Monte Carlo Simulation parameter settings

In Monte Carlo simulations, it is beneficial to explore a wide range of parameter settings, such as various sample sizes and combinations of variances. The parameter settings of Monte Carlo Simulation are set as follows.

As for **Simulated Type 1 Error Probability of Algorithm 1-2**, firstly, let the nominal significance level be 5%, the number of loops are 5,000, and $\mu = 0.95$. Secondly, as for 9 populations, we set bootstrap sample size as (N_1, N_2, N_3, N_4)=(30,30,40,40,50,50,60,70,70),(40,40,50,70, 70,90,100,110,120),(60,60,70,80,80,90,90,100,120),(80,80,80, 80,90,90,90,90,100),$(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_7^2, \sigma_8^2, \sigma_9^2)$=(1,1 ,1,1,1,1,1,1,1),(0.1,0.3,0.3,0.5,0.7,0.7,0.9,1.1,1.1),(0.1,0.3, 0.3,0.3,0.5,0.5,0.7,0.7,0.7),(0.3,0.3,0.5,0.5,0.5,0.7,0.7,0.8,0.9), (0.5, 0.6, 0.7, 0.8, 0.9, 0.9, 1, 1.1, 1.2),(0.5, 0.5, 0.6, 0.7, 0.7, 0.8, 0.9,1, 1.1),(0.7, 0.7, 0.8, 0.8, 0.9, 0.9, 1, 1.1, 1.1),(0.6, 0.6, 0.7, 0.8, 0.9, 1, 1.2, 1.3, 1.4),(0.8, 0.8, 0.9, 0.9, 1, 1, 1.1, 1.2, 1.2). For 10 populations, we set (N_1, N_2, N_3, N_4)=(40,40,40,50,50,60,60,80,80,80),(40,60, 60,60,60,80,80,80,90,90),(60,60,70,80,90,100,110,120,120, 120),(80,80,80,80,90,90,90,90,100,100), $(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_7^2, \sigma_8^2, \sigma_9^2)$=(1,1,1,1,1,1,1,1,1,1), (0.1,0.3,0.3,0.5,0.7,0.7,0.9,1.1,1.1,1.3),(0.2,0.4,0.4,0.6,0.8, 0.8,1,1,1.2,1.4),(0.3,0.3,0.5,0.6,0.7,0.7,0.9,1.3,1.5,1.7),(0.4, 0.5,0.6,0.6,0.8,0.8,1.0,1.2,1.4,1.6),(0.5,0.6,0.6,0.7,0.8,0.8,1, 1.3,1.5,1.9),(0.6,0.7,0.7,0.9,0.9,1.2,1.2,1.4,1.6,1.8),(0.7,0.9, 0.9,1.0,1.2,1.5,1.5,1.7,1.7,1.9),(0.8, 0.8, 0.8,1.2,1.2,1.4,1.4, 1.8,1.8,2).

For 15 populations, we set (N_1, N_2, N_3, N_4)=(40,40, 40,50,50,50,60,60,60,70,70,70,80,80,80),(50,50,50,60,60, 60,80,80,80,90,90,90,90,90,90),(60,60,60,70,70,80,80,90, 90,90,90,100,100,100,100),(70,70,70,70,80,80,80,80,100, 100,100,120,120,120,120),$(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_7^2, \sigma_8^2, \sigma_9^2)$= (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1),(0.1,0.3,0.3,0.3,0.5,0.5,0.5,0.7, 0.7,0.9,1.1,1.1,1.3,1.5,1.5),(0.3,0.3,0.4,0.4,0.5,0.5,0.6,0.7,0.8, 0.8,1,1,1.2,1.4,1.4),(0.4,0.4,0.4,0.6,0.6,0.6,0.8,0.8,0.8,0.9,0.9, 1.1,1.2,1.4,1.6),(0.5,0.5,0.5,0.6,0.6,0.6,0.8,0.8,0.8,0.9,0.9,1, 1.2,1.4,1.6),(0.7,0.7,0.7,0.8,0.8,0.8,0.9,0.9,0.9,0.9,1,1,1.2,1.5, 1.8),(0.8,0.8,0.9,0.9,1,1,1.2,1.4,1.5,1.6,1.7,1.8,1.9,2,2),(0.8,0.9, 0.9,0.9,1,1.1,1.2,1.3,1.4,1.5,1.5,1.5,1.6,1.7,1.9),(1,1.1,1.2, 1.2,1.3,1.3,1.4,1.4,1.5,1.5,1.5,1.6,1.7,1.8,2).

As for **Simulated Power of Algorithm 1-2**, for 9 populations, we set $(\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7)$=(0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.2, 0.2, 0.2),(0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.2, 0.3, 0.3),(0.1, 0.1, 0.1, 0.1,0.1, 0.2, 0.2, 0.3, 0.3),(0.1, 0.1, 0.1, 0.1, 0.2, 0.2, 0.3, 0.3, 0.3),(0.1, 0.1, 0.1, 0.2, 0.2, 0.2, 0.3, 0.3,0.4),(0.1, 0.1, 0.1, 0.2, 0.2, 0.3, 0.3, 0.4, 0.4),(0.1, 0.1, 0.1, 0.2, 0.3, 0.3, 0.4, 0.5, 0.5), (V_1, V_2, V_3)=(0.1, 0.3, 0.3, 0.5, 0.7, 0.7, 0.9, 1.1, 1.1),(0.3, 0.3, 0.5, 0.5, 0.5, 0.7, 0.7, 0.8, 0.9),(0.5, 0.6, 0.7, 0.8, 0.9, 0.9, 1, 1.1, 1.2). For 10 populations, we set $(\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7)$=(0.1, 0.1, 0.1, 0.1,0.1, 0.1, 0.2, 0.2, 0.2, 0.2),(0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.2, 0.2, 0.3, 0.3),(0.1, 0.1, 0.1, 0.1, 0.1, 0.2, 0.2,0.3, 0.3, 0.3),(0.1, 0.1, 0.1, 0.1, 0.2, 0.2, 0.3, 0.3, 0.3, 0.4),(0.1, 0.1, 0.1, 0.2, 0.2, 0.2, 0.3, 0.3, 0.4, 0.4),(0.1, 0.1, 0.2, 0.2, 0.3, 0.3, 0.4, 0.5, 0.5, 0.6),(0.1, 0.1, 0.2, 0.3, 0.3, 0.4, 0.4, 0.5, 0.6, 0.6), (V_1, V_2, V_3)=(0.2, 0.4, 0.4, 0.6, 0.8, 0.8, 1, 1, 1.2, 1.4),(0.3, 0.3, 0.5, 0.6, 0.7, 0.7, 0.9, 1.3, 1.5, 1.7),(0.5, 0.6, 0.6, 0.7, 0.8, 0.8, 1, 1.3, 1.5, 1.9). For 15 populations, we set $(\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7)$=(0.1, 0.1, 0.1, 0.1, 0.1, 0.1,0.1,0.1,0.1,0.1,0.2, 0.2, 0.2, 0.2, 0.2),(0.1, 0.1, 0.1, 0.1, 0.1, 0.1,0.1,0.1,0.1,0.2,0.2, 0.2, 0.2, 0.3, 0.3),(0.1, 0.1, 0.1, 0.1, 0.1, 0.1,0.1,0.2,0.2,0.2,0.2, 0.3, 0.3, 0.3, 0.4),(0.1, 0.1, 0.1, 0.1, 0.1, 0.1,0.2,0.2,0.2,0.3,0.3, 0.3, 0.4, 0.4, 0.4),(0.1, 0.1, 0.1, 0.1, 0.1, 0.2,0.2,0.2,0.3,0.3,0.4, 0.4, 0.4, 0.5, 0.5),(0.1, 0.1, 0.1, 0.1, 0.1, 0.2,0.2,0.2,0.3,0.3,0.5, 0.5, 0.5, 0.6, 0.6),(0.1, 0.1,0.1, 0.2, 0.3, 0.4,0.5,0.6,0.7,0.8,0.9, 1.0, 1.1, 1.2, 1.3), (V_1, V_2, V_3)=(0.1,0.3,0.3,0.3,0.5,0.5, 0.5,0.7,0.7,0.9,1.1,1.1,1.3,1.5,1.5),(0.4,0.4,0.4,0.6,0.6,0.6,0.8, 0.8,0.8,0.9,0.9,1.1,1.2,1.4,1.6),(0.7,0.7,0.7,0.8,0.8,0.8,0.9,0.9, 0.9,0.9,1,1,1.2,1.5,1.8). Here, δ represents the difference between the means of each client. As the performance differences of different clients' models increase, the power value quickly approaches 1, indicating that our proposed method can promptly detect unfair phenomena.

As for **Coverage Probability for Confidence Intervals of Algorithm 3**, we set the confidence level as 95%, $\mu = 0.95$. As for 9 populations, we set (N_1, N_2, N_3, N_4) = (30, 30, 40, 40, 50, 50, 60, 70, 70),(40, 40, 50, 70, 70, 90, 100, 110, 120),(60, 60, 70, 80, 80, 90, 90, 100, 120),(80, 80, 80, 80, 90, 90, 90, 90, 100),$(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_7^2, \sigma_8^2, \sigma_9^2)$=(1, 1, 1, 1, 1, 1, 1, 1, 1),(0.1, 0.3, 0.3, 0.5, 0.7, 0.7, 0.9, 1.1, 1.1),(0.1, 0.3, 0.3, 0.3, 0.5, 0.5, 0.7, 0.7, 0.7),(0.3, 0.3, 0.5, 0.5, 0.5, 0.7, 0.7, 0.8, 0.9),(0.5, 0.6, 0.7, 0.8, 0.9, 0.9, 1, 1.1, 1.2),(0.5, 0.5, 0.6, 0.7, 0.7, 0.8, 0.9, 1, 1.1),(0.7, 0.7, 0.8, 0.8, 0.9, 0.9, 1, 1.1, 1.1),(0.6, 0.6, 0.7, 0.8, 0.9, 1, 1.2, 1.3, 1.4),(0.8, 0.8, 0.9, 0.9, 1, 1, 1.1, 1.2, 1.2). For 10 populations, we set (N_1, N_2, N_3, N_4) = (40,40,40,50,50,60,60,80,80,80),(40,60,60,60,60,80,80,80,90, 90),(60,60,70,80,90,100,110,120,120,120),(80,80,80,80,90, 90,90,90,100,100), $(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_7^2, \sigma_8^2, \sigma_9^2)$=(1,1,1 ,1,1,1,1,1,1,1),(0.1, 0.3,0.3,0.5,0.7,0.7,0.9,1.1,1.1,1.3),(0.2,0.4, 0.4,0.6,0.8,0.8,1,1,1.2,1.4),(0.3,0.3,0.5,0.6,0.7,0.7,0.9,1.3,1.5, 1.7),(0.4,0.5,0.6,0.6,0.8,0.8,1.0,1.2,1.4,1.6),(0.5,0.6,0.6,0.7,0.8, 0.8,1,1.3,1.5,1.9),(0.6,0.7,0.7,0.9,0.9,1.2, 1.2,1.4,1.6,1.8),(0.7,0.9,0.9,1.0,1.2,1.5,1.5,1.7,1.7,1.9),(0.8, 0.8,0.8,1.2,1.2,1.4,1.4,1.8,1.8,2). For 15 populations, we set (N_1, N_2, N_3, N_4)=(40,40,40,50,50,50,60,60,60,70,70,70,80, 80,80),(50,50,50,60,60,60,80,80,80,90,90,90,90,90,90),(60,60, 60,70,70,80,80,90,90,90,90,100,100,100,100),(70,70,70,70,80, 80,80,80,100,100,100,120,120,120,120),

$(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_7^2, \sigma_8^2, \sigma_9^2)$=(1,1,1,1,1,1,1,1,1,1,1, 1,1,1,1), (0.1,0.3,0.3,0.3,0.5,0.5,0.5,0.7,0.7,0.9,1.1,1.1,1.3,1.5, 1.5),(0.3,0.3,0.4,0.4,0.5,0.5,0.6,0.7,0.8,0.8,1,1,1.2,1.4,1.4),(0.4, 0.4,0.4,0.6,0.6,0.6,0.8,0.8,0.8,0.9,0.9,1.1,1.2,1.4,1.6),(0.5, 0.5,0.5,0.6,0.6,0.6,0.8,0.8,0.8,0.9,0.9,1,1.2,1.4,1.6),(0.7,0.7, 0.7,0.8,0.8,0.8,0.9,0.9,0.9,0.9,1,1,1.2,1.5,1.8),(0.8,0.8,0.9,0.9, 1,1,1.2,1.4,1.5,1.6,1.7,1.8,1.9,2,2),(0.8,0.9,0.9,0.9,1,1.1,1.2, 1.3,1.4,1.5,1.5,1.5,1.6,1.7,1.9),(1,1.1,1.2,1.2,1.3,1.3,1.4,1.4, 1.5,1.5,1.5,1.6,1.7,1.8,2).