

$$x_1 \dots x_n$$

why $\frac{1}{n-1}$ rather than $\frac{1}{n}$:

$$S_x^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

unbiased estimator of σ^2 .

$$S_Y^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{Y})^2$$

$$= \frac{1}{n-1} \sum_{i=1}^n (aX_i + b - (a\bar{X} + b))^2$$

$$= \frac{1}{n-1} \sum_{i=1}^n (a(X_i - \bar{X}))^2$$

$$= a^2 \cdot \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

$$= a^2 \cdot S_x^2$$

linearity of summation

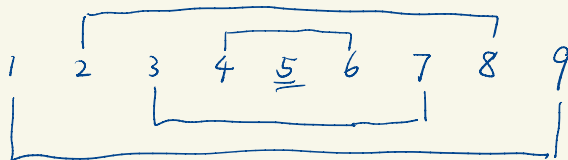
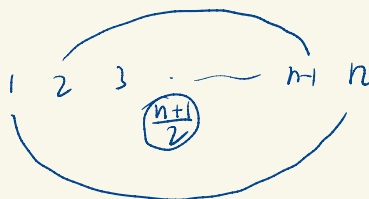
$$\textcircled{1} \quad \sum_{i=1}^n (a_i + b_i) = \sum_{i=1}^n a_i + \sum_{i=1}^n b_i$$

$$\textcircled{2} \quad \sum_{i=1}^n c \cdot a_i = c \cdot \sum_{i=1}^n a_i \quad (\text{c does not depend on } i)$$

$$\text{e.g.} \quad \sum_{i=1}^n a_i \cdot b_i$$

$$\text{e.g.} \quad \sum_{i=1}^{10} 5 = 5 \cdot \sum_{i=1}^{10} 1 = 5 \cdot 10 = 50$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$



if n even: $n/2$.

$$\sum_{i=1}^9 i = \sum_{i=1}^4 (i + (10-i)) + 5$$

$$= 40 + 5 = 45$$

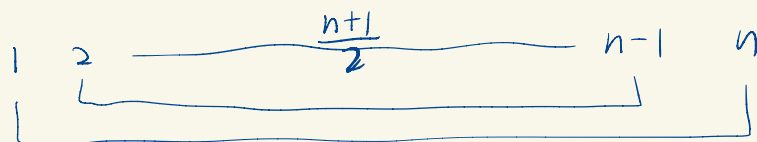
$$\sum_{j=1}^n \sum_{i=1}^n 1 = n^2$$

$$\sum_{j=1}^n \left(\sum_{i=1}^n 1 \right) = \sum_{j=1}^n n = n \cdot \sum_{j=1}^n 1 = n \cdot n = n^2$$

e.g. $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

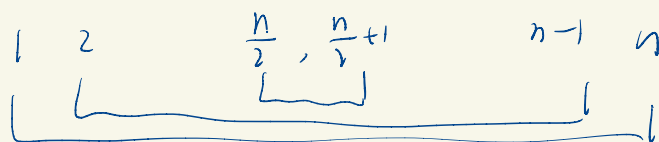
pf. ① if n is odd.

$n+1$ is even $k = \frac{n+1}{2}$



$$\sum_{i=1}^n i = \frac{n-1}{2} \cdot (n+1) + \frac{n+1}{2} = \frac{n+1}{2} (n-1+1) = \frac{(n+1)n}{2}$$

② if n is even.



$$\sum_{i=1}^n i = \frac{n}{2} \cdot (n+1)$$

$$2. \sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n \cdot \bar{x}^2$$

$$\text{pf.} \quad \text{LHS} = \sum_{i=1}^n (x_i^2 - 2\bar{x} \cdot x_i + \bar{x}^2)$$

$$= \sum_{i=1}^n x_i^2 - 2\bar{x} \cdot \sum_{i=1}^n x_i + n \cdot \bar{x}^2$$

$$= \sum_{i=1}^n x_i^2 - 2\bar{x} \cdot n \cdot \bar{x} + n \cdot \bar{x}^2$$

$$= \sum_{i=1}^n x_i^2 - n \cdot \bar{x}^2$$

$$\begin{aligned}
\sum_{i=1}^n (x_i - \bar{x})^2 &= \sum_{i=1}^n (x_i^2 - 2\bar{x} \cdot x_i + \bar{x}^2) \\
&= \sum_{i=1}^n x_i^2 - 2\bar{x} \cdot \sum_{i=1}^n x_i + n \cdot \bar{x}^2 \\
&= \sum_{i=1}^n x_i^2 - 2\bar{x} \cdot (n\bar{x}) + n \cdot \bar{x}^2 \\
&= \sum_{i=1}^n x_i^2 - n \cdot \bar{x}^2
\end{aligned}$$

play with 'Σ':

$$\sum_{i=1}^n a_i = a_1 + a_2 + \dots + a_n$$

$$\sum_{j=1}^n a_j = a_1 + \dots + a_n$$

dummy

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n 5 = 5 \cdot \sum_{i=1}^n 1 = 5 \cdot n$$

$$\sum_{i=1}^n x_i \cdot \bar{x} = \bar{x} \cdot \sum_{i=1}^n x_i = \bar{x} \cdot (n\bar{x}) = n \cdot \bar{x}^2$$

$$\sum_{j=1}^7 \sum_{i=1}^5 i$$

$$\sum_{j=1}^7 \sum_{i=1}^5 ij$$

e.g. discrete Cauchy-Schwarz inequality:

prop. $a_1, \dots, a_n, b_1, \dots, b_n \in \mathbb{R}$. then

$$\left(\sum_{i=1}^n a_i^2 \right) \cdot \left(\sum_{i=1}^n b_i^2 \right) \geq \left(\sum_{i=1}^n a_i \cdot b_i \right)^2$$

$$\begin{aligned}
\text{Pf. } \left(\sum_{i=1}^n a_i^2 \right) \left(\sum_{j=1}^n b_j^2 \right) &= \sum_{i=1}^n \sum_{j=1}^n a_i^2 b_j^2 = \sum_{i=1}^n \left(\sum_{j=1}^n a_i^2 b_j^2 \right) = \sum_{i=1}^n \left(a_i^2 \cdot \sum_{j=1}^n b_j^2 \right) = \left(\sum_{i=1}^n a_i^2 \right) \left(\sum_{j=1}^n b_j^2 \right) \\
&= \sum_{i=1}^n \sum_{j=1}^n a_i^2 b_j^2 \\
&= \frac{1}{2} \left(\sum_{i=1}^n \sum_{j=1}^n (a_i^2 b_j^2 + a_j^2 b_i^2) \right)
\end{aligned}$$

$$\left(\sum_{i=1}^n a_i b_i \right)^2 = \left(\sum_{i=1}^n a_i b_i \right) \left(\sum_{j=1}^n a_j b_j \right) = \sum_{i=1}^n \sum_{j=1}^n a_i b_i a_j b_j$$

$$\text{LHS} - \text{RHS} = \frac{1}{2} \cdot \sum_{i=1}^n \sum_{j=1}^n (a_i^2 b_j^2 + a_j^2 b_i^2 - 2a_i b_i a_j b_j)$$

$$= \frac{1}{2} \cdot \sum_{i=1}^n \sum_{j=1}^n (a_i b_j - a_j b_i)^2 \geq 0$$