Computing Diverse Optimal Stable Models

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Introduction

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- Impose preference relations among solutions to filter out optimal ones
- Certain preference relations and applications exhibit vast number of optimal solutions
  e.g. Pareto frontiers for multi-objective optimization problems in system synthesis, timetabling, configuration, planning

$\Rightarrow$ Further filtering necessary, e.g. diverse optimal stable models

Course timetabling (CTT)

- Courses a, b with two lessons each
- One room
- Two days with four periods
- Assign courses to days and periods

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Period	Day 1	Day 2
1	a	
2		a
3	b	b
4		

Preferences

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Combine both preferences in a Pareto preference relation.

Pareto preference

- Aggregates several preference relations
- Multi-objective optimization
- Generally no single optimal solution
- Set of Pareto optimal solutions (Pareto frontier)
  - y is dominated by x if x is at least as good as y in all objectives and better in one
  - x is Pareto optimal $\iff$ there is no y that dominates x

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  - x is Pareto optimal $\iff$ there is no y that dominates x
- e.g. "better" means $<$ for objectives in $\mathbb{Z}$:
  - $(2, 3, 1)$ *dominates* $(3, 3, 1)$
  - $(2, 3, 1)$ *is incomparable to* $(2, 1, 3)$

Preferences statements in *asprin*

```
#preference(compactness,less(cardinality)){
    assigned(C1,D,P), not assigned(C2,D,P+1)
}.

#preference(spread,less(cardinality)){
    assigned(C,D,P1), assigned(C,D,P2), P1>P2
}.

#preference(combine,pareto){
    name(compact); name(spread)
}.
```

Dominated solution

Period	Day 1	Day 2
1	a	
2		a
3	b	b
4		

Compactness violations: 3
Spread violations: 0

Optimal solution

Period	Day 1	Day 2
1		
2	a	a
3	b	b
4		

Compactness violations: 2
Spread violations: 0

Optimal solution

Period	Day 1	Day 2
1		
2	a	b
3	b	a
4		

Compactness violations: 2
Spread violations: 0

Optimal solution

Period	Day 1	Day 2
1	b	
2	a	
3	b	
4	a	

Compactness violations: 1
Spread violations: 2

Diverse optimal solution

- Many optimal solutions
- Select subset to present to user

Period	Day 1	Day 2
1		
2	a	a
3	b	b
4		

Period	Day 1	Day 2
1		
2	a	b
3	b	a
4		

Period	Day 1	Day 2
1	b	
2	a	
3	b	
4	a	

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Period	Day 1	Day 2
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Period	Day 1	Day 2
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3	b	a
4		

Period	Day 1	Day 2
1	b	
2	a	
3	b	
4	a	

Diverse optimal solution

- Many optimal solutions
- Select *diverse* subset to present to user

Period	Day 1	Day 2
1		
2	a	a
3	b	b
4		

Period	Day 1	Day 2
1		a
2		b
3		a
4		b

Period	Day 1	Day 2
1	a	
2	b	
3	a	
4	b	

Overview

- Main related work:
    - T. Eiter, E. Erdem, H. Erdogan, and M. Fink. Finding similar/diverse solutions in answer set programming. 2013.
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  ⇒ Provide comprehensive framework for computing diverse (or similar) optimal solutions

Outline

ASP Solving Schemes

1. Maxmin optimization

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Schemes are employed to implement diversification techniques, but are readily available for independent usage.

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1. Introduction and Motivation
2. ASP Solving Schemes
3. Diversification Techniques
4. Experiments

Diversification Techniques

1. Enumeration
   - Enumerate all optimal solutions and calculate most diverse subset
   - Express calculation of most diverse subset as logic program with preferences

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2. Replication
   - Replicate problem to produce *n* optimal solutions and optimize diversity
   - Uses *Guess and check automation* and *Maxmin optimization*
3. Approximation
   - Iterate optimal solutions distant to a previous set of optimal solutions
   - Uses *maxmin optimization*, *Querying programs with preferences* and *Preferences over optimal models*

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   - Uses *Guess and check automation* and *Maxmin optimization*
3. **Approximation**
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Approximation

Input: Logic program with preferences P, Set size $n \in \mathbb{N}$
Output: Set $\mathcal{X}$ approximating n most diverse optimal solutions

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$\mathcal{X} = \emptyset$

While $|\mathcal{X}| < n$ **and** $P \cup \mathcal{X}$ is satisfiable

$\mathcal{X} = \mathcal{X} \cup \mathit{solve}(P, \mathcal{X})$

return $\mathcal{X}$

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- Variants A-1 to A-3 implement different versions of $\mathit{solve}(P, \mathcal{X})$
- With increasing variant number: less exact, more performance

A-1

- *solve*$(P, \mathcal{X})$ returns optimal model of P most dissimilar to $\mathcal{X}$

A-1

- *solve*$(P, \mathcal{X})$ returns optimal model of P most dissimilar to $\mathcal{X}$
- Ensures next optimal solution is most distant to whole previous set $\mathcal{X}$ by using *Maxmin optimization* and *Preferences over optimal models*

A-1: Example

Enumerate three most diverse optimal solutions for CTT.

$\mathcal{X} =$

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1		
2	a	a
3	b	b
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3		a
4		b

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Enumerate three most diverse optimal solutions for CTT.

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Period	Day 1	Day 2
1		
2	a	a
3	b	b
4		

Period	Day 1	Day 2
1		a
2		b
3		a
4		b

Period	Day 1	Day 2
1		a
2	a	a,b
3	b	a,b
4		a

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Enumerate three most diverse optimal solutions for CTT.

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Period	Day 1	Day 2
1		
2	a	a
3	b	b
4		

Period	Day 1	Day 2
1		a
2		b
3		a
4		b

Period	Day 1	Day 2
1	a	
2	b	
3	a	
4	b	

A-2

- *solve*$(P, \mathcal{X})$ first computes partial interpretation I distant to $\mathcal{X}$

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A-2

- $solve(P, \mathcal{X})$ first computes partial interpretation I distant to $\mathcal{X}$
- Then returns optimal model of P closest to I
- Ensures next optimal solution is as close as possible to I by using *Maxmin optimization* and *Preferences over optimal models*
- Loses accuracy but gains performance since there is no direct comparison to earlier optimal solutions

A-2: Distant partial interpretation I

Given set of previous optimal models $\mathcal{X}$, select I

1. randomly
2. heuristically chosen
3. most diverse wrt. $\mathcal{X}$
4. complementary to last optimal solution

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3	b	b
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Period	Day 1	Day 2
1	a,b	a,b
2	b	b
3	a	a
4	a,b	a,b

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Period	Day 1	Day 2
1		a
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3	a	
4	b	

Period	Day 1	Day 2
1	a,b	b
2	a,b	a
3	b	a,b
4	a	a,b

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Enumerate three most diverse optimal solutions for CTT.

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Period	Day 1	Day 2
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1	a	
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Period	Day 1	Day 2
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3	a	
4	b	

Period	Day 1	Day 2
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3	b	
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A-3

- $solve(P, \mathcal{X})$ first computes partial interpretation I distant to $\mathcal{X}$

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- Approximation of A-2
- Purely heuristic, no guaranteed diversity; no optimization, less complex calculation

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Results

- Replication and Enumeration not efficient
- Approximation techniques display tradeoff between diversification and performance
- 816 instances enumerating four Pareto optimal diverse models with 16 Approximation configurations
- timeout of 600s

	A-3	A-1	A-3 most diverse diverse wrt. $\mathcal{X}$
T/TO	**165s/70**	482s/672	317s/351
Rank	16	**1**	4