Question 1.

1. FALSE

2. TRUE

3. TRUE

4. The difference is that strong converse for a DMS always have a typical sets ~~[illegible]~~ and have a more efficient source coding

Question 2

1. For this source, $H(x) = \sum_{x \in \mathcal{X}} P_x \log \frac{1}{P_x}$

$= 0.23 \log 0.23 + 0.04 \log 0.04 \cdots\cdots + 0.12 \log 0.12$

~~1.9818 bit~~ 2.1689 bit

~~= 1.982 bit~~

2. Huffman code

A → 00

E → 11101

I → 010

O → 110

U → 011

G → 1110000

K → 1110001

P → 11110

R → 100

S → 111001

T → 11111

V → 101

3. Shannon code

	P_x	$\sum P_x(x)$	LHS in binary	$\lceil \log_2 \frac{1}{P_x(x)} \rceil$	codeword
A	0.23	0	0.0000000	3	000
O	0.13	0.23	0.0011 ···	4	0011
U	0.12	0.36	0.01011 ···	5	01011
R	0.12	0.48	0.01111 ···	5	01111
V	0.12	0.6	0.10011 ···	5	10011
I	0.11	0.72	0.10111	5	10111
P	0.05	0.83	0.110101	6	110101
T	0.05	0.88	0.111000	6	111000
E	0.04	0.93		6	
S	0.02	0.97			
G	0.005	0.99			
K	0.005	0.995			
		1			

Question 3.

1. when $k=1$

$$E[l_k(x)] \leq \log |x| + 1$$

And then recall the property of huffman code.

$$E[l_k(x^k)] \leq k \log |x| + 1$$

2. $$\lim_{m \to \infty} \frac{1}{mk} \sum_{i=1}^{m} l_k(x_i^k) = R^*(x^k)$$

We use the upper bound to achieve, so

$$\left| \frac{1}{mk} \sum_{i=1}^{m} l_k(x_i^k) - H(x_i) \right| \leq \frac{2}{k}, \quad Z_i = \frac{1}{mk} \sum_{i=1}^{n} l_k(x_i^k) - H(x_i)$$ are i.i.d.

Then use weak law of large numbers, and change expression

we show $\lim_{m \to \infty} P_r\left[\frac{1}{mk} \sum_{i=1}^{m} l_k(x_i^k) \geq H(x) + \frac{2}{k}\right] = 0$

3. Encoder: $e(M|x^n) = \begin{cases} m(x^n|y^n) & (x^n \in \\ 0 & (x^n \end{cases}$

~~Decoder~~

FALCON