**Pb1：**

**（a）**The function below meet the requirement:

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 

**（b）**The pow（{a,b,c}）contain the follow 8 elements:

{∅}.{a,b,c},{a,b},{b,c},{a,c},{a},{b},{c}

And we can find the number of elements in pow{a,b,c} are exactly same as the number of function in question(a), furthermore, if we treat 0 in question(a) means not exist, and 1 means exist. The elements in question(b) can have follow relationship:

{a,b,c} match 1) in question(a)

{a,b} match 2) in question(a)

{a,c} match 3) in question(a)

{c,b} match 4) in question(a)

{b} match 5) in question(a)

{a} match 6) in question(a)

{c} match 7) in question(a)

{∅} match 8) in question(a)

**(c)**

1. The number of functions from A to B are **nm** , because each element in A can match the number of m in B, meaning that, if A have m elements, the number of possible functions are n\*n\*n…n(the length of n is m),so the result is nm
2. The number of relations between A and B are **2mn,** because the number of relations between two sets, is the same as to calculate the card (AB), so the result is 2mn
3. The number of symmetric relations on A is , the symmetric means (x,y) A ,and (y,x) A, to treat A as a matrix, and the number of elements in A is m2, when x,y are not equal ,the number of elements is , and we should add the element which x is equal to y, so the elements meet the symmetric is , to get the number of relation is the same as we do in question(ii) is to calculate the card(A), is 

**Pb2：**

**(a)**S2,-3={2m-3y:m,nZ}

When m =0, n=0, S2,-3=0

m=0, n=1, S2,-3=-3

m=1, n=0, S2,-3=2

m=1, n=1, S2,-3=-1

m=2, n=1, S2,-3=1

the elements can be:0,-3,2,-1,1

**(b)**S12,16={12m+16y:m,nZ},

When m =0, n=0, S12,16=0

m=0, n=1, S12,16=16

m=1, n=0, S12,16=12

m=1, n=1, S12,16=28

m=2, n=1, S12,16=40

the elements can be:0,16,12,28,40

**(c)**Proof: Sx,y = {mx + ny : m, n∈Z}

To prove Sx,y ⊆ {n : n ∈ Z and d|n}, we need every elements in Sx,y∈Z, and

Sx,y = kd,

Because m,n,x,y∈Z, therefore mx+ny∈Z

Since d=gcd(x,y) ,we have that: x=k1d,y=k2d

Therefore Sx,y=mx+ny=mk1d+nk2d, (k1,k2,m,n∈Z)

=(mk1+nk2)d ,(k1,k2,m,n∈Z)

Sx,y=k3d(k3∈Z)

Sx,y⊆ {n : n∈Z and d|n}

**(d)**Proof: Because n=kz , n∈Z, and z is the smallest positive number in Sx,y

Suppose there exist m,n satisfy that: z=m1x+n1y

therefore n=kz=k(m1x+n1y)=km1x+kn2y=(km1)x+(kn2)y

because km1,kn2∈Z

it follows that: (km1)x+(kn2)y⊆ {mx + ny : m, n∈Z}

**(e)**Proof: Because: d=gcd(x,y) ,therefore x=k1d,y=k2d

Suppose there exist m,n satisfy that:

z=m1x+n1y (m,n∈Z)

z=k1m1d+k2n2d

z=(k1m1+k2n2)d

d=Z/(k1m1+k2n2)

because d,z∈Z , k1m1+k2n2∈N and k1m1+k2n2>0

Z/(k1m1+k2n2) will get smaller than d, when k1m1+k2n2 get greater

Therefore d ≤ z

(f)Proof: Since d=gcd(x,y), therefore we have x=k1d, y=k2d

We know that Sx,y= mx + ny (m, n∈Z)

Sx,y=mk1d+n k2d=(mk1+nk2)d, because m,n∈Z,

Therefore mk1+nk2∈Z, Sx,y =kd(k∈Z)

Therefore d∈Sx,y

Because m,n∈Z , z is the smallest positive number in Sx,y

Therefore we have that z∈Sx,y

z∈(mk1+nk2)d, (m, n,k1,k2∈Z)

And we know that z is the smallest positive number, d∈Sx,y, d at least equal to z or greater than z.

Therefore we get z ≤ d

**Pb3：**

(a) (A\*B)\*(A\*B)

=(AcBc)\* (AcBc) (definition from given)

=(AcBc)(AcBc) (definition from given)

=(AcBc)c(AcBc)c  (de Morgan’s Laws)

=AB (Idempotence)

(b) Ac

=AcAc (Idempotence)

= A\*A (definition from given)

(c) ∅=AAc

=(AAc)( AAc) (Idempotence)

=(AcA)c(AcA)c (de Morgan’s Laws)

=(AcA)\* (AcA) (definition from given)

=(A\*Ac)\* (A\*Ac) (definition from given)

=(A\*(A\*A))\*(A\*(A\*A)) (definition from given)

(d) A\B=ABc

=(ABc)(ABc) (Idempotence)

=(AcB)c(AcB)c (de Morgan’s Laws)

=(AcB)\* (AcB) (definition from given)

=(A\*Bc)\* (A\*Bc) (definition from given)

=(A\*(B\*B))\*(A\*(B\*B)) (definition from given)

**Pb4：**

（a）if: w=a, v=b, v≠wz

w=b, v=a, v≠wz

(b) Because R←({aba}),

Since v=aba=wz

If: w=a, z=ba

w=ab, z=a

w=aba. z=

w=,z=aba

Therefore the answer is {a,ab,aba,}

(c) To show R is a partial order, we should prove R contain R, AS, T property.

For R:

If (w,w)∈R, w=wz, when z=,for all w satisfy the R

For AS:

Because (w,v)∈R, therefore v=wz1

Because (v,w)∈R, therefore w=vz2

For all z1=z2=, there exists v=w

Therefore (w,v), (w,v)∈R, v=w

We have AS

For T:

Because (w,v)∈R, Therefore v=wz1

Because (v,p)∈R, Therefore p=vz2

To combine p,w together ,we get that :p=wz1z2(z1z2∈Z)

Therefore p=kw(k∈Z)

Therefore (w,p)∈R

That prove the T

**Pb5：**

From Pb2, in function : Sx,y = {mx + ny : m, n∈Z},

To take z as the smallest positive number, d=gcd(x,y), we know that d=z, which satisfy the function: Sx,y = {mx + ny : m, n∈Z}

In problem 5:

When z=0, for all x: z=kx, therefore x|z

When z≠0:

Since x|yz

Therefore yz=kx

y=

From the conclusion in Problem 2 we know that: gcd(x,y)=1=d

Therefore for x,y, m,n, satisfy the function:

mx+ny=1 (m,n,x,y∈Z)

mx+ny=1

mx+n=1

mxz+nkx=1

(zm+nk)x=z

Since z,m,n,k∈Z

Therefore zm+nk∈Z

kx = z

Therefore x|z