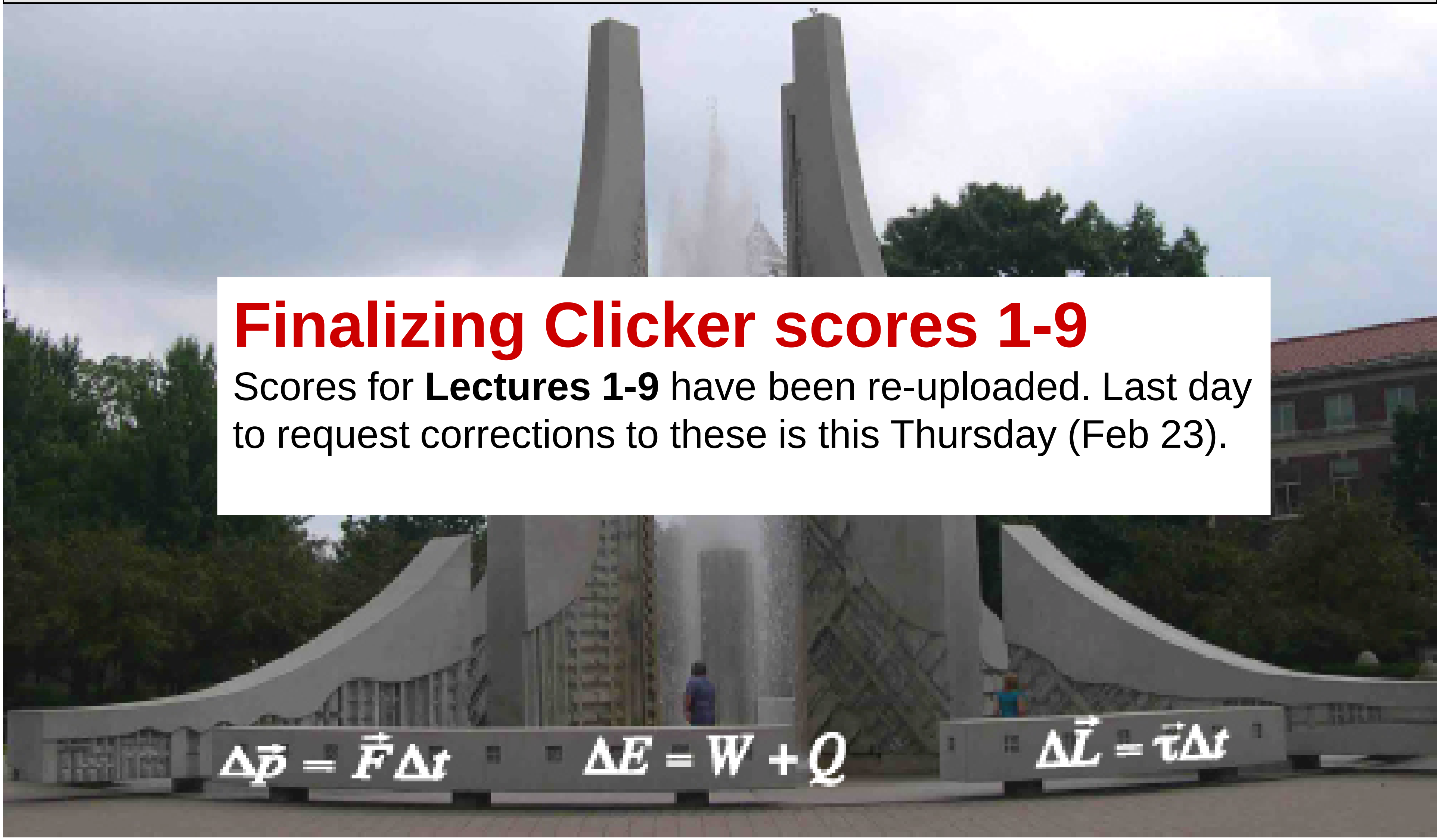
PHYS 172: Modern Mechanics **Spring 2012**

Finalizing Clicker scores 1-9

Scores for **Lectures 1-9** have been re-uploaded. Last day to request corrections to these is this Thursday (Feb 23).



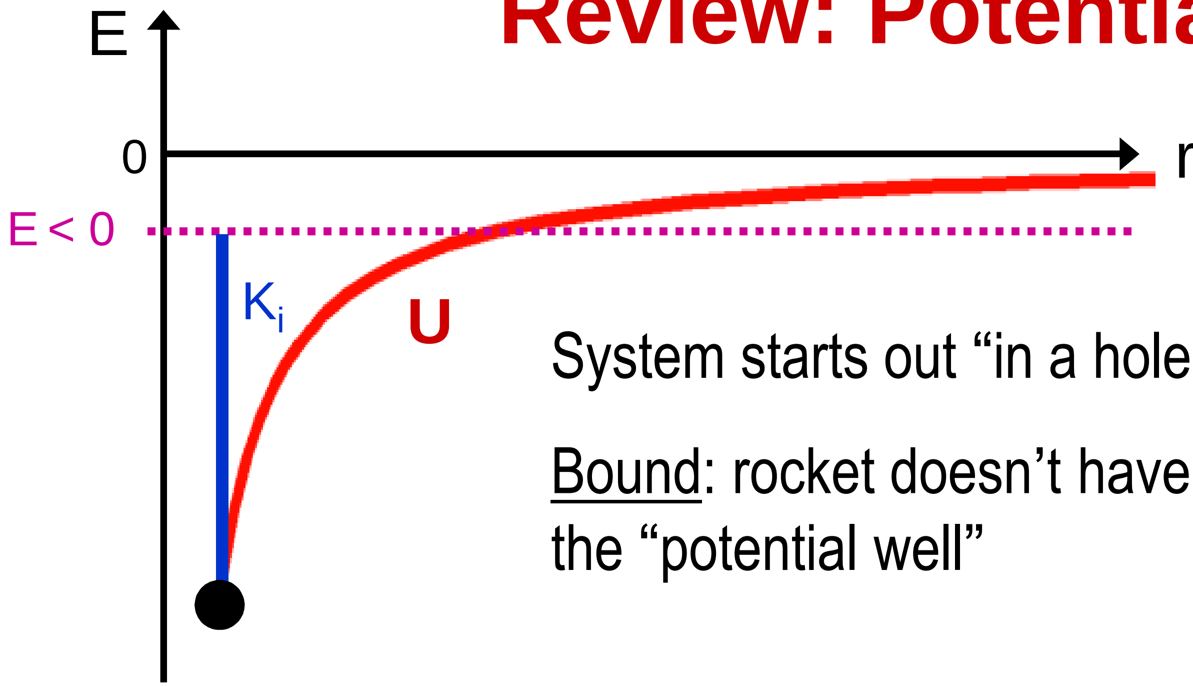
Lecture 12 – **The Energy Principle**

Read 6.15 – 6.17, 7.1 – 7.2

TODAY

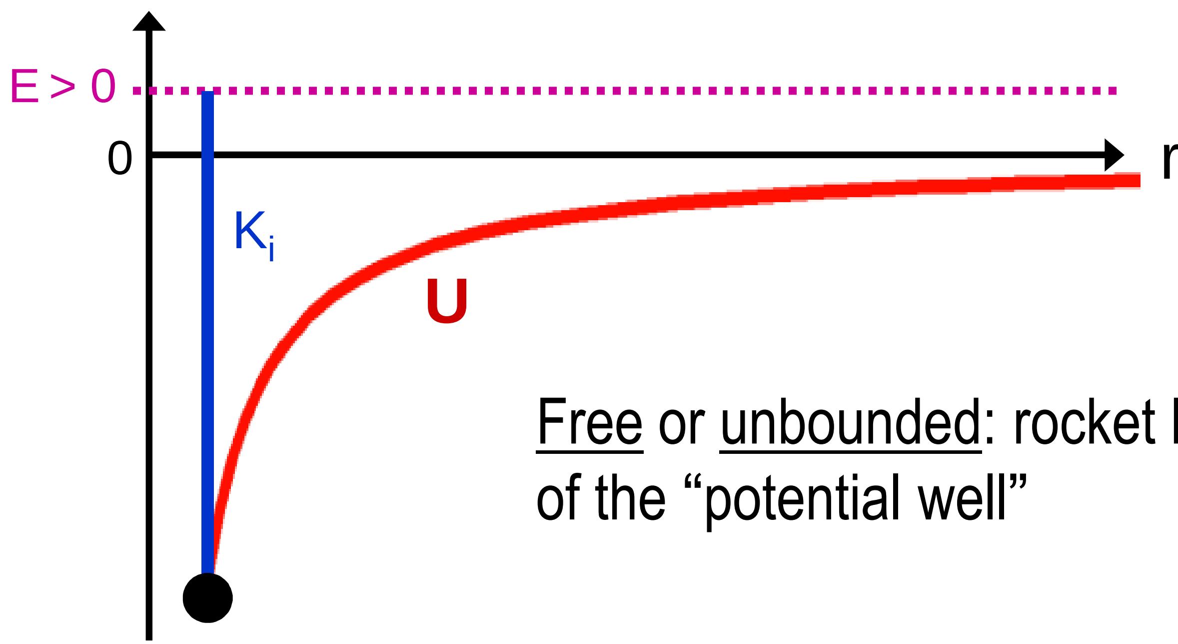
- Review: potential wells
- Mass and Energy
- Mass of a Multiparticle System
- Binding Energy
- Energy of a Mass-Spring System

Review: Potential Wells



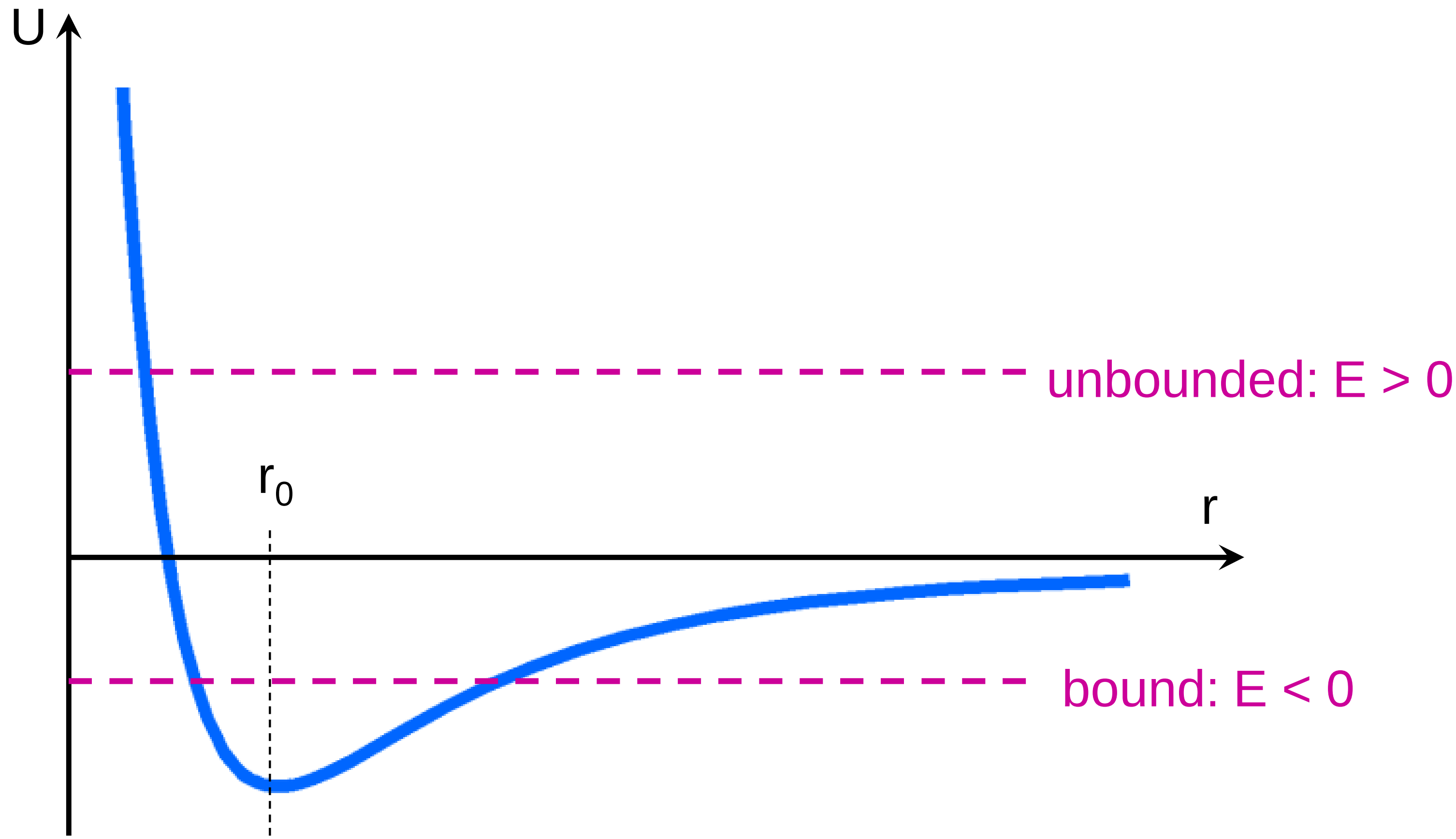
System starts out "in a hole" or a "well" since U<0

Bound: rocket doesn't have enough K_i to climb out of the "potential well"



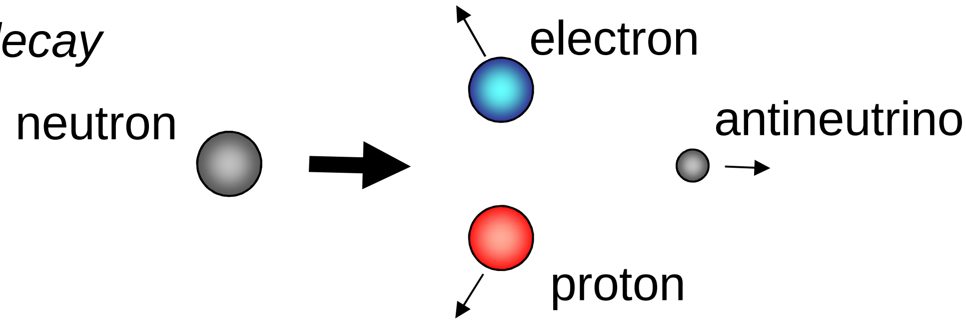
Free or unbounded: rocket has enough K_i to climb out of the "potential well"

Potential Energy for System of 2 Neutral Atoms



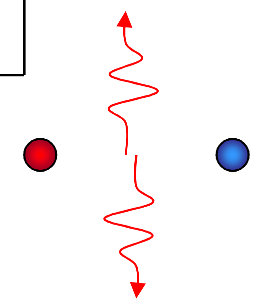
Mass and Energy

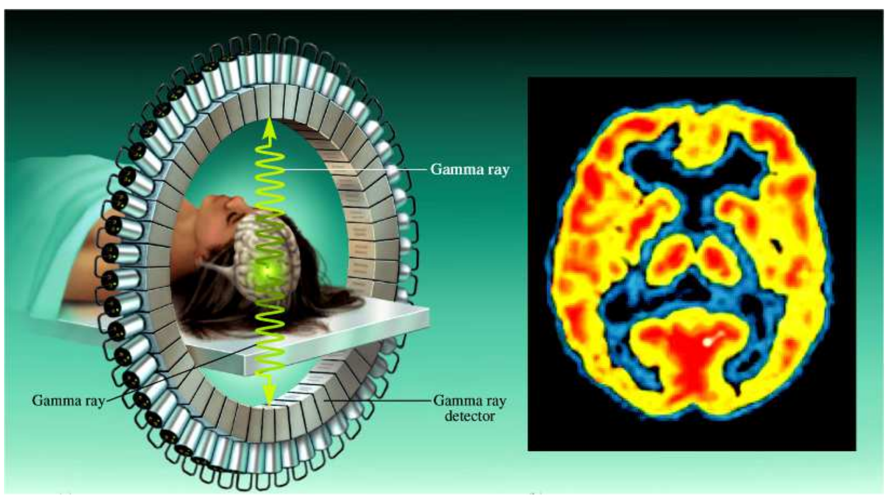
Example: neutron decay



Mass of products < mass of neutron!
Missing mass is converted into kinetic energy

Example: electron and positron (antielectron) annihilation:





Positron tomography

$$e^- + e^+ \to \gamma + \gamma$$

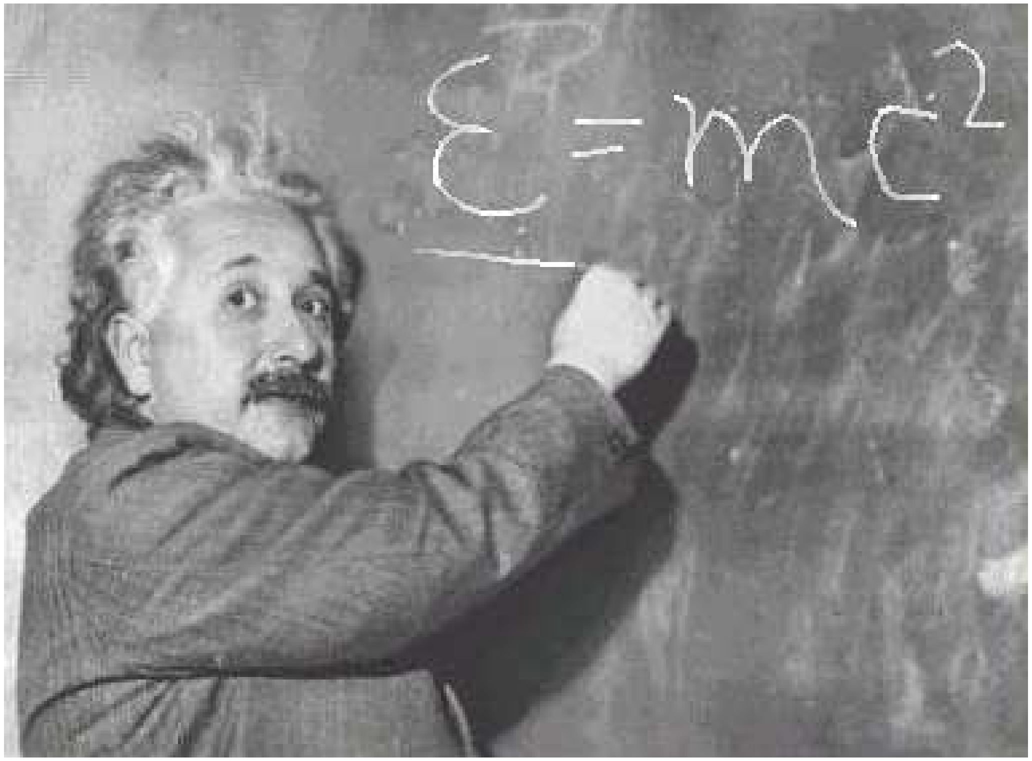
$$2m_e c^2 = 2E_\gamma$$

$$E_\gamma = 0.511\ \mathrm{MeV}$$

All mass is converted into electromagnetic radiation – gamma rays

Mass and Energy

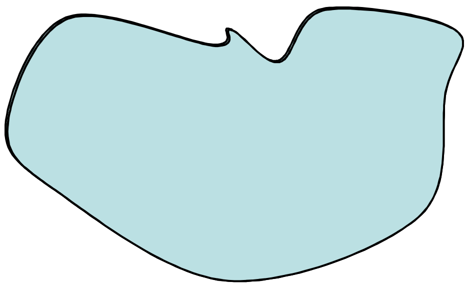
Mass and energy are the same thing!



There is no "mass conservation law"!
Energy, not mass is conserved!

Mass of a Multiparticle System

The System



$$E_{\text{system}} = \gamma M c^2 = M c^2 + K$$

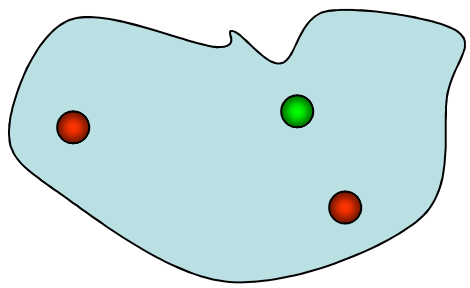
If the system is at rest, K=0 and:

$$M = \frac{E_{system}}{c^2} = \frac{m_1 c^2 + m_2 c^2 + \ldots + K_1 + K_2 + \ldots + U}{c^2}$$

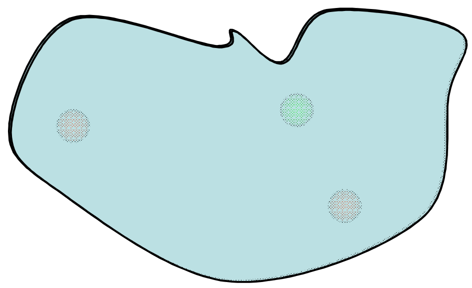
Mass of a multiparticle system:

$$\boxed{M = (m_1 + m_2 + \ldots) + \left(\frac{K_1 + K_2 + \ldots + U}{c^2} \right)}$$

Two ways of thinking



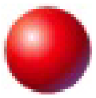
1. The energy of a multiparticle system consists of the individual particle energies plus their pair-wise interactions

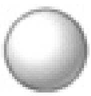


2. A system itself has energy, like a single particle, and if the system is at rest (not individual objects within the system!) its energy $E = Mc^2$, where M is mass of the system.

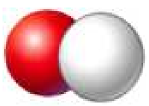
Binding energy: nuclear reactions

$$\boxed{1\,\mathrm{u} \equiv \frac{1}{12} m_{C^{12}} = 1.660539 \times 10^{-27}\ \mathrm{kg}}$$

 Proton: m = 1.007 276 5 u

 Neutron: m = 1.008 664 9 u

Adding these, get 2.015 941 4 u

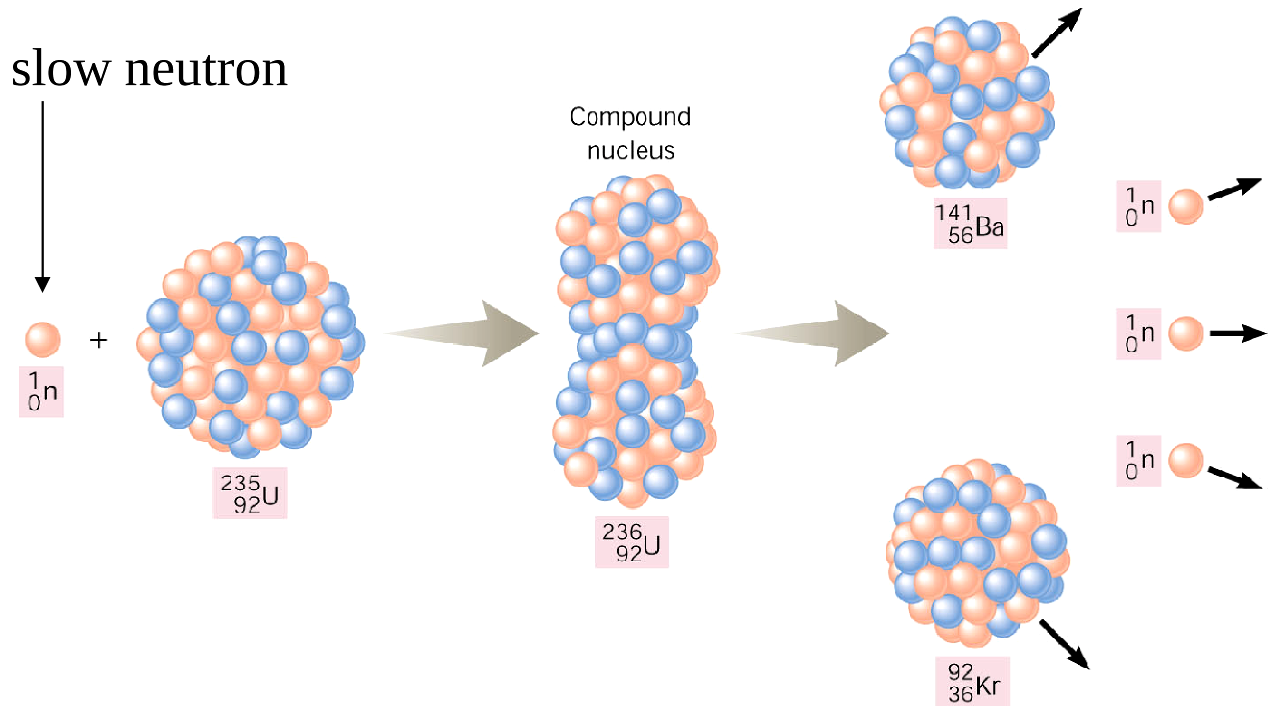
 Deuteron: m = 2.013 552 8 u

Difference is 0.002 388 6 u

Binding energy, Δmc^2 =2.2MeV

(1 eV = 1×10^{-19} J)

Nuclear fission



This process generates more than 1 neutron in product, they can initiate another reaction: *chain reaction*

Convert 1 pound into energy – enough to power one household for 100,000 years!

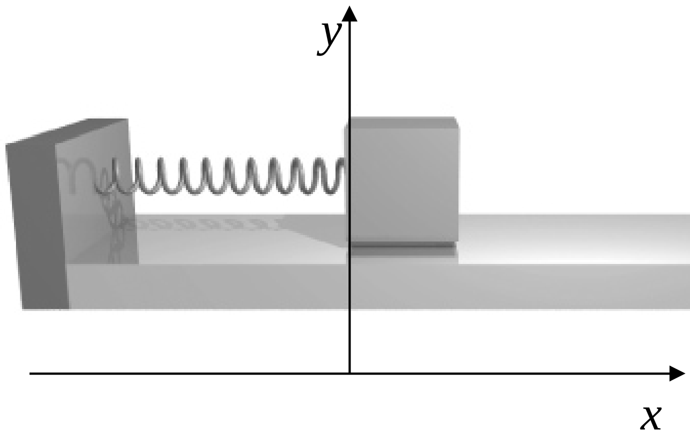
Energy versus forces

Energy: predict some aspects of motion

Limits of possible

Less details

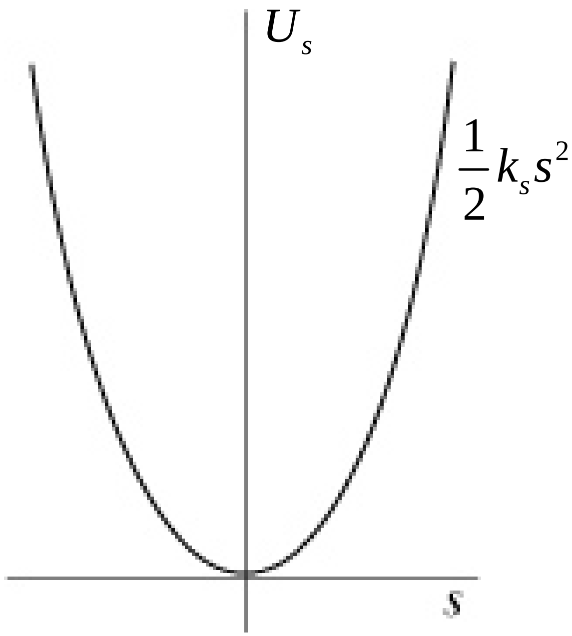
Potential energy of a spring



$$F_{s,x} = -k_s x$$

$$F_{s,x} = -\frac{\partial U_s}{\partial x}$$

$$U_s = \frac{1}{2} k_s x^2$$

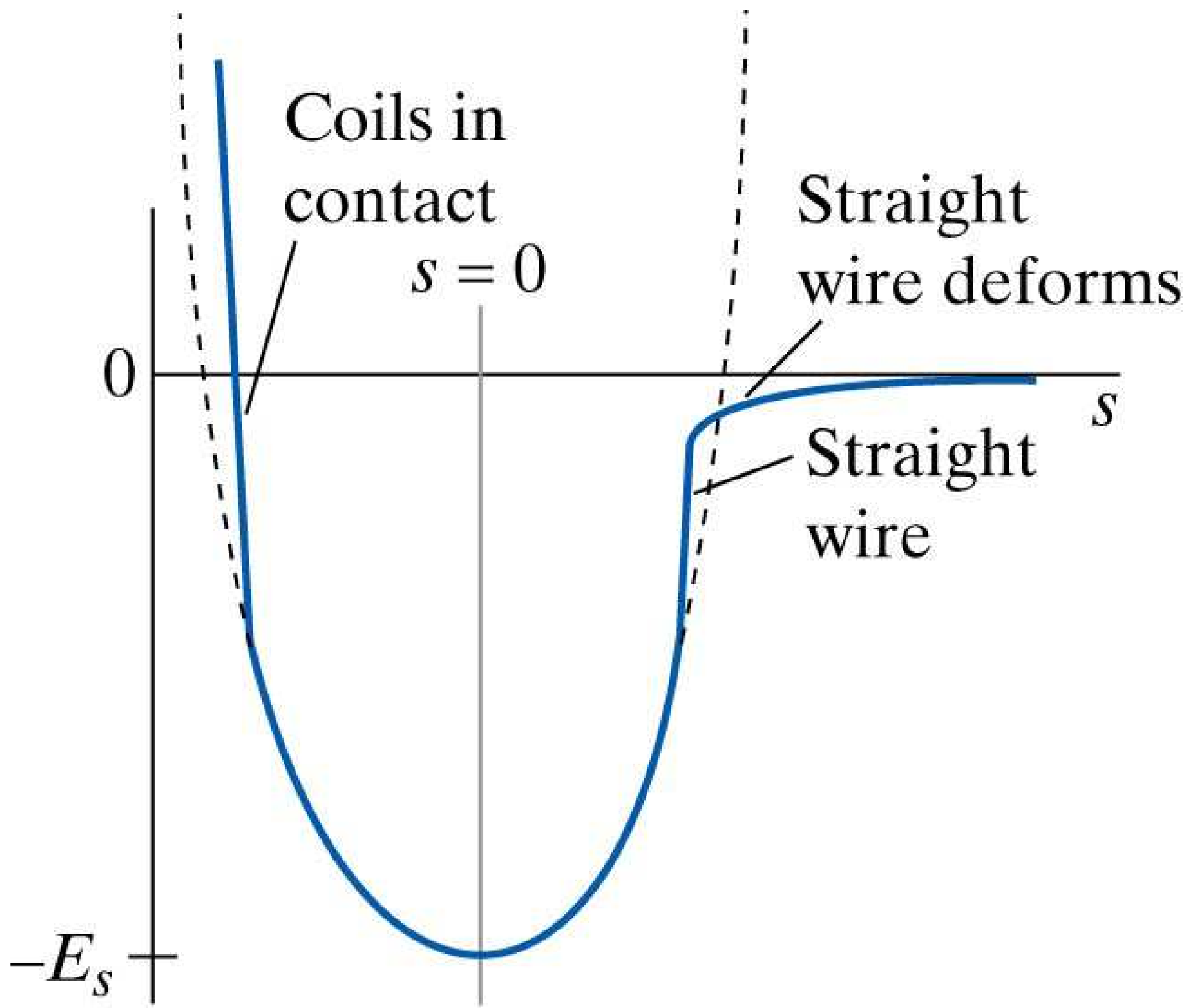


Potential energy of a spring:

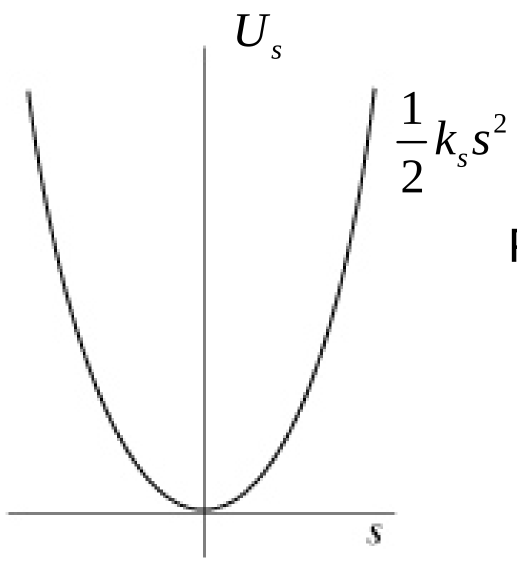
$$U_s = \frac{1}{2} k_s s^2$$

Assume U_s = 0 for relaxed spring

Real Life Physics: Potential energy of a spring

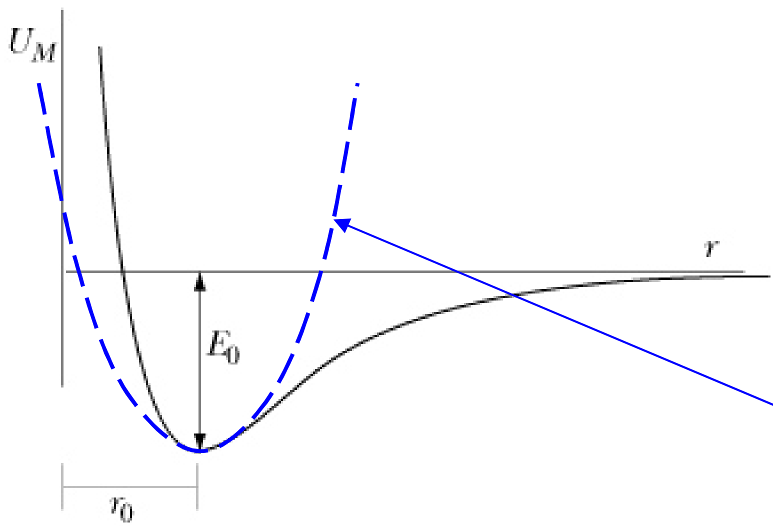


Interatomic potential energy between two atoms



Spring-mass model: $$U_s = \frac{1}{2}k_s s^2$$

Problem: Energy becomes infinite at large distances!



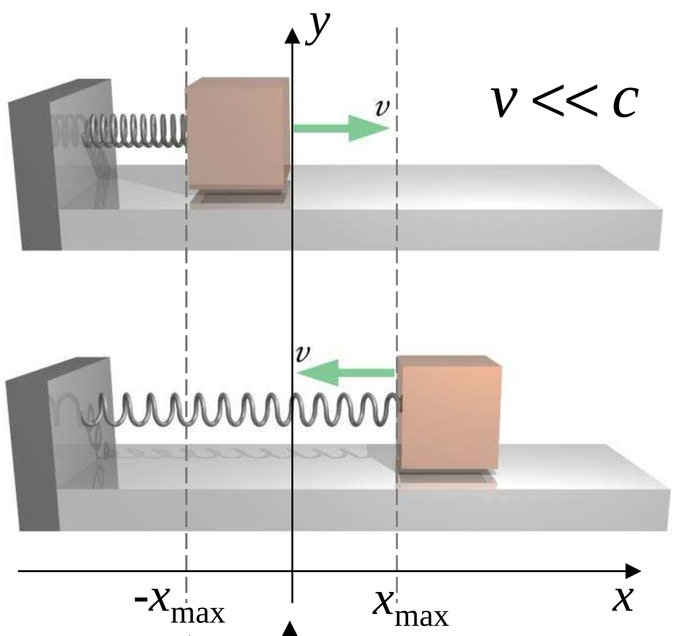
The **Morse potential** energy function is a realistic approximation

$$\boxed{U_M = E_0\left[1 - e^{\alpha(r-r_0)}\right]^2 - E_0}$$

Spring approximation:

$$U_M \approx \frac{1}{2}k_s r^2 - E_0$$

Energy of an oscillating spring-mass system



$$K = \frac{mv_{\max}^2}{2}$$
$$U = 0$$

$$K = 0$$
$$U = \frac{1}{2}k_s x_{\max}^2$$

Neglect friction:

$$\Delta E_{sys} = \Delta\left(mc^2 + K + U_s\right) = 0$$

$$\Delta\left(\frac{mv^2}{2} + \frac{1}{2}k_s x^2\right) = 0$$

Maximum speed:

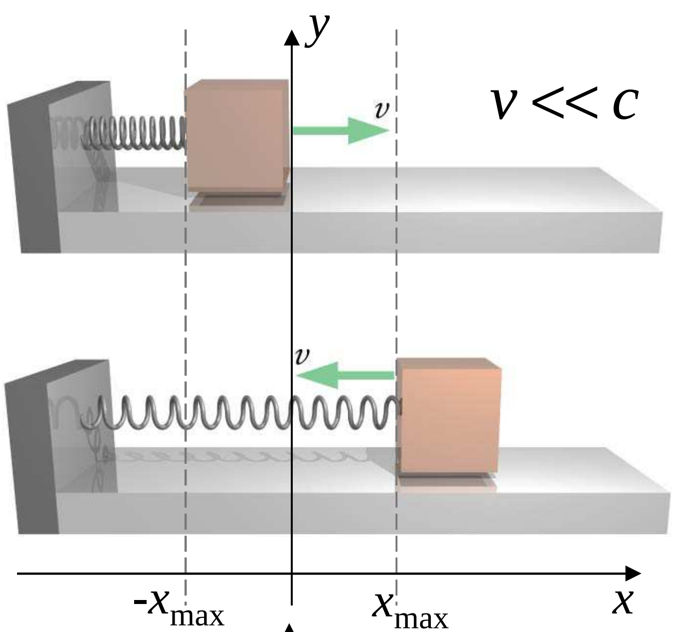
$$\frac{mv_{\max}^2}{2} = \frac{1}{2}k_s x_{\max}^2$$

Speed at any point:

$$\frac{mv^2}{2} + \frac{1}{2}k_s x^2 = \frac{1}{2}k_s x_{\max}^2$$

$$v = \sqrt{\left(k_s / m\right)\left(x_{\max}^2 - x^2\right)}$$

Energy of an oscillating spring-mass system



$v << c$

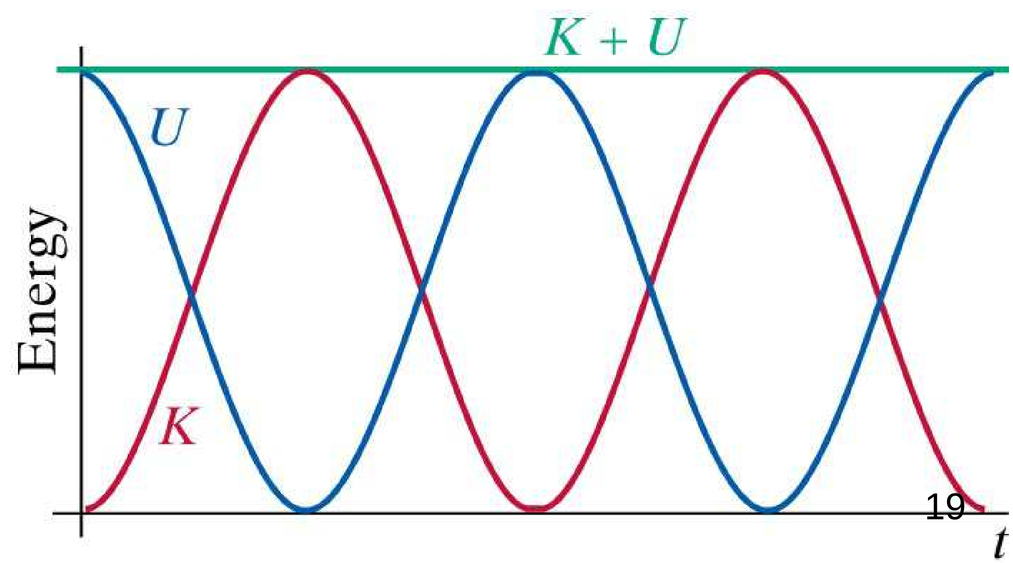
$$K = \frac{mv_{max}^2}{2}$$
$$U = 0$$

$$K = 0$$
$$U = \frac{1}{2} k_s x_{max}^2$$

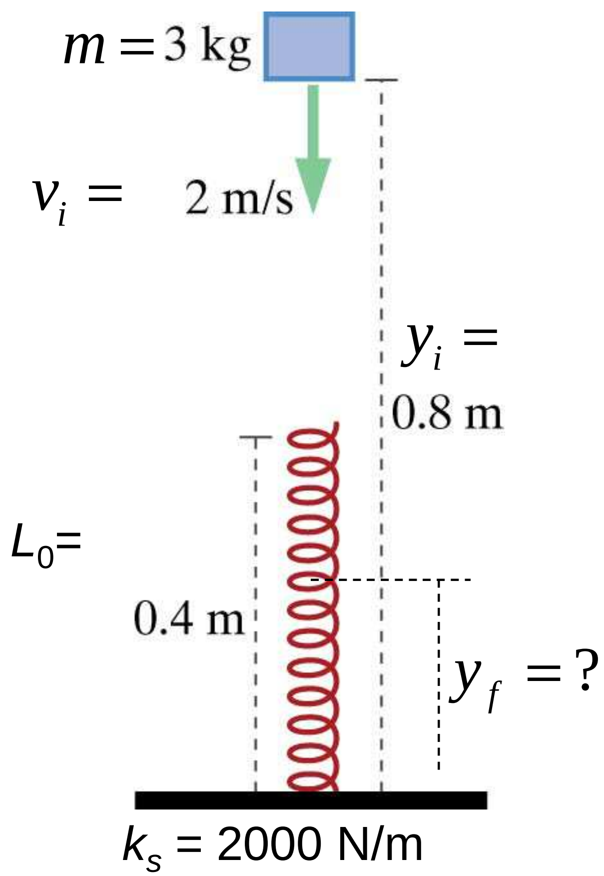
$$\Delta\left(\frac{mv^2}{2} + \frac{1}{2} k_s x^2\right) = 0$$

Same as

$$\Delta(K + U) = 0$$



HOME STUDY: Example, “a rebounding block”



1. What is the lowest point reached by the block?

System: block, spring, Earth
Assume: no interaction with surroundings

$$E_i = E_f$$

$$mgy_i + \frac{1}{2}mv_i^2 = mgy_f + \frac{1}{2}k_s\left(L_0 - y_f\right)^2$$

One equation, one unknown

2. What is the highest point reached by the block?

$$mgy_i + \frac{1}{2}mv_i^2 = mgy_f$$

One equation, one unknown

See details in the book (7.3), p. 295

WHAT WE DID TODAY

- Review: potential wells
- Mass and Energy
- Mass of a Multiparticle System
- Binding Energy
- Energy of a Mass-Spring System