The antennas location problem
project for the class “exploitation mathématique de simulateurs numériques”

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1 General problem statement

We study the influence of the location of antennas for the covering of a territory. One antenna is characterized by its location (longitude and latitude) and its radius of emission. The territory is a given polytope. The territory considered in this project is the Loire department in France. An illustration of the problem with 2 and 3 antennas is given in Figure 1. We will assume that measuring or calculating the surface covered by a set of antennas is a costly experiment so that only n_{expl} experiments are available. Even though a final goal is to maximize the surface of the territory covered by the antennas by changing their location, we wish to gather complementary analyses on the way : instead of providing a single response, we will progress in steady steps and apply all the methods studied in class to this question : design of experiments, metamodeling, sensitivity analysis, local and global optimization.

Problems descriptions
Two variants of the problem are going to be looked at : the first problem has 2 dimensions, the second 6 dimensions.

In the $d = 2$ dimensional problem, only the location of 1 antenna is changed. Its longitude and latitude are described by $x \coloneqq (x_1, x_2)$ (the inputs). The output is the covered surface, $S(x)$. Note that there is a second, fixed antenna with a small emission radius in the center of the Loire (see Figure 1 left) to make $S(x)$ multimodal. The location of the antenna is bounded within the red rectangle, $\text{LB}_i \leq x_i \leq \text{UB}_i$, $i = 1, 2$, $\text{LB} = (-6, -6.5)$, $\text{UB} = (5, 8.7)$. A 3D plot of what $S(x_1, x_2)$ might look like is provided in Figure 2. It is not the true $S()$ function, but an approximation of it through the kriging mean after 20 observations.

In the $d = 6$ dimensional problem, 3 antennas are free to move within $[\text{LB}, \text{UB}]$. The variables are $x \coloneqq (x_1, \ldots, x_6)$ where x_1, x_3 and x_5 are the longitudes of the 3 antennas, and x_2, x_4 and x_6 their latitudes. There is no other fixed antenna.

R and matlab implementations
Each problem comes with 2 data files :

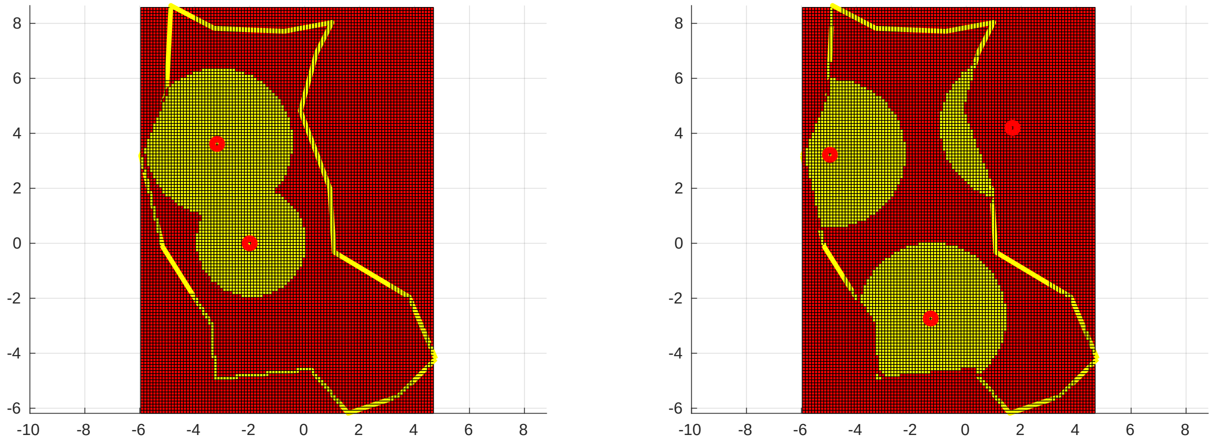

FIGURE 1 – The Loire Department in France with 2 and 3 antennas, left and right. The locations of the antennas are the red circles. The surface of the territory they cover is filled in yellow. An antenna can be outside of the territory (example on the right).

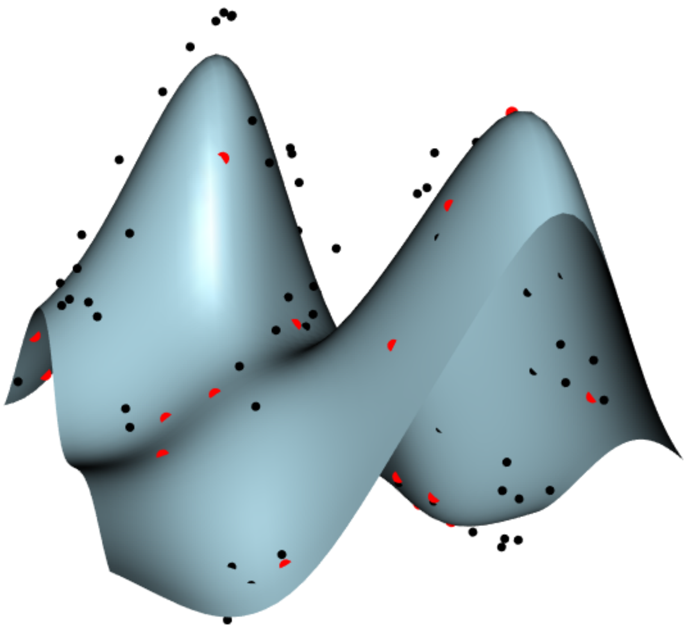

FIGURE 2 – Approximation of the Loire surface covered by one antenna for varying locations, $S(x_1, x_2)$. The approximation (silver surface) is the mean of a kriging model learned from 20 examples (the red bullets). The black bullets are test points (not learned), providing some insights into the imperfections of the approximation.

— `antennes_2d_train.Rdata`, `antennes_6d_train.Rdata` or their `.mat` versions (R or matlab versions) contain the training data : `C` is a $n_{expl} \times d$ matrix of inputs, each row is a data point, and `S` is the associated $n_{expl} \times 1$ vector of covered surfaces. When $d = 2$ there are $n_{expl} = 20$ data points, and $n_{expl} = 300$ when $d = 6$.
— `antennes_2d_test.Rdata`, `antennes_6d_test.Rdata` or their `.mat` versions (R or matlab versions) contain the test data : `C` is a 100×2 and 300×6 matrix when $d = 2$ and 6, respectively. No associated `S` matrix is provided. Making predictions about `S` will be part of the questions of the project.

2 Planning of the project and evaluation

Planning

— 26, 27, 28 November mornings : local optimization, support vector machines (SVM), Eric Touboul. Goal : have a working SVM to learn the mapping from x to S. Introduce the main data files for the project.
— 11 December : metamodeling, Xavier Bay. Goal : have a working kriging model.
— 12, 13 December, mornings : design of experiments, Mickaël Binois. Goal : know how to choose experiments and propose a complementary design.
— 17, 18 December, mornings : global sensitivity analysis, Espéran Padonou. Goal : measure the influence of each variables (coordinates of the antennas).
— 19 December, 6 January, mornings : global optimization, Rodolphe Le Riche. Goal : globally optimize the location of the antennas.
— 6 January 14 :00 : deadline to submit your projects
— 7 January, morning : presentation of the projects. All 3 members of the project team must be present.
— 7 January, afternoon : getting ready for the exam. The teachers provide a corrected exam to practice.
— 8 January, morning 9 :30-11 :30 : exam.

Project submission

Send by mail on January the 6 to `leriche@emse.fr`, `touboul@emse.fr`, `bay@emse.fr`, `mickael.binois@inria.fr` and `esperan.padonou@gmail.com`

1. the project report **as a file in pdf format** named `name1_name2_name3.pdf`.
2. the R code that has been used (you may want to zip the files into a single file)
3. A file in R format named `name1_name2_name3.Rdata` that contains the predictions at the test files locations as vectors with self-explicit names. The instructors will `load name1_name2_name3.Rdata` and expect to find, for example, vectors such as `Couv_2d_svr`, `Couv_2d_mean_krg`, `Couv_2d_sdv_krg`, which are vectors with $n_{expl} = 100$ predictions and variances. For the 6d problem, the vectors will have 300 components and 6d in their names.
   In addition, in the above `name1_name2_name3.Rdata` file, two optima for the 2d and two for the 6d problem will be proposed (CMA-ES and EGO approaches, see Section 7).

The solutions will be checked (with the true function) between the instructors and **all of the students** on January 7 morning.

Evaluation 60% of the grade based on the exam, 40% based on the project.

3 Local optimization, SVM (Eric Touboul)

4 Metamodeling (Xavier Bay)

In the 2d and 6d problems,

1. load the training data file (`antennes_2d_train.Rdata` or .mat, next the `6d` version),
2. build a kriging model using DiceKriging (if you work with R),
3. measure the quality of the kriging model by leave-one-out : calculate the Q2, plot the real versus predicted value, plot the standardizd residuals.
4. Load the testing data file (`antennes_2d_test.Rdata` or .mat, next the `6d` version), and make predictions with the kriging model at the testing locations. Add confidence intervals to the predictions.

5 Design of Experiments (Mickaël Binois)

Goal : Propose four new designs to improve the current designs of experiments (in 2D and 6D) for the subsequent tasks (e.g., optimization). Those can be sent in .csv format to obtain the corresponding evaluations.

1. a) Can you identify the type of design of experiments used to generate `antennes_2d_train$C`, `antennes_2d_test$C` and `antennes_6d_train$C`, `antennes_6d_test$C` ?
   b) Compare them with uniform designs of the same size and full-factorial designs with approximately the same size (5^2 in the 2D case, 3^6 in the 6D case). For instance, using the maximin distance, the centered L2 discrepancy (available in DiceDesign). Consider checking the `rss2d` function in that package too.
2. Propose one new design based on the maximin criteria and one based on the discrepancy criteria, for the 2D and 6D problems. In 2D represent the criteria.
3. Same as 2) but for the $IMSE$ and $maxVar$ criteria. The `DiceKriging` package can be used for fitting models.
4. You can send once two files with your 4 designs in 2D (resp. 6D) to `mickael.binois@inria.fr` in .csv format to get the corresponding outputs. (They might be useful for the subsequent tasks)

6 Global sensitivity analysis (Espéran Padonou)

6.1 Two dimensional case

Let (X_1, X_2) be a random variable, uniformly distributed over $[-6, 5] \times [-6.5, 8.7]$, corresponding to the position of the antenna. Then, $S(X_1, X_2)$ can be decomposed as $S_0 + S_1(X_1) + S_2(X_2) + S_{12}(X_1, X_2)$. Furthermore, the decomposition is unique under

Sobol conditions. The aim of this section is twofold : estimate the mean S_0, the main effects S_1 and S_2, and compute the normalized Sobol indices : $\text{var}\left(S_1\left(X_1\right)\right)$, $\text{var}\left(S_2\left(X_2\right)\right)$ and $\text{var}\left(S_{12}\left(X_1, X_2\right)\right)$ as percentages of $\text{var}\left(S\left(X_1, X_2\right)\right)$. Because a high number of points is needed for computations, the simulator S is approximated by a function $\hat{S}$ (the Kriging mean for instance ; section metamodeling). Then :

1. Simulate S_0 ; $S_1(t)$ for $t \in [-6, 5]$ and $S_2(t)$ for $t \in [-6.5, 8.7]$.
   Steps : Definition with conditional expectations, projection in $1D$, smoothing.
2. Compute the Sobol indices (function fast99, Package sensitivity).
3. Are the results consistent with the Morris method ?
4. By default, the fast99 function uses $q = "qunif"$. Comment this choice.

6.2 The 6 dimensional case

Given the 6d problem, explain why the Sobol decomposition of $S\left(X_1, X_2, X_3, X_4, X_5, X_6\right)$ would have 6 non nul terms at least. Implementation.

6.3 Bonus

The 2D antennas model is now applied to another territory (at choice : Benin, Morocco, Mali, Vietnam). The resulting dataset is given in the same fashion as for the Loire department. Propose a quantitative and/or qualitative sensibility analysis in this case.

7 Global optimization (Rodolphe Le Riche)

The addressed problem is

$$\begin{aligned} &\max_{x \in \mathbb{R}^d} S(x) \\ &\qquad \text{where } (x_{2i-1}, x_{2i}) \in [\text{LB}, \text{UB}]\ ,\ i = 1, \text{nb. of antennas} \end{aligned} \tag{1}$$

Two approaches are taken to guess where an optimal solution might be located.

7.1 CMA-ES optimization of the kriging mean

Problem (1), where $S(x)$ is unknown, is replaced with

$$\begin{aligned} &\max_{x \in \mathbb{R}^d} m(x) \\ &\qquad \text{where } (x_{2i-1}, x_{2i}) \in [\text{LB}, \text{UB}]\ ,\ i = 1, \text{nb. of antennas} \end{aligned} \tag{2}$$

where $m(x)$ is the kriging mean (i.e., kriging prediction) at x. This problem is solved with the CMA-ES algorithm provided in class.

7.2 One EGO iteration

In an attempt to both obtain a high performance point but also explore unknown regions of the design space, 1 iteration of the EGO algorithm is performed, i.e. the optimization problem

$$\begin{aligned} &\max_{x \in \mathbb{R}^d} EI(x) \\ &\quad \text{where } (x_{2i-1}, x_{2i}) \in [\text{LB}, \text{UB}] \ , \ i = 1, \text{nb. of antennas} \end{aligned} \tag{3}$$

is solved. The Expected Improvement (EI) function has been written in class. Again, the problem is solved with the CMA-ES algorithm.

Compare the solutions provided by Problems (2) and (3). Which one do you expect to perform best ? Why ? What would you do if you could afford 100 repetitions of the problems solving ?

Handout : A section in the report for the project. A code in R. R objects saved in the `name1_name2_name3.Rdata` file that contain the proposed solutions with their expected performance (kriging mean and standard deviation). Typically the objects could be vectors with self-explicit names such as

- `x_solution_cma_2d` : x solution vector of the problem of section 7.1 in 2d
- `y_solution_cma_2d` : scalar with the surface value associated to x=`x_solution_cma_2d`
- `x_solution_ego_2d` : x solution vector of the problem of section 7.2 in 2d
- `y_solution_ego_2d` : scalar with the surface value associated to x=`x_solution_ego_2d`

and similarly with the solution of the 6d versions of the problems with names ending in `..._6d`