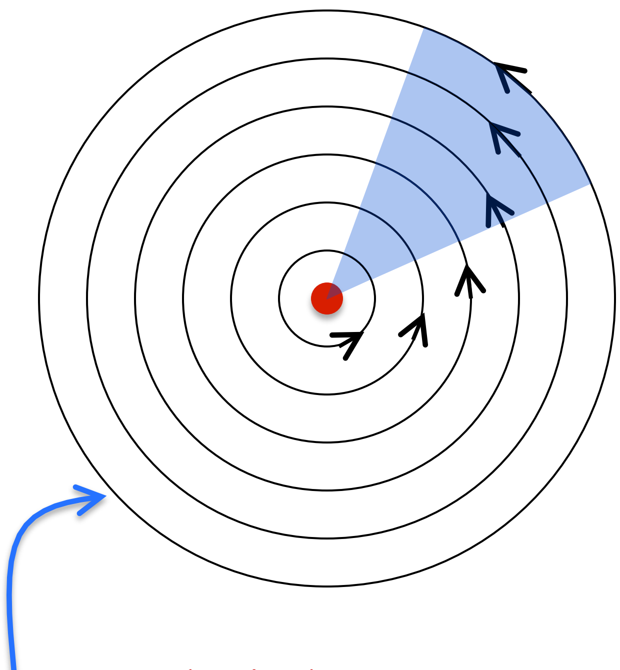

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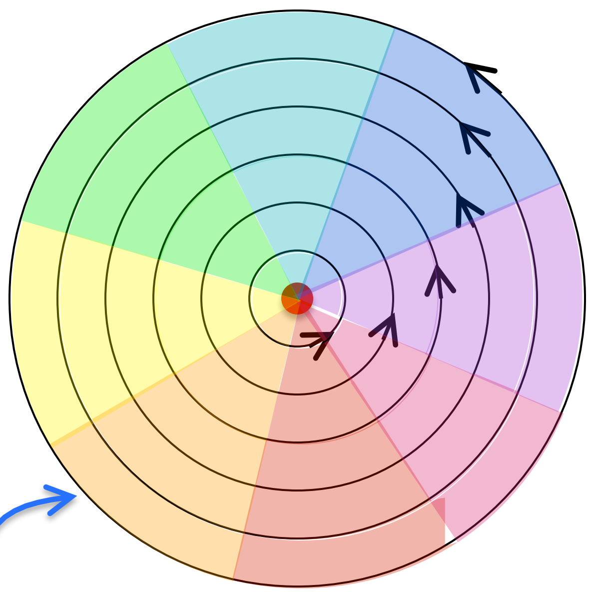

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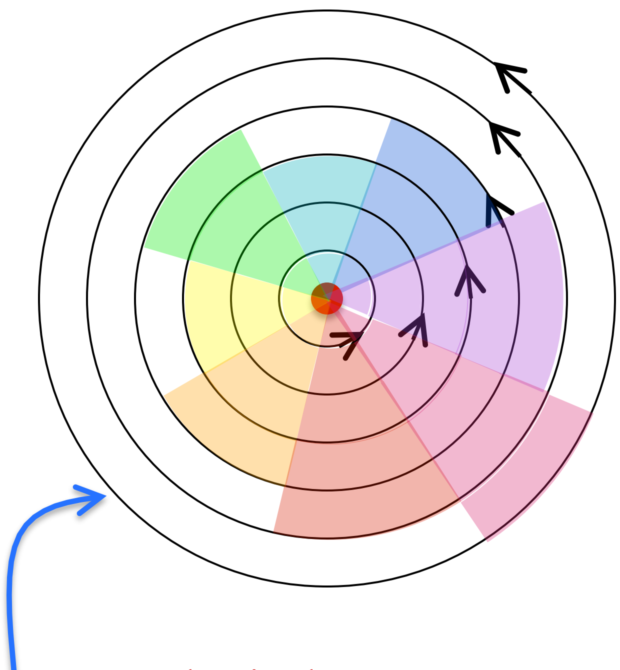

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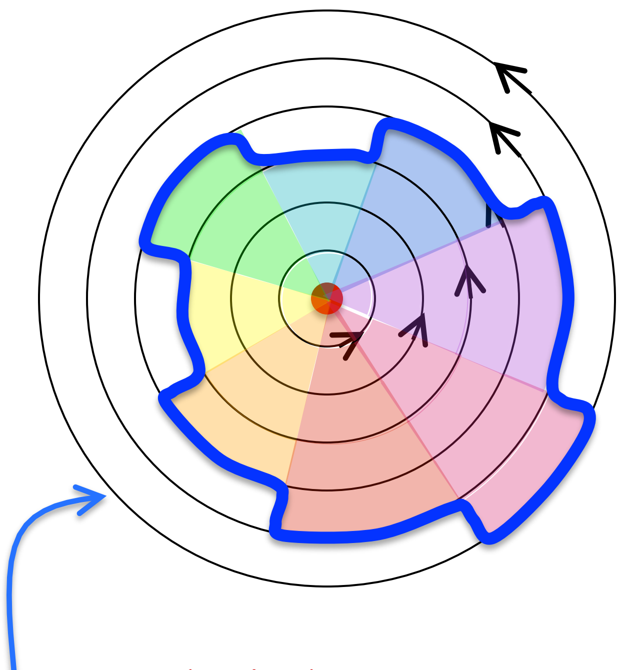

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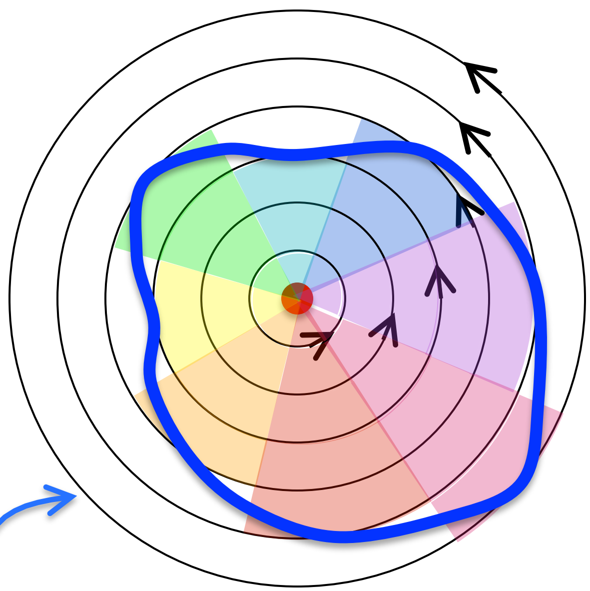

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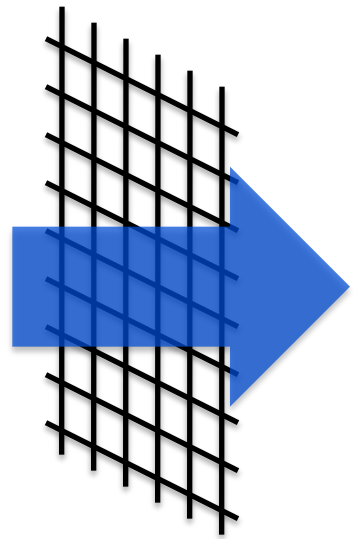
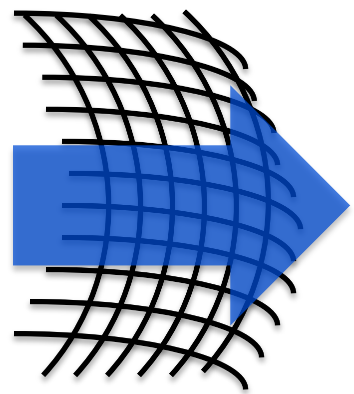
→ works for any smooth path

Electric Current is like Water Flow



Water flows through a net
at a certain rate.

Molecules/second
through each square



The net can deform,
but the flow rate is the same.

Same number of
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Ampere's Law

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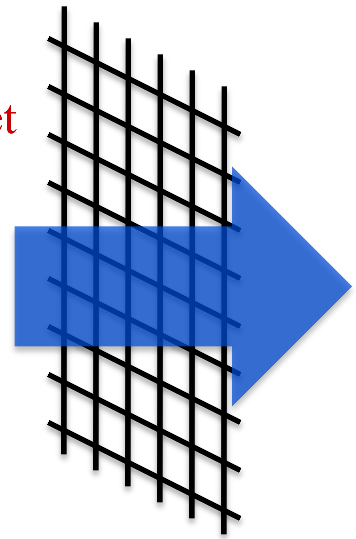

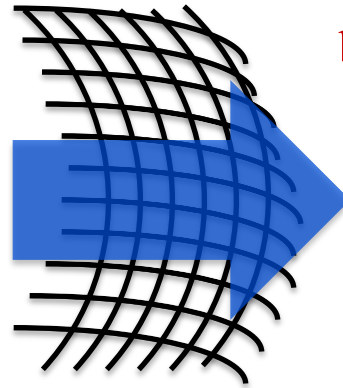

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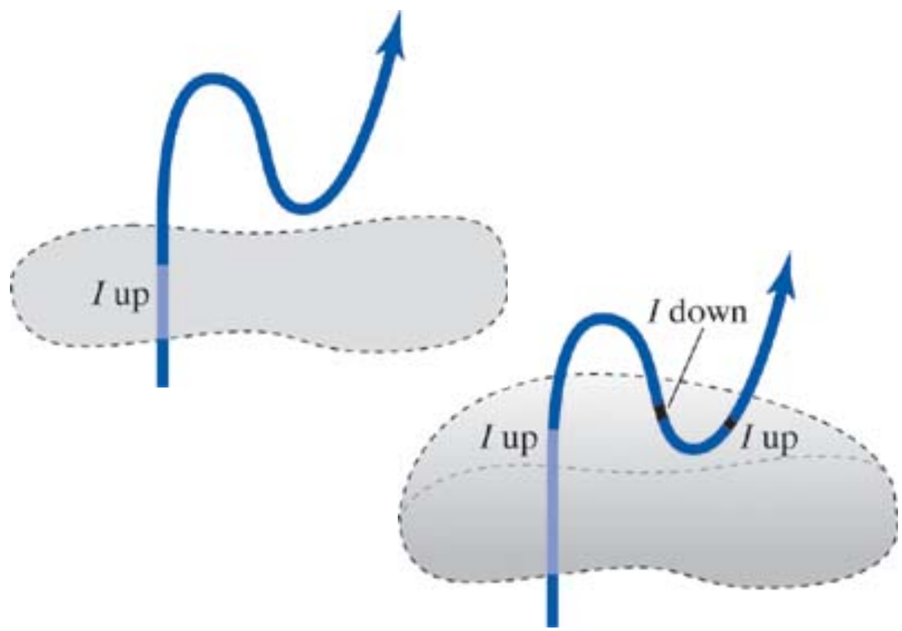
Likewise, Ampere's Law works
even when the surface through which
current I "pokes" is deformed,
even if it means the wire "pokes"
through in more places.



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