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Subject: Information Theory

Assignment: Homework Seven

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Exercise 7 Proof: As we know, Pr (Y) = EI T (YIX) PLX), Qr (Y) = ZI T (YIX) O(X) And for the definition of total variation distance, we can show that  $Stralpx, Qx) = \frac{1}{2}||Px - Qx|| = \frac{1}{2}||Px| - Qx||$ Stud (Pr. Qr) = = 11Por- Qr11 = = FEN P(+) -Q(+) = = ZENTEN [POS-OW]-TCHN With the definition of data processing inequality in KL D(Px IQx) > D(Pr IIQx) And then Stul (Px,Qx) - Stul(Pr,Qr) = = = = = [P(x) - Q(x)][1- Zi. T(y|x)] (1>Zi T(y|x)) >0. Finally, it is desired. Exercise 6027.2 Proof: As for the definition of empirical set, we know the empirical distribution: fxn(X):= 1/{i = {1, --- n}: xi=x} And use the repression of total variation distance, we an rewrite empirical typical see like that Stud (fxn, PX) < E = = 11 fx - P(x) | = E And then, we know X1, x2... In are a sequence of i.i.d. source. 1 [ie],...n): xi=x} -> u for n-> 00 In this case  $u = n \cdot p(x)$ . FALCON



And we use the tange law of large number po, which shows that Profilm 1 2 x; = u = 1 (almost sure convergence) In a word, the empirical distribution of a sequence should be close to the prior probability distribution Prp [xne A in) Pox) ] =1 Exercise 7.3 a). Symmetric error probabilities:  $C(P, Q, S) = -\min_{0 \le \lambda_1 \le 1} \log \overline{Z}_1 P(x) \overline{\lambda_1} Q(x) \overline{\lambda_2} \frac{1 - \lambda_1 - \lambda_2}{S(x)}$ especially when  $\lambda_1 = \lambda_2 = \frac{1}{3}$ , we an get minimum yalue  $C(p, 9, s) = -\frac{1}{3}$  $e^* q_{m,1} = \sum_{n=1}^{\infty} \frac{n}{2^{n}} = \frac{1}{2^{n}} = \frac{1}{2^{n}} = \frac{1}{2^{n}}$ 

h). In ternary hypothesis testing problem, especially when  $0 = 12 = \frac{3}{3}$ , all three have equal priors.

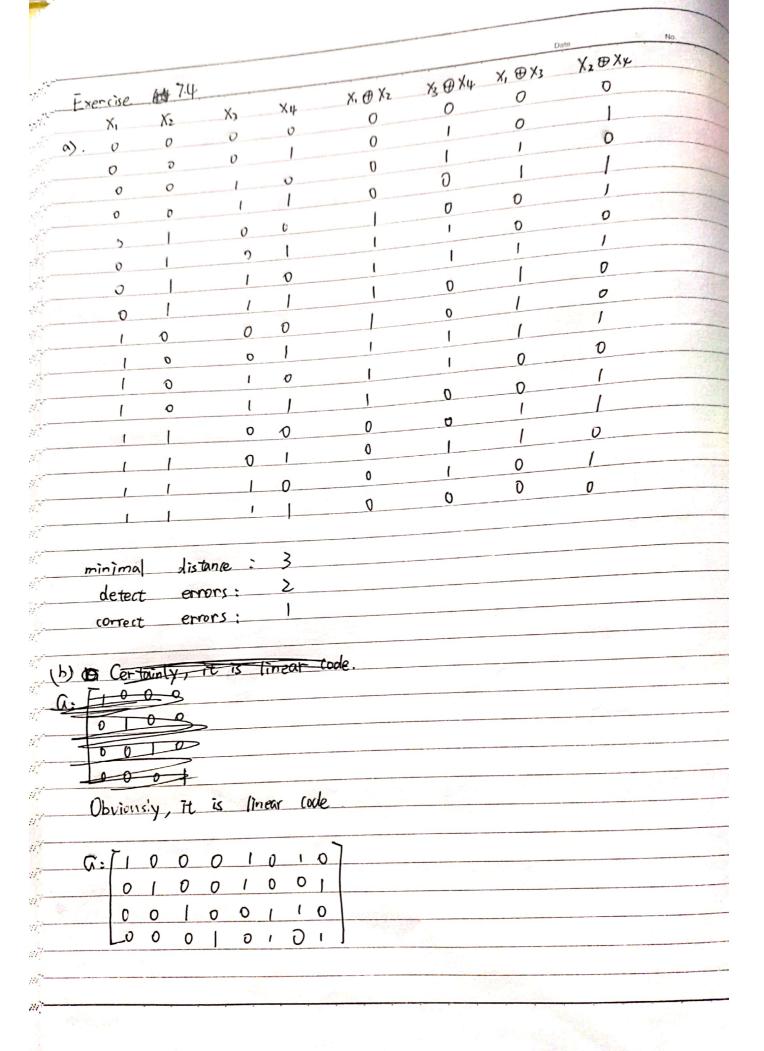
That is  $9(\frac{1}{3}, \frac{1}{3})$ Therefore, the minimal error probability: In a word. 9 is the optimal test.

$$D(P11s) = \sum_{i} s_{i} \log \frac{s_{i}}{p_{i}} = 1$$

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which means the most perfect code should be 7 bits	(the later) interfered party plant
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