Final Project

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Research Question

Do children differentiate the complexity of the information they teach based on the maturity of the audience?

Description of each variable in our preliminary analysis

Age: five-years-old and seven-years-old

Response: basic or complex information

Trials: number of trials per participant

Model and the priors

Model:

$$
\begin{aligned}
z_j^A &\sim \text{Bin}(N_j^A, \theta_j^A) \\
z_j^B &\sim \text{Bin}(N_j^B, \theta_j^B) \\
\theta_j^A &\sim \text{Beta2}(\mu, \kappa) \\
\theta_j^B &= \Phi(q_j) \\
q_j &= \Phi^{-1}(\theta_j^A) + \delta_j \\
\delta_j &\sim N(\mu_\delta, \tau_\delta)
\end{aligned}
$$

Prior:

$$
\begin{aligned}
\mu &\sim \text{Beta}(2, 2) \\
\kappa &\sim \text{Gamma}(0.01, 0.01) \\
\mu_\delta &\sim N(0, 1) \\
\tau_\delta &\sim N(0, 1)
\end{aligned}
$$

j:subject (subject 1, 2, ..., 68)

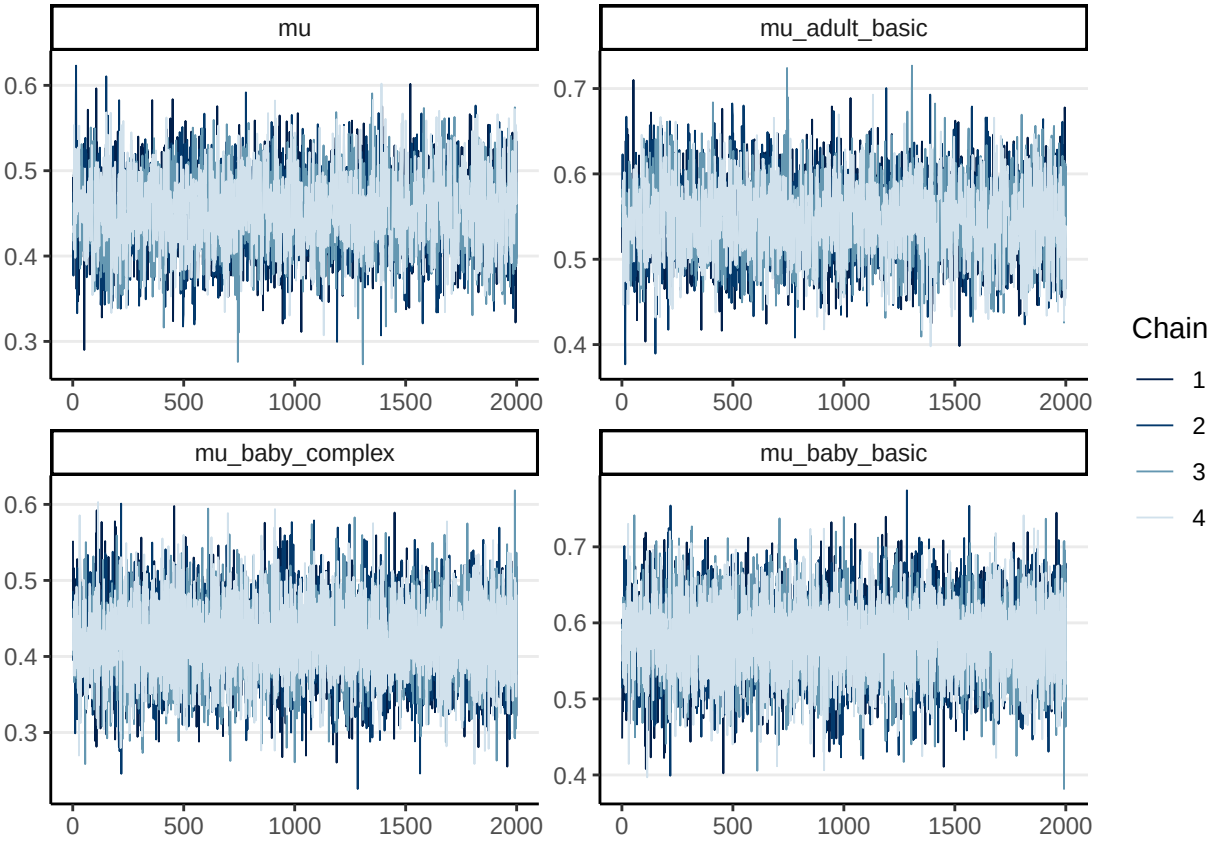
θ_j: individual subject's prob of transmitting complex info

N_j:the number of trails

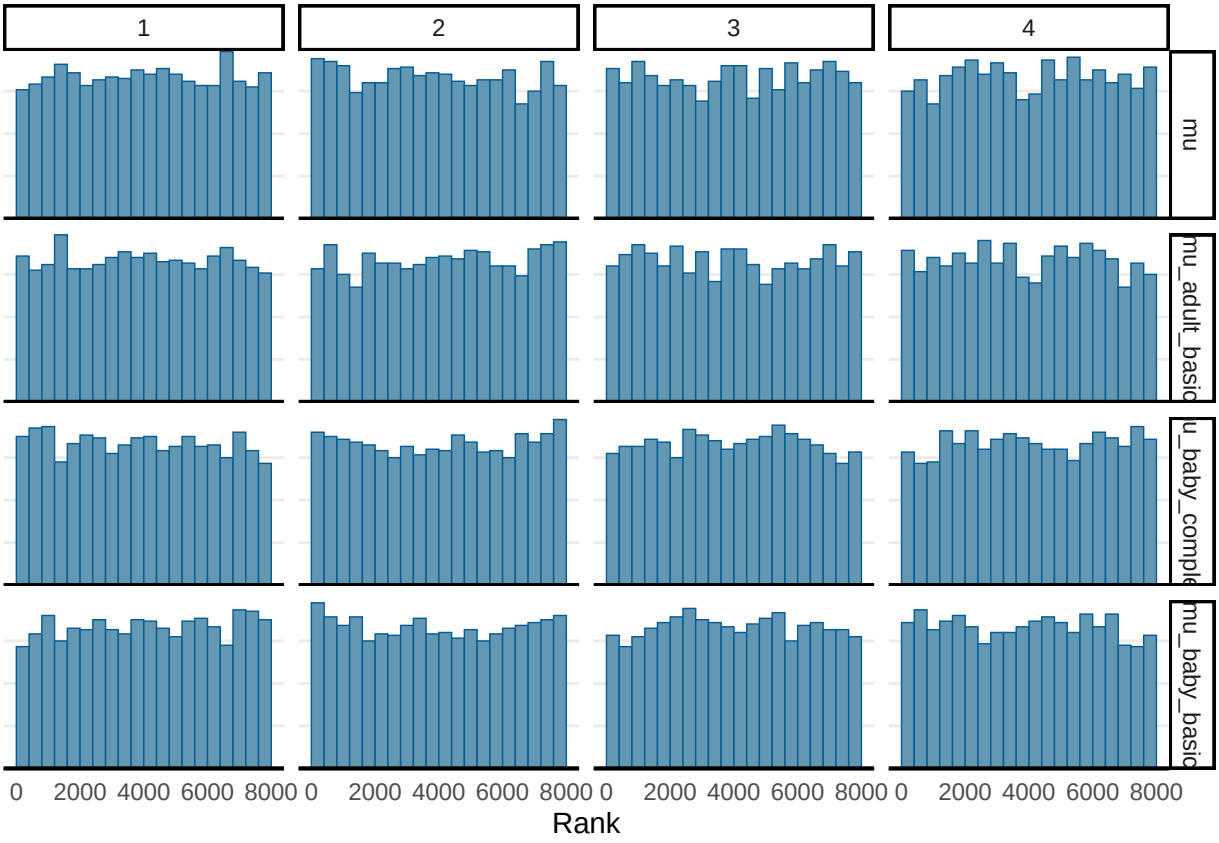
5-year-olds

Convergence Check for 5yos

Trace plots of key parameters



Rank histograms of key parameters



Posterior distribution of key model parameter mu:

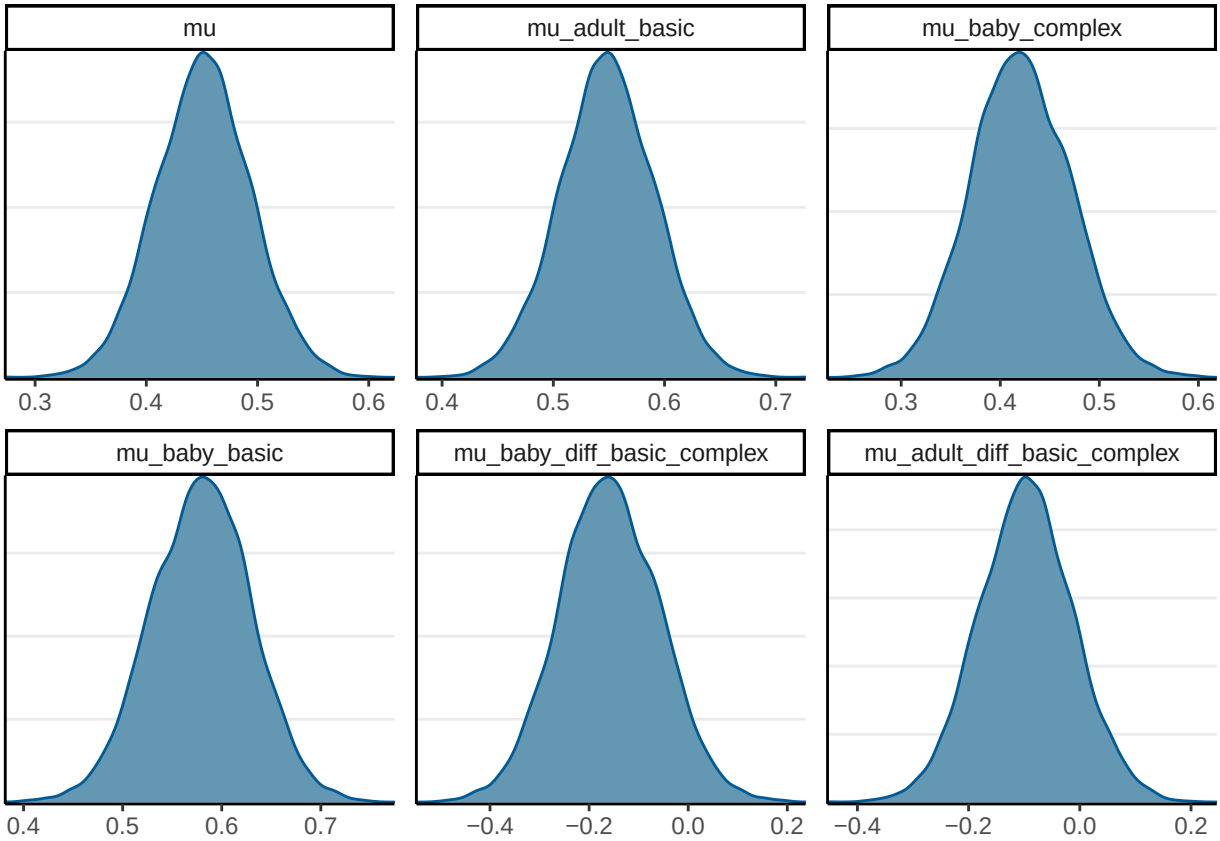


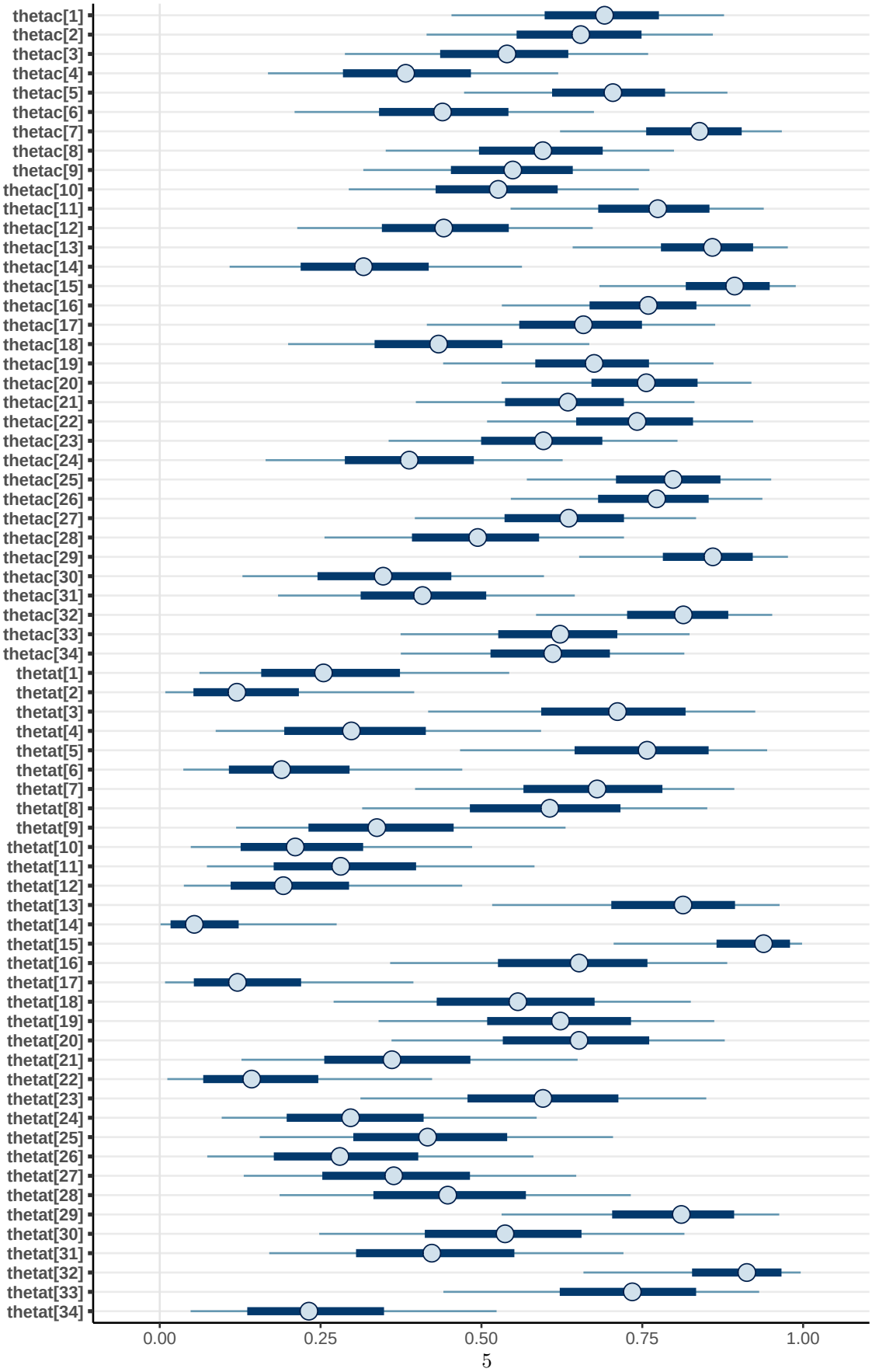
Table of the posterior distributions of key parameters

variable	mean	median	sd	mad	q5	q95	rhat	ess_bulk	ess_tail
mu	0.45	0.45	0.04	0.04	0.38	0.52	1	3034.21	5003.46
mu_adult_basic	0.55	0.55	0.04	0.04	0.48	0.62	1	3034.21	5003.46
mu_baby_complex	0.42	0.42	0.05	0.05	0.34	0.50	1	9786.65	5903.46
mu_baby_basic	0.58	0.58	0.05	0.05	0.50	0.66	1	9786.65	5903.46
mu_baby_diff_basic_complex	-0.16	-0.16	0.10	0.10	-0.32	0.01	1	9786.65	5903.46
mu_adult_diff_basic_complex	-0.10	-0.10	0.09	0.08	-0.24	0.05	1	3034.21	5003.46

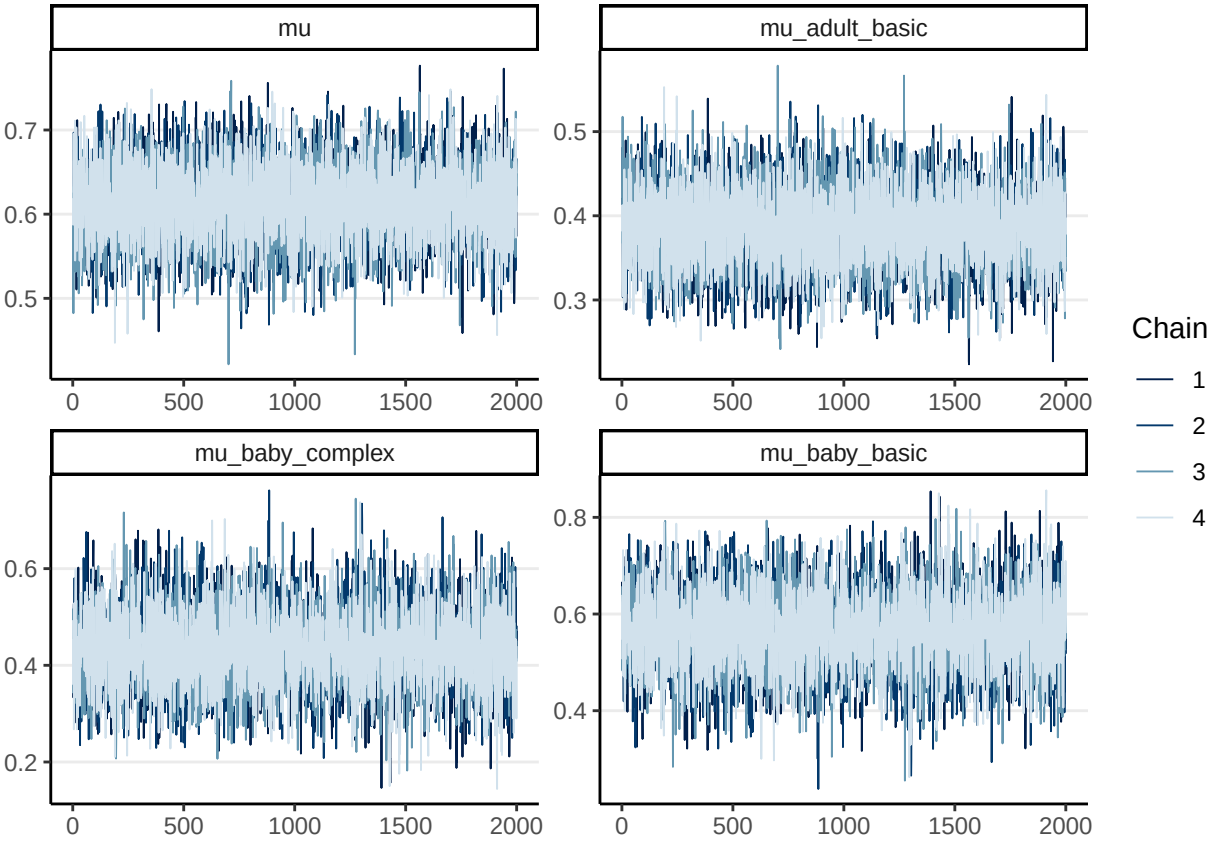
From the trace plots and the rank histograms, it seems like there is good mixing of the 4 chains as they frequently cross each other. The histograms for the four chains have similar average ranks and are roughly uniform, which also suggests good mixing. The ess_bulk for all of the key parameters is larger than 1000, and the rhat value for all key parameters are less than 1.01.

7-year-olds

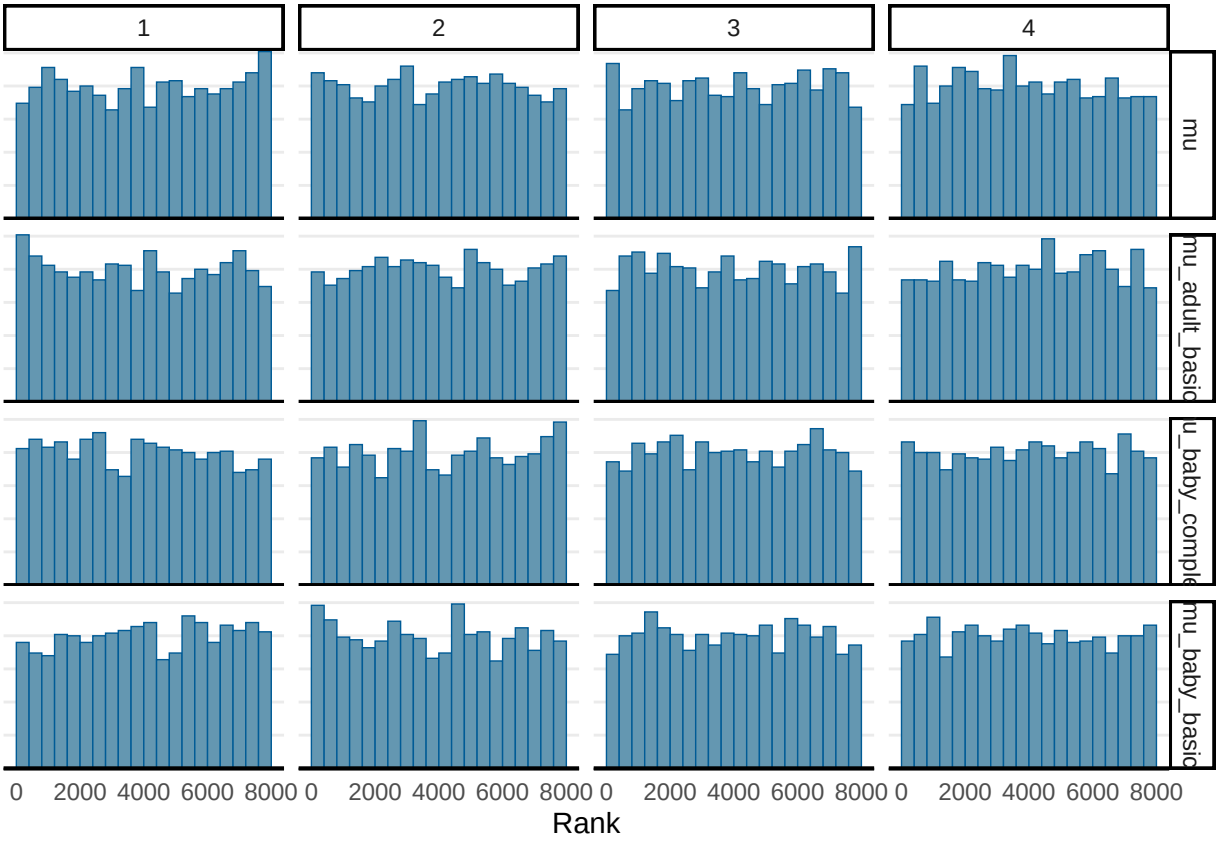
Convergence Check for 7yos



Trace plots of key parameters



Rank histograms of key parameters



Posterior distribution of key model parameter mu:

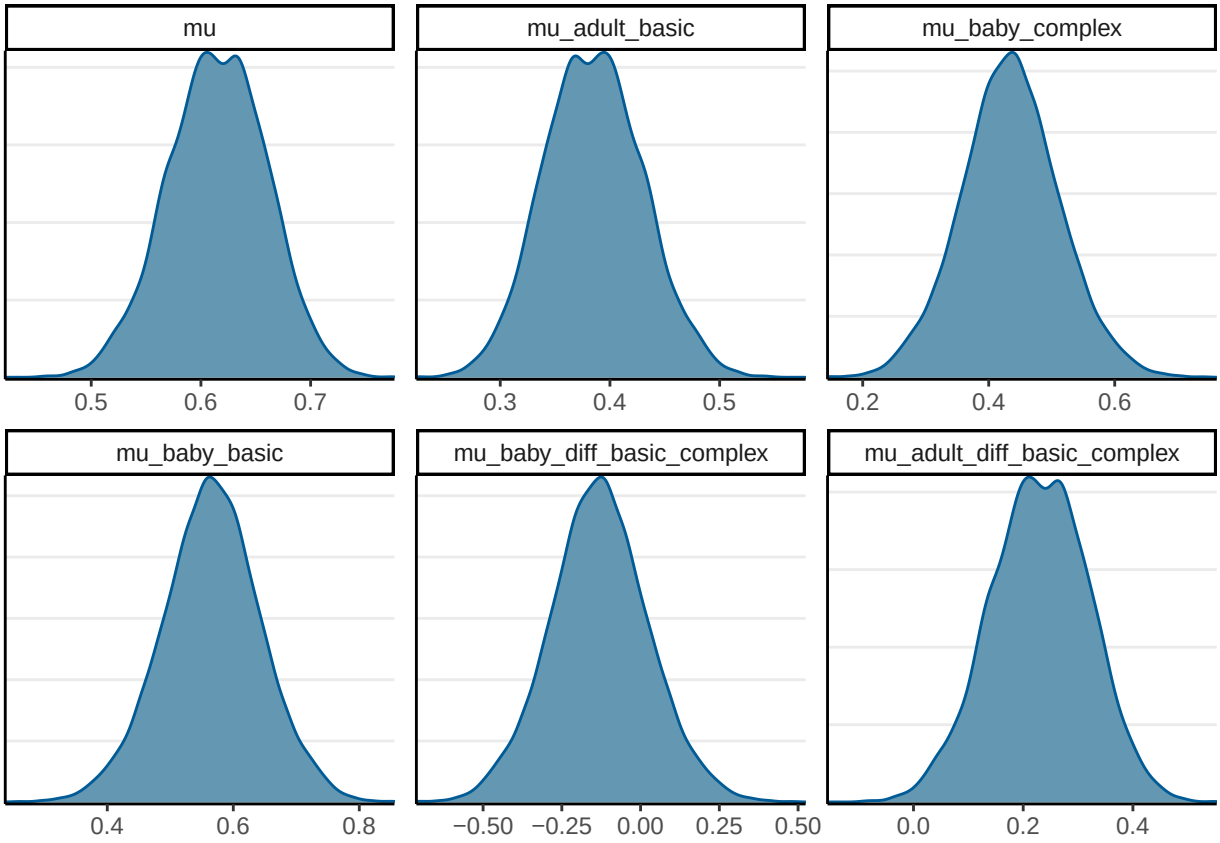


Table of the posterior distributions of key parameters

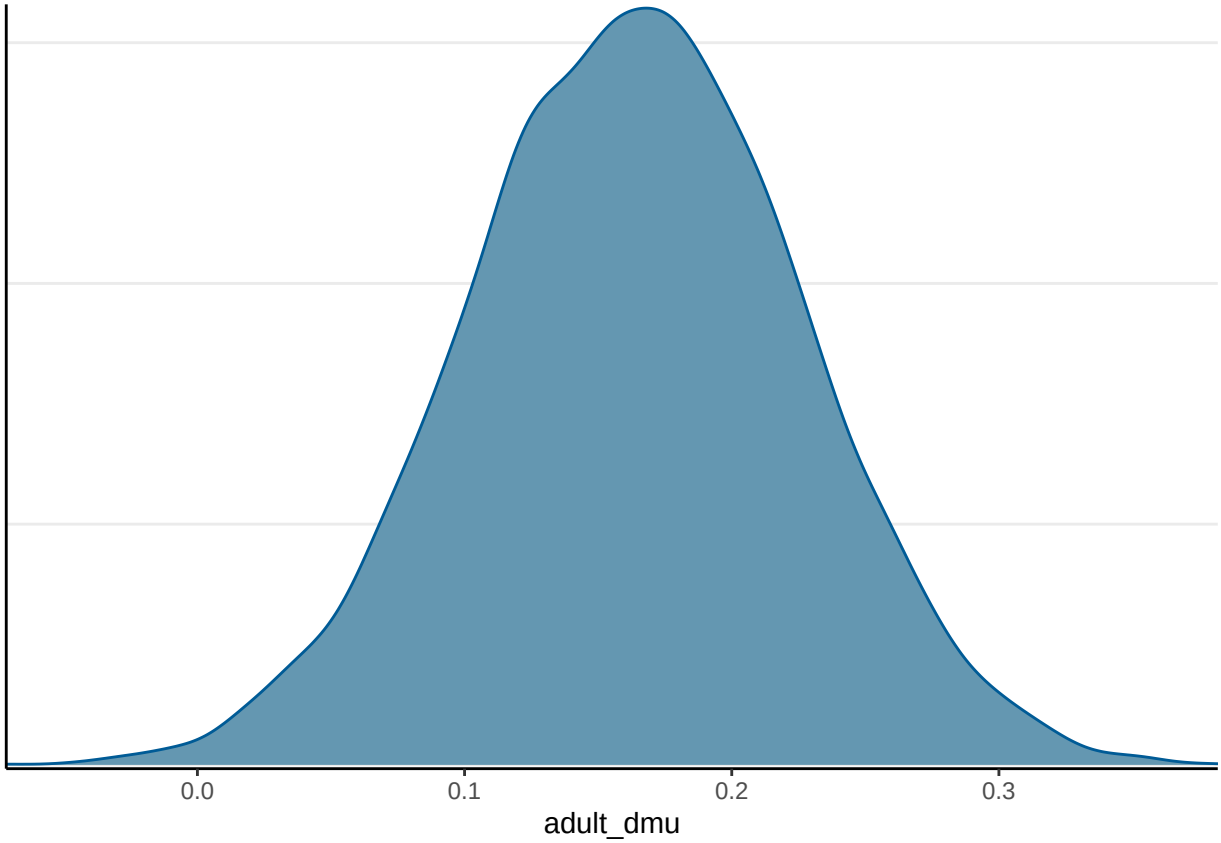
variable	mean	median	sd	mad	q5	q95	rhat	ess_bulk	ess_tail
mu	0.61	0.62	0.05	0.05	0.54	0.69	1	5405.97	5971.39
mu_adult_basic	0.39	0.38	0.05	0.05	0.31	0.46	1	5405.97	5971.39
mu_baby_complex	0.43	0.43	0.08	0.08	0.31	0.57	1	4865.76	5843.97
mu_baby_basic	0.57	0.57	0.08	0.08	0.43	0.69	1	4865.76	5843.97
mu_baby_diff_basic_complex	-0.13	-0.13	0.16	0.15	-0.39	0.13	1	4865.76	5843.97
mu_adult_diff_basic_complex	0.23	0.23	0.09	0.09	0.08	0.37	1	5405.97	5971.39

From the trace plots and the rank histograms, it seems like there is good mixing of the 4 chains as they frequently cross each other. The histograms for the four chains have similar average ranks and are roughly uniform, which also suggests good mixing. The ess_bulk for all of the key parameters is larger than 1000, and the rhat value for all key parameters are less than 1.01.

```
post_mu_five <- rstan::extract(fit1, pars = "mu")$mu
post_mu_seven <- rstan::extract(fit2, pars = "mu")$mu
# Compute the difference
dmu_seven_minus_five <- post_mu_seven - post_mu_five
# Summary
posterior::summarise_draws(
list(adult_dmu = dmu_seven_minus_five)
)%>%
  knitr::kable(digits=2)
```

variable	mean	median	sd	mad	q5	q95	rhat	ess_bulk	ess_tail
adult_dmu	0.16	0.16	0.06	0.06	0.06	0.27	1	8027.33	7383.85

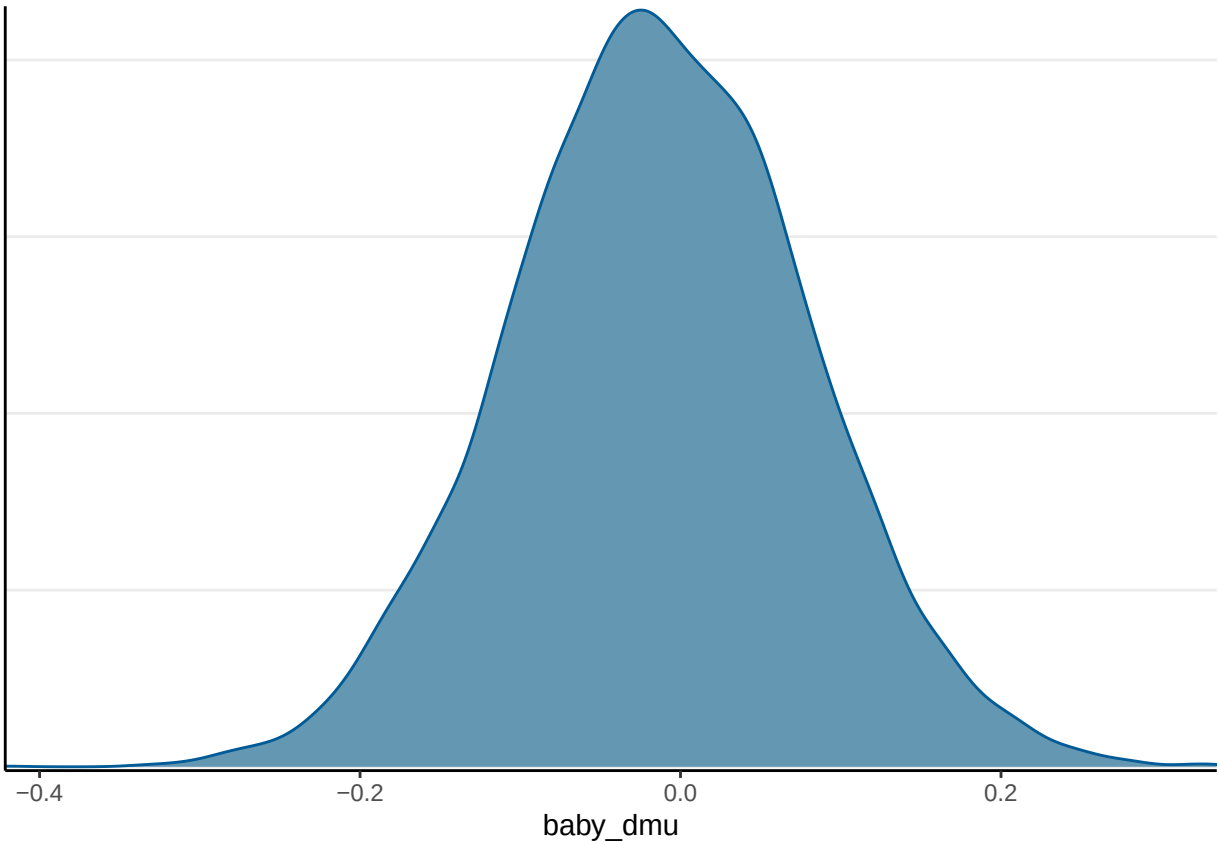
```
mcmc_dens(as_draws_array(list(adult_dmu=dmu_seven_minus_five)))
```



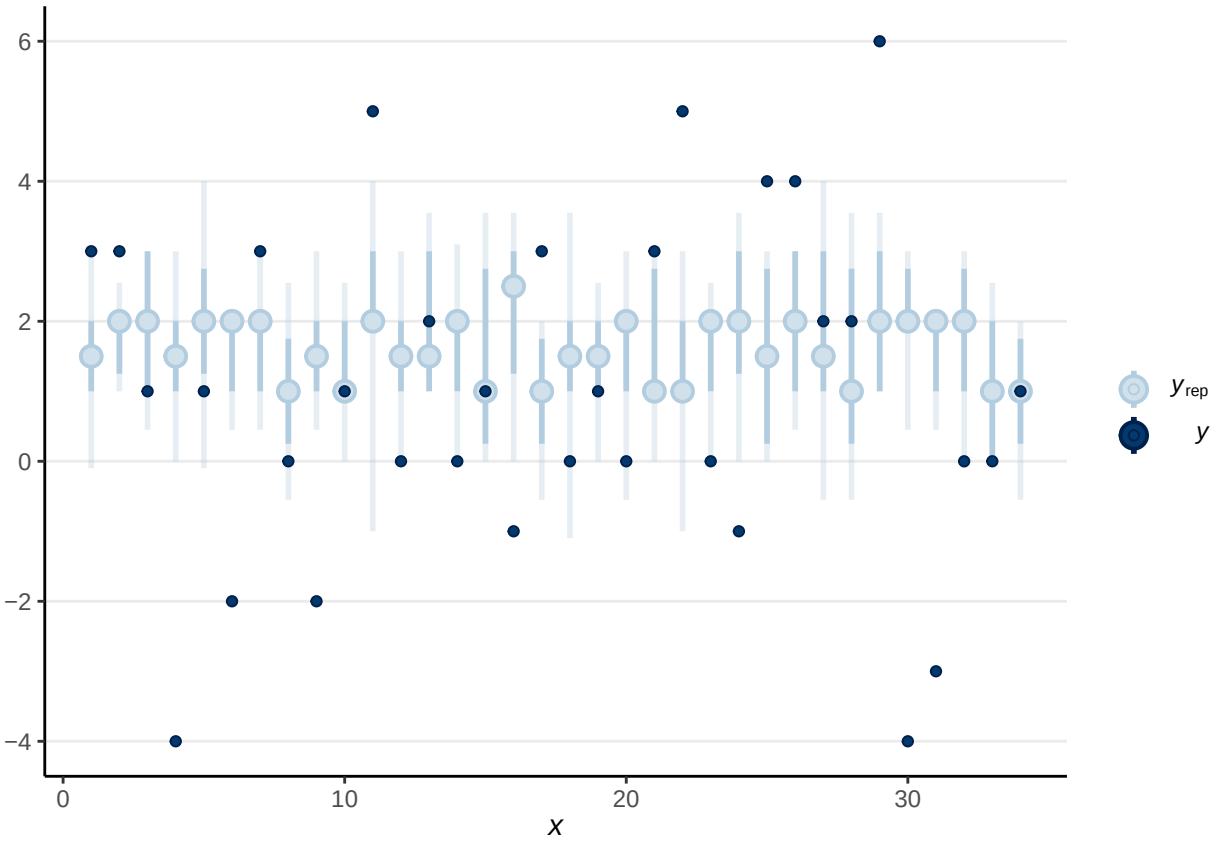

```
post_mu_bfive <- rstan::extract(fit1, pars = "mu_baby_basic")$mu_baby_basic
post_mu_bseven <- rstan::extract(fit2, pars = "mu_baby_basic")$mu_baby_basic
# Compute the difference
bdmu_seven_minus_five <- post_mu_bseven - post_mu_bfive
# Summary
posterior::summarise_draws(
list(baby_dmu = bdmu_seven_minus_five)
)%>%
  knitr::kable(digits=2)
```

variable	mean	median	sd	mad	q5	q95	rhat	ess_bulk	ess_tail
baby_dmu	-0.01	-0.01	0.09	0.09	-0.17	0.14	1	8168.75	7668.16

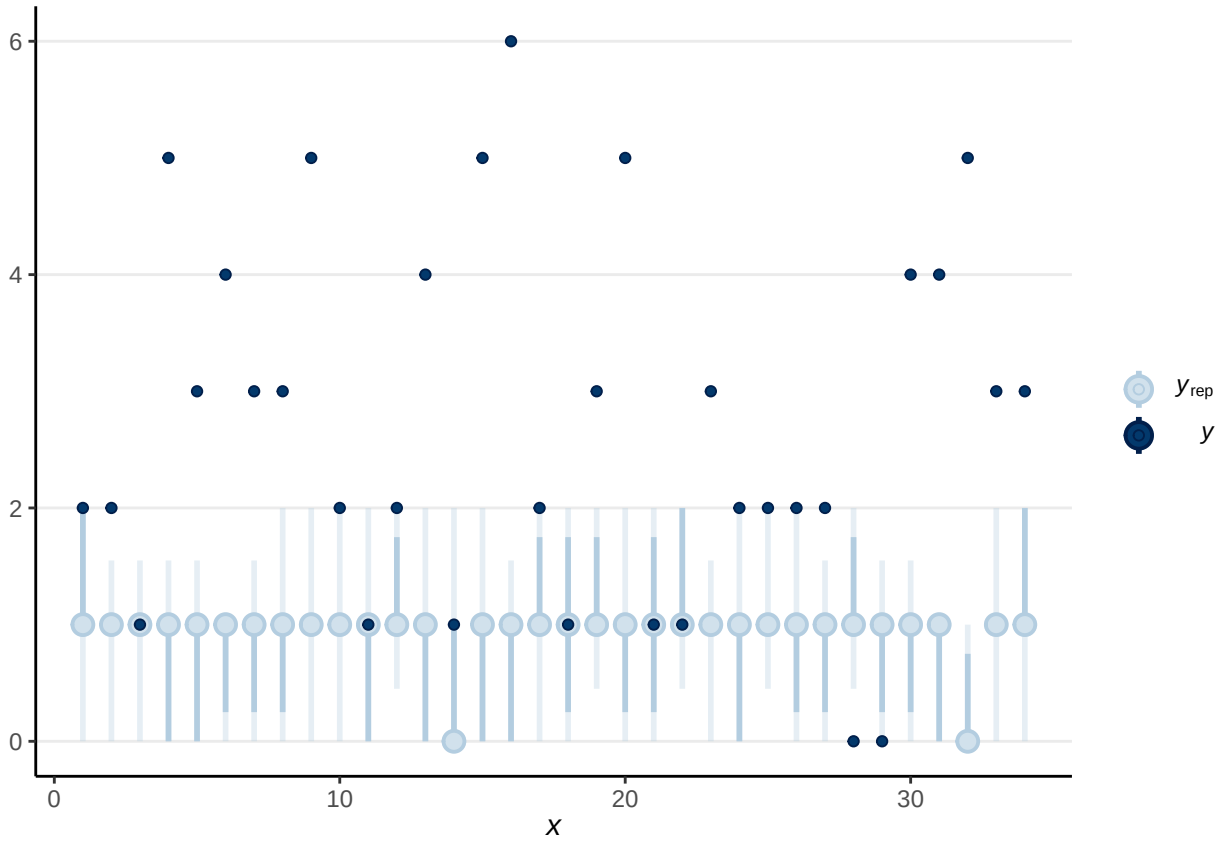
```
mcmc_dens(as_draws_array(list(baby_dmu=bdmu_seven_minus_five)))
```



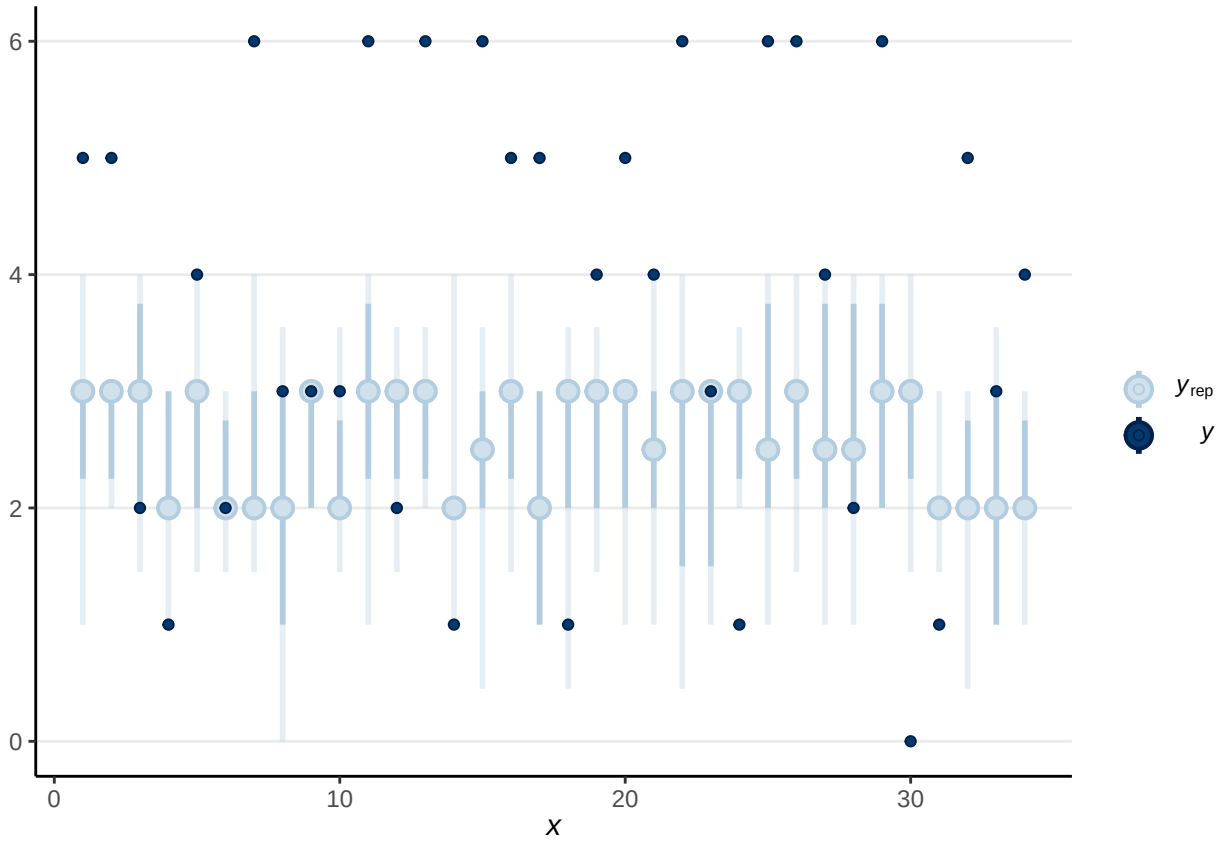

```
## Warning: The following arguments were unrecognized and ignored: bw
```



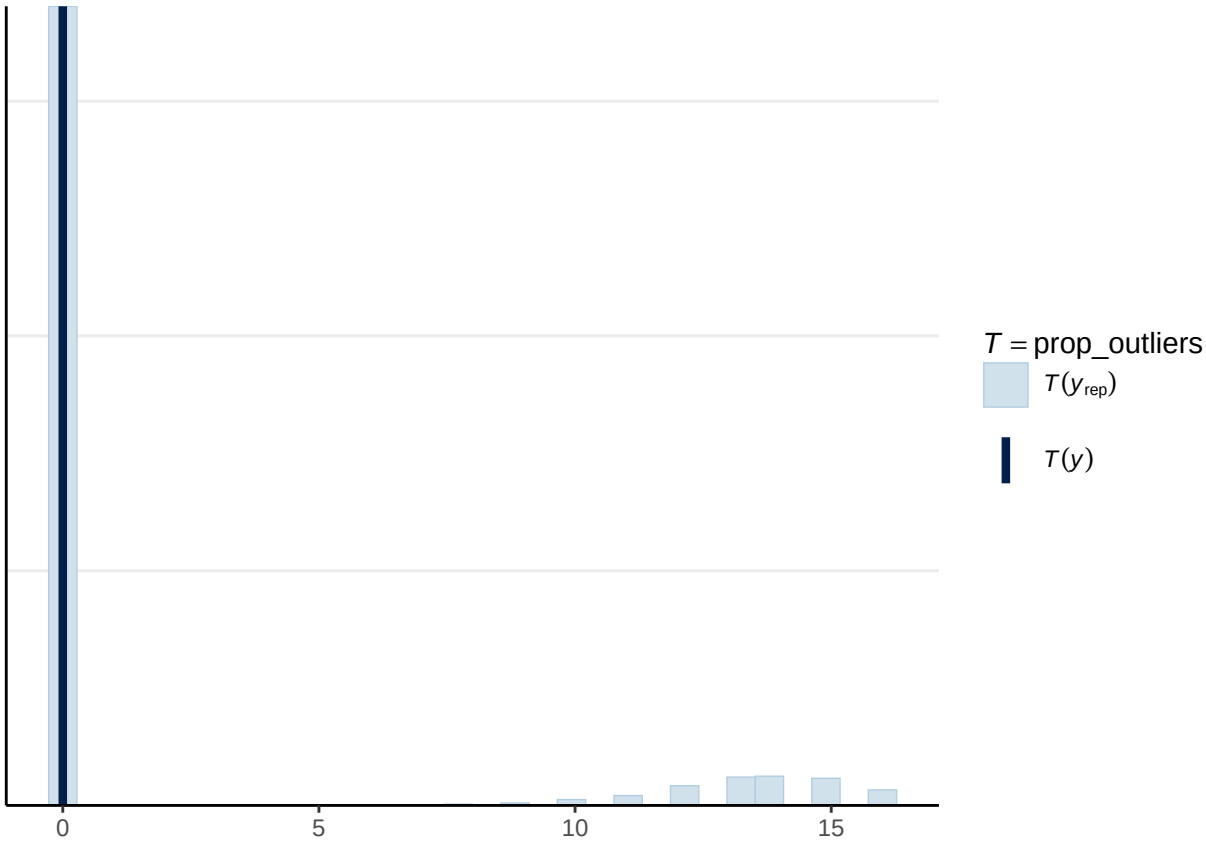

```
## Warning: The following arguments were unrecognized and ignored: bw
```



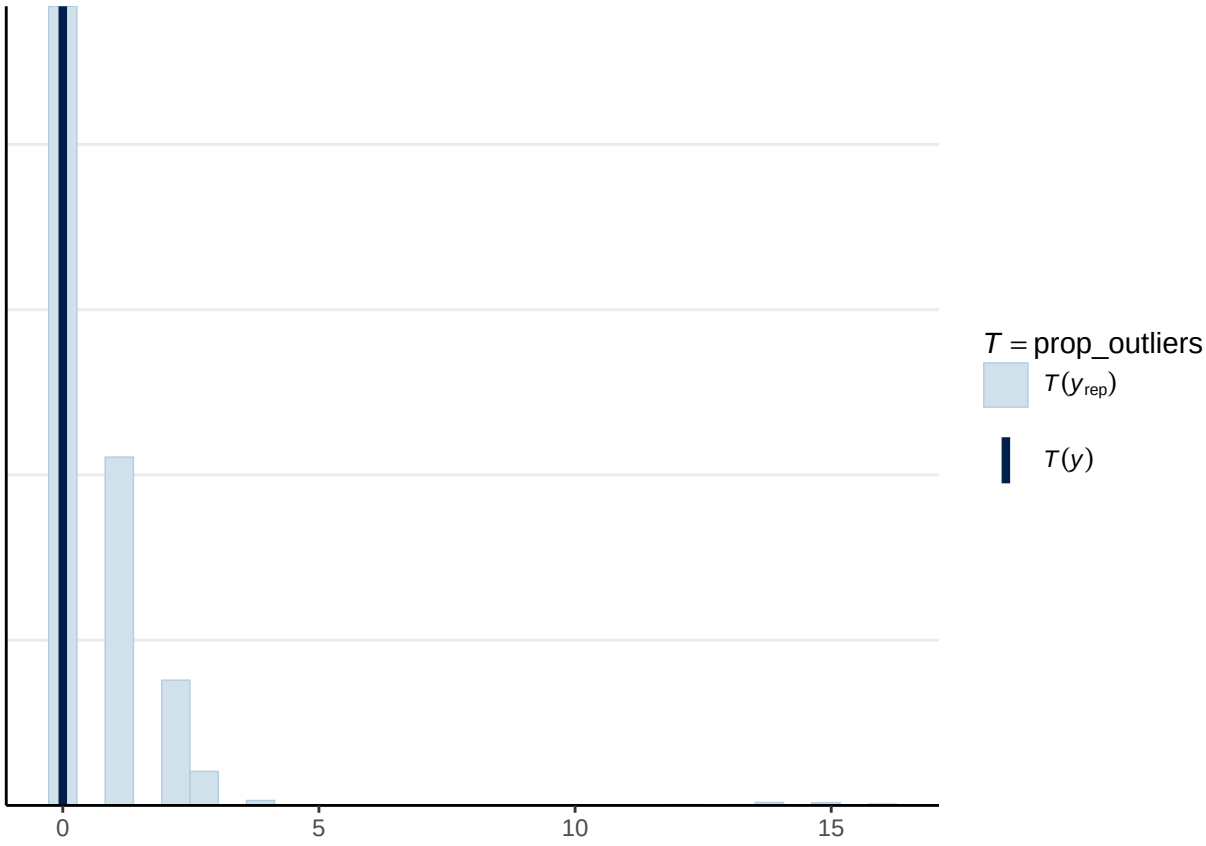

```
## Warning: The following arguments were unrecognized and ignored: bw
```




```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



Interpretation of Results

Within-Group Comparisons

5-year-olds When teaching an adult, 5-year-olds were not reliably different from zero, therefore, we don't have sufficient statistical evidence for them teaching more complex information than basic information. The estimated difference in probability is -0.09, 90% CI[-0.23, 0.04]. When teaching a baby, 5-year-olds' selective teaching was not reliably different from zero, therefore, we don't have sufficient statistical evidence for them choosing more basic information than complex information. The estimated difference in probability is -0.16, 90% CI[-0.32, 0.01].

The estimated probability for 5-year-olds teaching complex information to the adult is 0.45, 90% CI [0.38, 0.52]. Since this interval includes 0.5, it indicates that they are teaching complex information to the adult at chance. The estimated probability for 5-year-olds teaching basic information to the baby is 0.58, 90% CI [0.49, 0.66]. Since this interval includes 0.5, it indicates that they are also teaching basic information to the baby at chance.

7-year-olds When teaching an adult, 7-year-olds were more likely to teach complex information than basic information. The estimated difference in probability is 0.23, 90% CI[0.08, 0.37]. When teaching a baby, 7-year-olds' selective teaching was not reliably different from zero, therefore, we don't have sufficient statistical evidence for them choosing more basic information than complex information. The estimated difference in probability is -0.13, 90% CI[-0.39, 0.13].

The estimated probability for 7-year-olds teaching complex information to the adult is 0.61, 90% CI [0.54, 0.69]. Since this interval does not include 0.5, it indicates that they are teaching complex information to the adult above chance. The estimated probability for 7-year-olds teaching basic information to the baby is 0.57, 90% CI [0.43, 0.69]. Since this interval includes 0.5, it indicates that they are teaching basic information to the baby at chance.

Age Group Comparisons From the posterior distributions, it seems like 7-year-olds are better than 5-year-olds at teaching complex information to the adult. The mean amount of complex information taught to adults for 7-year-olds is 0.61, 90% CI [0.54, 0.69], whereas the mean amount of complex information taught to adults for 5-year-olds is 0.45, 90% CI[0.38, 0.52]. However, there does not seem to be an age difference in the probability of basic information taught to the baby audience. The estimated probability of 7-year-olds teaching basic information to the baby is 0.57, 90% CI [0.43, 0.69]. Similarly, the estimated probability of 5-year-olds teaching basic information to the baby is 0.58, 90% CI [0.49. 0.66].

This indicates that compared to 5-year-olds, 7-year-olds are significantly better at teaching complex information to the adult. On the other hand, 7-year-olds and 5-year-olds were similar in their probability of teaching basic information to the baby.

Model Fit From the posterior predictive check, it does not seem like the model is a great fit for the data, as there are about 15 data points that are outside of the predicted values range. We think this may be due to the fact that there are a lot of individual differences in our observed data.