Question 1

Consider a second-order system shown in the following transfer function, please study this second-order system and design the PID controller for the system.

Find a state-space model for the above system

For a second-order system,if a system is expressed as

$$G(s) = \frac{X(s)}{U(s)} = \frac{b}{s^2 + a_1 s + a_0} \tag{1}$$

or like this

$$\ddot{x} = -a_1\dot{x} - a_0 x + bu \tag{2}$$

In the state-space form, it should be

$$\dot{X}(t) = AX(t) + Bu(t) = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u \tag{3}$$

$$Y(t) = CX(t) + Du(t) \tag{4}$$

Therefore, we can get the parameters of this state-space model.

$$A = \begin{bmatrix} -12 & -3 \\ -1 & 0 \end{bmatrix} \tag{5}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{6}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{7}$$

$$D = 0 \tag{8}$$

Design a PID controller for the system using LQR and simulate

For LQR-assisted PID tuning method, the control performance specification measured in terms of

$$J = \int_0^\infty \left(x^\mathrm{T}(t)Qx(t) + u^\mathrm{T}(t)Ru(t)\right) \mathrm{d}t \tag{9}$$

The LQR fucntion is to find the optimal control $\mu(t)$ such that J in Eq. 9 is minimized.

$$u(t) = -R^{-1}B^\mathrm{T}Px(t) \tag{10}$$

where P is the positive definite solution of the Riccati equation:

$$A^\mathrm{T}P + PA - PBR^{-1}B^\mathrm{T}P + Q = 0 \tag{11}$$

we can get K gain through below equation

$$K = -R^{-1}B^\mathrm{T}P \tag{12}$$

If this linear process with time delay L, we can describe it by

$$\dot{X}(t) = AX(t) + Bu(t-L) \tag{13}$$

Through the idea of T.H.Lee's paper, LQR result for delay-free process also can be applied.

After rethinking the context of lecture notes, I use below function to calculate K gain by setting reasonable Q and R. Here is my part

Listing 1: Matlab script to calculate K

```
b=[1];
a=[1,12,3];
% Get the state-space model
[A,B,C,D] = tf2ss(b,a)
Q=[1000,0;0,1];
% To output the K gain from lqr function
R=[1];
N=zeros;
[K,S,e] = lqr(A,B,Q,R,N)
```

As for the overall system, the details are shown in Figure 1

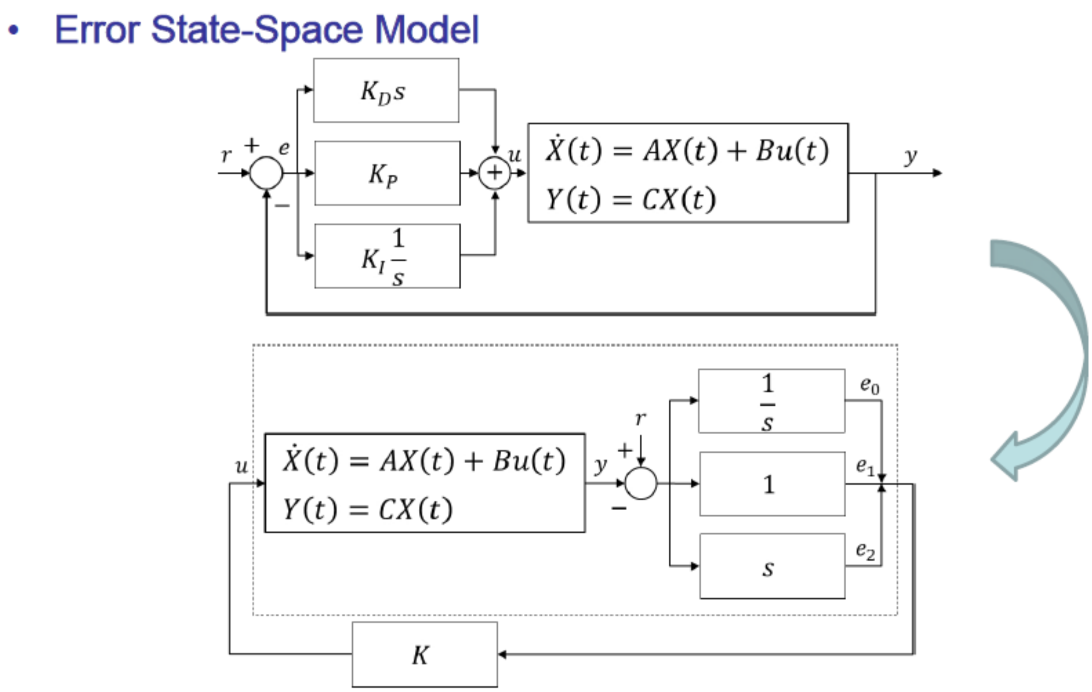

Figure 1: Error state model in the system

After calculating K, I use the simulink toolbox to show the designed control system. Firstly, the step signal is added into the sum box. And this result reduces the output from state-space model box. And then PID controller implements corresponding algorithm to the K gain amplifier. Finally the response from K gain amplifier direct into state-space model box again. This overall system is a close-loop system.

Here are the configurations of this system in Figure 2 3 4 5.

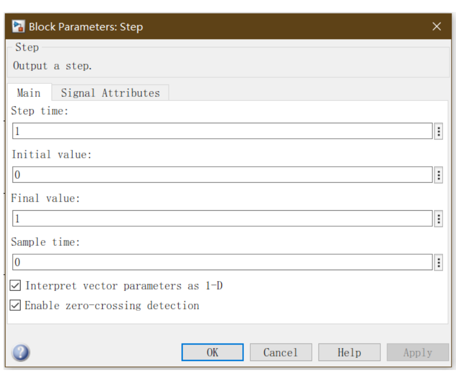

Figure 2: Configurations of step signal box

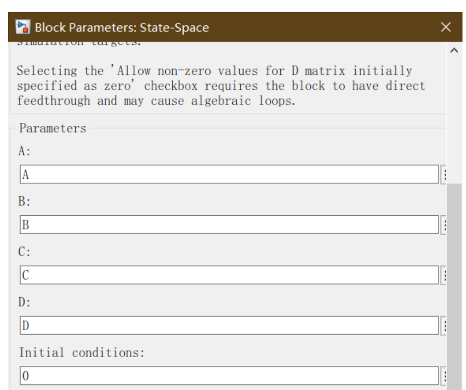

Figure 3: Configurations of state-space

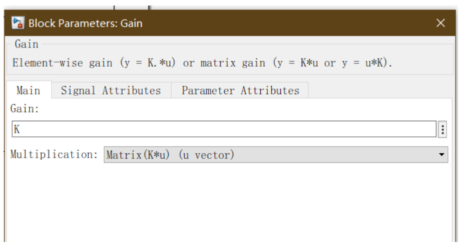

Figure 4: Configurations of K gain

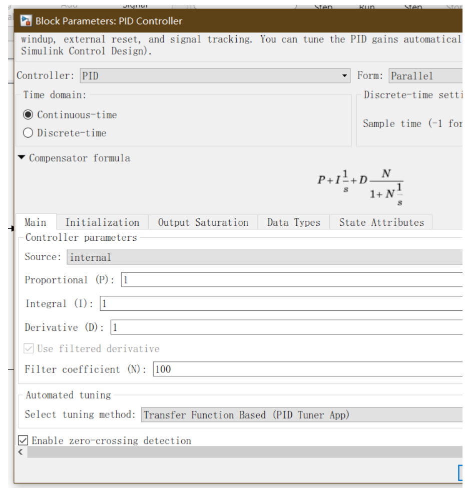

Figure 5: Configurations of PID controller

All variables from are kept in the working space,so they are loaded easily.

Now you can see the scope from below Firure 6. It shows it is a step response during this period of time(100s).

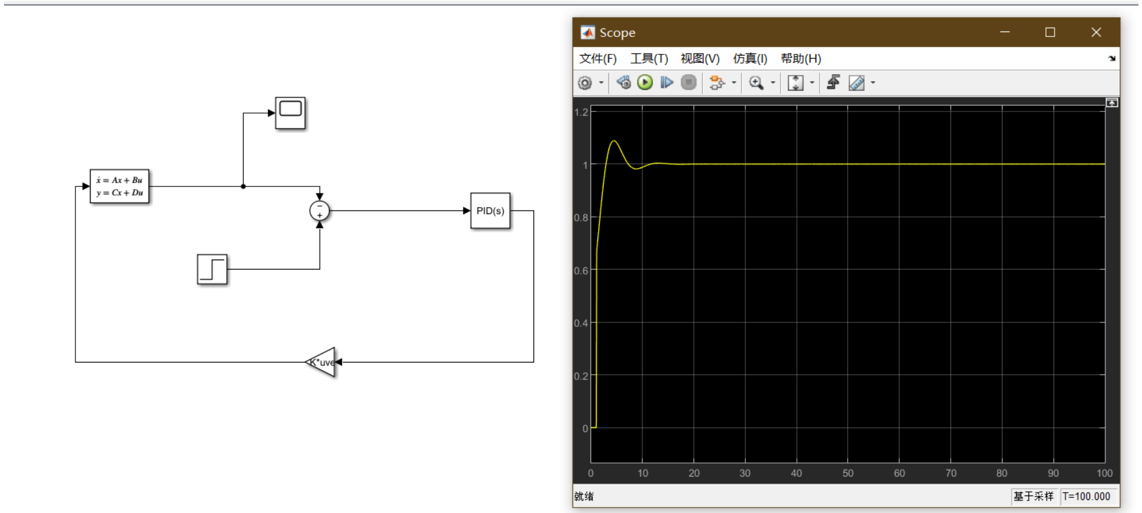

Figure 6: Simulation of designed overall system

As we said before, the delay of process has no effect to the step response. I set the delay parameter 10s and 50s respectively, the scope result shows they all work well despite of delay. Here are the scopes in Figure 7 8

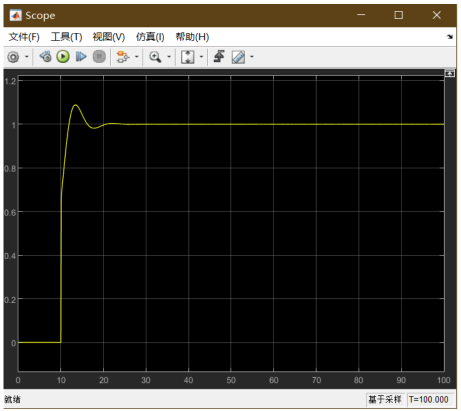

Figure 7: Scope in step signal with deley 10

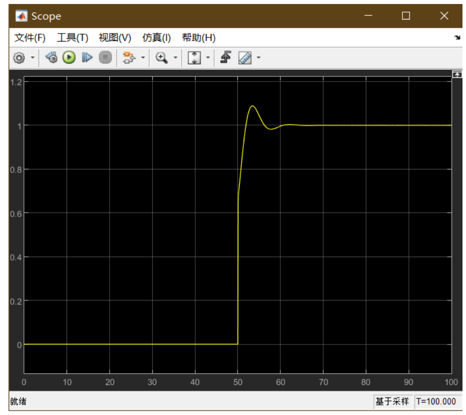

Figure 8: Scope in step signal with deley 50

Discuss the effects of weighting Q and R on system performance

In this part, firstly,I explore the effects of Q. For Q, there are two variable elements we can modify. I express them as Q1 and Q2.

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \tag{14}$$

If I increase Q1, the first element in K becomes larger and the first element in e becomes less. And the outcome of scope is more likely as step response.

If I increase Q2, the second element in K becomes larger and the second element in e becomes less. And the outcome of scope is more likely as step response.

Secondly,I explore the effects of R. For R, there is only one variable element we can modify. I express it as R.

$$R = \begin{bmatrix} R \end{bmatrix} \tag{15}$$

If I let R become m times than before, the element in K becomes $\frac{1}{m}$ than before. And the outcome of scope is more smooth and reach steady state with more time.

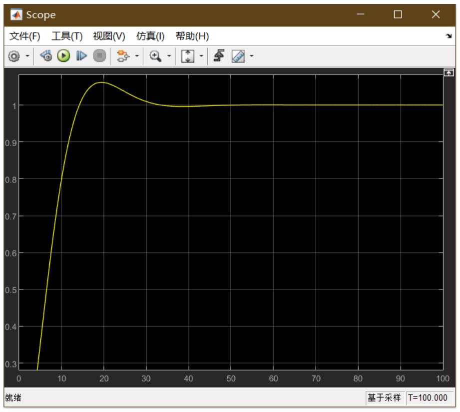

Figure 9: Scope(Q1=10,Q2=1,R=1)

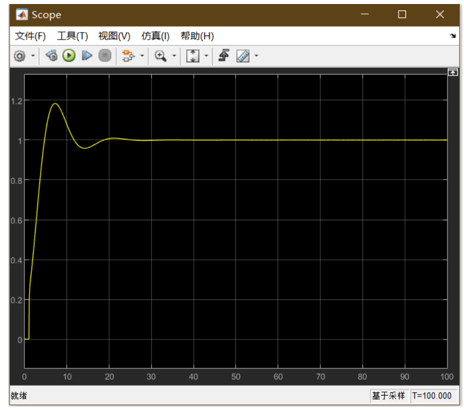

Figure 10: Scope(Q1=100,Q2=1,R=1)

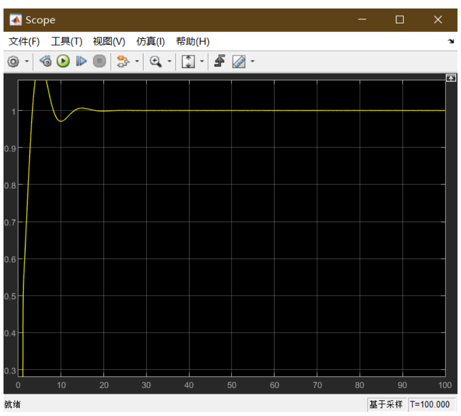

Figure 11: Scope(Q1=100,Q2=100,R=1)

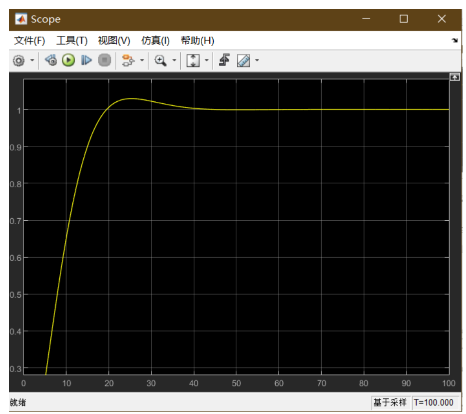

Figure 12: Scope(Q1=100,Q2=1,R=10)

Generally speaking, choosing a large Q value means that to make J small, then x(t) needs to be smaller, which means the state x(t) decays to steady at a faster rate. The larger the element, the more important the variable is in the performance function. The performance function is required to be minimized, which means that the constraints of this state are required to be high.

On the other hand, a large R means that more attention is paid to the decrease of input variables u(t), which means that the state decay will slow down. So, I believe that the R matrix is the weight of the control quantity. Similarly, the larger the corresponding element, which means the greater the control constraint.

Question 2

Consider a reactor vessel which process schematic. Please provide a diagram showing the connection of the devices to the PLC and provide the ladder diagram(LLD) implementing the logic.

After connection all the elements, the PLC graph is shown in Figure 13

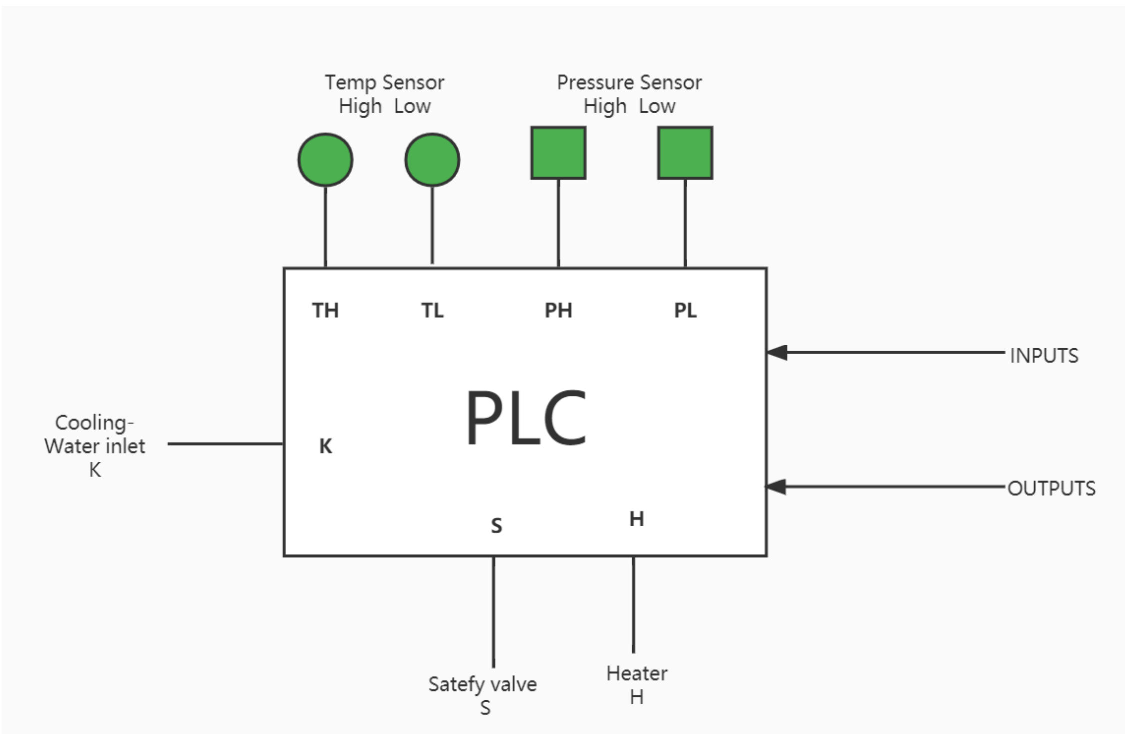

Figure 13: PLC connection in the system

The ladder graph is shown in Figure 14

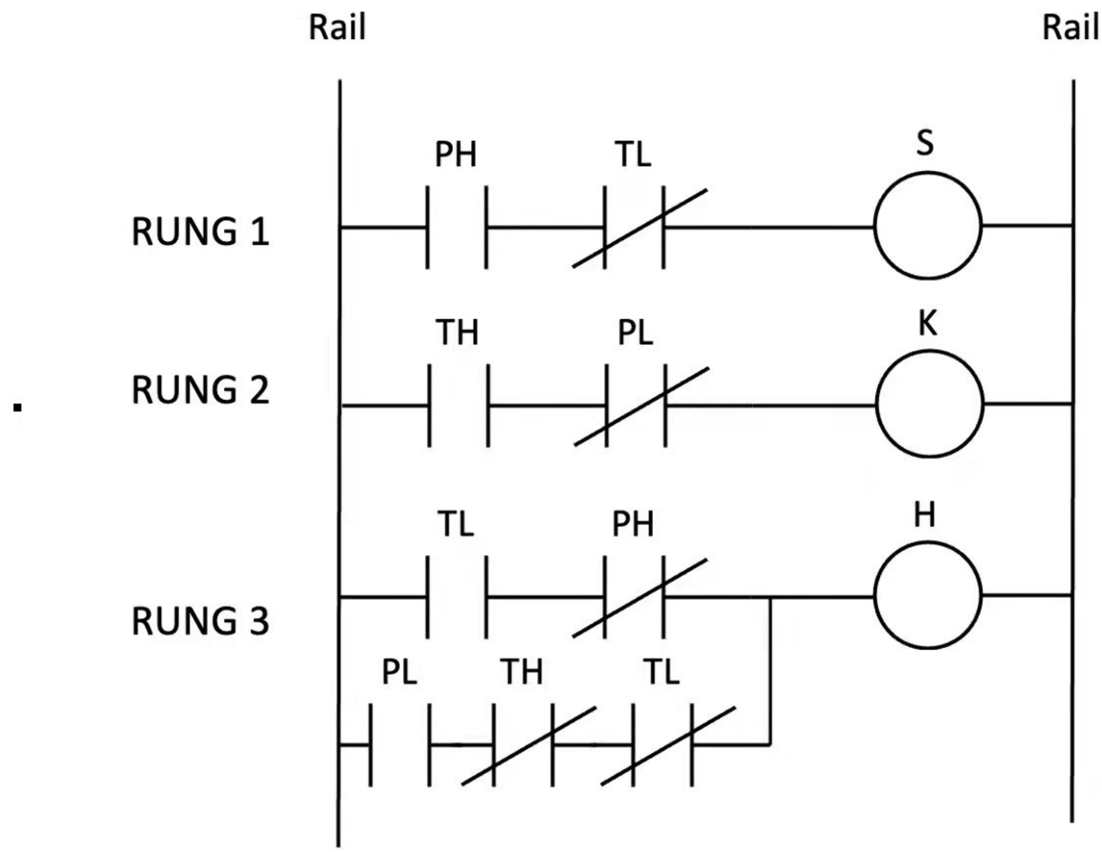

Figure 14: Ladder diagram in the system