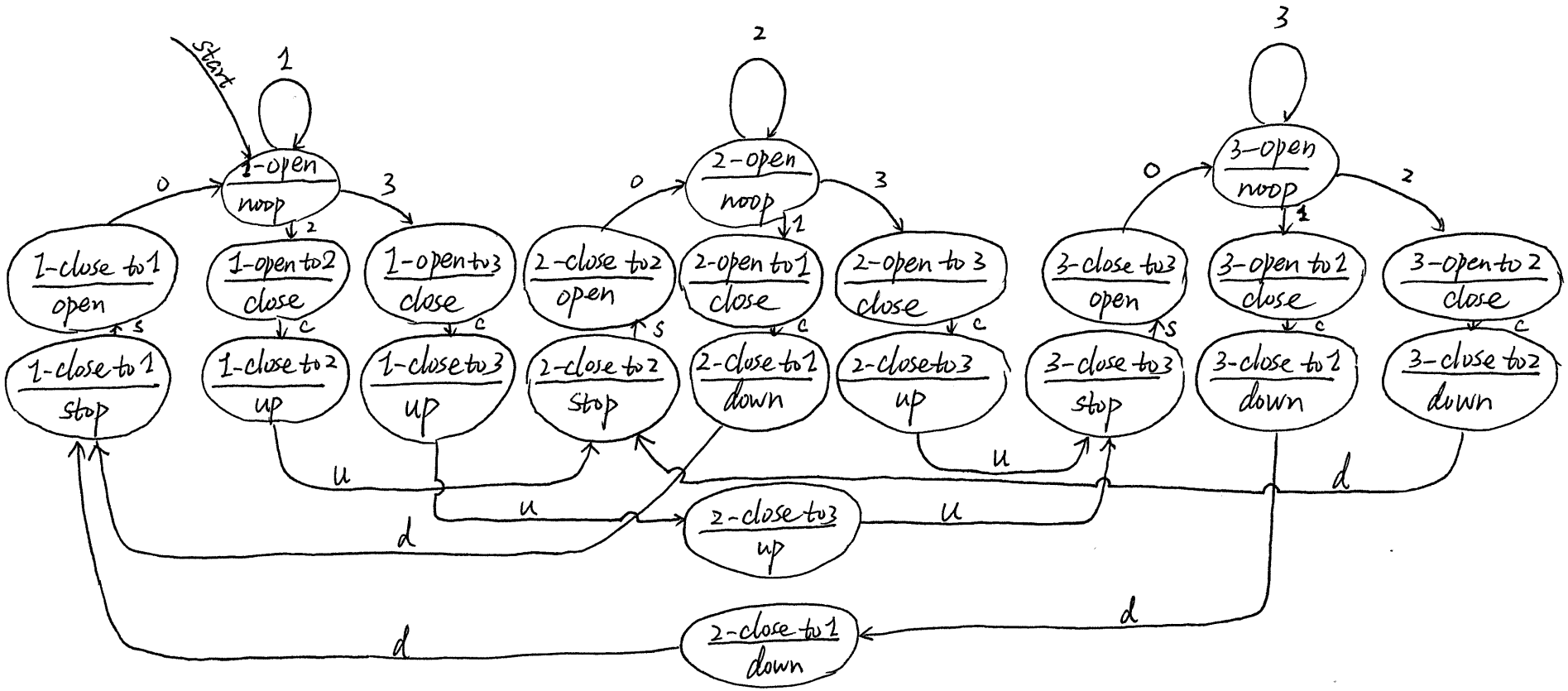
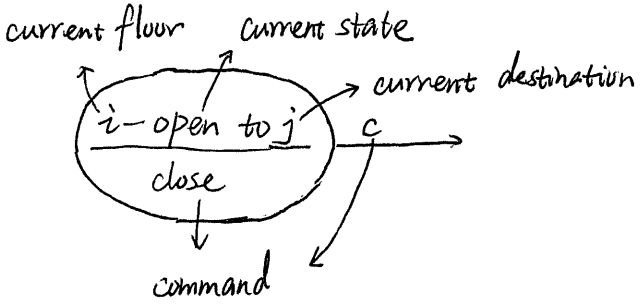


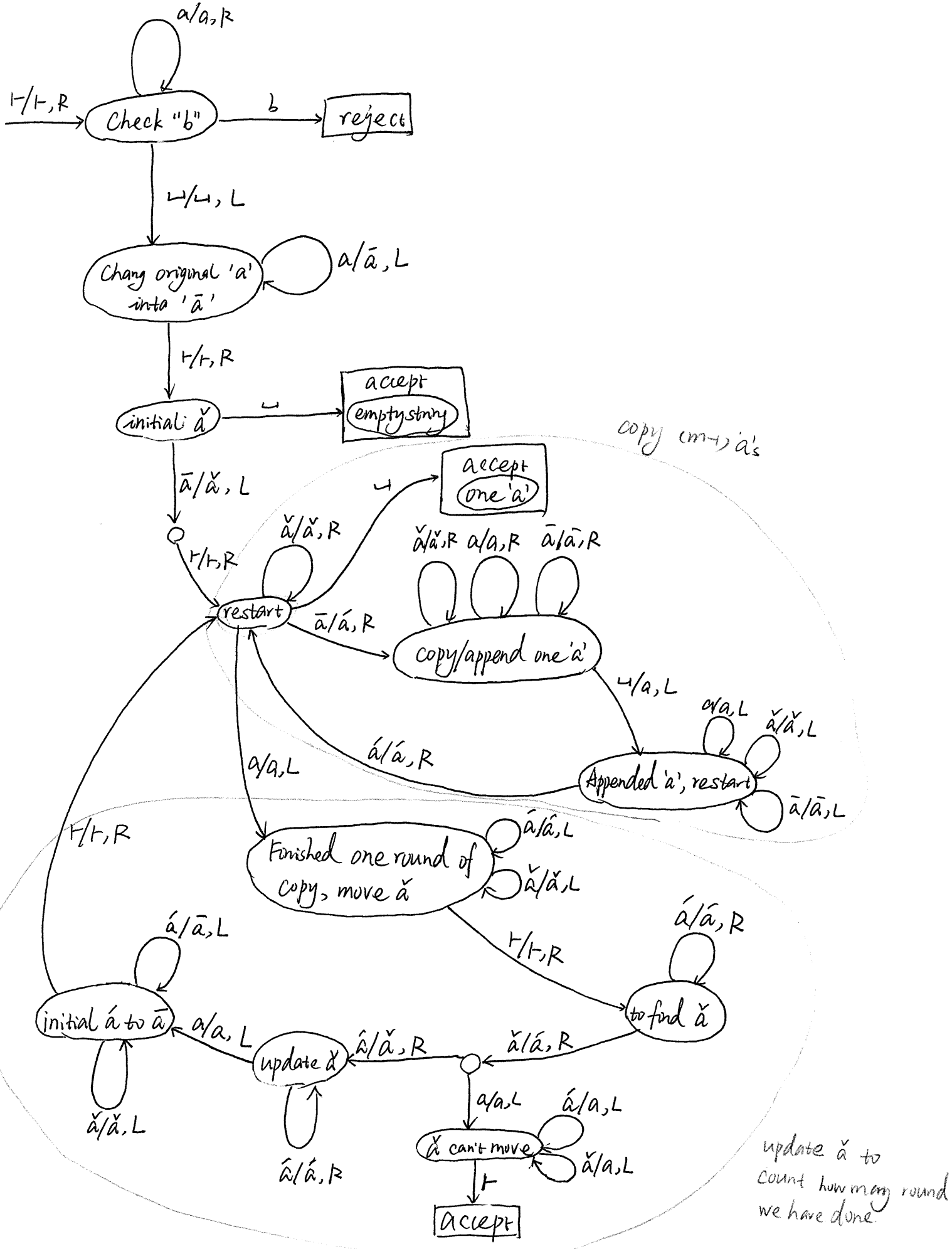


$\Sigma = \{1, 2, 3, 0, c, s, u, d\}$

$\Gamma = \{noop, open, close, up, down\}$

start state: 1-open / noop

Q states, δ transition function and $w: Q \to \Gamma$ output function are shown in the graph.


a/a, R
⊢/⊢, R
Check "b"
b
reject
␣/␣, L
Chang original 'a' into 'ā'
a/ā, L
⊢/⊢, R
initial ǎ
␣
accept
emptystring
copy (m-1) á's
ā/ǎ, L
␣
accept
one 'á'
⊢/⊢, R
ǎ/ǎ, R
restart
ā/á, R
ǎ/ǎ, R
a/a, R
ā/ā, R
copy/append one 'á'
␣/a, L
a/a, L
ǎ/ǎ, L
á/á, R
Appended 'á', restart
ā/ā, L
a/a, L
⊢/⊢, R
Finished one round of copy, move ǎ
á/á, L
ǎ/ǎ, L
á/ā, L
⊢/⊢, R
á/á, R
initial á to ā
to find ǎ
a/a, L
update ǎ
â/ǎ, R
ǎ/á, R
ǎ/ǎ, L
â/á, R
a/a, L
á/a, L
ǎ can't move
ǎ/a, L
⊢
accept
update ǎ to count how many round we have done.

Turing Machine:

First, recognize if there exists any 'b' in the string. If there is, reject.
If no 'b' exists, return the head back to the begining.
Two special cases: $\vdash\sqcup$ and $\vdash a$ accept and return directly.

Second, initial all 'a' into $\bar{a}$, and then, mark the first one as $\check{a}$

e.g. $\vdash aaaa \longrightarrow \vdash \bar{a}\bar{a}\bar{a}\bar{a} \longrightarrow \vdash \check{a}\bar{a}\bar{a}\bar{a}$

Third, do the copy for all $\bar{a}$:

e.g $\vdash \check{a}\bar{a}\bar{a}\bar{a} \rightarrow \vdash \check{a}\acute{a}\bar{a}\bar{a}\bar{a} \rightarrow \vdash \check{a}\acute{a}\acute{a}\bar{a}aa \rightarrow \vdash \check{a}\acute{a}\acute{a}\acute{a}aaa$

When there is no '$\bar{a}$' exists. Do next.

Forth, move $\check{a}$ to right by one character and change all $\acute{a}$ back to $\bar{a}$

e.g $\vdash \check{a}\acute{a}\acute{a}\acute{a}aaa \longrightarrow \vdash \acute{a}\check{a}\acute{a}\acute{a}aaa \longrightarrow \vdash \bar{a}\check{a}\bar{a}\bar{a}aaa$

Do this as the same as "Third" for copying all $\bar{a}$.

Move the $\check{a}$ until $\check{a}$ reaches the end of original string.

e.g: when $\vdash \acute{a}\acute{a}\acute{a}\check{a}\underbrace{aa\cdots a}_{3\times 4\text{ as}}$ is the condition to end.

$\check{a}$ has four loops, each loop copies 3 as. $4\times 3=12$. Total will be $3\times 4+4=16=4^2$

Finally, change all $\acute{a}$ and $\check{a}$ back to a. Return the result.