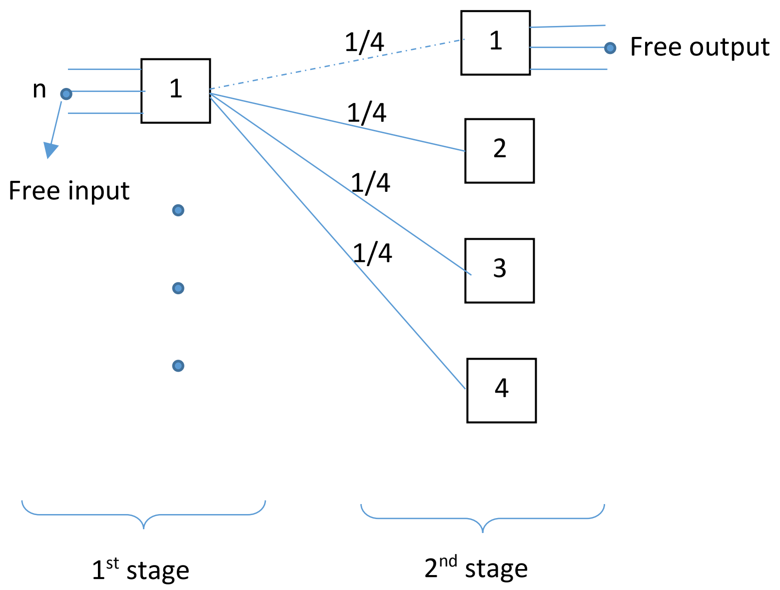
CLOS network

<u>Our objective</u>: To design a strictly non-blocking interconnection switch.

<u>Recall</u>: Two-stage was unable to meet this strictly non-blocking requirement. In addition, switch complexity would grow quadratically.

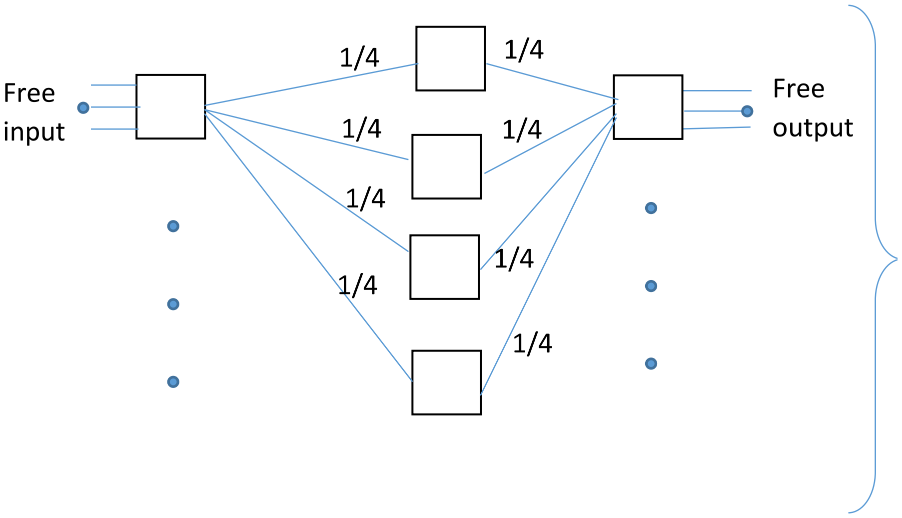
Bell labs paper – by Charles Clos

<u>Idea:</u> Extending from 2-stage networks



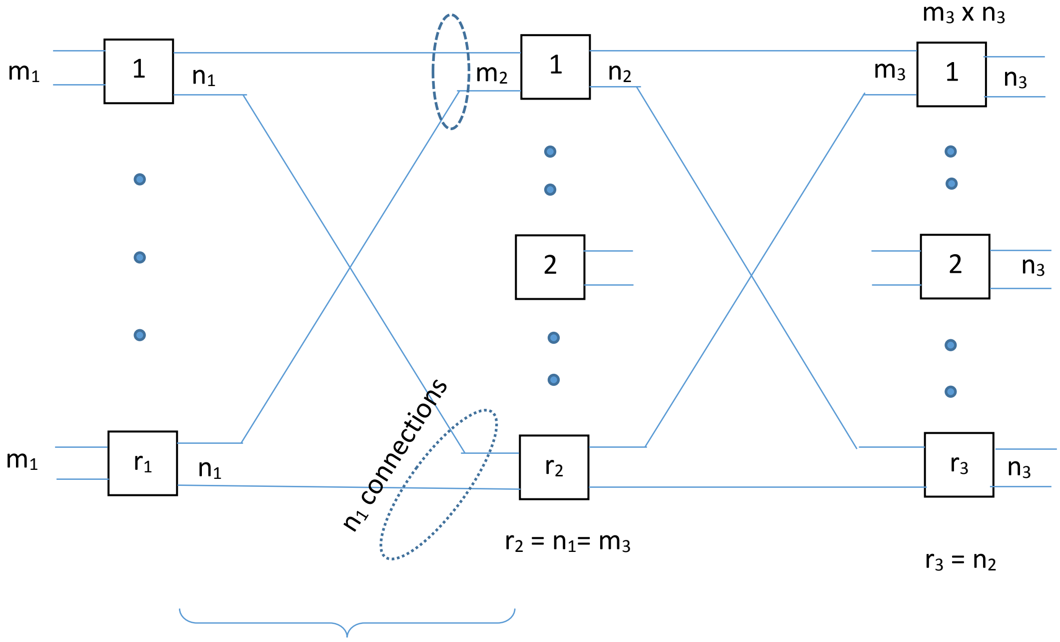
Issue is when there is one free input on sw1 (1^{st} stage) and 1 free output on sw1 (2^{nd} stage), there is no way a connection could be established as the ¼ link are currently occupied. This leads to blocking.

If we extend to 3 stages,



Then this solution may provide a path between any two free input-output line.

Actually, we can increase the number of switches in the middle stage. Clos came up with a condition under which three-stage becomes non-blocking.

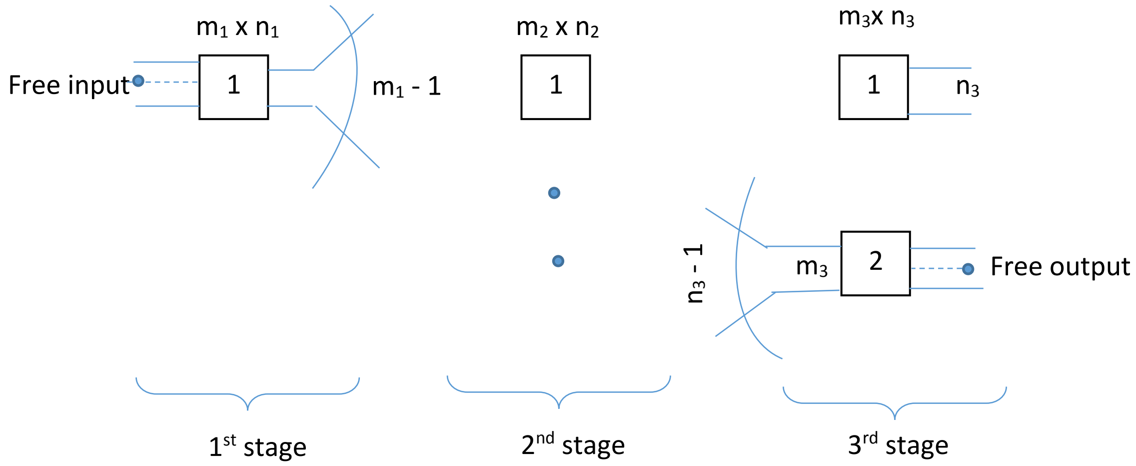


Exactly one connection between any pair of input-output

CLOS Network

<u>Note</u>: $m_1 < n_1$ usually.

Consider two free input-output pairs and let us see if they can be connected.



<u>3^{rd} stage</u>: Consider sw2. Out of m_3, only n_3 can be connected (max) since one output is free only (n_3-1) can be occupied or emanating from the input side. (m_3 could be greater than n_3)

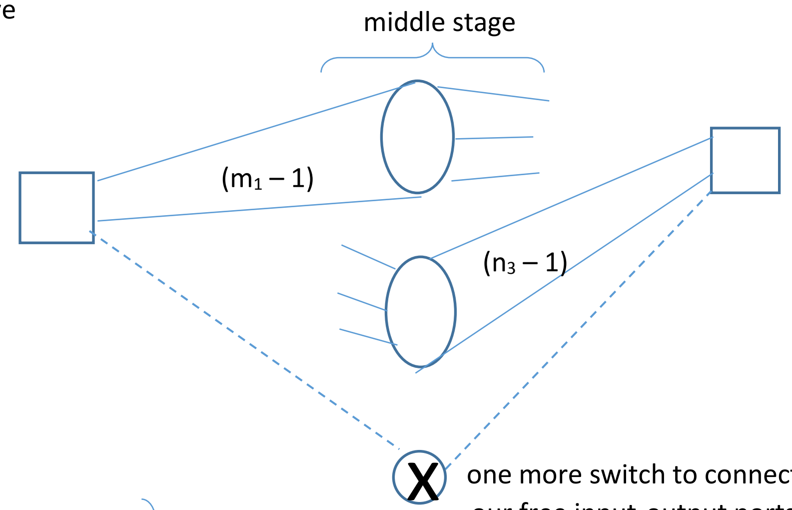
<u>1^{st} stage</u>: sw_1 – from sw_1 only (m_1-1) can be going out occupied (as one is free); that is out of m_1 input lines, $n_1 - (m_1 - 1)$ will be free.

So, if $(m_1 - 1)$ rows are occupied / coming out of stage 1, this means there are $(m_1 - 1)$ switches in the middle stage that are used for all other rows from sw_1 in stage 1.

Similarly, see sw_2 of 3^{rd} stage. <u>$(n_3 - 1)$ middle stage switches are used</u> to connect the requests to the 3^{rd} stage.

Note that we have our free link at switch 2 at stage 3.

So, we have



one more switch to connect our free input-output ports

So, if we have one more switch in the middle stage then we will be able to connect our free input and free output link.

Therefore,

$r_2 > (m_1 - 1) + (n_3 - 1)$

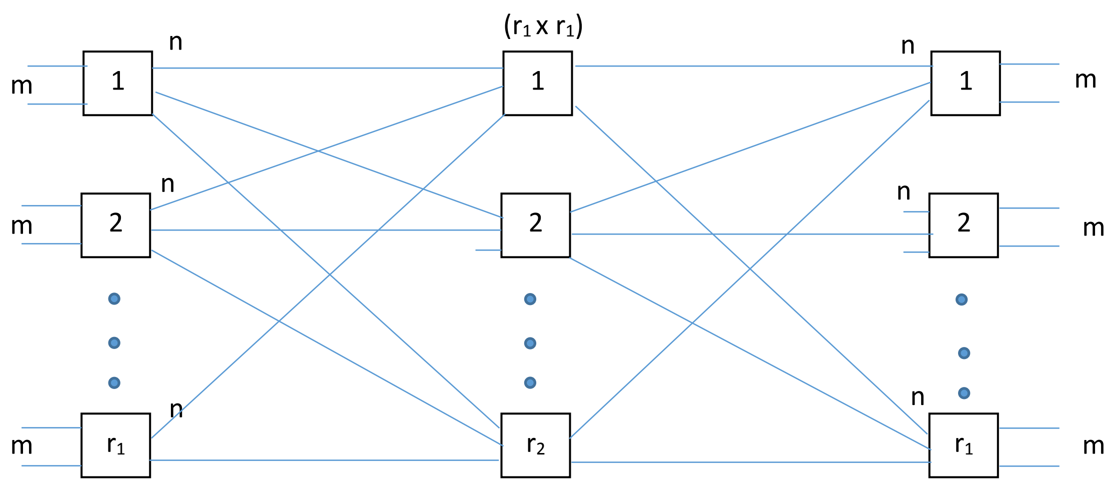
$\Rightarrow \boxed{r_2 > m_1 + n_3 - 2}$ → strictly non-blocking

$\boxed{\underline{\text{Note}}: r_2 \geq m_1 + n_3 - 1}$

Also, note that $m_3 = n_1 = r_2 \geq m_1 + n_3 - 1$

Q: How many <u>cross points required for this switch configuration</u>?

Let us consider a <u>symmetric configuration</u> $\rightarrow$ looks the same from either $\rightarrow$ ion



For symmetric config $m_1 = n_3$

$\therefore r_2 = n$

From our condition: $r_2 \geq m + m - 1$ i.e, $r_2 \geq 2m - 1$

so, $\boxed{n = r_2 \geq 2m - 1}$

<u>Number of input points</u>: $N = (m \times r_1) \Rightarrow r_1 = (N / m)$

Number of cross points needed: $(2m - 1)$ switches in the middle stage;

So, each switch in the first stage will be having $m(2m - 1)$ cross points.

There are r_1 switches in the first stage.

Therefore, number of crosspoints in the first stage is: $r_1 . m . (2m - 1)$, i.e $\overbrace{\left(\frac{N}{m}\right)}^{r_1} . m(2m - 1)$

<u>For middle stage:</u>

each switch: $n \; x \; n$

$$\& \; n = r_2 = (2m - 1)$$

$$\therefore n \; x \; n = (2m - 1)\left(\frac{N}{m}\right)\left(\frac{N}{m}\right)$$

\# switches in the middle

<u>Third stage:</u> same as first stage.

Therefore, number of crosspoint Q

$$Q = \underbrace{2m(2m-1)\frac{N}{m}}_{\text{1st \& 3rd stages}} + \underbrace{(2m-1)\frac{N}{m}.\frac{N}{m}}_{\text{Middle stage}}$$

We can further simplify. If we assume if we have one more middle switch, 2m - 1 becomes just 2m.

$$\boxed{\therefore Q = 4mN + 2\frac{N^2}{m}}$$

It is better to express complexity in terms of N & hence we will find an optimum m*.

$$Q = 4mN + 2\frac{N^2}{m}$$

$$\Longrightarrow \left(\frac{dQ}{dm}\right) = 0 = 4N - 2\frac{N^2}{m^2}$$

$$\Rightarrow 2N\left(2 - \frac{N}{m^2}\right) = 0, N > 0$$

$$\Longrightarrow \boxed{m^* = \frac{\sqrt{N}}{2}}$$

$$\therefore Q = 4.\sqrt{\frac{N}{2}}.N + \frac{2N^2}{\sqrt{\frac{N}{2}}}$$

$$Q = 2N^{3/2} + 2\sqrt{2}N^{3/2} = 4\sqrt{2}N^{3/2}$$

$$\therefore Complexity = O(N^{3/2})$$

So, number of crosspoints are not growing as N^2. We minimized to $O(N^{3/2})$.