Modelling a multiprocessor (multi-core processors) with independent queues and common queues - Performance comparison.

$\boxed{\text{Dual-core}}$

Criterion for performance evaluation is w.r.t. average response times

$E[R_S]$: Avg. Resp. time under separate Qs

$E[R_C]$: Avg. Resp. time under Common Qs.

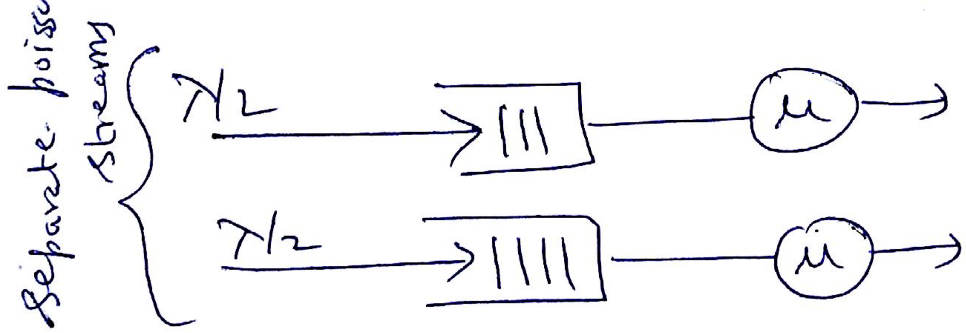
First design with separate Qs corresponds to 2 indep. M/M/1 system with $\rho = \left(\frac{\lambda}{2\mu}\right)$

- Using Little's formula : (for M/M/1)

$$E[R] = \lambda^{-1} \cdot \frac{\rho}{1-\rho} = \frac{1/\mu}{1-\rho} = \frac{\text{Avg. service time}}{(\text{prob. that the server is idle})}$$

For our separate Qs,

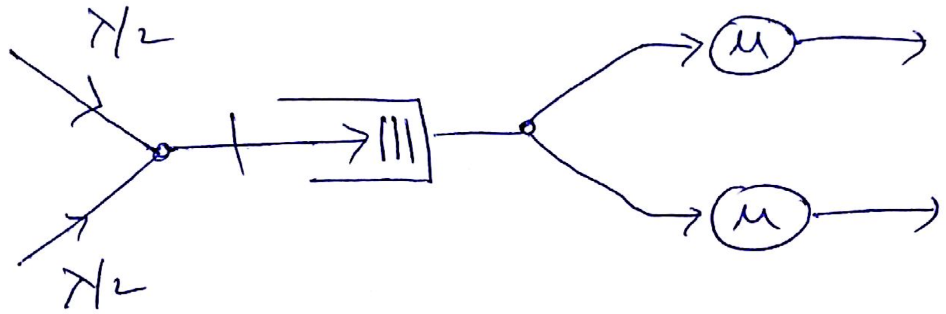
$$E[R_S] = \frac{1/\mu}{1 - \lambda/2\mu} = \left(\frac{2}{2\mu - \lambda}\right)$$



$$\boxed{E[R_S] = \frac{2}{2\mu - \lambda}}$$

(2)

For common Qs System;



we have M/M/2 & hence Using the relationship for M/M/m for E[N] we have:

$$E[N] = m\cdot\rho + \rho\frac{(m\rho)^m}{m!}\cdot\frac{p_0}{(1-\rho)^2}$$

where,

$$p_0 = \left[\sum_{k=0}^{m-1}\frac{(m\rho)^k}{k!} + \frac{(m\rho)^m}{m!}\cdot\frac{1}{(1-\rho)}\right]^{-1}$$

Thus, for our M/M/2 we have:

$$\begin{cases} E[N_c] = 2\rho + \rho\dfrac{(2\rho)^2}{2}\cdot\dfrac{p_0}{(1-\rho)^2} \quad \text{with} \\ p_0 = \left[1 + 2\rho + \dfrac{(2\rho)^2}{2!}\cdot\dfrac{1}{1-\rho}\right]^{-1} \end{cases}$$

simplifying $p_0 = \left(\dfrac{1-\rho}{1+\rho}\right)$

Thus,

$$E[N_c] = 2\rho + 2\rho^3 \cdot \frac{(1-\rho)}{(1+\rho)(1-\rho)^2}$$

$$= \frac{2\rho(1-\rho^2+\rho^2)}{1-\rho^2} = \frac{2\rho}{1-\rho^2}$$

Now using Little's formula,

$$E[R_c] = \frac{E[N_c]}{\lambda} = \frac{2 \cdot 1/2\mu}{1-(\lambda/2\mu)^2}$$

$$\boxed{E[R_c] = \frac{4\mu}{4\mu^2-\lambda^2}}$$

$$E[R_s] = \frac{2}{2\mu-\lambda} = \frac{2(2\mu+\lambda)}{4\mu^2-\lambda^2}$$

$$\therefore E[R_s] = \frac{4\mu+\lambda}{4\mu^2-\lambda^2} > E[R_c]$$

$\Rightarrow$ Common Q organization is better than a separate Q organization. This result can be generalized for m-cores case