Continuous Assessment for EE5138 Optimization for Electrical Engineering

Semester 2, 20/21

This continuous assessment has 40 marks in total. Answer all the questions. Please submit your answer in one single PDF file (with your name indicated as the file name) on the class website (under the folder "Student Submission") by **11:59pm April 18 2021** (a firm deadline with no extension allowed). Please show your working in all the answers and include all the used **Matlab codes** in your submitted PDF file.

[40 marks] Consider the following unconstrained problem,

$$\texttt{minimize} \quad f(x) = -\sum_{i=1}^{m} \log(1 - a_i^T x) - \sum_{i=1}^{n} \log(1 - x_i^2),$$

with variable $x \in \mathbf{R}^n$, and $\mathbf{dom}\ f = \{x | a_i^T x < 1, i = 1, \cdots, m, |x_i| < 1, i = 1, \cdots, n\}$. Note that this is the problem for computing the analytic center of the set of linear inequalities:

$$a_i^T x < 1, \quad i = 1, \cdots, m, \quad |x_i| \leq 1, \quad i = 1, \cdots, n.$$

(a) Show the gradient and Hessian of $f(x)$ are, respectively,

$$\nabla f(x) = \sum_{i=1}^{m} \frac{a_i}{1 - a_i^T x} - \left(\frac{1}{1 + x_1} - \frac{1}{1 - x_1}, \cdots, \frac{1}{1 + x_n} - \frac{1}{1 - x_n} \right)^T,$$

$$\begin{aligned}\nabla^2 f(x) = &\sum_{i=1}^{m} \frac{a_i a_i^T}{(1 - a_i^T x)^2} \\ &+ \mathrm{diag}\left(\frac{1}{(1 + x_1)^2} + \frac{1}{(1 - x_1)^2}, \cdots, \frac{1}{(1 + x_n)^2} + \frac{1}{(1 - x_n)^2} \right),\end{aligned}$$

where $\mathrm{diag}(z)$ denotes a diagonal matrix with its main diagonal given by vector z.

[10 marks]

(b) Use *gradient method* to solve this problem. Consider the case of $m = 300$ and $n = 200$. Note that the parameters a_i's are stored in the matrix $A = [a_1, \cdots, a_m]^T$, which is given in the attached A.txt file. Please read matrix A in matlab using the command "A=dlmread('A.txt')" and then solve the problem based on this given A. Choose $x^{(0)} = 0$ as your initial point, and $\|\nabla f(x^{(k)})\|_2 \le 10^{-3}$ as the stopping criterion for gradient method. Use the backtracking line search with each of the following four groups of parameters: $\alpha = 0.01$, $\beta = 0.1$; $\alpha = 0.01$, $\beta = 0.5$; $\alpha = 0.2$, $\beta = 0.1$; $\alpha = 0.2$, $\beta = 0.5$.

(b.1) Find the optimal value p^* of this problem obtained by gradient method.

(b.2) Plot $f(x^{(k)}) - p^*$ versus iteration for the given 4 sets of backtracking parameters in one figure and comment on the effect of backtracking parameters α and β on the total number of iterations required for convergence.

(b.3) Plot the step size $t^{(k)}$ versus iteration for the case of $\alpha = 0.01$, $\beta = 0.5$.

[15 marks]

(c) Use *Newton method* to solve this problem. Consider again the case of $m = 300$ and $n = 200$ and use the same matrix A as in part (b). Choose $x^{(0)} = 0$ as your initial point, and $\lambda(x^{(k)})^2 \le 10^{-8}$ as the stopping criterion for Newton method. Set $\alpha = 0.01$ and $\beta = 0.5$ for the backtracking line search.

(c.1) Find the optimal value p^* of this problem obtained by Newton method.

(c.2) Plot $f(x^{(k)}) - p^*$ versus iteration and comment on the quadratic local convergence observed.

(c.3) Plot the step size $t^{(k)}$ versus iteration.

[15 marks]

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