Adaptive Fixed-Time Control for Full State-Constrained Nonlinear Systems: Switched-Self-Triggered Case

Ziming Wang, Xin Wang and Ning Pang

***Abstract*—In this brief, an one-to-one asymmetric NM based auxiliary system is put forward to deal with the state transformation problem for nonstrict full state-constrained systems under the adaptive fixed-time control. Then one proposed a novel switched-self-triggered mechanism (SSTM) in the controller update, which is helpful to obtain a better system performance. In addition, one can show that under the condition of state-constraints, semiglobal practical fixed-time stability (SPFTS) standard can also preserved both the tracking performance and the stability of the closed-loop system in a specified time. Finally, the effectiveness of the controller design is thoroughly shown via an illustrative simulation example.**

***Index Terms*—Fixed-time control, full-state constraints, RBF NN, switched-self-triggered mechanism (SSTM).**

I. Introduction

THE theory of barrier Lyapunov function (BLF) has been a fundamental yet popular research topic in the control community over the last few decades. Combined with adaptive neural control and backstepping technology, many BLF-based strategies for full state-constrained systems with multifarious methods were raised, for instance, strict-feedback systems [1], pure-feedback systems [2] and stochastic systems [3]. Nevertheless, the state constraints must be converted into deviation constraints in the above results.

Considering the security and equipment constraints, researchers often set upper and lower bounds on the system states/output in practical system requirements. For example, the coordinated operation between the motor gears, if the bite force astriction between the gears is violated, it may cause serious deterioration of system capability or even system destruction. Therefore, state constrained nonlinear system has focused extensive efforts, see [4] [5] [6]. In [4], the robust stabilization in lower triangular form was considered upon state-constrained systems. Bounded state-feedback controllers were constructed to assure asymptotic stability. Study [6] proposed a novel backstepping stable controller by using the Integral Barrier Lyapunov Functionals (IBLF). and the output tracking is well achieved without violation of any constraint, offering researchers a different view of the full state constraint type. In [5], log-type quadratic Barrier Lyapunov Function (BLF) is proposed. These studies motivated the development of state-constrained nonlinear system research with different conditions.

It is worth noting that current results in controller design are never considered in terms of the control time. Indeed, most practical control systems are expected to achieve desired performances in a predetermined time in order to provide better robustness and shorter transient response time. In [7], researchers considered the adaptive fixed-time control for a class of nonlinear systems. In [8], contrast that the novel approach with the subsistent fixed-time control, the convergence time of the proposed fixed-time control approach does not depend on the original states. But what if the above works subject to full state constraints? It will be discussed in this paper.

It is worth that in wide engineering practice, high-frequency signal transmission and update commands may inevitably exacerbate the communication burden. So sampling the controller updating commands in real-time is pormising [9]. Fixed-threshold-based event-triggered scheme [10] and relative-threshold-based event-triggered scheme [11] are proposed to improve control efficiency as well as reduce reconfiguration and maintenance components. In [12], to avert the matter of unremitting monitoring threshold, the researchers developed a self-event-triggered control scheme to obtain the next trigger timing only through the present information. But the triggering interval produced by the event sampling mechanism may decrease the control accuracy. Thence, how to establish a balance between tracking performance and the availability of resources needs to be further researched.

The primary advantages of this brief can be outlined in the following three aspects. 1) Based on one-to-one NM, the system performs a log form transformation to meet the constrained conditions and converts into a fire-new pure-feedback system. 2) Compared with [13] [14] [19] [20], the initial states have no influence on the convergence time under the condition of state constraint. The application of new event-triggered control can effectively reduce resource utilization and improve control efficiency. 3) A new event-sampled scheme is proposed. Compared with [9] [10] [12], the novel switched-self-triggered mechanism (SSTM) is a more maneuverable one, which can meet the balance between the system capability and component safety better.

The structure of this brief is organized as follows. Section II describes the system model. The constrained system, auxiliary

This work was supported by Natural Science Foundation of China under Grant 62276214. (*Corresponding author: Xin Wang.*)

Z. M. Wang and N. Pang are with the Chongqing Key Laboratory of Nonlinear Circuits and intelligent Information Processing, College of Westa, Southwest University, 400715, Chongqing, China.

X. Wang is with the Chongqing Key Laboratory of Nonlinear Circuits and Intelligent Information Processing, College of Electronic and Information Engineering, Southwest University, Chongqing 400715, China.(e-mail: xinwangswu@163.com).

This article has been accepted for publication in IEEE Transactions on Circuits and Systems--II: Express Briefs. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TCSII.2023.3309400

system, and adaptive tracking controller are established in Section III, while Section IV proposed the simulation examples. Finally, the conclusion is presented in Section V.

II. PRELIMINARIES

A. System Model

Consider the following nonlinear time-varying system

$$\begin{cases} \dot{x}_i = g_i(\overline{x}_{i+1}) \\ \dot{x}_n = u(t) + g_n(\overline{x}_n) \\ y = x_1 \end{cases} \tag{1}$$

where $1 \leq i \leq n-1$, $\overline{x}_n = [x_1, x_2, ..., x_n]^T \in R^n$ denote the system states, $y(t) \in R$ expresses the system output, g_i is the unknown uncertain smooth function. All the states x_i observe: $-\varpi_{r1} < x_i < \varpi_{r2}$, where ϖ_{r1} and ϖ_{r2} are positive regulated constants with $r = 1, 2, ..., n$.

B. RBFNNs

It should be noted that radial basis function neural networks (RBF NNs) are a frequently-used method to solve the problem of a system containing unknown nonlinear functions. According to [15], for any unknown continuous equation $S(Z)$, it has $S(Z) = W^T\psi(Z) + \pi(Z)$, where $\psi(Z) = [\psi_1(Z), \psi_2(Z), ..., \psi_n(Z)]^T$ denotes the activation function, $\psi_i(Z) = exp(-||Z - U_i||/V)$, V stands for the Gaussian functions' width, U_i are the centers of the receptive field. W and $\pi(Z)$ are the weight vector and the approximation error, respectively.

At this point, the following lemmas are first provided to facilitate the derivation.

Lemma 1 [17] : Let j_1, j_2 and j_3 be positive parameters, for any real variable x and y, the following inequality holds

$$|x|^{j_1}|y|^{j_2} \leq \frac{j_1 j_3}{j_1 + j_2}|x|^{j_1+j_2} + \frac{j_1 j_3^{-\frac{j_1}{j_2}}}{j_1 + j_2}|y|^{j_1+j_2} \tag{2}$$

Lemma 2 [18] : Consider a generic dynamical system as $\dot{x}(t) = f(x(t))$, $x(0) = x_0$, where $x \in R^n$ and $f(\cdot) : R_f \times R^n \to R^n$, and the origin of the system is considered to be semiglobally practically fixed-time stability (SPFTS). Assuming the following positive parameters: $\alpha, \beta > 0$, $0 < h, q < 1$, $o > 1$ and $0 < \gamma < +\infty$. It obtained a positive definite function $\dot{V}(x) = -\alpha V^o(x) - \beta V^q(x) + \gamma$. The fixed time for system, which is stable at the origin, can be expressed as

$$T \leq T_{max} := \frac{1}{\alpha h(o-1)} + \frac{1}{\beta h(1-q)} \tag{3}$$

The residual set of the solution in the system is approximated by

$$m \in \{V(m) \leq min\{(\frac{\gamma}{(1-h)\alpha})^{\frac{1}{o}}, (\frac{\gamma}{(1-h)\beta})^{\frac{1}{q}}\}\} \tag{4}$$

Lemma 3 [16] : For any constants $\xi_1 \in R$ and $\xi_2 > 0$, the following holds, $0 \leq |\xi_1| - \xi tanh(\frac{\xi_1}{\xi_2}) \leq 0.2785\xi_2$.

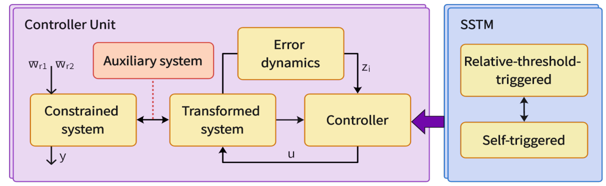

Fig. 1. Block scheme of the adaptive control framework.

III. THE CONTROLLER DESIGN

The considered full-state constraints are designed as:

$$\begin{cases} m_i = log\frac{\varpi_{r1}+x_i}{\varpi_{r2}-x_i} \\ \dot{m}_i = \frac{e^{m_i}+e^{-m_i}+2}{\varpi_{r1}+\varpi_{r2}}\dot{x}_i \end{cases} \tag{5}$$

with $i = 1, 2, ..., n$.

To processed, define

$$\Xi_i(m_i) = \frac{e^{m_i} + e^{-m_i} + 2}{\varpi_{r1} + \varpi_{r2}} \quad i = 1, 2, ..., n. \tag{6}$$

Based on (5) and (6), one has $\dot{m}_i = \Xi_i(m_i)\dot{x}_i$.

Since the subsequent process can be carried out according to the traditional method, the following auxiliary system is introduced

$$\begin{cases} G_i(\overline{m}_{i+1}) = \Xi_i(m_i)g_i(\overline{x}_{i+1}) - m_{i+1} \\ G_n(\overline{m}_n) = \Xi_n(m_n)g_i(\overline{x}_n) \end{cases} \tag{7}$$

Based on the above auxiliary system, the constrained system (1) can be rewritten as

$$\begin{cases} \dot{m}_i = m_{i+1} + G_i(\overline{m}_{i+1}) \\ \dot{m}_n = \Xi_n(m_n)u(t) + G_n(\overline{m}_n) \end{cases} \tag{8}$$

One chooses the following coordinate transformations:

$$\begin{cases} z_1 = m_1 - m_r \\ z_i = m_i - a_{i-1}, i = 2, 3, ..., n \end{cases} \tag{9}$$

where a_i is presented as the designed intermediary control law, and $m_r = log(\varpi_{r1} + x_r)/(\varpi_{r2} - x_r)$, x_r is the desired reference signal.

The above is the system setup process, the block scheme of the adaptive control framework with SSTM is displayed in Fig.1. The following is the specific theoretical derivation.

Assumption 1 [10]: $\Xi_i(m_i)$ satisfy the inequality $|\Xi_i(m_i)| \leq N$, where N is a constant to be determined. The real number N is intended to be deflated in the Young's inequality such that the final convergence condition holds.

Step 1 : From (12) and (13), one has

$$\dot{z}_1 = m_2 + G_1(\overline{m}_2) - \dot{m}_r = z_2 + a_1 + G_1(\overline{m}_2) - \dot{m}_r \tag{10}$$

Select the $1st$ Lyapunov function in the form of

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2\varrho_1}\widetilde{\lambda}_1^2 \tag{11}$$

where $\varrho_1 = 2p_1/(2p_1 - 1)$, p_1 satisfied the condition of $p_1 > \frac{1}{2}$, define $\widetilde{\lambda}_1 = \lambda_1 - \hat{\lambda}_1$, where $\hat{\lambda}_1$ denotes the estimation error, and it will be defined later.

Calculating the differentiation of (11), one has

$$V_1 = z_1(-k_{2,1}z_1^{2q-1} + z_2 + a_1 + S_1(Z_1)) - \frac{1}{\varrho_1}\widetilde{\lambda}_1\dot{\hat{\lambda}}_1 - \frac{1}{2}z_1^2 \tag{12}$$

$S_1(Z_1)$ can be described as

$$S_1(Z_1) = G_1(\overline{m}_2) + \frac{1}{2}z_1 + k_{2,1}z_1^{2q-1} \tag{13}$$

where $k_{2,1} > 0$ is a designed parameter. By applying the RBF NNs into $S_1(Z_1)$ as: $S_1(Z_1) = W_1^T\psi_1(Z_1) + \pi_1(Z_1)$, $||\pi_1(Z_1)|| \le \mu_1$, where $Z_i = [m_i, m_r, \dot{m}_r]$, μ_1 is a designed positive parameter.

On account of Young's inequality, one can obtain the following inequalities

$$z_1S_1(Z_1) \le \frac{1}{2e_1^2}z_1^2\lambda_1\psi_1^T(Z_1)\psi_1(Z_1) + \frac{e_1^2}{2} + \frac{z_1^2}{2} + \frac{\mu_1^2}{2} \tag{14}$$

where e_1 is a designed positive constant, $\lambda_i = ||W_i||^2, (i = 1, ..., n)$ are unknown constants to be estimated.

Choose the virtual controller a_1 and update law of parameter $\dot{\hat{\lambda}}_1$ as follows

$$a_1 = -k_{1,1}z_1^{2o-1} - \frac{1}{2e_1^2}z_1\hat{\lambda}_1\psi_1^T(Z_1)\psi_1(Z_1) + \dot{m}_r \tag{15}$$

$$\dot{\hat{\lambda}}_1 = \frac{\varrho_1}{2e_1^2}z_1^2\psi_1^T(Z_1)\psi_1(Z_1) - \delta_1\hat{\lambda}_1 \tag{16}$$

where $k_{1,1} > 0$, $\delta_1 > 0$ are designed constants.

Substituting (14)-(16) into (12), one get

$$\dot{V}_1 \le -k_{2,1}z_1^{2q} - k_{1,1}z_1^{2o} + z_1z_2 + \frac{e_1^2}{2} + \frac{\mu_1^2}{2} + \frac{\delta_1}{\varrho_1}\widetilde{\lambda}_1\hat{\lambda}_1 \tag{17}$$

Step i $(1 < i < n)$:

Construct the ist Lyapunov equation as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2\varrho_i}\widetilde{\lambda}_i^2 \tag{18}$$

where $\varrho_i = 2p_i/(2p_i - 1)$, $p_i > \frac{1}{2}$.

Similar to (13), $S_i(Z_i)$ is chosen by

$$S_1(Z_1) = -z_{i-1} + G_i(\overline{m}_{i+1}) - \dot{a}_{i-1} + \frac{1}{2}z_i + k_{2,i}z_i^{2q-1} \tag{19}$$

where $\dot{a}_{i-1} = \sum_{b=1}^{i-1}\frac{\partial a_{i-1}}{\partial m_b}(m_{b+1} + G_r(\overline{m}_{b+1})) + \sum_{b=1}^{i-1}\frac{\partial a_{i-1}}{\partial \hat{\lambda}_b}\dot{\hat{\lambda}}_b + \sum_{b=0}^{i-1}\frac{\partial a_{i-1}}{\partial m_r^b}m_r^{b+1}$.

The intermediate control equation and designed adaptation law are expressed as

$$a_i = -k_{1,i}z_i^{2o-1} - \frac{1}{2e_i^2}z_i\hat{\lambda}_i\psi_i^T(Z_i)\psi_i(Z_i) \tag{20}$$

$$\dot{\hat{\lambda}}_i = \frac{\varrho_i}{2e_i^2}z_i^2\psi_i^T(Z_i)\psi_i(Z_i) - \delta_i\hat{\lambda}_i \tag{21}$$

where $k_{1,i} > 0, \delta_i > 0, e_i > 0$ are design parameters.

Invoking the above inequality into the differentiation of V_i, one gets

$$\dot{V}_i \le -\sum_{b=1}^{i}k_{2,b}z_b^{2q} - \sum_{b=1}^{i}k_{1,b}z_b^{2o} + z_iz_{i+1} + \sum_{b=1}^{i}(\frac{e_b^2}{2} + \frac{\mu_b^2}{2}) + \sum_{b=1}^{i}\frac{\delta_b}{\varrho_b}\widetilde{\lambda}_b\hat{\lambda}_b \tag{22}$$

Step n:

Construct the final Lyapunov equation as

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2\varrho_n}\widetilde{\lambda}_n^2 \tag{23}$$

where $\varrho_n = 2p_n/(2p_n - 1)$, $p_n > \frac{1}{2}$.

The actual control function and adaptation law are designed as

$$a_n(t) = \frac{1}{\Xi_n(m_n)}[-k_{1,n}z_n^{2o-1} - \frac{1}{2e_n^2}z_n\hat{\lambda}_n\psi_n^T(Z_n)\psi_n(Z_n)] \tag{24}$$

$$\dot{\hat{\lambda}}_n = \frac{\varrho_n}{2e_n^2}z_n^2\psi_n^T(Z_n)\psi_n(Z_n) - \delta_n\hat{\lambda}_n \tag{25}$$

where $k_{1,n} > 0, \delta_n > 0, e_n > 0$ are positive regulated parameters.

The SSTM is constructed as

$$c(t) = d(t) - u(t) \tag{26}$$

$$u(t) = d(t_s) \quad \forall t \in [t_s, t_{s+1}) \tag{27}$$

$$t_{s+1} = \begin{cases} inf\{t \in R||c(t)| \ge \Delta|u(t)| + \Theta\}, \text{if } |u(t)| \ge Q \\ t_s + \dfrac{\varDelta|u(t)| + \varTheta}{max\{|\dot{d}(t)|, \varUpsilon\}} \qquad , \text{if } |u(t)| < Q \end{cases} \tag{28}$$

where $t_s, t_{s+1} \in Z^+$, $0 < \Delta < 1$, $0 < \varDelta < 1$, Θ, $\varTheta$ and $\varUpsilon$ are all designed positive constants. $c(t)$ denotes the measurement error. Q stands for the switching boundary.

Case $1: |u(t)| \ge Q$, the control protocol is devised by

$$d(t) = -(1 + \Delta)(a_n tanh(\frac{z_na_n}{\Gamma}) + \Lambda tanh(\frac{z_n\Lambda}{\Gamma})) \tag{29}$$

where $\Gamma > 0$ and $\Lambda > \Theta/(1 - \Delta)$ are all designed constants.

For the time-varying continuous functions $\Phi_1(t)$ and $\Phi_2(t)$, $|\Phi_1(t)| \le 1$, $|\Phi_2(t)| \le 1$, $\forall t \in [t_s, t_{s+1})$, in terms of (28), one has $d(t) = (1 + \Phi_1(t)\Delta)u(t) + \Phi_2(t)\Theta$. Then one can obtain $u(t) = (d(t)/(1 + \Phi_1(t)\Delta)) - (\Phi_2(t)\Theta/(1 + \Phi_1(t)\Delta))$.

One has $z_nd(t)/(1 + \Delta\Phi_1(t)) \le z_nd(t)/(1 + \Delta)$ and $\Phi_2(t)\Theta/(1 + \Delta\Phi_1(t)) \le \Theta/(1 - \Delta)$. Therefore, similar to ist and considering the above self-trigger scheme, it holds that

$$\dot{V}_n \le -\sum_{b=1}^{n}k_{2,b}z_b^{2q} - \sum_{b=1}^{n}k_{1,b}z_b^{2o} + \sum_{b=1}^{n}(\frac{e_b^2}{2} + \frac{\mu_b^2}{2}) + \sum_{b=1}^{n}\frac{\delta_b}{\varrho_b}\widetilde{\lambda}_b\hat{\lambda}_b + 0.557N\Gamma \tag{30}$$

Case $2: |u(t)| < Q$, the control protocol is devised by

$$d(t) = -(1 + \varDelta)(a_n tanh(\frac{z_na_n}{\varGamma}) + \varLambda tanh(\frac{z_n\varLambda}{\varGamma})) \tag{31}$$

where $\varGamma > 0$ and $\varLambda > \varTheta/(1 - \varDelta)$ are all designed constants.

Similar to the relative threshold strategy in Case 1, one can get the same inequation as (30). Combining the same terms in these two inequations, one can easily obtain

$$\dot{V}_n \le -\sum_{b=1}^{n}k_{2,b}z_b^{2q} - \sum_{b=1}^{n}k_{1,b}z_b^{2o} + \sum_{b=1}^{n}(\frac{e_b^2}{2} + \frac{\mu_b^2}{2}) + \sum_{b=1}^{n}\frac{\delta_b}{\varrho_b}\widetilde{\lambda}_b\hat{\lambda}_b + 1.114N\Gamma \tag{32}$$

This article has been accepted for publication in IEEE Transactions on Circuits and Systems--II: Express Briefs. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TCSII.2023.3309400

Theorem 1: Consider the closed-loop system with time-varying constraints (1). One develops the control scheme consisting of the virtual control laws (15), (20), the actual controller (24), and adaptive laws (16), (21), (25). Then, the stability of the closed-loop system is preserved, and The system output can track the target signal within a fixed time.

Proof: For any $P_b > 1/2$, one gets $\delta_b\widetilde{\lambda}_b\hat{\lambda}_b = -\delta_b\widetilde{\lambda}_b(\widetilde{\lambda}_b - \lambda_b) \leq -\frac{\delta_b}{\varrho_b}\widetilde{\lambda}_b^2 + \frac{P_b\delta_b}{2}\widetilde{\lambda}_b^2$, substituting the inequality into (33) yields

$$\begin{aligned}\dot{V}_n \leq & -\Upsilon(\sum_{b=1}^{n}\frac{z_b^2}{2})^o - \Upsilon(\sum_{b=1}^{n}\frac{z_b^2}{2})^q - \Upsilon(\sum_{b=1}^{n}\frac{\widetilde{\lambda}_b^2}{2\varrho_b})^o - \Upsilon(\sum_{b=1}^{n} \\ & \frac{\widetilde{\lambda}_b^2}{2\varrho_b})^q + \sum_{b=1}^{n}\delta_b(\frac{\widetilde{\lambda}_b^2}{2\varrho_b})^o + (\sum_{b=1}^{n}\frac{\widetilde{\lambda}_b^2}{2\varrho_b})^q - \sum_{b=1}^{n}\delta_b(\frac{\widetilde{\lambda}_b^2}{2\varrho_b})^o \\ & - \Upsilon\sum_{b=1}^{n}\frac{\widetilde{\lambda}_b^2}{2\varrho_b} + \sum_{b=1}^{n}(\frac{e_b^2}{2} + \frac{\mu_b^2}{2} + \frac{P_b\delta_b}{2}\lambda_b^2) + 1.114N\Gamma\end{aligned} \tag{33}$$

with $\Upsilon = min\{2^o k_{1,b}, 2^q k_{2,b}, \delta_b, b = 1, ..., n\}$.

In terms of Lemma 1, one gets $(\sum_{b=1}^{n}\frac{\widetilde{\lambda}_b^2}{2\varrho_b})^q \leq \sum_{b=1}^{n}\frac{\widetilde{\lambda}_b^2}{2\varrho_b} + q(1-q)q^{\frac{q}{1-q}}$, applying the above inequality into (33) and further simplification obtains

$$\dot{V}_n \leq -\alpha V_n^o - \beta V_n^q + \sum_{b=1}^{n}\delta_b(\frac{\widetilde{\lambda}_b^2}{2\varrho_b})^o - \sum_{b=1}^{n}\frac{\delta_b\widetilde{\lambda}_b^2}{2\varrho_b} + \gamma^* \tag{34}$$

where $\alpha = \Upsilon/(n+1)^o$, $\beta = \Upsilon$ and $\gamma^* = \sum_{b=1}^{n}(\frac{e_b^2}{2} + \frac{\mu_b^2}{2} + \frac{P_b\delta_b}{2}\widetilde{\lambda}_b^2) + (1-q)q^{\frac{q}{1-q}} + 1.114N\Gamma$.

Then assume that there are unknown parameters l_b satisfied $|\widetilde{\lambda}_b| < l_b$. Then one discusses the following two cases:

Case 1: $l_b < \sqrt{2\varrho_b}$. In this case, one has $\sum_{b=1}^{n}\delta_b(\frac{\widetilde{\lambda}_b^2}{2\varrho_b})^o - \sum_{b=1}^{n}\frac{\delta_b\widetilde{\lambda}_b^2}{2\varrho_b} < 0$. Then (34) can be rewritten as

$$\dot{V}_n \leq -\alpha V_n^o - \beta V_n^q + \gamma^* \tag{35}$$

Case 2: $l_b \geq \sqrt{2\varrho_b}$. In this case, one has $\sum_{b=1}^{n}\delta_b(\frac{\widetilde{\lambda}_b^2}{2\varrho_b})^o - \sum_{b=1}^{n}\frac{\delta_b\widetilde{\lambda}_b^2}{2\varrho_b} \leq \sum_{b=1}^{n}\delta_b(\frac{l_b^2}{2\varrho_b})^o - \sum_{b=1}^{n}\frac{\delta_b l_b^2}{2\varrho_b}$. Then (35) can be overwritten as

$$\dot{V}_n \leq -\alpha V_n^o - \beta V_n^q + \gamma^* + \sum_{b=1}^{n}\delta_b(\frac{l_b^2}{2\varrho_b})^o - \sum_{b=1}^{n}\frac{\delta_b l_b^2}{2\varrho_b} \tag{36}$$

Sum up the two cases obtains that

$$\dot{V}_n \leq -\alpha V_n^o - \beta V_n^q + \gamma \tag{37}$$

where

$$\gamma = \begin{cases}\gamma^* & , \text{if } l_b < \sqrt{2\varrho_b} \\ \gamma^* + \sum_{b=1}^{n}\delta_b(\frac{l_b^2}{2\varrho_b})^o - \sum_{b=1}^{n}\frac{\delta_b l_b^2}{2\varrho_b} & , \text{if } l_b \geq \sqrt{2\varrho_b}\end{cases} \tag{38}$$

Based on the [8] and Lemma 2, it is concluded that the considered closed-loop signals are all bounded. Moreover, they can converge to the set m, $m \in min\{V(m) \leq (\frac{\gamma}{(1-h)\alpha})^{\frac{1}{o}}, (\frac{\gamma}{(1-h)\beta})^{\frac{1}{q}}\}$.

Let $T_{max} = \frac{1}{h(o-1)\alpha} + \frac{1}{h(1-q)\beta}$. Then, one can conclude that the inequality $|y - y_b| \leq 2(\frac{\gamma}{(1-h)\alpha})^{\frac{1}{2o}}$ holds. Namely, the tracking deviation can be confined into an arbitrarily miniature region within a specified time by properly adjusting the design constants.

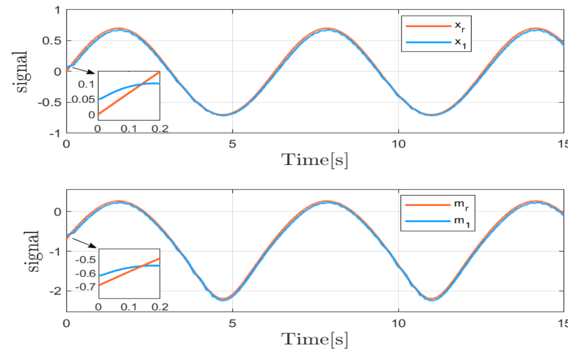

Fig. 2. Desired trajectories x_r, m_r and outputs x_1, m_1.

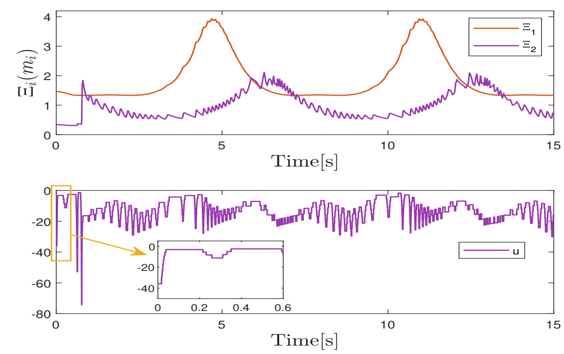

Fig. 3. Evolutionary trajectories of Ξ_i and event-triggered control signal $u(t)$.

Remark 1: In the early stage of the controller, the control signal amplitude changes greatly, so using the relative threshold can provide a more accurate trigger, while when the signal converges to a small region of the origin within a specified time, the control signal tends to stabilize, then using the self-triggered strategy to calculate the trigger time of the next moment can effectively improve the control performance. It also avoids the continuous monitoring of the threshold value and further reduces the system resource loss. Therefore, compared with [9] [10] [12], the SSTM can resultfully balance the relationship between tracking performance and resource utilization.

IV. ILLUSTRATIVE EXAMPLE

In this section, an illustrative example will be considered to proof the performance and the effectiveness of the proposed approach. Consider the nonlinear systems as follows

$$\begin{cases}\dot{x}_1 = cosx_2 \\ \dot{x}_2 = u + \frac{1}{14}x_1x_2 + \frac{1}{2}sin(x_1x_2)u + \frac{1}{14}(u + 0.14)^2 \\ y = x_1\end{cases} \tag{39}$$

The given desired trajectory $x_r = 0.7sint$. Based on the tracking control theory, it always expects that x_1 tracks the reference signal asap. This will be shown in Fig.2.

This article has been accepted for publication in IEEE Transactions on Circuits and Systems--II: Express Briefs. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TCSII.2023.3309400

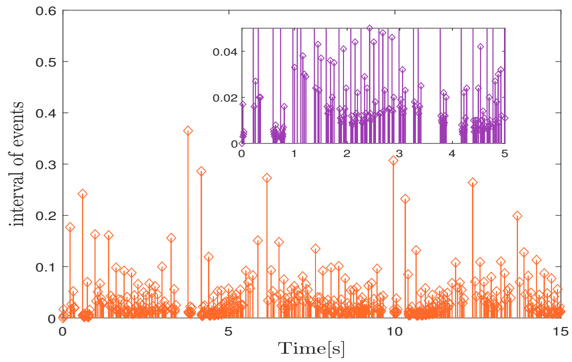

Fig. 4. Event-based release instants and release interval: SSTM.

In this brief, the design constants are taken as $\varpi_{11} = 1, \varpi_{12} = 2, \varpi_{21} = 7, \varpi_{22} = 6$ such that the system (1) is bounded. The parameters of the update law $\dot{\hat{\lambda}}_i$ are designed as $p_1 = 6, p_2 = 3, \delta_1 = \delta_2 = 10$, the gain coefficient parameters can be selected as $k_{1,1} = 800, k_{1,2} = 6$, and the other parameters can be selected as $e_1 = e_2 = 1, o = 1.05$. Under switched-self-triggered mechanism: $\Delta = 0.1, \Theta = 2, \Lambda = 5, \Gamma = 900, \mathit{\Delta} = 0.51, \mathit{\Theta} = 0.2, \mathit{\Lambda} = 2, \mathit{\Gamma} = 900, \mathit{\Upsilon} = 30, Q = 24$. Besides, the simulation is run with the initial values $x_1(0) = 0.05$ and $x_2(0) = 0.5$.

The corresponding simulation results are displayed in Figs.2-5. Fig.2 depicts the constrained system output and transformed system output x_1, m_1, respectively, with their desired trajectories x_r, m_r. It can be seen from the graph that the tracking error is controlled to a small extent. Fig.3 sketches the changes of evolutionary trajectories Ξ_1 and Ξ_2 and the veritable controller. The controller signal fluctuates a lot at first, and then tends to smooth out. The intervals of the SSTM are described in Fig.4. Moreover, the overall triggering times are 486, which is far below the time sampling strategy of 15000 times.

V. CONCLUSION

In this brief, one has coped with the adaptive fixed-time control for a full state-constrained nonlinear system. By introducing an auxiliary system with the help of the variation of coordinates and a one-to-one asymmetric NM, the problem of transformation between the restricted system and the unrestricted system can be effectively figured out. Moreover, the proposed switched-self-triggered mechanism can keep a balance between the availability of resources and tracking performance. One also verified, under the condition of state constraint, both the closed-loop stableness and tracking performance can be preserved in a specified time. The full states can abide by the desired constraints. In the end, a quintessential example has been exploited to reveal the availability and dependability of the proposed adaptive control approach.

REFERENCES

[1] K. Zhao, Y. Song, "Removing the feasibility conditions imposed on tracking control designs for state-constrained strict-feedback systems," *IEEE Trans. Automat. Contr.,* vol. 64, no. 3, pp. 1265-1272, Mar. 2019.

[2] B. S. Kim, S. J. Yoo, "Approximation-based adaptive control of uncertain non-linear pure-feedback systems with full state constraints," *IET Control.,* vol. 8, no. 17, pp. 2070-2081, Nov. 2014.

[3] K. Zhao, Y. Song, T. Ma and L. He, "Prescribed performance control of uncertain Euler-Lagrange systems subject to full-state constraints," *IEEE Trans Neural Netw Learn Syst,* vol. 29, no. 8, pp. 3478-3489, Aug. 2018.

[4] B. Niu, Z. R. Xiang, "State-constrained robust stabilisation for a class of high-order switched non-linear systems," *Iet Control Theory and Applications.,* vol. 9, no. 12, pp. 1901-1908, Aug. 2015.

[5] C. Guo, R. M. Xie, X. J. Xie, "Adaptive Control of Full-State Constrained High-Order Nonlinear Systems With Time-Varying Powers" *IEEE Trans. Syst. Man Cybern.* vol. 51, no. 8, pp. 5189-5197, Aug. 2021.

[6] D. J. Li, J. Li, S. Li, "Adaptive control of nonlinear systems with full state constraints using Integral Barrier Lyapunov Functionals," *Neurocomputing.,* vol. 186, pp. 90-96, Apr. 2016.

[7] Y. M. Sun, F. Wang, Z. Liu, Y. Zhang, C. L. P. Chen, "Fixed-Time Fuzzy Control for a Class of Nonlinear Systems" *IEEE Trans Cybern.,* vol. 52, no. 5, pp. 3880-3887, May. 2022.

[8] D. S. Ba, Y. X. Li, S. C. Tong, "Fixed-time adaptive neural tracking control for a class of uncertain nonstrict nonlinear systems," *Neurocomputing.,* vol. 363, pp. 273-280, Oct. 2019.

[9] Z. Y. Chen, B. Niu, X. D. Zhao, L. Zhang, and N. Xu, "Model-based adaptive event-triggered control of nonlinear continuous-time systems," *Appl. Math. Comput.,* vol. 408, Nov. 2021,

[10] N. Pang, X. Wang, and Z. M. Wang, "Observer-based event-triggered adaptive control for nonlinear multiagent systems with unknown states and disturbances" *IEEE Trans. Neural Netw. Learn. Syst.,* early access, Dec. 2021, doi: 10.1109/TNNLS.2021.3133440.

[11] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Trans. Autom. Control.,* vol. 52, no. 9, pp. 1680-1685, Sep. 2007.

[12] F. B. Cheng, H. J. Liang, H. Q. Wang, G. D. Zong, N. Xu, "Adaptive Neural Self-Triggered Bipartite Fault-Tolerant Control for Nonlinear MASs With Dead-Zone Constraints," *IEEE Trans. Autom. Sci. Eng.,* early access, Jun. 2022, doi: 10.1109/TASE.2022.3184022

[13] X. G. Liu, X. F. Liao, "Fixed-time stabilization control for port-Hamiltonian systems," *Nonlinear Dyn.* vol. 96, no. 2, pp. 1497-1509, Apr. 2019.

[14] Q. J. Yao, "Robust fixed-time trajectory tracking control of marine surface vessel with feedforward disturbance compensation," *Int J Syst Sci.,* vol. 53, no. 4, pp. 726-742, Mar. 2022.

[15] L.-X. Wang and J. M. Mendel, "Fuzzy basis functions, universal approximation, and orthogonal least-squares learning," *IEEE Trans. Neural Netw.,* vol. 3, no. 5, pp. 807-814, Sep. 1992.

[16] L. T. Xing, C. Y. Wen, Z. T. Liu, H. Y. Su, and J. P. Cai, "Event-Triggered Adaptive Control for a Class of Uncertain Nonlinear Systems," *IEEE Trans. Autom. Contrd,* vol. 62, no. 4, pp. 2071-2076, Apr. 2017.

[17] C. Qian and W. Lin, "Non-lipschitz continuous stabilizers for nonlinear systems with uncontrollable unstable linearization," *Syst. Control Lett.,* vol. 42, no. 3, pp. 185-200, Mar. 2001.

[18] Z. Zuo, B. Tian, M. Defoort, and Z. Ding, "Fixed-time consensus tracking for multiagent systems with high-order integrator dynamics," *IEEE Trans. Autom. Control.,* vol. 63, no. 2, pp. 563-570, Feb. 2018.

[19] X. Yuan, B. Chen, and C. Lin, "Neural Adaptive Fixed-Time Control for Nonlinear Systems With Full-State Constraints," *IEEE Trans Cybern,.* early access, Nov. 2021, doi: 10.1109/TCYB.2021.3125678.

[20] N. Pang, X. Wang and Z. M. Wang, "Event-Triggered Adaptive Control of Nonlinear Systems With Dynamic Uncertainties: The Switching Threshold Case," *IEEE Trans. Circuits Syst. II Express Briefs.,* vol. 69, no. 8, pp. 3540-3544, Aug. 2022.

[21] Y. C. Zhang, M. C. Ma, X. Y. Yang, and S. M. Song, "Disturbance-observer-based fixed-time control for 6-DOF spacecraft rendezvous and docking operations under full-state constraints," *Acta Astronaut.* vol. 205, pp. 225-238, Feb. 2023.

[22] Z. X. Du, H. J. Liang and C. K. Ahn, "Adaptive Fuzzy Control for Multi-Agent Systems With Unknown Measurement Sensitivity via a Simplified Backstepping Approach," *IEEE Trans. Circuits Syst. II Express Briefs.,* vol. 69, no. 6, pp. 2862-2866, Jun. 2022.