Notes on Book Theory of Games and Statistical Decisions

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September 16, 2018

1 Chapter 1

To read.

2 Chapter 2

Lemma 2.1. *For any set R, the set R^* of all points r^* which are centers of gravity of a finite set of points in R with weights, i.e., which are all representable as*

$$r^* = \lambda_1 r_1 + ... + \lambda_k r_k \tag{1}$$

where $\lambda_i \geq 0, \quad \sum_1^k \lambda_i = 1, \quad r_i \in R, \quad i = 1, ..., k, \quad k = 1, 2, ...,$
is a convex set containing R.

Proof. Suppose we have two points a and b in R^*. as long as we can prove any $\alpha a + (1-\alpha)b$ also in R^*, then by definition R^* will be a convex set.

$$\begin{aligned}
a &= \lambda_{i1} r_{i1} + ... + \lambda_{ik} r_{ik} \\
b &= \lambda_{j1} r_{j1} + ... + \lambda_{jk} r_{jk} \\
\alpha a &+ (1-\alpha) b = \\
&\alpha \lambda_{i1} r_{i1} + ... + \alpha \lambda_{ik} r_{ik} + (1-\alpha)\lambda_{j1} r_{j1} + ... + (1-\alpha)\lambda_{jk} r_{jk}
\end{aligned}$$

From the equation, $\alpha a + (1-\alpha)b$ is of the same representation of r^*, so it also belongs to R^*.

□

Proposition 2.1. *lemma 2.1 can also be rephased as R^*, which is all possible convex combinations of the subset of R, is a convex set.*

Lemma 2.2. *R^* is the convex hull of R, i.e., the minimum convex set which contains R.*

Proof. Since in lemma 2.1 we've already proved that R^* is a convex set, what's left is to prove the R^* is the mininum one. By proposition 2.1, we only need to prove that for any convex set C, which contains R, will contain all possible convex conbinations of subject of R.

Let's prove by induction. 2-way convex combinations of R is in C, which holds by convex set definition, i.e, any $\alpha a + (1 - \alpha)b$ will belongs to C if a and b belongs to R. If k-way convex combinations of R is in C, then k + 1 way convex combinations of R is also in C.

$$\begin{aligned}
&\lambda_1 r_1 + ... + \lambda_k r_k + \lambda_{k+1} r_{k+1} = \\
&\quad = \sum_1^k \lambda_i \left(\frac{\lambda_1 r_1 + ... + \lambda_k r_k}{\sum_1^k \lambda_i} \right) + \lambda_{k+1} r_{k+1} \\
&\quad = \sum_1^k \lambda_i r^* + \lambda_{k+1} r_{k+1}
\end{aligned}$$

$\square$