Week04_Report

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Question 1

I assume that the number of returns equals to 100000, sigma equals to 1, price at time t-1 equals to 100.

Simulation results:

1. Classical Brownian Motion

mean price at time t: 100.0050

standard deviation of price at time t: 1.0034

2. Arithmetic Return System

mean price at time t: 100.5012

standard deviation of price at time t: 100.3382

3. Log Return or Geometric Brownian Motion

mean price at time t: 164.9451

standard deviation of price at time t: 216.8931

Formula calculation:

r_t ~ N(0, sigma^2) ==> E[r_t] = 0, SD[r_t] = sigma = 1

1. Classical Brownian Motion

Expected value of price at time t:

E[P_t] = E[P_t-1] + E[r_t] = E[P_t-1] = 100

Standard deviation of price at time t:

SD[P_t] = SD[P_t-1 + r_t] = SD[r_t] = 1

VAR[P_t-1] SD[P_t-1] = 0 since price is a constant.

2. Arithmetic Return System

Expected value of price at time t:

E[P_t] = E[P_t-1 * (1 + r_t)] = P_t-1 * (1+E[r_t])= P_t-1 = 100

Standard deviation of price at time t:

SD[P_t] = SD[P_t-1 * (1 + r_t)] = SD[r_t] = P_t-1 * SD[1 + r_t] = P_t-1 * SD[1 + r_t] = P_t-1 * SD[1 + r_t] = 100 * 1 = 100

3. Log Return or Geometric Brownian Motion

Expected value of price at time t:

E[P_t] = E[P_t-1 * e^(r_t)] = P_t-1 * E[e^(r_t)] = P_t-1 * e^(0+sigma^2/2) = 100 * e^0.5 = 164.8721

Standard deviation of price at time t:

SD[P_t] = SD[P_t-1 * e^(r_t)] = P_t-1 * SD[e^(r_t)] = P_t-1 * sqrt((e^(sigma^2)-1) * e^(2*0+sigma^2)) = P_t-1 * sqrt((e -1) * e) = 100 * sqrt((e -1) * e) = 216.1197

Overall, the mean and standard deviation matches my expectations.

Question 2

1. VaR using normal distribution: -0.0542
2. VaR using normal distribution with exponentially weighted variance: -0.0287
3. VaR using T distribution: -0.0431
4. VaR using a fitted AR(1) model: -0.0537
5. VaR using historical distribution: -0.0395

VaR using normal distribution is the largest (-0.0542) and VaR using normal distribution with exponentially weighted variance is the smallest (-0.0287). Normal VaR provides a larger estimate of potential losses for the stock.

Question 3

I calculated portfolio values for each portfolio on each day first by stocks' price * holdings. Then, I calculated portfolio returns by p_t-p_(t-1) / p_(t-1) for each portfolio and moved mean returns from the portfolio returns. I got EWV VaR. Based on EWV VaR, I took an average on portfolio value and calculated dollar EWV VaR by EWV VaR * average portfolio value. I also obtained dollar EWV VaR by the latest EWV VaR * value.

I chose normal distribution because the calculation is easy to implement by getting mean and standard deviation of portfolio returns. It can give us a good sense of estimated VaR values. Compared with EWV VaR, normal VaR is larger for portfolios and thus having a more conservation risk estimation.

EWV VaR formula:

$$\mathrm{EWV}_{\mathrm{var}} = \sum_{t=1}^{n} \alpha(1-\alpha)^{t-1}(R_t - \bar{R})^2$$ where alpha = 1 - lambda

$$\mathrm{VaR}_{\mathrm{EWV}} = \mathrm{norm.ppf}(0.05) \times \mathrm{EWV}_{\mathrm{std\ dev}}$$

$\mathrm{EWV}_{\mathrm{std\ dev}} = \sqrt{\mathrm{EWV}_{\mathrm{var}}}$ given a 5% significant level

Normal VaR formula:

VaR_alpha = miu + Z_alpha * sigma where alpha = 0.05

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portfolio values
         Date              A              B             C         Total
0  2022-09-01  864511.243155  524138.141658  1.095803e+06  2.484453e+06
1  2022-09-02  855196.252739  519074.251166  1.081036e+06  2.455307e+06
2  2022-09-06  854111.899156  517844.336430  1.074900e+06  2.446856e+06
3  2022-09-07  868575.869512  527874.300107  1.100873e+06  2.497323e+06
4  2022-09-08  875384.565354  532238.524863  1.134243e+06  2.541866e+06

portfolio returns:
         Date         A         B         C     Total
0  2022-09-02 -0.010775 -0.009661 -0.013476 -0.011731
1  2022-09-06 -0.001268 -0.002369 -0.005677 -0.003442
2  2022-09-07  0.016935  0.019369  0.024163  0.020625
3  2022-09-08  0.007839  0.008268  0.030313  0.017837
4  2022-09-09  0.014413  0.016863  0.020010  0.017424

VaR_ewv:
         Date         A         B         C     Total
0  2022-09-02 -0.011711 -0.010085 -0.014428 -0.012567
1  2022-09-06 -0.002204 -0.002793 -0.006629 -0.004278
2  2022-09-07  0.015998  0.018945  0.023211  0.019789
3  2022-09-08  0.006902  0.007844  0.029361  0.017001
4  2022-09-09  0.013476  0.016440  0.019058  0.016588
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VaR_ewv:

A	-0.014031
B	-0.013552
C	-0.012849
Total	-0.012495

dollar VaR_ewv based on average portfolio value:

A	-$13274.62
B	-$7555.41
C	-$16233.20
Total	-$34573.47

dollar VaR_ewv based on the latest portfolio value:

A	-$15284.38
B	-$7786.28
C	-$17826.14
Total	-$38125.12

VaR_normal:

A	-0.018660
B	-0.020472
C	-0.018318
Total	-0.018098

dollar VaR_normal based on the average portfolio value:

A	-$17654.09
B	-$11413.48
C	-$23143.42
Total	-$50078.16

dollar VaR_normal based on the latest portfolio value:

A	-$20326.90
B	-$11762.24
C	-$25414.44
Total	-$55222.57