Q1 Answer all TEN (10) multiple choice questions. Each question has One (1) mark and has only ONE (1) correct answer. Write the answers on your own answer sheet instead of this exam paper. For example, you can write: 1. a, 2. b, etc.

1. Which of the following statements is True?
   a. Intersection of a positive semidefinite cone and a second-order cone is not always a convex set.
   b. Intersection of a positive semidefinite cone and a second-order cone is not always a convex cone.
   c. Intersection of a hyperplane and a halfspace is always an affine set.
   d. Intersection of two hyperplanes is always an affine set.

2. Which of the following sets is *not* convex?
   a. $\{(x_1, x_2)|(\sqrt{x_1} + \sqrt{x_2})^2 \geq 1\}$
   b. $\{(x_1, x_2)|(x_1 - 1)(x_2 - 2) \leq 0\}$
   c. $\{(x_1, x_2)|x_1 x_2 \geq 1, x_1 < 0, x_2 < 0\}$
   d. $\{(x_1, x_2)|x_1^2/x_2 \leq 1, x_2 > 0\}$

3. Which of the following functions is a convex function?
   a. $-\log \frac{1}{x^2}$, $x < 0$.
   b. $-x \log |x|$, $x < 0$.
   c. $-x^{\frac{1}{2}}$, $x < 0$.
   d. $-e^{-x}$, $x < 0$.

4. Which of the following functions is a convex function?
   a. $\sum_{i=1}^{m} \log \log (x_i + 1)$ on $\mathbf{R}^m_{++}$.
   b. $-\log \sum_{i=1}^{m} \exp(x_i^2)$ on $\mathbf{R}^m$.
   c. $\exp\left(-\sum_{i=1}^{m} \sqrt{x_i}\right)$ on $\mathbf{R}^m_{++}$.
   d. $-\frac{1}{\sum_{i=1}^{m} \log(x_i+1)}$ on $\mathbf{R}^m_{++}$.

5. Which of the following functions is a concave function?
   a. $x_1^2 - x_2^2 + 3x_1 - 4x_2 - 1$.
   b. $x_1^2 + x_2^2 + 2x_1 x_2 - 4x_1 + 3x_2 + 1$.
   c. $x_1^2 - x_1 x_3 + 4x_1 - x_2 + 5$.
   d. $-2x_1^2 - x_2^2 + 10x_1 - 4x_2 - 10$.

6. Which of the following statements is True?
   a. The maximum of two quasiconvex functions is always convex.
   b. The maximum of two quasiconvex functions is always quasiconvex.
   c. The minimum of two quasiconvex functions is always convex.
   d. The minimum of two quasiconvex functions is always quasiconvex.

7. Which of the following functions is quasiconcave?
   a. $x_1^2 x_2^2$ on $\mathbf{R}^2_{++}$.
   b. x_1^2/x_2^2 on $\mathbf{R}^2_{++}$.
   c. $\sqrt{|x_1 x_2|}$ on $\mathbf{R}^2$.
   d. $\frac{1}{x_1 x_2}$ on $\mathbf{R}^2_{++}$.

8. Which of the following functions is log-concave?
   a. $e^{x_1} + e^{-x_2}$ on $\mathbf{R}^2$.
   b. $e^{\sqrt{x_1 x_2}}$ on $\mathbf{R}^2_{++}$.
   c. $e^{(x_1^2 - x_2^2)}$ on $\mathbf{R}^2$.
   d. $e^{(e^{x_1} - e^{-x_2})}$ on $\mathbf{R}^2$.

9. Which of the following statements is True?
   a. A convex function cannot be quasiconcave.
   b. A concave function cannot be quasiconvex.
   c. A strictly positive concave function is always log-concave.
   d. A strictly positive convex function is always log-convex.

10. Which of the following statements is False for two positive and differentiable functions $f_1(x)$ and $f_2(x)$?
    a. $f_1(x)f_2(x)$ is always convex if $f_1(x)$ and $f_2(x)$ are both convex and both non-decreasing functions.
    b. $f_1(x)f_2(x)$ is always convex if $f_1(x)$ and $f_2(x)$ are both convex and both non-increasing functions.
    c. $f_1(x)f_2(x)$ is always concave if $f_1(x)$ and $f_2(x)$ are both concave, one of them is a non-increasing function, and the other is a non-decreasing function.
    d. $f_1(x)f_2(x)$ is always concave if $f_1(x)$ and $f_2(x)$ are both concave and both non-decreasing functions.

Q2 Show that $\{x_c + Au + Bv \mid \|u + v\|_2 \leq 1\}$ is a convex set, with given $x_c \in \mathbf{R}^n$, and $A, B \in \mathbf{S}^n$.

[5 marks]

Q3 Let $f(x) = 2x_1^2 + x_2^2 - ax_1x_2 - 2x_1 + 3x_2 + 1$ be a quadratic function with $a \in \mathbf{R}$. Find the range of the values for a such that this function is convex.

[5 marks]

END OF PAPER