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COMP9101 Ass04

2. You have n warehouses and n shops. At each warehouse, a truck is loaded with enough goods to supply one shop. There are m roads, each going from a warehouse to a shop, and driving along the ith road takes d_i hours, where d_i is an integer. Design a polynomial time algorithm to send the trucks to the shops, minimising the time until all shops are supplied. (25 pts)

   Hint: Combine a binary search with a max flow. By sorting you can assume that d_i form an increasing sequence. Fix i and consider only roads which take $\leq d_i$ hours to travel from a warehouse to the corresponding shop and use max flow to see if they are enough to obtain a matching of warehouses with shops which is of size n. Use a binary search on i to find the smallest d_i which meets the requirements.

Q2.

According to the hint, we can sort the d_i, which is the time spent on each road, in an increasing order. Then we can build a bipartite graph, which the left side is the warehouse, the right part is the shops. And the super source in this problem connect to the warehouse, the capacity of each edge is 1. The super sink connects to the shop, which the capacity of each edge is also 1. Suppose there is a time limit (dt), if the track can transfer from warehouse to shops with the ith road in the dt, which means dt >= di, we can set the capacity of current edge as 1. If not, set edge as 0. Use binary search to check the number of shops that can receive from the warehouse under dt time. Calculate the maximum flow, if the result equal to the number of shops. If not, we should increase the time dt, and repeating above.

For sorting step, the time complexity is O(mlogm);
For max flow algorithm, by using the Preflow push, the time complexity is O((2n+2)^3)=O(n^3);
The binary search in the last step is O(logm)
Therefore, the total time complexity is O(mlogm)+O(n^3logm)