Understanding and Attempting Proof of Goldbach Conjecture

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1 Introduction

In this article, we attempt to prove or provide a better understanding of this well known problem. Most computer science and mathematics graduates will have sufficient background to understand the content. We also ignore very edge cases and focus on the way of thinking. We also do not attempt to prove obvious facts.

2 Definitions

N-multiple

An integer has n repeatable prime integer factors. $n \in \mathbb{N} \setminus 1$. Examples, 2×3 is 2-multiple. $2 \times 2 \times 3$ is 3-multiple and $3 \times 5 \times 7$ is 3-multiple.

Odd Multiple

We only use nature numbers as the factors, in this case, 1 and 2 are removed.

3 Axioms

For convenience, we don't explicitly clarify n is even and it is large enough in all occurrences.

Axiom (Prime number):
Prime numbers are the holes in odd numbers such that odd multiples fail to fill in. It can also be understood as nature numbers subtract even numbers and odd multiples.

Axiom (Neighor Numbers are Co-prime):
Or 邻数因子互斥 in Chinese. Given any number n, n and $n+1$ are co-prime. If we put n in unique factorial format eg $p_1^{e_1} \cdot p_2^{e_2} \cdots p_l^{e_l}$, it will be more obvious $p_1^{e_1} \cdot p_2^{e_2} \cdots p_l^{e_l} + 1$ does not share any factor with n. Further $n-1$ does not share any factor with n.

Axiom (Neighboring Zone Factorization Analysis):
For number sequence n, $n-1$,$\cdots$, $n-i$, $\cdots$, $n-m$, $m < n$. We call i the δ. In this zone, a number only shares factors that are factors of current δ and previous δs' with larger numbers.

When δ is prime and n is subtracting this prime, we need fill in an odd-multiple, other wise we hit a prime and satisfy the conjecture. That creates two situations.

First, n and δ is not co-prime or δ is a factor of n. The upper bound of this happening is $\mathcal{O}(n)$. Compared to the total number of primes smaller than n, we can ignore this situation and continue.

Second, n and δ is co-prime. To create the odd-multiple, we can not use the factors from previous δs. Thus $n-\delta$ has to use a new prime, not in the factors set of δs, not in the factorization of any larger numbers in this zone.

4 Application in Goldbach Conjecture

Let's try with 85292 to show how to calculate the Goldbach prime pair 79 and 85213, partly manual with computer program aid. I also use Chatgpt aiding Latex writing. Pay attention to steps at $m = 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 47, 53, 67, 71, 73, 79$.

Proof of Goldbach Conjecture. In zone $[n-m, n]$, as δ is growing, the factorization in this zone will get all the primes smaller or equal to m sequentially. Then gradually m passes $\sqrt{n}$. We know between $n - \sqrt{n}$ and $\sqrt{n}$, there is a big gap of numbers. If situation one discussed in last section happens, we ignore it and continue. But it is certain situation two will follow, a new prime never used is forced in. The primes less equal to $\sqrt{n}$ has run out. We can only fill in one but ONLY ONE prime that is bigger than $\sqrt{n}$. n minus a prime is hitting a prime. $\square$

Table 1: n=85292

$n-m$	**Factorization of** $n-m$	$-m$
85212	2×2×3×3×3×3×263	-80
85213	85213	-79
85214	2×137×311	-78
85215	3×5×13×19×23	-77
85216	2×2×2×2×2×2663	-76
85217	11×61×127	-75
85218	2×3×7×2029	-74
85219	31×2749	-73
85220	2×2×5×4261	-72
85221	3×3×17×557	-71
85222	2×42611	-70
85223	85223	-69
85224	2×2×2×3×53×67	-68
85225	5×5×7×487	-67
85226	2×43×991	-66
85227	3×28409	-65
85228	2×2×11×13×149	-64
85229	85229	-63
85230	2×3×3×5×947	-62
85231	29×2939	-61
85232	2×2×2×2×7×761	-60
85233	3×28411	-59
85234	2×19×2243	-58
85235	5×17047	-57
85236	2×2×3×7103	-56
85237	85237	-55
85238	2×17×23×109	-54
85239	3×3×3×7×11×41	-53
85240	2×2×2×5×2131	-52
85241	13×79×83	-51
85242	2×3×14207	-50

$n-m$	**Factorization of** $n-m$	$-m$
85243	85243	-49
85244	2×2×101×211	-48
85245	3×5×5683	-47
85246	2×7×6089	-46
85247	85247	-45
85248	2×2×2×2×2×2×2×2×3×3×37	-44
85249	163×523	-43
85250	2×5×5×5×11×31	-42
85251	3×157×181	-41
85252	2×2×21313	-40
85253	7×19×641	-39
85254	2×3×13×1093	-38
85255	5×17×17×59	-37
85256	2×2×2×10657	-36
85257	3×3×9473	-35
85258	2×47×907	-34
85259	85259	-33
85260	2×2×3×5×7×7×29	-32
85261	11×23×337	-31
85262	2×89×479	-30
85263	3×97×293	-29
85264	2×2×2×2×73×73	-28
85265	5×17053	-27
85266	2×3×3×3×1579	-26
85267	7×13×937	-25
85268	2×2×21317	-24
85269	3×43×661	-23
85270	2×5×8527	-22
85271	71×1201	-21
85272	2×2×2×3×11×17×19	-20
85273	269×317	-19
85274	2×7×6091	-18
85275	3×3×5×5×379	-17
85276	2×2×21319	-16
85277	53×1609	-15
85278	2×3×61×233	-14
85279	107×797	-13
85280	2×2×2×2×2×5×13×41	-12
85281	3×7×31×131	-11
85282	2×42641	-10
85283	11×7753	-9
85284	2×2×3×3×23×103	-8
85285	5×37×461	-7
85286	2×42643	-6

$n-m$	**Factorization of** $n-m$	$-m$
85287	3×28429	-5
85288	2×2×2×7×1523	-4
85289	17×29×173	-3
85290	2×3×5×2843	-2
85291	19×67×67	-1
85292	2×2×21323	0

5 Final Words

It is still a little bit surreal for me to get here. I have sufficient background in computer science and mathematics in understanding what I am doing. But Number Theory is not my specialty and particularly I aimed too high for this conjecture. I hope the readers please do not be afraid to criticize any mistakes. I hope if the conjecture is not rigidly proved here, at least my article may provide any meaningful clues for further research. Though this is not my first academic writing, still these 'final words' may serve an 'excuse me' if there is any grieve mistake. Solute all the people dedicate to research in computer science and mathematics!