NUS

National University of Singapore

Name : LUO ZIJIAN

Matric.No: A0224725H

MUSNET: E0572844

Subject: Stochastic process

Assignment: Homework Eleven

Date: April 9th

Prof: Vincent Tan.

1. EXERCISE 8.15

(a) From the statement from part (a)

$f_{Y|X}$ is depent on the value of X

when $X=0$, Y is the range of $[-1, 1]$, when $X=1$, Y is the range of $[0, 2]$

so $f_{Y|X}(y|0) = \begin{cases} \frac{1}{2}, -1 \le y \le 1 \\ 0, \text{else} \end{cases}$, $f_{Y|X}(y|1) = \begin{cases} \frac{1}{2}, 0 \le y \le 2 \\ 0, \text{else} \end{cases}$

so we can get the ratio

$$\Lambda(y) = \frac{f_{Y|X}(y|1)}{f_{Y|X}(y|0)} = \begin{cases} 0; & -1 \le y < 0 \\ 1; & 0 \le y \le 1 \\ \infty; & 1 < y \le 2 \end{cases}$$

(b) There are three possible cases

① For $\eta > 1$, $\Lambda(y) \ge \eta \iff \hat{x} = 1$, $1 < y \le 2$

② For $\eta = 1$, $\hat{x} = 1 \iff \hat{x} = 1$, $0 \le y \le 2$

③ For $\eta < 1$, $\Lambda(y) \ge \eta \iff \hat{x} = 1$, $0 \le y \le 2$

As for $q_1(\eta)$, when $\eta > 1$, $\hat{x} = 1$ (no error) for $1 < y \le 2$,

to summarize, $q_1(\eta) = \frac{1}{2}$, for $\eta > 1$.

Also, for $\eta > 1$, $\hat{x} = 0$, for $-1 \le y \le 1$,

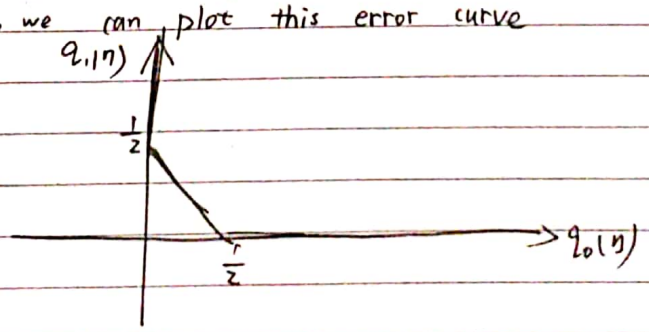
Thus $\begin{cases} q_0(\eta) = \frac{1}{2}, & q_1(\eta) = 0, \quad \eta \le 1 \\ q_0(\eta) = 0, & q_1(\eta) = \frac{1}{2}, \quad \eta > 1 \end{cases}$

(c) From the result from part (b), we know

for $\eta > 1$, $q_0(\eta) = 0$, $q_1(\eta) = \frac{1}{2}$

for $\eta \le 1$, $q_0(\eta) = \frac{1}{2}$, $q_1(\eta) = 0$

so, we can plot this error curve



(d) These don't care cases arise for $0 \le y \le 1$, when $\eta = 1$. With the decision rule of error curve, these don't care cases result in $\hat{x} = 1$. If half of those don't care cases are decided as $\hat{x} = 0$, then the error probability given $X=1$ is increased to 1/4 and that for $X=0$ is decreased to 1/4. This could be done by random choice, or more easily, by mapping $y > \frac{1}{2}$ into $\hat{x} = 1$ and $y \le 1/2$ into $\hat{x} = 0$