Graphical Abstract

Proof of Goldbach Conjecture Using Factorization Zone

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Highlights

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- Neighboring Zone Factorization Analysis
- Filling Odd-multiples into Odd Numbers

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Abstract

This article presents an elementary approach of proving the Goldbach Conjecture, one of the oldest unsolved problems in number theory. We attempt to understand the structure of the problem using factorization patterns in a given integer zone. This work may serve to inspire further exploration or critical examination of the conjecture from a new perspective.

Keywords:
Goldbach Conjecture, Co-prime, Zone Factorization, Odd-multiple

1. Introduction

In this article, we provide insights into the well-known Goldbach Conjecture. Most computer science and mathematics graduates will have sufficient background to follow the content. We focus on core reasoning patterns so we do not attempt to prove facts considered obvious in elementary number theory. In our proof, we try to put odd multiples into odd numbers. If it fails, a new prime is needed as exlained below.

2. Definitions

Remark 1. N-multiple
An integer composed of n repeatable prime integer factors, where $n \in \mathbb{N}\setminus\{1\}$.

Examples: 2×3 is a 2-multiple; $2 \times 2 \times 3$ is a 3-multiple; $3 \times 5 \times 7$ is also a 3-multiple.

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Remark 2. Odd Multiple
A product of odd primes. We exclude 1 and 2.

3. Axioms

Remark 3. Prime Number as Gaps
Prime numbers are the "gaps" in the distribution of odd numbers where odd multiples fail to fill in. Equivalently, primes can be viewed as natural numbers that are neither even nor odd multiples.

Remark 4. Neighbor Numbers are Co-prime
For any integer n, the numbers $n-1$ and $n+1$ are co-prime with n. This follows from the fundamental theorem of arithmetic.

Remark 5. Neighboring Zone Factorization Analysis
Consider a zone defined by the decreasing sequence $n, n-1, \ldots, n-m$. For an offset $\delta = m$, we examine how factorization properties evolve. In this zone, a number only shares factors that are factors of current δ and previous δs' with larger numbers. When δ is prime and n is subtracting this prime, we need fill in an odd-multiple, otherwise we hit a prime and satisfy the conjecture. That creates two situations.
First, n and δ are not co-prime or δ is a factor of n. The upper bound of this happening is $\mathcal{O}(logn)$. Compared to the total number of primes smaller than n, we can ignore this situation and continue.
Second, n and δ are co-prime. To create the odd-multiple, we can not use the factors from previous δs. Thus $n-\delta$ has to use a new prime, not in the factors set of δs, not in the factorization of any larger numbers in this zone.

4. Proof of Goldbach Conjecture

The intuition is acquired through finding the Goldbach prime pair eg $85292 = 79 + 85213$. We skip trivial details here. Upon request, we can provide code and samples that are on Github.

Proof. Goldbach Conjecture
In zone $[n - m, n]$, as δ is growing, the factorization in this zone will get all the primes smaller or equal to m sequentially. Then gradually m passes $\sqrt{n}$. We know between $n - \sqrt{n}$ and $\sqrt{n}$, there is a vast space of numbers including primes. When n is large enough, it is trivial to prove there exist primes that do not divide n in $[\sqrt{n}, n - \sqrt{n}]$. If situation one discussed in Remark No.5 happens, we ignore it and continue. But it is certain situation two will follow, a new prime never used is forced in. The primes less equal to $\sqrt{n}$ have run out. We can only fill in one but ONLY ONE prime that is bigger than $\sqrt{n}$. n minus a prime is hitting a prime. □

5. Final Words

The proof is presented at the strongest reasoning I can do. It is my sincere hope that the approach offers a meaningful clue or inspires scrutiny. I welcome constructive criticism, especially from those more experienced in number theory. I salute all those dedicated to advancing mathematical understanding.

Appendix A. Example Appendix Section

Appendix text.
Example citation, See Lamport (1994).

References

Leslie Lamport, *LaTeX: a document preparation system*, Addison Wesley, Massachusetts, 2nd edition, 1994.