$X_1 \cdots X_n$

$$S_X^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

why $\frac{1}{n-1}$ rather than $\frac{1}{n}$:

unbiased estimator of σ^2.

$$S_Y^2 = \frac{1}{n-1} \sum_{i=1}^{n} (Y_i - \bar{Y})^2$$

$$= \frac{1}{n-1} \sum_{i=1}^{n} \left((aX_i+b) - (a\bar{X}+b)\right)^2$$

$$= \frac{1}{n-1} \sum_{i=1}^{n} \left(a(X_i - \bar{X})\right)^2$$

$$= a^2 \cdot \frac{1}{n-1} \cdot \sum_{i=1}^{n} (X_i - \bar{X})^2$$

$$= a^2 \cdot S_X^2$$

linearity of summation

① $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

② $\sum_{i=1}^{n} c \cdot a_i = c \cdot \sum_{i=1}^{n} a_i$ (c does not depend on i)

e.g. $\sum_{i=1}^{n} a_i \cdot b_i$

e.g. $\sum_{i=1}^{10} 5 = 5 \cdot \sum_{i=1}^{10} 1 = 5 \cdot 10 = 50$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

1 2 3 . —— n-1 n

$\frac{n+1}{2}$

1 2 3 4 $\underline{5}$ 6 7 8 9

if n even: $n/2$.

$$\sum_{i=1}^{9} i = \sum_{i=1}^{4} \left(i + (10-i)\right) + 5$$

$$= 40 + 5 = 45$$

$$\sum_{j=1}^{n} \sum_{i=1}^{n} 1 = n^2$$

$$\sum_{j=1}^{n} \left(\sum_{i=1}^{n} 1 \right) = \sum_{j=1}^{n} n = n \cdot \sum_{j=1}^{n} 1 = n \cdot n = n^2$$

e.g. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

pf. ① if n is odd.

$n+1$ is even $\quad k = \frac{n+1}{2}$

$1 \quad 2 \quad \text{———} \quad \frac{n+1}{2} \quad \text{———} \quad n-1 \quad n$

$$\sum_{i=1}^{n} i = \frac{n-1}{2} \cdot (n+1) + \frac{n+1}{2} = \frac{n+1}{2}(n-1+1) = \frac{(n+1)n}{2}$$

② if n is even.

$1 \quad 2 \quad \frac{n}{2}, \frac{n}{2}+1 \quad n-1 \quad n$

$$\sum_{i=1}^{n} i = \frac{n}{2} \cdot (n+1)$$

2. $\sum_{i=1}^{n}(X_i-\bar{X})^2 = \sum_{i=1}^{n} X_i^2 - n\cdot\bar{X}^2$

pf. $LHS = \sum_{i=1}^{n}(X_i^2 - 2\bar{X}\cdot X_i + \bar{X}^2)$

$$= \sum_{i=1}^{n} X_i^2 - 2\bar{X}\cdot\sum_{i=1}^{n} X_i + n\cdot\bar{X}^2$$

$$= \sum_{i=1}^{n} X_i^2 - 2\bar{X}\cdot n\cdot\bar{X} + n\cdot\bar{X}^2$$

$$= \sum_{i=1}^{n} X_i^2 - n\cdot\bar{X}^2 .$$

$$\sum_{i=1}^{n}(x_i-\bar{x})^2 = \sum_{i=1}^{n}(x_i^2 - 2\bar{x}\cdot x_i + \bar{x}^2)$$
$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\cdot\sum_{i=1}^{n}x_i + n\cdot\bar{x}^2$$
$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\cdot(n\bar{x}) + n\cdot\bar{x}^2$$
$$= \sum_{i=1}^{n} x_i^2 - n\cdot\bar{x}^2$$

play with 'Σ':

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$$
$$\sum_{j=1}^{n} a_j = a_1 + \cdots\cdots + a_n$$

dummy

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
$$\sum_{i=1}^{n} 5 = 5\cdot\sum_{i=1}^{n} 1 = 5\cdot n$$
$$\sum_{i=1}^{n} x_i\cdot\bar{x} = \bar{x}\cdot\sum_{i=1}^{n}x_i = \bar{x}\cdot(n\cdot\bar{x}) = n\cdot\bar{x}^2$$
$$\sum_{j=1}^{7}\sum_{i=1}^{5} i$$
$$\sum_{j=1}^{7}\sum_{i=1}^{5} ij$$

e.g. discrete Cauchy-Schwartz inequality.

prop. $a_1,\cdots,a_n, b_1,\cdots,b_n \in \mathbb{R}$. then
$$\left(\sum_{i=1}^{n} a_i^2\right)\cdot\left(\sum_{i=1}^{n} b_i^2\right) \geq \left(\sum_{i=1}^{n} a_i\cdot b_i\right)^2$$

pf. $$\left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{j=1}^{n} b_j^2\right) = \underline{\sum_{i=1}^{n}\sum_{j=1}^{n} a_i^2 b_j^2} = \sum_{i=1}^{n}\left(\sum_{j=1}^{n} a_i^2 b_j^2\right) = \sum_{i=1}^{n}\left(a_i^2\cdot\sum_{j=1}^{n} b_j^2\right) = \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{j=1}^{n} b_j^2\right)$$
$$= \sum_{i=1}^{n}\sum_{j=1}^{n} a_j^2 b_i^2$$
$$= \frac{1}{2}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}(a_i^2 b_j^2 + a_j^2 b_i^2)\right)$$

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 = \left(\sum_{i=1}^{n} a_i b_i\right)\left(\sum_{j=1}^{n} a_j b_j\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_i b_i a_j b_j$$

$$\text{LHS} - \text{RHS} = \frac{1}{2}\cdot\sum_{i=1}^{n}\sum_{j=1}^{n}(a_i^2 b_j^2 + a_j^2 b_i^2 - 2a_i b_i a_j b_j)$$
$$= \frac{1}{2}\cdot\sum_{i=1}^{n}\sum_{j=1}^{n}(a_i b_j - a_j b_i)^2 \geq 0$$