1. FALSE. For $\{x \in S^n \mid X \preceq 0\}$, this is negative semidefinite. Use the definition of convex set, we can conclude that it is not convex set.

2. FALSE. For a positive semidefinite cone, it is convex set. For a second-order cone (Euclidean norm cone), it is convex set. Use the definition (the intersection of any number of convex sets is convex. Therefore, the intersection of a positive semidefinite cone and a second-order cone is always convex cone.

3. TRUE
For $\{x \in R^n, y \in R^m \mid \sum_{i=1}^{n} x_i A_i \succeq \sum_{j=1}^{m} y_j B_j, A_i, B_j \in S^P\}$ is the solution set of linear matrix inequality. It is the intersection of half space. Therefore, it is convex set.

4. TRUE
For $\{x \in R \mid x^2 - 4x + 5 \geq 0\}$ it is positive definite. Clearly, it is the convex set

5. FALSE
For $f(x) = x^2 \log x$ $f'(x) = x \log x + x^2 \cdot \frac{1}{x} = x \log x + x$ $f''(x) = \log x + x \cdot \frac{1}{x} + 1 = 2 + \log x$

~~when we take $x = \frac{1}{e}$, we get the global minimum~~ For $x \in (0, \frac{1}{e^2})$ $f''(x) < 0$

we ~~can plot this function~~ ~~For~~ $x \in (\frac{1}{e^2}, +\infty)$, $f''(x) > 0$. So it is not convex

Date No

6. TRUE

For $f(x_1,x_2)=\frac{x_1^2}{\sqrt{x_2}}$ $\quad \frac{\partial f'(x_1,x_2)}{\partial x_1}=\frac{2x_1}{\sqrt{x_2}}$ $\quad \frac{\partial f(x_1,x_1)}{\partial x_2}=-\frac{x_1^2}{2x_2^{\frac{3}{2}}}$

$\frac{\partial^2 f(x_1x_2)}{\partial x_1^2}=\frac{2}{\sqrt{x_2}}$ $\quad \frac{\partial^2 f(x_1,x_2)}{\partial x_1\partial x_2}=-\frac{x_1}{x_2^{\frac{3}{2}}}$ $\quad \frac{\partial^2 f(x_1,x_2)}{\partial x_2^2}=\frac{3}{4}x_1^2x_2^{-\frac{5}{2}}$

$H=\frac{1}{\sqrt{x_2}}\begin{bmatrix}2 & -\frac{x_1}{x_2}\\ -\frac{x_1}{x_2} & \frac{3}{4}\frac{x_1^2}{x_2^2}\end{bmatrix}$ $\quad H\succeq 0$, so $f(x_1,x_2)=\frac{x_1^2}{\sqrt{x_2}}$ is convex function

7. FALSE

For $f(x_1,x_2)=x_1^2+x_2^2+4x_1x_2-4x_1+3x_2+1$

$H=\begin{bmatrix}2 & 4\\ 4 & 2\end{bmatrix}$ $\quad$ H is not ~~positive definite or semipositive definite~~.

$[x_1\ x_2]\,H\begin{bmatrix}x_1\\x_2\end{bmatrix}$ $\quad$ So $f(x_1,x_2)$ is not convex function

$=2x_1^2+4x_1x_2+2x_2^2=2(x_1+2x_2)^2-6x_2^2$

8. TRUE

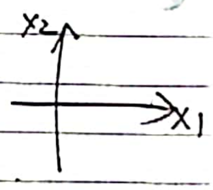
For $-\sum_{i=1}^m\log(x)$ is convex ~~concave~~ function, $\log(x+1)$ is a concave function

$-\sum_{i=1}^m\log(\log(x_i+1))$ is nonincreasing,

Therefore, $-\sum_{i=1}^m\log[\log(x_i+1)]$ is convex function

Inversly $\sum_{i=1}^m\log[\log(x_i+1)]$ is concave function.

9. TRUE for $f(x)=\frac{x_1^3}{\sqrt{x_2}}$ $\quad$ let $\frac{x_1^3}{\sqrt{x_2}}\geq\alpha$ $\quad x_2\leq\frac{x_1^6}{\alpha^2}$



$\because \frac{x_1^6}{\alpha^2}$ is power function, so $\frac{x_1^6}{\alpha^2}$ is convex function.

so $\{x_2\leq\frac{x_1^6}{\alpha^2}\}$ is superlevel set, then to equal $\frac{x_1^3}{\sqrt{x_2}}\geq\alpha$.

In conclusion, $f(x)=\frac{x_1^3}{\sqrt{x_2}}$ is quasiconcave function

10. FALSE

We can not find $0\leq\theta\leq1$, let it satisfy $f(\theta x+(1-\theta)y)\geq f(x)^\theta f(y)^{1-\theta}$

so it is not log-convex function