COMP3121/9101: Assignment 1

Due date: Tuesday 15 of June at Noon

In this assignment we review basic algorithms and data structures. You have **five problems**, marked out of a total of 100 marks.

NOTE: Your solutions must be typed, machine readable .pdf files. **All submissions will be checked for plagiarism!**

1. You are given an array A of n distinct positive integers.
   (a) Design an algorithm which decides in time $O(n^2 \log n)$ (in the worst case) if there exist four **distinct** integers m, s, k, p in A such that $m^2 + s = k + p^2$ (10 points)
   (b) Solve the same problem but with an algorithm which runs in the **expected time** of $O(n^2)$. (10 points)
2. You are given a set of n fractions of the form x_i/y_i $(1 \leq i \leq n)$, where x_i and y_i are positive integers. Unfortunately, all values y_i are incorrect; they are all of the form $y_i = c_i + E$ where numbers $c_i \geq 1$ are the correct values and E is a positive integer (equal for all y_i). Fortunately, you are also given a number S which is equal to the correct sum $S = \sum_{i=1}^{n} x_i/c_i$. Design an algorithm which finds all the correct values of fractions x_i/c_i and which runs in time $O(n \log \min\{y_i : 1 \leq i \leq n\})$. (20 points)
3. You are given an array A consisting of n positive integers, **not** necessarily all distinct. You are also given n pairs of integers (L_i, U_i) and have to determine for all $1 \leq i \leq n$ the number of elements of A which satisfy $L_i \leq A[m] \leq U_i$ by an algorithm which runs in time $O(n \log n)$. (20 points)
4. You are given an array containing a sequence of $2^n - 1$ consecutive positive integers starting with 1 except that one number was skipped; thus the sequence is of the form $1, 2, 3, \ldots, k-1, k+1, \ldots, 2^n$. You have to determine the missing term accessing at most $O(n)$ many elements of A. (20 points)
5. Read about the asymptotic notation in the review material and determine if $f(n) = O(g(n))$ or $g(n) = O(f(n)$ or both (i.e., $f(n) = \Theta(g(n))$) or neither of the two, for the following pairs of functions
   (a) $f(n) = \log_2(n)$; $\quad g(n) = \sqrt[10]{n}$; $\quad$ (6 points)

(b) $f(n) = n^n$; $\quad g(n) = 2^{n \log(n^2)}$; $\quad$ (6 points)

(c) $f(n) = n^{1+\sin(\pi n)}$; $\quad g(n) = n$. $\quad$ (8 points)

You might find useful L'Hôpital's rule: if $f(x), g(x) \to \infty$ and they are differentiable, then $\lim_{x\to\infty} f(x)/g(x) = \lim_{x\to\infty} f'(x)/g'(x)$