Pb1:

（a）The function below meet the requirement:

1) $f_{(a)}=f_{(b)}=f_{(c)}=1$
2) $f_{(a)}=f_{(b)}=1, f_{(c)}=0$
3) $f_{(a)}=f_{(c)}=1, f_{(b)}=0$
4) $f_{(c)}=f_{(b)}=1, f_{(a)}=0$
5) $f_{(a)}=f_{(c)}=0, f_{(b)}=1$
6) $f_{(b)}=f_{(c)}=0, f_{(a)}=1$
7) $f_{(a)}=f_{(b)}=0, f_{(c)}=1$
8) $f_{(a)}=f_{(b)}=f_{(c)}=0$

（b）The pow（{a,b,c}）contain the follow 8 elements:

{∅}.{a,b,c},{a,b},{b,c},{a,c},{a},{b},{c}

And we can find the number of elements in pow{a,b,c} are exactly same as the number of function in question(a), furthermore, if we treat 0 in question(a) means not exist, and 1 means exist. The elements in question(b) can have follow relationship:

{a,b,c} match 1) in question(a)
{a,b} match 2) in question(a)
{a,c} match 3) in question(a)
{c,b} match 4) in question(a)
{b} match 5) in question(a)
{a} match 6) in question(a)
{c} match 7) in question(a)
{∅} match 8) in question(a)

(c)

i. The number of functions from A to B are $\mathbf{n^m}$, because each element in A can match the number of m in B, meaning that, if A have m elements, the number of possible functions are n*n*n…n(the length of n is m),so the result is n^m

ii. The number of relations between A and B are $\mathbf{2^{mn}}$**,** because the number of relations between two sets, is the same as to calculate the card (A×B), so the result is 2^{mn}

iii. The number of symmetric relations on A is $2^{(\frac{m^2-m}{2}+m)}$, the symmetric means (x,y) ∈ A ,and (y,x) ∈A, to treat A as a matrix, and the number of elements in A is m^2, when x,y are not equal ,the number of elements is $\frac{m^2-m}{2}$, and we should add the element which x is equal to y, so the elements meet the symmetric is $\frac{m^2-m}{2}+m$, to get the number of relation is the same as we do in question(ii) is to calculate the card(A), is $2^{(\frac{m^2-m}{2}+m)}$

Pb2:

(a)$S_{2,-3}$={2m-3y:m,n∈Z}

When m =0, n=0, $S_{2,-3}$=0

m=0, n=1, $S_{2,-3}$=-3

$m=1$, $n=0$, $S_{2,-3}=2$

$m=1$, $n=1$, $S_{2,-3}=-1$

$m=2$, $n=1$, $S_{2,-3}=1$

the elements can be:0,-3,2,-1,1

(b)$S_{12,16}=\{12m+16y:m,n\in Z\}$,

When $m=0$, $n=0$, $S_{12,16}=0$

$m=0$, $n=1$, $S_{12,16}=16$

$m=1$, $n=0$, $S_{12,16}=12$

$m=1$, $n=1$, $S_{12,16}=28$

$m=2$, $n=1$, $S_{12,16}=40$

the elements can be:0,16,12,28,40

(c) Proof: $Sx,y = \{mx + ny : m, n\in Z\}$

To prove $Sx,y \subseteq \{n : n \in Z \text{ and } d|n\}$, we need every elements in $Sx,y\in Z$, and $Sx,y = kd$,

Because $m,n,x,y\in Z$, therefore $mx+ny\in Z$

Since $d=gcd(x,y)$,we have that: $x=k_1d,y=k_2d$

Therefore $Sx,y=mx+ny=mk_1d+nk_2d$, $(k_1,k_2,m,n\in Z)$

$=(mk_1+nk_2)d$,$(k_1,k_2,m,n\in Z)$

$Sx,y=k_3d(k_3\in Z)$

$Sx,y\subseteq \{n : n\in Z \text{ and } d|n\}$

(d) Proof: Because $n=kz$, $n\in Z$, and z is the smallest positive number in Sx,y

Suppose there exist m,n satisfy that: $z=m_1x+n_1y$

therefore $n=kz=k(m_1x+n_1y)=km_1x+kn_2y=(km_1)x+(kn_2)y$

because $km_1,kn_2\in Z$

it follows that: $(km_1)x+(kn_2)y\subseteq \{mx + ny : m, n\in Z\}$

(e) Proof: Because: $d=gcd(x,y)$,therefore $x=k_1d,y=k_2d$

Suppose there exist m,n satisfy that:

$z=m_1x+n_1y$ $(m,n\in Z)$

$z=k_1m_1d+k_2n_2d$

$z=(k_1m_1+k_2n_2)d$

$d=Z/(k_1m_1+k_2n_2)$

because $d,z\in Z$, $k_1m_1+k_2n_2\in N$ and $k_1m_1+k_2n_2>0$

$Z/(k_1m_1+k_2n_2)$ will get smaller than d, when $k_1m_1+k_2n_2$ get greater

Therefore $d \le z$

(f)Proof: Since $d=gcd(x,y)$, therefore we have $x=k_1d$, $y=k_2d$

We know that $S_{x,y}= mx + ny$ $(m, n\in Z)$

$S_{x,y}=mk_1d+n\,k_2d=(mk_1+nk_2)d$, because $m,n\in Z$,

Therefore $mk_1+nk_2\in Z$, $S_{x,y} =kd(k\in Z)$

Therefore $d\in S_{x,y}$

Because $m,n\in Z$, z is the smallest positive number in $S_{x,y}$

Therefore we have that $z\in S_{x,y}$

$z\in(mk_1+nk_2)d$, $(m, n,k_1,k_2\in Z)$

And we know that z is the smallest positive number, $d\in Sx,y$, d at least equal to z or greater than z.

Therefore we get $z \le d$

Pb3:

(a) $(A*B)*(A*B)$

$=(A^c \cup B^c)*(A^c \cup B^c)$ (definition from given)

$=(A^c \cup B^c) \cup (A^c \cup B^c)$ (definition from given)

$=(A^c \cup B^c)^c \cup (A^c \cup B^c)^c$ (de Morgan's Laws)

$=A \cap B$ (Idempotence)

(b) A^c

$=A^c \cup A^c$ (Idempotence)

$= A*A$ (definition from given)

(c) $\emptyset=A \cap A^c$

$=(A \cap A^c) \cup (A \cap A^c)$ (Idempotence)

$=(A^c \cup A)^c \cup (A^c \cup A)^c$ (de Morgan's Laws)

$=(A^c \cup A)*(A^c \cup A)$ (definition from given)

$=(A*A^c)*(A*A^c)$ (definition from given)

$=(A*(A*A))*(A*(A*A))$ (definition from given)

(d) $A\backslash B=A \cap B^c$

$=(A \cap B^c) \cup (A \cap B^c)$ (Idempotence)

$=(A^c \cup B)^c \cup (A^c \cup B)^c$ (de Morgan's Laws)

$=(A^c \cup B)*(A^c \cup B)$ (definition from given)

$=(A*B^c)*(A*B^c)$ (definition from given)

$=(A*(B*B))*(A*(B*B))$ (definition from given)

Pb4:

(a) if: $w=a$, $v=b$, $v \neq wz$

$w=b$, $v=a$, $v \neq wz$

(b) Because $R \leftarrow (\{aba\})$,

Since $v=aba=wz$

If: $w=a$, $z=ba$

$w=ab$, $z=a$

$w=aba$. $z=\lambda$

$w=\lambda$, $z=aba$

Therefore the answer is $\{a, ab, aba, \lambda\}$

(c) To show R is a partial order, we should prove R contain R, AS, T property.

For R:

If $(w,w) \in R$, $w=wz$, when $z=\lambda$, for all w satisfy the R

For AS:

Because $(w,v) \in R$, therefore $v=wz_1$

Because $(v,w) \in R$, therefore $w=vz_2$

For all $z_1=z_2=\lambda$, there exists $v=w$

Therefore (w,v), $(w,v) \in R$, $v=w$

We have AS

For T:

Because $(w,v) \in R$, Therefore $v=wz_1$

Because $(v,p) \in R$, Therefore $p=vz_2$

To combine p,w together ,we get that :$p=wz_1z_2(z_1z_2 \in Z)$

Therefore $p=kw(k \in Z)$

Therefore $(w,p) \in R$

That prove the T

Pb5:

From Pb2, in function : $S_{x,y} = \{mx + ny : m, n \in Z\}$,

To take z as the smallest positive number, d=gcd(x,y), we know that d=z, which satisfy the function: $S_{x,y} = \{mx + ny : m, n \in Z\}$

In problem 5:

When z=0, for all x: z=kx, therefore $x|z$

When $z \neq 0$:

Since $x|yz$

Therefore $yz=kx$

$$y=\frac{kx}{z}$$

From the conclusion in Problem 2 we know that: gcd(x,y)=1=d

Therefore for $\forall x,y$, $\exists m,n$, satisfy the function:

$mx+ny=1$ $(m,n,x,y \in Z)$

$mx+ny=1$

$mx+n\frac{kx}{z}=1$

$mxz+nkx=1$

$(zm+nk)x=z$

Since $z,m,n,k \in Z$

Therefore $zm+nk \in Z$

$kx = z$

Therefore $x|z$