

SPEED UP

$\psi(n, p)$ – Speedup achieved for a problem size n using p processors

$\sigma(n)$ – Sequential portion

$\varphi(n)$ – Parallel portion

$k(n, p)$ – Sequential portion 

$$\psi(n, p) \leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p} + k(n, p)}$$

EFFICIENCY

$$\text{Efficiency} = \frac{\text{Sequential Execution time}}{\text{Processors used} \times \text{parallel Execution time}}$$

$$\epsilon(n, p) \leq \frac{\sigma(n) + \varphi(n)}{p \left(\sigma(n) + \frac{\varphi(n)}{p} + k(n, p) \right)}$$

$$\epsilon(n, p) \leq \frac{\sigma(n) + \varphi(n)}{(p\sigma(n) + \varphi(n) + pk(n, p))}$$

$$\Rightarrow 0 \leq \epsilon(n, p) \leq 1$$

AMDHAL'S LAW

$$\psi(n, p) \leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p} + k(n, p)}$$

As $k(n, p) > 0$

$$\psi(n, p) \leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p}}$$

By Definition ,

$$\mathbf{F} = \frac{\sigma(n)}{\sigma(n) + \varphi(n)} \text{ (Sequential Fraction)}$$

So ,

$$\therefore \psi(n, p) \leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p}}$$

$$\psi(n, p) \leq \frac{\sigma(n)/F}{\sigma(n) + \frac{\sigma(n)(1/F - 1)}{p}}$$

$$\psi(n, p) \leq \frac{1/F}{1 + \frac{(1/F - 1)}{p}}$$

$$\mathbf{\psi(n, p) \leq \frac{1}{F + (1 - F)/p}}$$