Lecture 10
T-test

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What we learned...

- Sampling distribution
- Standard error of the mean
- Confidence interval

Hypothesis testing?

- Before we learn the JAMOVI implementations of *t*-test,

  it would be good to review the logic of hypothesis testing.

Hypotheses

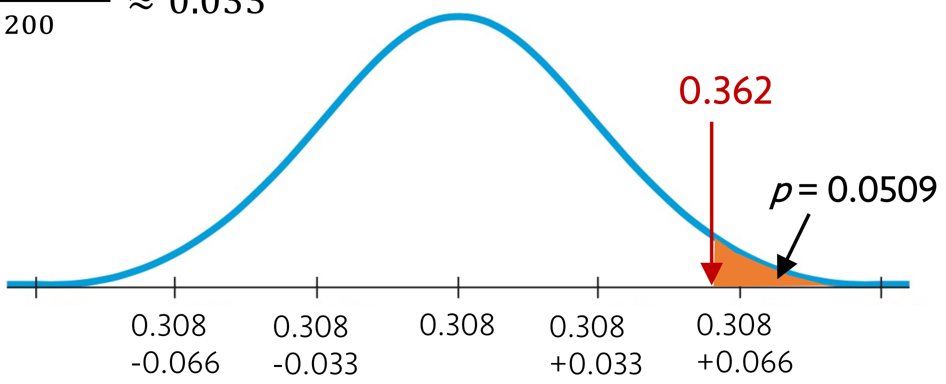
- **Hypothesis**: questions like, has the Facebook users who update their status daily increased since last month?

- **Null hypothesis:** null because it assumes no changes, thus p = 30.8%
- **Alternative hypothesis**: H_A: p > 30.8%

- We observed a new $\hat{p}$ from 200 respondents.

- Based on the null hypothesis, $SD(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.308 \times 0.692}{200}} \approx 0.033$

- Let's say the observed $\hat{p}$ = 36.2%
- Then, $z = \frac{0.362 - 0.308}{0.033} = 1.6364$
- p = 0.0509 (one-tail)



A Trial as a "Null Hypothesis Statistical Test" (NHST)

- It's the logic of jury trials.
  - The null hypothesis is that the defendant is *innocent*
  - Judge the evidence
  - Juries ask "Could these evidence plausibly have happened by chance if the defendant were in fact *innocent*?"
  - Make a decision

- In hypothesis testing:
  - We quantify *"how surprising the evidence would be if the null hypothesis were true."*

What's P-value?

- P-value = P(Data | H_0), not P(H_0 | Data)
  - The conditional probability of getting the data given that the null hypothesis is true
  - *NOT* the probability that the null hypothesis is true
  - *NOT* the conditional probability that the null hypothesis is true given the data
  - P-value = 0.03
    - does *NOT* mean "there is a 3% chance that the null hypothesis is true".
    - does mean "given the null hypothesis, there's a 3% chance of observing the observed statistic value.

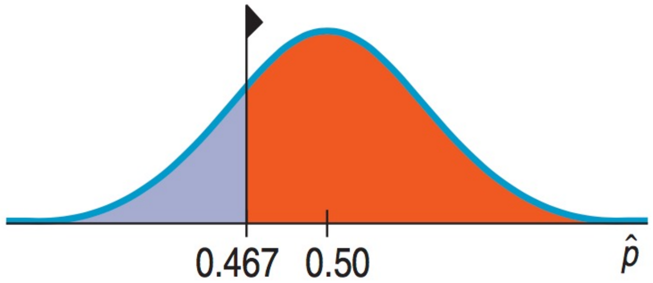
Small P-value

- First, yay!
- It means the result we just observed is unlikely to occur if the null hypothesis is true.
  - does *NOT* mean that the null hypothesis is "less true".

- How small the P-value should be?
  - depends on a lot of things, e.g., your prior belief in the null hypothesis, your trust in your data, in the experimental method, in the survey protocol, etc.
  - P-value serve as a measure of the strength of the evidence against the null hypothesis
  - should *NEVER* serve as a hard and fast rule for decisions.
  - *YOU* have to take the responsibility for the decision on yourself.

High P-value

- No evidence for rejecting H_0
- We cannot reject the null hypothesis.
- For one-sided test, if P-value is higher than 0.5, you know that your test statistic is on the "wrong" side.

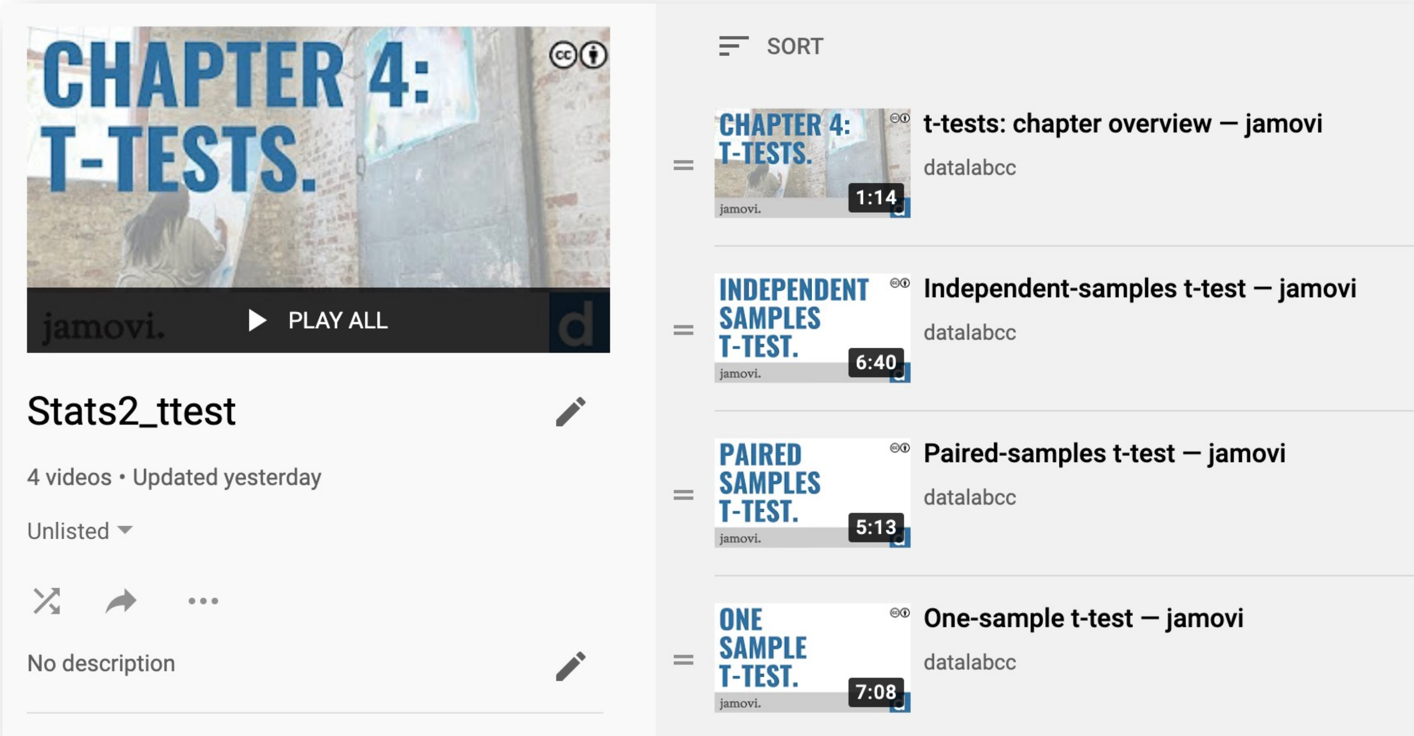


- High P-values mean
  - What we've observed is not surprising.
  - We have no reason to reject our null hypothesis.
  - Does *NOT* prove that the null hypothesis is true
  - Do *NOT* say that you "accept the null hypothesis".
  - You *should* say that "the data have failed to provide sufficient evidence to reject the null hypothesis".

T-tests

https://www.youtube.com/playlist?list=PLXCuLG6zw7mL5v44qpj4VuvcV22YyNc8x

- One sample t-test
- Paired t-test
- Two samples t-test



One sample t-test for the mean

- Null hypothesis, H0: $\mu = \mu_0$

- $t = \frac{\bar{y}-\mu_0}{SE(\bar{y})}$, where $SE(\bar{y}) = \frac{s}{\sqrt{n}}$

- When the conditions are met, this statistic follows a Student's t-model with n-1 degrees of freedom. We use that model to obtain a P-value.

Paired t-test

- Two-sample t-test?
  - NO. The races are run in pairs, so the columns are not independent.
- Instead, we should focus on the difference between each pair.
  - It's not a problem, paired data provides an opportunity!
  - We need to take advantage of the paired data structure.
- "Paired" t-test:
  - Use *pairwise* differences!
  - Ignore original two columns
  - One-sample t-test on the pairwise differences

Skating Pair	Inner Time	Outer Time	Inner – Outer
1	129.24		.
2	125.75	122.34	3.41
3	121.63	122.12	−0.49
4	122.24	123.35	−1.11
5	120.85	120.45	0.40
6	122.19	123.07	−0.88
7	122.15	122.75	−0.60
8	122.16	121.22	0.94
9	121.85	119.96	1.89
10	121.17	121.03	0.14
11	124.77	118.87	5.90
12	118.76	121.85	−3.09
13	119.74	120.13	−0.39
14	121.60	120.15	1.45
15	119.33	116.74	2.59
16	119.30	119.15	0.15
17	117.31	115.27	2.04
18	116.90	120.77	−3.87

Assumptions and conditions

- Paired data assumption
  - You should not use methods for paired data on independent data.
  - Or methods for independent data (e.g., two-sample *t*-test) should not be used on paired data.

- Independence assumption
  - The *differences* for pairs should be independent of each other.
  - Conditions
    - Randomization condition
    - 10% condition

- Normal population assumption
  - The population of *differences* should follows a Normal model.
  - Each group doesn't need to follow a Normal model.

Two-Sample t-test

- $H_0: \mu_1 - \mu_2 = \Delta_0$
  - many times $\Delta_0 = 0$

- $t = \frac{(\bar{y}_1 - \bar{y}_2) - \Delta_0}{SE(\bar{y}_1 - \bar{y}_2)}$

- $SE(\bar{y}_1 - \bar{y}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

$$\mathrm{df} = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

- When the conditions are met and the null hypothesis is true, the statistic can be closely modeled by a Student's t-model with a number of degrees of freedom (adjusted). We use that model to obtain P-value.

Confidence Interval for the Difference between two means

- $SD(\bar{y}_1 - \bar{y}_2) = \sqrt{Var(\bar{y}_1) + Var(\bar{y}_2)} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
- $SE(\bar{y}_1 - \bar{y}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
- Two-sample *t*-interval: The sampling model is Student's *t* with adjusted degrees-of-freedom value
- $(\bar{y}_1 - \bar{y}_2) \pm ME$, where $ME = t_{df}^* \times SE(\bar{y}_1 - \bar{y}_2)$

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