

CLOS Network

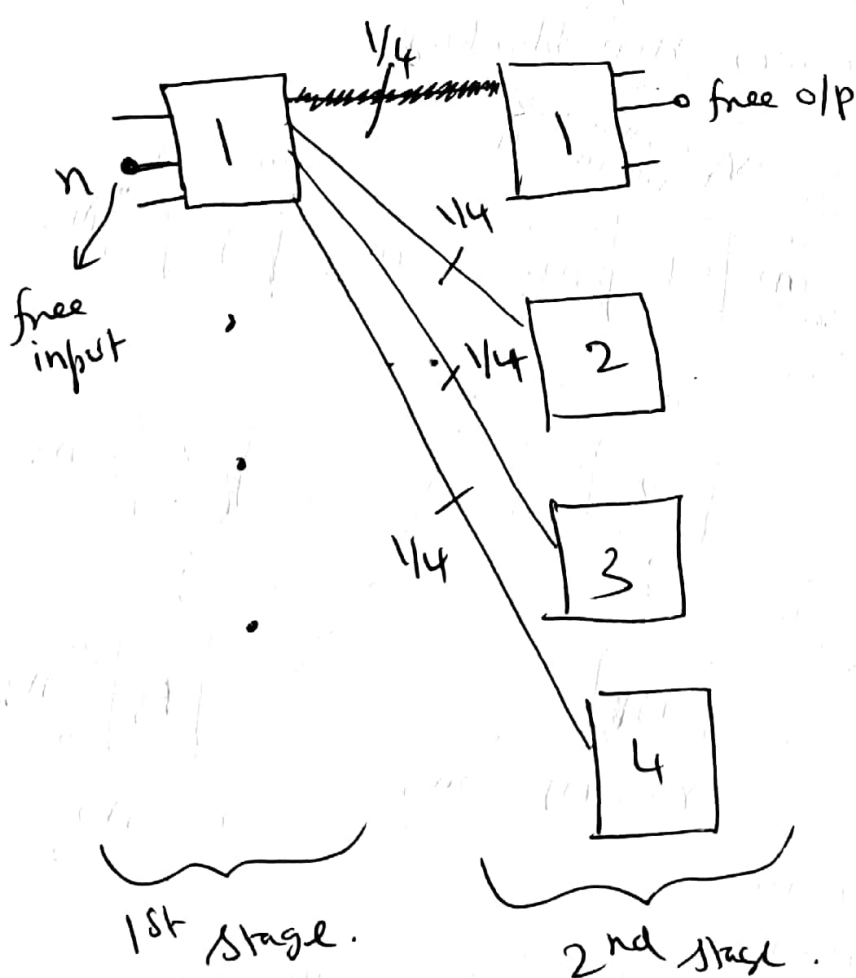
①

Our objective - To design a strictly non-blocking interconnection switch.

Recall: Two-stage was unable to meet this strictly non-blocking requirement; In addition, switch complexity would grow quadratically.

• Bell Labs paper - by Charles Clos

Idea Extending from 2-stage networks

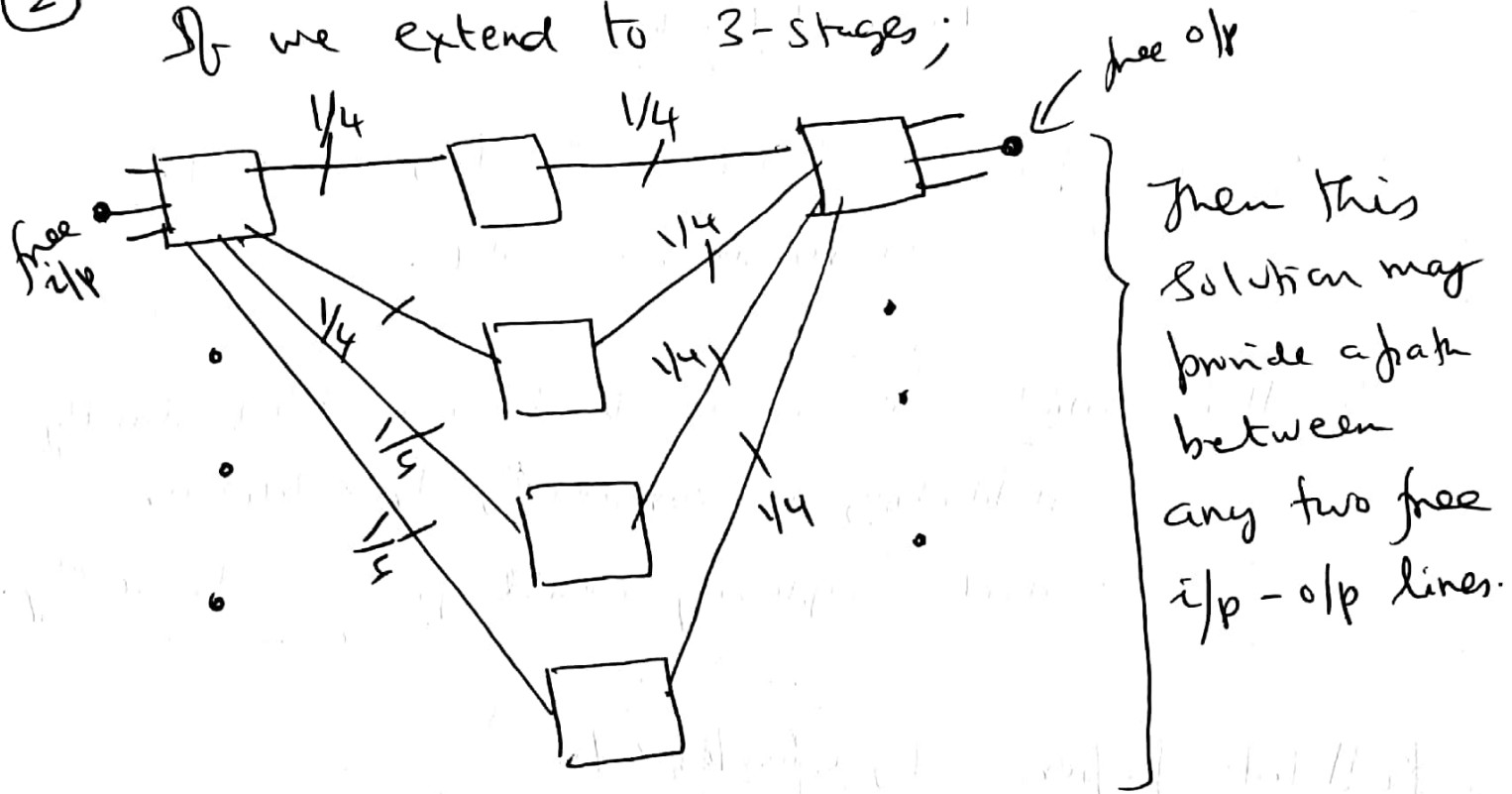


Issue is when there is one free i/p on SW₁ (1st stage) & 1 free o/p on SW₁ (2nd stage), there is no way a connection could be established as the $\frac{1}{4}$ th link are currently occupied.

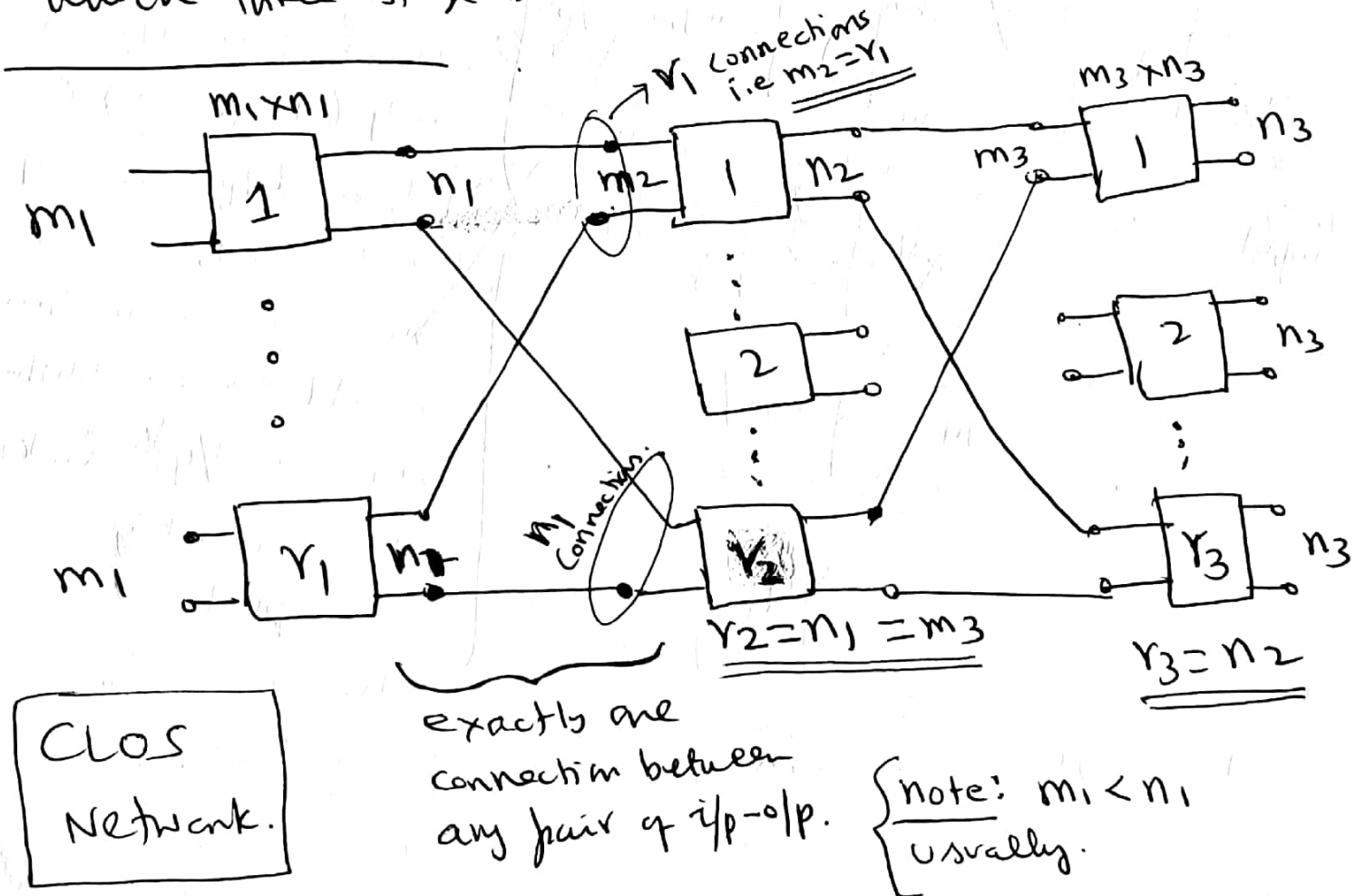
This leads to blocking.

②

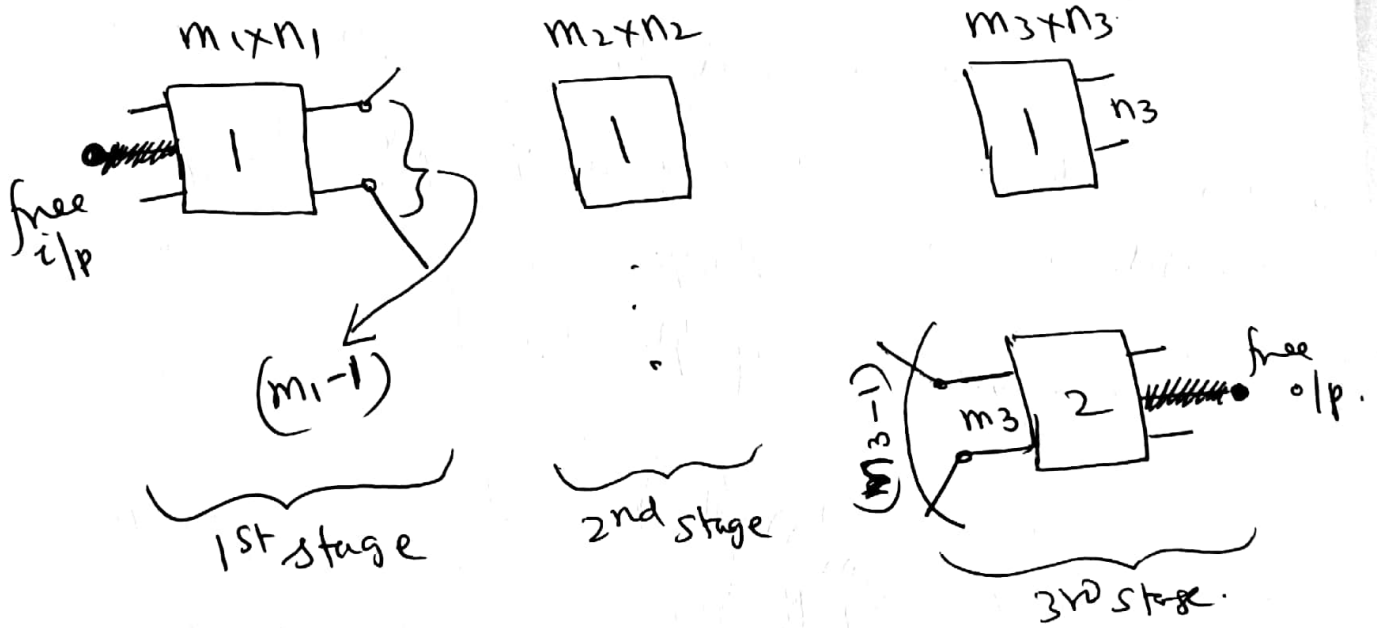
If we extend to 3-stages;



Actually, we can increase the # of switches in the middle stage! Clos came up with a condition under which three-stage becomes non-blocking.



Consider two free i/p - o/p pairs & let us see if they can be connected.



3rd stage: Consider SW_2 . Out of m_3 only n_3 can be connected (max $(m_3$ could be greater than $n_3)$)

Since one o/p is free only (n_3-1) can be occupied or emanating from the input side.

1st stage: SW_1 — From SW_1 only (m_1-1) can be going out occupied (as one is free); That is out of m_1 input lines, $m_1-(m_1-1)$ will be free.

So, if (m_1-1) lines are occupied / coming out of stage 1, this means there are (m_1-1) switches in the middle stage that are used for all other lines from SW_1 in stage 1.

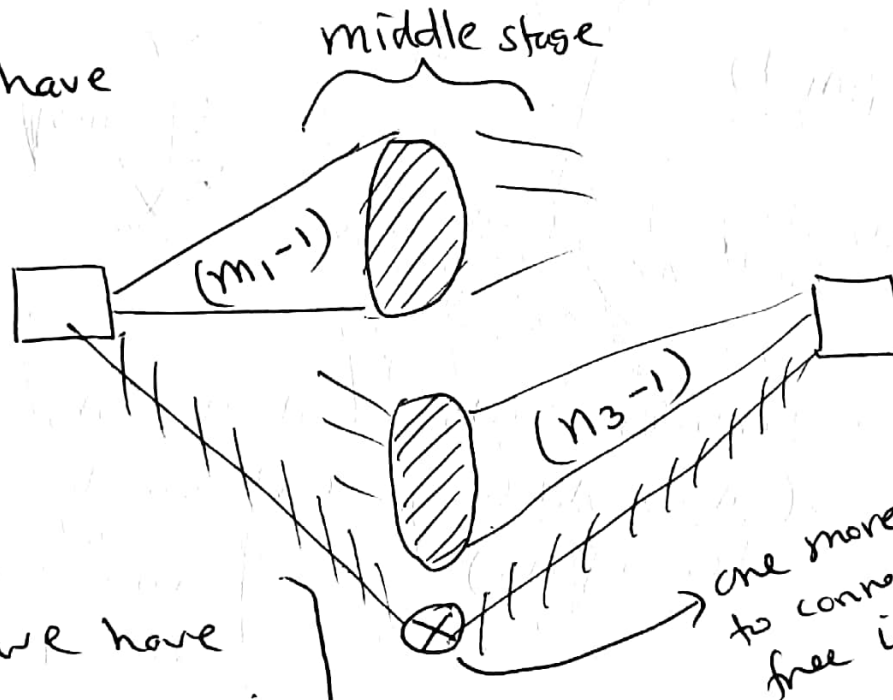
(4)

Similarly, see sw2 of 3rd stage.

$(n_3 - 1)$ middle stage switches are used
to connect the requests to the 3rd stage.

- Note that we have our free link at Switch 2 at stage 3.

So, we have



So, if we have one more switch in the middle stage then we will be able to connect our free i/p & free o/p link.

Therefore,

$$r_2 > (m_1 - 1) + (n_3 - 1)$$

$$\Rightarrow r_2 > m_1 + n_3 - 2$$

strictly non-blocking

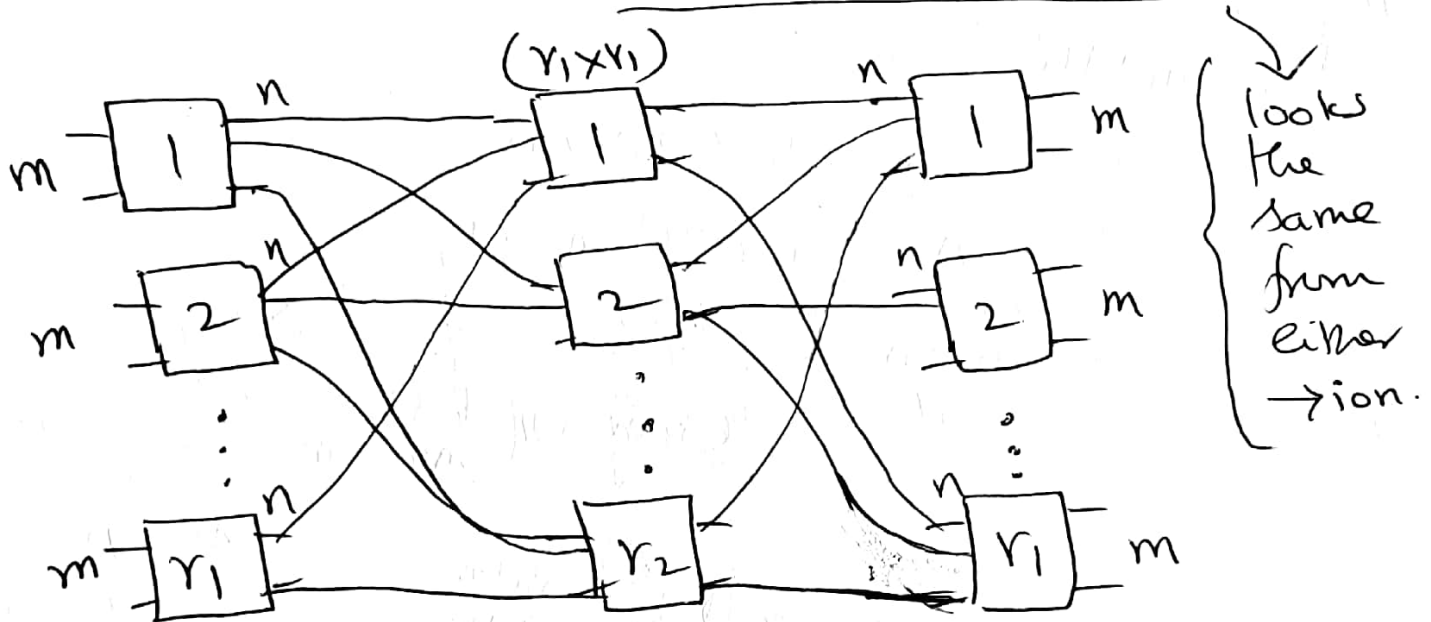
$$\text{note: } r_2 \geq m_1 + n_3 - 1$$

Also, note that,

$$m_3 = n_1 = r_2 \geq m_1 + n_3 - 1$$

Q: How many cross points required for this switch configuration?

Let us consider a symmetric configuration



• for symmetric confg. $m_1 = n_3$
 $\therefore r_2 = n$

from our condition:
 $r_2 \geq m + m - 1$ i.e., $r_2 \geq 2m - 1$
 So, $n = r_2 \geq 2m - 1$

of input points: $N = (m \times r_1) \Rightarrow r_1 = (N/m)$

of cross points needed: $(2m-1)$ switches in the middle stage;


So, each switch in the first stage will be having $m(2m-1)$ cross points.

There are r_1 switches in the first stage.

⑥ Therefore, # of crosspoints in the first stage is: $r_1 \cdot m \cdot (2m-1)$, i.e., $\left(\frac{N}{m}\right) \cdot m \cdot (2m-1)$

\nwarrow r_1

For middle stage:

each switch : $n \times n$ 

$$\& n = r_2 = (2m-1)$$

$$\therefore n \times n = (2m-1) \left(\frac{N}{m}\right) \left(\frac{N}{m}\right)$$

\nearrow
of switches
in the middle
stage.

Third stage: Same as first stage.

Therefore, # of cross point Q

$$Q = \underbrace{2m(2m-1) \frac{N}{m}}_{\text{1st \& 3rd stages}} + \underbrace{(2m-1) \frac{N}{m} \cdot \frac{N}{m}}_{\text{middle stage}}$$

We can further simplify. If we assume if we have one more middle switch $2m-1$ becomes just $2m$.

$$\therefore Q = 4mN + 2 \frac{N^2}{m}$$

It is better to express complexity in terms of N & hence we will find an optimum m^* .

$$Q = 4mN + \frac{2N^2}{m} \left. \vphantom{\frac{2N^2}{m}} \right\} \begin{array}{l} \text{\# of cross-points} \\ \text{in the system.} \end{array}$$

$$\Rightarrow \left(\frac{dQ}{dm} \right) = 0 = 4N - \frac{2N^2}{m^2}$$

$$\Rightarrow 2N \left(2 - \frac{N}{m^2} \right) = 0, \quad N > 0$$

$$\Rightarrow \boxed{m^* = \sqrt{N/2}}$$

$$\therefore Q = 4 \cdot \sqrt{\frac{N}{2}} \cdot N + \frac{2N^2}{\sqrt{N/2}}$$

$$Q = 2N^{3/2} + 2\sqrt{2} \cdot N^{3/2} = \underline{\underline{4\sqrt{2} N^{3/2}}}$$

$$\therefore \text{complexity} = O(N^{3/2})$$

So, # of cross points are not growing as N^2 !
We minimized to $O(N^{3/2})$.
