0

Begin inference!

(0) , $\overset{s}{\Rightarrow} p \vee \neg p$

(1) , $\overset{s}{\Rightarrow} \neg p, p$ $(by \Rightarrow \vee rule)$

(2) , $p, \overset{s}{\Rightarrow} p$ $(by \Rightarrow \neg rule)$

Result: T

1

Begin inference!

(0) , $\overset{s}{\Rightarrow} (P \rightarrow Q) \rightarrow ((P \wedge R) \rightarrow Q)$

(1) , $P \rightarrow Q, \overset{s}{\Rightarrow} (P \wedge R) \rightarrow Q$ $(by \Rightarrow \rightarrow rule)$

(2) , $P \rightarrow Q, P \wedge R, \overset{s}{\Rightarrow} Q$ $(by \Rightarrow \rightarrow rule)$

(3) , $P \rightarrow Q, R, P, \overset{s}{\Rightarrow} Q$ $(by \wedge \Rightarrow rule)$

(3a.1) , $R, P, Q, \overset{s}{\Rightarrow} Q$ $(by \rightarrow \Rightarrow rule)$

(3b.1) , $R, P, \overset{s}{\Rightarrow} Q, P$ $(by \rightarrow \Rightarrow rule)$

Result: T

2

Begin inference!

(0) , $\overset{s}{\Rightarrow} (P \rightarrow Q) \rightarrow (P \rightarrow (Q \vee R))$

(1) , $P \rightarrow Q, \overset{s}{\Rightarrow} P \rightarrow (Q \vee R)$ $(by \Rightarrow \rightarrow rule)$

(2) , $P \rightarrow Q, P, \overset{s}{\Rightarrow} Q \vee R$ $(by \Rightarrow \rightarrow rule)$

(3) , $P \rightarrow Q, P, \overset{s}{\Rightarrow} R, Q$ $(by \Rightarrow \vee rule)$

(3a.1) , $P, Q, \overset{s}{\Rightarrow} R, Q$ $(by \rightarrow \Rightarrow rule)$

(3b.1) , $P, \overset{s}{\Rightarrow} R, Q, P$ $(by \rightarrow \Rightarrow rule)$

Result: T

3

Begin inference!

(0) , $\overset{s}{\Rightarrow} P \rightarrow (\neg P \vee Q)$

(1) , $P, \overset{s}{\Rightarrow} \neg P \vee Q$ $(by \Rightarrow \rightarrow rule)$

(2) , $P, \overset{s}{\Rightarrow} Q, \neg P$ $(by \Rightarrow \vee rule)$

(3) , $P, P, \overset{s}{\Rightarrow} Q$ $(by \Rightarrow \neg rule)$

Result: F

4

Begin inference!

(0) , $\overset{s}{\Rightarrow} ((P \vee Q) \wedge (P \rightarrow Q)) \rightarrow (Q \rightarrow P)$

(1) , $(P \vee Q) \wedge (P \rightarrow Q), \overset{s}{\Rightarrow} Q \rightarrow P$ $(by \Rightarrow \rightarrow rule)$

(2) , $P \rightarrow Q, P \vee Q, \overset{s}{\Rightarrow} Q \rightarrow P$ $(by \wedge \Rightarrow rule)$

(3) , $P \rightarrow Q, P \vee Q, Q, \overset{s}{\Rightarrow} P$ $(by \Rightarrow \rightarrow rule)$

(3a.1) , $P \vee Q, Q, Q, \overset{s}{\Rightarrow} P$ $(by \rightarrow \Rightarrow rule)$

(3a.1a.1) , $Q, Q, P, \overset{s}{\Rightarrow} P$ $(by \vee \Rightarrow rule)$

(3a.1b.1) , $Q, Q, Q, \overset{s}{\Rightarrow} P$ $(by \vee \Rightarrow rule)$

Result: F

5

Begin inference!

(0) , $\overset{s}{\Rightarrow} P \rightarrow ((\neg Q \wedge P) \rightarrow R)$

(1) , $P, \overset{s}{\Rightarrow} (\neg Q \wedge P) \rightarrow R$ $(by \Rightarrow \rightarrow rule)$

(2) , $P, \neg Q \wedge P, \overset{s}{\Rightarrow} R$ $(by \Rightarrow \rightarrow rule)$

(3) , $P, P, \neg Q, \overset{s}{\Rightarrow} R$ $(by \wedge \Rightarrow rule)$

(4) , $P, P, \overset{s}{\Rightarrow} R, Q$ (by¬⇒ rule)

Result: F

6

 Begin inference!

(0) , $\overset{s}{\Rightarrow} ((P \to Q) \wedge (Q \to P)) \to (P \vee Q)$

(1) , $(P \to Q) \wedge (Q \to P), \overset{s}{\Rightarrow} P \vee Q$ (by⇒→ rule)

(2) , $Q \to P, P \to Q, \overset{s}{\Rightarrow} P \vee Q$ (by∧⇒ rule)

(3) , $Q \to P, P \to Q, \overset{s}{\Rightarrow} Q, P$ (by⇒∨rule)

(3a.1) , $P \to Q, P, \overset{s}{\Rightarrow} Q, P$ (by→⇒ rule)

(3a.1a.1) , $P, Q, \overset{s}{\Rightarrow} Q, P$ (by→⇒ rule)

(3a.1b.1) , $P, \overset{s}{\Rightarrow} Q, P, P$ (by→⇒ rule)

(3b.1) , $P \to Q, \overset{s}{\Rightarrow} Q, P, Q$ (by→⇒ rule)

(3b.1a.1) , $Q, \overset{s}{\Rightarrow} Q, P, Q$ (by→⇒ rule)

(3b.1b.1) , $\overset{s}{\Rightarrow} Q, P, Q, P$ (by→⇒ rule)

Result: F

7

 Begin inference!

(0) , $\overset{s}{\Rightarrow} ((P \vee Q) \to (P \vee \neg Q)) \to (\neg P \vee Q)$

(1) , $(P \vee Q) \to (P \vee \neg Q), \overset{s}{\Rightarrow} \neg P \vee Q$ (by⇒→ rule)

(2) , $(P \vee Q) \to (P \vee \neg Q), \overset{s}{\Rightarrow} Q, \neg P$ (by⇒∨rule)

(3) , $(P \vee Q) \to (P \vee \neg Q), P, \overset{s}{\Rightarrow} Q$ (by⇒¬rule)

(3a.1) , $P, P \vee \neg Q, \overset{s}{\Rightarrow} Q$ (by→⇒ rule)

(3a.1a.1) , $P, P, \overset{s}{\Rightarrow} Q$ (by∨⇒ rule)

Result: F

8

 Begin inference!

(0) , $\overset{s}{\Rightarrow} ((P \wedge Q) \vee (P \to Q)) \to (P \to Q)$

(1) , $(P \wedge Q) \vee (P \to Q), \overset{s}{\Rightarrow} P \to Q$ (by⇒→ rule)

(2) , $(P \wedge Q) \vee (P \to Q), P, \overset{s}{\Rightarrow} Q$ (by⇒→ rule)

(2a.1) , $P, P \wedge Q, \overset{s}{\Rightarrow} Q$ (by∨⇒ rule)

(2a.2) , $P, Q, P, \overset{s}{\Rightarrow} Q$ (by∧⇒ rule)

(2b.1) , $P, P \to Q, \overset{s}{\Rightarrow} Q$ (by∨⇒ rule)

(2b.1a.1) , $P, Q, \overset{s}{\Rightarrow} Q$ (by→⇒ rule)

(2b.1b.1) , $P, \overset{s}{\Rightarrow} Q, P$ (by→⇒ rule)

Result: T

9

 Begin inference!

(0) , $\overset{s}{\Rightarrow} ((P \wedge Q) \to R) \to ((P \to R) \wedge (Q \to R))$

(1) , $(P \wedge Q) \to R, \overset{s}{\Rightarrow} (P \to R) \wedge (Q \to R)$ (by⇒→ rule)

(1a.1) , $R, \overset{s}{\Rightarrow} (P \to R) \wedge (Q \to R)$ (by→⇒ rule)

(1a.1a.1) , $R, \overset{s}{\Rightarrow} P \to R$ (by⇒∧rule)

(1a.1a.2) , $R, P, \overset{s}{\Rightarrow} R$ (by⇒→ rule)

(1a.1b.1) , $R, \overset{s}{\Rightarrow} Q \to R$ (by⇒∧rule)

(1a.1b.2) , $R, Q, \overset{s}{\Rightarrow} R$ (by⇒→ rule)

(1b.1) , $\overset{s}{\Rightarrow} (P \to R) \wedge (Q \to R), P \wedge Q$ (by→⇒ rule)

(1b.1a.1) , $\overset{s}{\Rightarrow} P \wedge Q, P \to R$ (by⇒∧rule)

(1b.1a.2) , $P, \overset{s}{\Rightarrow} P \wedge Q, R$ $(by \Rightarrow \rightarrow rule)$

(1b.1a.2a.1) , $P, \overset{s}{\Rightarrow} R, P$ $(by \Rightarrow \wedge rule)$

(1b.1a.2b.1) , $P, \overset{s}{\Rightarrow} R, Q$ $(by \Rightarrow \wedge rule)$

Result: F

10

 Begin inference!

(0) , $\overset{s}{\Rightarrow} (((P \wedge Q) \rightarrow R) \wedge ((P \vee Q) \rightarrow \neg R)) \rightarrow ((P \wedge Q) \wedge R)$

(1) , $((P \wedge Q) \rightarrow R) \wedge ((P \vee Q) \rightarrow \neg R), \overset{s}{\Rightarrow} (P \wedge Q) \wedge R$ $(by \Rightarrow \rightarrow rule)$

(2) , $(P \vee Q) \rightarrow \neg R, (P \wedge Q) \rightarrow R, \overset{s}{\Rightarrow} (P \wedge Q) \wedge R$ $(by \wedge \Rightarrow rule)$

(2a.1) , $(P \wedge Q) \rightarrow R, \neg R, \overset{s}{\Rightarrow} (P \wedge Q) \wedge R$ $(by \rightarrow \Rightarrow rule)$

(2a.2) , $(P \wedge Q) \rightarrow R, \overset{s}{\Rightarrow} (P \wedge Q) \wedge R, R$ $(by \neg \Rightarrow rule)$

(2a.2a.1) , $R, \overset{s}{\Rightarrow} (P \wedge Q) \wedge R, R$ $(by \rightarrow \Rightarrow rule)$

(2a.2a.1a.1) , $R, \overset{s}{\Rightarrow} R, P \wedge Q$ $(by \Rightarrow \wedge rule)$

(2a.2a.1a.1a.1) , $R, \overset{s}{\Rightarrow} R, P$ $(by \Rightarrow \wedge rule)$

(2a.2a.1a.1b.1) , $R, \overset{s}{\Rightarrow} R, Q$ $(by \Rightarrow \wedge rule)$

(2a.2a.1b.1) , $R, \overset{s}{\Rightarrow} R, R$ $(by \Rightarrow \wedge rule)$

(2a.2b.1) , $\overset{s}{\Rightarrow} (P \wedge Q) \wedge R, R, P \wedge Q$ $(by \rightarrow \Rightarrow rule)$

(2a.2b.1a.1) , $\overset{s}{\Rightarrow} R, P \wedge Q, P \wedge Q$ $(by \Rightarrow \wedge rule)$

(2a.2b.1a.1a.1) , $\overset{s}{\Rightarrow} R, P \wedge Q, P$ $(by \Rightarrow \wedge rule)$

(2a.2b.1a.1a.1a.1) , $\overset{s}{\Rightarrow} R, P, P$ $(by \Rightarrow \wedge rule)$

Result: F

11

 Begin inference!

(0) , $\overset{s}{\Rightarrow} (P \rightarrow Q) \rightarrow ((P \rightarrow R) \rightarrow (Q \rightarrow R))$

(1) , $P \rightarrow Q, \overset{s}{\Rightarrow} (P \rightarrow R) \rightarrow (Q \rightarrow R)$ $(by \Rightarrow \rightarrow rule)$

(2) , $P \rightarrow Q, P \rightarrow R, \overset{s}{\Rightarrow} Q \rightarrow R$ $(by \Rightarrow \rightarrow rule)$

(3) , $P \rightarrow Q, P \rightarrow R, Q, \overset{s}{\Rightarrow} R$ $(by \Rightarrow \rightarrow rule)$

(3a.1) , $P \rightarrow R, Q, Q, \overset{s}{\Rightarrow} R$ $(by \rightarrow \Rightarrow rule)$

(3a.1a.1) , $Q, Q, R, \overset{s}{\Rightarrow} R$ $(by \rightarrow \Rightarrow rule)$

(3a.1b.1) , $Q, Q, \overset{s}{\Rightarrow} R, P$ $(by \rightarrow \Rightarrow rule)$

Result: F

12

 Begin inference!

(0) , $\overset{s}{\Rightarrow} ((P \vee Q) \vee R) \rightarrow (\neg P \rightarrow ((Q \vee R) \wedge \neg P))$

(1) , $(P \vee Q) \vee R, \overset{s}{\Rightarrow} \neg P \rightarrow ((Q \vee R) \wedge \neg P)$ $(by \Rightarrow \rightarrow rule)$

(2) , $(P \vee Q) \vee R, \neg P, \overset{s}{\Rightarrow} (Q \vee R) \wedge \neg P$ $(by \Rightarrow \rightarrow rule)$

(3) , $(P \vee Q) \vee R, \overset{s}{\Rightarrow} (Q \vee R) \wedge \neg P, P$ $(by \neg \Rightarrow rule)$

(3a.1) , $P \vee Q, \overset{s}{\Rightarrow} (Q \vee R) \wedge \neg P, P$ $(by \vee \Rightarrow rule)$

(3a.1a.1) , $P, \overset{s}{\Rightarrow} (Q \vee R) \wedge \neg P, P$ $(by \vee \Rightarrow rule)$

(3a.1a.1a.1) , $P, \overset{s}{\Rightarrow} P, Q \vee R$ $(by \Rightarrow \wedge rule)$

(3a.1a.1a.2) , $P, \overset{s}{\Rightarrow} P, R, Q$ $(by \Rightarrow \vee rule)$

(3a.1a.1b.1) , $P, \overset{s}{\Rightarrow} P, \neg P$ $(by \Rightarrow \wedge rule)$

(3a.1a.1b.2) , $P, P, \overset{s}{\Rightarrow} P$ $(by \Rightarrow \neg rule)$

(3a.1b.1) , $Q, \overset{s}{\Rightarrow} (Q \vee R) \wedge \neg P, P$ $(by \vee \Rightarrow rule)$

(3a.1b.1a.1) , $Q, \overset{s}{\Rightarrow} P, Q \vee R$ $(by \Rightarrow \wedge rule)$

(3a.1b.1a.2) , $Q, \overset{s}{\Rightarrow} P, R, Q$ $(by \Rightarrow \vee rule)$

(3a.1b.1b.1) , $Q, \overset{s}{\Rightarrow} P, \neg P$ $(by{\Rightarrow}\wedge rule)$

(3a.1b.1b.2) , $Q, P, \overset{s}{\Rightarrow} P$ $(by{\Rightarrow}\neg rule)$

(3b.1) , $R, \overset{s}{\Rightarrow} (Q \vee R) \wedge \neg P, P$ $(by\vee{\Rightarrow} rule)$

(3b.1a.1) , $R, \overset{s}{\Rightarrow} P, Q \vee R$ $(by{\Rightarrow}\wedge rule)$

(3b.1a.2) , $R, \overset{s}{\Rightarrow} P, R, Q$ $(by{\Rightarrow}\vee rule)$

(3b.1b.1) , $R, \overset{s}{\Rightarrow} P, \neg P$ $(by{\Rightarrow}\wedge rule)$

(3b.1b.2) , $R, P, \overset{s}{\Rightarrow} P$ $(by{\Rightarrow}\neg rule)$

Result: T

13

 Begin inference!

(0) , $\overset{s}{\Rightarrow} (\neg(P \to Q) \wedge (Q \to P)) \to (P \wedge \neg Q)$

(1) , $\neg(P \to Q) \wedge (Q \to P), \overset{s}{\Rightarrow} P \wedge \neg Q$ $(by{\Rightarrow}{\to} rule)$

(2) , $Q \to P, \neg(P \to Q), \overset{s}{\Rightarrow} P \wedge \neg Q$ $(by\wedge{\Rightarrow} rule)$

(3) , $Q \to P, \overset{s}{\Rightarrow} P \wedge \neg Q, P \to Q$ $(by\neg{\Rightarrow} rule)$

(4) , $Q \to P, P, \overset{s}{\Rightarrow} P \wedge \neg Q, Q$ $(by{\Rightarrow}{\to} rule)$

(4a.1) , $P, P, \overset{s}{\Rightarrow} P \wedge \neg Q, Q$ $(by{\to}{\Rightarrow} rule)$

(4a.1a.1) , $P, P, \overset{s}{\Rightarrow} Q, P$ $(by{\Rightarrow}\wedge rule)$

(4a.1b.1) , $P, P, \overset{s}{\Rightarrow} Q, \neg Q$ $(by{\Rightarrow}\wedge rule)$

(4a.1b.2) , $P, P, Q, \overset{s}{\Rightarrow} Q$ $(by{\Rightarrow}\neg rule)$

(4b.1) , $P, \overset{s}{\Rightarrow} P \wedge \neg Q, Q, Q$ $(by{\to}{\Rightarrow} rule)$

(4b.1a.1) , $P, \overset{s}{\Rightarrow} Q, Q, P$ $(by{\Rightarrow}\wedge rule)$

(4b.1b.1) , $P, \overset{s}{\Rightarrow} Q, Q, \neg Q$ $(by{\Rightarrow}\wedge rule)$

(4b.1b.2) , $P, Q, \overset{s}{\Rightarrow} Q, Q$ $(by{\Rightarrow}\neg rule)$

Result: T

14

 Begin inference!

(0) , $\overset{s}{\Rightarrow} ((P \to Q) \to (Q \to R)) \to ((R \to P) \to (Q \to P))$

(1) , $(P \to Q) \to (Q \to R), \overset{s}{\Rightarrow} (R \to P) \to (Q \to P)$ $(by{\Rightarrow}{\to} rule)$

(2) , $(P \to Q) \to (Q \to R), R \to P, \overset{s}{\Rightarrow} Q \to P$ $(by{\Rightarrow}{\to} rule)$

(3) , $(P \to Q) \to (Q \to R), R \to P, Q, \overset{s}{\Rightarrow} P$ $(by{\Rightarrow}{\to} rule)$

(3a.1) , $R \to P, Q, Q \to R, \overset{s}{\Rightarrow} P$ $(by{\to}{\Rightarrow} rule)$

(3a.1a.1) , $Q, Q \to R, P, \overset{s}{\Rightarrow} P$ $(by{\to}{\Rightarrow} rule)$

(3a.1a.1a.1) , $Q, P, R, \overset{s}{\Rightarrow} P$ $(by{\to}{\Rightarrow} rule)$

(3a.1a.1b.1) , $Q, P, \overset{s}{\Rightarrow} P, Q$ $(by{\to}{\Rightarrow} rule)$

(3a.1b.1) , $Q, Q \to R, \overset{s}{\Rightarrow} P, R$ $(by{\to}{\Rightarrow} rule)$

(3a.1b.1a.1) , $Q, R, \overset{s}{\Rightarrow} P, R$ $(by{\to}{\Rightarrow} rule)$

(3a.1b.1b.1) , $Q, \overset{s}{\Rightarrow} P, R, Q$ $(by{\to}{\Rightarrow} rule)$

(3b.1) , $R \to P, Q, \overset{s}{\Rightarrow} P, P \to Q$ $(by{\to}{\Rightarrow} rule)$

(3b.2) , $R \to P, Q, P, \overset{s}{\Rightarrow} P, Q$ $(by{\Rightarrow}{\to} rule)$

(3b.2a.1) , $Q, P, P, \overset{s}{\Rightarrow} P, Q$ $(by{\to}{\Rightarrow} rule)$

(3b.2b.1) , $Q, P, \overset{s}{\Rightarrow} P, Q, R$ $(by{\to}{\Rightarrow} rule)$

Result: T

15

 Begin inference!

(0) , $\overset{s}{\Rightarrow} (((P \to Q) \wedge (R \to Q)) \wedge (S \to Q)) \to (((P \wedge R) \wedge \neg S) \to Q)$

(1) , $((P \to Q) \wedge (R \to Q)) \wedge (S \to Q), \overset{s}{\Rightarrow} ((P \wedge R) \wedge \neg S) \to Q$ $(by{\Rightarrow}{\to} rule)$

(2) , $S \to Q, (P \to Q) \land (R \to Q), \overset{s}{\Rightarrow} ((P \land R) \land \neg S) \to Q$ $(by\land \Rightarrow rule)$

(3) , $S \to Q, R \to Q, P \to Q, \overset{s}{\Rightarrow} ((P \land R) \land \neg S) \to Q$ $(by\land \Rightarrow rule)$

(4) , $S \to Q, R \to Q, P \to Q, (P \land R) \land \neg S, \overset{s}{\Rightarrow} Q$ $(by\Rightarrow \to rule)$

(5) , $S \to Q, R \to Q, P \to Q, \neg S, P \land R, \overset{s}{\Rightarrow} Q$ $(by\land \Rightarrow rule)$

(6) , $S \to Q, R \to Q, P \to Q, P \land R, \overset{s}{\Rightarrow} Q, S$ $(by\neg \Rightarrow rule)$

(7) , $S \to Q, R \to Q, P \to Q, R, P, \overset{s}{\Rightarrow} Q, S$ $(by\land \Rightarrow rule)$

(7a.1) , $R \to Q, P \to Q, R, P, Q, \overset{s}{\Rightarrow} Q, S$ $(by\to \Rightarrow rule)$

(7a.1a.1) , $P \to Q, R, P, Q, Q, \overset{s}{\Rightarrow} Q, S$ $(by\to \Rightarrow rule)$

(7a.1a.1a.1) , $R, P, Q, Q, Q, \overset{s}{\Rightarrow} Q, S$ $(by\to \Rightarrow rule)$

(7a.1a.1b.1) , $R, P, Q, Q, \overset{s}{\Rightarrow} Q, S, P$ $(by\to \Rightarrow rule)$

(7a.1b.1) , $P \to Q, R, P, Q, \overset{s}{\Rightarrow} Q, S, R$ $(by\to \Rightarrow rule)$

(7a.1b.1a.1) , $R, P, Q, Q, \overset{s}{\Rightarrow} Q, S, R$ $(by\to \Rightarrow rule)$

(7a.1b.1b.1) , $R, P, Q, \overset{s}{\Rightarrow} Q, S, R, P$ $(by\to \Rightarrow rule)$

(7b.1) , $R \to Q, P \to Q, R, P, \overset{s}{\Rightarrow} Q, S, S$ $(by\to \Rightarrow rule)$

(7b.1a.1) , $P \to Q, R, P, Q, \overset{s}{\Rightarrow} Q, S, S$ $(by\to \Rightarrow rule)$

(7b.1a.1a.1) , $R, P, Q, Q, \overset{s}{\Rightarrow} Q, S, S$ $(by\to \Rightarrow rule)$

(7b.1a.1b.1) , $R, P, Q, \overset{s}{\Rightarrow} Q, S, S, P$ $(by\to \Rightarrow rule)$

(7b.1b.1) , $P \to Q, R, P, \overset{s}{\Rightarrow} Q, S, S, R$ $(by\to \Rightarrow rule)$

(7b.1b.1a.1) , $R, P, Q, \overset{s}{\Rightarrow} Q, S, S, R$ $(by\to \Rightarrow rule)$

(7b.1b.1b.1) , $R, P, \overset{s}{\Rightarrow} Q, S, S, R, P$ $(by\to \Rightarrow rule)$

Result: T

16

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \land (P \land P)$

(0a.1) , $\overset{s}{\Rightarrow} P$ $(by\Rightarrow \land rule)$

Result: F

17

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \lor (P \land P)$

(1) , $\overset{s}{\Rightarrow} P \land P, P$ $(by\Rightarrow \lor rule)$

(1a.1) , $\overset{s}{\Rightarrow} P, P$ $(by\Rightarrow \land rule)$

Result: F

18

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \to (P \land P)$

(1) , $P, \overset{s}{\Rightarrow} P \land P$ $(by\Rightarrow \to rule)$

(1a.1) , $P, \overset{s}{\Rightarrow} P$ $(by\Rightarrow \land rule)$

(1b.1) , $P, \overset{s}{\Rightarrow} P$ $(by\Rightarrow \land rule)$

Result: T

19

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \leftrightarrow (P \land P)$

(0a.1) , $P, \overset{s}{\Rightarrow} P \land P$ $(by\Rightarrow \leftrightarrow rule)$

(0a.1a.1) , $P, \overset{s}{\Rightarrow} P$ $(by\Rightarrow \land rule)$

(0a.1b.1) , $P, \overset{s}{\Rightarrow} P$ $(by\Rightarrow \land rule)$

(0b.1) , $P \land P, \overset{s}{\Rightarrow} P$ $(by\Rightarrow \leftrightarrow rule)$

(0b.2) , $P, P, \overset{s}{\Rightarrow} P$ $(by \wedge \Rightarrow rule)$

Result: T

20

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \wedge (P \vee P)$

(0a.1) , $\overset{s}{\Rightarrow} P$ $(by \Rightarrow \wedge rule)$

Result: F

21

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \vee (P \vee P)$

(1) , $\overset{s}{\Rightarrow} P \vee P, P$ $(by \Rightarrow \vee rule)$

(2) , $\overset{s}{\Rightarrow} P, P, P$ $(by \Rightarrow \vee rule)$

Result: F

22

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \rightarrow (P \vee P)$

(1) , $P, \overset{s}{\Rightarrow} P \vee P$ $(by \Rightarrow \rightarrow rule)$

(2) , $P, \overset{s}{\Rightarrow} P, P$ $(by \Rightarrow \vee rule)$

Result: T

23

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \leftrightarrow (P \vee P)$

(0a.1) , $P, \overset{s}{\Rightarrow} P \vee P$ $(by \Rightarrow \leftrightarrow rule)$

(0a.2) , $P, \overset{s}{\Rightarrow} P, P$ $(by \Rightarrow \vee rule)$

(0b.1) , $P \vee P, \overset{s}{\Rightarrow} P$ $(by \Rightarrow \leftrightarrow rule)$

(0b.1a.1) , $P, \overset{s}{\Rightarrow} P$ $(by \vee \Rightarrow rule)$

(0b.1b.1) , $P, \overset{s}{\Rightarrow} P$ $(by \vee \Rightarrow rule)$

Result: T

24

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \wedge (P \rightarrow P)$

(0a.1) , $\overset{s}{\Rightarrow} P$ $(by \Rightarrow \wedge rule)$

Result: F

25

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \vee (P \rightarrow P)$

(1) , $\overset{s}{\Rightarrow} P \rightarrow P, P$ $(by \Rightarrow \vee rule)$

(2) , $P, \overset{s}{\Rightarrow} P, P$ $(by \Rightarrow \rightarrow rule)$

Result: T

26

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \rightarrow (P \rightarrow P)$

(1) , $P, \overset{s}{\Rightarrow} P \rightarrow P$ $(by \Rightarrow \rightarrow rule)$

(2) , $P, P, \overset{s}{\Rightarrow} P$ $(by \Rightarrow \rightarrow rule)$

Result: T

27

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \leftrightarrow (P \rightarrow P)$

(0a.1) , $P, \overset{s}{\Rightarrow} P \to P$ $(by \Rightarrow \leftrightarrow rule)$

(0a.2) , $P, P, \overset{s}{\Rightarrow} P$ $(by \Rightarrow \to rule)$

(0b.1) , $P \to P, \overset{s}{\Rightarrow} P$ $(by \Rightarrow \leftrightarrow rule)$

(0b.1a.1) , $P, \overset{s}{\Rightarrow} P$ $(by \to \Rightarrow rule)$

(0b.1b.1) , $\overset{s}{\Rightarrow} P, P$ $(by \to \Rightarrow rule)$

Result: F

28

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \wedge (P \leftrightarrow P)$

(0a.1) , $\overset{s}{\Rightarrow} P$ $(by \Rightarrow \wedge rule)$

Result: F

29

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \vee (P \leftrightarrow P)$

(1) , $\overset{s}{\Rightarrow} P \leftrightarrow P, P$ $(by \Rightarrow \vee rule)$

(1a.1) , $P, \overset{s}{\Rightarrow} P, P$ $(by \Rightarrow \leftrightarrow rule)$

(1b.1) , $P, \overset{s}{\Rightarrow} P, P$ $(by \Rightarrow \leftrightarrow rule)$

Result: T

30

 Begin inference!

(0) , $\overset{s}{\Rightarrow} P \to (P \leftrightarrow P)$

(1) , $P, \overset{s}{\Rightarrow} P \leftrightarrow P$ $(by \Rightarrow \to rule)$

(1a.1) , $P, P, \overset{s}{\Rightarrow} P$ $(by \Rightarrow \leftrightarrow rule)$

(1b.1) , $P, P, \overset{s}{\Rightarrow} P$ $(by \Rightarrow \leftrightarrow rule)$

Result: T