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Subject: Information Theory

Assignment: Homework Two

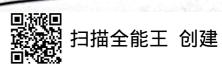
Date: Aug 22<sup>th</sup>

Prof: Marco Tomamichel

Exercise 2.1
Date No.
Proof for 7 = 80,1, R-14, we set 8p., B. PR be the pmf for X
Proof for 1 = $f(x) = -t \log t$ is concave function in $[0, 1]$
And following the $f(x) = -t \log C$ is interest.
And there is a constraint ZPO;= 1
so, H(x) = Zex Px log Px = E[log Px]
By using Jensen's inequality, we can rewrite as
By using Jensen's inequality, $H(x) = E \left[ \log \frac{1}{Px} \right] \leq \left[ \log \frac{1}{Px} \right]$
= 109 El Px · Px
J re-x
= log  X
As desired, we prove that is upper labound of entropy.
TO RESIDENT AS THE PROPERTY OF
Exercise 2.2
a) Firstly, we know $D(P_X    V_X) = \sum_{x \in X} P_X(x) \log \frac{P_X(x)}{V_X(x)} = \sum_{x \in X} P_X(x) \log \frac{P_X(x)}{ x }$
$V_{\star}(x) \times eX \longrightarrow \overline{ x }$
$\sim 21 \log \sqrt{2}$
XXX PAR
$H(x) = \overline{Z} P_{x}(x) \log \frac{1}{P_{x}(x)} = -\overline{Z} P_{x}(x) \log \frac{1}{P_{x}(x)} P_{x}(x)$ $\times \epsilon \chi \qquad \times \epsilon \chi$
$= -\sum_{x \in X} P_{x}(x) \log \left[ \frac{1}{ x } \cdot \frac{P_{x}(x)}{ x } \right]$
= - \(\sum_{\text{Nex}} P_{\text{Nex}} \right) \log \( \frac{1}{ \text{X} } - \sum_{\text{Px}} P_{\text{X}} \right) \log \( \frac{P_{\text{X}} \cdot \frac{1}{ \text{X} }}{ \text{T} } \)
X6X 1 X6X 1 X1
= - log   - P (Px 11 Ux)
$= -\log x  - \left( \frac{1}{ x } + \frac{1}{ x } \right)$
= log x1 - P(Px 11Ux)
b Firstly, we know H(XIT) = \(\Sigma\) \(\Sigma\)   \(\Si
XEX.TET P(XIT)
7. P. 11. [ ] PAIR 7
= - Z P(x,y) log [1 P(x)]
/h
/h
= - \(\sum_{\text{rest}} P(\text{rest}) \log \frac{1}{ \text{x} } - \sum_{\text{rest}} P(\text{rest}) \log \frac{P(\text{rest})}{ \text{x} } \log \fract{P(\text{rest})}{ \text{x} } \log \frac{P(\text{rest})}{ \text{x} } \log \fract{P(\text{rest})}{
= - \(\sum_{\text{rest}} P_{(\text{N} \text{v})} \log \(\frac{1}{\text{XI}} - \sum_{\text{N}} \P_{(\text{N} \text{v})} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{V})}} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{N} \text{V})}} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{N} \text{V})}} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{N} \text{N})}}{P_{(\text{N} \text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{N} \text{N})}}{P_{(\text{N} \text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{N} \text{N})}}{P_{(\text{N} \text{N} \text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{N} \text{N})}}{P_{(\text{N} \text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{N} \text{N})}}{P_{(\text{N} \text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{N} \text{N})}}{P_{(\tex
= - \(\sum_{\text{rest}} P_{(\text{N} \text{v})} \log \(\frac{1}{\text{XI}} - \sum_{\text{N}} \P_{(\text{N} \text{v})} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{V})}} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{T})}}{P_{(\text{N} \text{N} \text{N})}} \log \(\frac{P_{(\text{N} \text{N} \text{N})}}{P_{(\text{N} \text{N} \text{N} \text{N})}} \log \(P_{(\t
/h



c) <b>19</b>	Carrier place Allegation of the Carrier and Carrier an	
	d (b), we can get	lo.
1 (x:1)	$= H(x) - H(x Y)$ $= I_{1} I_{2} I_{3} I_{4} I_{4} I_{5} I_$	
	= log [x] - D(PxIIVx) - [log [x] - D(Pxx     V x x Px)]	
	= D(Pxr   Ux xPr) - D(Px   Ux)	
	= El P(x,y) log P(X,y) - El Px(x) log Px(x)  xex rer  Pr   x   Xex	
	PY   XI XEX   XX	
	= E P(x,y) log P(x,y) = 7, Rx,T) log Px(x)	
	= E P(x,y) log P(x)y) = E P(x,r) log P(x)y  *ex, ref   Pr	
	P(x,y)	
-	Xex. Ter P(x, y) log P(x, y)  Px · Py	
	= D(Pxx II Px x Px)	
	= P( PXY 11 / X / Y)	
	1 30 1 30 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1	
Exercise 2.3		
By setting th	the alphabet size at most 2 bits	
A). H(XITZ)=	0 ⇒ H(x YZ) = ZNZ P2 H(x Y=y, Z=z) =0.	
	=> Px   YZ = 0 or Px   YZ = 1	
H(XIT) = H	$H(x z) = 1 \Rightarrow \sum_{x \in X} P_x H(x x = x) = \sum_{x \in X} P_x H(x z = z) = 0$ $P_x   x = m^{-1} \text{ and } P_x   x = \frac{1}{2}$	
	$\Rightarrow P \times  Y = 0 = 1$ $\Rightarrow P \times  Y = 0 = 1$ $\Rightarrow P \times  Z = 1$ $\Rightarrow P \times  Z = 1$	
For example	x x x Z	
For example	0 0 0	
	2 1 0 1	
	3 1 1 0	
	0111	
1 27 - 1/2 / 10/2		.)-0
Comment of the second	= $ \Rightarrow H(Y Z) - H(Y XZ) =  \Rightarrow H(Y Z) =  and H(Y XZ) =  $	)=0
b). I(x: r 1 2):	$=   \Rightarrow H(Y Z) - H(Y Z) - H(Y Z) = 0 \text{ or } P_{Y XZ} = 1 $ $\Rightarrow P_{Y Z} = \frac{1}{2} \text{ and } \left[ P_{Y XZ} = 0 \text{ or } P_{Y XZ} = 1 \right]$	-
	$\Rightarrow PY z=\frac{1}{2}$ and $LY z=0$	
T(x:1)=0	$\Rightarrow H(X) - H(X) = 0$	
		- Carrie
For example		
101 example	0   0	
	3 0 1 0	
	0 0	ΔLCOΠ
	1 1 1	ДСОП



c)  $I(x:t)=1 \Rightarrow H(T)-H(T(x))=1 \Rightarrow H(T)=1$  and  $G_{abs}H(T(X))=0$ L(m) 12) - 17 (12) - 1 > Y is uniform dietributed => Prix = 1 or o And I (x: [ ] = H ( T | Z ) - H ( T | X Z ) = 0 for example: 0 01 (2) (3) 0 d) I(x:x) = I(x:z) = 1 => H(x)-H(x1x)=lond H(z)-H(z1x)=1 But for I(T: 2) = 0 => Parix = 0 or 1 / Pzix = 0 or 1 so, for example: 0 01 Exercise 2.5 For the first part, if we know X1, X2 -- Xn are mutually independent 50, I(x1, .... Xn = 1, .... Yn) = H(x1, x2 -... Xn) - H(x1, x2 -... xn | Y1, 12 --- Yn) = H(X1)+H(X2 | X1)+ ... H(X1 | Xn-1) - H(X1, X2 ... X1) Ti, Ti - Ti Because x1.... XT are mutual independent, we can rewrite above equation = H(x)+H(x)+... H(Xn)-H(X, x2-- x17, 12 .- x) = H(X1) +H (X2) +... H(XN) - H(X | Y1 /5 ... - Tn) + A H(X2 | X1 , T1 --- Tn) + -- - + H(Xn) X1 - Xn-1, + -- - Tn > During H(XI) = H(XI) = D & H (x1) - [H(X1) Ti --- Ta) -- -- + (Xn) Ti ... Ta)] Owing to H (X, 1), Tr ... rn) = H(X, 1), H(X2), ... rn) = H(X) ... H(xn/7, ... Tn) < H (xn/Tn) So jurabove  $\frac{1}{2}$  H(x;) - [H(x|Yi) +-- H(Xn|Yn)] =  $\frac{1}{2}$  H(x) -  $\frac{1}{2}$  H(x) -  $\frac{1}{2}$  H(x) | Yi) = \( \frac{7}{2} \ldots (x: \( \text{i} \)

Date No.
And for the second part, if we know hi is in conditional independent
of all the remaining random variables, Given Ti)
so. I (X1 ··· Xn: T1 ··· Tn) = H (X1, X2 ··· Xn) - H (X1, X2 ··· Xn   T1, 12 ··· Yn)
= H(X1) + H(X2/X1) + ++(x+ H(Xn/Xn-1)
-[H(X, 17, T2 Yn) + H(X2   X1, T1 Yn) + + H(Xn   X1 Xn-1, T1 Yn)
= +1 (X1) + H(X2  X1) + +1(Xn   Xn-1)
- [H(X1)Y1) + H(X2  Y2) + H(Xn1Yn)]
During to $H(x_2 x_1) \leq H(x_2)$
Uning 10 2
$H(X_n X_{n-1}) \leq H(X_n)$
Therefore, above equation, we can rewrite $= H(X_1) + H(X_2) H(X_1) - \left[ H(X_1 Y_1) + H(X_2 Y_2) + \cdots + H(X_n Y_n) \right]$
= +(x <sub>1</sub> ) + H(x <sub>2</sub> ) - Z H(x <sub>1</sub>   Y <sub>1</sub> )
= 1 7 (\(\lambda'\) 7=1 (\(\lambda'\) 7=1
= Z IN(xi = Yi)
C to Linished
In a word, the proof is finished.
FΔLCON

