

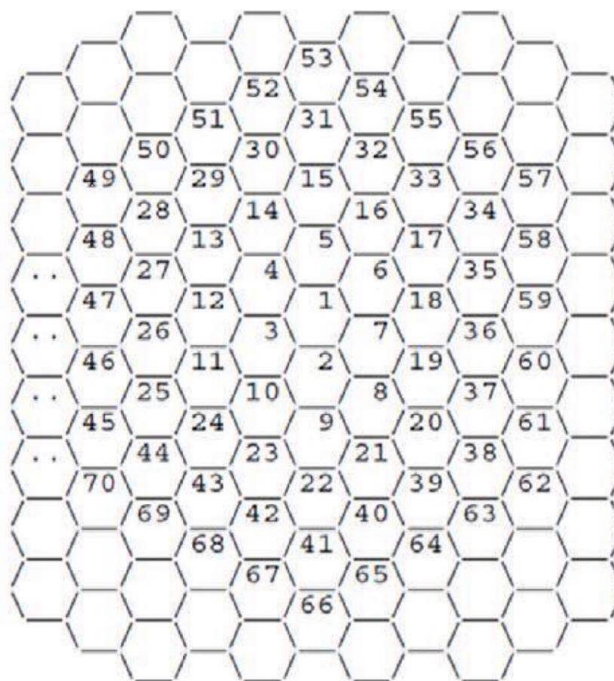
Pairwise shortest distance of honeycomb

Zhihao Wang

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1 Problem

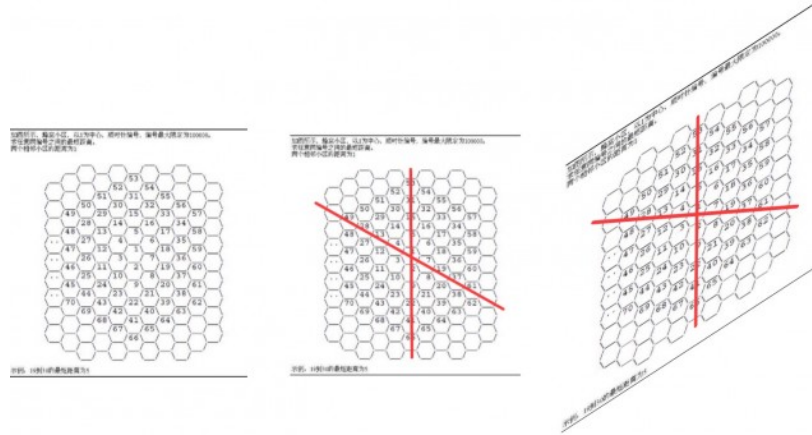
如图所示，蜂窝小区，以1为中心，顺时针编号，编号最大限定为100000。
求任意两编号之间的最短距离。
两个相邻小区的距离为1



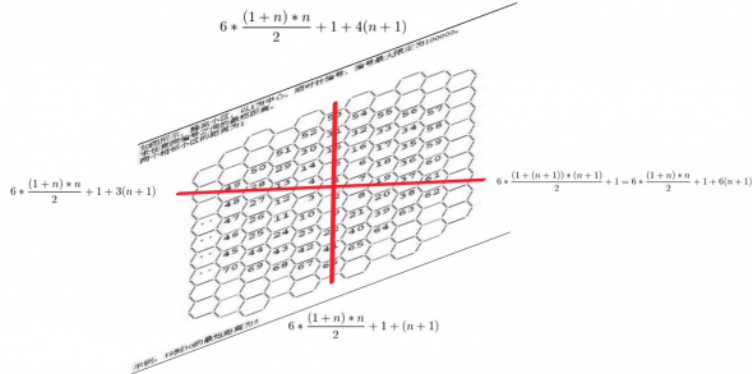
示例：19到30的最短距离为5

2 Solution

We can add two axes and skew the honeycomb to get a cartesian coordinate system.



Then we can calculate the coordinates as the following. Starting from some special coordinates, we can simulate the other coordinates case by case.



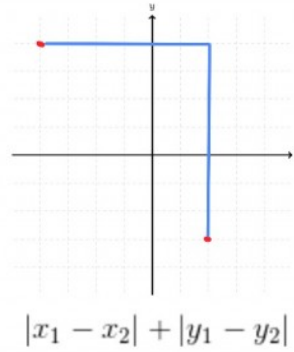
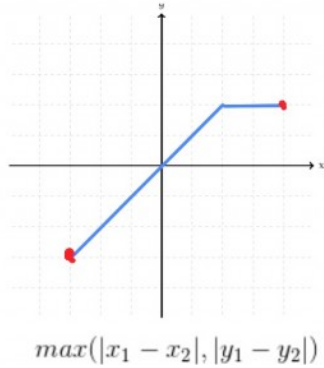
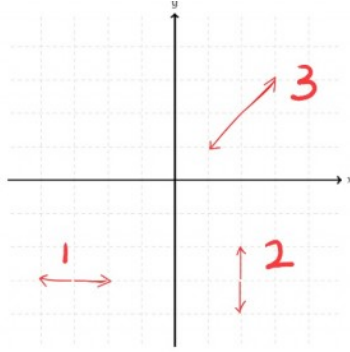
Find the minimum n and $0 \leq p \leq 5$ and $0 \leq q \leq n$

$$Number = 6 * \frac{(1+n) * n}{2} + 2 + p(n+1) + q$$

Then the coordinate will be

$$Coordinate = \begin{cases} (n - q, -1 - q), & \text{for } p = 0 \\ (-1 - q, -1 - n), & \text{for } p = 1 \\ (-1 - n, -n + q), & \text{for } p = 2 \\ (-n + q, 1 + q), & \text{for } p = 3 \\ (1 + q, 1 + n), & \text{for } p = 4 \\ (1 + n, n - q), & \text{for } p = 5 \end{cases}$$

We will have three operators in the new coordinate system, go horizontal, go vertical, and go 45 degree diagonal line, and there are two solutions.



Thanks to xinyu, he told me the answer are actually a mixture of Chebyshev distance and Manhattan distance, which is quite interesting especially when I notice Chebyshev distance is L_∞ distance and Manhattan distance is L_1 distance in L_p metrics.