CAR: Chapter 1

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Quick overview of basic regression and plots in R

Loading required package: carData

This script introduces a number of principles in R. Understanding these prin-

ciples goes a long way towards helping you become a productive user of R.

Warning: package 'car' was built under R version 3.6.2

The name of each principle is shown in **bold**. You can search through The R Language Definition to pursue questions that arouse your curiosity.

Post questions and comments on Piazza.

1 Arithmetic and basic objects

Operators grouped in reverse order of **precedence**

```
2 + 3 # addition
[1] 5
```

2 - 3 # subtraction

```
[1] -1
```

```
-3
      # unary minus (binary: two arguments, unary: one argument)
  [1] -3
2*3 # multiplication
  [1] 6
2/3 # division
  [1] 0.6666667
2<sup>3</sup> # exponentiation
  [1] 8
```

```
Examples
4^2 - 3*2
  [1] 10
4 ^ 2-3 * 2 # are spaces important in arithmetic expressions?
  [1] 10
1 - 6 + 4 # with equal precedence operations
  「1] -1
# are executed from left to right
1 + 4 - 6
```

```
[1] -1
4^2 - 3*2
  [1] 10
(4^2) - (3*2)
  [1] 10
4^ (2 - 3)*2 # what happens here?
  [1] 0.5
(4 + 3)^2
  [1] 49
```

```
4 + 3^2
  [1] 13
-2--3 # unary minus
  [1] 1
-2 - -3
  [1] 1
-2 - - 3
  [1] 1
```

-3*-2

[1] 6

Which has higher precedence: exponentiation or unary minus?

Some systems, e.g. Excel, follow the opposite order of precedence

-3^2

[1] -9

0-3^2

[1] -9

```
(-3)^2
  [1] 9
Integer division
10 %/% 3
  [1] 3
Modulo arithmetic: remainder
10 %% 3
  [1] 1
```

```
3 * (10 %/% 3) + 10 %% 3 # why?!
  [1] 10
1.1 Extended arithmetic
1/0
  [1] Inf
Inf - 1
  [1] Inf
```

```
Inf + Inf
  [1] Inf
Inf * 2
  [1] Inf
Inf - Inf
  [1] NaN
0/0
  [1] NaN
```

1.2 Complex numbers too

```
0i
  [1] 0+0i
1i
```

[1] 0+1i

exp(1)

Euler's identity

[1] 2.718282

```
exp(1)^(1i*pi)
  [1] -1+0i
\exp(1)^{(1i*pi)} + 1  # complex machine 0
  [1] 0+1.224606e-16i
```

1.3 How numbers are printed

1.23456789

[1] 1.234568

```
1.23456789 * 10^8
```

[1] 123456789

1.23456789e8 # same number in scientific notation

1.23456789 * 10^10

[1] 123456789

[1] 12345678900

1.23456789 * 10^12

[1] 1.234568e+12

```
1.23456789 * 10^-4
```

[1] 0.0001234568

1.23456789 * 10^-5

[1] 1.234568e-05

1.23456789 * 10^-10

[1] 1.234568e-10

1.4 Example of using options and restoring prior state

```
opts <- options(scipen = 15) # penalty 'against' scientific notati
1.23456789
  [1] 1.234568
1.23456789 * 10^8
  [1] 123456789
1.23456789 * 10^10
```

[1] 12345678900

```
1.23456789 * 10^12
```

[1] 1234567890000

1.23456789 * 10^-4

[1] 0.0001234568

1.23456789 * 10^-5

1.23456789 * 10^-10

[1] 0.00001234568

[1] 0.000000001234568

```
opts
```

```
$scipen [1] 0
```

```
options(opts) # restoring previous state
```

To see more options in base R use ?options

1.5 Printing with format

```
format(1.23456789 * 10^10)
```

```
[1] "12345678900"
```

```
format(1.23456789 * 10^10, big.mark = ',')
  [1] "12,345,678,900"
format(123456789.1234, big.mark = ',')
  [1] "123,456,789"
format(123456789.1234, big.mark = ',', nsmall = 2)
  [1] "123,456,789.12"
adding a dollar sign:
```

```
paste0('$',format(123456789.1234, big.mark = ',', nsmall = 2))
```

For more possibilities, see ?format and ?prettyNum. For people who know C, see ?formatC

1.6 typeof

[1] "\$123,456,789.12"

Every object in R has a 'type' that identifies its internal representation. You can find the type with the 'typeof' function.

Let's see the types of things we've seen so far:

```
typeof(1:4)
```

```
[1] "integer"
```

```
typeof(4) # looks like an integer
  [1] "double"
typeof(4L) # In 4L, L stands for L i.e. a 64 bit integer
  [1] "integer"
typeof(4L + 3L)
  [1] "integer"
typeof(4L + 3)
  [1] "double"
```

```
typeof(Inf)
  [1] "double"
typeof(options) # fancy term for a function with baggage
  [1] "closure"
typeof (1.23456789)
  [1] "double"
typeof(paste0)
  [1] "closure"
```

```
typeof('$')
  [1] "character"
typeof(typeof)
  [1] "closure"
typeof(opts)
  [1] "list"
```

2 Calling Functions

```
log(100)
  [1] 4.60517
log(100, base = 10) # why is this different?
  [1] 2
For the 'log' function, the second argument is optional because it has a default
value.
What do you think the default value is?
log10(100) # equivalent
```

[1] 2

```
Functions have arguments.
# the possible arguments and their names and defaults, if any
args("log") # the possible arguments and their
 function (x, base = exp(1))
  NUI.I.
# names and defaults, if any
args("+") # operators are functions too
 function (e1, e2)
  NUI.I.
args("format")
 function (x. ...)
  NULL
```

```
args("library")
  function (package, help, pos = 2, lib.loc = NULL, character.only =
      logical.return = FALSE, warn.conflicts, quietly = FALSE,
      verbose = getOption("verbose"), mask.ok, exclude, include.only
      attach.required = missing(include.only))
  NUI.I.
  • In a language like C, you need to supply every argument when you call a
     function. Thank goodness you don't need to do that in R.
  • Arguments can be supplied by position and/or by name
  • Names can be abbreviated as far as possible to avoid ambiguity
```

args("args")

NULL

function (name)

```
log(100, b=10) # 'b' will do. Why?
[1] 2
```

An argument can be optional even if it doesn't have a default value provided that value of the argument is not used as the function is evaluated.

```
log(100, 10) # arguments can be supplied by name or by position
[1] 2
```

3 Vectors and Variables

```
c(1, 2, 3, 4, NA, 6) # c: combine or catenate or concatenate
  [1] 1 2 3 4 NA 6
1:4 # integer sequence
  [1] 1 2 3 4
4:1 # descending
  [1] 4 3 2 1
-1:2 # negative to positive
  [1] -1 0 1 2
```

```
0-1:2 # why is this different? Could you explain why on a test???

[1] -1 -2
```

```
seq(1, 4) # equivalent to 1:4
```

```
[1] 1 2 3 4
```

```
seq(2, 8, by = 2) # specify interval between elements
```

```
[1] 2 4 6 8
```

```
seq(0, 1, by = 0.1) # noninteger sequence
```

[1] 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

```
seq(0, 1, length = 11) # specify number of elements
   [1] 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
rep(2, times = 10) # 'rep' for 'repeat'
   [1] 2 2 2 2 2 2 2 2 2 2 2
rep(c('a','b'), times= 5) # recycling repeat
   [1] "a" "b" "a" "b" "a" "b" "a" "b" "a" "b"
rep(c('a', 'b'), length.out = 5)
  [1] "a" "b" "a" "b" "a"
```

```
rep(c('a','b'), each = 5) # not recycling, each repeated 5 times
   rep(c('a','b'), times = c(5,2)) # like 'each' different times
  [1] "a" "a" "a" "a" "b" "b"
rep(c('a', 'b'), c(5,2)) # what's the default second argument?
  [1] "a" "a" "a" "a" "b" "b"
```

3.1 Operations on vectors

3 * c(1, 2, 3, 4) # scalar multiplication as in linear algebra

[1] 3 6 9 12

c(1, 2, 3, 4) / 2

[1] 0.5 1.0 1.5 2.0

Dividing a vector by a vector: Is this like linear algebra?

c(1, 2, 3, 4) / c(4, 3, 2, 1)

[1] 0.2500000 0.6666667 1.5000000 4.0000000

What happened? **element-wise operations**. Many operations are applied element by element.

Applying a function to a vector:

```
log(c(0, 0.1, 1, 10, 100), base=10)
  [1] -Inf -1 0 1 2
Many functions in R are vectorized. Sometimes for more than one argument:
log(c(2,10,100), base = c(2,10,100))
  \lceil 1 \rceil \ 1 \ 1 \ 1
What happens if you do something that doesn't make sense, like adding a long
vector to a short vector:
c(1, 2, 3, 4) + c(4, 3) # no warning! What happens?
```

[1] 5 5 7 7

R usually assumes that you must know what you are doing and tries to do something reasonable. Here R applied what's known as the **recycling** principle: recycle the shorter vector as many times as necessary to provide value to match the longer vector.

However, if the length of the longer vector is not a multiple of the length of the shorter vector, R starts to worry about you and gives you a warning:

$$c(1, 2, 3, 4) + c(4, 3, 2) \# R$$
 thinks maybe you made a mistake

Warning in c(1, 2, 3, 4) + c(4, 3, 2): longer object length is not a multiple of shorter object length

[1] 5 5 5 8

3.2 Creating and naming objects

```
x \leftarrow c(1, 2, 3, 4) # assignment -- does not print result x # prints value of x
```

[1] 1 2 3 4

 $(x \leftarrow c(1, 2, 3, 4))$ # assigns and prints at the same time

[1] 1 2 3 4

Why not = instead of <-? (pronouced 'gets').

We used = to assign arguments to parameters when calling functions.

Actually, = works \dots most of the time. But stick with <-. Your fingers will get used to it.

R uses three symbols for three distinct operations that are all represented with = in most other languages. The third, which we will see soon, is logical = which is denoted == in R.

I still regularly make the mistake of typing = when I should have typed == .

```
x/2 # equivalent to c(1, 2, 3, 4)/2

[1] 0.5 1.0 1.5 2.0
```

```
(y <- sqrt(x)) # assign AND print
```

```
[1] 1.000000 1.414214 1.732051 2.000000
```

3.3 Random number generation

```
set.seed(372291) # for reproducibility (chosen randomly)
(x <- rnorm(100)) # 100 standard normal random numbers</pre>
```

[1]	0.09833263	-0.89632065	-0.80499585	-0.23440421	
[5]	-0.57098627	-0.12431051	-0.35938865	1.19509408	
[9]	-0.51327643	-1.16888069	2.04501263	0.96774659	
[13]	-0.33892531	-0.02385635	-0.13606928	-0.26846724	
[17]	0.06233524	0.86995856	1.13066988	-0.34642095	
[21]	0.35064032	1.11546736	-0.51577251	0.12023891	
[25]	0.26794257	0.81975768	-0.05561499	0.78947930	
[29]	0.24476899	-0.23868740	0.72776887	0.24950307	
[33]	0.29120391	-0.98124553	0.04353949	-0.85734282	
[37]	2.63618842	-1.25254955	0.83030163	-1.95090213	
[41]	-2.68712959	-0.36168710	-0.58443713	0.81399991	
[45]	-0.02585634	-1.11835697	1.11836461	-1.29533607	
[49]	0.19998422	0.51477512	0.53101946	-0.03766582	
[53]	0.98327288	-1.57294475	1.51531106	1.04962419	
[57]	-0.88333128	2.13595738	1.44940962	-0.84459977	
[61]	-1.80824601	-0.09673322	-1.00007308	1.63933460	
[65]	-0.52798368	-0.24537308	-0.31443488	1.15314622	
[69]	-0.91751726	1.26672906	-1.59600134	0.16465603	
[73]	0.85672451	0.82863616	-0.42284364	0.79206310	

```
[77] -0.02569929 -0.19991971 1.51607308 0.50764612

[81] 1.28908616 1.03368892 -0.55391013 -1.29783452

[85] 1.56289281 0.51518221 0.69617559 -0.86324452

[89] -0.43990620 1.11514478 -1.40240347 0.10543920

[93] 0.12556730 1.88108615 1.45922041 -0.33864564

[97] 0.61101545 -0.23174691 -0.03237697 1.44987907
```

Many distributions are in base R: e.g. 'norm', 'exp', 'poisson', student', 't', 'cauchy', 'f', etc.

The 'extraDistr' package and others have many more.

To generate random numbers from a distribution, prepend the name with and ${f r}$:

```
rf(10, df1 = 2, df2 = 10)
```

```
[1] 1.31169785 0.07542327 0.44086623 4.30693205 9.62530417 [6] 1.31581038 0.09651469 0.05892881 0.63146620 0.29580129
```

To get the density for a continuous distribution) or the probability for a discrete distribution, prepend the name with a ${\bf d}$ for density. Note that the probability in the case of a discrete distribution is indeed a density with respect to counting measure.

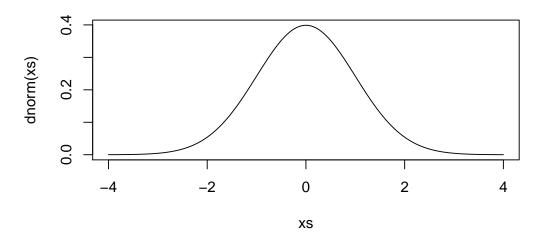
```
dnorm(0)
```

[1] 0.3989423

```
dnorm(seq(-3,3,1))
```

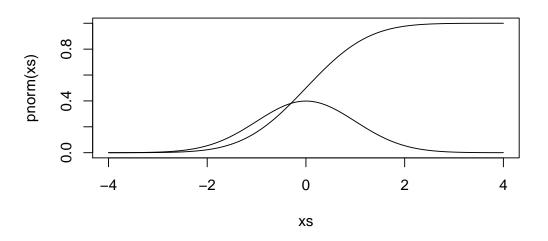
[1] 0.004431848 0.053990967 0.241970725 0.398942280 [5] 0.241970725 0.053990967 0.004431848

```
xs <- seq(-4, 4, by = .05)
plot(xs, dnorm(xs), type = 'l')</pre>
```



Prepend with a ${\bf p}$ for the cumulative distribution function

```
plot(xs, pnorm(xs), type = 'l')
lines(xs, dnorm(xs), type = 'l')
```



To get a quantile from a probability, the inverse CDF, prepend the distribution name with a ${\bf q}$

Note the ubiquitous summary function:

```
summary(x)
```

```
Min. 1st Qu. Median Mean 3rd Qu. Max. -2.68713 -0.51390 0.05294 0.12372 0.83691 2.63619
```

We will see much more about it when we take up OOP.

[1] "To" "be" "or" "not" "to" "be"

3.4 Character objects

```
(words <- c("To", "be", "or", "not", "to", "be"))
```

```
typeof(words)
  [1] "character"
paste(words, collapse=" ")
  [1] "To be or not to be"
paste(words)
  [1] "To" "be" "or" "not" "to" "be"
paste(words, words)
  [1] "To To" "be be" "or or" "not not" "to to"
  [6] "be be"
```

```
paste('Variable', 1:10)
   [1] "Variable 1" "Variable 2" "Variable 3" "Variable 4"
   [5] "Variable 5" "Variable 6"
                                  "Variable 7" "Variable 8"
   [9] "Variable 9" "Variable 10"
paste('Variable', 1:10, sep = ' ')
   [1] "Variable 1" "Variable 2"
                                  "Variable 3" "Variable 4"
   [5] "Variable 5" "Variable 6"
                                  "Variable 7" "Variable 8"
   [9] "Variable 9" "Variable 10"
paste('Var', 1:10, sep = '')
   [1] "Var1" "Var2" "Var3" "Var4" "Var5" "Var6" "Var7"
   [8] "Var8" "Var9" "Var10"
```

```
paste0('Var', 1:10) # same as sep = ''
   [1] "Var1" "Var2" "Var3" "Var4" "Var5" "Var6" "Var7"
   [8] "Var8" "Var9" "Var10"
Take a column of $ amounts and format accordingly
(amts \leftarrow c(123.45, 123456.78, 12345678.912))
  [1]
           123.45 123456.78 12345678.91
paste0('$', format(amts, big.mark = ',', nsmall = 2))
  [1] "$
               123.45" "$ 123,456.78" "$12,345,678.91"
```

OOPS: Look up help on format with '?format' and discover the 'trim' argument.

```
paste0('$', format(amts, big.mark = ',', nsmall = 2, trim = TRUE))
  [1] "$123.45"
                       "$123,456.78" "$12,345,678.91"
That's better!
3.5 Logical values
(logical.values <- c(TRUE, TRUE, FALSE, TRUE))</pre>
  Г1]
       TRUE TRUE FALSE TRUE
typeof(logical.values)
  [1] "logical"
```

```
Actually R uses '3-valued' logic
(logical.values <- c(TRUE, TRUE, FALSE, NA, TRUE))
  Г1]
      TRUE TRUE FALSE NA TRUE
!logical.values # unary not
  [1] FALSE FALSE TRUE NA FALSE
logical.values | FALSE # binary or -- note that FALSE gets recycle
  Г1]
      TRUE TRUE FALSE NA TRUE
logical.values & FALSE # binary and
  [1] FALSE FALSE FALSE FALSE
```

```
logical.values | TRUE # binary or -- note that FALSE gets recycled
  [1] TRUE TRUE TRUE TRUE TRUE
logical.values & TRUE # binary and
  [1]
           TRUE FALSE
      TRUE
                      NA TRUE
logical.values == FALSE # equals
  [1] FALSE FALSE TRUE NA FALSE
logical.values != FALSE # not equals
```

[1] TRUE TRUE FALSE NA TRUE

```
logical.values == TRUE # equals
```

[1] TRUE TRUE FALSE NA TRUE

logical.values != TRUE # not equals

[1] FALSE FALSE TRUE NA FALSE

Did the 'NA" always result in 'NA' in the corresponding position? If not, why not? Could you explain why on a test?

3.6 Truth tables

```
x <- c(TRUE, FALSE, NA) x
```

```
[1] TRUE FALSE NA
```

```
names(x) # the names attribute of x is NULL
```

NULL

```
names(x) <- as.character(x) # use 'names' replacement function
# to give x names
x</pre>
```

```
TRUE FALSE <NA>
TRUE FALSE NA
```

Truth tables

```
outer(x, x, '|') # note the value of "TRUE / NA"
```

TRUE FALSE <NA>
TRUE TRUE TRUE TRUE
FALSE TRUE FALSE NA
<NA> TRUE NA NA

```
outer(x, x, '&')
```

TRUE FALSE <NA>
TRUE TRUE FALSE NA
FALSE FALSE FALSE
<NA> NA FALSE NA

```
outer(x, x, '==')
```

TRUE FALSE <NA>

```
TRUE TRUE FALSE NA
FALSE FALSE TRUE NA
<NA> NA NA NA
```

Example of logical expressions

```
1 == 2
```

[1] FALSE

```
1 != 2
  [1] TRUE
1 <= 2
  [1] TRUE
1 < 1:3 # recycling
  [1] FALSE TRUE TRUE
3:1 > 1:3
  [1] TRUE FALSE FALSE
```

```
3:1 >= 1:3
```

[1] TRUE TRUE FALSE

```
TRUE & c(TRUE, FALSE) # logical AND
```

[1] TRUE FALSE

```
c(TRUE, FALSE, FALSE) | c(TRUE, TRUE, FALSE) # logical OR
```

[1] TRUE TRUE FALSE

Logical vectors used in 'ifelse' statement

$$(z \leftarrow x[1:10]) \# first 10 \ elements \ of \ x$$

z < -0.5 # is each element less than -0.5?

z > 0.5 # is each element greater than 0.5

 $z < -0.5 \mid z > 0.5 \# < and > are of higher precedence than /$

abs(z) > 0.5 # absolute value, equivalent to last expression

z[abs(z) > 0.5] # values of z for which |z| > 0.5

 z[!(abs(z) > 0.5)] # values z for which $|z| \le 0.5$

3.7 Special logical operators && and ||

- Work only on single expressions, not vectors
- Only evaluate what they need to determine the result

TRUE && FALSE # works only on single expressions, not vectors

[1] FALSE

```
TRUE || FALSE  # only evaluates what it needs to determine result
  [1] TRUE
QUESTION: Explain what happens here
log(-1)
  Warning in log(-1): NaNs produced
  [1] NaN
TRUE | | log(-1)
  [1] TRUE
```

```
FALSE || log(-1)
```

```
Warning in log(-1): NaNs produced
```

[1] NA

3.8 How coercion works in R

What happens when you try to do something that isn't quite right with an object?

```
sum(logical.values) # logical gets coerced to numeric
```

[1] NA

TRUE becomes 1 and FALSE becomes 0

```
sum(!logical.values)
```

[1] NA

What happens when you catenate things that are of different types? Vectors can only contain elements of the same type.

We'll soon see how to use **list** that can contain elements of different types.

```
c("A", FALSE, 3.0)
```

```
[1] "A" "FALSE" "3"
```

```
c(10, FALSE, -6.5, TRUE)
```

```
[1] 10.0 0.0 -6.5 1.0
```

3.9 The hierarchy of atomic types: promotion and demotion

- logical (lowest)
- numeric:
 - integer
 - double
 - complex
- character (highest)

All the elements of a vector must be of the same type so if you try to mix types in a vector, the 'lower' types get **coerced** (i.e. **promoted**) to the 'higher' types

```
c(TRUE, 1, 'one') # all get coerced to character
```

```
[1] "TRUE" "1" "one"
```

```
c(TRUE,1i) # all get coerced to complex

[1] 1+0i 0+1i

c(TRUE,1i, 'one') # what should happen here?

[1] "TRUE" "0+1i" "one"
```

QUESTION: Try to guess the difference between the result evaluated from the following 2 lines. Note that the innermost expressions need to be evaluated first.

```
c(c(TRUE, 2), 'dog')
c(TRUE, c(2, 'dog'))
```

3.10 Explicit coercion

```
as.logical(2)
```

```
[1] TRUE
```

```
as.character(2)
```

```
[1] "2"
```

as.integer(2.5)

```
[1] 2
```

```
as.integer(2.9) # truncation, not rounding
  [1] 2
round(2.5)
          # What's happening?
  [1] 2
round(3.5)
  [1] 4
typeof(round(2.5))
  [1] "double"
```

```
as.complex(2)
  [1] 2+0i
as.numeric('2')
  [1] 2
as.numeric('two')
 Warning: NAs introduced by coercion
  [1] NA
3.11
      Implicit coercion
```

```
4 + TRUE
  [1] 5
'TRUE' got promoted to numeric
4 & FALSE
  [1] FALSE
4 got demoted to logical
1 == TRUE # does 1 get demoted, or TRUE get promoted?
```

[1] TRUE

4 Selecting elements of a vector

4.1 Selecting by position with positive numbers

```
x[12]  # 12th element

<NA>
    NA

words[2]  # second element

[1] "be"
```

[1] FALSE

logical.values[3] # third element

x[6:15] # elements 6 through 15

< NA> < NA < NA < NA < NA < NA < NA < NA

x[c(1, 3, 5)] # 1st, 3rd, 5th elements

TRUE <NA> <NA>
TRUE NA NA

QUESTION: What happens if you go too far?

words[10]

[1] NA

```
x[100000]
```

<NA>

logical.values[9]

[1] NA

Although all these NA's look the same, they are subtly different

4.2 Selecting by omission of positions with negative numbers

```
x[-(11:100)] # omit elements 11 through 100
```

```
TRUE FALSE <NA>
TRUE FALSE NA
```

letters # this object comes with R

```
[1] "a" "b" "c" "d" "e" "f" "g" "h" "i" "j" "k" "l" "m" "n" [15] "o" "p" "q" "r" "s" "t" "u" "v" "w" "x" "y" "z"
```

letters [-c(1,5,9,15,21)] # what have we done?

```
[1] "b" "c" "d" "f" "g" "h" "j" "k" "l" "m" "n" "p" "q" "r" [15] "s" "t" "v" "w" "x" "y" "z"
```

4.3 Selecting with a logical vector

```
v <- 1:4
v[c(TRUE, FALSE, FALSE, TRUE)]

[1] 1 4

vowels <- c('a','e','i','o','u')
letters %in% vowels # a very useful operator</pre>
```

```
[1] TRUE FALSE FALSE FALSE TRUE FALSE FALSE TRUE
[10] FALSE FALSE FALSE FALSE TRUE FALSE FALSE
[19] FALSE FALSE TRUE FALSE FALSE FALSE FALSE
```

```
letters[!(letters %in% vowels)]
```

```
[1] "b" "c" "d" "f" "g" "h" "j" "k" "l" "m" "n" "p" "q" "r"
  [15] "s" "t" "v" "w" "x" "y" "z"
`[`(v,c(T,F,T,F))  # Everything that happens is a function!
  [1] 1 3
`[`(v,c(T,F)) # What's happening here??
  [1] 1 3
v[c(T,F)]
```

[1] 1 3

4.4 Selecting with names

Vectors can get named:

```
(age <- c(10, 9, 15, 16))
[1] 10 9 15 16
```

```
names(age) <- c('Paula','Imran','Angela','Jiwon') # replacement fn
age</pre>
```

```
Paula Imran Angela Jiwon
10 9 15 16
```

Selecting by name

```
age['Angela']
  Angela
      15
age['George']
  <NA>
    NA
age[c('Jiwon', 'Paula')]
  Jiwon Paula
     16
            10
```

```
age[c(NA, 'Paula')]
   <NA> Paula
     NA
          10
names (age)
  [1] "Paula" "Imran" "Angela" "Jiwon"
names(age) %in% c('Bob', 'Paula', 'Imran')
  [1]
      TRUE TRUE FALSE FALSE
age[ names(age) %in% c('Bob', 'Paula', 'Imran') ]
  Paula Imran
     10
```

```
age[ !names(age) %in% c('Bob', 'Paula', 'Imran') ]
  Angela Jiwon
      15
              16
Example of a regular expression
'A' matches a capital A at the beginning of a string
'grepl' stands for global regular expression print logical
grepl('^A', names(age))
  [1] FALSE FALSE TRUE FALSE
age[ grepl('^A', names(age)) ]
  Angela
      15
```

```
sort(names(age))
[1] "Angela" "Imran" "Jiwon" "Paula"
```

```
age[ sort(names(age)) ]
```

```
Angela Imran Jiwon Paula
15 9 16 10
```

4.5 Selecting elements of a matrix

A **matrix** is like a vector but with two dimensions. If the dimension is higher, it's called an **array**.

```
mat <- matrix(1:12, nrow = 3, ncol = 4)
mat # notice that it got filled column by column</pre>
```

```
[,1] [,2] [,3] [,4]
[1,] 1 4 7 10
[2,] 2 5 8 11
[3,] 3 6 9 12
```

A matrix can have rownames and columns names

```
rownames(mat) <- c('Algebra', 'Analysis', 'Geometry')
colnames(mat) <- c('2013', '2014', '2015', '2016')
mat</pre>
```



Selecting elements is much like vector except that we have two dimensions. Look at the following carefully to see what's happening. Notice what happens if a dimension is blank.

```
mat[2,4]
```

mat[2,5] # different from vectors

[1] 11

Error in mat[2, 5]: subscript out of bounds

mat[c(2,3),] # blank means: take everything

9

```
2013 2014 2015 2016
Analysis 2 5 8 11
Geometry 3 6
```

mat[c(2,3), 1:4] # blank means: take everything

```
2013 2014 2015 2016
Analysis 2 5 8 11
Geometry 3 6 9 12
```

mat[c(2,3),3] # Oops! a dimension got dropped

Analysis Geometry 8 9

Dropping a dimension might seem like no big deal but it's one of the things that early designers regret adopting as a default. If your matrix is buried in a function and it drops a dimension that might spell hidden trouble when the function tries to multiply it by another matrix.

```
mat[c(2,3), 3, drop = FALSE]
           2015
  Analysis 8
  Geometry 9
Using 'drop' is the safe way within functions where you don't know whether the
selecting vector might have length 1 or 0 when the function is called.
mat[c('Algebra', 'Geometry'), c(2014, 2015)] # Need characters
  Error in mat[c("Algebra", "Geometry"), c(2014, 2015)]: subscript o
mat[c('Algebra', 'Geometry'), c('2014', '2015')] # Good
           2014 2015
  Algebra 4 7
  Geometry 6 9
```

```
mat[c('Algebra','Geometry'),c(NA,'2014','2015')] # NA tilts
```

Error in mat[c("Algebra", "Geometry"), c(NA, "2014", "2015")]: sub

mat[c('Algebra','Geometry'),-3]

```
2013 2014 2016
Algebra 1 4 10
Geometry 3 6 12
```

mat[c('Algebra', 'Geometry'), -(1:4)] # a columnless matrix

Algebra Geometry

```
mat[-(1:3),]
```

2013 2014 2015 2016

```
mat[-(1:3),-(1:4)]
```

<0 x 0 matrix>

5 Operators are functions

Chambers' dictum (main early creator of S at Bell Labs)

- Everything that exists is an object
- Everything that happens is a function call

```
1 + 2 # `+` is a binary operator
[1] 3
`+`(1,2) # but really a function with two arguments
[1] 3
```

Note: to call an object with a weird name, just put its names in backticks.

6 Valid object names in R

What names are valid?

You can use:

- letters, case-sensitive
- numbers
- underline
- period
- must start with a letter or period but not a number or underline
- the first non-period character must not be a number

QUESTION: Which of the following assignments use valid names?

7 Writing your own functions

It's easy in R to write your own functions and to build up a toolbox over time. You can document your functions in a package and share it with other users.

```
mean(x)
```

```
sum(x)/length(x)
  Γ17 NA
Here's a very simple function that duplicates an existing one:
myMean <- function(x){</pre>
  sum(x)/length(x)
myMean(x)
  [1] NA
y # defined earlier as sqrt(c(1, 2, 3, 4))
  [1] 1.000000 1.414214 1.732051 2.000000
```

```
myMean(y)
  [1] 1.536566
myMean(1:100)
  [1] 50.5
myMean(sqrt(1:100))
  [1] 6.714629
mySD <- function(x){</pre>
  sqrt(sum((x - myMean(x))^2)/(length(x) - 1))
}
mySD(1:100)
```

```
[1] 29.01149
sd(1:100) # check
  [1] 29.01149
typeof(mySD)
  [1] "closure"
Functions are 'first-class' objects in R
mySD
  function(x){
    sqrt(sum((x - myMean(x))^2)/(length(x) - 1))
```

```
myMean
```

```
function(x){
  sum(x)/length(x)
}
<bytecode: 0x00000001243e8a0>
```

letters

```
[1] "a" "b" "c" "d" "e" "f" "g" "h" "i" "j" "k" "l" "m" "n" [15] "o" "p" "q" "r" "s" "t" "u" "v" "w" "x" "y" "z"
```

```
mySD(letters)
```

Error in sum(x): invalid 'type' (character) of argument

8 Quick overview of basic regression and plots in R

'Duncan' is a data frame (the term used in R to refer to the kind of object used to contain a data set) in the 'car' package. Since we loaded the 'car' package (with 'library(car)') we can use 'Duncan' typing its name.

head(Duncan, n=10) # U.S. data set from the 50s, first 10 lines

	type	income	education	prestige
${\tt accountant}$	prof	62	86	82
pilot	prof	72	76	83
architect	prof	75	92	90
author	prof	55	90	76
chemist	prof	64	86	90
minister	prof	21	84	87
professor	prof	64	93	93
dentist	prof	80	100	90

 reporter
 wc
 67
 87
 52

 engineer
 prof
 72
 86
 88

dim(Duncan)

[1] 45 4

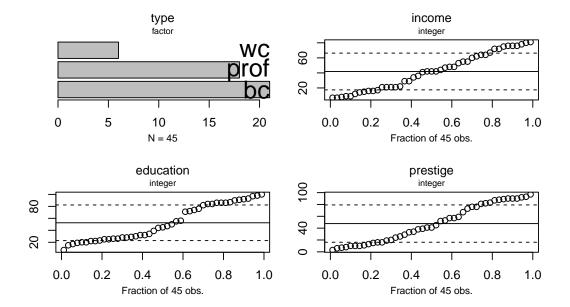
summary(Duncan)

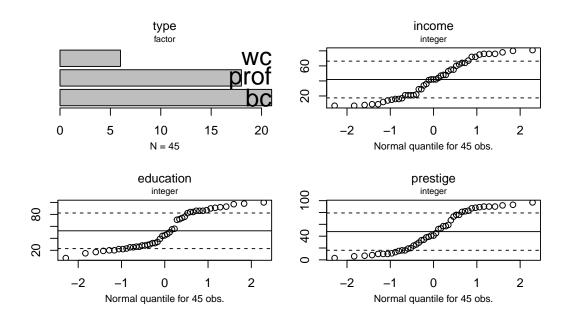
type	income	education	prestige
bc :21	Min. : 7.00	Min. : 7.00	Min. : 3.00
prof:18	1st Qu.:21.00	1st Qu.: 26.00	1st Qu.:16.00
wc : 6	Median :42.00	Median : 45.00	Median :41.00
	Mean :41.87	Mean : 52.56	Mean :47.69
	3rd Qu.:64.00	3rd Qu.: 84.00	3rd Qu.:81.00
	Max. :81.00	Max. :100.00	Max. :97.00

library(spida2) # install with devtools::install_github('gmonette/

 ${\tt spida2:\ development\ branch\ 0.2.0.9000.}$

xqplot(Duncan) # uniform quantile plots





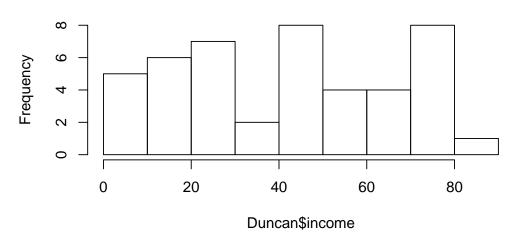
8.1 Referencing variables in data frames

We will see this again later

Fully qualified name

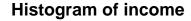
hist(Duncan\$income)

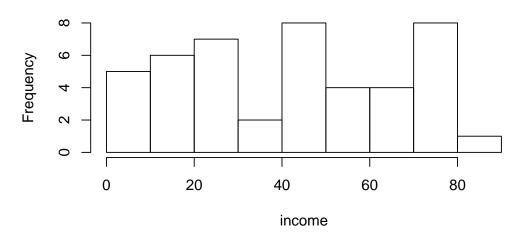
Histogram of Duncan\$income



using the 'with' function: the second argument is evaluated in the data frame

with(Duncan, hist(income))

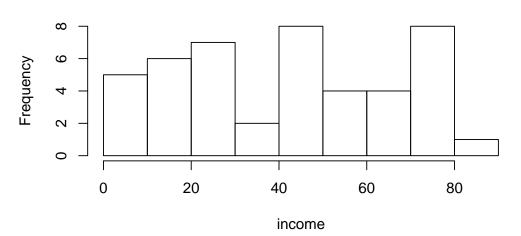




using 'attach' - ___ very highly deprecated___ e.g. this blog post attach(Duncan)

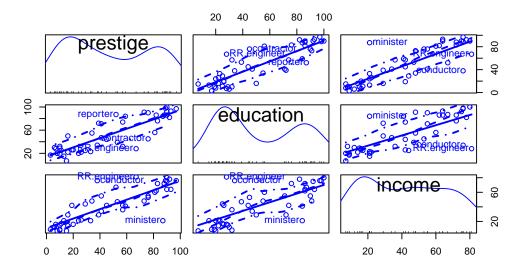
hist(income)

Histogram of income



detach(Duncan)

Many 'recent' (past 25 years) modelling functions and graphic functions in R use **formulas**. When a model or a graph is defined by a formula, the data frame in which the terms of the formula are to be found is an argument of the function.



Fitting a least-squares regression model

```
(Duncan.model <- lm(prestige ~ education + income,
data = Duncan))
```

```
Call:
lm(formula = prestige ~ education + income, data = Duncan)
Coefficients:
(Intercept) education income
    -6.0647    0.5458    0.5987
```

Estimated coefficients and other results of the regression:

```
summary(Duncan.model)
```

Call:

```
lm(formula = prestige ~ education + income, data = Duncan)
```

Residuals:

```
Min 1Q Median 3Q Max -29.538 -6.417 0.655 6.605 34.641
```

Coefficients:

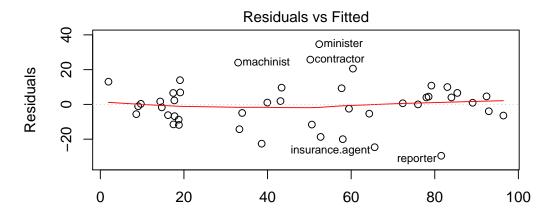
Signif. codes:

```
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

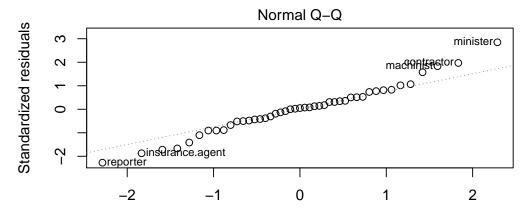
Residual standard error: 13.37 on 42 degrees of freedom Multiple R-squared: 0.8282, Adjusted R-squared: 0.82 F-statistic: 101.2 on 2 and 42 DF, p-value: < 2.2e-16

Four important plots to check the regression:

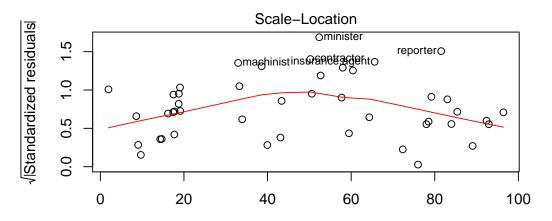
plot(Duncan.model, id.n = 5, ask = FALSE)



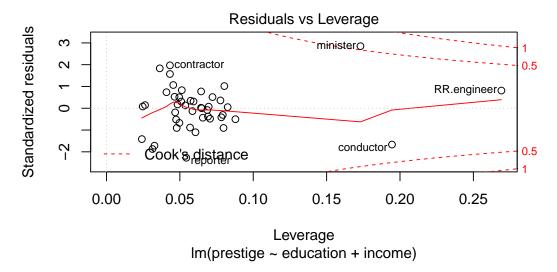
Fitted values Im(prestige ~ education + income)



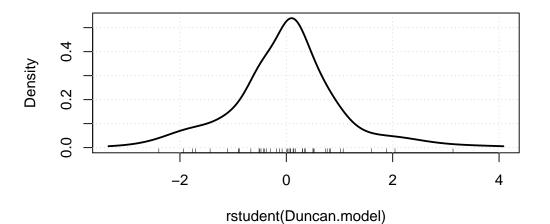
Theoretical Quantiles Im(prestige ~ education + income)



Fitted values Im(prestige ~ education + income)



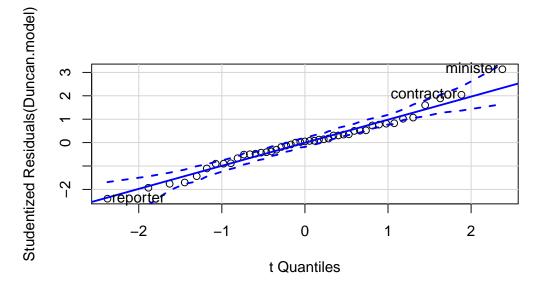
The density of the studentized residuals:

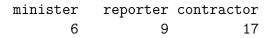


-- D1 -+ (D--- --- --- 1-1 - - 1 - 1 - - - 2))

qqPlot(Duncan.model, id = list(n = 3))

Quantile plot of residuals:



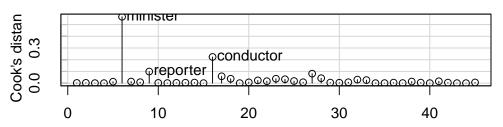


```
Outliers:
```

```
outlierTest(Duncan.model)
```

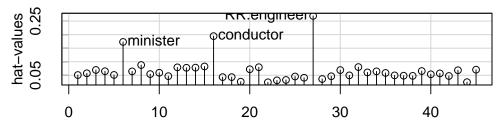
```
No Studentized residuals with Bonferroni p < 0.05 Largest |rstudent|: rstudent unadjusted p-value Bonferroni p minister 3.134519 0.0031772 0.14297
```

Diagnostic Plots



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Diagnostic Plots

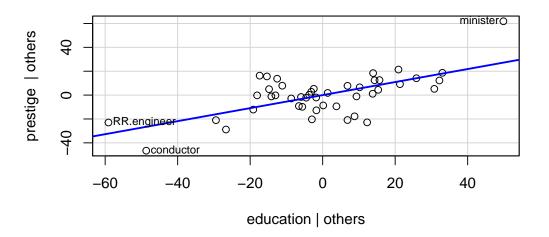


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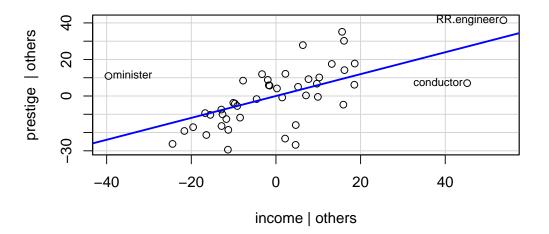
8.2 Added-variable plots

Also know in PROC REG in SAS as 'partial regression leverage plots' not to be confused with partial residual plots (better described as 'component plus residual plots') although the are often called that.

```
avPlots(Duncan.model, ~ education,
    id=list(cex=0.75, n=3, method="mahal"))
```



Imagine the following plot is a simple scatterplot between two variables. What do you see?

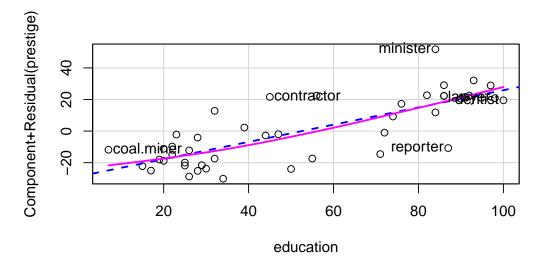


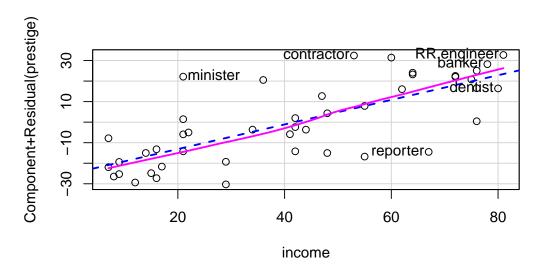
In multiple regression, the AVP is the closest you come to seeing what drives the estimation of each regression coefficient.

8.3 Component-plus-residual plots

Also known as partial residual plots. These tend to be more widely used but often less informative than AVPs

```
crPlots(Duncan.model, ~ education, id = list(n=3))
```





Non-constant variance test

ncvTest(Duncan.model)

Non-constant Variance Score Test Variance formula: ~ fitted.values Chisquare = 0.3810967, Df = 1, p = 0.53702

ncvTest(Duncan.model, var.formula = ~ income + education)

Non-constant Variance Score Test Variance formula: ~ income + education Chisquare = 0.6976023, Df = 2, p = 0.70553

Position of row names

```
whichNames(c("minister", "conductor"), Duncan)
   minister conductor
          6
                    16
duncan.model.2 <- update(Duncan.model, subset = -c(6, 16))
An alternative approach that uses the '%in%' operator to creae a logical vector
outliers <- names(Duncan) %in% c("minister", "conductor")
duncan.model.2 <- update(Duncan.model, subset = !outliers)</pre>
summary(duncan.model.2)
```

```
Call:
lm(formula = prestige ~ education + income, data = Duncan, subset
Residuals:
   Min 1Q Median 3Q Max
-29.538 -6.417 0.655 6.605 34.641
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) -6.06466 4.27194 -1.420 0.163
education 0.54583 0.09825 5.555 1.73e-06 ***
income 0.59873 0.11967 5.003 1.05e-05 ***
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 13.37 on 42 degrees of freedom
Multiple R-squared: 0.8282, Adjusted R-squared: 0.82
```

F-statistic: 101.2 on 2 and 42 DF, p-value: < 2.2e-16

```
Comparing coefficients with and without outliers:
```

```
compareCoefs(duncan.model, duncan.model.2)
```

```
Error in compareCoefs(duncan.model, duncan.model.2): object 'dunca
```

Type of predictor

21

```
summary(Duncan$type)
```

```
bc prof wc
   18 6
```

summary(Duncan\$prestige)

```
Min. 1st Qu. Median Mean 3rd Qu. Max.
3.00 16.00 41.00 47.69 81.00 97.00
```

summary(Duncan)

type	income	education	prestige
bc :21	Min. : 7.00	Min. : 7.00	Min. : 3.00
prof:18	1st Qu.:21.00	1st Qu.: 26.00	1st Qu.:16.00
wc : 6	Median :42.00	Median : 45.00	Median :41.00
	Mean :41.87	Mean : 52.56	Mean :47.69
	3rd Qu.:64.00	3rd Qu.: 84.00	3rd Qu.:81.00
	Max. :81.00	Max. :100.00	Max. :97.00

Model with interaction

```
summary(lm(prestige ~ education + income, data = Duncan))
```

```
Call:
lm(formula = prestige ~ education + income, data = Duncan)
```

```
Residuals:

Min 1Q Median 3Q Max
-29.538 -6.417 0.655 6.605 34.641
```

Coefficients:

Signif. codes:

```
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 13.37 on 42 degrees of freedom Multiple R-squared: 0.8282, Adjusted R-squared: 0.82 F-statistic: 101.2 on 2 and 42 DF, p-value: < 2.2e-16

```
class(Duncan$type)
  [1] "factor"
class(Duncan$prestige)
  [1] "integer"
class(Duncan)
  [1] "data.frame"
duncan.model <- lm(prestige ~ education + income, data=Duncan)</pre>
class(duncan.model)
  [1] "lm"
```

```
A generic function:
```

summary

```
function (object, ...)
UseMethod("summary")
<bytecode: 0x000000018c01c78>
<environment: namespace:base>
```

A **method** for the 'lm' **class** for the generic function

```
args(summary.lm)
```

The real work happens in the special function that is the method for the generic function 'summary' for the 'lm' class.

summary.lm

```
function (object, correlation = FALSE, symbolic.cor = FALSE,
    ...)
    z <- object
    p <- z$rank
    rdf <- z$df.residual
    if (p == 0) {
        r <- z$residuals
        n <- length(r)
        w <- z$weights
        if (is.null(w)) {
            rss \leftarrow sum(r^2)
```

```
else {
    rss \leftarrow sum(w * r^2)
    r \leftarrow sqrt(w) * r
resvar <- rss/rdf
ans <- z[c("call", "terms", if (!is.null(z$weights)) "weig
class(ans) <- "summary.lm"</pre>
ans$aliased <- is.na(coef(object))</pre>
ans$residuals <- r
ans$df <- c(OL, n, length(ans$aliased))
ans$coefficients <- matrix(NA real , OL, 4L, dimnames = li
    c("Estimate", "Std. Error", "t value", "Pr(>|t|)")))
ans$sigma <- sqrt(resvar)</pre>
ans$r.squared <- ans$adj.r.squared <- 0
ans$cov.unscaled <- matrix(NA real , OL, OL)
if (correlation)
    ans$correlation <- ans$cov.unscaled
return(ans)
```

```
if (is.null(z$terms))
    stop("invalid 'lm' object: no 'terms' component")
if (!inherits(object, "lm"))
    warning("calling summary.lm(<fake-lm-object>) ...")
Qr <- qr.lm(object)</pre>
n <- NROW(Qr$qr)</pre>
if (is.na(z$df.residual) || n - p != z$df.residual)
    warning("residual degrees of freedom in object suggest thi
r <- z$residuals
f <- z$fitted.values
w <- z$weights
if (is.null(w)) {
    mss <- if (attr(z$terms, "intercept"))</pre>
        sum((f - mean(f))^2)
    else sum(f^2)
    rss \leftarrow sum(r^2)
else {
    mss <- if (attr(z$terms, "intercept")) {</pre>
```

```
m \leftarrow sum(w * f/sum(w))
        sum(w * (f - m)^2)
    else sum(w * f^2)
    rss \leftarrow sum(w * r^2)
    r \leftarrow sqrt(w) * r
resvar <- rss/rdf
if (is.finite(resvar) && resvar < (mean(f)^2 + var(c(f))) *
    1e-30)
    warning("essentially perfect fit: summary may be unreliabl
p1 <- 1L:p
R <- chol2inv(Qr$qr[p1, p1, drop = FALSE])
se <- sqrt(diag(R) * resvar)</pre>
est <- z$coefficients[Qr$pivot[p1]]</pre>
tval <- est/se
ans <- z[c("call", "terms", if (!is.null(z$weights)) "weights"
ans$residuals <- r
ans$coefficients <- cbind(Estimate = est, `Std. Error` = se,
```

```
't value' = tval, Pr(>|t|) = 2 * pt(abs(tval), rdf,
        lower.tail = FALSE))
ans$aliased <- is.na(z$coefficients)</pre>
ans$sigma <- sqrt(resvar)</pre>
ans$df <- c(p, rdf, NCOL(Qr$qr))</pre>
if (p != attr(z$terms, "intercept")) {
    df.int <- if (attr(z$terms, "intercept"))</pre>
        1L
    else OL
    ans$r.squared <- mss/(mss + rss)</pre>
    ansadj.r.squared <-1 - (1 - ans<math>r.squared) * ((n -
        df.int)/rdf)
    ans$fstatistic <- c(value = (mss/(p - df.int))/resvar,</pre>
        numdf = p - df.int, dendf = rdf)
}
else ans$r.squared <- ans$adj.r.squared <- 0
ans$cov.unscaled <- R
dimnames(ans$cov.unscaled) <- dimnames(ans$coefficients)[c(1,
    1)]
```

```
if (correlation) {
        ans$correlation <- (R * resvar)/outer(se, se)
        dimnames(ans$correlation) <- dimnames(ans$cov.unscaled)
        ans$symbolic.cor <- symbolic.cor</pre>
    if (!is.null(z$na.action))
        ans$na.action <- z$na.action
    class(ans) <- "summary.lm"</pre>
    ans
<bytecode: 0x00000001262aac0>
```

This is R's basic form of Object-Oriented-Programming (OOP). It's what makes it possible for R to have grown through the work of different contributors working relatively independently.

<environment: namespace:stats>

If you create a new statistical method that produces a special kind of object, you don't have to request that 'summary' be changed to work on your object. You just give your object a class and write a summary method for it.

Here's latest discovery of a new statistical method: Use the median to 'fit' data!

silly <- function(x) {

```
silly <- function(x) {
  ret <- median(x)
  class(ret) <- 'silly'
  ret
}
fit <- silly(x)
fit</pre>
```

```
attr(,"class")
[1] "silly"
```

class(fit)

[1] "silly"

I write a method for the summary generic function for the class 'silly'.
summary.silly <- function(fit) {
 cat('This is the fitted value from the silly method: ')
 cat(fit,'\n')
 invisible(fit)</pre>

```
See what it does:
```

summary(fit)

```
This is the fitted value from the silly method: NA
```

When I called the 'summary' function, it checked the class of its first argument, 'fit'.

Finding that the class was 'silly', summary looks for a function called 'summary.silly' and uses that function. This is called **dispatching**. If 'summary.silly' hadn't existed summary would have used 'summary.default'.

```
For example, when you use 'glm'
mod.mroz <- glm(lfp ~ ., family=binomial, data=Mroz)</pre>
class(mod.mroz)
  [1] "glm" "lm"
summary.glm
  function (object, dispersion = NULL, correlation = FALSE, symbolic
      ...)
      est.disp <- FALSE
      df.r <- object$df.residual
      if (is.null(dispersion))
          dispersion <- if (object$family$family %in% c("poisson",
              "binomial"))
```

```
else if (df.r > 0) {
        est.disp <- TRUE
         if (any(object$weights == 0))
             warning("observations with zero weight not used fo
        sum((object$weights * object$residuals^2)[object$weigh
             01)/df.r
    else {
        est.disp <- TRUE
        NaN
aliased <- is.na(coef(object))</pre>
p <- object$rank</pre>
if (p > 0) {
    p1 <- 1L:p
    Qr <- qr.lm(object)</pre>
    coef.p <- object$coefficients[Qr$pivot[p1]]</pre>
    covmat.unscaled <- chol2inv(Qr$qr[p1, p1, drop = FALSE])</pre>
    dimnames(covmat.unscaled) <- list(names(coef.p), names(coe</pre>
```

```
covmat <- dispersion * covmat.unscaled</pre>
var.cf <- diag(covmat)</pre>
s.err <- sqrt(var.cf)</pre>
tvalue <- coef.p/s.err
dn <- c("Estimate", "Std. Error")</pre>
if (!est.disp) {
    pvalue <- 2 * pnorm(-abs(tvalue))</pre>
    coef.table <- cbind(coef.p, s.err, tvalue, pvalue)</pre>
    dimnames(coef.table) <- list(names(coef.p), c(dn,</pre>
         "z value", "Pr(>|z|)"))
else if (df.r > 0) {
    pvalue <- 2 * pt(-abs(tvalue), df.r)</pre>
    coef.table <- cbind(coef.p, s.err, tvalue, pvalue)</pre>
    dimnames(coef.table) <- list(names(coef.p), c(dn,</pre>
         "t value", "Pr(>|t|)"))
else {
    coef.table <- cbind(coef.p, NaN, NaN, NaN)</pre>
```

```
dimnames(coef.table) <- list(names(coef.p), c(dn,</pre>
             "t value", "Pr(>|t|)"))
    df.f <- NCOL(Qr$qr)</pre>
else {
    coef.table <- matrix(, OL, 4L)
    dimnames(coef.table) <- list(NULL, c("Estimate", "Std. Err</pre>
        "t value", "Pr(>|t|)"))
    covmat.unscaled <- covmat <- matrix(, OL, OL)</pre>
    df.f <- length(aliased)</pre>
keep <- match(c("call", "terms", "family", "deviance", "aic",</pre>
    "contrasts", "df.residual", "null.deviance", "df.null",
    "iter", "na.action"), names(object), OL)
ans <- c(object[keep], list(deviance.resid = residuals(object,
    type = "deviance"), coefficients = coef.table, aliased = a
    dispersion = dispersion, df = c(object$rank, df.r, df.f),
    cov.unscaled = covmat.unscaled, cov.scaled = covmat))
```

```
if (correlation && p > 0) {
    dd <- sqrt(diag(covmat.unscaled))
    ans$correlation <- covmat.unscaled/outer(dd, dd)
    ans$symbolic.cor <- symbolic.cor
}
class(ans) <- "summary.glm"
return(ans)</pre>
```

}

<bytecode: 0x000000018af6740>
<environment: namespace:stats>