NUS

National University of Singapore

Name : LUO ZIJIAN

Matric.No： A0224725H

MUSNET： E0572844

Subject： Stochastic process

Assignment: Homework Seven

Date: Mar 11th

Prof: Vincent Tan.

1. EXERCISE 4.1

From the above statement, we all know state is recurrent, which means state(i) can be accessible from all states that are accessible from i. $i \to k \to i$, two walk

But when we know $P_{ii} > 0$, means $i \to i$, one walk

The period of a state i, is the greatest common divisor (gcd). From other states, $d(i,k) \geq 2$, but from itself, $d(i,i) = 1$ so we can get $d(i) = 1$, therefore, the state is a aperiodic

2. EXERCISE 4.2

Proof:

Consider a finite Markov chains with r states, $S = \{1, 2, \cdots r\}$. Suppose that all states are transient. Then starting from the ~~starting~~ time, the chain might visit state 1 several times, but at some point - the chain will leave state 1 and will never return to it. That is - there exists an integer $M_1 > 0$ such that $X_n \neq 1$, for all $n \geq M_1$. Similarly, there exists an integar $M_2 > 0$ such that $X_n \neq 2$, for all $n \geq M_2$, and so on. Now, if you choose $n \geq \max\{M_1, M_2 \cdots M_r\}$ then X_n cannot be equal to any of the states $1, 2, \cdots r$. This is a contradiction, so we conclude that there must be at least one recurrent state.

ⓐ If there exists i_2 state, we can get $i_1 \to i_2$. However, we can imply $i_2 \to i_1$. So this statement is not sufficient

ⓑ This argument is sufficient. when i_2 is transient, Because $i_1 \to i_2$ and $i_2 \to i_3$, we can get $i_1 \to i_3$. If $i_3 \not\to i_1$, with $i_3 \to i_1$ and $i_1 \to i_2$, we can get $i_3 \to i_2$, this is self-contradict. And it is similar to $i_3 \not\to i_2$ ($i_3 \not\to i_2$ means $i_3 \neq i_2$, $i_3 \not\to i_1$ means $i_3 \neq i_1$)

ⓒ This statement is sufficient. If $i_k \to i_{k+1}$, and $i_{k+1} \not\to i_k$, ~~with $i_{k+1} \neq i_j$, we can not guarantee that $i_{k+1} \to i_{j+1} \to i_k$~~, when ~~When~~ $i_{k+1} = i_k$, in similar method like (b) it satisfy

(d) For transient states $i_1, \cdots, i_k$, using the state i_{k+1} is another state from ~~$i_1 \cdots i_k$~~ $\{i_j : j \le k\}$. Because there are only M states, there cannot be M transient states. (with $k=M$, a different state i_{M+1} would be generated, it is meaningless) Thus, it must satisfy $k<M$, it can leads to a recurrent state. So this statement is sufficient.

~~3. (a) As we all know, we can regard a~~

3. (a) In this problem, we just focus on the relative distance between this spider and this fly.

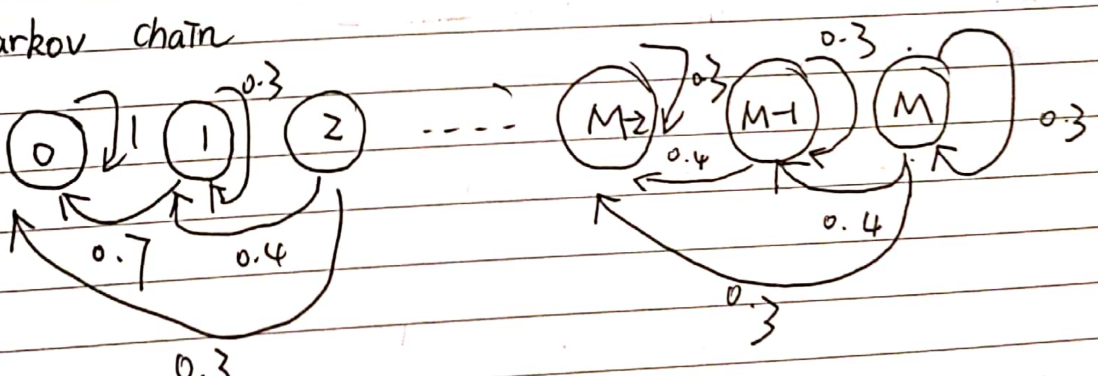
There are 3 cases

i). This fly moves towards the position of spider, which means the distance can be reduced 2 unit. (with 0.3 probability)

2) This fly stays in place ~~moves away from the spider~~, which means the distance can be ~~sta~~ reduced 1 unit (with 0.4 probability).

3) This fly moves away from the position of spider, which means the distance can be keep same. (with 0.3 probability)

Suppose the initial relative distance is M units, we can get this Markov chain



$Pr(i, i-2) = 0.3$ $P(i, i-1) = 0.4$ $P(i, i) = 0.3$

But for $P(1, 0) = 0.7$ (Because the spider captures the fly).
$P(0,0) = 1$

(b) 0 state is recurrent

1 ... M states are transient.