ECE 202

HW2 Solution

Fall '13

5\. a) $F_1(s) = \dfrac{1}{s^2+2s-8} = \dfrac{A}{s-2} + \dfrac{B}{s+4}$

$A = (s-2)F_1(s)\Big|_{s=2} = \dfrac{1}{6}$

$B = (s+4)F_1(s)\Big|_{s=-4} = -\dfrac{1}{6}$

$\Rightarrow F_1(s) = \dfrac{1}{6}\cdot\dfrac{1}{s-2} - \dfrac{1}{6}\cdot\dfrac{1}{s+4}$

$\Rightarrow \boxed{f_1(t) = \dfrac{1}{6}\cdot e^{2t} - \dfrac{1}{6}e^{-4t}}$

b). $F_2(s) = \dfrac{7s^2-19s-2}{s^3-4s^2+s+6} = \dfrac{A}{s-2} + \dfrac{B}{s-3} + \dfrac{C}{s+1}$

$A = (s-2)F_2(s)\Big|_{s=2} = 4$

$B = (s-3)F_2(s)\Big|_{s=3} = 1$

$C = (s+1)F_3(s)\Big|_{s=-1} = 2$

$\Rightarrow F_2(s) = \dfrac{4}{s-2} + \dfrac{1}{s-3} + \dfrac{2}{s+1}$

$\Rightarrow \boxed{f_2(t) = 4e^{2t} + e^{3t} + 2e^{-t}}$

c) $F_3(s) = \dfrac{s^4 - 3s^3 - 6s^2 + 20s - 12}{s^4 - 3s^3 + 2s^2} \doteq \dfrac{s^4 - 3s^3 - 6s^2 + 20s - 12}{s^2(s-2)(s-1)}$

$$= K + \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-2} + \frac{D}{s-1}$$

$$K = \lim_{s \to \infty} F_3(s) = 1$$

$$B = s^2 F_3(s)\Big|_{s=0} = -6$$

$$C = (s-2)F_3(s)\Big|_{s=2} = -1$$

$$D = (s-1)F_3(s)\Big|_{s=1} = 0$$

$$F_3(-1) = -\frac{17}{3} = 1 + \frac{A}{-1} - 6 - \frac{1}{-3} + 0$$

$$\Rightarrow \quad A = 1$$

$$\Rightarrow F_3(s) = 1 + \frac{1}{s} - \frac{6}{s^2} - \frac{1}{s-2}$$

$$\Rightarrow \boxed{f_3(t) = \delta(t) + u(t) - 6r(t) - e^{2t} u(t)}$$

d) $$F_4(s) = \frac{2s^2 + (a-6b)s + a^2 - 4ab}{(s^2-a^2)(s-2b)} = \frac{2s^2 + (a-6b)s + a^2 - 4ab}{(s-a)(s+a)(s-2b)}$$

$$= \frac{A}{s-a} + \frac{B}{s+a} + \frac{C}{s-2b}$$

$$A = (s-a)F_4(s)\Big|_{s=a} = \frac{2a-5b}{a-2b}$$

$$B = (s+a)F_4(s)\Big|_{s=-a} = \frac{a+b}{a+2b}$$

$$C = (s-2b)F_4(s)\Big|_{s=2b} = \frac{-a^2+2ab+4b^2}{a^2-4b^2}$$

$$\Rightarrow \boxed{f_4(t) = u(t)\left[\left(\frac{2a-5b}{a-2b}\right)\cdot e^{at} + \left(\frac{a+b}{a+2b}\right)\cdot e^{-at} + \left(\frac{-a^2+2ab+4b^2}{a^2-4b^2}\right)e^{2bt}\right]}$$

e) $$F_5(s) = \frac{2s^2+4s+12}{s^3+5s^2+17s+13} = \frac{2s^2+4s+12}{(s+1)(s^2+4s+13)}$$

$$= \frac{2s^2+4s+12}{(s+1)((s+2)^2+3^2)} = \frac{A}{s+1} + \frac{Bs+C}{(s+2)^2+3^2}$$

$$A = (s+1)F_5(s)\Big|_{s=-1} = 1$$

$$F_5(0) = \frac{12}{13} = 1 + \frac{0+C}{13} \Rightarrow C = -1$$

$$F_5(1) = \frac{1}{2} = \frac{1}{2} + \frac{B+C}{18} \Rightarrow B = -C = 1$$

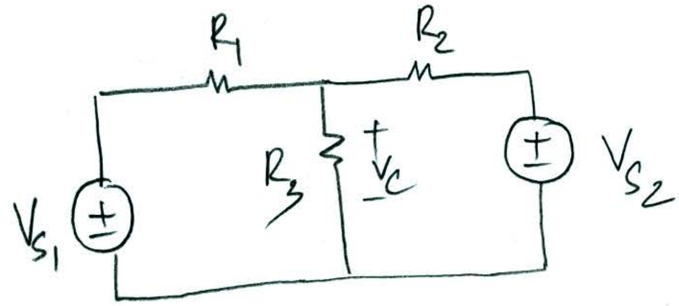
$$\Rightarrow F_5(s) = \frac{1}{s+1} + \frac{s-1}{(s+2)^2+3^2}$$

$$= \frac{1}{s+1} + \frac{s+2}{(s+2)^2+3^2} - \frac{3}{(s+2)^2+3^2}$$

$$\Rightarrow f_5(t) = e^{-t}u(t) + e^{-2t}\cdot\cos 3t\, u(t) - e^{-2t}\sin 3t \cdot u(t)$$

$$\boxed{f_5(t) = \left[e^{-t} + e^{-2t}(\cos 3t - \sin 3t)\right]u(t)}$$

(6) a)



$R_1 = 1,\ R_2 = 6,\ R_3 = 3$

Superposition: $V_C(s) = V_{S_1}(s) \cdot \frac{R_2 // R_3}{R_1 + R_2 // R_3} + V_{S_2}(s) \frac{R_1 // R_3}{R_1 // R_3 + R_2}$

$$\Rightarrow V_C(s) = V_{S_1}(s) \cdot \frac{2}{3} + V_{S_2}(s) \cdot \frac{1}{9}$$

$$= \frac{2}{3} \cdot \frac{3s+4}{s^2-16} + \frac{1}{9} \cdot \frac{12(s+1)}{s^2+6s+8}$$

$$= \frac{A}{s-4} + \frac{B}{s+4} + \frac{C}{s+2}$$

$$A = (s-4)V_C(s)\Big|_{s=4} = \frac{4}{3}$$

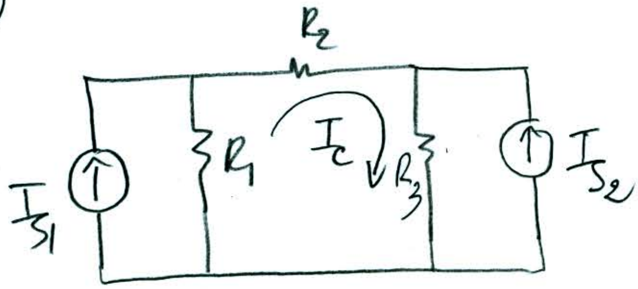
$$B = (s+4)V_C(s)\Big|_{s=-4} = \frac{8}{3}$$

$$C = (s+2)V_C(s)\Big|_{s=-2} = -\frac{2}{3}$$

$$V_c(s) = \frac{4}{3}\cdot\frac{1}{s-4} + \frac{8}{3}\cdot\frac{1}{s+4} - \frac{2}{3}\cdot\frac{1}{s+2}$$

$$\Rightarrow \boxed{V_c(t) = \frac{4}{3}e^{4t}u(t) + \frac{8}{3}e^{-4t}u(t) - \frac{2}{3}e^{-2t}u(t)}$$

b)



$R_1 = 12\Omega$
$R_2 = 24\Omega$
$R_3 = 48\Omega$

Superposition

$$I_c(s) = I_{S_1}\cdot\frac{R_1}{R_1+R_2+R_3} - I_{S_2}\cdot\frac{R_3}{R_1+R_2+R_3}$$

$$= I_S\cdot\frac{1}{7} - I_{S_2}\cdot\frac{4}{7}$$

$$= \frac{1}{7}\,\frac{20e^{-s}}{s^2+1} - \frac{4}{7}\cdot 90\,\frac{s^3-2s^2+16s-2}{(s^2+1)(s^2+16)}$$

let $F(s) = \frac{s^3-2s^2+16s-2}{(s^2+1)(s^2+16)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+16}$

$s=0 \Rightarrow -2 = 16B+D$
$s=1 \Rightarrow 13 = 17A+17B+2C+2D$
$s=-1 \Rightarrow -21 = -17A+17B-2C+2D$
$s=2 \Rightarrow 30 = 40A+20B+10C+5D$

$\Rightarrow A=1,\ B=0,\ C=0,\ D=-2$

$$\Rightarrow I_c(s) = \frac{1}{7}\,\frac{20e^{-s}}{s^2+1} - \frac{4}{7}.90\left[\frac{s}{s^2+1} - \frac{2}{s^2+16}\right]$$

$$= \frac{20}{7}\cdot\frac{e^{-s}}{s^2+1} - \frac{4}{7}.90.\ \frac{s}{s^2+1} + \frac{2}{7}.90.\ \frac{4}{s^2+4^2}$$

$$\Rightarrow \boxed{i_c(t) = \frac{20}{7}\sin(t-1)\,u(t-1) - \frac{360}{7}\cos(t)\,u(t) + \frac{180}{7}\sin(4t)\,u(t)}$$

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$$H(s) = \frac{24s}{s^2+64} = \frac{24s}{s^2+8^2}$$

a) $G(s) = L\left[e^{-4t}h(t)\right] = H(s+4) = \dfrac{24(s+4)}{(s+4)^2+64}$

b) $G(s) = L\left[t\,h(t)\right] = -\dfrac{d}{ds}H(s) = -\left[\dfrac{24(s^2+64)-48s^2}{(s^2+64)^2}\right]$

$$= \frac{24s^2 - 64\times 24}{(s^2+64)^2}$$

c) $G(s) = L\left[te^{-4t}h(t)\right] = -\dfrac{d}{ds}H(s+4)$

$$= \frac{24(s+4)^2 - 64\times 24}{\left[(s+4)^2+64\right]^2}$$

d) $G(s) = \mathcal{L}\left[\frac{d}{dt}\left(te^{-2t}h(t)\right)\right]$

let $g_1(t) = te^{-2t}h(t) \Rightarrow G_1(s) = \frac{24(s+2)^2 - 64\times 24}{\left((s+2)^2+64\right)^2}$

$$g(t) = \frac{d}{dt} g_1(t)$$

$$G(s) = sG_1(s) - g_1(0^-)$$

$$g_1(0^-) = 0.e^0 h(0) = 0$$

$$\Rightarrow \boxed{G(s) = sG_1(s) = \frac{24s(s+2)^2 - 64\times 24s}{\left[(s+2)^2+64\right]^2}}$$

e) $F(s) = \frac{s+4a}{(s+2a)^2}$, $a>0$

$$g(t) = e^{2at} f(t-2T)\, u(t-2T), \quad T>0$$

$$G(s) = \mathcal{L}[g(t)]$$

let $g_1(t) = f(t-2T)u(t-2T) \Rightarrow G_1(s) = e^{-2Ts}F(s)$

$$g(t) = e^{2at} g_1(t) \Rightarrow G(s) = G_1(s-2a)$$

$$\Rightarrow G(s) = e^{-2T(s-2a)} F(s-2a)$$

$$= e^{-2T(s-2a)} \frac{(s-2a)+4a}{(s-2a+2a)^2}$$

$$\boxed{G(s) = e^{-2T(s-2a)} \frac{s+2a}{s^2}}$$

8) a) i. $f(t) = -u(t+3) + 2u(t+2) - r(t+1) + r(t-3)$

ii. $L[g_1(t)] = L\left[\frac{d}{dt} f(t)\right] = sF(s) - f(0^-)$

$F(s) = L[f(t)]$

$= L[-r(t) + r(t-3)]$

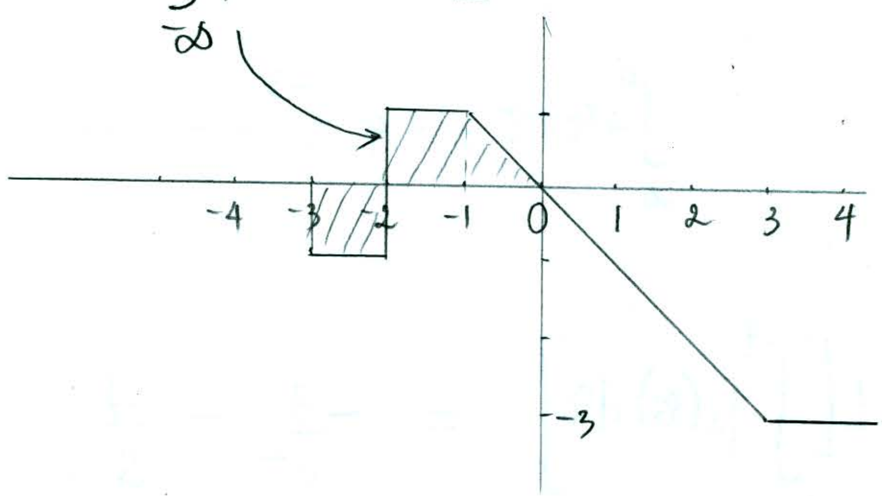
$= -\frac{1}{s^2} + \frac{e^{-3s}}{s^2}$

$f(0^-) = 0$

$\Rightarrow L[g_1(t)] = \frac{e^{-3s} - 1}{s}$

iii. $L(g_2(t)) = L\left[\int_{-\infty}^{t} f(\tau)\,d\tau\right] = \frac{F(s)}{s} + \frac{\int_{-\infty}^{0^-} f(\tau)\,d\tau}{s}$

$\int_{-\infty}^{0^-} f(\tau)\,d\tau = \frac{1}{2}$



$\Rightarrow G_2(s) = \frac{1}{s^3}(e^{-3s} - 1) + \frac{1}{2s}$

b. $f_2(t) = f(t+3)$

i. $f_2(t) = -u(t+6) + 2u(t+5) - r(t+4) + r(t)$

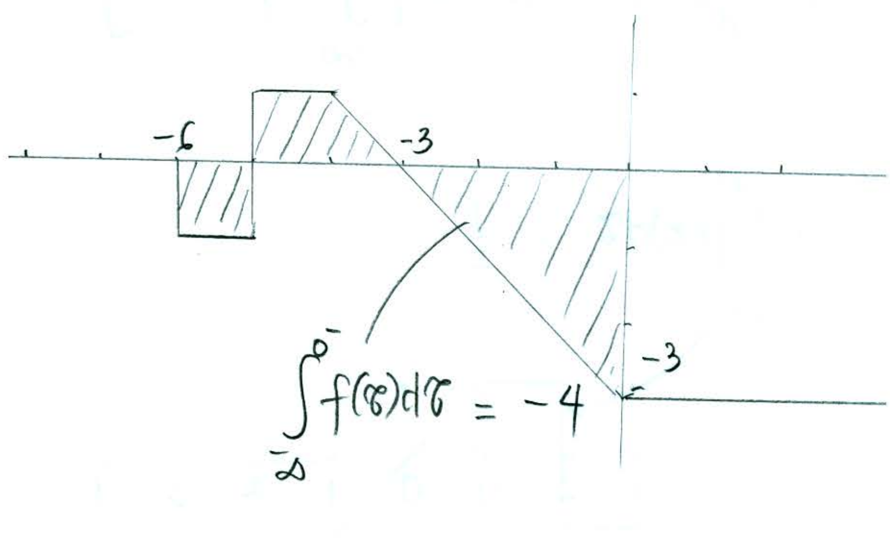
ii. $\mathcal{L}\left[\frac{d}{dt} f_2(t)\right] = sF_2(s) - f_2(0^-)$

$F_2(s) = \mathcal{L}[-3u(t)] = -\frac{3}{s}$

$f_2(0^-) = -3$

$\Rightarrow \mathcal{L}\left[\frac{d}{dt} f_2(t)\right] = s\left(-\frac{3}{s}\right) - (-3) = 0$

iii. $\mathcal{L}\left[\int_{-\infty}^{t} f_2(\tau)\,d\tau\right] = \frac{F_2(s)}{s} + \frac{\int_{-\infty}^{0^-} f(\tau)\,d\tau}{s}$



$\Rightarrow \mathcal{L}\left[\int_{-\infty}^{t} f_2(\tau)\,d\tau\right] = -\frac{3}{s^2} - \frac{4}{s}$