Trust Region-Guided Proximal Policy Optimization

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Problem

- **Reinforcement Learning**

Find a policy π which could maximize the accumulative reward

$$\eta(\pi) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t c(s_t, a_t)|\pi\right]$$

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- **Policy Gradient**

$$L_{\pi_{\rm old}}(\pi) = \mathbb{E}_{s\sim\rho^{\pi_{\rm old}}, a\sim\pi_{\rm old}}\left[\frac{\pi(a|s)}{\pi_{\rm old}(a|s)} A^{\pi_{\rm old}}_{s,a}\right] + \eta(\pi_{\rm old}),$$

where $A^{\pi}_{s,a} = \mathbb{E}[R^{\gamma}_t|s_t = s, a_t = a; \pi] - \mathbb{E}[R^{\gamma}_t|s_t = s; \pi]$ is the advantage function value and $R^{\gamma}_t = \Sigma_{k=0}^{\infty}\gamma^k c(s_{t+k}, a_{t+k})$;
$\rho^{\pi}(s) = (1-\gamma)\,\Sigma_{t=1}^{\infty}\gamma^{t-1}\rho^{\pi}_t(s)$, ρ^{π}_t is the visitation probability of π at t.

- **Trust Region Policy Optimization (TRPO)**

$$\max_{\pi} L_{\pi_{\rm old}}(\pi) \text{ subject to } \max_{s\in\mathcal{S}} D^{\rm s}_{\rm KL}(\pi_{\rm old}, \pi) \le \delta$$

Maximizing $L_{\pi_{\rm old}}(\pi)$ within the trust region guarantee non-decreasing of the performance.

Theorem 1. *Let $M_{\pi_{\rm old}}(\pi) = L_{\pi_{\rm old}}(\pi) - C\max_{s\in\mathcal{S}} D^{\rm s}_{\rm KL}(\pi_{\rm old}, \pi)$,*

$$\eta(\pi) \ge M_{\pi_{\rm old}}(\pi), \eta(\pi_{\rm old}) = M_{\pi_{\rm old}}(\pi_{\rm old}).$$

- **Proximal Policy Optimization (PPO)**

$$L^{\rm CLIP}_{\pi_{\rm old}}(\pi) = \mathbb{E}\left[\min\left(\frac{\pi(a|s)}{\pi_{\rm old}(a|s)}A^{\pi_{\rm old}}_{s,a}, clip\left(\frac{\pi(a|s)}{\pi_{\rm old}(a|s)}, l_{s,a}, u_{s,a}\right)A^{\pi_{\rm old}}_{s,a}\right)\right]$$

PPO attempts to enforce restriction on the policy

$$l_{s,a} \le \frac{\pi(a|s)}{\pi_{\rm old}(a|s)} \le u_{s,a}$$
$$\Downarrow$$
$$-\pi_{\rm old}(a|s)(1 - l_{s,a}) \le \pi(a|s) - \pi_{\rm old}(a|s) \le \pi_{\rm old}(a|s)(u_{s,a} - 1)$$

clipping range: $(l_{s,a}, u_{s,a})$;
feasible range: $(-\pi_{\rm old}(a|s)(1 - l_{s,a}), \pi_{\rm old}(a|s)(u_{s,a} - 1))$.

Motivation

Original PPO: ***clipping range***: $(1-\epsilon, 1+\epsilon)$;
feasible range: $(-\pi_{\rm old}(a|s)\epsilon, \pi_{\rm old}(a|s)\epsilon)$.

Consider an optimal action $a_{\rm opt}$ and a sub-optimal one $a_{\rm subopt}$. If $\pi_{\rm old}(a_{\rm opt}|s) < \pi_{\rm old}(a_{\rm subopt}|s)$, then $|(-\pi_{\rm old}(a_{\rm opt}|s)\epsilon, \pi_{\rm old}(a_{\rm opt}|s)\epsilon)| < |(-\pi_{\rm old}(a_{\rm subopt}|s)\epsilon, \pi_{\rm old}(a_{\rm subopt}|s)\epsilon)|$. This means that the feasible range of $\pi(a_{\rm opt}|s)$ is limited than that of $\pi(a_{\rm subopt}|s)$.

Note that $\pi(a_{\rm opt}|s)$ and $\pi(a_{\rm subopt}|s)$ are in a zero-sum competition. Such unequal preference may continuously weaken the likelihood of the optimal action and make the policy trapped in local optima.

Contributions

- We propose an enhanced PPO, TRGPPO, to improve the exploration ability while not harming the learning stability.
- Theoretically prove that PPO is prone to suffer from the risk of lack of exploration and TRGPPO has better exploration ability.
- Extensive experiments demonstrate the effectiveness of TRGPPO on benchmark tasks.

Method

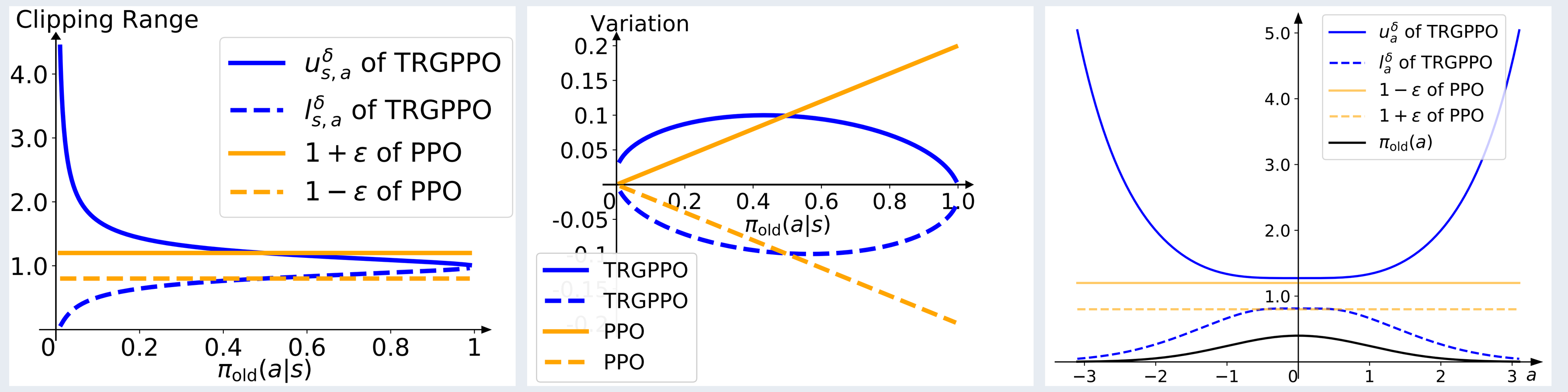
- **Improve exploration:** relax the restrictions on actions which are not preferred by $\pi_{\rm old}$.
- **Harmless to the learning stability:** make relaxation guided by trust region-based criterion.

The original PPO method:

$$l_{s,a} = 1-\epsilon,\ \text{for any } (s,a)$$
$$u_{s,a} = 1+\epsilon,\ \text{for any } (s,a)$$

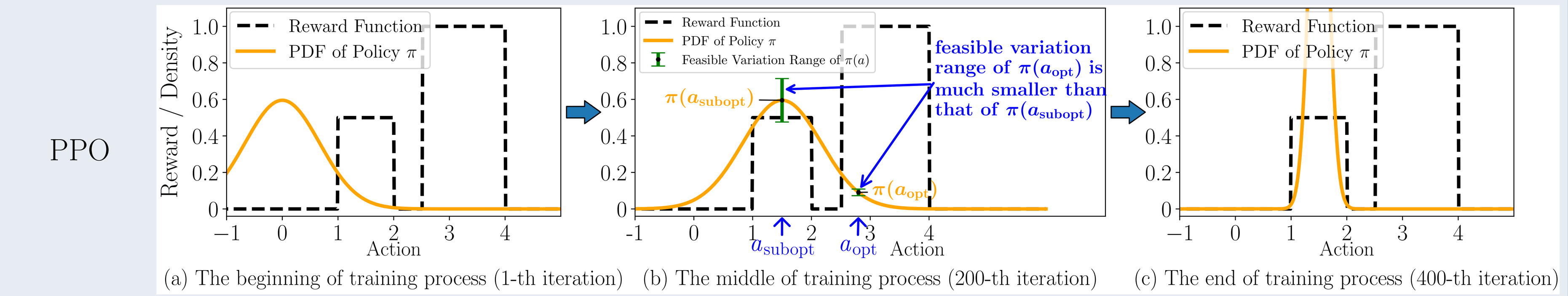
Our TRGPPO method:

$$l^{\delta}_{s,a} = \min_{\pi}\left\{\frac{\pi(a|s)}{\pi_{\rm old}(a|s)} : D^{s}_{\rm KL}(\pi_{\rm old}, \pi) \le \delta\right\}$$
$$u^{\delta}_{s,a} = \max_{\pi}\left\{\frac{\pi(a|s)}{\pi_{\rm old}(a|s)} : D^{s}_{\rm KL}(\pi_{\rm old}, \pi) \le \delta\right\}$$



(a) clipping range (discrete action space) (b) feasible range (discrete action space) (c) clipping range (continuous action space)

Didactic Example: A Continuous-armed Bandit Problem



(a) The beginning of training process (1-th iteration) (b) The middle of training process (200-th iteration) (c) The end of training process (400-th iteration)

Theoretical Analysis

1: Initialize a policy π_0, $t \leftarrow 0$.
2: **repeat**
3: Sample an action $\hat{a}_t \sim \pi_t$.
4: Get the new policy π_{t+1} by optimizing the surrogate objective function of PPO based on $\hat{a}_t$:

$$\hat{\pi}_{t+1}(a) = \begin{cases} \pi_t(a)u_a & a = \hat{a}_t \text{ and } c(a) > 0 \\ \pi_t(a)l_a & a = \hat{a}_t \text{ and } c(a) < 0 \\ \pi_t(a) - \frac{\pi_t(\hat{a}_t)u_{\hat{a}_t} - \pi_t(\hat{a}_t)}{|\mathcal{A}|-1} & a \ne \hat{a}_t \text{ and } c(\hat{a}_t) > 0 \\ \pi_t(a) + \frac{\pi_t(\hat{a}_t)(1-l_{\hat{a}_t})}{|\mathcal{A}|-1} & a \ne \hat{a}_t \text{ and } c(\hat{a}_t) < 0 \\ \pi_t(a) & c(\hat{a}_t) = 0 \end{cases}$$

5: $\pi_{t+1} = Normalize(\hat{\pi}_{t+1})$. $t \leftarrow t+1$.
6: **until** π_t converge

Algorithm 1: Simplified PPO for bandit problem

Notation:
$a_{\rm opt} = argmax_a\, c(a)$: the optimal action;
$a_{\rm subopt} \in \{a \in \mathcal{A} | c(a) > 0, a \ne a_{\rm opt}\}$: a sub-optimal action;
π^*: the optimal policy, where $\pi^*(a_{\rm opt}) = 1$, $\pi^*(a) = 0$ for $a \ne a_{\rm opt}$;
$\Delta_{\pi_0,t} \triangleq \mathbb{E}_{\pi_t}\left[\|\pi_t - \pi^*\|_{\infty}|\pi_0\right]$: the expected distance between π_t and π^*;

- **Convergence of PPO**

If the policy is initialized sufficiently far from the optimal one, PPO is expected to diverge from the optimal policy.

Theorem 2. *If $\pi_0^2(a_{\rm opt}) \cdot |\mathcal{A}| < \Sigma_{a_{\rm subopt}\in\mathcal{A}_{\rm subopt}}\pi_0^2(a_{\rm subopt}) - \Sigma_{a^-\in\mathcal{A}^-}\pi_0^2(a^-)$, then $\Delta_{\pi_0,0} < \Delta^{\rm PPO}_{\pi_0,1} < \cdots < \Delta^{\rm PPO}_{\pi_0,t}$.*

- **Characteristic of TRGPPO Compared to PPO**

1. **Better exploration:** larger clipping range for less explored actions; closer to the optimal policy.
2. **More sample efficient:** larger clipping range.
3. **Do not harm the learning stability:** not enlarging the policy divergence of the new policy with the old policy.
4. **Better performance guarantee:** larger empirical lower performance bound.

Lemma 3. $\frac{du^{\delta}_{s,a}}{d\pi_{\rm old}(a|s)} < 0$, $\frac{dl^{\delta}_{s,a}}{d\pi_{\rm old}(a|s)} > 0$.

Theorem 3. *If $\delta \le g(\max_{a\in\mathcal{A}_{\rm subopt}}\pi_t(a), 1+\epsilon)$ for all t, then $\Delta^{\rm TRGPPO}_{\pi_0,t} \le \Delta^{\rm PPO}_{\pi_0,t}$ for any t.*

Theorem 4. *If TRGPPO-ϵ and PPO have the same hyperparameter ϵ, then:*
(i) $u^{\delta}_{s_t,a_t} \ge 1+\epsilon$ and $l^{\delta}_{s_t,a_t} \le 1-\epsilon$ for all (s_t,a_t);
(ii) $\max_t D^{s_t}_{\rm KL}(\pi_{\rm old}, \pi^{\rm TRGPPO}_{\rm new}) = \max_t D^{s_t}_{\rm KL}(\pi_{\rm old}, \pi^{\rm PPO}_{\rm new})$;
(iii) $\hat{M}_{\pi_{\rm old}}(\pi^{\rm TRGPPO}_{\rm new}) \ge \hat{M}_{\pi_{\rm old}}(\pi^{\rm PPO}_{\rm new})$.

Experiments

Table: Reward function for Discrete Bandit

Action	$a_{\rm subopt}$	$a_{\rm opt}$	$a_{\rm worst}$
Reward	0.5	1	-50

Table: Reward function for Continuous Bandit

Action	$1 \leqslant a \leqslant 2$	$2.5 \leqslant a \leqslant 4$	otherwise
Reward	0.5	1	0

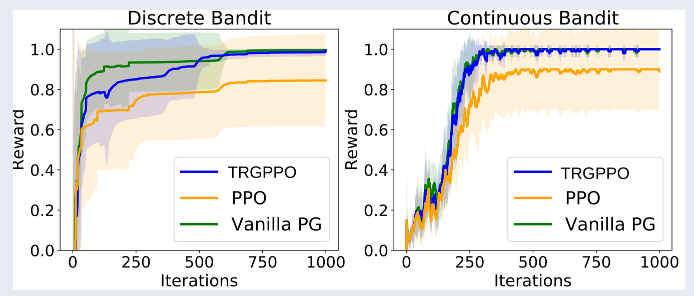


Figure: The performance on bandit problems.

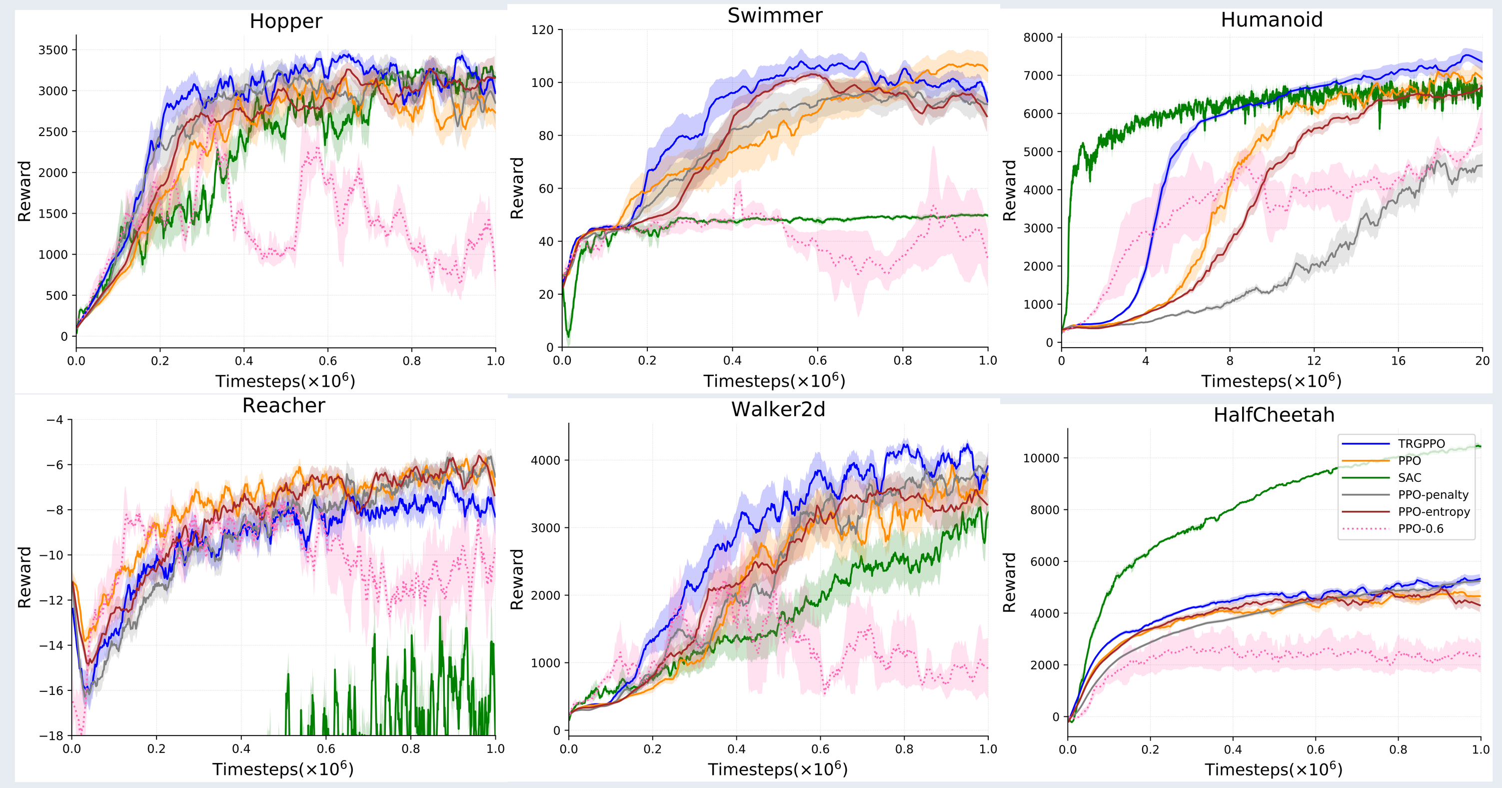
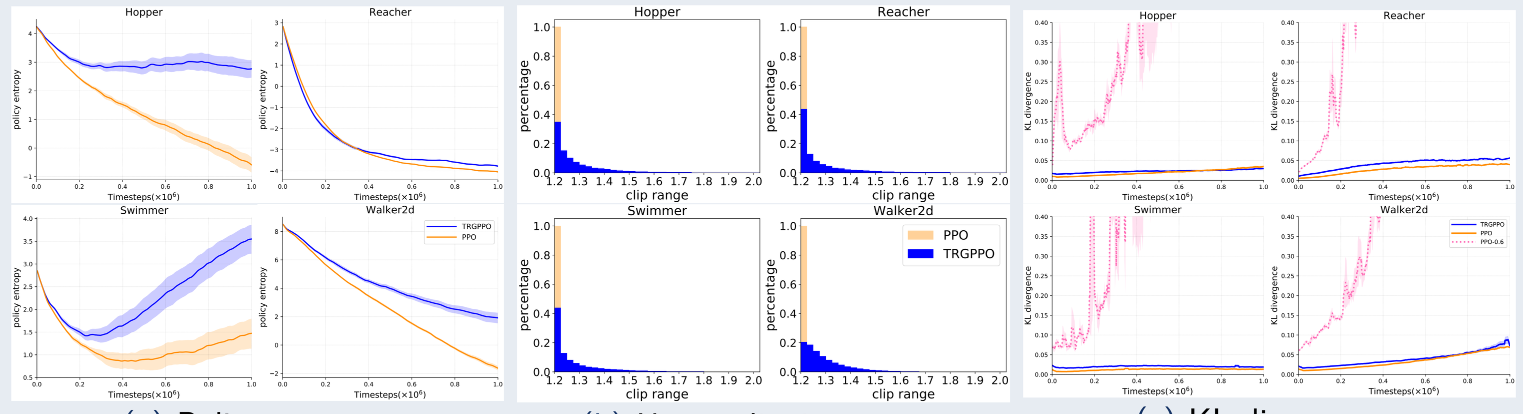


Figure: The performance on Mujoco during the training process averaged over 10 random seeds.



(a) Policy entropy (b) Upper clipping range (c) KL divergence

Figure: Characteristic of policy during the training process.

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