In the above experiments, our algorithms proposed for minimizing the bi-direcitonal connectivity have shown to be effective and efficient to retrieve the clusters given the ground truth labels. But it is still valuable to question whether these advantages come from the utilization of the bi-directional connections. We propose to the function NMcut over all the different clusters to give the answer. Firstly, a smaller NMcut implies that the pairs of vertices across different clusters tend to hold weaker bi-directional connecitivity. Secondly, if our proposed methods can achieve low NMcut over different synthetic and empirical datasets, the impacts of introducing the relaxation to formulate the problems are ensured to be limited and the effectiveness of the proposed algorithms can be validated.

The results are reported in I. $\mathcal{G}_1$ and $\mathcal{G}_2$ are following the default synthetic experimental settings with $N = 2600, p_k = 0.1, r_{42,31,41} = 0.8, r_{21,32,43} = 2, r_n = 0.2$ and $N = 2600, p_k = 0.1, r_{32,31,41,42} = 2, r_n = 0.2$ respectively. All the experiments are following the default settings in the above subsections. The results are rescaled from 0 to the maximum values among different approaches to the range between 0 and 100 for the ease of comparison. Oracle is obtained with the ground truth clusters. Basically, RGC achieves the smallest NMcut on all the datasets, which demonstrates that our algorithms are effective on decreasing the mutual connecitions between different clusters and thus retrieve plausible clusters with balanced volumes. Also, the results of Oracle keep at a low level, validating the importances of the bi-directional connections on digraph clustering. And it is worth noting that the Oracle results may not always be the smallest. This result reminds us that our proposal focused on minimizing the bi-directional connections may not be the optimal choice for all the digraphs. As an unsupervised machine learning scheme, the framework has its own limitations and the real-world communities can exist because of more complicated causes.

Methods	$\mathcal{G}_1$	$\mathcal{G}_2$	**Tra-mes1**	**Tra-mes2**	**Tra-tra1**	**Tra-tra2**	**Ema-Eu**	**Wik-cat**
Dir-rem	52.19	61.23	82.74	56.32	78.65	75.39	35.81	72.49
Deg-dis	12.69	50.84	39.38	100	42.89	36.52	61.22	100
NGA-LP	31.82	47.19	67.83	31.51	33.10	42.93	26.94	37.41
Ran-wal	100	28.51	100	79.14	100	100	100	48.56
Mut-lin	72.36	100	70.54	47.81	69.62	51.82	72.18	88.91
RGC (Proposed)	**2.13**	**4.38**	**8.27**	**6.51**	**7.92**	**7.61**	**9.91**	**9.14**
Oracle	1.42	1.97	13.17	12.61	5.70	6.72	21.51	7.82

Table I: NMcut on different datasets achieved by different methods.