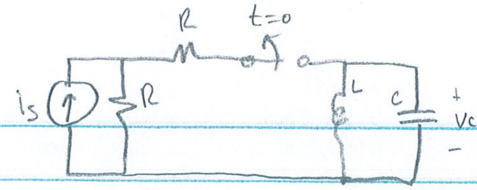
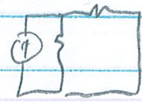
9.9



. At $t=0^-$:



a) $V_c(0^-) = V_c(0^+) = 0\,v.$

$i_L(0^-) = i_L(0^+) = i_s/2$ by current division.

General solution: $x(t) = A\cos\omega_0 t + B\sin\omega_0 t$

Plug in conditions: $V(0^+) = 0 = A_1\cos 0 + \cancel{B_1\sin 0}^{\,0}$

$\rightarrow A = 0.$

$i(0^+) = i_s/2 = A_2\cos 0 + \cancel{B_2\sin 0}^{\,0}$

$A_2 = i_s/2$

so $\boxed{i(t) = \frac{i_s}{2}\cos(\omega_d t)}$ $\boxed{\omega_d = 1/\sqrt{LC}}$

$V_c = V_L = L\,di/dt = \frac{-\omega_d L i_s}{2}\sin\omega_d t$

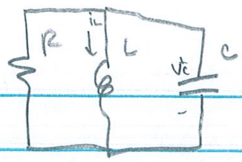
$V_c(t) = \frac{L}{\sqrt{LC}}\frac{i_s}{2}\sin(\omega_d t) = \boxed{\sqrt{\frac{L}{C}}\,\frac{I_s}{2}\sin(\omega_d t)}$

b) Energy at $t=0$:

$W_L = \frac{1}{2}L\,i(0)^2 = \frac{1}{2}L\left(i_s/2\cos 0\right)^2 = \boxed{\frac{L\,i_s^2}{8}}$

$W_C = \frac{1}{2}C\,V(0)^2 = \frac{1}{2}C\sin 0 = \boxed{0\ w}$

9.16



$R = 0.4\,\Omega$
$L = 0.5\,H$
$C = 0.5\,F$

a) $V_c(0) = -2v \qquad i_L(0) = 2.5A$

Parallel RLC: $\alpha = 1/2RC = 2.5$

$\omega_0 = 1/\sqrt{LC} = 2$

general solution, overdamped: $s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2.5 \pm 1.5 = -4, -1$

$V_c(t) = Ae^{-4t} + Be^{-t} \qquad V_c(0) = -2 = Ae^0 + Be^0 = A + B.$

$i_R(0) + i_L(0) + i_C(0) = 0$

$\frac{V_c(0)}{R} + 2.5 + C\,V'(0) = 0$

$-5 + 2.5 = -.5\,V'(0)$

$V'(0) = 5.$

$V_c'(0) = -4Ae^0 - Be^0 = 5$

$A + B = -2$
$-4A - B = 5$

$-3A = 3$

$A = -1 \qquad B = -1$

$$\boxed{V_c(t) = -e^{-4t} - e^{-t} \text{ volts.}}$$

b) $V_c(0) = +2 = A + B$

$\frac{+2}{R} + 2.5 + C\,V'(0) = 0$

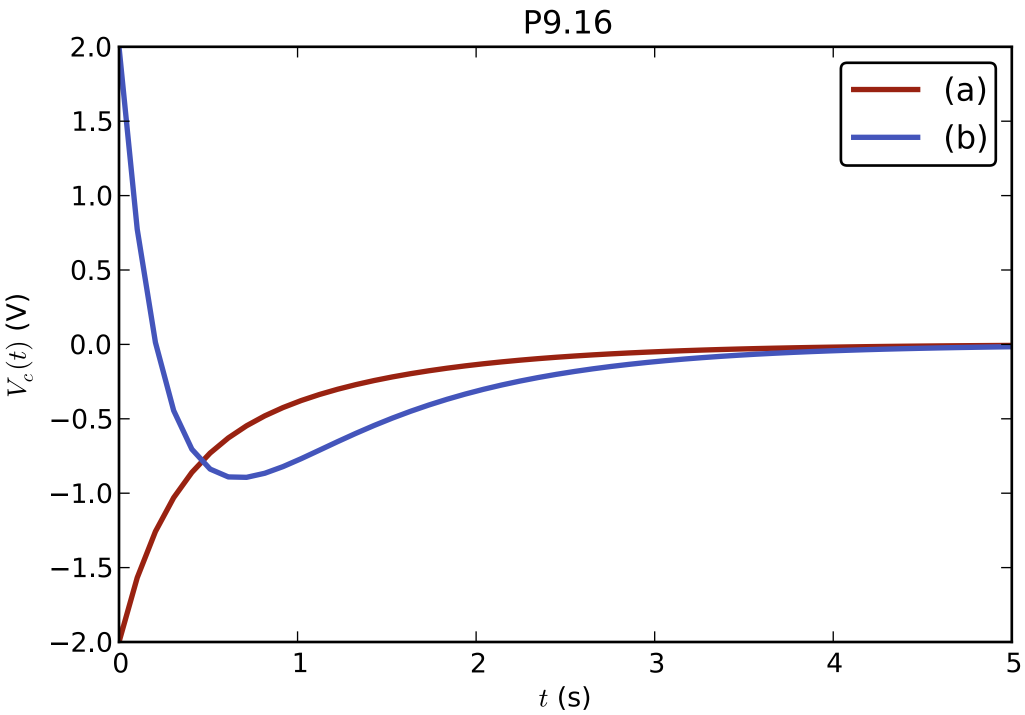
$V'(0) = -15\,v$

$A + B = 2$
$-4A - B = -15$

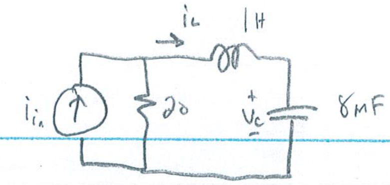
$-3A = -13$

$A = 13/3 \qquad B = -7/3$

$$\boxed{V_c(t) = \tfrac{13}{3}e^{-4t} - \tfrac{7}{3}e^{-t} \text{ Volts}}$$


P9.16
(a)
(b)
$V_c(t)$ (V)
t (s)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
1
2
3
4
5

$\boxed{9.18}$



$i_{in} = 0.5\,u(-t)$

a) @ $t=0^-$, $i_L(0^-) = i_L(0^+) = 0$

$V_c(0^-) = V_c(0^+) = i_{in}(0^-) \times 20 = \underline{10\,v}$

KCL @ $t=0^+$: $i_L(0^+) = C\,V_c'(0^+) = 0 \rightarrow V_c'(0^+) = \underline{0}$

KVL @ $t=0^+$: $\cancel{i_L R}^{\,0} + L\,i_L'(0^+) + V_c(0^+) = 0$

$$i_L'(0^+) = \frac{-V_c(0^+)}{L} = \underline{-10}$$

Series RLC: $\alpha = R/2L = 10$

$\omega_0 = 1/\sqrt{LC} = 11.2$ $\rightarrow$ underdamped. $\omega_d = \sqrt{\alpha^2 - \omega_0^2} = 5.04$

general solution: $V_c(t) = e^{-\alpha t}\left[A\cos\omega_d t + B\sin\omega_d t\right]$

$V_c(0^+) = A \rightarrow \underline{A = 10}$

$V_c'(0^+) = -\alpha A + \omega_d B \rightarrow 0 = -10(10) + 5.04\,B$

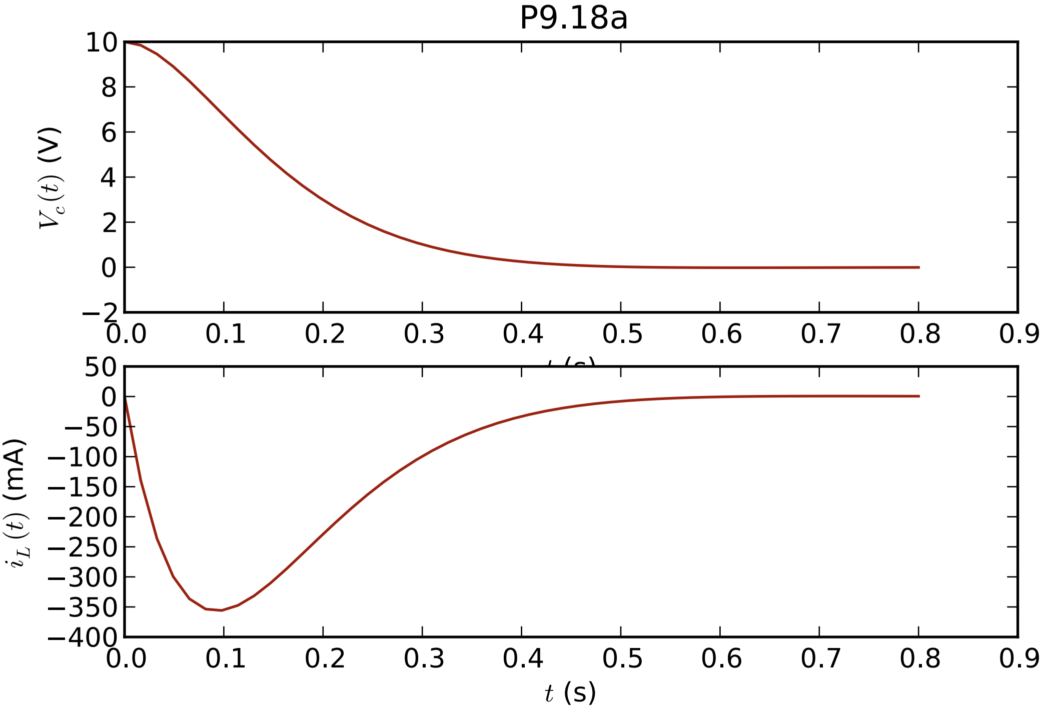
$\underline{B = 19.8}$

$$\boxed{V_c(t) = e^{-10t}\left[10\cos(5.04t) + 19.8\sin(5.04t)\right] \text{ Volts.}}$$

$i_L(0^+) = A = 0$

$i_L'(0^+) = -\cancel{\alpha A} + \omega_d B \rightarrow B = -10/\omega_d \cong \underline{-2}$

$$\boxed{i_L(t) = e^{-10t}\left[-2\sin(5.04t)\right] \text{ Amps}}$$



P9.18a
$V_c(t)$ (V)
t (s)
$i_L(t)$ (mA)
t (s)

b) Now, $R = 22.5\,\Omega$

$\alpha = R/2L = 11.25$

$w_0 = 11.2$ $\Big\} \rightarrow$ overdamped.

general solution: $s = -\alpha \pm \sqrt{\alpha^2 - w_0^2} = -11.25 \pm 1.06 = \underline{-10.2,\ -12.3}$

$x(t) = Ae^{-10.2t} + Be^{-12.3t}$

$x(0^+) = A + B$

$x'(0^+) = A(-10.2) + B(-12.3)$

Initial Conditions, similar to part, except $V_c(0)$ changes!

$V_c(0^+) = i_{in}(22.5) = 11.25$

$i_L(0^+) = 0$

$V_c'(0^+) = 0$

$i_L'(0^+) = -V_c(0^+)/L = -11.25$

$V_c(0^+) = 11.25 = A + B$

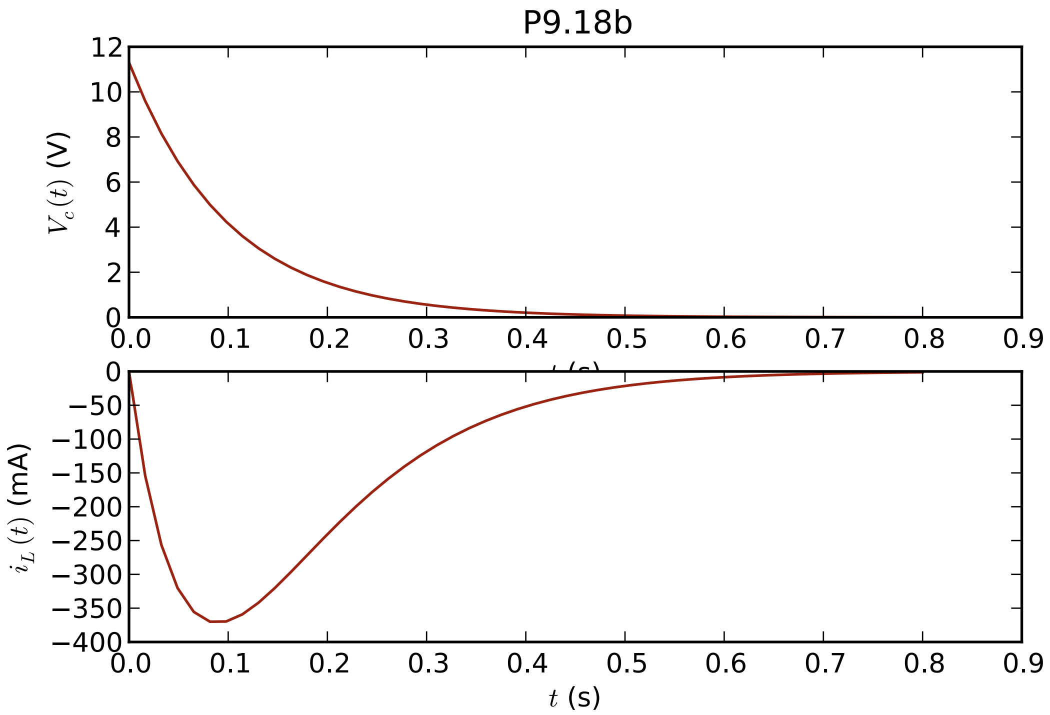
$V_c'(0^+) = 0 = -10.2A - 12.3B$ $\Big\} \rightarrow$ $\underline{A = 65.9,\ B = -54.6}$

$$\boxed{V_c(t) = 65.9\,e^{-10.2t} - 54.6\,e^{-12.3t} \text{ Volts}}$$

$i_L(0^+) = 0 = A + B$

$i_L'(0^+) = -11.25 = -10.2A - 12.3B$ $\Big\} \rightarrow$ $\underline{A = -5.4,\ B = +5.4}$

$$\boxed{i_L(t) = -5.4\,e^{-10.2t} + 5.4\,e^{-12.3t} \text{ Amps}}$$


P9.18b
$V_c(t)$ (V)
12
10
8
6
4
2
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
t (s)
$i_L(t)$ (mA)
0
−50
−100
−150
−200
−250
−300
−350
−400
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
t (s)