NUS National University of Singapore

EE5139/EE6139 Homework 7

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Exercise 7.1 More properties of linear codes (all)

Show the following properties of a (binary) linear $[n, k]$-code C.

a.) The minimal distance $d(C)$ is the minimal Hamming weight of all (non-zero) codewords.

b.) If H is the parity check matrix of C, then $d(C)$ equals the number of columns of H that are linearly dependent.

c.) Prove that (after permuting the coordinates if necessary) C has a generator matrix of the form $G = [I_k \ G']^T$ where I_k is the $k \times k$ identity matrix, and where G' is some $k \times (n - k)$ matrix.

Exercise 7.2 Modified linear codes (EE5139)

Some of the following operations on rows or columns of the generator matrix G or the parity-check matrix H may decrease the minimum distance of a linear block code? Which of the operations below can cause a reduction in the minimum weight? **Note: Here G is a $n \times k$ matrix.**

a.) Exchanging two rows of G.

b.) Exchanging two rows of H.

c.) Exchanging two columns of G.

d.) Exchanging two columns of H.

e.) Deleting a row of G.

f.) Deleting a row of H.

g.) Deleting a column of G and the corresponding column of H.

h.) Adding a column to G and a corresponding column to H.

i.) Adding one column of H to another column of H.

Exercise 7.3 Sudoku and the belief propagation algorithm (all)

Sudoku is a classical mathematical puzzle in which a player is asked to fill in missing numbers in a 9×9 array where each of the nine rows, nine columns, and nine 3×3 sub-arrays consists of numbers $\{1, \dots, 9\}$. An example is given in Figure 1.

a.) Denote the configuration of a Sudoku by $\{x_{i,j}\}_{i,j}$ where, for each $(i, j) \in \{1, \dots, 9\}^2$, $x_{i,j} \in \{1, \dots, 9\}$ is the number at the (i, j)-th location. Define the function $g : \{1, \dots, 9\}^{81} \to \{0, 1\}$ as

$$g(x_{i,j} : i, j \in \{1, \dots, 9\}) = \begin{cases} 1 & \text{if } \{x_{i,j}\}_{i,j} \text{ composes a valid Sudoku} \\ 0 & \text{otherwise} \end{cases}.$$

Represent g as a factorization of f over different arguments, where $f : (x_1, \dots, x_9) \mapsto 1\{\{x_1, \dots, x_9\} = \{1, \dots, 9\}\}$.

b.) Draw the factor graph corresponding to the factorization in a.).

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

Figure 1: A typical Sudoku and its solution (from Wikipedia)

c.) Suppose $\{x_{i,j}\}_{(i,j)\in\mathcal{A}}$ is known as the initial condition of the Sudoku, where $\mathcal{A}$ is a proper subset of $\{1,\ldots,9\}^2$. One could use the belief propagation algorithm to estimate the remaining positions. To do so, a partially finished MATLAB program has been provided. Please fill in the gaps in the program and run the program to solve the embedded Sudoku.