

EE5137: Quiz 2

Name: _____

Matriculation Number: _____

Total Score: _____

October 12, 2017

You have 1.0 hour for this quiz. You're allowed 1 sheet of handwritten notes (both sides). Please show provide *careful explanations* for all your solutions.

1. [Merged Processes] Let $N_1(t)$ and $N_2(t)$ be two independent Poisson processes with rates λ_1 and λ_2 respectively. Let $N(t) = N_1(t) + N_2(t)$ be the merged process.
 - (a) (5 points) Find the probability that $N(1) = 2$ and $N(2) = 5$. Express your answer in terms of λ_1 and λ_2 .

- (b) (5 points) Given that $N(1) = 2$, find the probability that $N_1(1) = 1$. Express your answer as a fraction and in terms of λ_1 and λ_2 .

2. [Waiting Time]

Patients arrive at the doctor's office according to a Poisson process with rate $\lambda = 1/10$ minutes. The doctor will not see a patient until at least three patients are in the waiting room.

- (a) (5 points) Find the expected waiting time in minutes until the first patient is admitted to see the doctor.

- (b) (5 points) What is the probability that nobody is admitted to see the doctor in the first hour (1 hour = 60 minutes)?

Write your answer as $c \times e^{-6}$ and find the constant c .

3. [Arrival Times] (5 points)

Let S_1, \dots, S_n be the arrival times of a Poisson process with rate λ . Find

$$\mathbb{E}[S_1 + S_2 + \dots + S_n | N(1) = n].$$

Hint: You may use the fact that $\sum_{i=1}^n i = n(n+1)/2$.