



An optimization model for a real-time flight scheduling problem

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Abstract

Although airlines plan aircraft routes and crew schedules in advance, perturbations occur everyday. As a result, flight schedules may become infeasible and would need to be updated. This Day of Operations Scheduling problem impacts the entire system of an airline as the decisions enforced are final. When perturbations are relatively small, the airline may be able to at least preserve the planned aircraft and crew itineraries. We propose a model that determines new flight schedules based on planned crew transfers, rest periods, passenger connections, and maintenance. Its dual is shown to be a network model, hence solvable in a real-time environment. In addition, it can be used in more sophisticated operational and planning systems. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

This paper considers the Day of Operations Scheduling (DAYOPS) problem which involves determining appropriate real-time changes to planned airline schedules when perturbations occur to minimize customer inconvenience and costs to the airline. Airlines must build aircraft routes and crew rotations to provide scheduled service while maximizing profits. This objective must be

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achieved in an environment that is difficult to predict. Hence, planning decisions – made in advance – may have to be altered by inevitable decisions made on the day of operations. Operational changes result from bad weather conditions, headwinds on route, technical difficulties with aircraft, crew and passenger delays, peak-hour congestion at airports, poorly calculated block and catering times, and strikes.

This complex problem is very important in practice since perturbations are costly in terms of rescheduling issues and especially in terms of loss of traveler goodwill. This is because they can lead to delaying or canceling flights, swapping aircraft among flights or using spare aircraft (if any exist), which in turn affect future deployment of aircraft and crews. Dispatchers usually adjust the planned schedules as soon as a perturbation occurs. They operate under stress and have little time to analyze cost-effective scheduling alternatives. Therefore it is important to find a good balance between the optimality of a proposed solution and the speed with which it is obtained. Historically, DAYOPS solutions have relied mainly on management information systems and graphical user interfaces, and more recently on simple heuristics to support the decision process. Rakshit et al. (1996) collected 251 cases of aircraft delays during one day. Using simple swaps between planned aircraft they saved 8495 min. For a conservative value of \$20 per minute of delay, this translates into \$169,900 savings in delay costs in the given period. Hence, the ability to find good solutions that keep changes to a minimum can significantly improve an airline's profitability and enhance its competitive position.

The literature on DAYOPS problems is rather recent. Teodorović and Guberinić (1984) stressed their importance and proposed a simplified model to minimize the total passenger waiting time. Later, Teodorović and Stojković (1990) developed a heuristic procedure that primarily minimizes the number of canceled flights and next, the total passengers waiting time. The approach of Teodorović and Stojković (1995) involves three modules that sequentially create crew rotations and aircraft itineraries while the last one verifies the maintenance feasibility, allowing for aircraft swaps if some itineraries are not feasible. Rakshit (1990), Krishnamurthy (1991), Jarrah et al. (1993) described two minimum-cost network models to determine aircraft swaps and flight cancellations, but not both simultaneously. Experimental results for the swap model were later presented by Rakshit et al. (1996). Similar, but richer in possible scheduling decisions, such as ferrying of spare aircraft, are the models proposed by Yan and Yang (1996), Yan and Ling (1997), Cao and Kanafani (1997a,b). These models are based on pure network problems or with side constraints, and on quadratic integer programs. None of these considers crew planning, maintenance requirements and passenger connections. Integration of several phases can be found in Lettovsky (1997), who proposed a formulation involving aircraft routing, crew assignment, and passenger flow in a single model that primarily focuses on crew aspects. Stojković et al. (1998) studied the *operational crew scheduling problem* where both the crew pairing and crew rostering problems are treated simultaneously using a column generation approach applied on a Set Partitioning model. Stojković (1999) integrated aircraft routing and crew pairing in a model solved by Benders decomposition. A different but closely related line of research is the time slot reassignment problem to determine the times at which aircraft from several airlines can land and take-off. Recent work can be found in Richetta and Odoni (1993, 1994), Bertsimas and Stock (1995), Luo and Yu (1997), Hoffman and Ball (1997).

The contribution of this paper is to model and solve optimally in real-time DAYOPS when minor perturbations occur. Specifically, we try to find schedules where the planned aircraft and

crew itineraries are preserved and only the arrival and departure times may be modified along with the duration of flights, ground service, maintenance scheduling and passenger connections. This has methodological value since our model represents an extension of simpler time-based models introduced in other contexts such as PERT/CPM. While previous models allow activity start times to vary, we also include activity durations as variables. We show that its dual reformulation is a network problem that can be solved in time linear in problem size. This has not only computational, but also especially practical importance since it underscores the real-time capabilities of our model. The magnitude of perturbations depends primarily on the factors that provoked them. For example, severe weather conditions usually trigger serious perturbations in planned flight schedules. Unexpected headwinds or air traffic congestion, however, generally create minor perturbations. Factors that may cause major disruptions notwithstanding, minor perturbations requiring only slight adjustments to the planned schedule occur daily. Using data from a major US carrier, Lettovsky et al. (2000) note that maintenance related disruptions are a common occurrence and some are small enough to be handled without disturbing routes or pairings. Accordingly, a dispatcher can first use our model to try to restore the schedule simply by delaying some flights without changing existing aircraft itineraries and crew rotations, before resorting to much more expensive changes.

2. A time-based formulation

Previous time-constrained models have been proposed by Sexton and Bodin (1985), Sexton and Choi (1986) and Dumas et al. (1990) to compute an optimal schedule for a single vehicle path. A parallel can also be established with activity compression in PERT/CPM project management studied by Foldes and Soumis (1993). We used ideas introduced in these simplified contexts as a basis for developing our DAYOPS model.

Our network consists of a set of origin–destination nodes, where every pair is linked by a directed arc to represent each flight leg. Let O and D be the sets of origin and destination nodes, respectively, and F be the arc subset of flight legs. Additional directed arcs are then used to represent other aircraft movements for maintenance (M) and ground service (G); crew movements for transfer between aircraft (T) and rest (R); and passenger connections (C). We also define reverse arc set (L) so as to impose limits on aircraft and crew itineraries length. Let N be the node set and A the arc set.

Fig. 1 shows a network made of two aircraft and five crew itineraries. Aircraft itineraries from node 1 to node 2 and node 3 to node 4 are composed of flight legs and ground service arcs involving airports A , B , C , and D , and, E , B , F , A , and C , respectively; reverse arcs $(2, 1)$ and $(4, 3)$ complete them. To identify the crew itineraries we find the corresponding five reverse arcs. There is a short crew itinerary between nodes 1 and 5 followed by a second starting at node 6 and ending at node 7. The crew on the second itinerary starts on the first aircraft but transfers on the other between nodes 8 and 9. The crew itinerary from node 10 to node 4 and that from node 1 to node 5 actually belong to the same crew pairing and a rest arc separates them from node 5 to node 10. The two remaining crew itineraries originate at nodes 3 and 12, respectively, and are easily identified. The network also contains a passenger connection between nodes 11 and 12 and

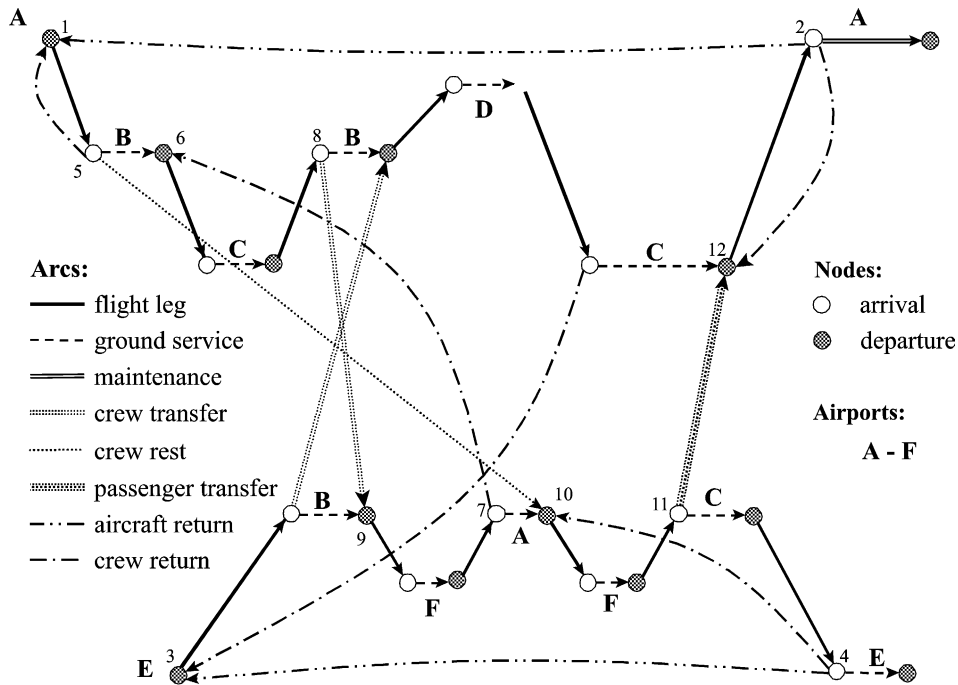


Fig. 1. A network comprising two aircraft and five crew itineraries.

a maintenance arc after node 2. Note that maintenance arcs are imposed similarly to the crew rest arcs – each of the itinerary portions needs a reverse arc so as to impose the maximum elapsed time restriction.

Generally, a bounded time variable $a_i \leq t_i \leq b_i$, $i \in O \cup D$ is associated with each arrival and departure node. Duration variables $a_{ij} \leq t_{ij} \leq b_{ij}$, $(i, j) \in F \cup M \cup G \cup C$ are set on flight legs, maintenance, ground service, and passenger connection arcs. Minimum times a_{ij} , $(i, j) \in T \cup R$ are imposed on crew transfers and rest periods, while b_{ij} , $(i, j) \in L$ set limits on the duration of aircraft and crew itineraries.

We also assume a linear objective function with appropriate non-negative node and arc coefficients, c_i and c_{ij} . Cost functions on flight, ground service, maintenance and passenger connection arcs penalize any acceleration; upper bounds represent planned schedules. Penalties are imposed since accelerated flights involve additional aircraft costs (e.g., fuel over-consumption and mechanical stress), and crew costs (e.g., psychological stress). Accelerated ground service costs arise from the need for additional employees or facilities. If maintenance is accelerated, costs are incurred earlier and more often than necessary. Finally, reductions in passenger transfer times are also penalized since they can lead to stress for both passengers and airline personnel. Cost functions on departure nodes are increasing as they account for passenger inconvenience, crew waiting, and catering costs on delayed flights. Similarly, ground service costs are imposed on arrival nodes and increase with the length of the delays due to the additional working hours, personnel or facilities required. Note that the penalties on activity duration considered here can be extended to account for any modification to an initial schedule by allowing them to also be non-

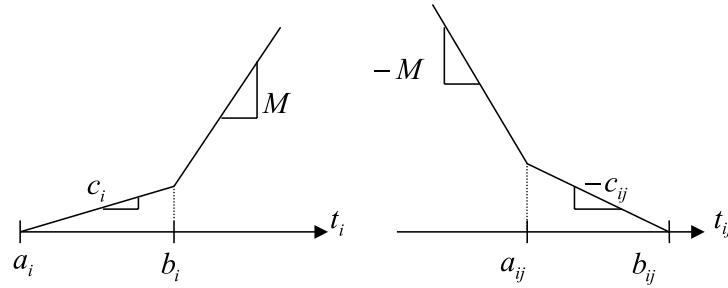


Fig. 2. Piecewise convex cost functions using big-M artificial variables.

positive (see also Fig. 2). Hence, an airline has the option to decide that certain activity changes are not penalized, and may even choose to favor others. Solely time restrictions are imposed on crew transfers and rest periods, and on the span of aircraft and crew itineraries. Then, the proposed time-based formulation is as follows:

$$\text{Minimize} \quad \sum_{i \in O \cup D} c_i t_i - \sum_{(i,j) \in F \cup G \cup M \cup C} c_{ij} t_{ij} \quad (1)$$

$$\text{subject to} \quad t_j \geq t_i + t_{ij} \quad \forall (i,j) \in F \cup G \cup M \cup C \quad [x_{ij}], \quad (2)$$

$$t_j \geq t_i + a_{ij} \quad \forall (i,j) \in T \cup R \quad [y_{ij}], \quad (3)$$

$$t_j \geq t_i - b_{ij} \quad \forall (i,j) \in L \quad [z_{ij}], \quad (4)$$

$$a_i \leq t_i \leq b_i \quad \forall i \in O \cup D \quad [\alpha_i \text{ and } \beta_i], \quad (5)$$

$$a_{ij} \leq t_{ij} \leq b_{ij} \quad \forall (i,j) \in F \cup G \cup M \cup C \quad [\alpha_{ij} \text{ and } \beta_{ij}]. \quad (6)$$

The objective function (1) minimizes total cost while constraints (2) and (3) impose precedence relations for aircraft, crew and passenger itineraries, respectively. Set (4) gives maximum duration restrictions on aircraft and crew itineraries by using the reverse arcs. Finally, restrictions (5) and (6) define time windows on all variables. The dual variables shown in brackets on the right-hand side are non-negative if we assume all relations to be less than or equal inequalities. Clarke (1961) provided an interesting mechanical analog to this model: activities on arcs are springs and events at nodes are plates that are free to move left and right within a defined region. The model is a giant shock absorber gently pushed to one side. Springs and plates move to a new equilibrium position so that the reaction, i.e., the movement to the other side of the shock absorber, is as small as possible.

The assumption of a linear objective function in (1) is realistic. Indeed, any convex cost function can be approximated by a piecewise linear function. In this case, additional arcs must be created that only increase the size of the problem. This property can be useful to ensure that the model is always feasible. As shown in Fig. 2, we only have to append a portion to the time window intervals that are governed by big-M artificial variables. In a first phase solution process, these artificial variables need to be eliminated. If not, they give essential information to modify the time window widths, to augment the length of the time horizon, or simply to use a different solution approach to the current DAYOPS problem.

3. Dual formulations

It is well known that any system of difference constraints, that is, a linear program for which the constraint matrix contains at most a single +1 and a single –1 in each row, can be transformed into a network flow problem (see Ahuja et al., 1993). The above formulation does not fulfill these requirements in constraint set (2). However, model (1)–(6) can be transformed into a network problem without arc capacities by defining $s_{ij} = t_i + t_{ij} \forall (i, j) \in F \cup G \cup M \cup C$ in (2) and replacing t_{ij} in (6). Another approach involves a smaller, but capacitated network. Let $\Gamma(i) = \{j | (i, j) \in A\}$ and $\Gamma^{-1}(i) = \{j | (j, i) \in A\}$. Given the dual variables introduced earlier, the usual dual formulation is as follows:

$$\text{Maximize} \quad \sum_{i \in O \cup D} (a_i \alpha_i - b_i \beta_i) + \sum_{(i,j) \in F \cup G \cup M \cup C} (a_{ij} \alpha_{ij} - b_{ij} \beta_{ij}) + \sum_{(i,j) \in T \cup R} a_{ij} y_{ij} - \sum_{(i,j) \in L} b_{ij} z_{ij} \quad (7)$$

$$\text{subject to} \quad \alpha_i - \beta_i + \sum_{j \in \Gamma^{-1}(i)} (x_{ji} + y_{ji} + z_{ji}) - \sum_{j \in \Gamma(i)} (x_{ij} + y_{ij} + z_{ij}) = c_i \quad \forall i \in O \cup D, \quad (8)$$

$$x_{ij} - \alpha_{ij} + \beta_{ij} = c_{ij} \quad \forall (i, j) \in F \cup G \cup M \cup C, \quad (9)$$

$$\alpha_i, \beta_i \geq 0 \quad \forall i \in O \cup D, \quad x_{ij}, y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij} \geq 0 \quad \forall (i, j) | i \in N, j \in \Gamma(i). \quad (10)$$

To obtain the network formulation (11)–(14) with arc capacities given below, define $x'_{ij} = x_{ij} - \alpha_{ij}$, consider β_{ij} as a slack variable and substitute for $\beta_{ij} = c_{ij} - x'_{ij}$ in the objective function (7) and for $x_{ij} = x'_{ij} - \alpha_{ij}$ in constraints (8). We must also impose $x'_{ij} \geq 0 \forall (i, j) \in F \cup G \cup M \cup C$, otherwise the dual function (11) is always unbounded. Indeed, if $x'_{ij} < 0$, we can set $\alpha_{ij} = -x'_{ij}$ so that (12) is satisfied and therefore, the partial sum $a_{ij} \alpha_{ij} - b_{ij} x'_{ij} = -x'_{ij}(a_{ij} + b_{ij}) > 0$ is unbounded. Hence, the resulting dual model is:

$$\begin{aligned} \text{Maximize} \quad & \sum_{i \in O \cup D} (a_i \alpha_i - b_i \beta_i) + \sum_{(i,j) \in F \cup G \cup M \cup C} (a_{ij} \alpha_{ij} - b_{ij} x'_{ij} - b_{ij} c_{ij}) \\ & + \sum_{(i,j) \in T \cup R} a_{ij} y_{ij} - \sum_{(i,j) \in L} b_{ij} z_{ij} \end{aligned} \quad (11)$$

$$\begin{aligned} \text{subject to} \quad & \alpha_i - \beta_i + \sum_{j \in \Gamma^{-1}(i)} (\alpha_{ji} + x'_{ji} + y_{ji} + z_{ji}) - \sum_{j \in \Gamma(i)} (\alpha_{ij} + x'_{ij} + y_{ij} + z_{ij}) = c_i \\ & \forall i \in O \cup D, \end{aligned} \quad (12)$$

$$0 \leq x'_{ij} \leq c_{ij} \quad \forall (i, j) \in F \cup G \cup M \cup C, \quad (13)$$

$$\alpha_i, \beta_i \geq 0 \quad \forall i \in O \cup D, \quad x'_{ij}, y_{ij}, z_{ij}, \alpha_{ij} \geq 0 \quad \forall (i, j) | i \in N, j \in \Gamma(i). \quad (14)$$

Comparing the two dual formulations, without and with arc capacities, one can observe that the second comprises $|F \cup G \cup M \cup C|$ fewer nodes (constraints) and arcs (variables). All dual variables x'_{ij} , y_{ij} , z_{ij} , and α_{ij} are placed on the arcs with the corresponding ij indices. The one-index variables, α_i and β_i can also be considered as arc variables by connecting all nodes to an artificial zero-node, hence defining α_{0i} and β_{i0} variables. The time-based primal formulation (1)–(6) is natural and more understandable than the dual formulation (11)–(14). However, the latter is much more efficient to solve. Given the complementary slackness conditions, the solution to the dual indicates which of the time constraints is binding on the primal formulation.

4. Computational results

To validate our approach, we generated 10 problems having linear cost functions on arcs and nodes. The data were increased linearly from one problem size to the next as illustrated in Table 1. The schedule variables on the nodes and the duration variables on the arcs were subject to time window constraints. Our test problems are realistic since the larger examples exceed the dimension of a one-day flight plan at the largest airline. These large size problems could also represent a flight plan extending over a few days at a mid-sized airline, or a one-day flight plan at a mid-sized airline with linear piecewise convex cost functions on arcs and nodes.

The runtime analysis was carried out using the CPLEX Linear Optimizer 3.0 on a HP9000/715 workstation (115.1 specint92, 138.7 specfp92). The simplex algorithm was used to solve the primal formulation. A network simplex algorithm was used to solve the dual formulation. We compared the execution times for the primal and dual formulations. Our results, presented in Table 2, demonstrate that both formulations are efficient. The CPU times for the primal ranged from 0.02 to 29.60 s, while the dual times increased from 0.02 to 6.71 s. Overall, execution times for the primal increased polynomially in problem size, while those for the dual increased linearly.

5. Model applicability

An important use of the model arises during the planning process. Large airlines usually have four independent phases of planning: flight scheduling, fleet assignment, aircraft routing, and crew scheduling. This model could be used as a fifth phase to reoptimize the flight schedule when the previous four phases have been completed and the aircraft routes and crew schedules are available. Similarly, it could also be useful as a module in a more general model with two levels of optimization. At the higher level, some flights could be canceled, fleet assignment and aircraft itineraries changed, crew constraints relaxed, and crew rotations modified. Then, at the lower level, the model would find a new best feasible flight schedule for the given scenario. In the same

Table 1
Dimensions of test problems

	Aircraft itineraries	Crew rotations	Nodes		Legs			Connections	
			Arrival	Departure	Flight	Ground service	Mainte- nance	Crews	Passengers
<i>A</i>	2	5	12	14	12	11	1	3	1
<i>B</i>	100	250	600	700	600	550	50	150	50
<i>C</i>	200	500	1200	1400	1200	1100	100	300	100
<i>D</i>	300	750	1800	2100	1800	1650	150	450	150
<i>E</i>	400	1000	2400	2800	2400	2200	200	600	200
<i>F</i>	500	1250	3000	3500	3000	2750	250	750	250
<i>G</i>	600	1500	3600	4200	3600	3300	300	900	300
<i>H</i>	700	1750	4200	4900	4200	3850	350	1050	350
<i>I</i>	800	2000	4800	5600	4800	4400	400	1200	400
<i>J</i>	900	2250	5400	6300	5400	4950	450	1350	450

Table 2

Primal and dual problem sizes and CPU times (in seconds)

	Primal				Dual			
	Variables	Constraints	Bounds	CPU times	Variables	Constraints	Bounds	CPU times
<i>A</i>	51	35	51	0.02	112	26	25	0.02
<i>B</i>	2550	1750	2550	0.60	5600	1300	1250	0.58
<i>C</i>	5100	3500	5100	1.60	11,200	2600	2500	1.33
<i>D</i>	7650	5250	7650	3.51	16,800	3900	3750	2.14
<i>E</i>	10,200	7000	10,200	5.50	22,400	5200	5000	2.84
<i>F</i>	12,750	8750	12,750	9.04	28,000	6500	6250	3.64
<i>G</i>	15,300	10,500	15,300	12.35	33,600	7800	7500	4.49
<i>H</i>	17,850	12,250	17,850	16.70	39,200	9100	8750	5.18
<i>I</i>	20,400	14,000	20,400	21.74	44,800	10,400	10,000	5.79
<i>J</i>	22,950	15,750	22,950	29.60	50,400	11,700	11,250	6.71

manner, the model could generate contingency plans for airports that are known to experience certain types of recurrent perturbations such as fog. Hence, at the planning stage, the model can provide maneuvering capabilities at critical airports by producing scheduling alternatives consisting of delaying flights within given time windows. These uses of the model are further supported by the recent work of Jarrah et al. (2000) who indicate that once a fleet assignment has been generated, planners dedicate considerable time to examining various scenarios prior to “freezing” the assignments.

The model could also be beneficial to smaller regional airlines, where often the planning process consists of only one phase. Aircraft itineraries and crew rotations are traditionally predefined by the geography of market the airlines serve and the small number of aircraft and crews. The model could then establish the best flight schedule for the existing aircraft routes and crew rotations by imposing one or more passenger connection constraints.

While the above off-line applications are apparent, its stand-alone on-line use may not be immediate. This is because the model has a realistic objective function that requires large amounts of data. These data are available during the planning process but may not be presently available during the day of operations of many airlines. Given the current trend in enterprise resource planning where one core database feeds all business applications, it is likely that airlines will shortly move in this direction. Hence, information regarding aircraft itineraries and crew rotations could be used in the operational process.

Given accessibility of the necessary data, the model should be useful before considering more complex and expensive changes such as new fleet assignments, new aircraft routes or flight cancelations. It could also be utilized as an optimization-based heuristic where only a subset of flights is considered and delay costs are approximated based on historical data. In stand-alone mode, if the use of the model does not lead to satisfactory results, one can cancel all flights following the violated constraint and make some changes in aircraft itineraries and crew rotations manually. Embedded in an operational system, it could be helpful prior to using other heuristics. For example, in the framework just suggested by Lettovsky et al. (2000), our model could determine whether some crews might not need reassignment. Similarly, it is likely that using a heuristic, such as Rakshit et al. (1996) Delay and Swap Adviser at a hub, could result in infeasible

crew rotations and violated maintenance constraints. Our model would then verify the proposed solution and find optimal departure times for the complete or part of the airline network, taking into consideration crew, passenger and maintenance constraints. This would certainly result in additional savings in delay costs.

6. Conclusions

This paper has presented a model that attempts to reinstate planned airline service following an unexpected perturbation in airline operations without changes to aircraft itineraries and crew rotations. The model extends prior time-based models by considering not only activity start time variables, but also activity duration variables. The model reoptimizes departure times to take into account the sequences of activities that have to be carried out within all aircraft routes and crew rotations. Fixed rotations allow complicated resource constraints to become simple backward arcs in the proposed network. The new schedule is obtained by reducing flying, ground service, maintenance, or passenger transfer time. The costs of time reductions, elements of the crew costs and passenger inconvenience are included in the objective function. This makes the model simple and realistic. Given the linear behavior of its dual with respect to problem size, it can be used to solve problems with linear, piecewise convex or smooth convex objective functions in real time for the largest airlines. It can also be used to cover several consecutive days.

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