1 Matrix Multiplication

$A \rightarrow n \times n$; $B \rightarrow n \times n$

$C = (A \times B)$

n n n

- $i = 1$ to n
  - $j = 1$ to n
    - $k = 1$ to n
      - $C_{ij} + A_{ik} \cdot B_{kj} \Rightarrow C_{ij}$

Using 1 processor: $O(n^3)$

Using n processors : ?

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$\begin{cases} c_{11} = a_{11}b_1 + a_{12}b_{21} + a_{13}b_{31} \\ c_{12} \\ c_{13} \end{cases}$$

$\rightarrow$ 1 processor

Similarly, c_{ij}, $j = 1, 2, 3$ will be computed by processor P_i

$\Rightarrow$ each row $\{c_{ij}, j = 1, 2, \ldots, n\}$ is computed by one processor

$\Rightarrow O(n^2)$

Using n^2 processors : ?

Verify : $O(n)$

- Mesh Architecture : $n \times n$ processors
  - Claim: $O(n)$
  - Describe the algorithm.
  - What is the PRAM model?

Prefix - Sum Computation

Let us assume that the set

X — natural numbers

$\oplus$ — operation

$X = \{x_0, x_1, \ldots, x_{n-1}\}$ input set

need to find:

$$\underbrace{s_i}_{\text{prefix sum}} = x_0 + x_1 + \cdots + x_i \quad , \quad 0 \le i \le n-1$$

On a RAM this can be done in $O(n)$

We are given n processors, n is a power of 2.

$m_0, m_1, \ldots, m_{n-1}$: memory locations

initially $m_i \leftarrow x_i$

When our algorithm terminates

$$\boxed{m_i \leftarrow s_i}$$

for $j = 0$ to $\log n - 1$ do

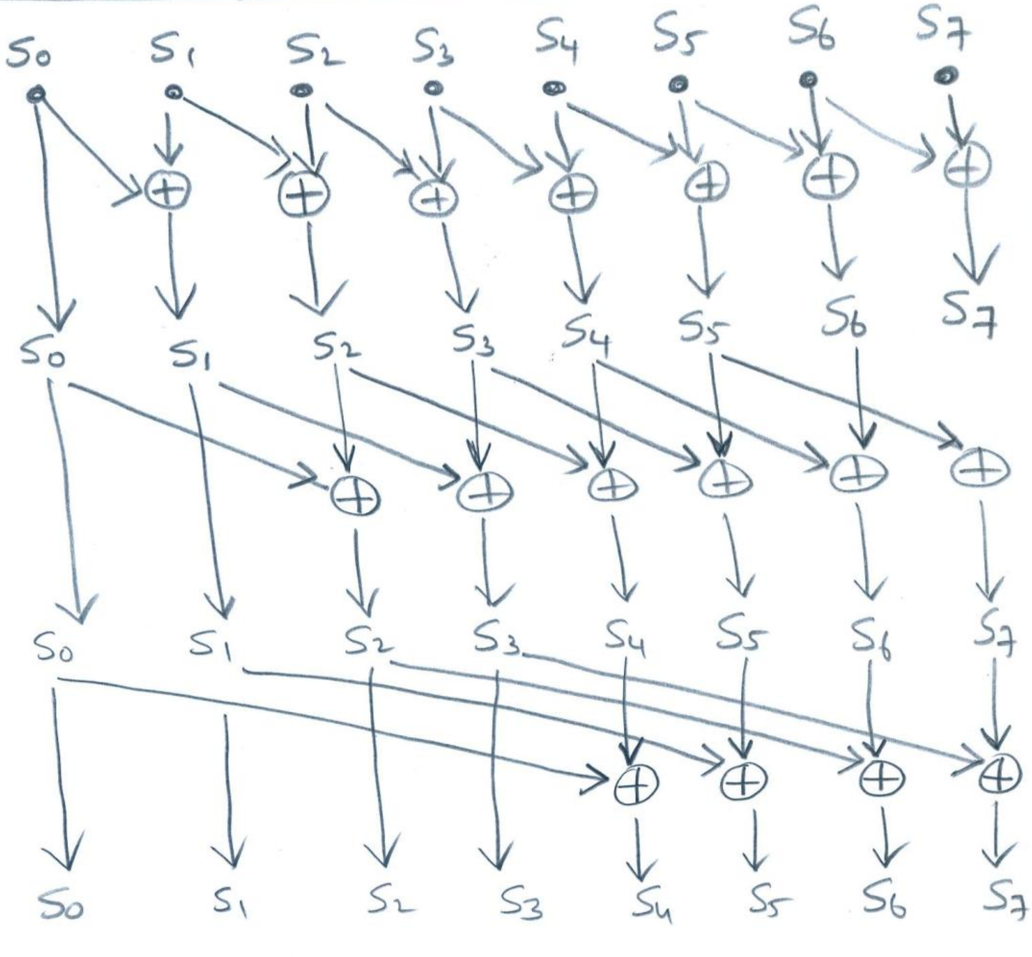
  for $i = 2^j$ to $n-1$ doPar

    $s_i = s_{i-2^j} + s_i$

  endfor

endfor

Complexity : $O(\quad)$



Prefix Computation on the PRAM