> with(student);

> 
$$fx1 := k \cdot \left( \operatorname{sqrt} \left( \left( x_1 \right)^2 + \left( y_1 \right)^2 \right) - L \right) \cdot \frac{-x_1}{\operatorname{sqrt} \left( \left( x_1 \right)^2 + \left( y_1 \right)^2 \right)} + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - x_1 \right)^2 + \left( y_2 - x_1 \right)^2 \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) \right) + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) \right) \right)$$

$$fx1 := -\frac{k \left( \sqrt{x_1^2 + y_1^2} - L \right) \left( x_1 - x_1 \right) + k \cdot \left( \sqrt{\left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 - L \right) \left( x_2 - x_1 \right)}{\sqrt{\left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2}}$$

$$fx1 := -\frac{k \left( \sqrt{x_1^2 + y_1^2} - L \right) \left( x_1 - x_1 \right) + k \cdot \left( \sqrt{\left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 - L \right) \left( x_2 - x_1 \right)}{\sqrt{\left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2}}$$

$$(2)$$

$$\left(\left(\left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+y_{1}^{2}-2 \ y_{2} \ y_{1}+y_{2}^{2}\right) \ \left(\left(-2 \ x_{1}^{2}-2 \ y_{1}^{2}\right) \ \sqrt{x_{1}^{2}+y_{1}^{2}}\right) \right. \\
\left.+L \ y_{1}^{2}\right) \sqrt{x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}} +L \ \left(x_{1}^{2}+y_{1}^{2}\right)^{3/2} \left(-y_{2}+y_{1}\right)^{2}\right) \\
k\right) \left/\left(\left(x_{1}^{2}+y_{1}^{2}\right)^{3/2} \left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}\right)^{3/2}\right)\right.$$

 $\rightarrow diff(fx1, y_1) : simplify(%);$ 

 $\rightarrow$  diff(fx1,  $x_2$ ): simplify(%);

$$-\frac{1}{\left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}\right)^{3}/2}\left(\left(\left(-y_{1}^{2}+2 \ y_{2} \ y_{1}-y_{2}^{2}-\left(-x_{2}\right)\right)^{2}+x_{1}^{2}\right)^{2}}+\left(-y_{2}+y_{1}^{2}\right)^{2}+\left(-y_{2}+y_{1}^{2}\right)^{2}+x_{1}^{2}\left(-y_{2}+y_{1}^{2}\right)^{2}+x_{1}^{2}\left(-y_{2}+y_{1}^{2}\right)^{2}+x_{1}^{2}\right)^{2}$$

>  $diff(fx1, y_2) : simplify(\%);$ 

$$\frac{k \left(-x_2 + x_1\right) \left(-y_2 + y_1\right) L}{\left(x_1^2 - 2 \ x_2 \ x_1 + x_2^2 + \left(-y_2 + y_1\right)^2\right)^{3/2}}$$
 (6)

$$fy1 := k \cdot \left( \operatorname{sqrt} \left( \left( x_1 \right)^2 + \left( y_1 \right)^2 \right) - L \right) \cdot \frac{-y_1}{\operatorname{sqrt} \left( \left( x_1 \right)^2 + \left( y_1 \right)^2 \right)} + k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) - L \right) \cdot \frac{y_2 - y_1}{\operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right)} - m \cdot g;$$

$$fy1 := -\frac{k \left(\sqrt{x_1^2 + y_1^2} - L\right) y_1}{\sqrt{x_1^2 + y_1^2}} + \frac{k \left(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} - L\right) (y_2 - y_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$
(7)

-mg

 $\rightarrow$  diff(fy1,  $x_1$ ): simplify(%);

$$-\left(\left(x_{1} \ y_{1} \left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+y_{1}^{2}-2 \ y_{2} \ y_{1}\right)\right.\right.$$

$$\left.+y_{2}^{2}\right) \sqrt{x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}}+\left(x_{1}^{2}+y_{1}^{2}\right)^{3/2} \left(-y_{2}+y_{1}\right) \left(-x_{2}+y_{1}\right)^{3/2} \left(\left(x_{1}^{2}+y_{1}^{2}\right)^{3/2} \left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}\right)^{3/2}\right)$$

$$\left.+x_{1}\right) k L \left.\right/\left(\left(x_{1}^{2}+y_{1}^{2}\right)^{3/2} \left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}\right)^{3/2}\right)$$

diff(fy1, y<sub>1</sub>): simplify(%);

$$\left(\left(\left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+y_{1}^{2}-2 \ y_{2} \ y_{1}+y_{2}^{2}\right) \ \left(\left(-2 \ x_{1}^{2}-2 \ y_{1}^{2}\right) \ \sqrt{x_{1}^{2}+y_{1}^{2}}\right) \right.$$

$$\left.+L \ x_{1}^{2}\right) \sqrt{x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}} +L \ \left(x_{1}^{2}+y_{1}^{2}\right)^{3/2} \left(-x_{2}+x_{1}\right)^{2}\right)$$

$$\left.k\right) \left/\left(\left(x_{1}^{2}+y_{1}^{2}\right)^{3/2} \left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}\right)^{3/2}\right)\right.$$

 $\rightarrow$  diff(fy1,  $x_2$ ): simplify(%);

$$\frac{k \left(-x_2 + x_1\right) \left(-y_2 + y_1\right) L}{\left(x_1^2 - 2 \ x_2 \ x_1 + x_2^2 + \left(-y_2 + y_1\right)^2\right)^{3/2}}$$
 (10)

 $\rightarrow$  diff(fy1,  $y_2$ ): simplify(%);

$$-\frac{1}{\left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}\right)^{3/2}}\left(\left(\left(-x_{1}^{2}+2 \ x_{2} \ x_{1}-x_{2}^{2}-\left(-y_{2}\right)\right)^{2}+y_{1}^{2}\right)^{2}\left(\left(-x_{1}^{2}+2 \ x_{2} \ x_{1}-x_{2}^{2}-\left(-y_{2}\right)\right)^{2}+y_{1}^{2}\right)^{2}+y_{1}^{2}\left(\left(-x_{1}^{2}+2 \ x_{2} \ x_{1}-x_{2}^{2}-\left(-y_{2}\right)\right)^{2}+y_{1}^{2}\right)^{2}+y_{1}^{2}\left(\left(-x_{1}^{2}+2 \ x_{2} \ x_{1}-x_{2}^{2}-\left(-y_{2}\right)\right)^{2}+y_{1}^{2}\right)^{2}+y_{1}^{2}\left(\left(-x_{1}^{2}+2 \ x_{2} \ x_{1}-x_{2}^{2}-\left(-y_{2}+x_{1}\right)^{2}\right)^{2}+y_{1}^{2}\right)^{2}+y_{1}^{2}\left(\left(-x_{1}^{2}+2 \ x_{2} \ x_{1}-x_{2}^{2}+\left(-y_{2}+x_{1}\right)^{2}\right)^{2}+y_{1}^{2}\right)^{2}+y_{1}^{2}\left(\left(-x_{1}^{2}+2 \ x_{2} \ x_{1}-x_{2}^{2}+\left(-y_{2}+x_{1}\right)^{2}\right)^{2}+y_{1}^{2}\right)^{2}+y_{1}^{2}\left(-x_{1}^{2}+x_{1$$

>  $fx2 := k \cdot (\operatorname{sqrt}((x_2 - x_1)^2 + (y_2 - y_1)^2) - L) \cdot \frac{x_1 - x_2}{\operatorname{sqrt}((x_2 - x_1)^2 + (y_2 - y_1)^2)};$ 

$$fx2 := \frac{k \left(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} - L\right) (-x_2 + x_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$
(12)

 $\rightarrow$  diff(fx2,  $x_1$ ): simplify(%);

$$-\frac{1}{\left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}\right)^{3}/2}\left(\left(\left(-y_{1}^{2}+2 \ y_{2} \ y_{1}-y_{2}^{2}-\left(-x_{2}\right)^{2}\right)^{3}/2}\right)$$

$$(13)$$

$$+ x_1)^2$$
  $\sqrt{x_1^2 - 2 x_2 x_1 + x_2^2 + (-y_2 + y_1)^2} + L (-y_2 + y_1)^2) k$ 

 $\rightarrow$  diff(fx2,  $y_1$ ): simplify(%);

$$\frac{k \left(-x_2 + x_1\right) \left(-y_2 + y_1\right) L}{\left(x_1^2 - 2 \ x_2 \ x_1 + x_2^2 + \left(-y_2 + y_1\right)^2\right)^{3/2}}$$
 (14)

 $\rightarrow$  diff(fx2,  $x_2$ ): simplify(%);

$$\frac{1}{\left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}\right)^{3/2}}\left(\left(\left(-y_{1}^{2}+2 \ y_{2} \ y_{1}-y_{2}^{2}-\left(-x_{2}\right)\right)^{2}+x_{1}^{2}\right)^{2}\left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}+L \left(-y_{2}+y_{1}\right)^{2}\right)^{2}\right) (15)$$

 $\rightarrow$  diff(fx2,  $y_2$ ): simplify(%);

$$-\frac{k \left(-x_2+x_1\right) \left(-y_2+y_1\right) L}{\left(x_1^2-2 x_2 x_1+x_2^2+\left(-y_2+y_1\right)^2\right)^{3/2}}$$
 (16)

 $fy2 := k \cdot \left( \operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right) - L \right) \cdot \frac{y_1 - y_2}{\operatorname{sqrt} \left( \left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2 \right)} - m \cdot g;$ 

$$fy2 := \frac{k \left(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} - L\right) (-y_2 + y_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} - m g$$
 (17)

 $\rightarrow$  diff(fy2,  $x_1$ ): simplify(%);

$$\frac{k \left(-x_2 + x_1\right) \left(-y_2 + y_1\right) L}{\left(x_1^2 - 2 \ x_2 \ x_1 + x_2^2 + \left(-y_2 + y_1\right)^2\right)^{3/2}}$$
 (18)

>  $diff(fy2, y_1) : simplify(\%);$ 

$$-\frac{1}{\left(x_{1}^{2}-2 \ x_{2} \ x_{1}+x_{2}^{2}+\left(-y_{2}+y_{1}\right)^{2}\right)^{3/2}}\left(\left(\left(-x_{1}^{2}+2 \ x_{2} \ x_{1}-x_{2}^{2}-\left(-y_{2}\right)\right)^{3/2}\right)^{3/2}$$

$$+y_1$$
)<sup>2</sup>)  $\sqrt{x_1^2 - 2 x_2 x_1 + x_2^2 + (-y_2 + y_1)^2} + L (-x_2 + x_1)^2) k$ )

 $\rightarrow$  diff(fy2,  $x_2$ ): simplify(%);

$$-\frac{k \left(-x_2+x_1\right) \left(-y_2+y_1\right) L}{\left(x_1^2-2 \ x_2 \ x_1+x_2^2+\left(-y_2+y_1\right)^2\right)^{3/2}}$$
 (20)

 $\rightarrow$  diff(fy2,  $y_2$ ): simplify(%);