Belt Velocity Initialization

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Contents

1 Input situation: ω_I, M_O, F_{C_O}

1.1 Condition

Input parameters: ω_I, M_O, F_{C_O}
Unknowns: ω_O, v_0

1.2 Simplification

From 2.216 we can get

$$\left(L_{O_{in}} - K\right) e^{\mu^* \Phi_O} = \frac{M_O}{r_O} + \left(L_{O_{in}} - K\right) \tag{1}$$

For equation 2.214, we can substitute $\left(L_{O_{in}} - K\right) e^{\mu^* \Phi_O}$ by (1) and then insert equation 2.218. So we get

$$r_I \omega_I = \frac{v_0 M_O}{EA r_O} + r_O \omega_O \tag{2}$$

From 2.216 we also can get

$$\left(L_{O_{in}} - K\right) \left(e^{\mu^* \Phi_O} - 1\right) = \frac{M_O}{r_O} \tag{3}$$

From 2.218 and the formula of K we can get

$$\frac{\left(L_{O_{in}} - K\right)}{EA} = \frac{r_O \omega_O}{v_0} - \frac{EA}{EA - m^* {v_0}^2} \tag{4}$$

For equation 2.217, we can substitute $\left(L_{O_{in}} - K\right) \left(e^{\mu^* \Phi_O} - 1\right)$ by (3) and $\left(L_{O_{in}} - K\right)/EA$ by (4), then insert 2.218. So we get

$$2F_{C_O} \tan(\delta_0) = \left(EA - m^* {v_0}^2\right) \left[2\varphi \left(\frac{r_O \omega_O}{v_0} - 1\right) + \frac{M_O}{\mu^* r_O EA} - \left(\frac{r_O \omega_O}{v_0} - \frac{EA}{EA - m^* {v_0}^2}\right) \Phi_O\right] - 2m^* {v_0}^2 \varphi \tag{5}$$

Then we can solve equations (2) and (5) with unknowns ω_O and v_0 with the starting vaulue provided by 2.219 and 2.220.

1.3 How to calculate v_0 when $M_O = 0$

Condition: $\Phi_I = \Phi_O$, $F_{C_O} \neq 0$, $L_{O_{in}} \neq 0$
Insert $\Phi_O = 0$ and 2.219 in to 2.217 and simplify it, we can get

$$r_I \omega_I \varphi m^* {v_0}^2 + (F_{C_O} \tan(\delta_0) + EA\varphi) v_0 - EA r_I \omega_I \varphi = 0 \tag{6}$$

And as $v_0 \geq 0$, solving the quadratic equation in v_0 (6),we can get v_0 descripted in 2.220.

2 Input situation: ω_O, M_I, F_{C_I}

2.1 Condition

Input parameters: ω_O, M_I, F_{C_I}
Unknowns: ω_I, v_0

2.2 Basic formulas

From 2.214,we can get

$$L_{I_{in}} = \frac{EAr_I\omega_I}{v_0} - EA \ . \tag{7}$$

By 2.211 and the relationship of $L_{I_{out}}$ and $L_{I_{in}}$ we can get

$$\begin{aligned} L_{I_{out}} - L_{I_{in}} &= [L_{I_{in}} - K]e^{-\mu^*\Phi_I} + K - L_{I_{in}} \\ &= (L_{I_{in}} - K)(e^{-\mu^*\Phi_I} - 1). \end{aligned} \tag{8}$$

As the longitudinal force decreases from $L_{I_{in}}$ from $L_{I_{out}}$, we choose the negative sign when using the formula 2.211.
The equality of torques are

$$M_O = r_O(L_{O_{out}} - L_{O_{in}}) \tag{9}$$

$$\begin{aligned} M_I &= r_I(L_{I_{out}} - L_{I_{in}}) \\ &= r_I(L_{I_{in}} - K)(e^{-\mu^*\Phi_I} - 1) \ . \end{aligned} \tag{10}$$

As $L_{O_{in}} = L_{I_{out}}$ and $L_{O_{out}} = L_{I_{in}}$, we can get

$$M_O = -M_I\frac{r_O}{r_I} \ . \tag{11}$$

And the axial equality of force on the input pulley is

$$\begin{aligned} F_{C_I} &= \int_{\varphi_I} S'\mathrm{d}\theta \\ &= \int_{\varphi_I} \frac{L(EA - m^*{v_0}^2) - m^*{v_0}^2EA}{2\tan(\delta_0)EA}\,\mathrm{d}\theta \\ &= \frac{EA - m^*{v_0}^2}{2\tan(\delta_0)EA}\left[\int_{-(\pi-\varphi)}^{\pi-\varphi-\Phi_I} L\mathrm{d}\theta + \int_{\pi-\varphi-\Phi_I}^{\pi-\varphi} L\mathrm{d}\theta\right] - \frac{m^*{v_0}^2(\pi-\varphi)}{\tan(\delta_0)} \\ &= \frac{EA - m^*{v_0}^2}{2\tan(\delta_0)EA}\left[L_{I_{in}}(2\pi - 2\varphi - \Phi_I) + \int_{\pi-\varphi-\Phi_I}^{\pi-\varphi}\Big((L_{I_{in}} - K)e^{-\mu^*(\theta-(\pi-\varphi-\Phi_I))} + K\Big)\,\mathrm{d}\theta\right] - \frac{m^*{v_0}^2(\pi-\varphi)}{\tan(\delta_0)} \\ &= \frac{EA - m^*{v_0}^2}{2\tan(\delta_0)EA}\left[L_{I_{in}}(2\pi - 2\varphi - \Phi_I) + \left|\left((L_{I_{in}} - K)\frac{1}{-\mu^*}e^{\mu^*(\pi-\varphi-\Phi_I-\theta)} + K\theta\right)\right|_{\pi-\varphi-\Phi_I}^{\pi-\varphi}\right] - \frac{m^*{v_0}^2(\pi-\varphi)}{\tan(\delta_0)} \\ &= \frac{EA - m^*{v_0}^2}{2\tan(\delta_0)EA}\left[2L_{I_{in}}(\pi-\varphi) + (L_{I_{in}} - K)\left(\frac{e^{-\mu^*\Phi_I} - 1}{-\mu^*} - \Phi_I\right)\right] - \frac{m^*{v_0}^2(\pi-\varphi)}{\tan(\delta_0)}. \end{aligned} \tag{12}$$

Equation (12) can be simplified the same way as (5). So we get

$$2F_{C_I}\tan(\delta_0) = \left(EA - m^* {v_0}^2\right)\left[2(\pi-\varphi)\left(\frac{r_I\omega_I}{v_0}-1\right) + \frac{M_I}{(-\mu^*)r_I EA} - \left(\frac{r_I\omega_I}{v_0} - \frac{EA}{EA - m^*{v_0}^2}\right)\Phi_I\right] - 2m^*{v_0}^2(\pi-\varphi) \tag{13}$$

2.3 How to calculate v_0 when $M_O = 0$

Condition: $\Phi_I = \Phi_O$, $F_{C_I} \neq 0$, $L_{I_{in}} \neq 0$
Insert $\Phi_O = 0$ and 2.219 in to (12) and simplify it, we can get

$$r_O\omega_O(\pi-\varphi)m^*{v_0}^2 + (F_{C_I}\tan(\delta_0) + EA(\pi-\varphi))v_0 - EAr_O\omega_O(\pi-\varphi) = 0 \tag{14}$$

And as $v_0 \geq 0$, solving the equation (14),we can get

$$v_0 = \frac{\sqrt{\left(F_{C_I}\tan(\theta_0) + EA(\pi-\varphi)\right)^2 + 4EAm^* {r_O}^2{\omega_O}^2(\pi-\varphi)^2} - EA(\pi-\varphi) - F_{C_I}\tan(\delta_0)}{2m^* r_O\omega_O(\pi-\varphi)} \tag{15}$$

2.4 How to calculate v_0 when $M_O \neq 0$

If $M_O \neq 0$, it is necessarily $\Phi_I = \Phi_O$ and $L_{O_{in}} \neq K$. So

$$\Phi_I = -\frac{1}{\mu^*}\ln\left[\frac{M_I}{r_I(L_{I_{in}} - K)} + 1\right] \tag{16}$$

is only defined for $M_I \geq r_I(K - L_{I_{in}})$. With the help of Equation (11), Equation (2) and Equation (13) only depend on ω_I, v_0.They can be solved by NEWTON method with starting values from the $M_O = 0$ case.

3 Input situation: ω_I, ω_O, F_{C_O}

3.1 Condition

Input parameters: ω_I, ω_O, F_{C_O}
Unknowns: M_O, v_0

3.2 How to calculate v_0 when $M_O = 0$

Condition: $\Phi_I = \Phi_O$, $F_{C_O} \neq 0$, $L_{O_{in}} \neq 0$
If $\omega_O/\omega_I = r_I/r_O$, it means that the output torque $M_O = 0$. Then we can get v_0 by 2.220.

3.3 How to calculate v_0 when $M_O \neq 0$

If $\omega_O/\omega_I \neq r_I/r_O$, it means that the output torque $M_O \neq 0$. We can solve equations (2) and (5) in unknowns M_O and v_0. We can choose 2.220 as the starting value of v_0. From (2), we can get:

$$M_O = \frac{(r_I\omega_I - r_O\omega_O)r_O EA}{v_0}. \tag{17}$$

By inserting the starting value of v_0 in to (17), we can get the starting value for M_O.

4 Input situation: ω_I, ω_O, F_{C_I}

4.1 Condition

Input parameters: ω_I, ω_O, F_{C_I}
Unknowns: M_I, v_0

4.2 How to calculate v_0 when $M_O = 0$

Condition: $\Phi_I = \Phi_O$, $F_{C_I} \neq 0$, $L_{I_{in}} \neq 0$
If $\omega_O/\omega_I = r_I/r_O$, it means that the output torque $M_O = 0$. Then we can get v_0 by (15).

4.3 How to calculate v_0 when $M_O \neq 0$

If $\omega_O/\omega_I \neq r_I/r_O$, it means that the output torque $M_O \neq 0$. With the help of Equation (11), (2) can be writen in the form of

$$r_I\omega_I = -\frac{v_0 M_I}{EAr_I} + r_O\omega_O. \tag{18}$$

we can solve equations (18) and (13) in unknowns M_I and v_0. We can choose (15) as the starting value of v_0.
From (18), we can get:

$$M_I = -\frac{(r_I\omega_I - r_O\omega_O)r_I EA}{v_0}. \tag{19}$$

By inserting the starting value of v_0 in to (19), we can get the starting value for M_I.

5 Input situation: ω_I, M_O, F_{C_I}

5.1 Condition

Input parameters: ω_I, M_O, F_{C_I}
Unknowns: ω_O, v_0

5.2 How to calculate v_0 when $M_O = 0$

Condition: $\Phi_I = \Phi_O$, $F_{C_I} \neq 0$, $L_{I_{in}} \neq 0$
If the output torque $M_O = 0$, then we can get v_0 by (15).

5.3 How to calculate v_0 when $M_O \neq 0$

From equation (11), we can get

$$M_I = -M_O\frac{r_I}{r_O} . \tag{20}$$

If the output torque $M_O \neq 0$. With the help of Equation (20), we can solve equations (2) and (13) with unknowns ω_O and v_0. We can choose (15) as the starting value of v_0.
From (2), we can get:

$$\omega_O = \frac{r_I\omega_I}{r_O} - \frac{v_0 M_O}{r_I r_O EA} \tag{21}$$

By inserting the starting value of v_0 in to (21), we can get the starting value for ω_O.

6 Input situation: ω_I, M_I, F_{C_O}

6.1 Condition

Input parameters: ω_I, M_I, F_{C_O}
Unknowns: ω_O, v_0

6.2 How to calculate v_0 when $M_O = 0$

Condition: $\Phi_I = \Phi_O$, $F_{C_O} \neq 0$, $L_{O_{in}} \neq 0$
If the input torque $M_I = 0$,it means that the output torque $M_O = 0$. Then we can get v_0 by 2.220.

6.3 How to calculate v_0 when $M_O \neq 0$

If the input torque $M_I \neq 0$. With the help of Equation (11), we can solve equations (2) and (5) in unknowns ω_O and v_0. We can choose 2.220 and (21) as the starting value of v_0 and ω_O.

7 Input situation: ω_I, M_I, F_{C_I}

7.1 Condition

Input parameters: ω_I, M_I, F_{C_I}
Unknowns: ω_O, v_0

7.2 How to calculate v_0 when $M_O = 0$

Condition: $\Phi_I = \Phi_O$, $F_{C_I} \neq 0$, $L_{I_{in}} \neq 0$
If the input torque $M_I = 0$, it means that the output torque $M_O = 0$. Then we can get v_0 by (15).

7.3 How to calculate v_0 when $M_O \neq 0$

If the input torque $M_I \neq 0$. it means that the output torque $M_O \neq 0$. We can solve equations (18) and (13) with unknowns ω_O and v_0. We can choose (15) and (21) as the starting value of v_0 and ω_O.

A Futher Simplifiction

A.1 Φ_O

Insert (17) and (4) in to 2.221, we can get

$$\Phi_O = \frac{1}{\mu^*} \ln \left[\frac{\frac{(r_I\omega_I - r_O\omega_O) r_O EA}{v_0}}{r_O EA(\frac{r_O\omega_O}{v_0} - \frac{EA}{EA - m^* {v_0}^2})} + 1 \right] = \frac{1}{\mu^*} \ln \left[\frac{r_I\omega_I (EA - m^* {v_0}^2) - EAv_0}{r_O\omega_O (EA - m^* {v_0}^2) - EAv_0} \right] \quad (22)$$

A.2 F_{C_O}

Insert (17) and (22) in to (5), we can get

$$2F_{C_O} \tan(\delta_0) = \left(EA - m^* {v_0}^2\right) \left[2\varphi \left(\frac{r_O\omega_O}{v_0} - 1 \right) + \frac{r_I\omega_I - r_O\omega_O}{\mu^* v_0} - \left(\frac{r_O\omega_O}{v_0} - \frac{EA}{EA - m^* {v_0}^2} \right) \frac{1}{\mu^*} \ln \left[\frac{r_I\omega_I (EA - m^* {v_0}^2) - EAv_0}{r_O\omega_O (EA - m^* {v_0}^2) - EAv_0} \right] \right] - 2m^* {v_0}^2 \varphi \quad (23)$$

A.3 Φ_I

From (7) and the formula of K we can get

$$(L_{O_{in}} - K) = \frac{r_I\omega_I}{v_0} - \frac{EA}{EA - m^* {v_0}^2} EA \quad (24)$$

Insert (19) and (**??**) in to 2.221, we can get

$$\Phi_I = \frac{1}{-\mu^*} \ln \left[- \frac{\frac{(r_I\omega_I - r_O\omega_O) r_I EA}{v_0}}{r_I EA(\frac{r_I\omega_I}{v_0} - \frac{EA}{EA - m^* {v_0}^2})} + 1 \right] = \frac{1}{-\mu^*} \ln \left[\frac{r_O\omega_O (EA - m^* {v_0}^2) - EAv_0}{r_I\omega_I (EA - m^* {v_0}^2) - EAv_0} \right] \quad (25)$$

A.4 F_{C_I}

Insert (19) and 25 in to (13), we can get

$$2F_{C_I} \tan(\delta_0) = \left(EA - m^* {v_0}^2\right) \left[2(\pi - \varphi) \left(\frac{r_I\omega_I}{v_0} - 1 \right) + \frac{r_O\omega_O - r_I\omega_I}{-\mu^* v_0} - \left(\frac{r_I\omega_I}{v_0} - \frac{EA}{EA - m^* {v_0}^2} \right) \frac{1}{-\mu^*} \ln \left[\frac{r_O\omega_O (EA - m^* {v_0}^2) - EAv_0}{r_I\omega_I (EA - m^* {v_0}^2) - EAv_0} \right] \right] - 2m^* {v_0}^2 (\pi - \varphi) \quad (26)$$