COMP9020 assignment 2

November 20, 2019

1 Problem 1

a

let $\varphi_1, \varphi_2, \varphi_3 \in F$

φ_1	φ_2	φ_3	$\varphi_1 \leftrightarrow \varphi_2$	$\varphi_2 \leftrightarrow \varphi_1$	$\varphi_2 \leftrightarrow \varphi_3$	$\varphi_1 \leftrightarrow \varphi_3$
F	F	F	T	T	T	T
F	F	T	T	T	F	F
F	T	F	F	F	F	T
F	T	T	F	F	T	F
T	F	F	F	F	T	F
T	F	T	F	F	F	T
T	T	F	T	T	F	F
T	T	T	T	T	T	T

$\varphi_1 \equiv \varphi_1$
For all $\varphi_1 \equiv \varphi_2$, there is $\varphi_2 \equiv \varphi_1$
For all $\varphi_1 \equiv \varphi_2 and \varphi_2 \equiv \varphi_3$, there is $\varphi_1 \equiv \varphi_3$
Based on the truth assignment table we can see the relation $\equiv$ is reflexive, symmetric and transive. Hence logical equivalence relation, $\equiv$, is an equivalence relation on F.

b

1. $(\perp \vee \perp)$
2. $(\perp \wedge \top)$
3. $(\perp \wedge \perp)$
4. $\neg\neg\perp$

c

Since $\varphi \equiv \varphi^{'}, \psi \equiv \psi^{'}$, we can get
$\varphi \leftrightarrow \varphi^{'} \Rightarrow (\neg\varphi \vee \varphi^{'}) \wedge (\neg\varphi^{'} \vee \varphi)$
$\psi \leftrightarrow \psi^{'} \Rightarrow (\neg\psi \vee \psi^{'}) \wedge (\neg\psi^{'} \vee \psi)$

φ	ψ	$\varphi^{'}$	$\psi^{'}$	$\neg\varphi^{'}$	$\varphi \wedge \psi$	$\varphi^{'} \wedge \psi^{'}$
F	F	F	F	T	F	F
F	T	F	T	T	F	F
T	F	T	F	F	F	F
T	T	T	T	F	T	T

(i) $(\neg\varphi \vee \varphi^{'}) \wedge (\neg\varphi^{'} \vee \varphi)$

$\Rightarrow (\neg\varphi^{'} \vee \varphi) \wedge (\neg\varphi \vee \varphi^{'})$

$\Rightarrow (\varphi \vee \neg\varphi^{'}) \wedge (\varphi^{'} \vee \neg\varphi)$

$\Rightarrow (\neg\neg\varphi \vee \neg\varphi^{'}) \wedge (\neg\neg\varphi^{'} \vee \neg\varphi)$

$\Rightarrow \neg\varphi \leftrightarrow \neg\varphi^{'}$

$\Rightarrow \neg\varphi \equiv \neg\varphi^{'}$

d

see picture how to form a boolean algebra
zero = $\perp$, one = $\top$

2 Problem 2

a

let $v_0 - > v_5, v_1 - > v_6, v_2 - > v_7, v_3 - > v_8, v_4 - > v_9$

b

As graph shows.

3 Problem 3

a

vertices stand for the subjects, edges stand for the adjcent vertices are non-conflict.
The graph problem here is to find maximum clique.

b

From the graph we can see that the maximum clique is 2, hence the maximum number of classes he can take is 2.

4 Problem 4

a

From the definition of T(n) we can derive a recurrence equation
Base case:
If it is an empty tree T(0) = 0
If it has only one node then it only has 1 structure T(1) = 0
Recurrence:
For an arbitrary tree with n nodes, the root has a left subtree of i nodes and a right subtree of n-i-1 nodes.
then we can have T(n) = $\sum_{i=0}^{n-1} T(i)T(n-i-1)$

b

1. total = leaves + internals. leaves = internals + 1. Hence total = internals * 2 + 1 which is odd.
2. Induction

c

B(n) = T((n-1)/2)

d

n = leaves
internals = leaves -1 = n-1

$F(n) = T(n-1) * n! * 2^n * 2^(n-1)$

5 Problem 5

a

$p_1(n+1) = \frac{1}{3}p_2(n) + \frac{1}{3}p_4(n)$

$p_2(n+1) = \frac{1}{2}p_1(n) + \frac{1}{2}p_3(n) + \frac{1}{3}p_4$

$p_3(n+1) = \frac{1}{3}p_2(n) + \frac{1}{3}p_4(n)$

$p_4(n+1) = \frac{1}{2}p_1(n) + \frac{1}{2}p_3(n) + \frac{1}{3}p_2$

b

When $p_i(n+1) = p_i(n)$ we can get
$p_1 = \frac{1}{3}p_2 + \frac{1}{3}p_4$

$p_2 = \frac{1}{2}p_1 + \frac{1}{2}p_3 + \frac{1}{3}p_4$

$p_3 = \frac{1}{3}p_2 + \frac{1}{3}p_4$

$p_4 = \frac{1}{2}p_1 + \frac{1}{2}p_3 + \frac{1}{3}p_2$

$p_1 + p_2 + p_3 + p_4 = 1$
Solve the equation
$p_1 = p_3 = 0.2, p_2 = p_4 = 0.3$