EE5137 Stochastic Processes: Problem Set 1
Assigned: 15/01/21, Due: 22/01/21

There are five non-optional problems in this problem set.

1. Exercise 1.1 (Gallager's book)
2. Exercise 1.2 (Gallager's book)
3. Exercise 1.3 (Gallager's book)
4. **Probability Review**: Flip a fair coin four times. Let X be the number of Heads obtained, and let Y be the position of the first Heads i.e. if the sequence of coin flips is TTHT, then $Y = 3$, if it is THHH, then $Y = 2$. If there are no heads in the four tosses, then we define $Y = 0$.
   (a) Find the joint PMF of X and Y;
   (b) Using the joint PMF, find the marginal PMF of X
5. (Strengthened Union Bound) Let $A_1, \ldots, A_n$ be arbitrary events. Prove that
$$\Pr\left\{\bigcup_{i=1}^{n} A_i\right\} \le \min_{1\le k\le n} \left(\sum_{i=1}^{n} \Pr\{A_i\} - \sum_{i=1\,:\, i\neq k}^{n} \Pr\{A_i \cap A_k\} \right).$$
   Hint: For any two sets C and D,
$$C = (C \cap D) \cup (C \cap D^c)$$

---

6. (Optional) Suppose there are n different types of coupons, and each day we acquire a single coupon uniformly at random from the n types. The coupon collector problem asks: "How many days before we collect *at least one* of each type?"

   Let's formulate this precisely. We will count the time before seeing each new coupon type. Let X_i be the random variable that denotes the number of days to see new type after setting the i-th type. The quantity
$$c_n = \mathbb{E}\left[\sum_{i=0}^{n-1} X_i\right]$$
   gives us the total number of days before we see all n types on average. Show that $c_n \approx n \ln n$. Make this precise.