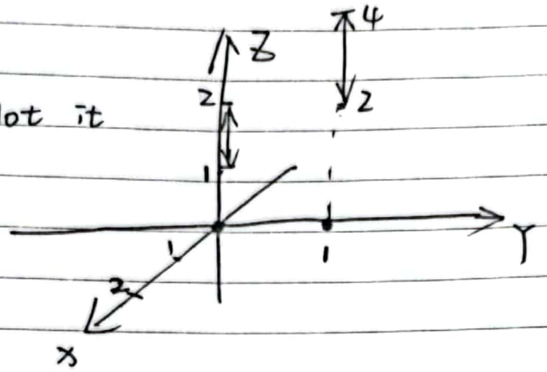
Exercise 1.5

a) From the statement, when $\begin{cases} Y=1 & Z = X + \frac{1}{2}\cdot X + \frac{1}{2}\cdot X = 2X \\ Y=0 & Z = X \end{cases}$

so we rewrite as

$$Z = 1\{Y=1\}\cdot X + X \qquad \begin{cases} Y \in Bern(\frac{1}{2}) \\ X \in U[1,2] \end{cases}$$

• And I plot it



b). For the conclusion from part a, we can get

$$Pr(Y \mid Z=z) = \frac{Pr(Y, Z=z)}{Pr(z)} = \frac{Pr(Y, Z=z)}{Pr(Y=0)Pr(Z \mid Y=0) + Pr(Y=1)\cdot Pr(Z \mid Y=1)}$$

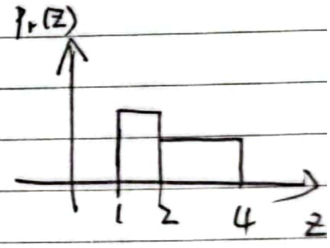
$$= \frac{2Pr(Y, Z=z)}{Pr(Z \mid Y=0) + Pr(Z \mid Y=1)} \qquad Z \in [z-\varepsilon, z+\varepsilon]$$

For $1 \le z < 2$ $\quad Pr(Y=0 \mid Z=z) = \dfrac{2\cdot\frac{1}{2\varepsilon}}{\frac{1}{2\varepsilon}\cdot 2} = 1$

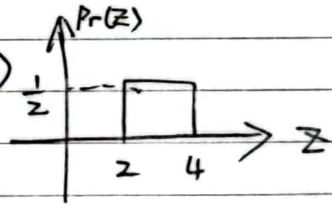
$Pr(Y=1 \mid Z=z) = 0$

For $2 < z \le 4$ $\quad Pr(Y=1 \mid Z=z) = \dfrac{2\cdot\frac{1}{2\varepsilon}}{\frac{1}{2\varepsilon}\cdot 2} = 1$

$Pr(Y=0 \mid Z=z) = 0$



Then we can get Plot $Pr(Y=0 \mid Z=z) \Rightarrow$



$=$

$= \frac{1}{2}$

Exercise 1.6

(a) $W = \begin{bmatrix} 1-\varepsilon & \varepsilon & 0 \\ 0 & 1-\varepsilon & \varepsilon \\ \varepsilon & 0 & 1-\varepsilon \end{bmatrix}$

(b) output symbols: $pW = \left[\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right] \begin{bmatrix} 1-\varepsilon & \varepsilon & 0 \\ 0 & 1-\varepsilon & \varepsilon \\ \varepsilon & 0 & 1-\varepsilon \end{bmatrix}$

$= \left[\frac{1}{2} - \frac{\varepsilon}{4}, \frac{1}{4} + \frac{\varepsilon}{4}, \frac{1}{4}\right]$

(c). $Pr(X=0 \mid Y=1) = \dfrac{P_r(X=0, Y=1)}{P_r(Y=1)} = \dfrac{\varepsilon}{P_0 \cdot \varepsilon + P_1 \cdot (1-\varepsilon)}$

suppose p vector

$p = [p_0, P_1, P_2]$

$Pr(X=1 \mid Y=1) = \dfrac{Pr(X=1, Y=1)}{Pr(Y=1)} = \dfrac{1-\varepsilon}{P_0 \cdot \varepsilon + P_1(1-\varepsilon)}$

$Pr(X=2 \mid Y=1) = \dfrac{Pr(X=2, Y=1)}{Pr(Y=1)} = \dfrac{0}{P_0 \cdot \varepsilon + P_1(1-\varepsilon)} = 0$