1.

valid probability function.

① $p(x) \geq 0 \quad \forall x.$ ✓

② $\sum_{x \in S} p(x) = 1$

$$\sum_{x=1}^{k} \frac{2x}{k(k+1)} = \frac{2}{k(k+1)} \cdot \sum_{x=1}^{k} x = \frac{2}{k(k+1)} \cdot \frac{k(k+1)}{2} = 1$$

2. $\int_1^{+\infty} \frac{1}{x^2}\,dx$ improper integral.

$$:= \lim_{M\to+\infty} \int_1^M \frac{1}{x^2}\,dx$$

$$= \lim_{M\to+\infty} \left(-\frac{1}{x}\right)\Big|_1^M$$

$$= \lim_{M\to+\infty} \left(1-\frac{1}{M}\right) = 1.$$

expected value = mean

$$E[X] = \int_{-\infty}^{\infty} x\cdot f(x)\,dx = \int_1^{+\infty} x\cdot\frac{1}{x^2}\,dx$$

$$= \int_1^{+\infty} \frac{1}{x}\,dx$$

$$= \lim_{M\to+\infty} \int_1^M \frac{1}{x}\,dx$$

$$= \lim_{M\to+\infty} (\ln M - \ln 1)$$

does not exist.

3. exponential distribution with parameter λ.

pdf.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

cdf.

$$F(x) = \mathbb{P}(X \leq x) = 1 - e^{-\lambda x}$$

$\mathbb{E}[X] =$

$\mathbb{E}[X^2] =$ ______ integral by part.

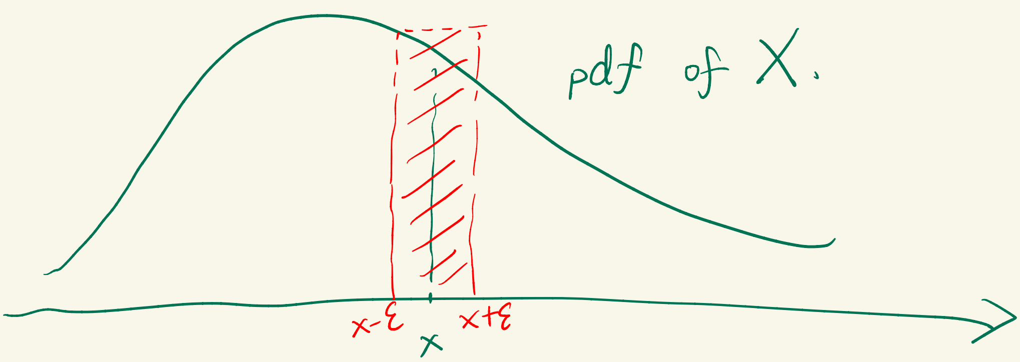
$$\text{var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2.$$

memoriless. property.

$$\mathbb{P}(X > t+s \mid X > s) = \mathbb{P}(X > t) \quad \forall\, s, t > 0.$$

why. Continuous. r.v. X. has property.

$$\mathbb{P}(X = x) = 0.$$



$$\mathbb{P}(X=x) \leq \mathbb{P}(x-\varepsilon < X < x+\varepsilon)$$

$$= \int_{x-\varepsilon}^{x+\varepsilon} f(x)\,dx$$

$$\leq \int_{x-\varepsilon}^{x+\varepsilon} \sup_{t\in[x-\varepsilon,x+\varepsilon]} f(t)\,dx$$

$$= [(x+\varepsilon)-(x-\varepsilon)] \cdot \sup_{t\in(x-\varepsilon,x+\varepsilon)} f(t).$$

$$= 2\varepsilon \cdot \sup \underline{\qquad\qquad}$$

$\varepsilon \to 0^+$

$$\mathbb{P}(X=x) \leq 0.$$

$$\mathbb{P}(X=x) = 0.$$