Lecture 16: Feedback channel and source-channel separation

- Feedback channel
- Source-channel separation theorem

Feedback channel

- in wireless communication, receiver estimate channel and sends channel state information (CSI) to transmitter
- feeding back CSI is a fundamental way to combat multipath and fading
- feedback enables many wireless communication techniques: adaptive modulation, power allocation, multiple-input-multiple-output (MIMO) communication
- ARQ protocol (Automatic Retransmission Request) uses feedback to coordinate reliable communication

Feedback capacity

- Assume feedback has no delay, infinite feedback capacity: all received symbols are sent back immediately and noiselessly to the transmitter

- Fundamental question:

  can we do better with feedback?

- Shannon's answer: No.

  Theorem (Feedback channel capacity)**.**

$$C_{FB} = C = \max_{p(x)} I(X;Y)$$

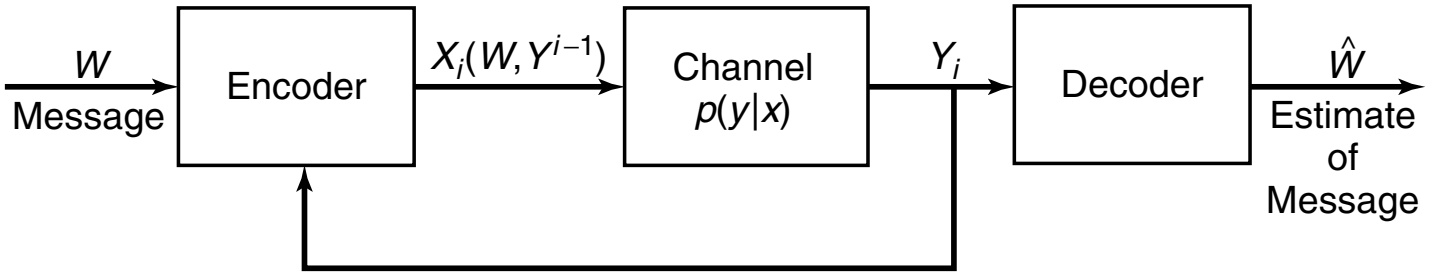


FIGURE 7.13. Discrete memoryless channel with feedback.

Feedback really has no value?

- in binary erasure channel, feedback can help simplifying encoding and decoding

- feedback cannot improve capacity of a channel, but under the assumptions:
  - infinite feedback capacity - what if we can only send back limited number of bits? What if feedback is erroneous?
  - zero delay - what if feedback has delay?
  - infinite code length - what if code length is finite?

- feedback can improve capacity in some cases: multiuser interference channel

  New view: Feedback capacity of the Gaussian inference channel to within 2 bits, C. Suh and D. Tse, 2011, IEEE Trans IT.

Source-channel separation theorem

- It's time to combine the two main results we had so far:
  - Data compression $R > H$
  - Data transmission: $R < C$

- is the condition $H < C$ necessary and sufficient for sending a source over a channel?

Source-channel separation

- sending digital music or speech
- two-stage method:
- Step 1: compress music into its most efficient form
- Step 2: map the sequence of music code into channel codes
- Are we loosing anything in two-stage method?
  data compression does not depend on channel, channel coding does not depend on source distribution

Two-stage method is optimal

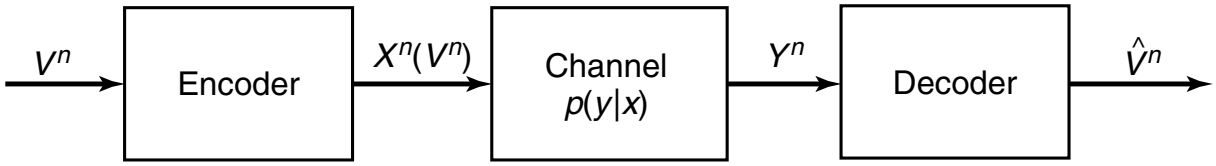


FIGURE 7.14. Joint source and channel coding.

Theorem (Source-channel coding theorem). *If $V_1, V_2, \ldots, V_n$ satisfies AEP and $H(\mathcal{V}) \leq C$, there exists a source-channel code with $p(\hat{V}^n \neq V^n) \to 0$. Conversely, for stationary process, if $H(\mathcal{V}) > C$, probability of error is bounded away from 0.*

Implication of source-channel separation

- keep designs of source and channel coding separate
- greatly simplifies communication system design
- source coding: find the most efficient representation of the source (removes redundancy)
- channel coding: encodes the message to combat the noise and errors (introduces designed redundancy)

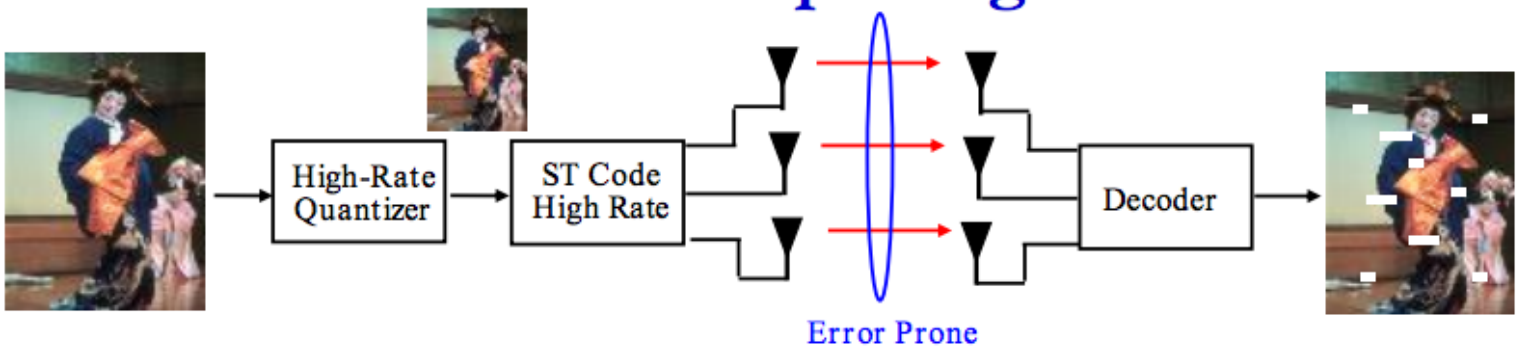
Two-stage method is not always optimal

Theorem assumes: $n \to \infty$; point-to-point DMC
Source-channel coding should not be separated in:

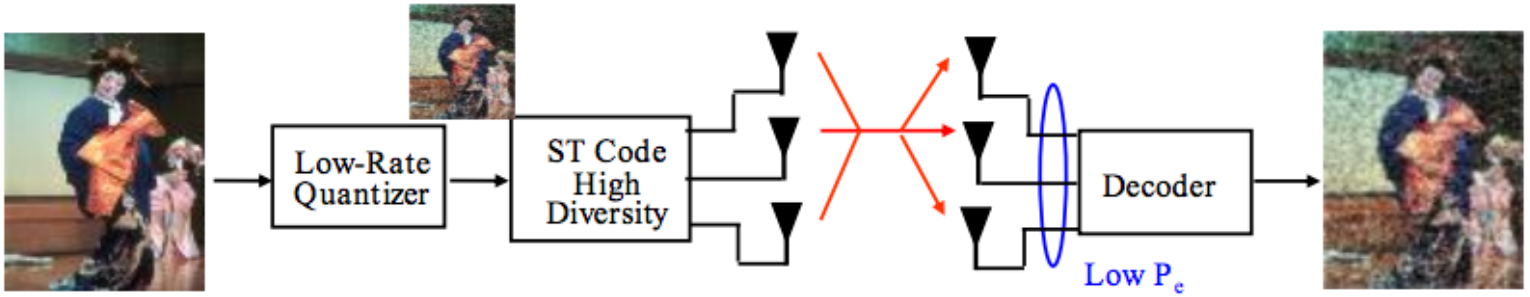
- Multiuser channel
- sending English text over erasure channel: sending binary sequences, the corrupted bit would be extremely hard to recover. if we directly send English through channel, must easier to recover.
- redundancy in the source is suited to the channel
- in speech and video transmissions, joint source-channel coding is valuable (since early 90s)

Joint source and channel coding for MIMO systems, T. Holliday and A. Goldsmith, 2008.

- **Use antennas for multiplexing:**



- **Use antennas for diversity**



Diversity/Multiplexing/Delay Tradeoff at Links with ARQ

example by A. Goldsmith

Take-home message

- Feedback does not increase capacity
- Source coding and channel coding can usually be separately designed
- Caveat: not always true, has to specify assumptions