ASSIGNMENT - 2

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ECE - IV YEAR
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① Given state representation for the truck

$$X_K = \begin{bmatrix} P(K) \\ V(K) \end{bmatrix}$$

where $P(K)$ represents the position of truck at time $t=K$

$V(K)$ represents the velocity of the truck at time $t=K$

Since in the question; we are only able to track the position of the truck (through the GPS); velocity information is not made available to the end user.

Given $a(K)$: random acceleration at time K assumed to be constant for the next 1 second interval with a variance of 1.0

$e(K)$: GPS measurement error which has a variance of 0.25

Since acceleration is assumed to be constant for a one second interval; we can use the following kinematics equations ~~for~~ since they hold for a one second interval [constant acceleration]

$$x(t) = x(t_0) + u(t_0) + \frac{1}{2}at^2$$

$$v(t) = u(t_0) + at$$

Process Equation :-

$$X_K = FX_{K-1} + Ga_K \qquad F = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} \Delta t^2/2 \\ \Delta t \end{bmatrix}$$

represent Ga_K by $v_1(n)$

estimate the covariance of Ga_K by $E[(Ga_K)^*(Ga_K)^H]$

$$E[(Ga_K)^*(Ga_K)^H] \Rightarrow E[G^* G^T a_K^* a_K] \Rightarrow G^* G^T \sigma_a^2$$

$\Rightarrow$ Covariance of $v_1(n)$ is given by $G^TG\sigma_a^2$ where σ_a^2 is the variance of the random acceleration

$\Rightarrow v_1(n) \sim N(0, G^TG\sigma_a^2)$ as defined by the question

$$\cancel{G^TG\sigma_a^2 = [\Delta t^2/2 \quad \Delta t]^T \begin{bmatrix} \Delta t^2/2 \\ \Delta t \end{bmatrix}^T \sigma_a^2 = \begin{bmatrix} \Delta t^4/4 & \Delta t^2 \end{bmatrix}}$$

$$GG^T\sigma_a^2 = \begin{bmatrix}\Delta t^2/2 \\ \Delta t\end{bmatrix}\begin{bmatrix}\Delta t^2/2 & \Delta t\end{bmatrix}\sigma_a^2 = \begin{bmatrix}\Delta t^4/4 & \Delta t^3/2 \\ \Delta t^3/2 & \Delta t^2\end{bmatrix}\sigma_a^2$$

For the given dynamics of this question

$\Delta t = 1$; $\sigma_a^2 = 1$

$$\Rightarrow GG^T\sigma_a^2 = \begin{bmatrix}1/4 & 1/2 \\ 1/2 & 1\end{bmatrix} \times 1 \qquad F = \begin{bmatrix}1 & 1 \\ 0 & 1\end{bmatrix}$$

Final process equation is given by :

$$\cancel{X_K = \begin{bmatrix}1 & 1 \\ 0 & 1\end{bmatrix} X_{K-1}} \qquad \boxed{\begin{bmatrix}P(K) \\ V(K)\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix}P(K-1) \\ V(K-1)\end{bmatrix} + V_1(K)}$$

where $V_1(K) \sim N\left[0, \begin{bmatrix}1/4 & 1/2 \\ 1/2 & 1\end{bmatrix}\right]$

<u>Measurement Equation :</u>

$\cancel{X_K}$ $Y_K = C X_K + V_2(K)$

since we track only the position :- $C = [1 \quad 0]$

$V_2(K) \sim N(0, \sigma_g^2)$ where σ_g^2 is the variance of the GPS measurement error.

So the final measurement equation can be represented by

$$P'(K) = [1 \quad 0]\begin{bmatrix}P(K) \\ V(K)\end{bmatrix} + V_2(K)$$

$\boxed{P'(K) = P(K) + V_2(K)}$ where $V_2(K) \sim N(0, 0.25)$ and $P'(K)$ represents the position of the truck as measured by the GPS