NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ONLINE EXAMINATION

Matriculation No.:	A0224725H
Module Code:	EE 5139
Number of pages in this PDF file (including this cover page and Declaration Form): i.e. 2+no. of answer pages	9

INSTRUCTIONS TO CANDIDATES

1. Follow the instructions for online examination and invigilation.
2. Write your answers on **A4 size paper** with black or dark blue ink. Put page number on every page.
3. Write the question number at the top left comer of each page. **Start the answer to each question on a new page**. Indicate the part, e.g. "(a)", on the left margin.
4. At the end of the exam:
   a) scan or take photographs of your answers (make sure your writing and/or drawings can be seen clearly);
   b) enter your matriculation number, module code and the total number of pages (including the cover and declaration pages, i.e. 2+ scanned pages) on the cover page;
   c) **merge the following documents in that order**: (1) Completed cover page, (2) signed declaration form, (3) scanned answer pages into a single PDF file named **<matric_no>-<module code>.pdf** (e.g.**A1234567R- EE5902. pdf**)
   d) **Important -** open the PDF file to ensure that it has been generated without error and the contents are correct;
   e) upload your PDF file into the stated LumiNUS exam submission folder within the stipulated deadline. Late submissions will not be accepted.

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Question	Mark	Remarks
Section A Q1		
TOTAL		

Exam Declaration Form

Please read sections A, B and C below. Sign and submit this declaration form together with your answers.

A. Academic, Professional and Personal Integrity

1. *The University is committed to nurturing an environment conducive for the exchange of ideas, advancement of knowledge and intellectual development. Academic honesty and integrity are essential conditions for the pursuit and acquisition of knowledge, and the University expects each student to maintain and uphold the highest standards of integrity and academic honesty at all times.*

2. *The University takes a strict view of cheating in any form, deceptive fabrication, plagiarism and violation of intellectual property and copyright laws. Any student who is found to have engaged in such misconduct will be subject to disciplinary action by the University.*

3. *It is important to note that all students share the responsibility of protecting the academic standards and reputation of the University. This responsibility can extend beyond each student' s own conduct, and can include reporting incidents of suspected academic dishonesty through the appropriate channels. Students who have reasonable grounds to suspect academic dishonesty should raise their concerns directly to the relevant Head of Department, Dean of Faculty, Registrar, Vice Provost or Provost.*

B. I have read and understood the rules of the assessments stated below:

a. *Students should attempt the assessments on their own. There should be no discussion or communication, via face to face or communication devices, with any other person during the assessment.*

b. *Students should not reproduce any assessment materials, e.g. by photo graph y, videography, screenshots, copying down of questions, etc. Posting on public forums, e.g. social media and websites, is prohibited.*

C. I understand that by breaching any of the rules above, I would have committed offences under clause 3(1) of the NUS Statute 6, Discipline with Respect to Students, which is punishable with disciplinary action under clause 10 or clause 11 of the said statute.

3) Any student who is alleged to have committed or attempted to commit, or caused or attempted to cause any other person to commit any of the following offences, may be subject to disciplinary proceedings:
(l) plagiarism, giving or receiving unauthorized assistance in academic work, or other forms of academic dishonest y.

I have read and will abide by the NUS Code of Student Conduct (in particular, (A) Academic, Professional and Personal Integrity), B and C when attempting this assessment.

Signature: LVO ZIJIAN Date: 29th Nov. 2021

Matric. No.: A0224725H

Question 1.

1. FALSE

2. FALSE

3. TRUE

4. FALSE

5. TRUE

6. FALSE

7. $P(X=0)=\frac{1}{2}$
   $P(X=1)=\frac{1}{4}$ $\Rightarrow H(X)=\frac{3}{2}$, $H_{min}(X)=1$
   $P(X=2)=\frac{1}{4}$

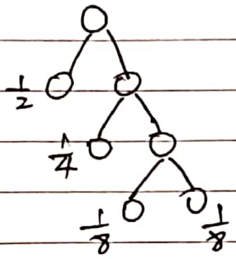
8. $P(X=0, Y=0)=\frac{1}{4}$ $\quad P(X=0, Y=1)=0$
   $P(X=1, Y=0)=\frac{1}{4}$ $\quad P(X=1, Y=1)=0$ $\Rightarrow H(X)=H(X|Y)+I(X;Y)=2$
   $P(X=2, Y=0)=0$ $\quad P(X=2, Y=1)=\frac{1}{4}$ $\quad H(X|Y)=1$
   $P(X=3, Y=0)=0$ $\quad P(X=3, Y=1)=\frac{1}{4}$

9. P(X=0

Question 2

1. $H(X) = \frac{1}{2}\log\left(\frac{1}{\frac{1}{2}}\right) + \frac{1}{4}\times\log\left(\frac{1}{\frac{1}{4}}\right) + 2\times\frac{1}{8}\times\log\left(\frac{1}{\frac{1}{8}}\right) = \frac{7}{4}$ bit

2.



$1 \rightarrow 0$
$2 \rightarrow 10$
$3 \rightarrow 110$
$4 \rightarrow 111$

~~codeword length = L(X) =~~

3. average codeword length $\bar{l} = \frac{1}{2}\times 1 + \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{8}\times 3 = \frac{7}{4}$ bit
$\because \bar{l} = H(X)$
$\therefore$ this code is optimal without referring to the fact that is a Huffman code.

4. expected number of 0's $= \frac{1}{2}\times 1 + \frac{1}{4}\times 1 + \frac{1}{8}\times 1 + \frac{1}{8}\times 0 = \frac{7}{8}$
expected number of 1's $= \frac{1}{2}\times 0 + \frac{1}{4}\times 1 + \frac{1}{8}\times 2 + \frac{1}{8}\times 3 = \frac{7}{8}$

5. We know $Y = C(X_1)\cdot C(X_2)\cdots$
and from (4), we know 0's and 1's are equal in codeword.
Therefore Y producing perfectly random bits

And Y_i is uniformly distributed because 0's and 1's are equal in codeword.

For $Y_1, Y_2, \ldots Y_{i-1}$ any $i \in N$, we can conclude that they are independent because they can effert other codeword.

As desired, proof is finished.

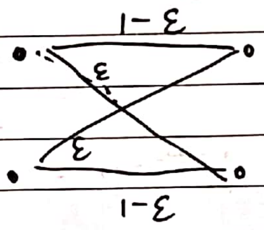
Question 3.

1. $P_Y(y) = \begin{cases} P_0 + P_1 & y=0 \\ P_2 \cdot \varepsilon & y=1 \\ P_2 \cdot (1-\varepsilon) & y=2 \end{cases}$ maximising input distribution is the uniform distribution ~~$P_0 = P_1 = P_2 = \frac{1}{3}$~~

$\Rightarrow P_Y(y) = \begin{cases} \frac{2}{3} & y=0 \\ \frac{1}{3}\cdot\varepsilon & y=1 \\ \frac{1}{3}(1-\varepsilon) & y=2 \end{cases}$

$$I(W) = \max H(Y) - H(Y|X)$$
$$= \max H(Y) - \sum_{x=0,1,2} H(Y|X=x) P_X(x)$$
$$= \max H([(1-\varepsilon)P_2]) - h(\varepsilon)$$
$$= 1 - h(\varepsilon)$$

2. By symmetry, $\Rightarrow$



$$W(Y|x) = (1-\varepsilon)\cdot\{x=y\} + \varepsilon\cdot\{x\neq y\}$$
$$I(W) = \max I(X:Y)$$
$$= H(Y) - h(\varepsilon)$$
$$= -2\times\frac{1}{2}\log\frac{1}{2} - h(\varepsilon) \quad \text{by uniform distributed}$$
$$= 1 - h(\varepsilon)$$

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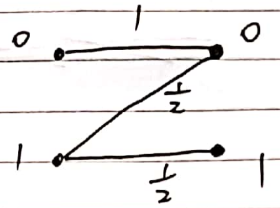
3. We can use symmetry in this,



It looks like Z- channel

let $P_X(0) = \alpha$, $P_X(1) = 1-\alpha$

$$I(W) = \max I(X;Y) = H(Y) - H(Y|X)$$

$$= \max H(Y) - \sum_{x=0,1} H(Y|X=x) \cdot P_X(x)$$

$$= \max H_b\left(\tfrac{1}{2}(1-\alpha)\right) - H(Y|X=1) \cdot P_X(1)$$

$$= \max_{\alpha} H_b\left(\tfrac{1}{2}(1-\alpha)\right) - (1-\alpha)$$

Taking derivative of above expression

$$\alpha = \alpha^* = 1 - \frac{1}{\frac{1}{2}(1+2^2)} = \frac{3}{5}$$

$$I(W) = H_b\left(\tfrac{1}{5}\right) - \tfrac{2}{5}$$

Question 4.

1. From Fano's inequality, we can get $H(M|\hat{M}) \le 1 + \varepsilon_n \cdot n \cdot R$
And we notice $\hat{M}'$ is the result from decoder,
so, in this place, we use the data processing Inequalition (DPI)
$$H(M|Y^n s^n) \le H(M|M^n)$$
Combined these two inequation,
$$H(M|Y^n s^n) \le H(M|M^n) \le 1 + \varepsilon_n \cdot n \cdot R$$

2. Using the definition of mutual information
$$I(M:Y^n s^n) = H(M|s^n) - H(M|Y^n s^n)$$
$$= H(M) - H(M|Y^n s^n) \qquad (H(M|s^n) = H(M))$$
we know $M_n = [2^{nR}]$, so $H(M) \ge nR$
$$\Downarrow \text{ continue}$$
$$\ge nR - H(M|Y^n s^n)$$
Thus $nR \le I(M:Y^n|s^n) - H(M|Y^n s^n)$

3. Since e_n is injective, we can define its inverse $e_n^{-1}: X \times S^n \to [M_n]$ for each $s^n \in S^n$.
Then we use the data processing inequation (DPI)
$$\Downarrow$$
$$H(Y^n|X^n s^n) \le H(Y^n|M s^n)$$
Therefore we can get $I(X^n:Y^n|s^n) = H(Y^n|s^n) - H(Y^n|X^n s^n)$
$$\ge H(Y^n|s^n) - H(Y^n|M s^n)$$
$$= I(M:Y^n|s^n)$$
As for $I(X^n:Y^n|s^n) = I(W^n)$
Combined these constraints, we finally get
$$I(M:Y^n|s^n) \le I(X^n:Y^n|s^n) = I(W^n)$$

As desired, its proof is finished.

Date No.

4. For $n=2$

$$I(w^2) = I(X_1 : Y_1, Y_2 \mid S_1, S_2) + I(X_2 : Y_1, Y_2 \mid X_1, S_1, S_2)$$
$$= I(X_1 : Y_1 \mid S_1) + I(X_2 : Y_2 \mid X_1, S_2)$$
$$\le \max_{P_{X_1}} I(X_1 : Y_1 \mid S_1) + \max_{P_{X_2}} I(X_2 : Y_2 \mid S_2)$$
$$= 2 I(W)$$

And similarly, we can use this chain argument in $n \geq 2$

Thefore, $I(w^n) \le n \cdot I(W)$

5. Summarizing above inequalities, we have

$$nR \le 1 + \varepsilon_n \cdot nR + n I(W)$$

Then $\varepsilon_n \geq \dfrac{R - I(W)}{R} - \dfrac{1}{nR}$

and thus $\lim_{n \to \infty} \varepsilon_n \geq \dfrac{R - I(W)}{R} > 0$ by assumption on R.

Question 5.

Date No.

1. Hypothesis test

$H_0: Q = P_0 = (0.6, 0.385, 0.013, 0.001, 0.001)$

$H_1: Q = P_1 = (0.4, 0.55, 0.036, 0.006, 0.008)$

$$\delta_{tvd}(P_0, P_1) = \frac{1}{2}\sum_{x \in \mathcal{X}} |P_0 - P_1| = \frac{1}{2}(0.2 + 0.165 + 0.023 + 0.005 + 0.007) = 0.2$$

$$\varepsilon^*_{sym,1} = \frac{1}{2}(1 - \delta_{tvd}(P_0, P_1)) = \frac{1}{2}(1 - 0.2) = 0.4$$

2. If the priori probability of being vaccinated is $p = 0.84$

So the minimal probability of error:

$$\varepsilon^*_{p,1} = \frac{1}{2}(1 - \|pP_0 - (1-p)P_1\|)$$

$H_0: Q = P_0 = (0.6p, 0.385p, 0.013p, 0.001p, 0.001p)$

$H_1: Q = P_1 = (0.4(1-p), 0.55(1-p), 0.036(1-p), 0.006(1-p), 0.008(1-p))$

$$\delta_{tvd}(P_0, P_1) = \frac{1}{2}\sum_{x \in \mathcal{X}} |P_0 - P_1| = \frac{1}{2}\Big[(p - 0.4) + (0.935p - 0.55) + (0.049p - 0.036) - (0.007p - 0.006) - (0.009p - 0.008)\Big]$$

$$= \frac{1}{2}\left[1.968p - 0.968\right]$$

$$= 0.34256$$

$$\varepsilon^*_{p,1} = \frac{1}{2}(1 - \delta_{tvd}(P_0, P_1)) = 0.32872$$

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