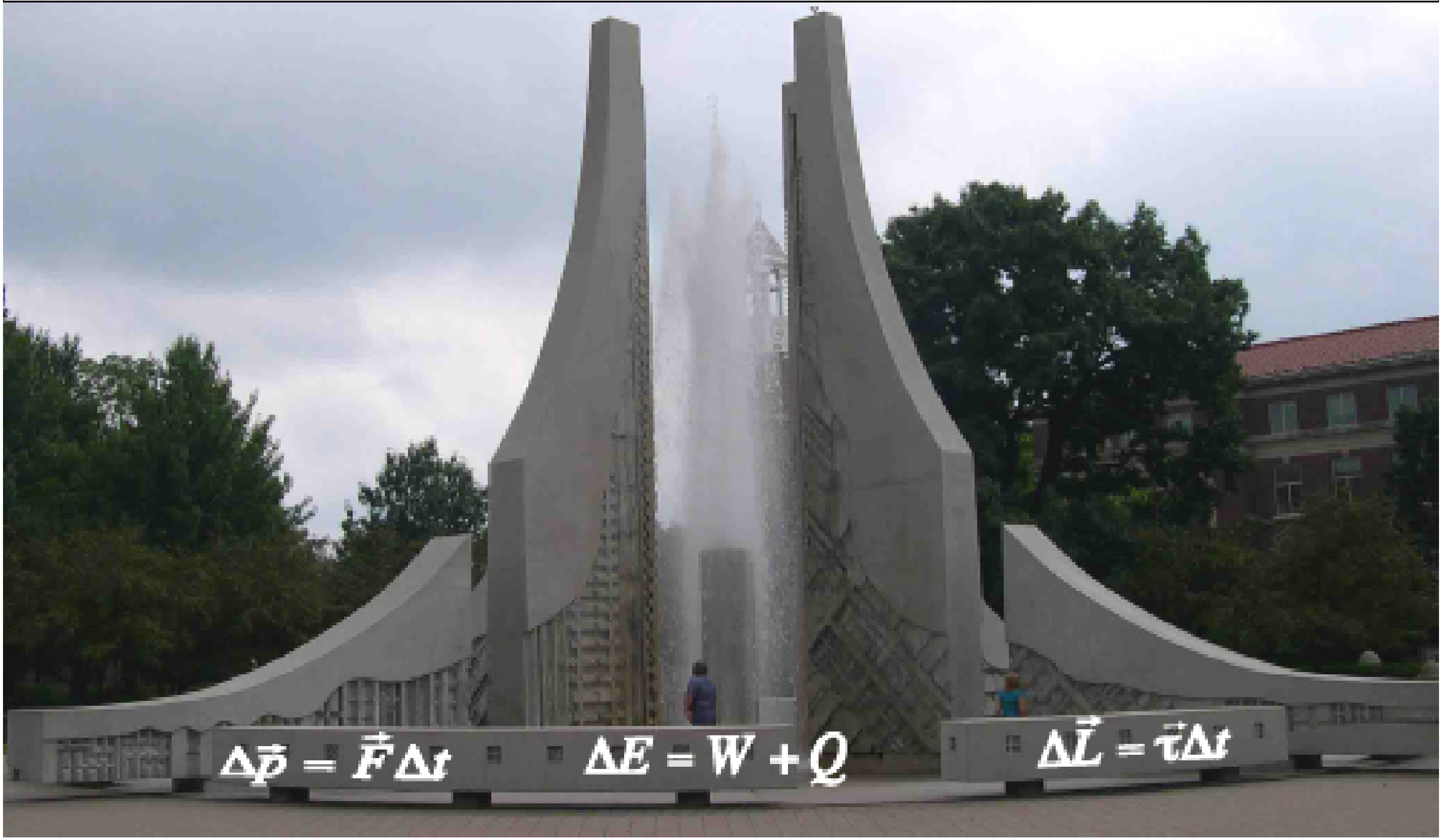
PHYS 172: Modern Mechanics — Spring 2012



Lecture 3 - **The Momentum Principle**

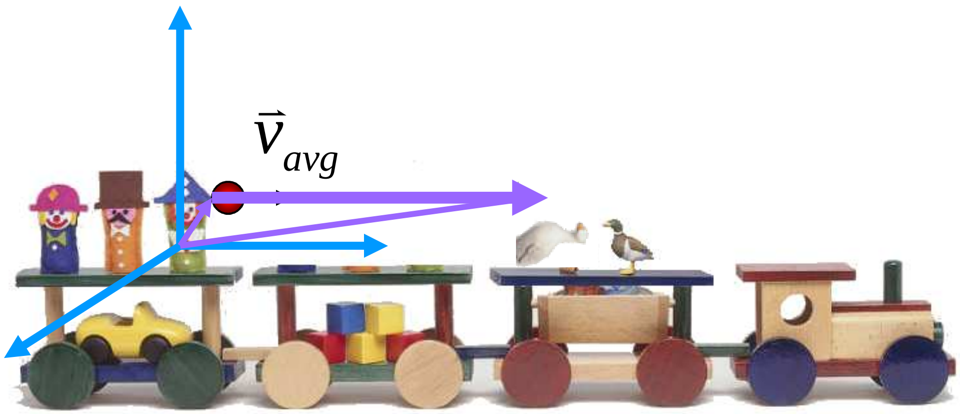
Read 2.1-2.6

TODAY

- Relativity
- Momentum Principle
- Impulse
- Net Force
- Predicting Motion

RELATIVITY

"Physical laws work in the same way for observers in uniform motion as for observers at rest."
(=in all inertial reference frames)



The position update formula

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\left(t_f - t_i\right)$$

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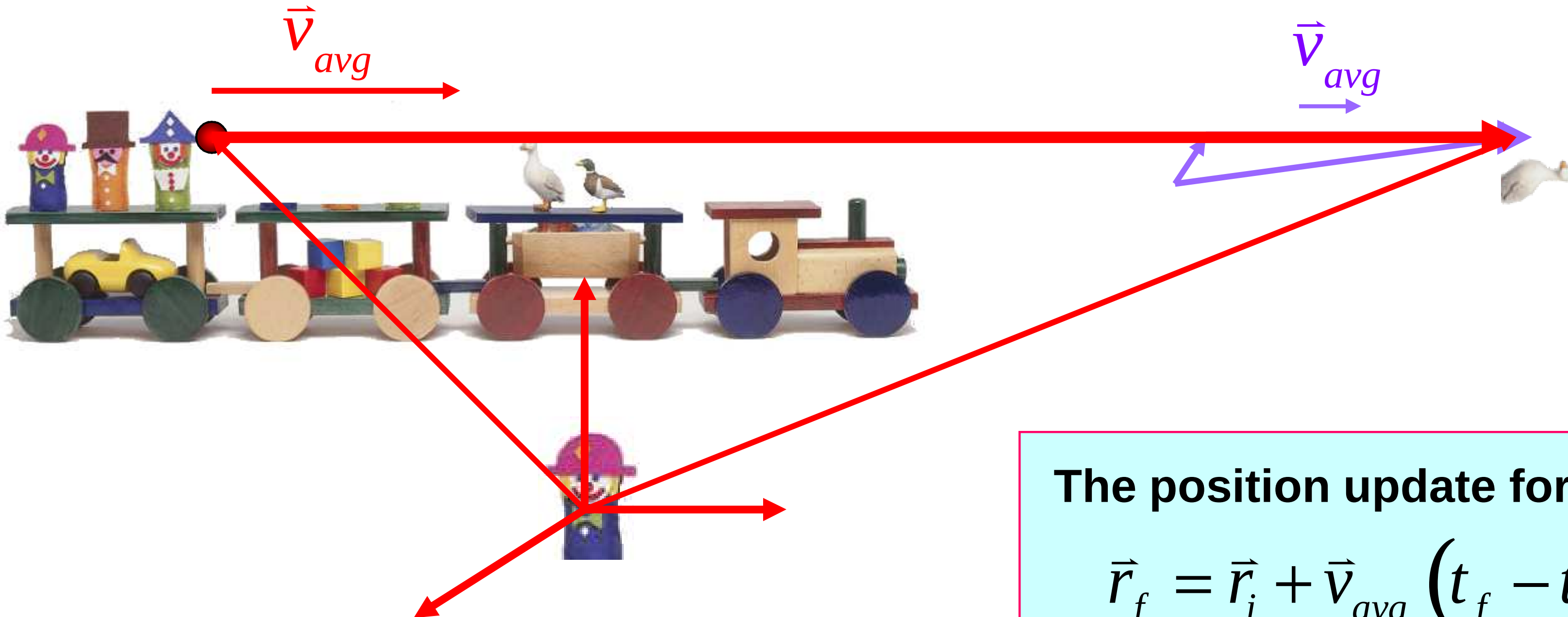


The position update formula

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\left(t_f - t_i\right)$$

Note: all parameters must be measured with respect to the selected reference frame to predict motion with respect to that reference frame

The Momentum Principle

An object moves in a straight line and at constant speed except to the extent that it interacts with other objects

$$\vec{p} = \gamma m \vec{v}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

The Momentum Principle

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

Change of momentum is equal to the net force acting on an object times the duration of the interaction

- Assume that $\vec{F}$ does not change during Δt

$$\text{F units: N} \equiv \text{m}\cdot\text{kg/s}^2$$

(Newton)

What is "force" $\vec{F}$?

- measure of interaction.
- defined by the momentum principle.

Impulse

The Momentum Principle

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

Definition of impulse

$$\text{Impulse} \equiv \vec{F}_{net}\Delta t$$

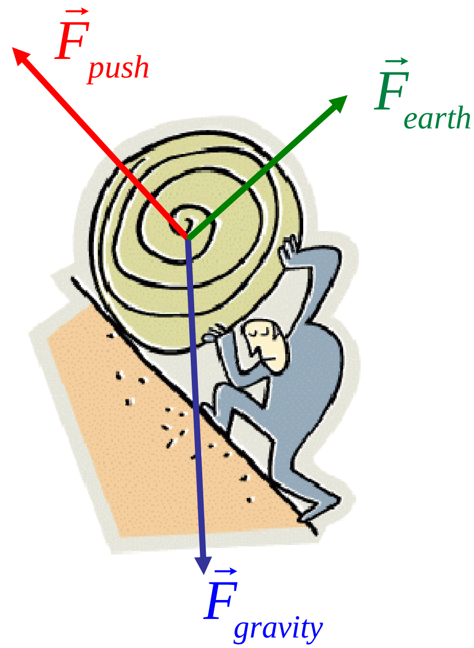
Note: small Δt
F_{net} ~ const

Momentum principle:
The change of the momentum of an object is equal to the net impulse applied to it

The principle of superposition

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

Net force



The Superposition Principle:

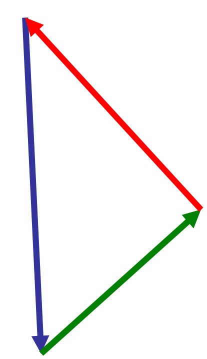
The net force on an object is the vector sum of all the individual forces exerted on it by all other objects

Each individual interaction is unaffected by the presence of other interacting objects

Ignored friction!

Definition of net force:

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \ldots$$



Misconception: need constant force to maintain motion ? Why planets move ?

Predictions using the Momentum Principle

The Momentum Principle

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

$$\vec{p}_f - \vec{p}_i = \vec{F}_{net}\Delta t$$

Update form of the momentum principle

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

Short enough, *F~const*

$$\langle p_{fx}, p_{fy}, p_{fz} \rangle = \langle p_{ix}, p_{iy}, p_{iz} \rangle + \langle F_{net,x}, F_{net,y}, F_{net,z} \rangle \Delta t$$

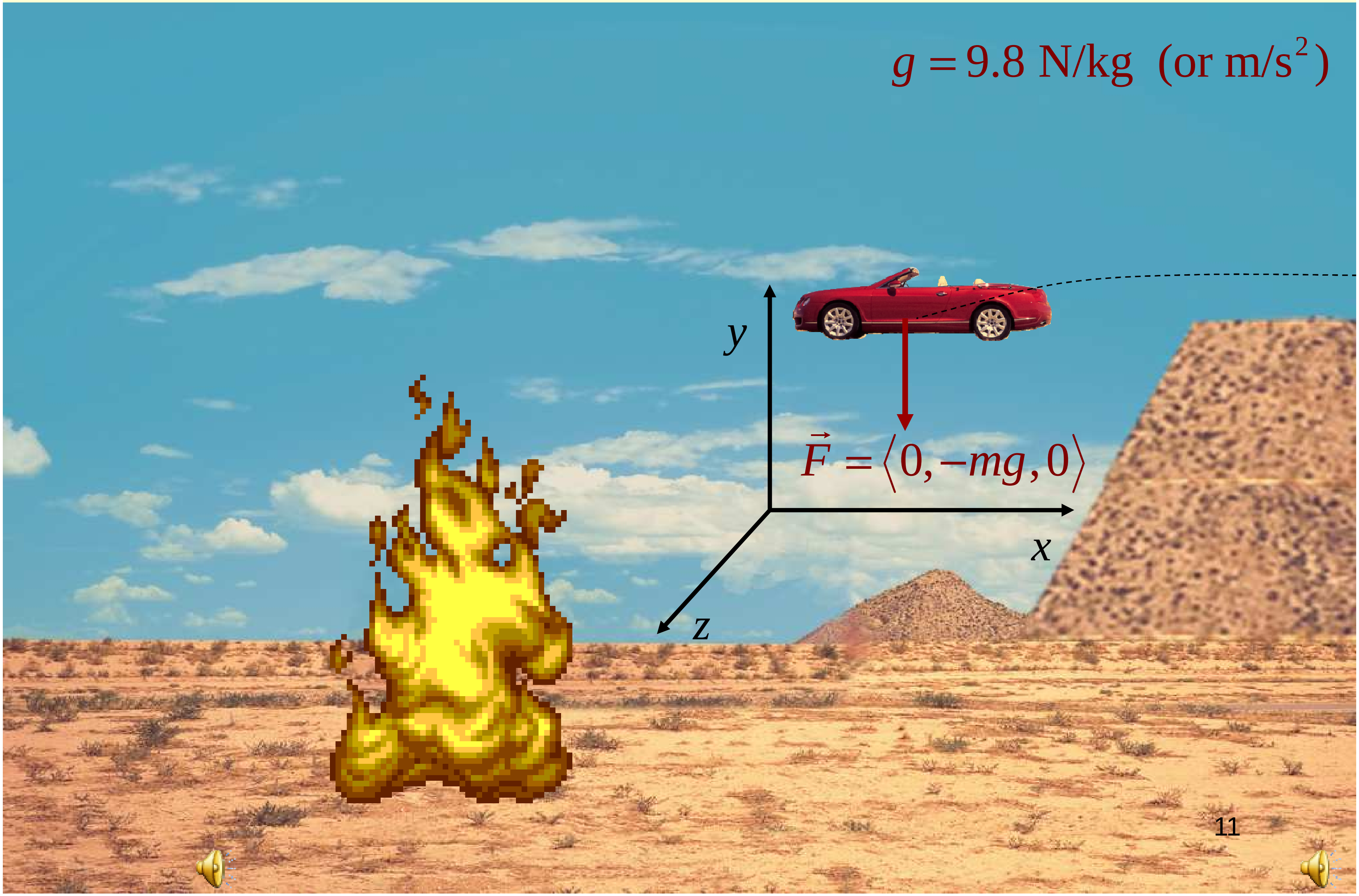
For components:

$$p_{fx} = p_{ix} + F_{net,x}\Delta t$$

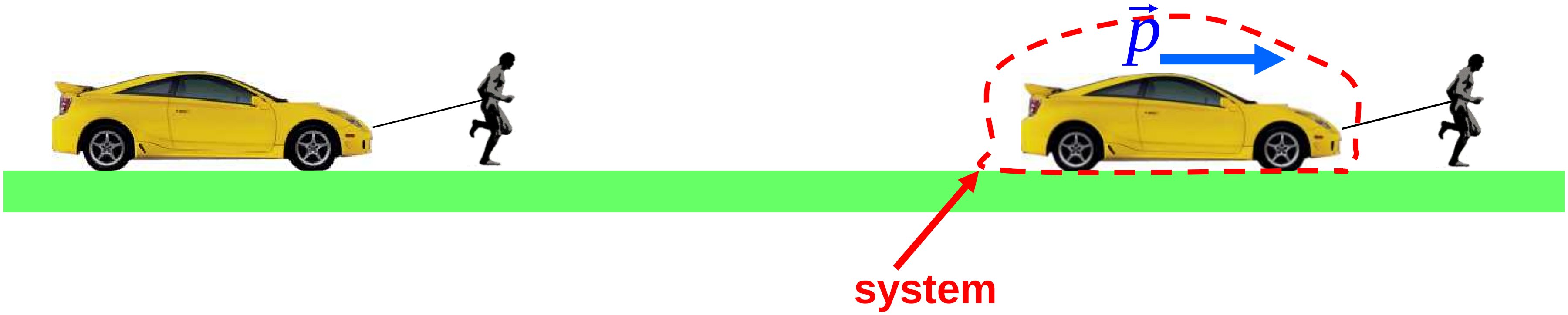
$$p_{fy} = p_{iy} + F_{net,y}\Delta t$$

$$p_{fz} = p_{iz} + F_{net,z}\Delta t$$

Constant Gravitational Field



System and surroundings



System: an object for which we calculate momentum (car)
a system can consist of several objects

Surroundings: objects which interact with system (earth, man, air…)

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

! Only external forces matter !

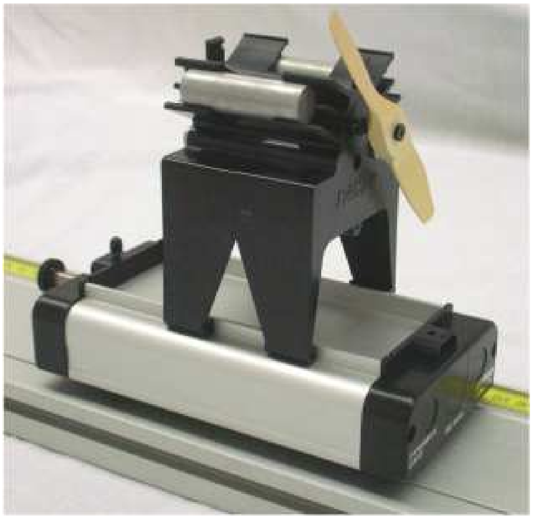
Internal forces cancel

animated slide

Applying the Momentum Principle to a system: predicting motion

1. Choose a system and surroundings
2. Make a list of objects in surroundings that exert significant forces on system
3. Apply the Momentum Principle $\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$
4. Apply the position update formula if needed $\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$
5. Check for reasonableness (units, etc.)

Motion of an object under constant force



System: cart and fan. One dimension.

air

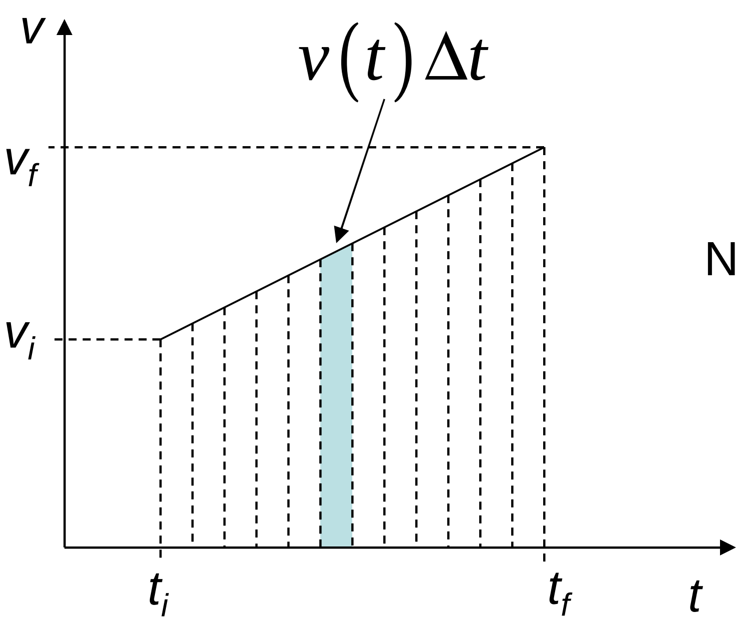
$$p_f = p_i + F_{net}\Delta t$$

$$\boxed{p \approx mv}$$

Assume: $v << c$ Then: $v_f = v_i + \frac{F_{net}}{m}\Delta t$

Position update: $r_f = r_i + v_{avg}\left(t_f - t_i\right)$

Divide *t* into small intervals so that *v* does not change



$$r_f = r_i + \sum\left[v(t)\Delta t\right]$$

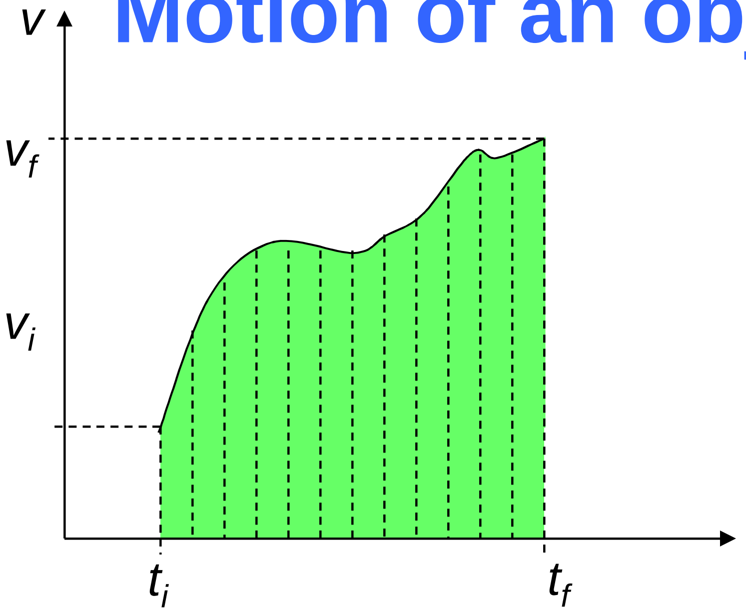
Need to find area under *v*(t) !

Only if *v* changes at constant rate

$$r_f = r_i + \underbrace{\boxed{\frac{v_i + v_f}{2}}}_{=v_{avg}}\left(t_f - t_i\right)$$

See numerical example in text book

Motion of an object under varying force



$$r_f = r_i + \sum \left[v(t)\,\Delta t \right]$$

Area under the curve:

$$r_f = r_i + \lim_{\Delta t \to 0} \sum \left(v(t)\,\Delta t \right)$$

$$r_f = r_i + \int_{t_i}^{t_f} v(t)\,dt$$

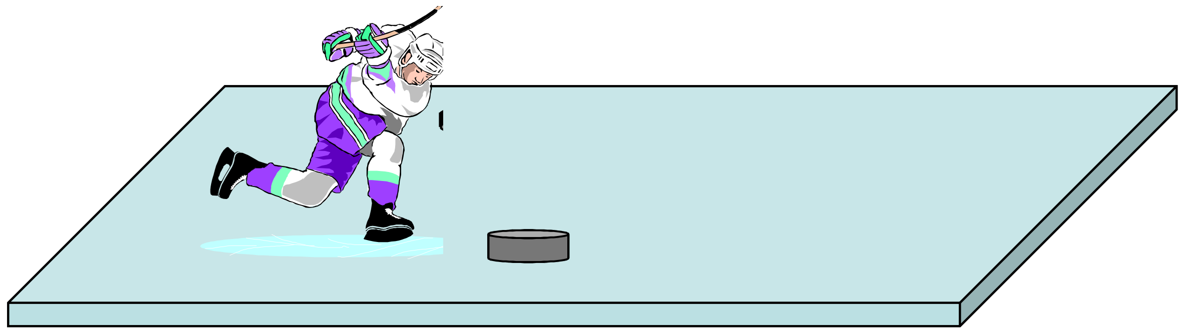
integral

Example: a hockey puck

A hockey puck with a mass of 0.16 kg is initially at rest. A player hits it applying a force $\vec{F} = \langle 400, 400, 0 \rangle$ N during Δt = 4 ms. Where would the puck be 2 seconds after it loses contact with hockey stick?

Solution:

1. Choose a system and surroundings:



2. Make a list of objects in surroundings that exert significant forces on system

Hockey stick
Gravity
Normal force opposing gravity
Friction

3. Apply the Momentum Principle

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$\vec{p}_f = \langle 0,0,0 \rangle \ (\mathrm{m\ kg/s}) + \langle 400,400,0 \rangle \mathrm{N} \cdot \left(4 \cdot 10^{-3} s\right)$$

$$\vec{p}_f = \langle 1.6,1.6,0 \rangle \ \mathrm{m \cdot kg/s}$$

Example: a hockey puck

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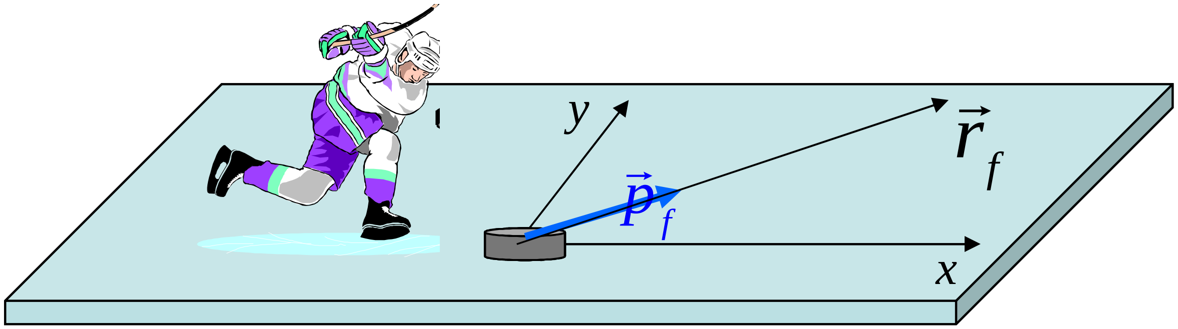
Solution:

3. Momentum

$$\vec{p}_f = \langle 1.6, 1.6, 0 \rangle \ \mathrm{m \cdot kg/s}$$



4. The position update formula

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} \Delta t$$

$$\vec{p} \approx m\vec{v} \rightarrow \vec{v} \approx \frac{\vec{p}}{m} = \langle 10, 10, 0 \rangle \mathrm{m/s}$$

* Choose coordinate system origin: position of puck at the end of interaction

$$\vec{r}_f = \langle 0,0,0 \rangle \mathrm{m} + \langle 10,10,0 \rangle \mathrm{m/s} \cdot (2\ \mathrm{s})$$

$$\vec{r}_f = \langle 20, 20, 0 \rangle \mathrm{m}$$

How Do You Measure force

1. Use the momentum principle – not convenient

The Momentum Principle

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

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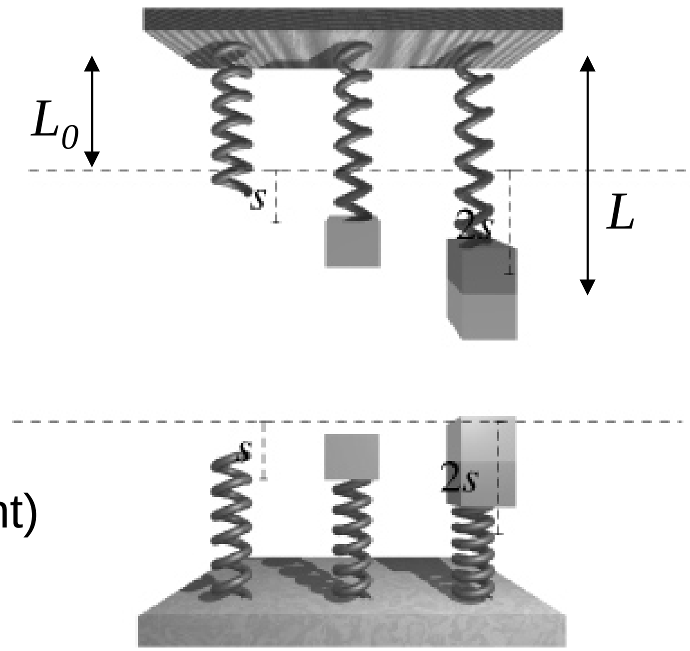
2. Using Hooke's spring force law

$$\boxed{\left|\vec{F}_{spring}\right| = k_S\left|s\right|}$$

$$s = \Delta L = L - L_0$$

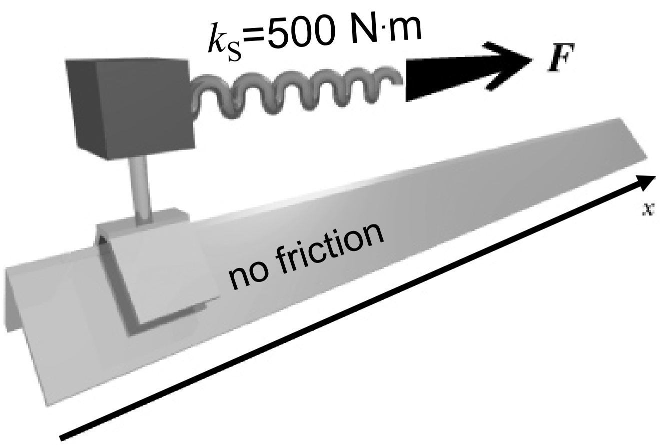


Robert Hooke
1635-1702

k_S – spring stiffness (spring constant)
units: N/m

Direction of force: toward equilibrium

Example $\Delta\vec{p} = \vec{F}_{net}\Delta t$



NB: force must not change during Δt

Force: provided by a spring stretched by ΔL=4 cm
interaction duration: 1 s

? Find momentum p_f if p_i=<0,0,0> kg·m/s

1. Force: $\left|\vec{F}_{spring}\right| = k_S\left|\Delta L\right|$

$$\left|\vec{F}_{spring}\right| = 500(\text{N/m})0.04(\text{m})=20\ \text{N}$$

$$\vec{F}_{spring} = \langle 20,0,0\rangle\ \text{N}$$

2. Momentum: $\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$

$$\vec{p}_f =< 0,0,0 > \text{kg}\cdot\text{m/s} + \langle 20,0,0\rangle \text{N}\cdot(1\ \text{s})$$

N·s = kg·m/s^2·s
= kg·m/s

$$\boxed{\vec{p}_f =< 20,0,0 > \text{kg}\cdot\text{m/s}}$$

WHAT WE DID TODAY

- Relativity
- Momentum Principle
- Impulse
- Net Force
- Predicting Motion