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part2
1
(a) A^+ = ABCDEFGH Because A is a superkey, A \to CF does not violate BCNF BCG^+ = BCDG Because BCG is NOT a superkey, BCG \to D violates BCNF CF^+ = ABCDEFGH Because CF is a superkey, CF \to AH does not violate BCNF D^+ = DB Because D is NOT a superkey, D \to B violates BCNF H^+ = BDEGH Because H is NOT a superkey, H \to DEG violates BCNF (b)
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- Decompose R using FD  $BCG \rightarrow D$ .  $BCG^+ = BCDG$ , so this yields two relations: R1 = BCDG and R2 = ABCEFGH.
- Project the FDs onto R1 = BCDG.  $B^+ = B$ ,  $C^+ = C$ ,  $G^+ = G$ ,  $D^+ = DB$ , so we need to decompose further because  $D \to B$  violates BCNF.
- Decompose R1 using FD  $D \rightarrow B$ .  $D^+ = DB$ , so this yields two relations: R3 = BD, R4 = CDG
- Project the FDs onto R3 = BD.  $D \rightarrow B$ , so D is a superkey. This relation satisfies BCNF.
- Project the FDs onto R4 = CDG.  $C^+ = C$ ,  $G^+ = G$ ,  $D^+ = DB$ ,  $CG^+ = CG$ ,  $CD^+ = CD$ ,  $DG^+ = DG$ , so there is no more FDs that we can use to decompose. This relation satisfies BCNF.
- Project the FDs onto R2 = ABCEFGH. We know that  $C^+ = C$ ,  $G^+ = G$ ,  $D^+ = DB$ ,  $CG^+ = CG$ ,  $H^+ = HBDEG$ , and those FDs violates BCNF. So this yields two relations: R5 = BEGH, R6 = ACFH
- Project the FDs onto R5 = BEGH. We know that  $B^+ = B, E^+ = E, G^+ = G, H^+ = BDEGH$ , so H is the superkey. This satisfies BCNF.
- Project the FDs onto R6 = ACFH. We know  $A^+ = ACFH$ ,  $C^+ = C$ ,  $F^+ = F$ ,  $H^+ = BDEG$ , A is the superkey so this FD satisfies BCNF.
- Final decomposition:
  - (a) R3 = BD with FD  $D \rightarrow B$
  - (b) R4 = CDG with no FDs
  - (c) R5 = BEGH with FD  $H \rightarrow BEG$
  - (d) R6 = ACFH with FD  $A \rightarrow CFH$

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2. (a) S = \{AB \rightarrow EF, B \rightarrow CEF, BCD \rightarrow AF, BCDE \rightarrow A, BCE \rightarrow D, DF \rightarrow C\} AB^+ = ABCDEF B^+ = ABCDEF BCD^+ = ABCDEF BCDE^+ = ABCDEF BCE^+ = ABCDEF BCE^+ = ABCDEF
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So the key is B because B is the smallest superkey

(b)Set
$$AB \rightarrow E \qquad As (1)$$

$$AB \rightarrow F \qquad As (2)$$

$$B \rightarrow C \qquad As (3)$$

$$B \rightarrow E \qquad As (4)$$

$$B \rightarrow F \qquad As (5)$$

$$BCDE \rightarrow A \qquad As (6)$$

$$BCD \rightarrow F \qquad As (7)$$

$$BCD \rightarrow A \qquad As (8)$$

$$BCE \rightarrow D \qquad As (9)$$

$$DF \rightarrow C \qquad As (10)$$

Then we can use the FDs above to compute the closure:

$$AB^{+}_{S-(1)} = ABE$$
 we can get rid of (1)

$$AB^{+}_{S-(1)-(2)} = ABF$$
 we can get rid of (2)

$$B^+_{S-(1)-(2)-(3)} = B$$

$$B^+_{S-(1)-(2)-(4)} = B$$

$$B^+_{S-(1)-(2)-(5)} = B$$

$$BCDE^{+}_{S-(1)-(2)-(6)} = BCDEA$$
 we can get rid of (6)

$$BCD^{+}_{S-(1)-(2)-(6)-(7)} = BCDF$$
 we can get rid of (7)

$$BCD^{+}_{S-(1)-(2)-(6)-(7)-(8)} = BCD$$

$$BCE^{+}_{S-(1)-(2)-(6)-(7)-(9)} = BCE$$

$$DF^{+}_{S-(1)-(2)-(6)-(7)-(10)} = DF$$

As a result we have:

$$B \rightarrow C$$
 As (3)  
 $B \rightarrow E$  As (4)  
 $B \rightarrow F$  As (5)  
 $BCD \rightarrow A$  As (8)  
 $BCE \rightarrow D$  As (9)  
 $DF \rightarrow C$  As (10)

We can simplify this result furthermore by reducing multiple attributes on the LHS:

We can reduce the LHS to (8) because  $B^+ = ABCDEF$ 

$$R \rightarrow A$$

We can reduce the LHS to (9) because  $B^+ = ABCDEF$ 

$$B \to D$$

After the change:

$$B \rightarrow C$$

$$B \rightarrow E$$

$$B \rightarrow F$$

$$B \rightarrow A$$

$$B \rightarrow D$$

$$DF \rightarrow C$$

Without  $B \to C$ , we can also get C by other FDs. Remove  $B \to C$ .

In conclusion, the minimal basis is:

$$B \rightarrow E$$

$$B \rightarrow F$$

$$B \rightarrow A$$

$$B \rightarrow D$$

$$DF \rightarrow C$$

(c)

Recall the revised FDs:

$$B \rightarrow ADEF$$

$$DF \rightarrow C$$

The set of relations that would result would have these attributes:

So the final set of relation is

$$R1(A, B, D, E, F)$$
  $R2(C, D, F)$ 

(D)

We cannot find a relation that violates BCNF after doing all the full projections:

$$B^+ = ABCDEF$$

B is the key of R1

$$DF^+ = CDF$$

DF is the superkey of R2.

So this schema does not allow redundancy.