A-G.

$$\sqrt[n]{x_1\cdots x_n} \le \frac{x_1+\cdots+x_n}{n}$$

$$x_1\cdots x_n \le \left(\frac{x_1+\cdots+x_n}{n}\right)^n$$

1. (1) $n! \le \left(\frac{n+1}{2}\right)^n$

$$\underbrace{1\cdot 2\cdots n}_{x_1 \ \cdots \ x_n} \le \left(\frac{1+2+\cdots+n}{n}\right)^n$$

$$\left(\frac{(1+n)\cancel{n}}{2\cancel{n}}\right)^n$$

(2) $\sqrt[n]{n}-1 < \frac{2}{\sqrt{n}}$ $\quad n\in N_+$

Thinking: convert L or R to the form of n-product or n-sum.

Method 1:

$$a^n-1=(a-1)(\underbrace{a^{n-1}+a^{n-2}+\cdots+1}_{A})$$

$$A=\frac{1(a^n-1)}{a-1}=\frac{a^n-1}{a-1}$$

$$(\sqrt[n]{n})^n=n$$

$$a=n^{\frac{1}{n}}$$

$$L=a-1$$

$$L=\frac{a^n-1}{a^{n-1}+\cdots+1} \qquad \frac{n-a}{n}\to 1-\frac{a}{n}$$

$$=\frac{n-1}{n^{\frac{n-1}{n}}+n^{\frac{n-2}{n}}+\cdots+n^{\frac{n-n}{n}}}$$

$$=\frac{n-1}{[n^{\frac{n-1}{n}}+n^{\frac{n-2}{n}}+\cdots+n^{\frac{n-n}{n}}]}$$

$$L\le\frac{n-1}{n\cdot(n^{\frac{(n-1)+(n-2)+\cdots+(n-n)}{n}\cdot\frac{1}{n}}} \qquad n-1 \quad 0 \qquad \frac{n(n+1)}{2}$$

$$=\frac{n-1}{n\cdot n^{\frac{\frac{n(n-1)}{2}}{n^2}}} \qquad \sqrt[n]{a}=a^{\frac{1}{n}}$$

$$=\frac{n-1}{n\cdot n^{\frac{n-1}{2n}}}$$

$L<\frac{2}{\sqrt{n}}$

$$=\frac{1}{\sqrt{n}}\cdot\frac{n-1}{n^{\frac{n-1}{2n}+\frac{n}{2n}}}=\frac{1}{\sqrt{n}}\cdot\frac{n-1}{n^{1-\frac{1}{2n}}} \nearrow$$

$$<\frac{1}{\sqrt{n}}\quad n^{\frac{1}{2n}}$$

• $\frac{1}{2\sqrt{n}}<\frac{(2n-1)!!}{(2n)!!}<\frac{1}{\sqrt{2n+1}}$ $\quad n\ge 2.$ $\quad 0 \sim 0$ $\quad (2n)!!$

$$x_n=\frac{1}{2}\times\frac{3}{4}\times\frac{5}{6}\times\cdots\times\frac{2n-1}{2n}=\frac{(2n-1)!!}{(2n)!!}$$

$$b_n=\frac{2n-1}{2n}$$

$$x_n=b_1\cdot b_2\cdots b_n$$

$$0 \quad \frac{n-2}{n-1}<\frac{n-1}{n}<\frac{n}{n+1}$$

$$b_n<\frac{2n}{2n+1} \quad \to n+1$$

$$x_n<\frac{2}{3}\times\frac{4}{5}\times\cdots\times\frac{2n}{2n+1}=\frac{(2n)!!}{(2n-1)!!}\times\frac{1}{2n+1}$$

$$=\frac{1}{x_n}\times\frac{1}{2n+1}$$

$$x_n^2<\frac{1}{2n+1}$$

$$x_n<\frac{1}{\sqrt{2n+1}}$$

$$x_n=1\times\frac{3}{2}\times\frac{5}{4}\times\frac{7}{6}\times\cdots\times\frac{2n-1}{2n-2}\times\frac{1}{2n}$$

$$\frac{2}{2}\times\frac{4}{3}\times\frac{6}{5}\times\cdots\times\frac{2n}{2n-1}\times\frac{1}{2n}$$

$$=\frac{1}{x_n}\times\frac{1}{4n} \qquad x_n^2>\frac{1}{4n} \qquad x_n>\frac{1}{2\sqrt{n}}$$

• $x+y=3$

$x^2-4x+y^2=0$

$x^2-y^2=1$

$(x^2+y^2)^{\frac{3}{2}}=x^2-y^2$

$\vec{x_1},\vec{x_2}$ $\quad \vec{x_3}\quad\vec{x_4}$

(y_1,y_2) $\quad \vec{x_3}=a\vec{x_1}+b\vec{x_2}$, $\vec{x_4}=c\vec{x_1}+d\vec{x_2}$

$(x,y)\to(r,\theta)$ $\quad (y_1' \quad y_2')$

$x=r\cos\theta$ $\quad y=r\sin\theta$

$r\ge 0$

$r\cos\theta+r\sin\theta=3$

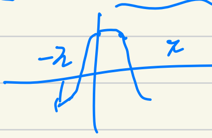
$r=f(\theta)\ge 0$ $\quad \frac{3}{\cos\theta+\sin\theta}\ge 0$

$\theta\in ?$ $\quad \theta\in(-\frac{\pi}{4},\frac{3}{4}\pi)$

$\pm 2k\pi \quad \pm 2k\pi$

$r^2\cos^2\theta-4r\cos\theta+r^2\sin^2\theta=0$

$r=4\cos\theta$ $\quad [-\frac{\pi}{2},\frac{\pi}{2}]$ $\quad 2\pi$



$x^2-y^2=1$

$r=\frac{1}{\sqrt{\cos 2\theta}}$ $\quad \theta(-\frac{\pi}{4},\frac{\pi}{4})\cup(\frac{3}{4}\pi,\frac{5}{4}\pi)$

4) $r=\cos 2\theta$ $\quad \theta\in[-\frac{\pi}{4},\frac{\pi}{4}]\cup[\frac{3}{4}\pi,\frac{5}{4}\pi]$

极限是否存在?

$f(x)=\frac{1}{x}$ $\quad x\to 0 \quad f(x)\to\infty$

$f(x)=\sin\frac{1}{x-1}$ $\quad x\to 1 \quad f(x)$ 震荡

$f(x)=\frac{|x|}{x}$ $\quad f(x)=\arctan\frac{1}{x}$ $\quad f(x)=\frac{1}{1+2^{\frac{1}{x}}}$

$x\to 0$

无界 → 震荡 $\quad +\infty,\ -\infty,\ \infty,\ 0,\ a$

无穷大 $\quad \lim_{x\to x_0} f(x)\Rightarrow\infty$

无穷小

$$x_n=\begin{cases}\frac{n^2+\sqrt{n}}{n} & n\in 2k+1,\ k\in Z^+\cup\{0\}\\ \frac{1}{n} & n\in 2k\quad k\in Z^+\cup\{0\}\end{cases}$$

$n\to\infty \quad x_n$

① $x_n=\underset{\infty}{n}+\underset{0}{\frac{1}{\sqrt{n}}}=+\infty$

② $x_n=\frac{1}{n}=0$

$f(x)=\frac{x^2-1}{x-1}e^{\frac{1}{x-1}}$ $\quad x\to 1 \quad f(x)\to$?

$f(x)=(x+1)e^{\frac{1}{x-1}}\to 2e^{\frac{1}{x-1}}$

$\lim_{x\to 1^-} 2e^{\frac{1}{x-1}}\to 0$

$\lim_{x\to 1^+} 2e^{\frac{1}{x-1}}\to +\infty$

$f(x)$ 无极限

$\underbrace{\lim_{x\to+\infty}[(ax+b)e^{\frac{1}{x}}-x]}_{f(x)}=2$ $\quad$ 求 a,b

$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty} b e^{\frac{1}{x}}+\lim_{x\to+\infty}(axe^{\frac{1}{x}}-x)$ $\quad e^x=1+x+\frac{1}{2}x^2+\cdots$

$=b+x(ae^{\frac{1}{x}}-1)$ $\quad$ 2.

$a(1+\frac{1}{x})-1$

$=b+x(a(1+\frac{1}{x})-1)$ $\quad a=1$

$=$ $\quad x\cdot\frac{1}{x}=1$

$=b+(a-1)x+a$

$a=b=1$

$\sin^2 x$	$1+\tan x$	$x\cos x$	$\cos(\omega t+\theta)$
π	π	x	$\frac{2\pi}{\omega}$

0

$e^x = 1+x+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!}+o(x^n)$

$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots+(-1)^{m+1}\frac{x^{2m-1}}{(2m-1)!}+o(x^{2m})$

$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots+(-1)^m\frac{x^{2m}}{(2m)!}+o(x^{2m+1})$

$\ln(1+x) \sim x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots+(-1)^{n+1}\frac{x^n}{n}+o(x^n)$

$(1+x)^a = 1+ax+\frac{a(a-1)}{2!}x^2+\cdots+\frac{a(a-1)\cdots(a-n+1)}{n!}x^n+o(x^n)$

$\frac{1}{1-x} = 1+x+x^2+\cdots+x^n+o(x^n)$

$1-\cos x \sim \frac{1}{2}x^2$

$\sqrt[n]{1+x} \sim 1+\frac{1}{n}x$

$\ln(1+x) \sim x$

$a^x-1 \sim x\ln a \rightarrow a^x = e^{x\ln a}$

$\lim \frac{a_n x^n+\cdots+a_1x+a_0}{b_m x^m+\cdots+b_1x+b_0}$

$n\to\infty \begin{cases} n=m & \frac{a_n}{b_m} \\ n>m & \infty \\ n<m & 0 \end{cases}$

$\alpha\sim\alpha_1,\quad \beta\sim\beta_1$

$\lim\frac{\alpha}{\beta} = \lim\frac{\alpha_1}{\beta_1}$

$\lim \alpha-\beta \overset{?}{\sim} \lim \alpha_1-\beta_1$

- $\lim\limits_{n\to\infty}\frac{n^{n+1}}{(n+1)^n}\sin\frac{1}{n}$

- $\lim\limits_{x\to}\boxed{\frac{x-\sin x}{x^3}} = A$ $\quad \sin 2\frac{x}{2}$

$= \frac{x-\sin 2\frac{x}{2}}{x^3}$

$= \frac{x-2\sin\frac{x}{2}\cos\frac{x}{2}}{x^3}$

$= \frac{x-x\cos\frac{x}{2}+x\cos\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}{x^3}$

x^3

$\boxed{\frac{x-\sin x}{x^3}} = \frac{1}{6}$

$= \frac{\boxed{x(1-\cos\frac{x}{2})}+\cos\frac{x}{2}(x-2\sin\frac{x}{2})}{x^3}$ $\quad 3$ $\quad 2\frac{x}{2}$

$= \frac{\frac{1}{8}x^3}{x^3}+\frac{\cos\frac{x}{2}(x-2\sin\frac{x}{2})}{x^3}$ $\quad 8(\frac{x}{2})^3$

$= \frac{1}{8}+\lim\limits_{x\to 0}\frac{2\cos\frac{x}{2}(\frac{x}{2}-\sin\frac{x}{2})}{8(\frac{x}{2})^3}$

$x\to 0$

$\frac{x}{2}\to 0$

- $\lim\limits_{x\to 0}\frac{\ln(\sin^2x+e^x)-x}{\ln(e^{2x}-x^2)-2x}$

$\rightarrow \ln e^x$

$\ln x-\ln y = \ln\frac{x}{y}$

$\ln x \sim x-1$

$= \frac{\ln\frac{\sin^2x+e^x}{e^x}}{\ln\frac{e^{2x}-x^2}{e^{2x}}}$

$= \frac{\frac{\sin^2 x}{e^x}}{-\frac{x^2}{e^{2x}}}$ $\quad 2$

$= -\frac{\sin^2x\cdot e^x}{x^2}$ $\quad 1$

$= -\frac{x^2}{x^2} = -1$ $\quad \frac{1}{x}$

$\lim\limits_{x\to 0}(3e^{\frac{x}{x-1}}-2)^{\frac{1}{x}}$ $\rightarrow$ $(1+0)^\infty$ $\quad (1+\frac{1}{x})^x = e$ $\quad x\to\infty$

$= (1+3e^{\frac{x}{x-1}}-3)^{\frac{u}{u}\cdot\frac{1}{x}}$

u

$= \left(1+3e^{\frac{x}{x-1}}-3\right)^{\frac{1}{3e^{\frac{x}{x-1}}-3}\cdot\boxed{\frac{3e^{\frac{x}{x-1}}-3}{x}}}$

$= ((\quad)^{(\quad)\cdot})$

$= e^{\boxed{\frac{3e^{\frac{x}{x-1}}-3}{x}}}$

$\Rightarrow \lim\limits_{x\to 0}\frac{3e^{\frac{x}{x-1}}-3}{x} = \frac{3(e^{\boxed{\frac{x}{x-1}}}-1)}{x}$

$= 3\frac{\frac{x}{x-1}}{x}$

$= 3\frac{1}{x-1}$

$= -3$

Original problem:

$= e^{-3}$

$$= \frac{1}{8} + \frac{1}{4} \lim_{x \to 0} \frac{x - \sin x}{x^3}$$

$$A = \frac{1}{8} + \frac{1}{4} A$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\lim_{x \to 0} \frac{(1+x^2)(1-\cos 2x) - 2x^2}{x^4}$$

$$= \frac{(1+x^2) \cdot 2\sin^2 x - 2x^2}{x^4}$$

$$= \frac{2x^2 \sin^2 x - 2x^2 + 2\sin^2 x}{x^4}$$

$$= \frac{2x^2 \sin^2 x + 2\sin^2 x - 2x^2}{x^4}$$

$$= \frac{2(\sin x + x)(\sin x - x) + 2x^2 \sin^2 x}{x^4}$$

$$\left(x - \frac{x^3}{6}\right)^2$$

$$x$$

3 4 6

5 6 8

$x \to o(x)$

$$= \frac{2(\sin x + x)}{x} \cdot \frac{(\sin x - x)}{x^3} + \frac{2x^2 \sin^2 x}{x^4}$$

$$= 2 \cdot \frac{2x}{x} \cdot \left(-\frac{1}{6}\right) + 2$$

$$= -\frac{2}{3} + 2$$

$$= \frac{4}{3}$$