Pairwise shortest distance of honeycomb

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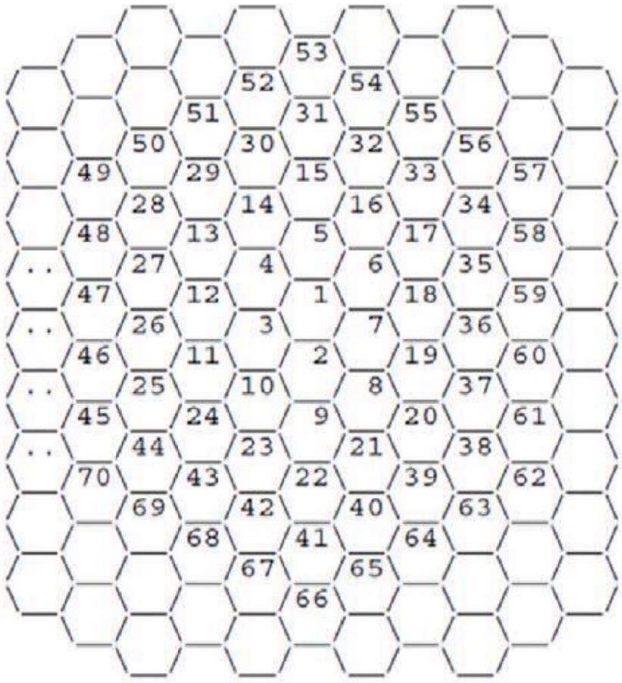
1 Problem

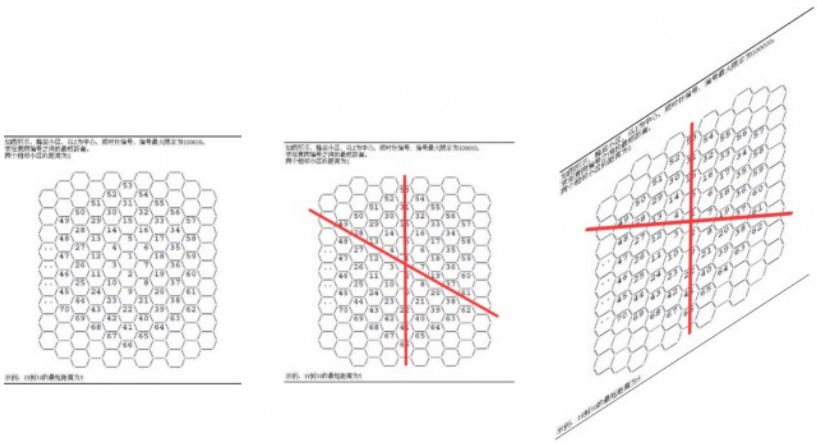
如图所示，蜂窝小区，以1为中心，顺时针编号，编号最大限定为100000。
求任意两编号之间的最短距离。
两个相邻小区的距离为1



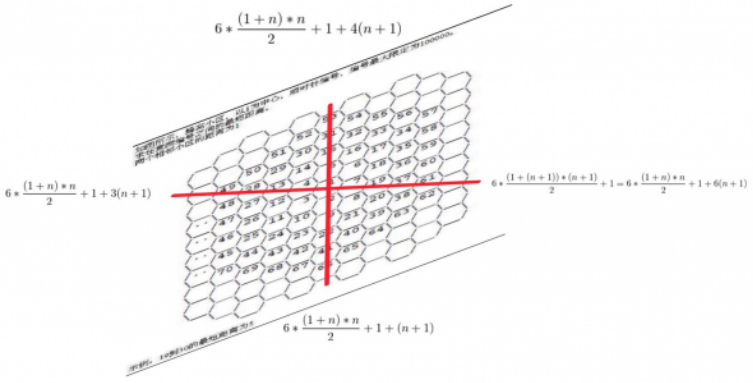
示例：19到30的最短距离为5

2 Solution

We can add two axies and skew the honeycomb to get a cartesian coordinate system.



Then we can calculate the coordinates as the following. Starting from some special coordinates, we can simulate the other coordinates case by case.



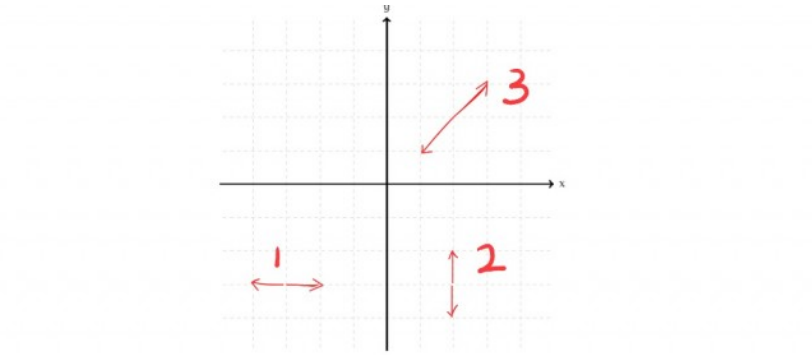
Find the mininum n and $0 \leq p \leq 5$ and $0 \leq q \leq n$

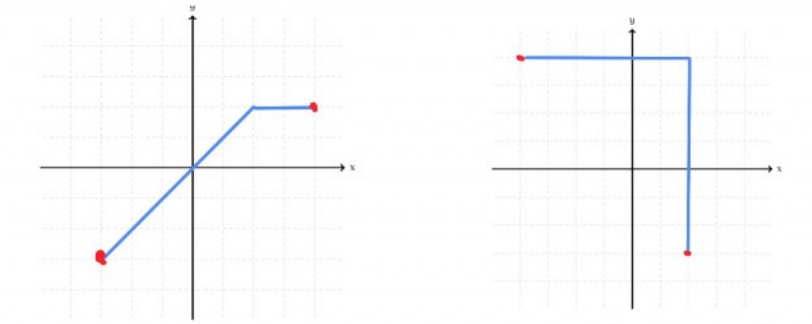
$$Number = 6 * \frac{(1+n)*n}{2} + 2 + p(n+1) + q$$

Then the coordinate will be

$$Coordinate = \begin{cases} (n-q, -1-q), & \text{for } p = 0 \\ (-1-q, -1-n), & \text{for } p = 1 \\ (-1-n, -n+q), & \text{for } p = 2 \\ (-n+q, 1+q), & \text{for } p = 3 \\ (1+q, 1+n), & \text{for } p = 4 \\ (1+n, n-q), & \text{for } p = 5 \end{cases}$$

We will have three operators in the new coordinate system, go horizontal, go veritical, and go 45 degree diagonal line, and there are two solutions.





$max(|x_1 - x_2|, |y_1 - y_2|)$ $\quad$ $|x_1 - x_2| + |y_1 - y_2|$

Thanks to xinyu, he told me the answer are actually a mixture of Chebyshev distance and Manhattan distance, which is quite interesting especially when I found out Chebyshev distance is L_∞ distance and Manhattan distance is L_1 distance.