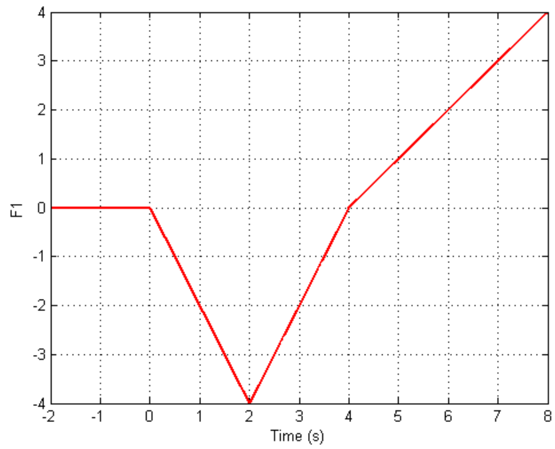
HW1 Solutions

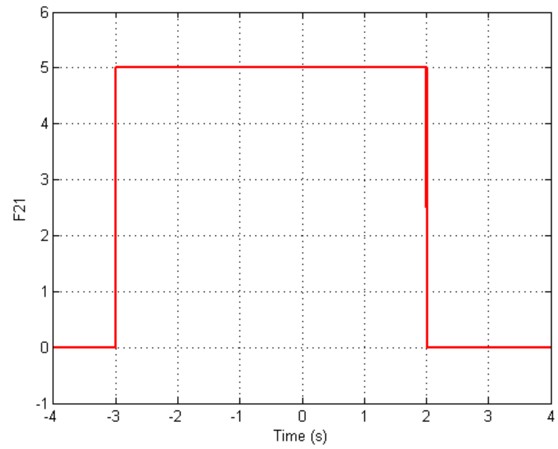
ECE 202 Fall 2013

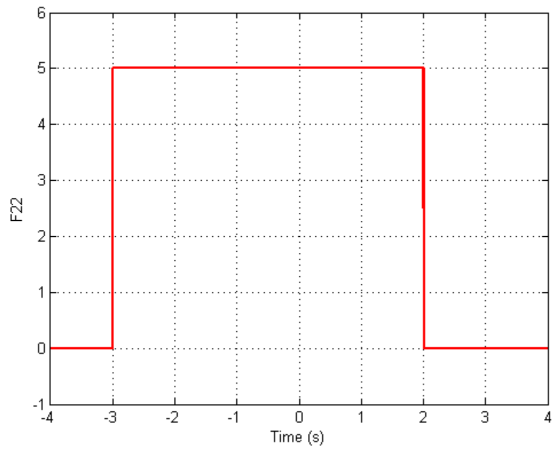
MK
MN

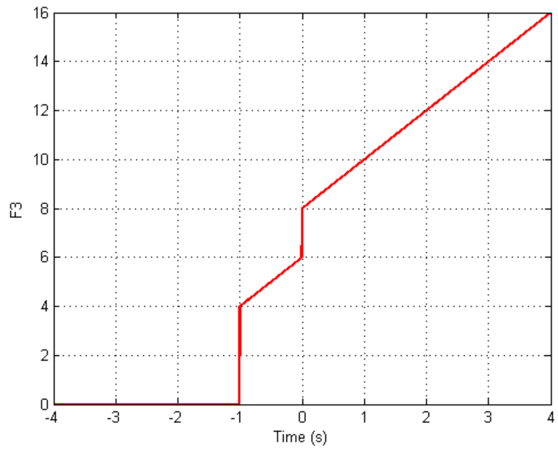
1. a) In Matlab:

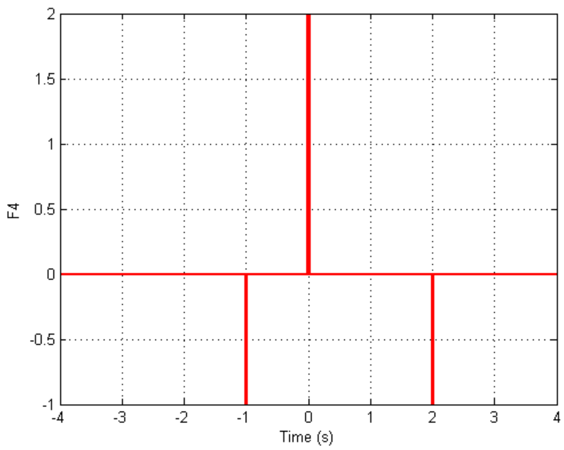
```
clear all
clc
t1=-2:0.0001:8;
t=-4:0.0001:4;
f1=4*(t1-2).*heaviside(t1-2)-2*t1.*heaviside(t1)-(t1-4).*heaviside(t1-4);
f21=5*heaviside(t+3)-5*heaviside(t-2);
f22=5*heaviside(t+3).*heaviside(2-t);
f3=4*(t+2).*heaviside(t+2).*heaviside(t+1)-2*(t+1).*heaviside(t+1)+2*heavisid(t);
f4=2*double(t==0)-double(t==2)-double(t==-1);
figure(1)
p=plot(t1,f1)
% Change the line color to red and
% set the line width to 2 points
set(p,'Color','red','LineWidth',2)
grid on
xlabel('Time (s)')
ylabel('F1')
figure(2)
p=plot(t,f21)
axis([-4 4 -1 6])
set(p,'Color','red','LineWidth',2)
grid on
xlabel('Time (s)')
ylabel('F21')
figure(3)
p=plot(t,f22)
axis([-4 4 -1 6])
set(p,'Color','red','LineWidth',2)
grid on
xlabel('Time (s)')
ylabel('F22')
figure(4)
p=plot(t,f3)
set(p,'Color','red','LineWidth',2)
grid on
xlabel('Time (s)')
ylabel('F3')
figure(5)
p=plot(t,f4)
set(p,'Color','red','LineWidth',2)
grid on
xlabel('Time (s)')
ylabel('F4')
```


F1
Time (s)

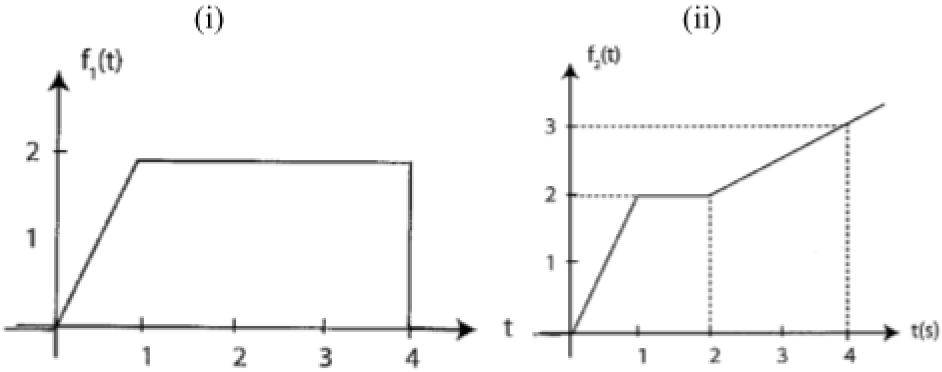

F21
Time (s)


F22
Time (s)


F3
Time (s)


F4
Time (s)

b)



$f_1(t) = 2r(t) - 2r(t-1) - 2u(t-4)$

$f_2(t) = 2r(t) - 2r(t-1) + \frac{1}{2}r(t-2)$

2.
Note: Since Laplace transform is a one-sided integral, we need to represent all of the above functions by looking only at the portion which is above zero (t>=0)
a)
(i)

$$f_1(t) = -2r(t) + 2r(t-2) + 2r(t-2) - 2r(t-4) + r(t-4)$$
$$f_1(t) = -2r(t) + 4r(t-2) - r(t-4)$$

(ii)

$$f_2(t) = 5u(t) - 5u(t-2)$$

(iii)

$$f_3(t) = 8u(t) + 2r(t)$$

(iv)

$$f_4(t) = 2\delta(t) - \delta(t-2)$$

Now we can find the Laplace transform of each function using the time-shift property:

$$F_1(s) = -\frac{2}{s^2} + \frac{4e^{-2s}}{s^2} - \frac{e^{-4s}}{s^2}$$
$$F_2(s) = \frac{5}{s} + \frac{5e^{-2s}}{s}$$
$$F_3(s) = \frac{8}{s} + \frac{2}{s^2}$$
$$F_4(s) = 2 - e^{-2s}$$

In Matlab:

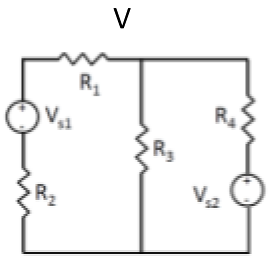
```
clear all
clc
syms s t f1 f21 f22 f3 f4
f1=4*(t-2).*heaviside(t-2)-2*t.*heaviside(t)-(t-4).*heaviside(t-4);
f21=5*heaviside(t+3)-5*heaviside(t-2);
f22=5*heaviside(t+3).*heaviside(2-t);
f3=4*(t+2).*heaviside(t+2).*heaviside(t+1)-
2*(t+1).*heaviside(t+1)+2*heaviside(t);
f4=2*dirac(t)-dirac(t-2)-dirac(t+1);
pretty(laplace(f1))
pretty(laplace(f21))
pretty(laplace(f3))
pretty(laplace(f4))
```

b) Again using the time-shift property:

$$F_1(s) = \frac{2}{s^2} + \frac{2e^{-s}}{s^2} - \frac{2e^{-4s}}{s}$$

$$F_2(s) = \frac{2}{s^2} - \frac{2e^{-s}}{s^2} - \frac{\frac{1}{2}e^{-2s}}{s^2}$$

3.



KCL at V:

$$\frac{V-V_{s1}}{R_1+R_2}+\frac{V}{R_3}+\frac{V-V_{s2}}{R_4}=0$$

In Matlab:

```
clear all
clc
syms V Vs1 Vs2
R1=10;R2=5;R3=30;R4=40;
solve((V-Vs1)/(R1+R2)+V/R3+(V-Vs2)/R4,V)
pretty(V)
```

$$V=\frac{8}{15}V_{s1}+\frac{V_{s2}}{5}$$

Voltage Division:

$$V_{R_1}=\frac{R_1}{R_1+R_2}(V-V_{s1})=\frac{10}{15}(V-V_{s1})$$

$$V_{R_1}(s)=\frac{10}{15}\left(\frac{8}{15}V_{s1}(s)+\frac{V_{s2}(s)}{5}-V_{s1}(s)\right)=-\frac{70}{225}V_{s1}(s)+\frac{10}{75}V_{s2}(s)$$

Let's find the Laplace transform of v_{s1} and v_{s2}. To do so we will use the time shift and also frequency shift properties of the Laplace transform.

$$V_{s1}=L\{v_{s1}\}=L\{3u(t)-3u(t-2)\}$$

$$V_{s1}=\frac{3}{s}-\frac{3e^{-2s}}{s}$$

$$V_{s2}=L\{v_{s2}\}=L\{109e^{-2t}r(t-2)\}=L\left\{109e^{-2(t-2+2)}r(t-2)\right\}$$

$$V_{s2}=L\left\{109e^{-4}.e^{-2(t-2)}r(t-2)\right\}$$

$$V_{s2}=\frac{109e^{-4}e^{-2s}}{(s+2)^2}=\frac{109e^{-2(s+2)}}{(s+2)^2}$$

$$V_{R_1}(s) = \frac{2}{5s} - \frac{2e^{-2s}}{5s} - \frac{1526e^{-2(s+2)}}{45(s+2)^2}$$

4.

a) $f_1(t) = 3\sin(t)\,u(t-\pi) - 3\cos(t)u(t-2\pi)$

$$f_1(t) = 3\sin(t-\pi+\pi)\,u(t-\pi) - 3\cos(t-2\pi+2\pi)\,u(t-2\pi)$$

$$f_1(t) = -3\sin(t-\pi)\,u(t-\pi) - 3\cos(t-2\pi)\,u(t-2\pi)$$

Using the time shift property:

$$F_1(s) = -\frac{3e^{-\pi s}}{s^2+1} - 3e^{-2\pi s}\frac{s}{s^2+1}$$

b)$f_2(t) = 3\delta(t) - 2\delta(t-2)$

From the time shift property:

$$F_2(s) = 3 - 2e^{-2s}$$

c)$f_3(t) = \sinh(2t)\,u(t+1)\delta(t) + 10t\delta(2t+4) - 2tu(t)\delta(t-2)$

Using the sifting property of Dirac function:

$$f_3(t) = \sinh(0)\,u(1)\delta(t) + 10\times -2\times\delta(2t+4) - 2\times 2u(2)\delta(t-2)$$

$$f_3(t) = -20\delta(2t+4) - 4\delta(t-2)$$

Note that $\delta(2t+4)$ is below zero (t=0)

$$F_3(s) = -4e^{-2s}$$

d)$f_4(t) = 12e^{4t}u(t) + 4te^{-3t}u(t-1)$

$$f_4(t) = 12e^{4t}u(t) + 4(t-1+1)e^{-3(t-1+1)}u(t-1)$$

$$f_4(t) = 12e^{4t}u(t) + 4(t-1)e^{-3}.\,e^{-3(t-1)}u(t-1) + 4e^{-3}.\,e^{-3(t-1)}u(t-1)$$

Using the time and frequency shift properties:

$$F_4(s) = \frac{12}{s-4} + \frac{4e^{-3}e^{-s}}{(s+3)^2} + \frac{4e^{-3}e^{-s}}{s+3}$$

$$F_4(s) = \frac{12}{s-4} + \frac{4e^{-(s+3)}}{(s+3)^2} + \frac{4e^{-(s+3)}}{s+3}$$