Lecture 1: EE5137.

	Subject: Date:
- 44.5	Reading: Sections. 1-1-1-4-1 of Gallagers Book.
	Ω: sample space, i.e. the set of all sample ponto for a given experiment. Egi Coin tous Ω= EH, T? Dice throw Ω= ?1,2,3,4,5,6%.
Lagr	Events: Subsets (Legitimite) of the sample space. Esi Odd outcomes E=21,3,53 C.S.
	If we have events A, Az, -A, C. Sz, their union is denoted as Vi=, Ai or A, U U An; this consists of all pts in at least
<u> </u>	one of the events A, Az, An.
	Interaction Ai or AI () Az Now () An in the pot of mints
	Intersection Ai or AI NA2 non NAn in the set of points in It that are contained in all of the Ais
	The complement of a set A, denoted as A; is the set of points not in A. AC= SC/A or AC= SC/A or AC=
	not in A. AcassiA or Anasse Je MA.
	A series of the set in the set of
	A countable set is one in which the objects can be placed in one-to-one correspondence with the northead numbers N = \$1,2,3,?
	Eg: The Jet of even numbers is countable.
	The set of all integers #= T, -2-10, 1,23) is countable.
	The set of all rational numbers $Q = [-1, -1, 2, 3, -1]$ is countable.
- y y ⁽¹) - z = 1	Some a large to be a marked and the sound an
	Axioms to Events.
	Disample space. The class of all subsets (of D) that constitute the set of Chaptimite) events (i.e., the o-algebra) satisfies
	the set of (lightmite) events (i.e., the T-algebra) sortisties
	i) Si u an event so on event she union U Ai - us an event.
	THE WHILE IT IN WHICH I TO US UN EVAT.
	TiD & A evat, AC = DIA is an evert.

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	The class of all subsets that vatishes (i) - (iii) is called a J-algebra
Fact	Why? the empty set, is on event. Why? the Sec. Now you (i) & (iii)
	Why? PZ SEC: Now yor (i) & (iii)
Fact	Ay-, An is a frite collection of evals. Then U Ai is an eval.
	17 1 1 (·) () () () () () () () () (
	Why? Use (ii) & let Anti=Anti==p.
fact:	Every finite or countable intersection of events is also an event
	PR: (UA;) = A; C = (UA; C) = (A;
Thay	Ti Cim in
	event 2 event
	ent
Fact.	The class of all subsets of SL (if & SL is uncountable) does
100	not allow to probability axiom to be vativated in a "sevible" way-
	But this is not needed for our module
	Axiom of probability.
konst	
3/4	For any sample space I , and any class of legitimate evots
	For any sample upace Ω , and any class of legitimate events E (a σ -algebra which satisfies the axiom of events), a probability rule $Pr\bar{r}$ \bar{r} then; $E \to t_0, 1\bar{t}$ s.t.
Viol	two of the site of the state of view shows the
	1) Pr (N) = 1
	ii) $\forall A \in \mathcal{E}$, $P_r(A) \geq 0$ iii) $\forall A, A_2, \dots$ dojoint eveto $\in \mathcal{E}$, $P_r(A_n) = \sum_{n \geq 1} P_r(A_n) = \sum_{n \geq 1} P_r(A_n)$
	This leads to the fillowing faction

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Fact 1)	P-(\$)=0
	Pf: let An= & the N. The And are disjoint. Unes An= \$.
	$P(\phi)=P(\mathcal{V}, A\lambda)=\lim_{m\to\infty}P(A_n)=\lim_{m\to\infty}P(\phi)=\lim_{n\to\infty}mP(\phi),$
	Since P(4) is a real non-negative number, P-(4)=0.
fact il)	If An., Am disjoint, Pr (DAn) = I Pr (An)
	Apply axion (iii) to the sequence Ai, Az,, Am, Anti=Antz== 4.
Facf (iii)	
	Now use (i) & Fact (ii).
	Now use (i) & tact (ii).
Fact (iv)	P(A) EP(B) YA,BEEJ. ASB.
_{WC1_O-J_	11 (1) (1) VA, DCC JA. 11 26
	B= AU(B\A) A& B\A are disjoint.
	P_(B) = P_(A) + P_(B\A) 2 P_(A)
Fact (V)	$P(A) \leq A + B = \Omega$
10((1 (0)	- A R 75 A 1 A 1
Fact (vs)	LPCAN SI Y siquence of disjoint evals As As.
	Pf. Let A'n Fort (i) by Az DAN Men we Fact 1(11).
Fact (vin)	P-(MAn)= man Pr(MAn) (continuity of measure)
	for any requerce of not recovarily disjoint events AvAz, ∈ E.
	To the second se

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	Pf: DAn= L Bn. where Bi=Ai				
	B) = A2 \ A1				
	M / 1				
(4/3.5	$B_n = A_n \setminus \bigcup A_i$				
3, 4, 1.1					
	Hence Flothate Reliable				
	= TP(B)- lin TP(B)				
	MEI KAM MEI				
	But I. Pr(Bn) = Pr(DBn) = Pr(DAn)				
	Dut Lir(Dn) - r(Dn) - tr(Nz)				
4	J P (DA) - Im P (D A)				
	- IN LADO F WEI				
Fact ((iii) (An) 21 not necrusarily disjoint				
1917 (MIN CAN 1,21 HOT PREZIDENTY VIDININ				
	P(UA) & IP(An) (union-of-events bound)				
	MIN - 01-(VESIS 100MA)				
	TC - La dische Ha was bou 17				
	[Can you tighten the win bound?]				
	Probability Review.				
	CALLES CARRIED AND MILLION				
not.	En and to exect ARES in a national the conditional note of				
1) (For any two events ABEE in a prob. model the conditional prob. of A, conditional on B, is defined if Pr (B) so by				
	Draan)				
	P-(AIB)= P-(AAB)				
	Bayes Lav. P(AIB) P(B)=P(BIA)P(A)				
	4 2				
Def:	Two events are independent if P-(A(B)=P-(A)P-(B) If P-(B)>O, this is equivalent to P-(A B)=P-(A)				
In C	TE P(R)>0, this in particulant to P(AIR)=R(A)				
Rmki	A&B are conditionally indep. of each other given Cit P(ABBC)= P(ACBC).				
Killo	PLACBICI= PLAIC) PLBICI				

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De f:	The events A, Az, -, An (n=2) are independent if $\forall S \in \mathbb{N}$,, n3				
VC1'	and loss				
	5+. 5 72,				
M	Pr (Ai) = Tr Pr(Ai).				
	Pr (IES Pr(A))				
RMK;	This includes the entire collection, i.e., S= ?1,2, , n) but the statement				
	P ((Ai) = T P-(Ai)				
	LEELY) LEELY				
	does not imply statistical independence among the Ai's				
	COST HOL MAIN MAIN MAIN MAIN IN THE MAIN MAIN MAIN MAIN MAIN MAIN MAIN MAIN				
Eg:	Sample point Il Az Az Az				
<u> </u>	Sample point IZ A, Az Az				
	2				
	3 0				
	4 0 0 0				
	5				
	6				
	7 0 0 1				
	8 - 1 - 1 - 0 - 0 - 1				
	P-(A, A) - P-(Ai) = I VI=1,2,3.				
V	(C(1) (1110 C(113)) (111) - 2 (11) - 2 (11) - 2				
	$\frac{1}{1}$				
	Howeve, Pr(A2 (A2) = \$ # Pr(A2) Pr(A2) = A2 & A3 dependent! P(A) (A2) (A3) = 0 # Pr(A) Pr(A2) Pr(A3) = 1/8				
	1-(A) (1A) 1= 0 7 1- (A) 1 1- (A) 1 - 18				
•					
	Note that P-(A, MAz) = F2 P-(A) P-(Az) A, I Az				
	P(A, nA3) = 4 = P(A,). P(A3). Aill A3.				
	the state of the second				
	Control of the state of the sta				

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\ \mathrea{\gamma_{} \]	Random Variables			
- 0 0	to the second se			
Def:	A riv. X is a function that maps the sample space SL to the			
	real line IK 5-1.			
	{w∈D: X(w) ≤x) ∈ E is an event ∀x∈R.			
Tripostal)	ext for In III a section were set retaining and			
E9:	She la, b, c'y			
	$X(a) = \frac{1}{2}, X(b) = \frac{1}{2}, X(c) = \frac{3}{2}$			
	let xolil, Then Zwesl: X(w) Ex) = Za, b) is an evert.			
	let x=0.4. Then [weil= X(w) < x) = p ju also an event			
Def:	The cymulative distribution function (cdf) of X is			
	Fx(x)= Pr[] we so: X(w) < x }) = Pr(X < x)			
Fact	i) X H Fx(x) is non-decreasing			
	i) $x \mapsto f_{x}(x)$ is non-decreasing ii) $x \mapsto f_{x}(x) = 0$, $x \mapsto f_{x}(x) = 1$.			
:	iii) Fx() is right-continuous, i.e.,			
	The following th			
	Special case: If X has a finite or countable it of pavible values,			
	Say XIXI XI the mole P(X=Xi) of each grante Xi			
- Jaharan	say XIX2, X3,, the prob. P-(X=Xi) of each sample Xi is called the probability mais function evaluated at Xi.			
	2 = (241-1124)-8 (3413 +) = (34134)-3 (3114)			
	$p(x_i) = p_x(x_i) = P_r(X = x_i), f_x(x)$			
	ALA E CONSCALS ED CON A FIZA - THE TOWN			
	If I Fx(x) has a derivative at x,			
	the derivative to called the rapidility			
	the derivative is called the probability denisty for (pdf) of X. p(x)?			
-	$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{4} F_{\mathbf{X}}(\mathbf{x})$.			
-	X XZ X3 X.			
-				

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	$f_X(x) f_i \approx i \int_{-\infty}^{\infty} (ix \xi X \xi X + f) = f_X(x) dx \approx f_X(x) f$
I	= Fx(x+s) - Fx(x)
100 57	A ry is continuous if $\exists f_{x}(x) \ \forall x \in \mathbb{R}$ in which case
()	$F_{X}(x) = \int_{-\infty}^{x} f_{X}(y) dy$ $X = \frac{1}{2} \int_{-\infty}^{x} f_{X}(y) dy$
	Multiple random variables.
	Joint cof fxxn(xyxn)=R(Xj\xx,, Xn\xxn) =R(\frac{7}{4}w\xxx\frac{2}{5}\text{Xj\w}\xxy\frac{1}{5}\text{Xj\w}\xxy\xxy\xxy\xxy\xxy\xxy\xxy\xxy\xxy\xx
	Given F_{X_1} X_n , how do we get the cdf of a single rv (marginal cdf) [EiEn: $F_{X_i}(X_i) = F_{X_1} X_n(\infty_1,,\infty_n, X_i,\infty_1,,\infty_n)$.
	Joint prf: Px1xn(x1,,xn)=Pr(X1=X1,,Xn=xn). Independence of 2 ns. Two rus are independent if
	Fxx(x,y)=Fx(x)Fy(y), xyeR. If x,7 are discrete, this is equivalent to
	PXY (xi, y) = R(xi) Py(y) Vxie X, y, 16 y.
	where X (resp. y) is the set of values that X (resp. Y) takes on.

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	Example of a stochastic process (the Bernoulli process)
Pef:	A stochastic prices is an infinite collection of vis defined on a
	mmon mob. model.
	The rus are usually indexed by an integer (n) or a red-valued parameter (t) that is interpreted as time.
	parameter (t) that is interpreted as time.
9-0	A Bosselli account in a consular of hid broad (or Becomette)
Def:	A Bernoulli process is a sequence of i.i.d. binary (or Bernoulli)
	LIMPLY TOWN SIGNAL
	p= P= (Zi=1) 0=1-p \ \ i \in N.
	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
()	[Zi=1) => customer arrives at time i. 2 Zi=0 > → no customer arrives at time i.
C the low in	1 -1 5 T7.100
	Based on [Zisizi, we can define another collection of the rus.
	Sn= ZZi
	The state of the s
	(X2) Xis interarrival
	EX. sime
	1012345678
	7. 4. 0 1 1 2 5 5 1 0 0
	Si 0 1 2 2 3 3 3
	Consider Xi first interarrival time. What is its dist??
	$X_{1} = 1 \text{iff} Z_{1} = 1 P_{1}(1) = 0$
nj gde	$X_{1}=1$ iff $Z_{1}=1$ $f_{X_{1}}(1)=p$. $X_{1}=2$ iff $Z_{1}=0$, $Z_{2}=1$ $f_{X_{2}}=1$ $f_{X_{1}}(2)=(1-p)p$. $X_{1}=3$ iff $Z_{1}=Z_{2}=0$, $Z_{3}=1$ $f_{X_{1}}(3)=(1-p)^{2}p$
	X1=3 iff Z1=Z2=0 Z3=1 Px (3)= (1-p)2p

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=> R(j)= p(1-p)j-1;	jz	
Geometric dist?	505	
GEOMETRIC OBJET		
In fact, the dist of PXX	k=123 in the case	al Ox
a geometric distribution.	13 110 001110	<u> </u>
What's the dist of the	partial sums?	
$S_n = \sum_{i=1}^{n} Z_i$		
JN - 2 G		
Each In is the # of arin	rals up to an including time	n.
Binomial distribution Binla	p)	
Pol Soek = Polk = p	106. that Kout of n Zi's e	a. d. 1
		qua I.
$=$ $\binom{k}{k} p^{k} (1-$	-p)n-k OEKEn.	
*	r	
Often, we want to study po	alk) for larger n & ICE	Lan John
Jarra april XELVII JEB	book to details.	