NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ONLINE EXAMINATION

Matriculation No.:	A0224725H
Module Code:	EE5904 / ME5404
Number of pages in this PDF file (including this cover page and Declaration Form): i.e. 2+no. of answer pages	10

INSTRUCTIONS TO CANDIDATES

1. Follow the instructions for online examination and invigilation.
2. Write your answers on A4 size paper with black or dark blue ink.
3. Write the question number at the top left corner of each page. Start the answer to each question on a new page. Indicate the part, e.g. "(a)", on the left margin.
4. At the end of the exam:
   a) scan or take photographs of your answers (make sure your writing and/or drawings can be seen clearly);
   b) enter your matriculation number, module code and the total number of pages (including the cover and declaration pages, i.e. 2+number of answer pages) on the cover page;
   c) merge the completed cover page, signed declaration form and your answers into a single PDF file named <matric_no>-<module code>.pdf (e.g. A1234567R-EExxxx.pdf)
   d) open the PDF file to ensure that it has been generated without error and the contents are correct;
   e) upload your PDF file into the stated LumiNUS exam submission folder within the stipulated deadline. Late submissions will not be accepted.

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Question	Mark	Remarks
TOTAL		

Exam Declaration Form

Please read sections A, B and C below. Sign and submit this declaration form together with your answers.

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I have read and will abide by the NUS Code of Student Conduct (in particular, (A) Academic, Professional and Personal Integrity), B and C when attempting this assessment.

Signature: LUO ZIJIAN

Date: ~~2021~~ 3rd May, 2021

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Q1 (a)

for hidden neurons.

$$y' = w_1 x_1 + w_2 x_2 + b_1$$

for output neurons

$$y = w_5 \varphi(y') + w_3 x_1 + w_4 x_2 + b_2$$
$$= w_5 \varphi(w_1 x_1 + w_2 x_2 + b_1) + w_3 x_1 + w_4 x_2 + b_2$$

$$E(n) = \frac{1}{2} e(n)^2 = \frac{1}{2}[d(n) - y_o(n)]^2$$

for gradient descent, $w(n+1) = w(n) - \eta g(n)$

$$g'(n) = \frac{\partial E(n)}{\partial w(n)}$$

In this problem, we need use chain rule,

for the out neurons.

$$\frac{\partial E(n)}{\partial y(n)} = -[d(n) - y(n)]$$

As for $\frac{\partial y(n)}{\partial w^{(2)}(n)} = \begin{cases} \varphi(w_1 x_1 + w_2 x_2 + b_1) \\ x_1 \text{ or } x_2 \end{cases}$

Therefore $\frac{\partial E(n)}{\partial w^{(2)}(n)} = \sum \frac{\partial E(n)}{\partial y(n)} \frac{\partial y(n)}{\partial w^{(2)}(n)} = -[d(n) - y(n)][x_1 + x_2 + \varphi(w_1 x_1 + w_2 x_2 + b_1)]$

for the hidden neurons.

$$\frac{\partial \varphi(n)}{\partial v^{(1)}(n)} = \frac{e^{-v}(-1)}{(1+e^{-v})^2} = \frac{-e^{-v}}{(1+e^{-v})^2}$$

$$\frac{\partial v^{(1)}(n)}{\partial w(n)} = \begin{cases} x_1 \text{ or } x_2 \end{cases}$$

In total, $\frac{\partial E(n)}{\partial w(n)} = [d(n) - y(n)][x_1 + x_2 + \varphi(w_1 x_1 + w_2 x_2 + b_1)] \frac{e^{-v}}{(1+e^{-v})^2}(x_1 + x_2)$

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(b) In order to solve XOR problem

x_1	x_2	d
0	0	0
0	1	1
1	0	1
1	1	0

$$\begin{cases} x_1=0 \quad x_2=0 \quad y'=b_1 \\ x_1=0 \quad x_2=1 \quad y'=b_1+w_2 \\ x_1=1 \quad x_2=0 \quad y'=b_1+w_1 \\ x_1=1 \quad x_2=1 \quad y'=b_1+w_1+w_2 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} w_5 b_1 + b_2 = 0 \\ w_5(b_1+w_2)+w_4+b_2=1 \\ w_5(b_1+w_1)+w_3+b_2=1 \\ w_5(b_1+w_1+w_2)+w_3+w_4+b_2=0 \end{cases}$$

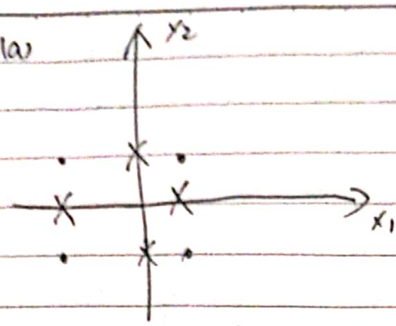
To solve these equations, we can get

$$w_5 = -\frac{b_2}{b_1}, \quad w_1=w_2=b_1 \quad w_3=w_4=b_2$$

If it satisfy this requirement, that MLP can solve XOR.

Q2

(a)



This two-class pattern classification problem is nonlinearly separable.

let type I denote 1, type II denote 0

assume that this problem is linearly separable, then there must exist a weight vector w, such that the perceptron $y = \varphi(v) = \begin{cases} 0, & v \le 0 \\ 1, & v > 0 \end{cases}$

where $v = w_1 x_1 + w_2 x_2 + b$ can correctly perform the classification.

From the example nodes,

$$\begin{cases} x_1 = 0,\ x_2 = 1,\ y = 0 & w_2 + b \le 0 \\ x_1 = 0,\ x_2 = -1,\ y = 0 & -w_2 + b \le 0 \\ x_1 = 1,\ x_2 = 1,\ y = 1 & w_1 + w_2 + b \ge 1 \\ x_1 = -1,\ x_2 = -1,\ y = 1 & -w_1 - w_2 + b \ge 1 \\ \vdots \end{cases}$$

for example, $1 - b \le w_1 + w_2 \le b - 1$

Therefore, it can not guarantee these equation all correct.

so this problem is not linearly seperable.

(b). Design parameters are set below:

the number of centers: 8, $\overset{\text{type II}}{\overbrace{(0,1)(1,0)(0,-1)(-1,0)}}$, $\overset{\text{type I}}{\overbrace{(1,1)(1,-1)(-1,1)(-1,}}$

the form of the radial basis function:

Gaussian RBF with $\sigma = 1$, $\varphi_i(x) = \exp\left(-\frac{\|x - x_i\|^2}{2}\right)$

the weight of the outer layer

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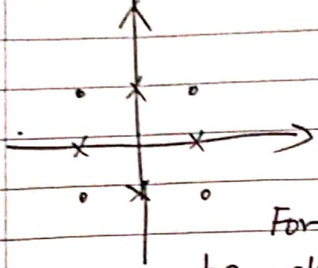
(c) Based the definition of RBFN, I think the minimal number of centers to solve this pattern recognition problem is 5.



For type I, there are 4 centers at least. (-1,-1), (-1,1), (1,-1), (1,1)
if less 4 there are wrong classification.
For example, if we remove (1,1), this node would be classifiled into type II.
Therefore, there are at least 4 centers.
As for type II, I think only one center (0,0) is ok, because it can successfully classifed (1,0) (-1,0) (0,-1) (0,1) into type II.

In a word, at least 5 centers.

(d). A simple example: contextual maps by SOM
SOM can recoanize the data owing to emerged similarity from neighhoods. groups.
The advantage of SOM is to let high-dimension information to be visilaze into 2-D dimension.
Obviously, K-means clustering algorithm is not avaliable at the high-dimension condition.

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Q3

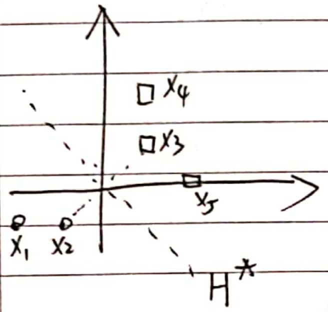
(a). For example x_5, we know the hyperplane H_1 is $w[1\ 0]^T$, $b=0$

$$\gamma_{5f} = d\{[1\ 0]^T[2\ 0]+0\} = 2$$

As for the functional margin of the training set

$$\gamma_f = \min\{\gamma_i f\} = \gamma_2 f = \{[1\ 0]^T[-1\ -1]+0\}(-1) = 1$$

(b). During the analysis of this training set, we know the optimal hyperplane is the maxinum functional margin one.



The hyperplane is the optimal one

$w=[1\ 1]^T$, $b=0$

$$\gamma_f = \min(\gamma_i f) = \gamma_3 f = \{[1\ 1]^T[1\ 1]+0\}(1) = 2$$

$$\gamma_g = \frac{\gamma_f}{|w|} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

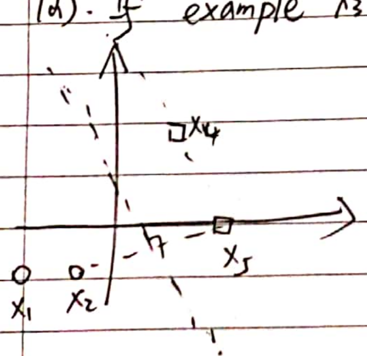
so this margin of training set: $\begin{cases} \text{functional margin} : 2 \\ \text{geometric margin} : \sqrt{2} \end{cases}$

(c). If example x_5 is removed from the training set, actually, the hyperplane is still same as part (b). Therefore the result is same.

this margin of resulting training set $\begin{cases} \text{functional margin} : 2 \\ \text{geometric margin} : \sqrt{2} \end{cases}$

(d). If example x_3 is removed from the original training set.

This optimal hyperplane is ploted.



Therefore, we can get this support vector of this optimal hyperplane

$w=[3\ 1]^T$ $b=-1$

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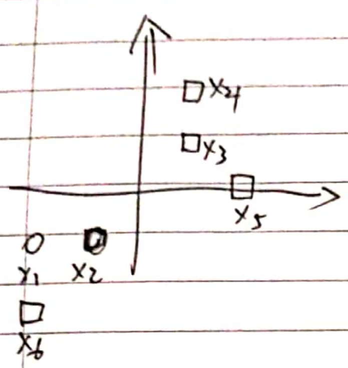
(e) Based on the statement from question.

$x_1 = [-2, -1]$ $x_2 = [-1\ -1]$ $x_3 = [1\ 1]$ $x_4 = [1\ 2]$ $x_5 = [2\ 0]$ $x_6 = [-2, -2]$



$k(x_i, x_j) = (x_i^T x_j + 1)^2$

$k(x_1, x_1) = 36$ $k(x_1, x_2) = 16$ $k(x_1, x_3) = 4$ $k(x_1, x_4) = 9$ $k(x_1, x_5) = 9$

$k(x_1, x_6) = 49$

$k(x_2, x_2) = 9$ $k(x_2, x_3) = 1$ $k(x_2, x_4) = 4$ $k(x_2, x_5) = 1$ $k(x_2, x_6) = 25$

$k(x_3, x_3) = 9$ $k(x_3, x_4) = 16$ $k(x_3, x_5) = 9$ $k(x_3, x_6) = 9$

$k(x_4, x_4) = 36$ $k(x_4, x_5) = 9$ $k(x_4, x_6) = 25$

$k(x_5, x_5) = 25$ $k(x_5, x_6) = 9$

$k(x_6, x_6) = 81$

Therefore Gram matrix is here

$$\begin{bmatrix} 36 & 16 & 4 & 9 & 9 & 49 \\ 16 & 9 & 1 & 4 & 1 & 25 \\ 4 & 1 & 9 & 16 & 9 & 9 \\ 9 & 4 & 16 & 36 & 9 & 25 \\ 9 & 1 & 9 & 9 & 25 & 9 \\ 49 & 25 & 9 & 25 & 9 & 81 \end{bmatrix}$$

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Q4

(a) a. for sequence $a_2 a_1 a_1$

$R_1 = 10 + \gamma \cdot 4 + \gamma^2 \cdot 4$

for sequence $a_1 a_2 a_2$

$R_2 = 4 + \gamma \cdot 10 + \gamma^2 \cdot 10$

$a_2 a_1 a_1$ is preferred over $a_1 a_2 a_2$

$10 + 4\gamma + 4\gamma^2 > 4 + 10\gamma + 10\gamma^2$

$\gamma^2 + \gamma - 1 < 0$

$\frac{-\sqrt{5}-1}{2} < \gamma < \frac{\sqrt{5}-1}{2}$

$\because \gamma > 0$, so $0 < \gamma < \frac{\sqrt{5}-1}{2}$,

9.

b. for sequence $a_1 a_1 \cdots a_1$

$R = 4 + 4\cdot\gamma + 4\gamma^2 + \cdots 4\cdot\gamma^n$

$= 4[1 + \gamma + \gamma^2 + \cdots \gamma^n]$

$= 4 \cdot \frac{1-\gamma^{n+1}}{1-\gamma}$

$\gamma = 0.9$ so $\lim_{n\to\infty} \gamma^{n+1} \to 0$.

Therefore $R = 4 \cdot \frac{1-0}{1-0.9} = 40$

(b). $Q_1(s_1, a_1) = Q_0(s_1, a_1) + \alpha[\rho(s_1, a_1, s_1) + \gamma \max_{a'} Q_0(s_1, a') - Q_0(s_1, a_1)]$

$= 0 + 0.5[-10 + 0.5 \times 0 - 0] = -5$

state	Action	New state	$Q(s_1,a_1)$	$Q(s_1,a_2)$	$Q(s_2,a_1)$	$Q(s_2,a_2)$
s_1	a_1	s_1	-5	0	0	0

$Q_2(s_1, a_2) = Q_1(s_1, a_2) + \alpha[\rho(s_1, a_2, s_2) + \gamma \max_{a'} Q_1(s_2, a') - Q_1(s_1, a_2)]$

$= 0 + 0.5[-10 + 0.5 \times 0 - 0] = -5$

state	Action	New state	$Q(s_1,a_1)$	$Q(s_1,a_2)$	$Q(s_2,a_1)$	$Q(s_2,a_2)$
s_1	a_1	s_1	-5	-5	0	0

$Q_3(s_2, a_1) = Q_2(s_2, a_1) + \alpha[\rho(s_2, a_1, s_1) + \gamma \max_{a'} Q_2(s_1, a') - Q_2(s_2, a_1)]$

$= 0 + 0.5[20 + 0.5 \times (-5) - 0] = 8.75$

state	Action	New state	$Q(s_1,a_1)$	$Q(s_1,a_2)$	$Q(s_2,a_1)$	$Q(s_2,a_2)$
s_2	a_1	s_1	-5	-5	8.75	0

$Q_4(s_1, a_2) = Q_3(s_1, a_2) + \alpha[\rho(s_1, a_2, s_2) + \gamma \max_{a'} Q_3(s_2, a') - Q_3(s_1, a_2)]$

$= -5 + 0.5[-10 + 0.5 \times 8.75 - (-5)] = -5.3125$

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②

state	Action	new state	$Q(S_1,a_1)$	$Q(S_1,a_2)$	$Q(S_2,a_1)$	$Q(S_2,a_2)$
S_1	a_2	S_2	-5	-5.3125	8.75	0

In a word, the total table is here

State	Action	new state	$Q(S_1,a_1)$	$Q(S_1,a_2)$	$Q(S_2,a_1)$	$Q(S_2,a_2)$
S_1	a_1	S_1	-5	0	0	0
S_1	a_2	S_2	-5	-5	0	0
S_2	a_1	S_1	-5	-5	8.75	0
S_1	a_2	S_2	-5	-5.3125	8.75	0

(c) For this problem, the optimal policy is [3, 5]

which means $1 \xrightarrow[+0]{up} 3 \xrightarrow[+10]{up} 5$

Because, in ~~this~~ state 1, to ~~be~~ state 3 is the optimal action, which means can get reward. And at state 3 is near to state 5, which means can get reward. And at the same time, this direction is the largest probability.

And then, in state 3, to state 5 is the optimal action, State 5 can get reward, as for state 4, can get negative reward from state 6, so this robot should move to state 5.

In a word, this optimal policy is [3 5]