# ECN-614: Assignment #1

An auto-regressive process is defined by the difference equation

$$x[n] = a_1\, x[n-1] \;+\; a_2\, x[n-2] \;+\; w[n]$$

where $w[n]$ is a zero-mean white-noise process with variance $\sigma_w^2$.

(1) Consider the linear prediction of the above AR process $x[n]$, with $a_1 = 1.2$, $a_2 = -0.8$ and $\sigma_w^2 = 0.3$, using a Wiener filter. Determine

(a) the autocorrelation function values $r_{xx}(0)$, $r_{xx}(1)$ and $r_{xx}(2)$,

(b) the optimum tap-weight vector $\mathbf{w}_{\text{opt}}$ of the forward predictor,

(c) the minimum mean-square prediction error $J_{\min}$ produced by the predictor,

(d) the mean-square prediction error $J(\mathbf{w})$ when the filter tap-weight vector $\mathbf{w} = \mathbf{0}$. Hence, plot the contour of the error-performance surface corresponding to the mean-square prediction error $J(\mathbf{0})$.

(2) Given the input $x[n]$, an LMS filter is used to recursively estimate the unknown AR parameters $a_1$ and $a_2$. Write a MATLAB program for the same.

Generate a sequence of 10,000 samples of $x[n]$, $1 \le n \le 10{,}000$, taking $a_1 = 1.2$, $a_2 = -0.8$ and $\sigma_w^2 = 0.3$. Assume $x[n] = 0$ for all $n < 1$. Using this $x[n]$ as input and starting with zero initial parameter values, *i.e.*, $a_1[1] = 0$ and $a_2[1] = 0$, run the above program with step-size parameter $\mu = 0.05$. Plot the estimated parameter values $a_1[n]$ and $a_2[n]$ as a function of the iteration number $n$.

Repeat the above for 10 trials using different noise sequences. Hence, plot the learning curve for the average MSE by averaging over these 10 trials. Comment on your results.

**– OR –**

Given the input $x[n]$, an LMS filter is used to recursively estimate the unknown AR parameters $a_1$ and $a_2$. Write a pseudo-code for the same.

Determine the first 5 samples of $x[n]$, $1 \le n \le 5$, taking $a_1 = 1.2$, $a_2 = -0.8$ and the noise sequence $w[n] = \{0.30,\ 1.00,\ -1.24,\ 0.47,\ 0.18,\ \dots\}$ with variance $\sigma_w^2 = 0.3$. Assume $x[n] = 0$ for all $n < 1$. Using this $x[n]$ as input and starting with zero initial parameter values, *i.e.*, $a_1[1] = 0$ and $a_2[1] = 0$, calculate the steepest-descent and the LMS estimates of the parameter values in the first 5 iterations, *i.e.*, $a_1[n]$ and $a_2[n]$ for $2 \le n \le 6$. Take step-size parameter $\mu = 0.05$.

Sketch the loci of $a_2[n]$ versus $a_1[n]$ for $1 \le n \le 6$, as obtained in both the cases above. Hence, comment on the performance of the LMS algorithm compared to that of the steepest-descent algorithm.

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