Version 3.1

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Part I Introduction

Introduction

Getting Help

The latest release of wannier90 and documentation can always be found at http://www.wannier.org.

The development version may be cloned/downloaded from the official repository of the wannier90 code on GitHub (see https://github.com/wannier-developers/wannier90).

There is a wannier90 mailing list for discussing issues in the development, theory, coding and algorithms pertinent to MLWF. You can register for this mailing list by following the links at http://www.wannier.org/forum.html. Alternatively, for technical issues about the wannier90 code, check the official repository of wannier90 on GitHub where you may raise issues or ask questions about about its functionalities.

Finally, many frequently asked questions are answered in Appendix B. An expanded FAQ session may be found on the Wiki page of the GitHub repository at https://github.com/wannier-developers/wannier90/wiki/FAQ.

Citation

We ask that you acknowledge the use of wannier90 in any publications arising from the use of this code through the following reference

[ref] G. Pizzi, V. Vitale, R. Arita, S. Blügel, F. Freimuth, G. Géranton, M. Gibertini, D. Gresch, C. Johnson, T. Koretsune, J. Ibañez-Azpiroz, H. Lee, J.M. Lihm, D. Marchand, A. Marrazzo, Y. Mokrousov, J.I. Mustafa, Y. Nohara, Y. Nomura, L. Paulatto, S. Poncé, T. Ponweiser, J. Qiao, F. Thöle, S.S. Tsirkin, M. Wierzbowska, N. Marzari, D. Vanderbilt, I. Souza, A.A. Mostofi, J.R. Yates,

Wannier90 as a community code: new features and applications, J. Phys. Cond. Matt. 32, 165902 (2020)

https://doi.org/10.1088/1361-648X/ab51ff

If you are using versions 2.x of the code, cite instead:

[ref] A. A. Mostofi, J. R. Yates, G. Pizzi, Y.-S. Lee, I. Souza, D. Vanderbilt and N. Marzari, An updated version of wannier90: A Tool for Obtaining Maximally-Localised Wannier Functions, *Comput. Phys. Commun.* **185**, 2309 (2014) http://dx.doi.org/10.1016/j.cpc.2014.05.003

It would also be appropriate to cite the original articles:

Maximally localized generalized Wannier functions for composite energy bands, N. Marzari and D. Vanderbilt, *Phys. Rev. B* **56**, 12847 (1997)

Maximally localized Wannier functions for entangled energy bands, I. Souza, N. Marzari and D. Vanderbilt, *Phys. Rev. B* **65**, 035109 (2001)

Credits

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Part II

wannier90.x

Chapter 1

Methodology

wannier90 computes maximally-localised Wannier functions (MLWF) following the method of Marzari and Vanderbilt (MV) [1]. For entangled energy bands, the method of Souza, Marzari and Vanderbilt (SMV) [2] is used. We introduce briefly the methods and key definitions here, but full details can be found in the original papers and in Ref. [3].

First-principles codes typically solve the electronic structure of periodic materials in terms of Bloch states, $\psi_{n\mathbf{k}}$. These extended states are characterised by a band index n and crystal momentum \mathbf{k} . An alternative representation can be given in terms of spatially localised functions known as Wannier functions (WF). The WF centred on a lattice site \mathbf{R} , $w_{n\mathbf{R}}(\mathbf{r})$, is written in terms of the set of Bloch states as

$$w_{n\mathbf{R}}(\mathbf{r}) = \frac{V}{(2\pi)^3} \int_{BZ} \left[\sum_{m} U_{mn}^{(\mathbf{k})} \psi_{m\mathbf{k}}(\mathbf{r}) \right] e^{-i\mathbf{k}.\mathbf{R}} d\mathbf{k} , \qquad (1.1)$$

where V is the unit cell volume, the integral is over the Brillouin zone (BZ), and $\mathbf{U^{(k)}}$ is a unitary matrix that mixes the Bloch states at each \mathbf{k} . $\mathbf{U^{(k)}}$ is not uniquely defined and different choices will lead to WF with varying spatial localisations. We define the spread Ω of the WF as

$$\Omega = \sum_{n} \left[\langle w_{n\mathbf{0}}(\mathbf{r}) | r^2 | w_{n\mathbf{0}}(\mathbf{r}) \rangle - |\langle w_{n\mathbf{0}}(\mathbf{r}) | \mathbf{r} | w_{n\mathbf{0}}(\mathbf{r}) \rangle|^2 \right]. \tag{1.2}$$

The total spread can be decomposed into a gauge invariant term $\Omega_{\rm I}$ plus a term $\tilde{\Omega}$ that is dependent on the gauge choice $\mathbf{U^{(k)}}$. $\tilde{\Omega}$ can be further divided into terms diagonal and off-diagonal in the WF basis, $\Omega_{\rm D}$ and $\Omega_{\rm OD}$,

$$\Omega = \Omega_{\rm I} + \tilde{\Omega} = \Omega_{\rm I} + \Omega_{\rm D} + \Omega_{\rm OD} \tag{1.3}$$

where

$$\Omega_{\rm I} = \sum_{n} \left[\langle w_{n\mathbf{0}}(\mathbf{r}) | r^2 | w_{n\mathbf{0}}(\mathbf{r}) \rangle - \sum_{\mathbf{R}m} |\langle w_{n\mathbf{R}}(\mathbf{r}) | \mathbf{r} | w_{n\mathbf{0}}(\mathbf{r}) \rangle|^2 \right]$$
(1.4)

$$\Omega_{\rm D} = \sum_{n} \sum_{\mathbf{R} \neq \mathbf{0}} |\langle w_{n\mathbf{R}}(\mathbf{r}) | \mathbf{r} | w_{n\mathbf{0}}(\mathbf{r}) \rangle|^2$$
(1.5)

$$\Omega_{\rm OD} = \sum_{m \neq n} \sum_{\mathbf{R}} |\langle w_{m\mathbf{R}}(\mathbf{r}) | \mathbf{r} | w_{n\mathbf{0}}(\mathbf{r}) \rangle|^2$$
(1.6)

The MV method minimises the gauge dependent spread $\tilde{\Omega}$ with respect the set of $\mathbf{U^{(k)}}$ to obtain MLWF.

wannier90 requires two ingredients from an initial electronic structure calculation.

1. The overlaps between the cell periodic part of the Bloch states $|u_{nk}\rangle$

$$M_{mn}^{(\mathbf{k},\mathbf{b})} = \langle u_{m\mathbf{k}} | u_{n\mathbf{k}+\mathbf{b}} \rangle, \tag{1.7}$$

where the vectors **b**, which connect a given k-point with its neighbours, are determined by wannier90 according to the prescription outlined in Ref. [1].

2. As a starting guess the projection of the Bloch states $|\psi_{n\mathbf{k}}\rangle$ onto trial localised orbitals $|g_n\rangle$

$$A_{mn}^{(\mathbf{k})} = \langle \psi_{m\mathbf{k}} | g_n \rangle, \tag{1.8}$$

Note that $\mathbf{M^{(k,b)}}$, $\mathbf{A^{(k)}}$ and $\mathbf{U^{(k)}}$ are all small, $N \times N$ matrices¹ that are independent of the basis set used to obtain the original Bloch states.

To date, wannier90 has been used in combination with electronic codes based on plane-waves and pseudopotentials (norm-conserving and ultrasoft [4]) as well as mixed basis set techniques such as FLAPW [5].

1.1 Entangled Energy Bands

The above description is sufficient to obtain MLWF for an isolated set of bands, such as the valence states in an insulator. In order to obtain MLWF for entangled energy bands we use the "disentanglement" procedure introduced in Ref. [2].

We define an energy window (the "outer window"). At a given k-point \mathbf{k} , $N_{\text{win}}^{(\mathbf{k})}$ states lie within this energy window. We obtain a set of N Bloch states by performing a unitary transformation amongst the Bloch states which fall within the energy window at each k-point:

$$|u_{n\mathbf{k}}^{\text{opt}}\rangle = \sum_{m \in N_{\text{win}}^{(\mathbf{k})}} U_{mn}^{\text{dis}(\mathbf{k})} |u_{m\mathbf{k}}\rangle \tag{1.9}$$

where $\mathbf{U}^{\mathrm{dis}(\mathbf{k})}$ is a rectangular $N_{\mathrm{win}}^{(\mathbf{k})} \times N$ matrix². The set of $\mathbf{U}^{\mathrm{dis}(\mathbf{k})}$ are obtained by minimising the gauge invariant spread Ω_{I} within the outer energy window. The MV procedure can then be used to minimise $\tilde{\Omega}$ and hence obtain MLWF for this optimal subspace.

It should be noted that the energy bands of this optimal subspace may not correspond to any of the original energy bands (due to mixing between states). In order to preserve exactly the properties of a system in a given energy range (e.g., around the Fermi level) we introduce a second energy window. States lying within this inner, or "frozen", energy window are included unchanged in the optimal subspace.

¹Technically, this is true for the case of an isolated group of N bands from which we obtain N MLWF. When using the disentanglement procedure of Ref. [2], $\mathbf{A}^{(\mathbf{k})}$, for example, is a rectangular matrix. See Section 1.1.

²As $\mathbf{U}^{\mathrm{dis}(\mathbf{k})}$ is a rectangular matrix this is a unitary operation in the sense that $(\mathbf{U}^{\mathrm{dis}(\mathbf{k})})^{\dagger}\mathbf{U}^{\mathrm{dis}(\mathbf{k})} = \mathbf{1}_{N}$.

Chapter 2

Parameters

2.1 Usage

wannier90.x can be run in parallel using MPI libraries to reduce the computation time.

For serial execution use: wannier90.x [-pp] [seedname]

- seedname: If a seedname string is given the code will read its input from a file seedname.win. The default value is wannier. One can also equivalently provide the string seedname.win instead of seedname.
- -pp: This optional flag tells the code to generate a list of the required overlaps and then exit. This information is written to the file seedname.nnkp.

For parallel execution use: mpirun -np NUMPROCS wannier90.x [-pp] [seedname]

• NUMPROCS: substitute with the number of processors that you want to use.

Note that the mpirun command and command-line flags may be different in your MPI implementation: read your MPI manual or ask your computer administrator.

Note also that this requires that the wannier90.x executable has been compiled in its parallel version (follow the instructions in the file README.install in the main directory of the wannier90 distribution) and that the MPI libraries and binaries are installed and correctly configured on your machine.

2.2 seedname.win File

The wannier90 input file seedname.win has a flexible free-form structure.

The ordering of the keywords is not significant. Case is ignored (so num_bands is the same as Num_Bands). Characters after !, or # are treated as comments. Most keywords have a default value that is used unless the keyword is given in seedname.win. Keywords can be set in any of the following ways

```
num_wann 4
num_wann = 4
num_wann : 4
```

A logical keyword can be set to true using any of the following strings: T, true, .true..

For further examples see Section 10.1 and the the wannier90 Tutorial.

2.3 Keyword List

Keyword Type		Description	
	Syste	m Parameters	
NUM_WANN	I	Number of WF	
NUM_BANDS	I	Number of bands passed to the code	
UNIT_CELL_CART	Р	Unit cell vectors in Cartesian coor-	
		dinates	
ATOMS_CART *	Р	Positions of atoms in Cartesian co-	
		ordinates	
ATOMS_FRAC *	R	Positions of atoms in fractional co-	
		ordinates with respect to the lattice	
		vectors	
MP_GRID	I	Dimensions of the Monkhorst-Pack	
		grid of k-points	
KPOINTS	R	List of k-points in the Monkhorst-	
		Pack grid	
GAMMA_ONLY	L	Wavefunctions from underlying ab	
_		initio calculation are manifestly real	
SPINORS	L	WF are spinors	
SHELL_LIST	I	Which shells to use in finite differ-	
_		ence formula	
SEARCH SHELLS I		The number of shells to search when	
		determining finite difference formula	
SKIP_B1_TESTS	L		
NNKPTS	I	Explicit list of nearest-neighbour k-	
		points.	
KMESH_TOL	R	The tolerance to control if two	
		kpoint belong to the same shell	

Table 2.1: seedname.win file keywords defining the system. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string. * ATOMS_CART and ATOMS_FRAC may not both be defined in the same input file.

Keyword	Type	Description
	Job C	Control
POSTPROC SETUP	L	To output the seedname.nnkp file
EXCLUDE_BANDS	I	List of bands to exclude from the
		calculation
SELECT_PROJECTIONS	I	List of projections to use in Wan-
	_	nierisation
AUTO_PROJECTIONS	L	To automatically generate initial
	a	projections
RESTART	S	Restart from checkpoint file
IPRINT	I	Output verbosity level
LENGTH_UNIT	S	System of units to output lengths
WVFN_FORMATTED	L	Read the wavefunctions from a
		(un)formatted file
SPIN	S	Which spin channel to read
DEVEL_FLAG	S	Flag for development use
TIMING_LEVEL	I	Determines amount of timing infor-
		mation written to output
OPTIMISATION	I	Optimisation level
TRANSLATE_HOME_CELL	L	To translate final Wannier centres to
		home unit cell when writing xyz file
WRITE_XYZ	L	To write atomic positions and final
_		centres in xyz file format
WRITE_VDW_DATA	L	To write data for futher processing
		by w90vdw utility
WRITE_HR_DIAG	L	To write the diagonal elements of
		the Hamiltonian in the Wannier ba-
		sis to seedname.wout (in eV)

Table 2.2: seedname.win file keywords defining job control. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string. TRANSLATE_HOME_CELL only relevant if WRITE_XYZ is .true.

Keyword	Type	Description
Plot	Parame	eters
WANNIER_PLOT	L	Plot the WF
WANNIER_PLOT_LIST	I	List of WF to plot
WANNIER_PLOT_SUPERCELL	I	Size of the supercell for plotting the
		WF
WANNIER_PLOT_FORMAT	S	File format in which to plot the WF
WANNIER_PLOT_MODE	S	Mode in which to plot the WF,
		molecule or crystal
WANNIER_PLOT_RADIUS	R	Cut-off radius of WF*
WANNIER_PLOT_SCALE	R	Scaling parameter for cube files
WANNIER_PLOT_SPINOR_MODE	S	Quantity to plot for spinor WF

WANNIER_PLOT_SPINOR_PHASE	L	Include the "phase" when plotting spinor WF
BANDS PLOT	L	Plot interpolated band structure
KPOINT_PATH	P	K-point path for the interpolated band structure
BANDS_NUM_POINTS	I	Number of points along the first section of the k-point path
BANDS_PLOT_FORMAT	S	File format in which to plot the interpolated bands
BANDS_PLOT_PROJECT	I	WF to project the band structure onto
BANDS_PLOT_MODE	S	Slater-Koster type interpolation or Hamiltonian cut-off
BANDS_PLOT_DIM	I	Dimension of the system
FERMI SURFACE PLOT	L	Plot the Fermi surface
FERMI SURFACE NUM POINTS	I	Number of points in the Fermi sur-
FERMI_SORFACE_NOW_FORMES	1	face plot
FERMI_ENERGY	P	The Fermi energy
FERMI_ENERGY_MIN	P	Lower limit of the Fermi energy
		range
FERMI_ENERGY_MAX	P	Upper limit of the Fermi energy range
FERMI_ENERGY_STEP	R	Step for increasing the Fermi energy in the specified range
FERMI_SURFACE_PLOT_FORMAT	S	File format for the Fermi surface plot
HR_PLOT	L	This parameter is not used anymore. Use WRITE HR instead.
WRITE_HR	L	Write the Hamiltonian in the WF basis
WRITE_RMN	L	Write the position operator in the WF basis
WRITE_BVEC	L	Write to file the matrix elements of the byectors and their weights
WRITE_TB	L	Write lattice vectors, Hamiltonian, and position operator in WF basis
HR_CUTOFF	Р	Cut-off for the absolute value of the
DIGE CHEODE	P	Hamiltonian Cut-off for the distance between WF
DIST_CUTOFF	S	Dimension in which the distance be-
DIST_CUTOFF_MODE		tween WF is calculated
TRANSLATION_CENTRE_FRAC	R	Centre of the unit cell to which final WF are translated
USE_WS_DISTANCE	L	Improve interpolation using minimum distance between WFs, see Chap. 9
WS_DISTANCE_TOL	R	Absolute tolerance for the distance to equivalent positions.

WS_SEARCH_SIZE	I	Maximum extension in each direc-
		tion of the super-cell of the Born-von
		Karmann cell to search for points in-
		side the Wigner-Seitz cell
WRITE_U_MATRICES	L	Write $\mathbf{U^{(k)}}$ and $\mathbf{U}^{\mathrm{dis(k)}}$ matrices to
		files

Table 2.5: seedname.win file keywords controlling the plotting. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string. * Only applies when WANNIER_PLOT_FORMAT is cube.

Keyword	Type	Description
Disenta	nglemer	nt Parameters
DIS_WIN_MIN	P	Bottom of the outer energy window
DIS_WIN_MAX	P	Top of the outer energy window
DIS_FROZ_MIN	P	Bottom of the inner (frozen) energy
		window
DIS_FROZ_MAX	P	Top of the inner (frozen) energy win-
		dow
DIS_NUM_ITER	I	Number of iterations for the minimi-
		sation of $\Omega_{\rm I}$
DIS_MIX_RATIO	R	Mixing ratio during the minimisa-
		tion of Ω_{I}
DIS CONV TOL	R	The convergence tolerance for find-
		$\log \Omega_{ m I}$
DIS_CONV_WINDOW	I	The number of iterations over which
		convergence of $\Omega_{\rm I}$ is assessed.
DIS_SPHERES_NUM	I	Number of spheres in k-space where
		disentaglement is performed
DIS SPHERES FIRST WANN	I	Index of the first band to be consid-
		ered a Wannier function
DIS SPHERES	R	List of centres and radii, for disen-
		tanglement only in spheres

Table 2.3: seedname.win file keywords controlling the disentanglement. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description		
Wannierise Parameters				
NUM_ITER	I	Number of iterations for the minimi-		
_		sation of Ω		
NUM_CG_STEPS	I	During the minimisation of Ω the		
		number of Conjugate Gradient steps		
		before resetting to Steepest Descents		
CONV_WINDOW	I	The number of iterations over which		
	ъ	convergence of Ω is assessed		
CONV_TOL	Р	The convergence tolerance for find-		
PPPGOVE	т	$\operatorname{ing} \Omega$		
PRECOND	L R	Use preconditioning The amplitude of random noise ap-		
CONV_NOISE_AMP	n	plied towards end of minimisation		
		procedure		
CONV_NOISE_NUM	I	The number of times random noise		
	_	is applied		
NUM_DUMP_CYCLES	I	Control frequency of check-pointing		
NUM PRINT CYCLES	I	Control frequency of printing		
WRITE_R2MN	L	Write matrix elements of r^2 between		
		WF to file		
GUIDING_CENTRES	L	Use guiding centres		
NUM_GUIDE_CYCLES	I	Frequency of guiding centres		
NUM_NO_GUIDE_ITER	I	The number of iterations after which		
J-	ъ	guiding centres are used		
TRIAL_STEP *	R	The trial step length for the		
		parabolic line search during the minimisation of Ω		
FIXED_STEP *	R	The fixed step length to take dur-		
FIXED_SIEF	16	ing the minimisation of Ω , instead		
		of doing a parabolic line search		
USE_BLOCH_PHASES **	L	To use phases for initial projections		
SITE_SYMMETRY***	L	To construct symmetry-adapted		
_		Wannier functions		
SYMMETRIZE_EPS***	R	The convergence tolerance used in		
		the symmetry-adapted mode		
SLWF_NUM	I	The number of objective WFs for se-		
		lective localization		
SLWF_CONSTRAIN	L	Whether to constrain the centres of		
	D	the objective WFs		
SLWF_LAMBDA	R	Value of the Lagrange multiplier for		
CIME CEMEDEC	Р	constraining the objective WFs		
SLWF_CENTRES	r	The centres to which the objective WFs are to be constrained		
		WES are to be constrained		

Table 2.4: seedname.win file keywords controlling the wannierisation. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string. * fixed_step and trial_step may not both be defined in the same input file. **Cannot be used in conjunction with disentanglement. ***Cannot be used in conjunction with the inner (frozen) energy window.

Keyword	Type	Description		
Transport Parameters				
TRANSPORT	L	Calculate quantum conductance and		
		density of states		
TRANSPORT_MODE	S	Bulk or left-lead_conductor_right-		
		lead calculation		
TRAN_WIN_MIN	P	Bottom of the energy window for transport calculation		
TRAN_WIN_MAX	Р	Top of the energy window for transport calculation		
TRAN_ENERGY_STEP	R	Sampling interval of the energy values		
FERMI ENERGY	R	The Fermi energy		
TRAN_NUM_BB	I	Size of a bulk Hamiltonian		
TRAN_NUM_LL	I	Size of a left-lead Hamiltonian		
TRAN_NUM_RR	I	Size of a right-lead Hamiltonian		
TRAN_NUM_CC	I	Size of a conductor Hamiltonian		
TRAN_NUM_LC	I	Number of columns in a left-		
TRAN_NUM_CR	I	lead_conductor Hamiltonian Number of rows in a conductor_right-lead Hamilto-		
TRAN_NUM_CELL_LL	I	nian Number of unit cells in PL of left lead		
TRAN_NUM_CELL_RR	I	Number of unit cells in PL of right lead		
TRAN_NUM_BANDC	I	Half-bandwidth+1 of a band- diagonal conductor Hamiltonian		
TRAN_WRITE_HT	L	Write the Hamiltonian for transport calculation		
TRAN_READ_HT	L	Read the Hamiltonian for transport calculation		
TRAN_USE_SAME_LEAD	L	Left and right leads are the same		
TRAN_GROUP_THRESHOLD	R	Distance that determines the grouping of WFs		
HR_CUTOFF	Р	Cut-off for the absolute value of the Hamiltonian		
DIST CUTOFF	Р	Cut-off for the distance between WF		
DIST_CUTOFF_MODE	S	Dimension in which the distance between WF is calculated		
ONE_DIM_AXIS	S	Extended direction for a one- dimensional system		
TRANSLATION_CENTRE_FRAC	R	Centre of the unit cell to which final WF are translated		

Table 2.6: seedname.win file keywords controlling transport. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

2.4 System

2.4.1 integer :: num_wann

Number of WF to be found.

No default.

2.4.2 integer :: num_bands

Total number of bands passed to the code in the seedname.mmn file.

Default num_bands=num_wann

2.4.3 Cell Lattice Vectors

The cell lattice vectors should be specified in Cartesian coordinates.

begin unit_cell_cart
[units]

$$\begin{array}{cccc} A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \\ A_{3x} & A_{3y} & A_{3z} \end{array}$$

end unit_cell_cart

Here A_{1x} is the x-component of the first lattice vector \mathbf{A}_1 , A_{2y} is the y-component of the second lattice vector \mathbf{A}_2 , etc.

[units] specifies the units in which the lattice vectors are defined: either Bohr or Ang.

The default value is Ang.

2.4.4 Ionic Positions

The ionic positions may be specified in fractional coordinates relative to the lattice vectors of the unit cell, or in absolute Cartesian coordinates. Only one of atoms_cart and atoms_frac may be given in the input file.

Cartesian coordinates

begin atoms_cart
[units]

$$\begin{array}{cccc} P & R_x^P & R_y^P & R_z^P \\ Q & R_x^Q & R_y^Q & R_z^Q \\ \vdots & & & \end{array}$$

end atoms_cart

The first entry on a line is the atomic symbol. The next three entries are the atom's position $\mathbf{R} = (R_x, R_y, R_z)$ in Cartesian coordinates. The first line of the block, [units], specifies the units in which the coordinates are given and can be either bohr or ang. If not present, the default is ang.

Fractional coordinates

begin atoms_frac

$$\begin{array}{cccc} P & F_{1}^{P} & F_{2}^{P} & F_{3}^{P} \\ Q & F_{1}^{Q} & F_{2}^{Q} & F_{3}^{Q} \\ & \cdot & \end{array}$$

end atoms_frac

The first entry on a line is the atomic symbol. The next three entries are the atom's position in fractional coordinates $\mathbf{F} = F_1 \mathbf{A}_1 + F_2 \mathbf{A}_2 + F_3 \mathbf{A}_3$ relative to the cell lattice vectors \mathbf{A}_i , $i \in [1, 3]$.

2.4.5 integer, dimension :: mp_grid(3)

Dimensions of the regular (Monkhorst-Pack) k-point mesh. For example, for a $2 \times 2 \times 2$ grid:

mp_grid : 2 2 2

No default.

2.4.6 K-points

Each line gives the coordinate $\mathbf{K} = K_1\mathbf{B}_1 + K_2\mathbf{B}_2 + K_3\mathbf{B}_3$ of a k-point in relative (crystallographic) units, i.e., in fractional units with respect to the primitive reciprocal lattice vectors \mathbf{B}_i , $i \in [1,3]$. The position of each k-point in this list assigns its numbering; the first k-point is k-point 1, the second is k-point 2, and so on.

begin kpoints

$$\begin{array}{cccc} K_1^1 & K_2^1 & K_3^1 \\ K_1^2 & K_2^2 & K_3^2 \\ \vdots & & & \end{array}$$

end kpoints

There is no default.

Note: There is an utility provided with wannier90, called kmesh.pl, which helps to generate the explicit list of k points required by wannier90. See Sec. A.1.

2.4.7 logical :: gamma_only

If gamma_only=true, then wannier90 uses a branch of algorithms for disentanglement and localisation that exploit the fact that the Bloch eigenstates obtained from the underlying ab initio calculation are manifestly real. This can be the case when only the Γ -point is used to sample the Brillouin zone. The localisation procedure that is used in the Γ -only branch is based on the method of Ref. [6].

The default value is false.

2.4.8 logical :: spinors

If spinors=true, then wannier90 assumes that the WF correspond to singularly occupied spinor states and num_elec_per_state=1.

The default value is false.

2.4.9 Shells

The MV scheme requires a finite difference expression for $\nabla_{\mathbf{k}}$ defined on a uniform Monkhorst-Pack mesh of k-points. The vectors $\{\mathbf{b}\}$ connect each mesh-point \mathbf{k} to its nearest neighbours. $N_{\rm sh}$ shells of neighbours are included in the finite-difference formula, with M_s vectors in the $s^{\rm th}$ shell. For $\nabla_{\mathbf{k}}$ to be correct to linear order, we require that the following equation is satisfied (Eq. B1 of Ref. [1]):

$$\sum_{s}^{N_{\rm sh}} w_s \sum_{i}^{M_s} b_{\alpha}^{i,s} b_{\beta}^{i,s} = \delta_{\alpha\beta} , \qquad (2.1)$$

where $\mathbf{b}^{i,s}$, $i \in [1, M_s]$, is the i^{th} vector belonging to the s^{th} shell with associated weight w_s , and α and β run over the three Cartesian indices.

2.4.10 integer :: shell_list(:)

shell_list is vector listing the shells to include in the finite difference expression. If this keyword is absent, the shells are chosen automatically.

2.4.11 integer :: search_shells

Specifies the number of shells of neighbours over which to search in attempting to determine an automatic solution to the B1 condition Eq. 2.1. Larger values than the default may be required in special cases e.g. for very long thin unit cells.

The default value is 36.

2.4.12 logical :: skip_B1_tests

If set to .true., does not check the B1 condition Eq. 2.1. This should *only* be used if one knows why the B1 condition should not be verified. A typical use of this flag is in conjunction with the Z2PACK code: http://www.physics.rutgers.edu/z2pack/.

The default value is .false..

2.4.13 integer, dimension(:, 5) :: nnkpts

Specifies the nearest-neighbour k-points which are written to the .nnkp file. This can be used to explicitly specify which overlap matrices should be calculated.

```
begin nnkpts
1 2 0 0 0
.
.
end nnkpts
```

Each nearest neighbour $\mathbf{k} + \mathbf{b}$ is given by a line of 5 integers. The first specifies the k-point number \mathbf{k} of \mathbf{k} . The second is the k-point number of the neighbour. The final three integers specify the reciprocal lattice vector which brings the k-point specified by the second integer to $\mathbf{k} + \mathbf{b}$.

This format is the same as in the .nnkp file, except that the number of neighbours per k-point is not specified. However, the number of neighbours still needs to be a multiple of the number of k-points.

This input parameter can be used only if postproc_setup = .true., and is not intended to be used with a full Wannier90 run. It can be used also if the k-points do not describe a regular mesh.

2.4.14 real(kind=dp) :: kmesh_tol

Two kpoints belong to the same shell if the distance between them is less than kmesh_tol. Units are Ang.

The default value is 0.000001 Ang.

2.5 Projection

The projections block defines a set of localised functions used to generate an initial guess for the unitary transformations. This data will be written in the **seedname.nnkp** file to be used by a first-principles code.

```
begin projections
.
.
end projections
```

If guiding_centres=true, then the projection centres are used as the guiding centres in the Wannierisation routine.

For details see Section 3.1.

2.6 Job Control

2.6.1 logical :: postproc_setup

If postproc_setup=true, then the wannier code will write seedname.nnkp file and exit. If wannier90 is called with the option -pp, then postproc_setup is set to true, over-riding its value in the seedname.win file.

The default value is false.

2.6.2 integer :: iprint

This indicates the level of verbosity of the output from 0 ("low"), the bare minimum, to 3 ("high"), which corresponds to full debugging output.

The default value is 1.

2.6.3 integer :: optimisation

This indicates the level of optimisation used in the code. This is a trade between speed and memory. A positive number indicates fastest execution time at the cost of more memory. Zero or negative numbers indicates a smaller memory footprint - at increased execution time.

At the moment the only values that have an effect are optimisation<=0 (low memory) and optimisation>0 (fast)

The default value is 3.

2.6.4 character(len=20) :: length_unit

The length unit to be used for writing quantities in the output file seedname.wout.

The valid options for this parameter are:

- Ang (default)
- Bohr

2.6.5 character(len=50) :: devel_flag

Not a regular keyword. Its purpose is to allow a developer to pass a string into the code to be used inside a new routine as it is developed.

No default.

2.6.6 integer :: exclude_bands(:)

A k-point independent list of states to excluded from the calculation of the overlap matrices; for example to select only valence states, or ignore semi-core states. This keyword is passed to the first-principles code via the seedname.nnkp file. For example, to exclude bands 2, 6, 7, 8 and 12:

```
exclude_bands : 2, 6-8, 12
```

2.6.7 integer :: select_projections(:)

A list of projections to be included in the wannierisation procedure. In the case that num_proj is greater than num_wann, this keyword allows a subset of the projections in the projection matrices to be used. For example, to select the projections given by the indices 2, 6, 7, 8 and 12:

```
select_projections : 2, 6-8, 12
```

2.6.8 logical :: auto_projections

If .true. and no projections block is defined, then wannier90 writes an additional block in the .nnkp file during the pre-processing step, to instruct the interface code to automatically generate the $A_{mn}^{(k)}$.

For additional information on the behavior and on the added block, see Sec. 5.1.9.

Note: the interface code (e.g. pw2wannier90.x) must have at least one implementation of a method to automatically generate initial projections in order for this option to be usable.

The default value of this parameter is false.

2.6.9 character(len=20) :: restart

If restart is present the code will attempt to restart the calculation from the seedname.chk file. The value of the parameter determines the position of the restart

The valid options for this parameter are:

- default. Restart from the point at which the check file seedname.chk was written
- wannierise. Restart from the beginning of the wannierise routine
- plot. Go directly to the plotting phase
- transport. Go directly to the transport routines

2.6.10 character(len=20) :: wvfn_formatted

If wvfn_formatted=true, then the wavefunctions will be read from disk as formatted (ie ASCII) files; otherwise they will be read as unformatted files. Unformatted is generally preferable as the files will take less disk space and I/O is significantly faster. However such files will not be transferable between all machine architectures and formatted files should be used if transferability is required (i.e., for test cases).

The default value of this parameter is false.

2.6.11 character(len=20) :: spin

For bands from a spin polarised calculation spin determines which set of bands to read in, either up or down.

The default value of this parameter is up.

2.6.12 integer :: timing_level

Determines the amount of timing information regarding the calculation that will be written to the output file. A value of 1 produces the least information.

The default value is 1.

2.6.13 logical :: translate_home_cell

Determines whether to translate the final Wannier centres to the home unit cell at the end of the calculation. Mainly useful for molecular systems in which the molecule resides entirely within the home unit cell and user wants to write an xyz file (write_xyz=.true.) for the WF centres to compare with the structure.

The default value is false.

2.6.14 logical :: write_xyz

Determines whether to write the atomic positions and final Wannier centres to an xyz file, seedname_centres.xyz, for subsequent visualisation.

The default value is false.

2.6.15 logical :: write_vdw_data

Determines whether to write seedname.vdw for subsequent post-processing by the w90vdw utility (in the utility/w90vdw/ directory of the distribution) for calculating van der Waals energies. Brillouin zone sampling must be at the Gamma-point only.

The default value is false.

2.7 Disentanglement

These keywords control the disentanglement routine of Ref. [2], i.e., the iterative minimisation of $\Omega_{\rm I}$. This routine will be activated if num_wann < num_bands.

2.7.1 real(kind=dp) :: dis_win_min

The lower bound of the outer energy window for the disentanglement procedure. Units are eV.

The default is the lowest eigenvalue in the system.

2.7.2 real(kind=dp) :: dis_win_max

The upper bound of the outer energy window for the disentanglement procedure. Units are eV.

The default is the highest eigenvalue in the given states (i.e., all states are included in the disentanglement procedure).

2.7.3 real(kind=dp) :: dis_froz_min

The lower bound of the inner energy window for the disentanglement procedure. Units are eV.

If dis_froz_max is given, then the default for dis_froz_min is dis_win_min.

2.7.4 real(kind=dp) :: dis_froz_max

The upper bound of the inner (frozen) energy window for the disentanglement procedure. If dis_froz_max is not specified, then there are no frozen states. Units are eV.

No default.

2.7.5 integer :: dis_num_iter

In the disentanglement procedure, the number of iterations used to extract the most connected subspace.

The default value is 200.

2.7.6 real(kind=dp) :: dis_mix_ratio

In the disentanglement procedure, the mixing parameter to use for convergence (see pages 4-5 of Ref. [2]). A value of 0.5 is a 'safe' choice. Using 1.0 (i.e., no mixing) often gives faster convergence, but may cause the minimisation of $\Omega_{\rm I}$ to be unstable in some cases.

Restriction: $0.0 < dis_mix_ratio \le 1.0$

The default value is 0.5

2.7.7 real(kind=dp) :: dis_conv_tol

In the disentanglement procedure, the minimisation of Ω_I is said to be converged if the fractional change in the gauge-invariant spread between successive iterations is less than dis_conv_tol for dis_conv_window iterations. Units are Å².

The default value is 1.0E-10

2.7.8 integer :: dis_conv_window

In the disentanglement procedure, the minimisation is said to be converged if the fractional change in the spread between successive iterations is less than dis_conv_tol for dis_conv_window iterations.

The default value of this parameter is 3.

2.7.9 integer :: dis_spheres_num

Number of spheres in reciprocal space where the k-dependent disentanglement is performed. No disentanglement is performed for those k-points that are not included in any of the spheres.

The default is 0, which means disentangle at every k-point in the full BZ (the standard mode in Wannier90).

2.7.10 integer :: dis_spheres_first_wann

Index of the first band that has to be considered as a Wannier function. Used only if dis_spheres_num is greater than zero. At k-points where disentanglement is not performed the bands from dis_spheres_first_wann to dis_spheres_first_wann+num_wann are used to wannierise. The bands excluded using exclude_bands should not be counted.

The default is 1, the band at the lowest energy.

2.7.11 dis spheres

Each line gives the coordinate $\mathbf{K} = K_1 \mathbf{B}_1 + K_2 \mathbf{B}_2 + K_3 \mathbf{B}_3$ of a k-point representing the center of one of the spheres used for k-dependent disentanglement. The same crystallographic units as for kpoints are used here. Each k-point coordinate \mathbf{K}^i must the followed by the respectice sphere radius r_i in inverse angstrom (on the same line).

The number of lines must be equal to dis_spheres_num.

begin dis_spheres

$$\begin{array}{ccccc} K_1^1 & K_2^1 & K_3^1 & r_1 \\ K_1^2 & K_2^2 & K_3^2 & r_2 \\ \vdots & & & & \end{array}$$

end dis_spheres

There is no default.

2.8 Wannierise

Iterative minimisation of $\widetilde{\Omega}$, the non-gauge-invariant part of the spread functional.

2.8.1 integer :: num_iter

Total number of iterations in the minimisation procedure. Set num_iter=0 if you wish to generate projected WFs rather than maximally-localized WFs (see Example 8 in the Tutorial).

The default value is 100

2.8.2 integer :: num_cg_steps

Number of conjugate gradient steps to take before resetting to steepest descents.

The default value is 5

2.8.3 integer :: conv_window

If $conv_window > 1$, then the minimisation is said to be converged if the change in Ω over $conv_window$ successive iterations is less than $conv_tol$. Otherwise, the minimisation proceeds for num_iter iterations (default).

The default value is -1

2.8.4 real(kind=dp) :: conv_tol

If conv_window > 1, then this is the convergence tolerance on Ω , otherwise not used. Units are $Å^2$.

The default value is 1.0E-10

2.8.5 logical :: precond

Whether or not to use preconditioning to speed up the minimization of the spreads. This is based on the same idea as the classical Tetter-Payne-Allan preconditionning for DFT and dampens the high-frequency oscillations of the gradient due to contributions from large real lattice vectors. It is useful when the optimization is slow, especially on fine grids. When optimisation<3, this uses a slower algorithm to save memory.

The default value is false.

2.8.6 real(kind=dp) :: conv_noise_amp

If conv_noise_amp > 0, once convergence (as defined above) is achieved, some random noise f is added to the search direction, and the minimisation is continued until convergence is achieved once more. If the same value of Ω as before is arrived at, then the calculation is considered to be converged. If not, then random noise is added again and the procedure repeated up to a maximum of conv_noise_num times. conv_noise_amp is the amplitude of the random noise f that is added to the search direction: $0 < |f| < \text{conv_noise_amp}$. This functionality requires conv_window > 1. If conv_window is not specified, it is set to the value 5 by default.

If $conv_noise_amp \le 0$, then no noise is added (default).

The default value is -1.0

2.8.7 integer :: conv_noise_num

If conv_noise_amp > 0, then this is the number of times in the minimisation that random noise is added.

The default value is 3

2.8.8 integer :: num_dump_cycles

Write sufficient information to do a restart every num_dump_cycles iterations.

The default is 100

2.8.9 integer :: num_print_cycles

Write data to the master output file seedname.wout every num_print_cycles iterations.

The default is 1

2.8.10 logical :: write_r2mn

If write_r2mn = true, then the matrix elements $\langle m|r^2|n\rangle$ (where m and n refer to WF) are written to file seedname.r2mn at the end of the Wannierisation procedure.

The default value of this parameter is false.

2.8.11 logical :: guiding_centres

Use guiding centres during the minimisation, in order to avoid local minima.

wannier90 uses a logarithm definition of the spread functional. As we are taking the log of a complex argument there is a possibility that the algorithm might make inconsistent choices for the branch cut. This manifests itself as complex WF with a large spread. By using guiding centres the code will attempt to make a consistent choice of branch cut. Experience shows that with guiding_centres set to true this problem is avoided and doing so does not cause any problems. For this reason we recommend setting guiding_centres to true where possible (it is only not possible if an explicit projection block is not defined).

The default value is false.

2.8.12 integer :: num_guide_cycles

If guiding_centres is set to true, then the guiding centres are used only every num_guide_cycles.

The default value is 1.

2.8.13 integer :: num_no_guide_iter

If guiding_centres is set to true, then the guiding centres are used only after num_no_guide_iter minimisation iterations have been completed.

The default value is 0.

2.8.14 real(kind=dp) :: trial_step

The value of the trial step for the parabolic fit in the line search minimisation used in the minimisation of the spread function. Cannot be used in conjunction with fixed_step (see below). If the minimisation procedure doesn't converge, try decreasing the value of trial_step to give a more accurate line search.

The default value is 2.0

2.8.15 real(kind=dp) :: fixed_step

If this is given a value in the input file, then a fixed step of length fixed_step (instead of a parabolic line search) is used at each iteration of the spread function minimisation. Cannot be used in conjunction

with trial_step. This can be useful in cases in which minimisation with a line search fails to converge.

There is no default value.

2.8.16 logical :: use_bloch_phases

Determines whether to use the Bloch functions as the initial guess for the projections. Can only be used if disentanglement = false.

The default value is false.

2.8.17 logical :: site_symmetry

Construct symmetry-adapted Wannier functions. For the detail of the theoretical background, see Ref. [7]. Cannot be used in conjunction with the inner (frozen) energy window.

The default value is false.

2.8.18 real(kind=dp) :: symmetrize_eps

Convergence threshold to check whether the symmetry condition (Eq. (19) in Ref. [7]) on the unitary matrix $\mathbf{U}^{(\mathbf{k})}$ is satisfied or not. See also Eq. (29) in Ref. [7]. Used when $\mathtt{site_symmetry} = .\mathsf{true}$.

The default value is 1.0E-3.

2.8.19 integer :: slwf_num

The number of objective Wannier functions for selective localisation in the selectively localised Wannier function (SLWF) method of Ref. [8]. These functions are obtained by minimising the spread functional only with respect to the degrees of freedom of a subset of slwf_num < num_wann functions. At convergence, the objective WFs will have a minimum cumulative spread, whereas the remaining num_wann - slwf_num functions are left unoptimised. The initial guesses for the objective WFs are given by the first slwf_num orbitals in the projections block. If slwf_num = num_wann no selective minimisation is performed. In this case, wannier90 will simply generate a set of num_wann MLWFs.

The default is num_wann.

2.8.20 logical :: slwf_constrain

If slwf_constrain=true, then the centres of the objective Wannier functions are constrained to either the centres of the first slwf_num orbitals in the projections block or to new positions specified in the slwf_centres block (see Sec. 2.8.22). In this case, a modified spread functional, Ω_c , with the addition of a constraint term, as described in Ref. [8].

The default is false

2.8.21 real(kind=dp) :: slwf_lambda

The value of the Lagrange multiplier λ for the constraint term in term added to modify the spread functional: $\lambda \sum_{n=1}^{J'} (\bar{\mathbf{r}}_n - \mathbf{r}_{0n})^2$, where J' is slwf_num, and $\bar{\mathbf{r}}_n$ and \mathbf{r}_{0n} are the centre and target centre, respectively, for the n^{th} objective WF.

The default is 0.0.

2.8.22 Constraints on centres

If slwf_constrain=true, then by default the centres to which the slwf_num objective Wannier function centres are constrained are given by the first slwf_num rows of the projections block.

Optionally, the slwf_centres block may be used to define alternative target centres for some or all of the slwf_num objective Wannier functions.

The block below shows an example of how to set the constraints:

```
begin slwf_centres
2 0.0 0.0 0.0
4 0.25 0.0 0.0
end slwf_centres
```

- The first line sets the constraint for the centre of objective WF number 2 (as defined by the order of WFs in the projections block) to (0.0,0.0,0.0) in fractional co-ordinates.
- The second line sets the constraint for the centre of objective WF number 4 (as defined by the order of WFs in the projections block) to (0.25,0.0,0.0) in fractional co-ordinates.
- The target centres of all other objective Wannier functions remain as the centres given in the corresponding rows of the projections block.

2.9 Post-Processing

Capabilities:

- Plot the WF
- Plot the interpolated band structure
- Plot the Fermi surface
- Output the Hamiltonian in the WF basis
- Transport calculation (quantum conductance and density of states)

2.9.1 logical :: wannier_plot

If wannier_plot = true, then the code will write out the Wannier functions in a format specified by wannier_plot_format

The default value of this parameter is false.

2.9.2 integer :: wannier_plot_list(:)

A list of WF to plot. The WF numbered as per the **seedname.wout** file after the minimisation of the spread.

The default behaviour is to plot all WF. For example, to plot WF 4, 5, 6 and 10:

```
wannier_plot_list : 4-6, 10
```

2.9.3 integer :: wannier_plot_supercell

The code generates the WFs on a grid corresponding to a 'super-unit-cell'. If wannier_plot_supercell is provided as a single integer, then the size of the super-unit-cell is wannier_plot_supercell times the size of the unit cell along all three linear dimensions (the 'home' unit cell is kept approximately in the middle); otherwise, if three integers are provided, the size of the super-unit-cell is wannier_plot_supercell(i) times the size of the unit cell along the i-th linear dimension.

The default value is 2.

2.9.4 character(len=20) :: wannier_plot_format

WF can be plotted in either XCrySDen (xsf) format or Gaussian cube format. The valid options for this parameter are:

```
- xcrysden (default)
```

- cube

If wannier_plot_format=xsf: the code outputs the WF on the entire super-unit-cell specified by wannier_plot_supercell.

If wannier_plot_format=cube: the code outputs the WF on a grid that is smaller than the super-unit-cell specified by wannier_plot_supercell. This grid is determined by wannier_plot_mode, wannier_plot_radius and wannier_plot_scale, described in detail below.

The code is able to output Gaussian cube files for systems with non-orthogonal lattice vectors. Many visualisation programs (including XCrySDen), however, are only able to handle cube files for systems with *orthogonal* lattice vectors. One visualisation program that is capable of dealing with non-orthogonal lattice vectors is VESTA (http://jp-minerals.org/vesta/en/).¹

2.9.5 character(len=20) :: wannier_plot_mode

Choose the mode in which to plot the WF, either as a molecule or as a crystal.

The valid options for this parameter are:

```
- crystal (default)
```

¹It's worth noting that another visualisation program, VMD (http://www.ks.uiuc.edu/Research/vmd), is able to deal with certain special cases of non-orthogonal lattice vectors; see http://www.ks.uiuc.edu/Research/vmd/plugins/molfile/cubeplugin.html for details.

- molecule

If wannier_plot_format=cube:

• if wannier_plot_mode = molecule, then wherever the WF centre sits in the supercell, the origin of the cube is shifted (for the purpose of plotting only, ie, nothing is done to the U matrices etc) to coincide with the centre of mass of the atomic positions specified by the user in the *.win input file. These atomic positions are also written to the cube file, so when it is visualised, the WF appears superimposed on the molecular structure.

- if wannier_plot_mode = crystal, then the WF is not shifted, but instead the code searches for atoms that are within a radius of wannier_plot_scale × wannier_plot_radius of the WF centre and writes the coordinates of these atoms to the cube file. In this way, when the cube file is visualised, the WF appears superimposed on the nearest atoms to the WF centre.
- crystal mode can be used for molecules, and molecule mode can be used for crystals.

2.9.6 real(kind=dp) :: wannier_plot_radius

If wannier_plot_format=cube, then wannier_plot_radius is the radius of the sphere that must fit inside the parallelepiped in which the WF is plotted. wannier_plot_radius must be greater than 0. Units are Å.

The default value is 3.5.

2.9.7 real(kind=dp) :: wannier_plot_scale

If wannier_plot_format=cube and wannier_plot_mode=crystal, then the code searches for atoms that are within a radius of wannier_plot_scale × wannier_plot_radius of the WF centre and writes the coordinates of these atoms to the cube file. In this way, when the cube file is visualised, the WF appears superimposed on the nearest atoms to the WF centre. wannier_plot_scale must be greater than 0. This parameter is dimensionless.

The default value is 1.0.

2.9.8 character(len=20) :: wannier_plot_spinor_mode

If spinors = true then this parameter controls the quantity to plot. For a spinor WF with components $[\phi, \psi]$ the quantity plotted is

- total (default). $\sqrt{|\phi|^2 + |\psi|^2}$
- up. $|\phi| \times sign(Re\{\phi\})$ if wannier_plot_spinor_phase = true, otherwise $|\phi|$
- down. $|\psi| \times sign(Re\{\psi\})$ if wannier_plot_spinor_phase = true, otherwise $|\psi|$

Note: making a visual representation of a spinor WF is not as straightforward as for a scalar WF. While a scalar WF is typically a real valued function, a spinor WF is a complex, two component spinor. wannier90 is able to plot several different quantities derived from a spinor WF which should give you a good idea of the nature of the WF.

2.9.9 logical :: wannier_plot_spinor_phase

If wannier_plot_spinor_phase = true phase information will be taken into account when plotting a spinor WF.

2.9.10 logical :: bands_plot

If bands_plot = true, then the code will calculate the band structure, through Wannier interpolation, along the path in k-space defined by bands_kpath using bands_num_points along the first section of the path and write out an output file in a format specified by bands_plot_format.

The default value is false.

2.9.11 kpoint path

Defines the path in k-space along which to calculate the bandstructure. Each line gives the start and end point (with labels) for a section of the path. Values are in fractional coordinates with respect to the primitive reciprocal lattice vectors.

begin kpoint_path

```
G 0.0 0.0 0.0 L 0.0 0.0 1.0 L 0.1 1.0 L 0.0 0.0 1.0 L 0.0 0.0 1.0 1.0 1.0 :
```

end kpoint_path

There is no default

2.9.12 integer :: bands_num_points

If bands_plot = true, then the number of points along the first section of the bandstructure plot given by kpoint_path. Other sections will have the same density of k-points.

The default value for bands_num_points is 100.

2.9.13 character(len=20) :: bands_plot_format

Format in which to plot the interpolated band structure. The valid options for this parameter are:

```
- gnuplot (default)
```

- xmgrace

Note: it is possible to request output in both formats eg bands_format = gnuplot xmgrace

2.9.14 integer :: bands_plot_project(:)

If present wannier90 will compute the contribution of this set of WF to the states at each point of the interpolated band structure. The WF are numbered according to the seedname.wout file. The result is

 $written \ in \ the \ {\tt seedname_band.dat} \ file, \ and \ a \ corresponding \ gnuplot \ script \ to \ {\tt seedname_band_proj.dat}$

For example, to project on to WFs 2, 6, 7, 8 and 12:

```
bands_plot_project : 2, 6-8, 12
```

2.9.15 character(len=20) :: bands_plot_mode

To interpolate the band structure along the k-point path, either use the Slater-Koster interpolation scheme or truncate the Hamiltonian matrix in the WF basis. Truncation criteria are provided by hr_cutoff and dist_cutoff.

The valid options for this parameter are:

```
- s-k (default)
```

- cut

2.9.16 integer :: bands_plot_dim

Dimension of the system. If bands_plot_dim < 3 and bands_plot_mode = cut, lattice vector $\mathbf{R} = N_1 \mathbf{A}_1 + N_2 \mathbf{A}_2 + N_3 \mathbf{A}_3$, where $N_i = 0$ if \mathbf{A}_i is parallel to any of the confined directions specified by one_dim_axis, are exclusively used in the band structure interpolation.

The valid options for this parameter are:

```
- 3 (default)
```

-2

- 1

2.9.17 logical :: fermi_surface_plot

If fermi_surface_plot = true, then the code will calculate, through Wannier interpolation, the eigenvalues on a regular grid with fermi_surface_num_points in each direction. The code will write a file in bxsf format which can be read by XCrySDen in order to plot the Fermi surface.

The default value is false.

2.9.18 integer :: fermi_surface_num_points

If fermi_surface_plot = true, then the number of divisions in the regular k-point grid used to calculate the Fermi surface.

The default value for fermi_surface_num_points is 50.

2.9.19 real(kind=dp) :: fermi_energy

The Fermi energy in eV. This parameter is written into the bxsf file. If fermi_energy is specified, fermi_energy_min, fermi_energy_max, and fermi_energy_step should not be specified, and viceversa.

The default value is 0.0

2.9.20 real(kind=dp) :: fermi_energy_min

Instead of specifyfing a single Fermi energy, it is possible to scan the Fermi level over a range of values, and recompute certain quantities for each ε_F .² This is the minimum value in the range (in eV).

There is no default value.

2.9.21 real(kind=dp) :: fermi_energy_max

The maximum value in the range of Fermi energies. Units are eV.

The default value is fermi_energy_min+1.0.

2.9.22 real(kind=dp) :: fermi_energy_step

Difference between consecutive values of the Fermi energy when scanning from fermi_energy_min to fermi_energy_max. Units are eV.

The default value is 0.01.

2.9.23 character(len=20) :: fermi_surface_plot_format

Format in which to plot the Fermi surface. The valid options for this parameter are:

- xcrysden (default)

2.9.24 logical :: write_hr

If write_hr = true, then the Hamiltonian matrix in the WF basis will be written to a file seedname_hr.dat.

The default value is false.

2.9.25 logical :: write_rmn

If write_rmn = true, then the position operator in the WF basis will be written to a file seedname_r.dat.

The default value is false.

²Scanning the Fermi level is currently supported only by the postw90 module berry, for berry_task=ahc,morb. For all other functionalities that require a knowledge of ε_F , use fermi_energy instead.

2.9.26 logical :: write_bvec

If write_bvec = true, then the matrix elements of bvector and their weights will be written to a file seedname.bvec.

The default value is false.

2.9.27 logical :: write_tb

If write_tb = true, then the lattice vectors, together with the Hamiltonian and position-operator matrices in the WF basis, will be written to a file seedname_tb.dat, in units of Angstrom and eV.

The default value is false.

2.9.28 logical :: transport

If transport = true, then the code will calculate quantum conductance and density of states of a one-dimensional system. The results will be written to files seedname_qc.dat and seedname_dos.dat, respectively. Since both quantities are a function of energy, they will be evaluated from tran_win_min to tran_win_max with an interval of tran_energy_step.

The default value of this parameter is false.

2.9.29 character(len=20) :: transport_mode

If transport_mode = bulk, quantum conductance and density of states are calculated for a perfectly-periodic one-dimensional system. In this case, the transport part can either use the Hamiltonian matrix in the WF basis generated by wannier90 or a Hamiltonian matrix provided by the external file seedname_htB.dat.

If transport_mode = lcr, quantum conductance and density of states are calculated for a system where semi-infinite, left and right leads are connected through a central conductor region. In this case, the transport part will work independently from the disentanglement and wannierise procedure. Details of the method is described in Ref. [9].

If tran_read_ht = true then the Hamiltonian matrices must be provided by the five external files: seedname_htL.dat, seedname_htC.dat, seedname_htC.dat, seedname_htCR.dat, seedname_htR.dat. If tran_read_ht = false then the Hamiltonian matrices are found automatically provided the supercell adheres to conditions outlined in Section 7.3.

The valid options for this parameter are:

- bulk (default)
- lcr

2.9.30 real(kind=dp) :: tran_win_min

The lower bound of the energy window for the transport calculation. Units are eV.

The default value is -3.0.

2.9.31 real(kind=dp) :: tran_win_max

The upper bound of the energy window for the transport calculation. Units are eV.

The default value is 3.0.

2.9.32 real(kind=dp) :: tran_energy_step

Sampling interval of the energy values from tran_win_min to tran_win_max. Units are eV.

The default value is 0.01.

2.9.33 real(kind=dp) :: fermi_energy

The Fermi energy in eV. The energy axis of the quantum conductance and density of states data will be shifted rigidly by this amount.

The default value is 0.0

2.9.34 integer :: tran_num_bb

Size of a bulk Hamiltonian matrix. This number is equal to the number of WFs in one principal layer.

A one-dimensional system can be viewed as an array of principal layers which are defined in a way that localized basis functions inside a certain principal layer only interact with those in the nearest neighbor principal layer. In wannier90 a principal layer will be an integer multiple of a unit cell, and the size is determined by hr_cutoff and/or dist_cutoff. The criterion is rather arbitrary when WFs are adopted as a localized basis set, and it is up to a user's choice.

The default value is 0.

2.9.35 integer :: tran_num_ll

Size of a left-lead Hamiltonian matrix. If transport_mode = lcr and tran_read_ht = false then tran_num_ll is the number of Wannier functions in a principal layer.

The default value is 0.

2.9.36 integer :: tran_num_rr

Size of a right-lead Hamiltonian matrix.

The default value is 0.

2.9.37 integer :: tran_num_cc

Size of a conductor Hamiltonian matrix.

The default value is 0.

2.9.38 integer :: tran_num_lc

Number of columns in a left-lead_conductor Hamiltonian matrix. Number of rows must be equal to tran_num_ll.

The default value is 0.

2.9.39 integer :: tran_num_cr

Number of rows in a conductor_right-lead Hamiltonian matrix. Number of columns must be equal to tran_num_rr.

The default value is 0.

2.9.40 integer :: tran_num_cell_ll

Number of unit cells in one principal layer of left lead. Used if transport_mode = lcr and tran_read_ht = false.

The default value is 0.

2.9.41 integer :: tran_num_cell_rr

Number of unit cells in one principal layer of right lead. Not used at present.

The default value is 0.

2.9.42 integer :: tran_num_bandc

Half-bandwidth+1 of a band-diagonal conductor Hamiltonian matrix.

The Hamiltonian matrix of a central conductor part, which is read from seedname_htC.dat, will be diagonally dominant when tran_num_cc is very large. tran_num_bandc is used to construct a compact matrix which contains the non-zero band-diagonal part of a full conductor Hamiltonian matrix. Setting this parameter is only meaningful when tran_num_bandc is greater than tran_num_lc and tran_num_cr.

The default value is 0.

2.9.43 logical :: tran_write_ht

If tran_write_ht = true, then the Hamiltonian matrix formatted for the transport calculation will be written to a file seedname_htB.dat.

The default value is false.

2.9.44 logical :: tran_read_ht

If tran_write_ht = true, then the Hamiltonian matrix formatted for the transport calculation will be read from a set of files described in the parameter transport_mode. Set tran_write_ht = false to perform automated lcr calculations (see Section 7.3).

The default value is false.

2.9.45 logical :: tran_use_same_lead

If tran_use_same_lead = true, then the left and the right leads are the same. In this case, seedname_htR.dat is not required.

The default value is true.

2.9.46 real(kind=dp) :: tran_group_threshold

Used to group and sort Wannier functions according to the positions of their centres. Wannier functions in a group are within $tran_group_threshold$ from one another in x,y and z directions. Units are Å

The default is 0.15

2.9.47 real(kind=dp) :: translation_centre_frac(3)

Centre of the unit cell to which the final Wannier centres are translated. Numbers are in fractional coordinates with respect to the lattice vectors.

The default value is (0.0,0.0,0.0).

2.9.48 logical :: use_ws_distance

Improves the interpolation of the k-space Hamiltonian, by applying a translation to each WF by a basis vector of the super-lattice that minimises the distance between their centres. The translation is dependent on both WF and on the unit cell vector to which they belong, i.e., translate function $W_i(\mathbf{r} - \mathbf{R})$ inside the Wigner-Seitz cell centred on WF $W_i(\mathbf{r})$.

For a longer explanation, see Chapter 9.

If false the code puts all the WF in the home cell, only possible choice until wannier90 v2.0.1.

The default value is **true** (default changed since v.3.0). Introduced in v2.1.

2.9.49 real(kind=dp) :: ws_distance_tol

Tolerance when determining whether two values $\|\mathbf{d}_{ij\mathbf{R}} + \tilde{\mathbf{R}}_{nml}\|$ and $\|\mathbf{d}_{ij\mathbf{R}} + \tilde{\mathbf{R}}_{n'm'l'}\|$ (as defined in chapter 9) for the shortest distance between two Wannier functions are equivalent. If the difference in distance (in Angstrom) is less than ws_distance_tol, they are taken to be equivalent.

The default value is 10^{-5} .

2.9.50 :: ws_search_size

Maximum absolute value for the integers n, m, l that identify the super-lattice vectors $\tilde{\mathbf{R}}_{nml}$ (see chapter 9) when searching for points inside the Wigner-Seitz cell. If ws_search_size is provided as a single integer, then the number of repetitions of the Born-von Karman cell is the same along all three linear dimensions; otherwise, if three integers are provided, the number of repetitions along the i-th linear dimension is $ws_search_size(i)$. The variable is used both in hamiltonian.F90 and in $ws_distance.F90$. In the latter case, its value is incremented by one in order to account for WFs whose centre wanders away from the original reference unit cell.

The default value is generally sufficient, but might need to be increased in case of elongated cells.

The default value is 2.

2.9.51 logical :: write_u_matrices

Write the $U^{(k)}$ and $U^{dis(k)}$ matrices obtained at the end of wannierization to files seedname_u.mat and seedname_u_dis.mat, respectively.

The default value is false.

2.9.52 real(kind=dp) :: hr_cutoff

The absolute value of the smallest matrix element of the Hamiltonian in the WF basis. If $h_{mn}(\mathbf{R}) > \text{hr_cutoff}$, then the matrix element $h_{mn}(\mathbf{R})$ is retained and used in the band structure interpolation (when bands_plot_mode = cut) or in the transport calculation. Otherwise it is deemed to be insignificant and is discarded. Units are eV.

The default value is 0.0.

2.9.53 real(kind=dp) :: dist_cutoff

The largest distance between two WFs for which the Hamiltonian matrix element is retained and used in the band interpolation (when bands_plot_mode = cut) or in the transport calculation. Units are Å.

The default value is 1000.0.

2.9.54 character(len=20) :: dist_cutoff_mode

Dimension in which the distance between two WFs is calculated. The vector connecting two WFs may be projected to a line (one_dim) or a plane (two_dim). The size of the projected vector is calculated, and dist_cutoff is applied. When one_dim or two_dim is used, one_dim_axis must be given to specify extended or confined direction.

The valid options for this parameter are:

```
- three_dim (default)
```

- two_dim
- one_dim

2.9.55 character(len=20) :: one_dim_axis

Extended direction for a one-dimensional system or confined direction for a two-dimensional system. This direction must be parallel to one of the Cartesian axes.

The valid options for this parameter are:

- x
- у
- z

No default.

Chapter 3

Projections

3.1 Specification of projections in seedname.win

Here we describe the projection functions used to construct the initial guess $A_{mn}^{(\mathbf{k})}$ for the unitary transformations.

Each projection is associated with a site and an angular momentum state defining the projection function. Optionally, one may define, for each projection, the spatial orientation, the radial part, the diffusivity, and the volume over which real-space overlaps A_{mn} are calculated.

The code is able to

- 1. project onto s,p,d and f angular momentum states, plus the hybrids sp, sp², sp³, sp³d, sp³d².
- 2. control the radial part of the projection functions to allow higher angular momentum states, e.g., both 3s and 4s in silicon.

The atomic orbitals of the hydrogen atom provide a good basis to use for constructing the projection functions: analytical mathematical forms exist in terms of the good quantum numbers n, l and m; hybrid orbitals (sp, sp², sp³, sp³d etc.) can be constructed by simple linear combination $|\phi\rangle = \sum_{nlm} C_{nlm} |nlm\rangle$ for some coefficients C_{nlm} .

The angular functions that use as a basis for the projections are not the canonical spherical harmonics Y_{lm} of the hydrogenic Schrödinger equation but rather the real (in the sense of non-imaginary) states Θ_{lm_r} , obtained by a unitary transformation. For example, the canonical eigenstates associated with $l=1, m=\{-1,0,1\}$ are not the real p_x , p_y and p_z that we want. See Section 3.4 for our mathematical conventions regarding projection orbitals for different n, l and m_r .

We use the following format to specify projections in <seedname>.win:

units:

Optional. Either Ang or Bohr to specify whether the projection centres specified in this block (if given in Cartesian co-ordinates) are in units of Angstrom or Bohr, respectively. The default value is Ang.

site:

C, Al, etc. applies to all atoms of that type

f=0,0.50,0 - centre on (0.0,0.5,0.0) in fractional coordinates (crystallographic units) relative to the direct lattice vectors

c=0.0,0.805,0.0 – centre on (0.0,0.805,0.0) in Cartesian coordinates in units specified by the optional string units in the first line of the projections block (see above).

ang_mtm:

Angular momentum states may be specified by 1 and mr, or by the appropriate character string. See Tables 3.1 and 3.2. Examples:

1=2,mr=1 or dz2 - a single projection with l=2, $m_r=1$ (i.e., d_{z^2})

1=2,mr=1,4 or dz2,dx2-y2 - two functions: d_{z^2} and d_{xz}

1=-3 or $sp3 - four sp^3$ hybrids

Specific hybrid orbitals may be specified as follows:

1=-3,mr=1,3 or sp3-1,sp3-3 – two specific sp^3 hybrids

Multiple states may be specified by separating with ';', e.g.,

sp3;1=0 or 1=-3;1=0 - four sp^3 hybrids and one s orbital

zaxis (optional):

z=1,1,1- set the z-axis to be in the (1,1,1) direction. Default is z=0,0,1

xaxis (optional):

x=1,1,1- set the x-axis to be in the (1,1,1) direction. Default is x=1,0,0

radial (optional):

r=2 – use a radial function with one node (ie second highest pseudostate with that angular momentum). Default is r=1. Radial functions associated with different values of r should be orthogonal to each other.

zona (optional):

zona=2.0 – the value of $\frac{Z}{a}$ for the radial part of the atomic orbital (controls the diffusivity of the radial function). Units always in reciprocal Angstrom. Default is zona=1.0.

Examples

1. CuO, s,p and d on all Cu; sp³ hybrids on O:

```
Cu:1=0;1=1;1=2
```

2. A single projection onto a p_z orbital orientated in the (1,1,1) direction:

```
c=0,0,0:l=1,mr=1:z=1,1,1 or c=0,0,0:pz:z=1,1,1
```

3. Project onto s, p and d (with no radial nodes), and s and p (with one radial node) in silicon:

```
Si:1=0;1=1;1=2
```

3.2 Spinor Projections

When spinors=.true. it is possible to select a set of localised functions to project onto 'up' states and a set to project onto 'down' states where, for complete flexibility, it is also possible to set the local spin quantisation axis.

Note, however, that this feature requires a recent version of the interface between the ab-initio code and Wannier90 (i.e., written after the release of the 2.0 version, in October 2013) supporting spinor projections.

```
Begin Projections
[units]
site:ang_mtm:zaxis:xaxis:radial:zona(spin)[quant_dir]
End Projections
spin (optional):
Choose projection onto 'up' or 'down' states
u – project onto 'up' states.
d – project onto 'down' states.
Default is u,d
quant_dir (optional):
1,0,0 – set the spin quantisation axis to be in the (1,0,0) direction. Default is 0,0,1
Examples
   • 18 projections on an iron site
     Fe:sp3d2;dxy;dxx;dyz
   • same as above
     Fe:sp3d2;dxy;dxx;dyz(u,d)
   • same as above
     Fe:sp3d2;dxy;dxz;dyz(u,d)[0,0,1]
   • same as above but quantisation axis is now x
     Fe:sp3d2;dxy;dxz;dyz(u,d)[1,0,0]
   • now only 9 projections onto up states
     Fe:sp3d2;dxy;dxz;dyz(u)
   • 9 projections onto up-states and 3 on down
     Fe:sp3d2;dxy;dxz;dyz(u)
     Fe:dxy;dxz;dyz(d)
```

• projections onto alternate spin states for two lattice sites (Cr1, Cr2)

Cr1:d(u) Cr2:d(d)

3.3 Short-Cuts

3.3.1 Random projections

It is possible to specify the projections, for example, as follows:

Begin Projections random C:sp3

End Projections

in which case wannier90 uses four sp³ orbitals centred on each C atom and then chooses the appropriate number of randomly-centred s-type Gaussian functions for the remaining projection functions. If the block only consists of the string random and no specific projection centres are given, then all of the projection centres are chosen randomly.

3.3.2 Bloch phases

Setting use_bloch_phases = true in the input file absolves the user of the need to specify explicit projections. In this case, the Bloch wave-functions are used as the projection orbitals, namely $A_{mn}^{(\mathbf{k})} = \langle \psi_{m\mathbf{k}} | \psi_{n\mathbf{k}} \rangle = \delta_{mn}$.

3.4 Orbital Definitions

The angular functions $\Theta_{lm_r}(\theta,\varphi)$ associated with particular values of l and m_r are given in Tables 3.1 and 3.2.

The radial functions $R_r(r)$ associated with different values of r should be orthogonal. One choice would be to take the set of solutions to the radial part of the hydrogenic Schrödinger equation for l = 0, i.e., the radial parts of the 1s, 2s, 3s... orbitals, which are given in Table 3.3.

l	$m_{ m r}$	Name	$\Theta_{lm_{ m r}}(heta,arphi)$
0	1	S	$\frac{1}{\sqrt{4\pi}}$
1	1	pz	$\sqrt{\frac{3}{4\pi}}\cos\theta$ $\sqrt{\frac{3}{4\pi}}\sin\theta\cos\varphi$
1	2	px	$\sqrt{rac{3}{4\pi}}\sin heta\cosarphi$
1	3	ру	$\sqrt{rac{3}{4\pi}}\sin heta\sinarphi$
2	1	dz2	$\sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$
2	2	dxz	$\sqrt{\frac{15}{4\pi}}\sin\theta\cos\theta\cos\varphi$
2	3	dyz	$\sqrt{\frac{15}{4\pi}}\sin\theta\cos\theta\sin\varphi$
2	4	dx2-y2	$\sqrt{\frac{15}{16\pi}}\sin^2\theta\cos2\varphi$
2	5	dxy	$\sqrt{\frac{15}{16\pi}}\sin^2\theta\sin2\varphi$
3	1	fz3	$\frac{\sqrt{7}}{4\sqrt{\pi}}(5\cos^3\theta - 3\cos\theta)$
3	2	fxz2	$\frac{\sqrt{21}}{4\sqrt{2\pi}} (5\cos^2\theta - 1)\sin\theta\cos\varphi$
3	3	fyz2	$\frac{\sqrt{21}}{4\sqrt{2\pi}} (5\cos^2\theta - 1)\sin\theta\sin\varphi$ $\frac{\sqrt{105}}{4\sqrt{\pi}}\sin^2\theta\cos\theta\cos2\varphi$
3			
3	5	fxyz	$\frac{\sqrt{105}}{4\sqrt{\pi}}\sin^2\theta\cos\theta\sin2\varphi$
3	6	fx(x2-3y2)	$\frac{\sqrt{35}}{4\sqrt{2\pi}}\sin^3\theta(\cos^2\varphi - 3\sin^2\varphi)\cos\varphi$ $\frac{\sqrt{35}}{4\sqrt{2\pi}}\sin^3\theta(3\cos^2\varphi - \sin^2\varphi)\sin\varphi$
3	7	fy(3x2-y2)	$\frac{\sqrt{35}}{4\sqrt{2\pi}}\sin^3\theta(3\cos^2\varphi-\sin^2\varphi)\sin\varphi$
3.1: Angular fu	unctio	ons $\Theta_{lm_{ m r}}(heta,arphi)$ a	ssociated with particular values of l

l	$m_{ m r}$	Name	$\Theta_{lm_{ m r}}(heta,arphi)$
-1	1	sp-1	$\frac{1}{\sqrt{2}}$ s $+\frac{1}{\sqrt{2}}$ px
-1	2	sp-2	$\frac{1}{\sqrt{2}}$ s $-\frac{1}{\sqrt{2}}$ px
-2	1	sp2-1	$\frac{1}{\sqrt{3}}s - \frac{1}{\sqrt{6}}px + \frac{1}{\sqrt{2}}py$
-2	2	sp2-2	$\frac{1}{\sqrt{3}}s - \frac{1}{\sqrt{6}}px - \frac{1}{\sqrt{2}}py$
-2	3	sp2-3	$\frac{1}{\sqrt{3}}$ s $+\frac{2}{\sqrt{6}}$ px
-3	1	sp3-1	$\frac{1}{2}(s + px + py + pz)$
-3	2	sp3-2	$\frac{1}{2}(\mathtt{s}+\mathtt{px}-\mathtt{py}-\mathtt{pz})$
-3	3	sp3-3	$\frac{1}{2}(\mathtt{s}-\mathtt{px}+\mathtt{py}-\mathtt{pz})$
-3	4	sp3-4	$\frac{1}{2}(\mathtt{s}-\mathtt{px}-\mathtt{py}+\mathtt{pz})$
-4	1	sp3d-1	$\frac{1}{\sqrt{3}}s - \frac{1}{\sqrt{6}}px + \frac{1}{\sqrt{2}}py$
-4	2	sp3d-2	$\frac{1}{\sqrt{3}}$ s $-\frac{1}{\sqrt{6}}$ px $-\frac{1}{\sqrt{2}}$ py
-4	3	sp3d-3	$\frac{1}{\sqrt{3}}$ s $+\frac{2}{\sqrt{6}}$ px
-4	4	sp3d-4	$\frac{1}{\sqrt{2}}$ pz $+\frac{1}{\sqrt{2}}$ dz2
-4	5	sp3d-5	$-rac{1}{\sqrt{2}}$ pz $+rac{1}{\sqrt{2}}$ dz2
-5	1	sp3d2-1	$\frac{1}{\sqrt{6}}s - \frac{1}{\sqrt{2}}px - \frac{1}{\sqrt{12}}dz2 + \frac{1}{2}dx2-y2$
-5	2	sp3d2-2	$\frac{1}{\sqrt{6}}s + \frac{1}{\sqrt{2}}px - \frac{1}{\sqrt{12}}dz^2 + \frac{1}{2}dx^2 - y^2$
-5	3	sp3d2-3	$\frac{1}{\sqrt{6}}s - \frac{1}{\sqrt{2}}py - \frac{1}{\sqrt{12}}dz2 - \frac{1}{2}dx2-y2$
-5	4	sp3d2-4	$\frac{1}{\sqrt{6}}s + \frac{1}{\sqrt{2}}py - \frac{1}{\sqrt{12}}dz2 - \frac{1}{2}dx2-y2$
-5	5	sp3d2-5	$rac{1}{\sqrt{6}}$ s $-rac{1}{\sqrt{2}}$ pz $+rac{1}{\sqrt{3}}$ dz2
-5	6	sp3d2-6	$\frac{1}{\sqrt{6}}s + \frac{1}{\sqrt{2}}pz + \frac{1}{\sqrt{3}}dz2$

Table 3.2: Angular functions $\Theta_{lm_r}(\theta,\varphi)$ associated with particular values of l and m_r for l<0, in terms of the orbitals defined in Table 3.1.

r	$R_{ m r}(r)$
1	$2\alpha^{3/2}\exp(-\alpha r)$
2	$\frac{1}{2\sqrt{2}}\alpha^{3/2}(2-\alpha r)\exp(-\alpha r/2)$
3	$\sqrt{\frac{4}{27}}\alpha^{3/2}(1 - 2\alpha r/3 + 2\alpha^2 r^2/27)\exp(-\alpha r/3)$

Table 3.3: One possible choice for the radial functions $R_r(r)$ associated with different values of r: the set of solutions to the radial part of the hydrogenic Schrödinger equation for l=0, i.e., the radial parts of the 1s, 2s, 3s... orbitals, where $\alpha=Z/a=$ **zona**.

3.5 Projections via the SCDM-k method in pw2wannier90

For many systems, such as aperiodic systems, crystals with defects, or novel materials with complex band structure, it may be extremely hard to identify a-priori a good initial guess for the projection functions used to generate the $A_{mn}^{(\mathbf{k})}$ matrices. In these cases, one can use a different approach, known as the SCDM- \mathbf{k} method[10], based on a QR factorization with column pivoting (QRCP) of the density matrix from the self-consistent field calculation, which allows one to avoid the tedious step of specifying a projection block altogether, hence to avoid. This method is robust in generating well localised function with the correct spatial orientations and in general in finding the global minimum of the spread functional Ω . Any electronic-structure code should in principle be able to implement the SCDM- \mathbf{k} method within their interface with Wannier90, however at the moment (develop branch on the GitHub repository July 2019) only the Quantum ESPRESSO package has this capability implemented in the pw2wannier90 interface program. Moreover, the pw2wannier90 interface program supports also the SCDM- \mathbf{k} method for spin-noncollinear systems. The SCDM- \mathbf{k} can operate in two modes:

- 1. In isolation, i.e., without performing a subsequent Wannier90 optimisation (not recommended). This can be achieved by setting num_iter=0 and dis_num_iter=0 in the <seedname>.win input file. The rationale behind this is that in general the projection functions obtained with the SCDM-k are already well localised with the correct spatial orientations. However, the spreads of the resulting functions are usually larger than the MLWFs ones.
- 2. In combination with the Marzari-Vanderbilt (recommended option). In this case, the SCDM- \mathbf{k} is only used to generate the initial $A_{mn}^{(\mathbf{k})}$ matrices as a replacement scheme for the projection block.

The following keywords need to be specified in the pw2wannier90.x input file <seedname>.pw2wan: scdm_proj scdm_entanglement scdm_mu scdm_sigma

Chapter 4

Code Overview

wannier90 can operate in two modes:

- 1. Post-processing mode: read in the overlaps and projections from file as computed inside a first-principles code. We expect this to be the most common route to using wannier90, and is described in Ch. 5;
- 2. Library mode: as a set of library routines to be called from within a first-principles code that passes the overlaps and projections to the wannier90 library routines and in return gets the unitary transformation corresponding to MLWF. This route should be used if the MLWF are needed within the first-principles code, for example in post-LDA methods such as LDA+U or SIC, and is described in Ch. 6.



Figure 4.1: Schematic overview of the module structure of wannier90. Modules may only use data and subroutines from lower modules.

Chapter 5

wannier90 as a post-processing tool

This is a description of how to use wannier90 as a post-processing tool.

The code must be run twice. On the first pass either the logical keyword postproc_setup must be set to .true. in the input file seedname.win or the code must be run with the command line option -pp. Running the code then generates the file seedname.nnkp which provides the information required to construct the $M_{mn}^{(\mathbf{k},\mathbf{b})}$ overlaps (Ref. [1], Eq. (25)) and $A_{mn}^{(\mathbf{k})}$ (Ref. [1], Eq. (62); Ref. [2], Eq. (22)).

Once the overlaps and projection have been computed and written to files seedname.mmn and seedname.amn, respectively, set postproc_setup to .false. and run the code. Output is written to the file seedname.wout.

5.1 seedname.nnkp file

OUTPUT, if postproc_setup = .true.

The file seedname.nnkp provides the information needed to determine the required overlap elements $M_{mn}^{(\mathbf{k},\mathbf{b})}$ and projections $A_{mn}^{(\mathbf{k})}$. It is written automatically when the code is invoked with the -pp command-line option (or when postproc_setup=.true. in seedname.win. There should be no need for the user to edit this file.

Much of the information in seedname.nnkp is arranged in blocks delimited by the strings begin block_name ... end block_name, as described below.

5.1.1 Keywords

The first line of the file is a user comment, e.g., the date and time:

File written on 12Feb2006 at 15:13:12

The only logical keyword is calc_only_A, eg,

calc_only_A : F

5.1.2 Real_lattice block

begin real_lattice

```
2.250000 0.000000 0.000000
0.000000 2.250000 0.000000
0.000000 0.000000 2.250000
end real_lattice
```

The real lattice vectors in units of Angstrom.

5.1.3 Recip_lattice block

```
begin recip_lattice
2.792527 0.000000 0.000000
0.000000 2.792527 0.000000
0.000000 0.000000 2.792527
end recip_lattice
```

The reciprocal lattice vectors in units of inverse Angstrom.

5.1.4 Kpoints block

```
begin kpoints
8
0.00000 0.00000 0.00000
0.00000 0.50000 0.00000
.
.
.
0.50000 0.50000 0.50000
end kpoints
```

The first line in the block is the total number of k-points num_kpts. The subsequent num_kpts lines specify the k-points in crystallographic co-ordinates relative to the reciprocal lattice vectors.

5.1.5 Projections block

```
begin projections
  n_proj
  centre l mr r
    z-axis x-axis zona
  centre l mr r
    z-axis x-axis zona
  .
  .
end projections
```

Notes:

n_proj: integer; the number of projection centres, equal to the number of MLWF num_wann.

centre: three real numbers; projection function centre in crystallographic co-ordinates relative to the direct lattice vectors.

1 mr r: three integers; l and m_r specify the angular part $\Theta_{lm_r}(\theta, \varphi)$, and r specifies the radial part $R_r(r)$ of the projection function (see Tables 3.1, 3.2 and 3.3).

z-axis: three real numbers; default is 0.0 0.0 1.0; defines the axis from which the polar angle θ in spherical polar coordinates is measured.

x-axis: three real numbers; must be orthogonal to z-axis; default is 1.0 0.0 0.0 or a vector perpendicular to z-axis if z-axis is given; defines the axis from with the azimuthal angle φ in spherical polar coordinates is measured.

zona: real number; the value of $\frac{Z}{a}$ associated with the radial part of the atomic orbital. Units are in reciprocal Angstrom.

5.1.6 spinor_projections block

```
begin spinor_projections
   n_proj
   centre l mr r
   z-axis x-axis zona
   spin spn_quant
   centre l mr r
   z-axis x-axis zona
   spin spn_quant
   .
   .
end spinor_projections
```

Notes: Only one of projections and spinor_projections should be defined. Variables are the same as the projections block with the addition of spin and spn_quant.

spin: integer. '1' or '-1' to denote projection onto up or down states.

spn_quant: three real numbers. Defines the spin quantisation axis in Cartesian coordinates.

5.1.7 nnkpts block

```
begin nnkpts
    10
    1    2    0    0    0
    .
    .
end nnkpts
```

First line: nntot, the number of nearest neighbours belonging to each k-point of the Monkhorst-Pack mesh

Subsequent lines: nntot×num_kpts lines, ie, nntot lines of data for each k-point of the mesh.

Each line of consists of 5 integers. The first is the k-point number nkp. The second to the fifth specify it's nearest neighbours $\mathbf{k} + \mathbf{b}$: the second integer points to the k-point that is the periodic image of the $\mathbf{k} + \mathbf{b}$ that we want; the last three integers give the G-vector, in reciprocal lattice units, that brings the k-point specified by the second integer (which is in the first BZ) to the actual $\mathbf{k} + \mathbf{b}$ that we need.

5.1.8 exclude_bands block

```
begin exclude_bands
   8
   1
   2
   .
   .
end exclude_bands
```

To exclude bands (independent of k-point) from the calculation of the overlap and projection matrices, for example to ignore shallow-core states. The first line is the number of states to exclude, the following lines give the states for be excluded.

5.1.9 auto_projections block

```
begin auto_projections

8

0
end auto_projections
```

This block is only printed if auto_projections=true in the input. The choice of an additional block has been made in order to maintain back-compatibility with codes that interface with wannier90, e.g. pw2wannier90. The first entry in the block (in the example above, 8) is the total number of target projections and it is equal to the number of sought Wannier functions.

The second entry is a reserved flag with the value of zero. The implementations of the interface codes MUST check for this value to be zero and stop otherwise. In the future, one possible extension that we plan is to combine the automatic generation of initial projections with the selection of projections via a projections block. This will allow the user to specify only a subset of initial projections in the projections block and leave the interface code to automatically generate the remaining ones. In that case the constraint on the second entry will be lifted, so that it can take on the meaning of the number of projections that need to be generated automatically.

The selected columns of the density matrix (SCDM) method [10] is one way of generating the initial $A_{mn}^{(\mathbf{k})}$ in an automatic way. This has been implemented in the pw2wannier90 interface code (you need v6.3 with the files provided in the pwscf folder of Wannier90, or v6.4), see for instance Example 27 in the wannier90 tutorial that shows how to use it.

Moreover, also the automatic generation of initial projections with spinor WFs is implemented in the pw2wannier90 interface. See Example 31 in the wannier90 tutorial that shows how to use it.

5.1.10 An example of projections

As a concrete example: one wishes to have a set of four sp^3 projection orbitals on, say, a carbon atom at (0.5,0.5,0.5) in fractional co-ordinates relative to the direct lattice vectors. In this case $\mathrm{seedname.win}$ will contain the following lines:

```
begin projections
C:l=-1
end projections
```

and seedname.nnkp, generated on the first pass of wannier90 (with postproc_setup=T), will contain:

```
begin projections
   0.50000
              0.50000
                         0.50000
                                    -1
                                        1
                                           1
     0.000
                                  0.000 0.000
           0.000 1.000
                           1.000
                                                 2.00
   0.50000
              0.50000
                         0.50000
                                    -1
                                        2 1
     0.000 0.000
                  1.000
                           1.000
                                  0.000 0.000
                                                 2.00
   0.50000
              0.50000
                         0.50000
                                    -1
                                        3
     0.000
           0.000
                   1.000
                           1.000
                                  0.000 0.000
                                                 2.00
   0.50000
              0.50000
                         0.50000
                                    -1
                                        4 1
                                  0.000 0.000
     0.000 0.000 1.000
                           1.000
                                                 2.00
end projections
```

where the first line tells us that in total four projections are specified, and the subsquent lines provide the projection centre, the angular and radial parts of the orbital (see Section 3.4 for definitions), the z and x axes, and the diffusivity and cut-off radius for the projection orbital.

PWSCF, or any other *ab initio* electronic structure code, then reads **seedname.nnkp** file, calculates the projections and writes them to **seedname.amn**.

5.2 seedname.mmn file

INPUT.

The file seedname.mmn contains the overlaps $M_{mn}^{(\mathbf{k},\mathbf{b})}$.

First line: a user comment, e.g., the date and time

Second line: 3 integers: num_bands, num_kpts, nntot

Then: num_kpts × nntot blocks of data:

First line of each block: 5 integers. The first specifies the \mathbf{k} (i.e., gives the ordinal corresponding to its position in the list of k-points in seedname.win). The 2nd to 5th integers specify $\mathbf{k} + \mathbf{b}$. The 2nd integer, in particular, points to the k-point on the list that is a periodic image of $\mathbf{k} + \mathbf{b}$, and in particular is the image that is actually mentioned in the list. The last three integers specify the \mathbf{G} vector, in reciprocal lattice units, that brings the k-point specified by the second integer, and that thus lives inside the first BZ zone, to the actual $\mathbf{k} + \mathbf{b}$ that we need.

Subsequent num_bands × num_bands lines of each block: two real numbers per line. These are the real and imaginary parts, respectively, of the actual scalar product $M_{mn}^{(\mathbf{k},\mathbf{b})}$ for $m,n\in[1,\text{num_bands}]$. The order of these elements is such that the first index m is fastest.

5.3 seedname.amn file

INPUT.

The file seedname.amn contains the projection $A_{mn}^{(\mathbf{k})}$.

First line: a user comment, e.g., the date and time

Second line: 3 integers: num_bands, num_kpts, num_wann

Subsequently num_bands \times num_wann \times num_kpts lines: 3 integers and 2 real numbers on each line. The first two integers are the band indices m and n. The third integer specifies the k by giving the ordinal corresponding to its position in the list of k-points in seedname.win. The real numbers are the real and imaginary parts, respectively, of the actual $A_{mn}^{(k)}$.

5.4 seedname.dmn file

INPUT.

The file seedname.dmn contains the data needed to construct symmetry-adapted Wannier functions [7]. Required if site_symmetry = .true.

First line: a user comment, e.g., the date and time

Second line: 4 integers: num_bands, nsymmetry, nkptirr, num_kpts.

nsymmetry: the number of symmetry operations
nkptirr: the number of irreducible k-points

Blank line

num_kpts integers: Mapping between full k- and irreducible k-points. Each k-point is related to some k-point in the irreducible BZ. The information of this mapping is written. Each entry corresponds to a k-point in the full BZ, in the order in which they appear in the k-point list in seedname.win file. The (integer) value of each entry is the k-point index in the IBZ to which the k-point maps. The number of unique values is equal to the number of k-points in the IBZ. The data is written 10 values per line.

Blank line

nkptirr integers: List of irreducible k-points. Each entry corresponds to a k-point of the IBZ. The (integer) value of each entry is the k-point index corresponding to the k-point list in seedname.win file. The values should be between 1 and num_kpts. The data is written 10 values per line.

Blank line

nkptirr blocks of nsymmetry integer data (each block separated by a blank line): List of k-points obtained by acting the symmetry operations on the irreducible k-points. The data is written 10 values per line.

Blank line

$nsymmetry \times nkptirr blocks of data:$

The information of D matrix in Eq. (15) of Ref. [7]. Each block contains $num_wann \times num_wann$ lines and is separated by a blank line. The data are stored in $d_matrix_wann(m,n,isym,ikirr)$ with $m,n \in [1,num_wann]$, $isym \in [1,nsymmetry]$, and $ikirr \in [1,nkptirr]$. The order of the elements is such that left indices run faster than right indices (m: fastest, ikirr: slowest).

Blank line

nsymmetry × nkptirr blocks of data:

The information of d matrix in Eq. (17) of Ref. [7]. Each block contains num_bands \times num_bands lines and is separated by a blank line. The data are stored in d_matrix_band(m,n,isym,ikirr) with $m, n \in [1, num_bands]$, isym $\in [1, nsymmetry]$, and ikirr $\in [1, nkptirr]$. The order of the elements is such that left indices run faster than right indices (m: fastest, ikirr: slowest).

5.5 seedname.eig file

INPUT.

Required if any of disentanglement, plot_bands, plot_fermi_surface or write_hr are .true.

The file seedname.eig contains the Kohn-Sham eigenvalues $\varepsilon_{n\mathbf{k}}$ (in eV) at each point in the Monkhorst-Pack mesh.

Each line consist of two integers and a real number. The first integer is the band index, the second integer gives the ordinal corresponding to the k-point in the list of k-points in **seedname.win**, and the real number is the eigenvalue.

E.g.,

1	1	-6.43858831271328
2	1	19.3977795287297
3	1	19.3977795287297
4	1	19 3977795287298

5.6 Interface with PWSCF

Interfaces between wannier90 and many ab-initio codes such as PWSCF, ABINIT (http://www.abinit.org), SIESTA (http://www.icmab.es/siesta/), FLEUR, VASP and WIEN2K (http://www.wien2k.at) are available. Here we describe the seamless interface between wannier90 and PWSCF, a plane-wave DFT code that comes as part of the QUANTUM ESPRESSO package (see http://www.quantum-espresso.org). You will need to download and compile PWSCF (i.e., the pw.x code) and the post-processing interface pw2wannier90.x. Please refer to the documentation that comes with the QUANTUM ESPRESSO distribution for instructions.

- 1. Run 'scf'/'nscf' calculation(s) with pw
- 2. Run wannier90 with postproc_setup = .true. to generate seedname.nnkp
- 3. Run pw2wannier90. First it reads an input file, e.g., seedname.pw2wan, which defines prefix and outdir for the underlying 'scf' calculation, as well as the name of the file seedname.nnkp, and

does a consistency check between the direct and reciprocal lattice vectors read from seedname.nnkp and those defined in the files specified by prefix. pw2wannier90 generates seedname.mmn, seedname.amn and seedname.eig. seedname.dmn and seedname.sym files are additionally created when write_dmn = .true. (see below).

4. Run wannier90 with postproc_setup = .false. to disentangle bands (if required), localise MLWF, and use MLWF for plotting, bandstructures, Fermi surfaces etc.

Examples of how the interface with PWSCF works are given in the wannier90 Tutorial.

5.6.1 seedname.pw2wan

A number of keywords may be specified in the pw2wannier90 input file:

- outdir Location to write output files. Default is './'
- prefix Prefix for the PWSCF calculation. Default is ' '
- seedname Seedname for the wannier90 calculation. Default is 'wannier'
- spin_component Spin component. Takes values 'up', 'down' or 'none' (default).
- wan_mode Either 'standalone' (default) or 'library'
- write_unk Set to .true. to write the periodic part of the Bloch functions for plotting in wannier90. Default is .false.
- reduce_unk Set to .true. to reduce file-size (and resolution) of Bloch functions by a factor of 8. Default is .false. (only relevant if write_unk=.true.)¹
- wvfn_formatted Set to .true. to write formatted wavefunctions. Default is .false. (only relevant if write_unk=.true.)
- write_amn Set to .false. if $A_{mn}^{(k)}$ not required. Default is .true.
- write_mmn Set to .false. if $M_{mn}^{(\mathbf{k},\mathbf{b})}$ not required. Default is .true.
- write_spn Set to .true. to write out the matrix elements of S between Bloch states (non-collinear spin calculation only). Default is .false.
- spn_formatted Set to .true. to write spn data as a formatted file. Default is .false. (only relevant if write_spn=.true.)
- write_uHu Set to .true. to write out the matrix elements

$$\langle u_{n\mathbf{k}+\mathbf{b}_1}|H_{\mathbf{k}}|u_{m\mathbf{k}+\mathbf{b}_2}\rangle.$$

Default is .false.

• uHu_formatted - Set to .true. to write uHu data as a formatted file. Default is .false. (only relevant if write_uHu=.true.)

¹Note that there is a small bug with this feature in v3.2 (and subsequent patches) of quantum-espresso. Please use a later version (if available) or the CVS version of pw2wannier90.f90, which has been fixed.

• write_uIu - Set to .true. to write out the matrix elements of

$$\langle u_{n\mathbf{k}+\mathbf{b}_1}|u_{m\mathbf{k}+\mathbf{b}_2}\rangle$$
.

Default is .false.

- uIu_formatted Set to .true. to write uIu data as a formatted file. Default is .false. (only relevant if write_uIu=.true.)
- write_unkg Set to .true. to write the first few Fourier components of the periodic parts of the Bloch functions.
- write_dmn Set to .true. to construct symmetry-adapted Wannier functions. Default is .false.
- read_sym Set to .true. to customize symmetry operations to be used in symmetry-adapted mode. When read_sym = .true., an additional input seedname.sym is required. Default is .false. (only relevant if write_dmn=.true.).

For examples of use, refer to the wannier90 Tutorial.

5.6.2 seedname.sym

If read_sym = .true., then this additional input file is required for pw2wannier90.x if read_sym = .false., then this file is written by pw2wannier90.x (only for reference - it is not used in subsequent calculations)

The file seedname.sym contains the information of symmetry operations used to create symmetry-adapted Wannier functions. If read_sym = .false. (default), pw2wannier90.x uses the full symmetry recognized by pw.x. If read_sym = .true., you can specify symmetry operations to be used in symmetry-adapted mode.

First line: an integer: nsymmetry (number of symmetry operations)

Second line: blank

Then: nsymmetry blocks of data. Each block (separated by a blank line) consists of four lines. The order of the data in each block is as follows:

```
R(1,1) R(2,1) R(3,1) R(1,2) R(2,2) R(3,2) R(1,3) R(2,3) R(3,3) t(1) t(2) t(3)
```

Here, R is the rotational part of symmetry operations (3×3 matrix), and ${\bf t}$ is the fractional translation in the unit of "alat" (refer the definition of "alat" to the manual of PWSCF). Both data are given in Cartesian coordinates. The symmetry operations act on a point ${\bf r}$ as ${\bf r}R - {\bf t}$.

Chapter 6

wannier90 as a library

This is a description of the interface between any external program and the wannier code. There are two subroutines: wannier_setup and wannier_run. Calling wannier_setup will return information required to construct the $M_{mn}^{(\mathbf{k},\mathbf{b})}$ overlaps (Ref. [1], Eq. (25)) and $A_{mn}^{(\mathbf{k})} = \langle \psi_{m\mathbf{k}} | g_n \rangle$ projections (Ref. [1], Eq. (62); Ref. [2], Eq. (22)). Once the overlaps and projection have been computed, calling wannier_run activates the minimisation and plotting routines in wannier90.

IMPORTANT NOTE: the library mode ONLY works in serial. Please call it from a serial code, or if compiled in parallel, make sure to run it from a single MPI process.

You can find a minimal example of how the library mode can be used among the tests, in the file test-suite/library-mode-test/test_library.F90 in the Wannier90 git repository.

6.1 Subroutines

6.1.1 wannier_setup

- character(len=*), intent(in) :: seed_name The seedname of the current calculation.
- integer, dimension(3), intent(in) :: mp_grid The dimensions of the Monkhorst-Pack k-point grid.
- integer, intent(in) :: num_kpts
 The number of k-points on the Monkhorst-Pack grid.
- real(kind=dp), dimension(3,3), intent(in) :: real_lattice The lattice vectors in Cartesian co-ordinates in units of Angstrom.
- real(kind=dp), dimension(3,3), intent(in) :: recip_lattice

 The reciprocal lattice vectors in Cartesian co-ordinates in units of reciprocal Angstrom.

• real(kind=dp), dimension(3,num_kpts), intent(in) :: kpt_latt

The positions of the k-points in fractional co-ordinates relative to the reciprocal lattice vectors.

- integer, intent(in) :: num_bands_tot
 The total number of bands in the first-principles calculation (note: including semi-core states).
- integer, intent(in) :: num_atoms

 The total number of atoms in the system.
- character(len=20), dimension(num_atoms), intent(in) :: atom_symbols The elemental symbols of the atoms.
- real(kind=dp), dimension(3,num_atoms), intent(in) :: atoms_cart The positions of the atoms in Cartesian co-ordinates in Angstrom.
- logical, intent(in) :: gamma_only Set to .true. if the underlying electronic structure calculation has been performed with only Γ -point sampling and, hence, if the Bloch eigenstates that are used to construct $A_{mn}^{(\mathbf{k})}$ and $M_{mn}^{(\mathbf{k},\mathbf{b})}$ are real.
- logical, intent(in) :: spinors

 Set to .true. if underlying electronic structure calculation has been performed with spinor wavefunctions.
- integer, intent(out) :: nntot
 The total number of nearest neighbours for each k-point.
- integer, dimension(num_kpts,num_nnmax), intent(out) :: nnlist The list of nearest neighbours for each k-point.
- integer, dimension (3, num_kpts, num_nnmax), intent (out) :: nncell The vector, in fractional reciprocal lattice co-ordinates, that brings the nnth nearest neighbour of k-point nkp to its periodic image that is needed for computing the overlap $M_{mn}^{(\mathbf{k},\mathbf{b})}$.
- integer, intent(out) :: num_bands

 The number of bands in the first-principles calculation used to form the overlap matricies (note: excluding eg. semi-core states).
- integer, intent(out) :: num_wann
 The number of MLWF to be extracted.
- real(kind=dp), dimension(3,num_bands_tot), intent(out) :: proj_site

 Projection function centre in crystallographic co-ordinates relative to the direct lattice vectors.
- integer, dimension(num_bands_tot), intent(out) :: proj_l l specifies the angular part $\Theta_{lm_r}(\theta,\varphi)$ of the projection function (see Tables 3.1, 3.2 and 3.3).
- integer, dimension(num_bands_tot), intent(out) :: proj_m $m_{\rm r}$ specifies the angular part $\Theta_{lm_{\rm r}}(\theta,\varphi)$, of the projection function (see Tables 3.1, 3.2 and 3.3).
- integer, dimension(num_bands_tot), intent(out) :: proj_radial r specifies the radial part $R_r(r)$ of the projection function (see Tables 3.1, 3.2 and 3.3).
- real(kind=dp), dimension(3,num_bands_tot), intent(out) :: proj_z Defines the axis from which the polar angle θ in spherical polar coordinates is measured. Default is 0.0 0.0 1.0.

real(kind=dp), dimension(3,num_bands_tot), intent(out) :: proj_x
 Must be orthogonal to z-axis; default is 1.0 0.0 0.0 or a vector perpendicular to proj_z if proj_z is given; defines the axis from with the azimuthal angle φ in spherical polar coordinates is measured.

- real(kind=dp), dimension(num_bands_tot), intent(out) :: proj_zona The value of $\frac{Z}{a}$ associated with the radial part of the atomic orbital. Units are in reciprocal Angstrom.
- integer, dimension(num_bands_tot), intent(out) :: exclude_bands Kpoints independant list of bands to exclude from the calculation of the MLWF (e.g., semi-core states).
- integer, dimension(num_bands_tot), optional,intent(out) :: proj_s '1' or '-1' to denote projection onto up or down spin states
- real(kind=dp), dimension(3,num_bands_tot), intent(out) :: proj_s_qaxisx Defines the spin quantisation axis in Cartesian coordinates.

Conditions:

```
\star \  \, \texttt{num\_kpts} = \texttt{mp\_grid(1)} \times \texttt{mp\_grid(2)} \times \texttt{mp\_grid(3)}.
```

 \star num_nnmax = 12

This subroutine returns the information required to determine the required overlap elements $M_{mn}^{(\mathbf{k},\mathbf{b})}$ and projections $A_{mn}^{(\mathbf{k})}$, i.e., M_matrix and A_matrix, described in Section 6.1.2.

For the avoidance of doubt, real_lattice(1,2) is the y-component of the first lattice vector \mathbf{A}_1 , etc.

The list of nearest neighbours of a particular k-point nkp is given by nnlist(nkp,1:nntot).

Additionally, the parameter shell_list may be specified in the wannier90 input file.

6.1.2 wannier_run

- character(len=*), intent(in) :: seed_name
 The seedname of the current calculation.
- integer, dimension(3), intent(in) :: mp_grid
 The dimensions of the Monkhorst-Pack k-point grid.
- integer, intent(in) :: num_kpts

 The number of k-points on the Monkhorst-Pack grid.
- real(kind=dp), dimension(3,3), intent(in) :: real_lattice The lattice vectors in Cartesian co-ordinates in units of Angstrom.

• real(kind=dp), dimension(3,3), intent(in) :: recip_lattice

The reciprical lattice vectors in Cartesian co-ordinates in units of inverse Angstrom.

- real(kind=dp), dimension(3,num_kpts), intent(in) :: kpt_latt

 The positions of the k-points in fractional co-ordinates relative to the reciprocal lattice vectors.
- integer, intent(in) :: num_bands

 The total number of bands to be processed.
- integer, intent(in) :: num_wann
 The number of MLWF to be extracted.
- integer, intent(in) :: nntot
 The number of nearest neighbours for each k-point.
- integer, intent(in) :: num_atoms
 The total number of atoms in the system.
- character(len=20), dimension(num_atoms), intent(in) :: atom_symbols The elemental symbols of the atoms.
- real(kind=dp), dimension(3,num_atoms), intent(in) :: atoms_cart The positions of the atoms in Cartesian co-ordinates in Angstrom.
- logical, intent(in) :: gamma_only Set to .true. if the underlying electronic structure calculation has been performed with only Γ -point sampling and, hence, if the Bloch eigenstates that are used to construct $A_{mn}^{(\mathbf{k})}$ and $M_{mn}^{(\mathbf{k},\mathbf{b})}$ are real.

- real(kind=dp), dimension(num_bands,num_kpts), intent(in) :: eigenvalues The eigenvalues $\varepsilon_{n\mathbf{k}}$ corresponding to the eigenstates, in eV.
- complex(kind=dp), dimension(num_bands,num_wann,num_kpts),
 optional, intent(out) :: U_matrix_opt
 The unitary matrices that describe the optimal sub-space at each k-point (see Ref. [2], Section IIIA). The array is packed (see below)
- logical, dimension(num_bands,num_kpts), optional, intent(out) :: lwindow The element lwindow(nband,nkpt) is .true. if the band nband lies within the outer energy window at kpoint nkpt.

- real(kind=dp), dimension(3,num_wann), optional, intent(out) :: wann_centres The centres of the MLWF in Cartesian co-ordinates in Angstrom.
- real(kind=dp), dimension(num_wann), optional, intent(out) :: wann_spreads The spread of each MLWF in \mathring{A}^2 .
- real(kind=dp), dimension(3), optional, intent(out) :: spread The values of Ω , Ω_I and $\tilde{\Omega}$ (Ref. [1], Eq. (13)).

Conditions:

```
\star num_wann \leq num_bands
```

```
* num_kpts = mp_grid(1) \times mp_grid(2) \times mp_grid(3).
```

If num_bands = num_wann then U_matrix_opt is the identity matrix and lwindow=.true.

For the avoidance of doubt, real_lattice(1,2) is the y-component of the first lattice vector \mathbf{A}_1 , etc.

```
\begin{array}{lcl} \texttt{M\_matrix(m,n,nn,nkp)} &=& \langle u_{m\mathbf{k}} | u_{n\mathbf{k}+\mathbf{b}} \rangle \\ \\ \texttt{A\_matrix(m,n,nkp)} &=& \langle \psi_{m\mathbf{k}} | g_n \rangle \\ \\ \texttt{eigenvalues(n,nkp)} &=& \varepsilon_{n\mathbf{k}} \end{array}
```

where

```
\mathbf{k} = \text{kpt\_latt(1:3,nkp)}

\mathbf{k} + \mathbf{b} = \text{kpt\_latt(1:3,nnlist(nkp,nn))} + \text{nncell(1:3,nkp,nn)}
```

and $\{|g_n\rangle\}$ are a set of initial trial orbitals. These are typically atom or bond-centred Gaussians that are modulated by appropriate spherical harmonics.

Additional parameters should be specified in the wannier90 input file.

Chapter 7

Transport Calculations with wannier90

By setting transport = TRUE, wannier90 will calculate the quantum conductance and density of states of a one-dimensional system. The results will be written to files seedname_qc.dat and seedname_dos.dat, respectively.

The system for which transport properties are calculated is determined by the keyword transport_mode.

7.1 transport_mode = bulk

Quantum conductance and density of states are calculated for a perfectly periodic one-dimensional conductor. If tran_read_ht = FALSE the transport properties are calculated using the Hamiltonian in the Wannier function basis of the system found by wannier90. Setting tran_read_ht = TRUE allows the user to provide an external Hamiltonian matrix file seedname_htB.dat, from which the properties are found. See Section 2.9 for more details of the keywords required for such calculations.

7.2 transport_mode = lcr

Quantum conductance and density of states are calculated for a system where semi-infinite, left and right leads are connected through a central conductor region. This is known as the *lcr* system. Details of the method is described in Ref. [9].

In wannier90 two options exist for performing such calculations:

- If tran_read_ht = TRUE the external Hamiltonian files seedname_htL.dat, seedname_htLC.dat, seedname_htC.dat, seedname_htC.dat, seedname_htR.dat are read and used to compute the transport properties.
- If tran_read_ht = FALSE, then the transport calculation is performed automatically using the Wannier functions as a basis and the 2c2 geometry described in Section 7.3.

7.3 Automated lcr Transport Calculations: The 2c2 Geometry

Calculations using the 2c2 geometry provide a method to calculate the transport properties of an lcr system from a single wannier90 calculation. The Hamiltonian matrices which the five external files provide in the tran_read_ht = TRUE case are instead built from the Wannier function basis directly. As such, strict rules apply to the system geometry, which is shown in Figure 7.1. These rules are as follows:

- Left and right leads must be identical and periodic.
- Supercell must contain two principal layers (PLs) of lead on the left, a central conductor region and two principal layers of lead on the right.
- The conductor region must contain enough lead such that the disorder does not affect the principal layers of lead either side.
- A single k-point (Gamma) must be used.

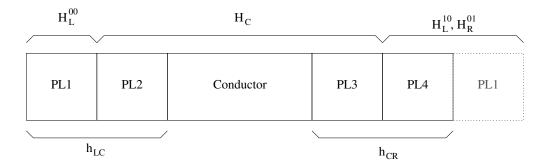


Figure 7.1: Schematic illustration of the supercell required for 2c2 lcr calculations, showing where each of the Hamiltonian matrices are derived from. Four principal layers (PLs) are required plus the conductor region.

In order to build the Hamiltonians, Wannier functions are first sorted according to position and then type if a number of Wannier functions exist with a similar centre (eg. d-orbital type Wannier functions centred on a Cu atom). Next, consistent parities of Wannier function are enforced. To distingiush between different types of Wannier function and assertain relative parities, a signature of each Wannier function is computed. The signature is formed of 20 integrals which have different spatial dependence. They are given by:

$$I = \frac{1}{V} \int_{V} g(\mathbf{r}) w(\mathbf{r}) d\mathbf{r}$$
 (7.1)

where V is the volume of the cell, $w(\mathbf{r})$ is the Wannier function and $g(\mathbf{r})$ are the set of functions:

$$g(\mathbf{r}) = \left\{ 1, \sin\left(\frac{2\pi(x-x_c)}{L_x}\right), \sin\left(\frac{2\pi(y-y_c)}{L_y}\right), \sin\left(\frac{2\pi(z-z_c)}{L_z}\right), \sin\left(\frac{2\pi(x-x_c)}{L_x}\right) \sin\left(\frac{2\pi(y-y_c)}{L_y}\right), \\ \sin\left(\frac{2\pi(x-x_c)}{L_x}\right) \sin\left(\frac{2\pi(z-z_c)}{L_z}\right), \dots \right\}$$

$$(7.2)$$

upto third order in powers of sines. Here, the supercell has dimension (L_x, L_y, L_z) and the Wannier function has centre $\mathbf{r}_c = (x_c, y_c, z_c)$. Each of these integrals may be written as linear combinations of the following sums:

$$S_n(\mathbf{G}) = e^{i\mathbf{G}.\mathbf{r}_c} \sum_m U_{mn} \tilde{u}_{m\Gamma}^*(\mathbf{G})$$
(7.3)

where n and m are the Wannier function and band indexes, \mathbf{G} is a G-vector, U_{mn} is the unitary matrix that transforms from the Bloch reopresentation of the system to the maximally-localised Wannier function basis and $\tilde{u}_{m\Gamma}^*(\mathbf{G})$ are the conjugates of the Fourier transforms of the periodic parts of the Bloch states at the Γ -point. The complete set of $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$ are often outputted by plane-wave DFT codes. However, to calculate the 20 signature integrals, only 32 specific $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$ are required. These are found in an additional file (seedname.unkg) that should be provided by the interface between the DFT code and wannier90. A detailed description of this file may be found in Section 8.32.

Additionally, the following keywords are also required in the input file:

- tran_num_11: The number of Wannier functions in a principal layer.
- tran_num_cell_ll: The number of unit cells in one principal layer of lead

A further parameter related to these calculations is tran_group_threshold.

Examples of how 2c2 calculations are preformed can be found in the wannier90 Tutorial.

Chapter 8

Files

8.1 seedname.win

INPUT. The master input file; contains the specification of the system and any parameters for the run. For a description of input parameters, see Chapter 2; for examples, see Section 10.1 and the wannier90 Tutorial.

8.1.1 Units

The following are the dimensional quantities that are specified in the master input file:

- Direct lattice vectors
- Positions (of atomic or projection) centres in real space
- Energy windows
- Positions of k-points in reciprocal space
- Convergence thresholds for the minimisation of Ω
- zona (see Section 3.1)
- wannier_plot_cube: cut-off radius for plotting WF in Gaussian cube format

Notes:

- The units (either ang (default) or bohr) in which the lattice vectors, atomic positions or projection centres are given can be set in the first line of the blocks unit_cell_cart, atoms_cart and projections, respectively, in seedname.win.
- Energy is always in eV.
- Convergence thresholds are always in Å²
- Positions of k-points are always in crystallographic coordinates relative to the reciprocal lattice vectors.

- zona is always in reciprocal Angstrom ($Å^{-1}$)
- The keyword length_unit may be set to ang (default) or bohr, in order to set the units in which the quantities in the output file seedname.wout are written.
- wannier_plot_radius is in Angstrom

The reciprocal lattice vectors $\{\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3\}$ are defined in terms of the direct lattice vectors $\{\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3\}$ by the equation

$$\mathbf{B}_1 = \frac{2\pi}{\Omega} \mathbf{A}_2 \times \mathbf{A}_3 \quad \text{etc.}, \tag{8.1}$$

where the cell volume is $V = \mathbf{A}_1 \cdot (\mathbf{A}_2 \times \mathbf{A}_3)$.

8.2 seedname.mmn

INPUT. Written by the underlying electronic structure code. See Chapter 5 for details.

8.3 seedname.amn

INPUT. Written by the underlying electronic structure code. See Chapter 5 for details.

8.4 seedname.dmn

INPUT. Read if site_symmetry = .true. (symmetry-adapted mode). Written by the underlying electronic structure code. See Chapter 5 for details.

8.5 seedname.eig

INPUT. Written by the underlying electronic structure code. See Chapter 5 for details.

8.6 seedname.nnkp

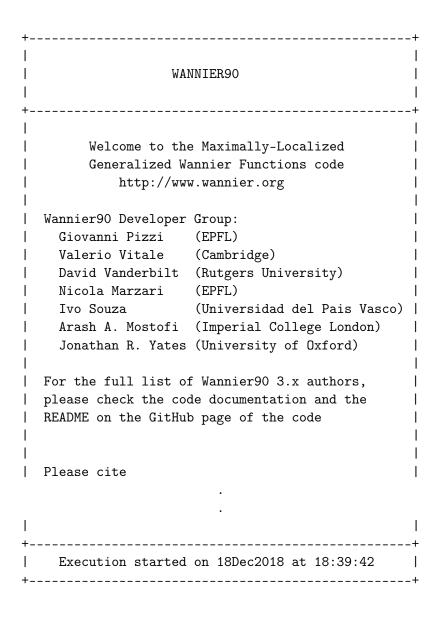
OUTPUT. Written by wannier90 when postproc_setup=.TRUE. (or, alternatively, when wannier90 is run with the -pp command-line option). See Chapter 5 for details.

8.7 seedname.wout

OUTPUT. The master output file. Here we give a description of the main features of the output. The verbosity of the output is controlled by the input parameter **iprint**. The higher the value, the more detail is given in the output file. The default value is 1, which prints minimal information.

8.7.1 Header

The header provides some basic information about wannier90, the authors, the code version and release, and the execution time of the current run. The header looks like the following different (the string might slightly change across different versions):



8.7.2 System information

This part of the output file presents information that wannier90 has read or inferred from the master input file seedname.win. This includes real and reciprocal lattice vectors, atomic positions, k-points, parameters for job control, disentanglement, localisation and plotting.

SYSTEM

				Lattice	Vectors	(Ang)			
		a	_1 3.	938486	0.0000	00 0	.0000	00	
		a	_2 0.	.000000	3.9384	86 0	.0000	00	
			_3 0.						
		Un	it Cell \	/olume:	61.	09251	(Ang	^3)	
			Recipi	rocal-Spa	ce Vect	ors (A	ng^-1)	
		b	_1 1.	595330	0.0000	00 0	.0000	00	
		b	_2 0.	.000000	1.5953	30 0	.0000	00	
		b	_3 0.	.000000	0.0000	00 1	.5953	30	
*-	 								 *
		Fract							
		0.00000							
		0.50000							
					•				
*-	 								 *
				K-POIN	T GRID				
		Grid siz	e = 4 x	4 x 4	То	tal no	ints	= 64	
		4114 DID	1		10	od to	_1100	0 -	

GIIU SIZ	C - 4	A 4.	A 4	Iotai	points -	04	

*-	MAI	N		*
	Number of Wannier Functions	:	9	
	Number of input Bloch states	:	9	1
	Output verbosity (1=low, 5=high)	:	1	1
	Length Unit	:	Ang	1
	<pre>Post-processing setup (write *.nnkp)</pre>	:	F	1

8.7.3 Nearest-neighbour k-points

This part of the output files provides information on the b-vectors and weights chosen to satisfy the condition of Eq. 2.1.

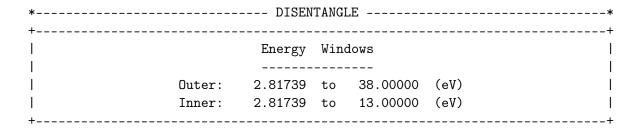
*	K-MF.SH		*
	11 112011		
+			+
1	Distance to Nearest-Neighbour	Shells	1
I			- 1
Shell	Distance (Ang^-1)	Multiplicity	1

				1
1	0.398833		6	
1 2	0.564034		12	1
	•			
	•			
+				+
The b-vectors are ch	sen automatically			- 1
The following shells	are used: 1			1
+				+
1	Shell # Nearest-	Neighbours		1
1				1
1	1	6		1
+				+
Completeness relation	n is fully satisfied	-	RB 56, 12847 (19	97)]

8.7.4 Disentanglement

Then (if required) comes the part where $\Omega_{\rm I}$ is minimised to disentangle the optimally-connected subspace of states for the localisation procedure in the next step.

First, a summary of the energy windows that are being used is given:



Then, each step of the iterative minimisation of $\Omega_{\rm I}$ is reported.

Extraction of optimally-connected subspace

<<< Delta < 1.000E-10 over 3 iterations >>>
<<< Disentanglement convergence criteria satisfied >>>

Final Omega_I 3.66268867 (Ang^2)

+-----+

The first column gives the iteration number. For a description of the minimisation procedure and expressions for $\Omega_{\rm I}^{(i)}$, see the original paper [2]. The procedure is considered to be converged when the fractional difference between $\Omega_{\rm I}^{(i)}$ and $\Omega_{\rm I}^{(i-1)}$ is less than dis_conv_tol over dis_conv_window iterations. The final column gives a running account of the wall time (in seconds) so far. Note that at the end of each line of output, there are the characters "<- DIS". This enables fast searching of the output using, for example, the Unix command grep:

my_shell> grep DIS wannier.wout | less

8.7.5 Wannierisation

The next part of the output file provides information on the minimisation of $\widetilde{\Omega}$. At each iteration, the centre and spread of each WF is reported.

```
*----*
+-----+<-- CONV
               RMS Gradient Spread (Ang^2) Time | <-- CONV
| Iter Delta Spread
+-----+<-- CONV
Initial State
 WF centre and spread 1 (0.000000, 1.969243, 1.969243) 1.52435832
 WF centre and spread 2 ( 0.000000, 1.969243, 1.969243)
                                            1.16120620
   0 0.126E+02 0.000000000 12.6297685260 0.29 <-- CONV
           Cycle:
        1
 WF centre and spread 1 ( 0.000000, 1.969243, 1.969243 ) 1.52414024
WF centre and spread 2 ( 0.000000, 1.969243, 1.969243 ) 1.16059775
 Sum of centres and spreads (11.815458, 11.815458, 11.815458) 12.62663472
                             12.6266347170 0.34 <-- CONV
                0.0697660962
       -0.313E-02
     Delta: O_D= -0.4530841E-18 O_OD= -0.3133809E-02 O_TOT= -0.3133809E-02 <-- DLTA
______
Cycle:
 WF centre and spread 1 ( 0.000000, 1.969243, 1.969243)
                                            1.52414866
 WF centre and spread 2 ( 0.000000, 1.969243, 1.969243)
                                            1.16052405
 Sum of centres and spreads (11.815458, 11.815458, 11.815458) 12.62646411
      -0.171E-03
                0.0188848262
                            12.6264641055
                                         0.38 <-- CONV
           Delta: O_D= -0.2847260E-18 O_OD= -0.1706115E-03 O_TOT= -0.1706115E-03 <-- DLTA
```

•

Final State

WF centre and spread 1 (0.000000, 1.969243, 1.969243) 1.52416618 WF centre and spread 2 (0.000000, 1.969243, 1.969243) 1.16048545

Sum of centres and spreads (11.815458, 11.815458, 11.815458) 12.62645344

It looks quite complicated, but things look more simple if one uses grep:

my_shell> grep CONV wannier.wout

gives

+				+<	CONV
Iter	Delta Spread	RMS Gradient	Spread (Ang^2)	Time <	CONV
+				+<	CONV
0	0.126E+02	0.000000000	12.6297685260	0.29 <	CONV
1	-0.313E-02	0.0697660962	12.6266347170	0.34 <	CONV
			•		
			•		
50	0.000E+00	0.000000694	12.6264534413	2.14 <	CONV

The first column is the iteration number, the second is the change in Ω from the previous iteration, the third is the root-mean-squared gradient of Ω with respect to variations in the unitary matrices $\mathbf{U}^{(\mathbf{k})}$, and the last is the time taken (in seconds). Depending on the input parameters used, the procedure either runs for num_iter iterations, or a convergence criterion is applied on Ω . See Section 2.8 for details.

Similarly, the command

my_shell> grep SPRD wannier.wout

gives

0_D=	0.0000000	0.1491718 O_TOT=	12.6297685 < SPRD
0_D=	0.0000000	0.1460380 O_TOT=	12.6266347 < SPRD
		•	
0_D=	0.0000000	0.1458567 O_TOT=	12.6264534 < SPRD

which, for each iteration, reports the value of the diagonal and off-diagonal parts of the non-gauge-invariant spread, as well as the total spread, respectively. Recall from Section 1 that $\Omega = \Omega_{\rm I} + \Omega_{\rm D} + \Omega_{\rm OD}$.

Wannierisation with selective localization and constrained centres

For full details of the selectively localised Wannier function (SLWF) method, the reader is referred to Ref. [8]. When using the SLWF method, only a few things change in the output file and in general the same principles described above will apply. In particular, when minimising the spread with respect to the degrees of freedom of only a subset of functions, it is not possible to cast the total spread functional Ω as a sum of a gauge-invariant part and a gauge-dependent part. Instead, one has $\Omega' = \Omega_{\text{IOD}} + \Omega_{\text{D}}$, where

$$\Omega' = \sum_{n=1}^{J' < J} \left[\langle r^2 \rangle_n - \overline{\mathbf{r}}_n^2 \right]$$

and

$$\Omega_{\mathrm{IOD}} = \sum_{n=1}^{J' < J} \left[\langle r_n^2 \rangle - \sum_{\mathbf{R}} |\langle \mathbf{R} n | \mathbf{r} | n \mathbf{R} \rangle|^2 \right].$$

The total number of Wannier functions is J, whereas J' is the number functions to be selectively localized (so-called *objective WFs*). The information on the number of functions which are going to be selectively localized (Number of Objective Wannier Functions) is given in the MAIN section of the output file:

*-	MAIN			*
-	Number of Wannier Functions	:	4	1
	Number of Objective Wannier Functions	:	1	- 1
	Number of input Bloch states	:	4	

Whether or not the selective localization procedure has been switched on is reported in the WANNIERISE section as

```
| Perform selective localization : T
```

The next part of the output file provides information on the minimisation of the modified spread functional:

```
*----*
+-----+<-- CONV
                                 Spread (Ang^2)
| Iter Delta Spread
                   RMS Gradient
                                                 Time | <-- CONV
Initial State
                   1 (-0.857524, 0.857524,
WF centre and spread
                                         0.857524)
                                                     1.80463310
                   2 ( 0.857524, -0.857524,
WF centre and spread
                                         0.857524)
                                                     1.80463311
                                0.857524, -0.857524)
                   3 (0.857524,
WF centre and spread
                                                     1.80463311
                   4 ( -0.857524, -0.857524, -0.857524 )
WF centre and spread
                                                     1.80463311
Sum of centres and spreads ( -0.000000, -0.000000, 0.000000)
                                                     7.21853243
       -0.317E+01
                   0.000000000
                                  -3.1653368719
                                                  0.00 <-- CONV
            0.0000000 O_IOD= -3.1653369 O_TOT=
    0_D =
                                              -3.1653369 <-- SPRD
```

```
Cycle:
            1
 WF centre and spread
                         1 (-0.853260, 0.853260,
                                                    0.853260)
                                                                   1.70201498
  WF centre and spread
                         2 ( 0.857352, -0.857352,
                                                    0.862454)
                                                                   1.84658331
  WF centre and spread
                         3 ( 0.857352, 0.862454, -0.857352 )
                                                                   1.84658331
                         4 ( -0.862454, -0.857352, -0.857352 )
  WF centre and spread
                                                                   1.84658331
  Sum of centres and spreads ( -0.001010, 0.001010, 0.001010)
                                                                   7.24176492
          -0.884E-01
                         0.2093698260
                                                               0.00 <-- CONV
      1
                                            -3.2536918930
      0_{IDD}=
                 -3.2536919 O_D=
                                      -3.2536919 <-- SPRD
Delta: O_IOD= -0.1245020E+00 O_D= 0.0000000E+00 O_TOT= -0.8835502E-01 <-- DLTA
 Final State
                         1 (-0.890189, 0.890189,
                                                    0.890189)
  WF centre and spread
                                                                   1.42375495
  WF centre and spread
                         2 ( 0.895973, -0.895973, 0.917426 )
                                                                   2.14313664
                         3 ( 0.895973, 0.917426, -0.895973 )
  WF centre and spread
                                                                   2.14313664
                         4 ( -0.917426, -0.895973, -0.895973 )
  WF centre and spread
                                                                   2.14313664
  Sum of centres and spreads ( -0.015669, 0.015669, 0.015669)
                                                                   7.85316486
         Spreads (Ang^2)
                              Omega IOD
                                                1.423371553
                              Omega D
                                                0.000383395
                              Omega Rest
                                           =
                                                 9.276919811
   Final Spread (Ang^2)
                              Omega Total =
                                                 1.423754947
```

When comparing the output from an SLWF calculation with a standard wannierisation (see Sec. 8.7.5), the only differences are in the definition of the spread functional. Hence, during the minimization O_OD is replaced by O_IDD and O_IDD now reflects the fact that the new total spread functional is Ω' . The part on the final state has one more item of information: the value of the difference between the global spread functional and the new spread functional given by Olega Rest

$$\Omega_R = \sum_{n=1}^{J-J'} \left[\langle r^2 \rangle_n - \overline{\mathbf{r}}_n^2 \right]$$

If adding centre-constraints to the SLWFs, you will find the information about the centres of the original projections and the desired centres in the SYSTEM section

*									*
1	√annier	r# 01	riginal Cer	ntres		Constra	ained centi	res	-
+									+
1	1	0.25000	0.25000	0.25000	I	0.00000	0.00000	0.00000	- 1
*									*

As before one can check that the selective localization with constraints is being used by looking at the WANNIERISE section:

```
| Perform selective localization : T | Use constrains in selective localization : T | Value of the Lagrange multiplier : 0.100E+01
```

which also gives the selected value for the Lagrange multiplier. The output file for the minimisation section is modified as follows: both O_IOD and O_TOT now take into account the factors coming from the new term in the functional due to the constraints, which are implemented by adding the following penalty functional to the spread functional,

$$\lambda_c \sum_{n=1}^{J'} \left(\overline{\mathbf{r}}_n - \mathbf{r}_{0n}\right)^2,$$

where \mathbf{r}_{0n} is the desired centre for the n^{th} Wannier function, see Ref. [8] for details. The layout of the output file at each iteration is unchanged.

```
1 -0.884E-01 0.2093698260 -3.2536918930 0.00 <-- CONV
```

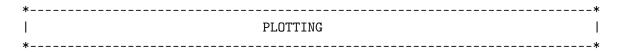
As regarding the final state, the only addition is the information on the value of the penalty functional associated with the constraints (Penalty func), which should be zero if the final centres of the Wannier functions are at the target centres:

```
Final State
WF centre and spread 1 (-1.412902, 1.412902, 1.412902)
                                                              1.63408756
WF centre and spread 2 ( 1.239678, -1.239678, 1.074012)
                                                              2.74801593
WF centre and spread 3 ( 1.239678, 1.074012, -1.239678)
                                                              2.74801592
WF centre and spread 4 (-1.074012, -1.239678, -1.239678)
                                                              2.74801592
Sum of centres and spreads (-0.007559, 0.007559, 0.007559)
                                                              9.87813534
                           Omega IOD_C =
       Spreads (Ang^2)
                                            -4.261222001
      ==========
                           Omega D
                                             0.00000000
                           Omega Rest =
                                           5.616913337
                           Penalty func =
                                             0.000000000
  Final Spread (Ang^2)
                           Omega Total_C =
                                            -4.261222001
```

8.7.6 Plotting

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After WF have been localised, wannier90 enters its plotting routines (if required). For example, if you have specified an interpolated bandstucture:



Calculating interpolated band-structure

8.7.7 Summary timings

At the very end of the run, a summary of the time taken for various parts of the calculation is given. The level of detail is controlled by the timing_level input parameter (set to 1 by default).

===========		===	========	=======
	TIMING INFORMATION			
Tag			Ncalls	Time (s)
kmesh: get		:	1	0.212
overlap: read		:	1	0.060
wann: main		:	1	1.860
plot: main		:	1	0.168
*				*

All done: wannier90 exiting

8.8 seedname.chk

INPUT/OUTPUT. Information required to restart the calculation or enter the plotting phase. If we have used disentanglement this file also contains the rectangular matrices $\mathbf{U}^{\mathrm{dis}(\mathbf{k})}$.

8.9 seedname.r2mn

OUTPUT. Written if write_r2mn = true. The matrix elements $\langle m|r^2|n\rangle$ (where m and n refer to MLWF)

8.10 seedname_band.dat

OUTPUT. Written if bands_plot=.TRUE.; The raw data for the interpolated band structure.

8.11 seedname_band.gnu

OUTPUT. Written if bands_plot=.TRUE. and bands_plot_format=gnuplot; A gnuplot script to plot the interpolated band structure.

8.12 seedname_band.agr

OUTPUT. Written if bands_plot=.TRUE. and bands_plot_format=xmgrace; A grace file to plot the interpolated band structure.

8.13 seedname_band.kpt

OUTPUT. Written if bands_plot=.TRUE.; The k-points used for the interpolated band structure, in units of the reciprocal lattice vectors. This file can be used to generate a comparison band structure from a first-principles code.

8.14 seedname.bxsf

OUTPUT. Written if fermi_surface_plot=.TRUE.; A Fermi surface plot file suitable for plotting with XCrySDen.

8.15 seedname_w.xsf

OUTPUT. Written if wannier_plot=.TRUE. and wannier_plot_format=xcrysden. Contains the wth WF in real space in a format suitable for plotting with XCrySDen or VMD, for example.

8.16 seedname_w.cube

OUTPUT. Written if wannier_plot=.TRUE. and wannier_plot_format=cube. Contains the wth WF in real space in Gaussian cube format, suitable for plotting in XCrySDen, VMD, gopenmol etc.

8.17 UNKp.s

INPUT. Read if wannier_plot=.TRUE. and used to plot the MLWF. Read if transport_mode=lcr and tran_read_ht=.FALSE. for use in automated lcr transport calculations.

The periodic part of the Bloch states represented on a regular real space grid, indexed by k-point p (from 1 to num_kpts) and spin s ('1' for 'up', '2' for 'down').

The name of the wavefunction file is assumed to have the form:

```
write(wfnname,200) p,spin
200 format ('UNK',i5.5,'.',i1)
```

The first line of each file should contain 5 integers: the number of grid points in each direction (ngx, ngy and ngz), the k-point number ik and the total number of bands num_band in the file. The full file will be read by wannier90 as:

```
read(file_unit) ngx,ngy,ngz,ik,nbnd
do loop_b=1,num_bands
  read(file_unit) (r_wvfn(nx,loop_b),nx=1,ngx*ngy*ngz)
end do
```

If spinors=true then s='NC', and the name of the wavefunction file is assumed to have the form:

```
write(wfnname,200) p
200 format ('UNK',i5.5,'.NC')
```

and the file will be read by wannier90 as:

```
read(file_unit) ngx,ngy,ngz,ik,nbnd
do loop_b=1,num_bands
    read(file_unit) (r_wvfn_nc(nx,loop_b,1),nx=1,ngx*ngy*ngz) ! up-spinor
    read(file_unit) (r_wvfn_nc(nx,loop_b,2),nx=1,ngx*ngy*ngz) ! down-spinor
end do
```

All UNK files can be in formatted or unformatted style, this is controlled by the logical keyword wvfn_formatted.

8.18 seedname_centres.xyz

OUTPUT. Written if write_xyz=.TRUE.; xyz format atomic structure file suitable for viewing with your favourite visualiser (jmol, gopenmol, vmd, etc.).

8.19 seedname_hr.dat

OUTPUT. Written if write_hr=.TRUE.. The first line gives the date and time at which the file was created. The second line states the number of Wannier functions num_wann. The third line gives the number of Wigner-Seitz grid-points nrpts. The next block of nrpts integers gives the degeneracy of each Wigner-Seitz grid point, with 15 entries per line. Finally, the remaining num_wann² × nrpts lines each contain, respectively, the components of the vector \mathbf{R} in terms of the lattice vectors $\{\mathbf{A}_i\}$, the indices m and n, and the real and imaginary parts of the Hamiltonian matrix element $H_{mn}^{(\mathbf{R})}$ in the WF basis, e.g.,

```
Created on 24May2007 at 23:32:09
        20
        17
   4
        1
            2
                              1
                                          2
                                                1
                                                           6
                                                                 1
                                                                       1
                                                                            1
                                                                                2
                   1
                        4
                                    1
        2
   1
   0
        0
           -2
                  1
                        1
                             -0.001013
                                            0.00000
           -2
   0
        0
                  2
                        1
                              0.000270
                                            0.00000
   0
        0
           -2
                  3
                             -0.000055
                                            0.00000
                        1
           -2
   0
        0
                  4
                        1
                              0.000093
                                            0.000000
   0
           -2
                  5
                        1
                             -0.000055
                                            0.00000
```

8.20 seedname_r.dat

OUTPUT. Written if write_rmn = true. The matrix elements $\langle m\mathbf{0}|\mathbf{r}|n\mathbf{R}\rangle$ (where $n\mathbf{R}$ refers to MLWF n in unit cell \mathbf{R}). The first line gives the date and time at which the file was created. The second line states the number of Wannier functions num_wann. The third line states the number of \mathbf{R} vectors nrpts. Similar to the case of the Hamiltonian matrix above, the remaining num_wann²× nrpts lines each contain, respectively, the components of the vector \mathbf{R} in terms of the lattice vectors $\{\mathbf{A}_i\}$, the indices m and n, and the real and imaginary parts of the position matrix element in the WF basis.

8.21 seedname_tb.dat

OUTPUT. Written if write_tb=.TRUE.. This file is essentially a combination of seedname_hr.dat and seedname_r.dat, plus lattice vectors. The first line gives the date and time at which the file was created. The second to fourth lines are the lattice vectors in Angstrom unit.

The next part is the same as seedname_hr.dat. The fifth line states the number of Wannier functions num_wann. The sixth line gives the number of Wigner-Seitz grid-points nrpts. The next block of nrpts integers gives the degeneracy of each Wigner-Seitz grid point, with 15 entries per line. Then, the next num_wann²× nrpts lines each contain, respectively, the components of the vector \mathbf{R} in terms of the lattice vectors $\{\mathbf{A}_i\}$, the indices m and n, and the real and imaginary parts of the Hamiltonian matrix element $H_{mn}^{(\mathbf{R})}$ in the WF basis, e.g.,

```
7
         93
       6
                     2
                                         2
                                                                                              2
 4
              2
                           2
                                  1
                                               2
                                                      1
                                                                                2
                                                                                       2
       2
              2
 6
                     4
                           1
                                  1
                                         1
                                               4
                                                      1
                                                             1
                                                                   1
                                                                                2
                                                                                              1
       2
              2
                                               2
                                                                                              2
 1
                     1
                                  2
                                         4
                                                      1
                                                             2
                                                                   1
                                                                                1
                           1
                                                                          1
 1
       1
              1
                     2
                                  1
                                         1
                                               1
                                                      2
                                                             1
                                                                   2
                                                                          4
                                                                                2
                                                                                              1
                           1
                                                                                       1
 2
       2
                                  2
                                                                   4
              1
                     1
                           1
                                         1
                                               1
                                                      1
                                                             1
                                                                          1
                                                                                1
                                                                                              4
                                                                                       1
 2
       2
                                  2
                                               2
                                                                   2
                                                                          2
              6
                     2
                            2
                                         6
                                                      1
                                                             1
                                                                                       2
                                                                                              2
 2
       6
              4
-3
       1
              1
       1
              0.42351556E-02 -0.95722060E-07
 1
 2
       1
              0.69481480E-07 -0.20318638E-06
 3
              0.10966508E-06 -0.13983284E-06
```

Finally, the last part is the same as seedname_r.dat. The num_wann²× nrpts lines each contain, respectively, the components of the vector \mathbf{R} in terms of the lattice vectors $\{\mathbf{A}_i\}$, the indices m and n,

and the real and imaginary parts of the position matrix element in the WF basis (the float numbers in columns 3 and 4 are the real and imaginary parts for $\langle m\mathbf{0}|\mathbf{r}_x|n\mathbf{R}\rangle$, columns 5 and 6 for $\langle m\mathbf{0}|\mathbf{r}_y|n\mathbf{R}\rangle$, and columns 7 and 8 for $\langle m\mathbf{0}|\mathbf{r}_z|n\mathbf{R}\rangle$), e.g.

```
-3
      1
1
      1
           0.32277552E-09
                           0.21174901E-08 -0.85436987E-09
                                                             0.26851510E-08
2
      1
          -0.18881883E-08
                           0.21786973E-08
                                            0.31123076E-03
                                                             0.39228431E-08
 3
           0.31123242E-03 -0.35322230E-09
                                           0.70867281E-09
                                                             0.10433480E-09
```

8.22 seedname.byec

OUTPUT. Written if write_bvec = true. This file contains the matrix elements of bvector and their weights. The first line gives the date and time at which the file was created. The second line states the number of k-points and the total number of neighbours for each k-point nntot. Then all the other lines contain the b-vector (x,y,z) coordinate and weights for each k-points and each of its neighbours.

8.23 seedname_wsvec.dat

OUTPUT. Written if write_hr = true or write_rmn = true or write_tb = true. The first line gives the date and time at which the file was created and the value of use_ws_distance. For each pair of Wannier functions (identified by the components of the vector \mathbf{R} separating their unit cells and their indices) it gives: (i) the number of lattice vectors of the periodic supercell \mathbf{T} that bring the Wannier function in \mathbf{R} back in the Wigner-Seitz cell centred on the other Wannier function and (ii) the set of superlattice vectors \mathbf{T} to make this transformation. These superlattice vectors \mathbf{T} should be added to the \mathbf{R} vector to obtain the correct centre of the Wannier function that underlies a given matrix element (e.g. the Hamiltonian matrix elements in seedname_hr.dat) in order to correctly interpolate in reciprocal space.

```
## written on 20Sep2016 at 18:12:37 with use_ws_distance=.true.
    0
           0
                 0
                       1
                             1
    1
     0
           0
                 0
           0
                             2
    0
                 0
                       1
     1
    0
           0
                 0
    0
           0
                 0
                       1
                             3
     1
    0
           0
                 0
           0
     0
                 0
                       1
                             4
     1
           0
                 0
    0
    0
           0
                 0
                       1
                             5
     1
```

```
0 0 0
0 0 0 1 6
2
0 -1 -1
1 -1 -1
```

8.24 seedname_qc.dat

OUTPUT. Written if transport = .TRUE.. The first line gives the date and time at which the file was created. In the subsequent lines, the energy value in units of eV is written in the left column, and the quantum conductance in units of $\frac{2e^2}{h}$ ($\frac{e^2}{h}$ for a spin-polarized system) is written in the right column.

```
## written on 14Dec2007 at 11:30:17
-3.000000 8.999999
-2.990000 8.999999
-2.980000 8.999999
-2.970000 8.999999
.
.
```

8.25 seedname_dos.dat

OUTPUT. Written if transport = .TRUE.. The first line gives the date and time at which the file was created. In the subsequent lines, the energy value in units of eV is written in the left column, and the density of states in an arbitrary unit is written in the right column.

```
## written on 14Dec2007 at 11:30:17
-3.000000 6.801199
-2.990000 6.717692
-2.980000 6.640828
-2.970000 6.569910
.
.
```

8.26 seedname_htB.dat

INPUT/OUTPUT. Read if transport_mode = bulk and tran_read_ht = .TRUE.. Written if tran_write_ht = .TRUE.. The first line gives the date and time at which the file was created. The second line gives tran_num_bb. The subsequent lines contain tran_num_bb \times tran_num_bb H_{mn} matrix, where the indices m and n span all tran_num_bb WFs located at 0^{th} principal layer. Then tran_num_bb is recorded

again in the new line followed by H_{mn} , where m^{th} WF is at 0^{th} principal layer and n^{th} at 1^{st} principal layer. The H_{mn} matrix is written in such a way that m is the fastest varying index.

1Dec2007 at	11:30:17			
-2.941054	0.052673	-0.032926	0.010738	-0.009515
-0.016325	0.051863	-0.170897	-2.170467	0.202254
-0.571967	-0.691431	0.015155	-0.007859	0.000474
-0.001141	-0.002126	0.019188	-0.686423	-10.379876
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	-0.001576
-0.000143	-0.001264	0.002278	0.000000	0.000000
	-2.941054 -0.016325 -0.571967 -0.001141 0.000000 0.000000	-0.016325	-2.941054	-2.941054 0.052673 -0.032926 0.010738 -0.016325 0.051863 -0.170897 -2.170467 -0.571967 -0.691431 0.015155 -0.007859 -0.001141 -0.002126 0.019188 -0.686423 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000

8.27 seedname_htL.dat

INPUT. Read if transport_mode = lcr and tran_read_ht = .TRUE.. The file must be written in the same way as in seedname_htB.dat. The first line can be any comment you want. The second line gives tran_num_ll tran_num_ll in seedname_htL.dat must be equal to that in seedname.win. The code will stop otherwise.

Created by a	WANNIER use	r			
0.316879	0.000000	-2.762434	0.048956	0.000000	-0.016639
0.000000	0.000000	0.000000	0.000000	0.000000	-2.809405
•					
•					
•					
0.000000	0.078188	0.000000	0.000000	-2.086453	-0.001535
0.007878	-0.545485	-10.525435			
105					
0.000000	0.000000	0.000315	-0.000294	0.000000	0.000085
0.000000	0.000000	0.000000	0.000000	0.000000	0.000021
•					
	0.000000	0 000000	0 000000	0 000000	0 000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000			

8.28 seedname_htR.dat

INPUT. Read if transport_mode = lcr and tran_read_ht = .TRUE. and tran_use_same_lead = .FALSE.. The file must be written in the same way as in seedname_htL.dat. tran_num_rr in seedname_htR.dat must be equal to that in seedname.win.

8.29 seedname_htC.dat

INPUT. Read if transport_mode = lcr and tran_read_ht = .TRUE.. The first line can be any comment you want. The second line gives tran_num_cc. The subsequent lines contain tran_num_cc \times tran_num_cc H_{mn} matrix, where the indices m and n span all tran_num_cc WFs inside the central conductor region. tran_num_cc in seedname_htC.dat must be equal to that in seedname.win.

```
Created by a WANNIER user
   99
 -10.499455
               -0.541232
                            0.007684
                                        -0.001624
                                                     -2.067078
                                                                  -0.412188
   0.003217
               0.076965
                            0.000522
                                        -0.000414
                                                      0.000419
                                                                  -2.122184
  -0.003438
               0.078545
                                         0.757343
                                                                 -0.001632
                            0.024426
                                                     -2.004899
   0.007807
               -0.542983
                          -10.516896
```

8.30 seedname_htLC.dat

INPUT. Read if transport_mode = lcr and tran_read_ht = .TRUE.. The first line can be any comment you want. The second line gives tran_num_ll and tran_num_lc in the given order. The subsequent lines contain tran_num_ll \times tran_num_lc H_{mn} matrix. The index m spans tran_num_ll WFs in the surface principal layer of semi-infinite left lead which is in contact with the conductor region. The index n spans tran_num_lc WFs in the conductor region which have a non-negligible interaction with the WFs in the semi-infinite left lead. Note that tran_num_lc can be different from tran_num_cc.

```
Created by a WANNIER user
  105
         99
   0.000000
                0.000000
                            0.000000
                                         0.000000
                                                      0.000000
                                                                   0.000000
   0.000000
                0.000000
                            0.000000
                                         0.000000
                                                      0.000000
                                                                   0.000000
  -0.00003
               0.000009
                                         0.000001
                                                     -0.000007
                            0.000290
                                                                  -0.000008
   0.000053
               -0.000077
                           -0.000069
```

8.31 seedname_htCR.dat

INPUT. Read if transport_mode = 1cr and tran_read_ht = .TRUE.. The first line can be any comment you want. The second line gives tran_num_cr and tran_num_rr in the given order. The subsequent lines contain tran_num_cr \times tran_num_rr H_{mn} matrix. The index m spans tran_num_cr WFs in the conductor region which have a non-negligible interaction with the WFs in the semi-infinite right lead. The index n spans tran_num_rr WFs in the surface principal layer of semi-infinite right lead which is in contact with the conductor region. Note that tran_num_cr can be different from tran_num_cc.

```
Created by a WANNIER user
   99
        105
  -0.000180
               0.000023
                            0.000133
                                        -0.00001
                                                     0.000194
                                                                  0.00008
  -0.000879
              -0.000028
                            0.000672
                                        -0.000257
                                                     -0.000102
                                                                 -0.000029
                                         0.000000
                                                     0.000000
                                                                  0.000000
   0.00000
               0.00000
                            0.000000
   0.000000
               0.000000
                            0.00000
```

8.32 seedname.unkg

INPUT. Read if transport_mode = 1cr and tran_read_ht = .FALSE.. The first line is the number of G-vectors at which the $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$ are subsequently printed. This number should always be 32 since 32 specific $\tilde{u}_{m\mathbf{k}}$ are required. The following lines contain the following in this order: The band index m, a counter on the number of G-vectors, the integer co-efficient of the G-vector components a, b, c (where $\mathbf{G} = a\mathbf{b}_1 + b\mathbf{b}_2 + c\mathbf{b}_3$), then the real and imaginary parts of the corresponding $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$ at the Γ -point. We note that the ordering in which the G-vectors and $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$ are printed is not important, but the specific G-vectors are critical. The following example displays for a single band, the complete set of $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$ that are required. Note the G-vectors (a, b, c) needed.

```
32
                 0
                                         0.000000
1
     1
           0
                      0
                           0.4023306
1
     2
           0
                 0
                          -0.0000325
                                         0.000000
                      1
     3
1
           0
                 1
                      0
                          -0.3043665
                                         0.000000
     4
                 0
                          -0.3043665
                                         0.000000
1
           1
                      0
     5
1
           2
                 0
                      0
                           0.1447143
                                         0.0000000
1
     6
           1
                -1
                           0.2345179
                                         0.000000
                      0
1
     7
           1
                 1
                      0
                           0.2345179
                                         0.000000
1
     8
           1
                 0
                      -1
                           0.0000246
                                         0.000000
     9
1
           1
                 0
                      1
                           0.0000246
                                         0.0000000
1
    10
           0
                 2
                           0.1447143
                                         0.000000
                      0
1
    11
           0
                 1
                      -1
                           0.0000246
                                         0.000000
1
    12
           0
                 1
                           0.0000246
                                         0.000000
                      1
                 0
                                         0.000000
1
    13
           0
                      2
                           0.0000338
           3
1
    14
                 0
                      0
                          -0.0482918
                                         0.0000000
1
    15
                          -0.1152414
                                         0.000000
                -1
```

```
1
    16
           2
                 1
                      0
                          -0.1152414
                                         0.0000000
    17
           2
                 0
1
                      -1
                          -0.0000117
                                         0.000000
1
    18
           2
                 0
                          -0.0000117
                                         0.000000
                      1
    19
                -2
                                         0.0000000
1
           1
                      0
                          -0.1152414
                 2
1
    20
           1
                      0
                          -0.1152414
                                         0.000000
1
    21
           1
                          -0.0000190
                                         0.000000
                -1
                      -1
                -1
1
    22
           1
                       1
                          -0.0000190
                                         0.000000
1
    23
                          -0.0000190
                                         0.000000
           1
                 1
                      -1
1
    24
           1
                 1
                      1
                          -0.0000190
                                         0.000000
1
    25
                 0
                      -2
                          -0.0000257
                                         0.0000000
           1
                      2
1
    26
           1
                 0
                          -0.0000257
                                         0.0000000
1
                 3
                          -0.0482918
                                         0.0000000
    27
           0
                      0
                 2
                          -0.0000117
                                         0.000000
1
    28
           0
                      -1
1
    29
           0
                 2
                          -0.0000117
                                         0.000000
                      1
1
    30
           0
                 1
                      -2
                          -0.0000257
                                         0.0000000
    31
           0
                      2
                          -0.0000257
                                         0.000000
1
                 1
1
    32
           0
                 0
                      3
                           0.0000187
                                         0.000000
2
     1
           0
                 0
                      0
                          -0.0000461
                                         0.000000
```

8.33 seedname_u.mat

OUTPUT. Written if write_u_matrices = .TRUE.. The first line gives the date and time at which the file was created. The second line states the number of kpoints num_kpts and the number of wannier functions num_wann twice. The third line is empty. Then there are num_kpts blocks of data, each of which starts with a line containing the kpoint (in fractional coordinates of the reciprocal lattice vectors) followed by $num_wann * num_wann$ lines containing the matrix elements (real and imaginary parts) of $\mathbf{U}^{(k)}$. The matrix elements are in column-major order (ie, cycling over rows first and then columns). There is an empty line between each block of data.

```
written on 15Sep2016 at 16:33:46
          64
                       8
                                   8
                               +0.000000000
  0.000000000
                +0.000000000
  0.4468355787
                +0.1394579978
 -0.0966033667
                +0.4003934902
 -0.0007748974
                +0.0011788678
 -0.0041177339
                +0.0093821027
  0.1250000000
                 0.000000000
                               +0.000000000
  0.4694005589
                +0.0364941808
 +0.2287801742
                -0.1135511138
 -0.4776782452
                -0.0511719121
```

```
+0.0142081014 +0.0006203139
.
```

8.34 seedname_u_dis.mat

OUTPUT. Written if write_u_matrices = .TRUE. and disentanglement is enabled. The first line gives the date and time at which the file was created. The second line states the number of kpoints num_kpts, the number of wannier functions num_bands and the number of num_bands. The third line is empty. Then there are num_kpts blocks of data, each of which starts with a line containing the kpoint (in fractional coordinates of the reciprocal lattice vectors) followed by num_wann * num_bands lines containing the matrix elements (real and imaginary parts) of $\mathbf{U}^{\mathrm{dis}(\mathbf{k})}$. The matrix elements are in column-major order (ie, cycling over rows first and then columns). There is an empty line between each block of data.

```
written on 15Sep2016 at 16:33:46
                                 16
  0.0000000000
               +0.000000000
                              +0.000000000
  1.0000000000
               +0.000000000
 +0.0000000000
               +0.000000000
 +0.000000000
               +0.000000000
 +0.0000000000
               +0.0000000000
 0.1250000000
                0.000000000
                              +0.000000000
  1.0000000000
               +0.000000000
 +0.0000000000
               +0.000000000
 +0.0000000000
               +0.000000000
 +0.000000000
              +0.000000000
```

Chapter 9

Some notes on the interpolation

In wannier90 v.2.1, a new flag use_ws_distance has been introduced (and it is set to .true. by default since version v3.0). Setting it to .false. reproduces the "standard" behavior of wannier90 in v.2.0.1 and earlier, while setting it to .true. changes the interpolation method as described below. In general, this allows a smoother interpolation, helps reducing (a bit) the number of k-points required for interpolation, and reproduces the band structure of large supercells sampled at Γ only (setting it to .false. produces instead flat bands, which might instead be the intended behaviour for small molecules carefully placed at the centre of the cell).

The core idea rests on the fact that the Wannier functions $w_{n\mathbf{R}}(\mathbf{r})$ that we build from $N \times M \times L$ k-points are actually periodic over a supercell of size $N \times M \times L$, but when you use them to interpolate you want them to be *zero* outside this supercell. In 1D it is pretty obvious want we mean here, but in 3D what you really want that they are zero outside the Wigner-Seitz cell of the $N \times M \times L$ superlattice.

The best way to impose this condition is to check that every real-space distance that enters in the $R \to k$ Fourier transform is the shortest possible among all the $N \times M \times L$ -periodic equivalent copies.

If the distances were between unit cells, this would be trivial, but the distances are between Wannier functions which are not centred on $\mathbf{R} = 0$. Hence, when you want to consider the matrix element of a generic operator \mathbf{O} (i.e., the Hamiltonian) $\langle w_{i\mathbf{0}}(\mathbf{r})|\mathbf{O}|w_{j\mathbf{R}}(\mathbf{r})\rangle$ you must take in account that the centre τ_i of $w_{i\mathbf{0}}(\mathbf{r})$ may be very far away from \mathbf{O} and the centre τ_j of $w_{j\mathbf{R}}(\mathbf{r})$ may be very far away from \mathbf{R} .

There are many way to find the shortest possible distance between $w_{i0}(\mathbf{r})$ and $w_{j\mathbf{R}}(\mathbf{r}-\mathbf{R})$, the one used here is to consider the distance $\mathbf{d}_{ij\mathbf{R}} = \boldsymbol{\tau}_i - (\boldsymbol{\tau}_j + \mathbf{R})$ and all its superlattice periodic equivalents $\mathbf{d}_{ij\mathbf{R}} + \tilde{\mathbf{R}}_{nml}$, with $\tilde{\mathbf{R}}_{nml} = (Nn\mathbf{a}_1 + Mm\mathbf{a}_2 + Ll\mathbf{a}_3)$ and n, l, m = -L, -L+1, ...0, ..., L-1, L, with L controlled by the parameter ws_search_size.

Then,

- 1. if $\mathbf{d}_{ij\mathbf{R}} + \tilde{\mathbf{R}}_{nml}$ is inside the $N \times M \times L$ super-WS cell, then it is the shortest, take it and quit
- 2. if it is outside the WS, then it is not the shortest, throw it away
- 3. if it is on the border/corner of the WS then it is the shortest, but there are other choices of (n, m, l) which are equivalent, find all of them

In all distance comparisons, a small but finite tolerance is considered, which can be controlled with the parameter ws_distance_tol.

Because of how the Fourier transform is defined in the wannier90 code (not the only possible choice) it is only $\mathbf{R} + \tilde{\mathbf{R}}_{nml}$ that enters the exponential, but you still have to consider the distance among the actual centres of the Wannier functions. Using the centres of the unit-cell to which the Wannier functions belong is not enough (but is easier, and saves you one index).

Point 3 is not strictly necessary, but using it helps enforcing the symmetry of the system in the resulting band structure. You will get some small but evident symmetry breaking in the band plots if you just pick one of the equivalent $\tilde{\mathbf{R}}$ vectors.

Note that in some cases, all this procedure does absolutely nothing, for instance if all the Wannier function centres are very close to 0 (e.g., a molecule carefully placed in the periodic cell).

In some other cases, the effect may exist but be imperceptible. E.g., if you use a very fine grid of k-points, even if you don't centre each functions perfectly, the periodic copies will still be so far away that the change in centre applied with use ws_distance does not matter.

When instead you use few k-points, activating the use_ws_distance may help a lot in avoiding spurious oscillations of the band structure even when the Wannier functions are well converged.

Chapter 10

begin kpoints

Sample Input Files

10.1 Master input file: seedname.win

```
: 4
num_wann
                 : 4 4 4
mp_grid
num_iter
                : 100
postproc_setup : true
begin unit_cell_cart
ang
-1.61 0.00 1.61
0.00 1.61 1.61
-1.61 1.61 0.00
end unit_cell_cart
begin atoms_frac
  -0.125 -0.125 -0.125
    0.125 0.125 0.125
end atoms_frac
bands_plot : true
bands_num_points : 100
bands_plot_format : gnuplot
begin kpoint_path
L 0.50000 0.50000 0.50000 G 0.00000 0.00000 0.00000
G 0.00000 0.00000 0.00000 X 0.50000 0.00000 0.50000
X 0.50000 0.00000 0.50000 K 0.62500 0.25000 0.62500
end kpoint_path
begin projections
C:1=0,1=1
end projections
```

```
0.00 0.00 0.00
0.00 0.00 0.25
0.00 0.50 0.50
.
.
.
0.75 0.75 0.50
0.75 0.75 0.75
end kpoints
```

10.2 seedname.nnkp

Running wannier90 on the above input file would generate the following nnkp file:

```
File written on 9Feb2006 at 15:13: 9
calc_only_A
begin real_lattice
 -1.612340
            0.000000
                        1.612340
  0.000000
             1.612340
                        1.612340
  -1.612340
             1.612340
                        0.000000
end real_lattice
begin recip_lattice
 -1.951300 -1.951300
                        1.951300
            1.951300
  1.951300
                        1.951300
 -1.951300
            1.951300 -1.951300
end recip_lattice
begin kpoints
     64
 0.00000
           0.00000
                     0.00000
 0.00000
           0.25000
                     0.00000
 0.00000
           0.50000
                     0.00000
 0.00000
           0.75000
                     0.00000
 0.25000
           0.00000
                     0.00000
 0.50000
           0.75000
                     0.75000
 0.75000
           0.00000
                     0.75000
 0.75000
           0.25000
                     0.75000
 0.75000
           0.50000
                     0.75000
 0.75000
           0.75000
                     0.75000
end kpoints
```

begin projections 8 -0.12500 -0.12500 -0.12500 0 1 1 0.000 0.000 1.000 1.000 0.000 0.000 2.00 -0.12500 -0.12500 -0.12500 1 1 1 0.000 0.000 1.000 0.000 0.000 1.000 2.00 -0.12500 -0.12500 -0.12500 1 2 1 0.000 0.000 1.000 2.00 1.000 0.000 0.000 -0.12500 -0.12500 -0.12500 1 3 1 0.000 0.000 0.000 0.000 1.000 1.000 2.00 0.12500 0.12500 0.12500 0 1 1 0.000 0.000 1.000 1.000 0.000 0.000 2.00 0.12500 0.12500 0.12500 1 1 1 0.000 0.000 1.000 1.000 0.000 0.000 2.00 0.12500 0.12500 0.12500 1 2 1 0.000 0.000 0.000 0.000 1.000 1.000 2.00 0.12500 0.12500 0.12500 1 3 1 0.000 0.000 1.000 1.000 0.000 0.000 2.00

begin nnkpts

end projections

8	;			
1	2	0	0	0
1	4	0	-1	0
1	5	0	0	0
1	13	-1	0	0
1	17	0	0	0
1	22	0	0	0
1	49	0	0	-1
1	64	-1	-1	-1
2	1	0	0	0
2	3	0	0	0
2 2 2	6	0	0	0
	14	-1	0	0
2 2	18	0	0	0
2	23	0	0	0
2	50	0	0	-1
2	61	-1	0	-1
•				
64	1	1	1	1
64	16	0	0	1
64	43	0	0	0
64	48	0	0	0
64	52	1	0	0
64	60	0	0	0
64	61	0	1	0
		_	_	_

64

63

0

0

end nnkpts

begin exclude_bands

4

1

2

3 4

end exclude_bands

Part III

postw90.x

Chapter 11

Parameters

11.1 Introduction

The wannier90.x code described in Part II calculates the maximally-localized Wannier functions.

The postw90.x executable contains instead a series of modules that take the Wannier functions calculated by wannier90.x and use them to calculate different properties. This executable is parallel (by means of MPI libraries), so it can be run on multiple CPUs. The information on the calculated Wannier functions is read from the checkpoint seedname.chk file. Note that this is written in an unformatted machine-dependent format. If you need to use this file on a different machine, or you want to use a version of postw90.x compiled with a different compiler, refer to Sec. A.2 in the Appendices for a description of how to export/import this file.

11.2 Usage

postw90.x can be run in parallel using MPI libraries to reduce the computation time.

For serial execution use: postw90.x [seedname]

• seedname: If a seedname string is given the code will read its input from a file seedname.win. The default value is wannier. One can also equivalently provide the string seedname.win instead of seedname.

For parallel execution use: mpirun -np NUMPROCS postw90.x [seedname]

• NUMPROCS: substitute with the number of processors that you want to use.

Note that the mpirun command and command-line flags may be different in your MPI implementation: read your MPI manual or ask your computer administrator.

Note also that this requires that the postw90.x executable has been compiled in its parallel version (follow the instructions in the file README.install in the main directory of the wannier90 distribution) and that the MPI libraries and binaries are installed and correctly configured on your machine.

11.3 seedname.win File

The postw90.x uses the same seedname.win input file of wannier90.x. The input keywords of postw90.x must thus be added to this file, using the same syntax described in Sec. 2.2.

Note that wannier90.x checks if the syntax of the input file is correct, but then ignores the value of the flags that refer only to modules of postw90.x, so one can safely run wannier90.x on a file that contains also postw90.x flags.

Similarly, postw90.x ignores flags that refer only to wannier90.x (as number of iterations, restart flags, ...). However, some parts of the input file must be there, as for instance the number of Wannier functions, etc.

The easiest thing to do is therefore to simply add the postw90 input keywords to the seedname.win file that was used to obtain the Wannier functions.

11.4 List of available modules

The currently available modules in postw90.x are:

- dos: Calculation of the density of states (DOS), projected density of states (PDOS), net spin etc.
- kpath: Calculation of k-space quantities such as energy bands, Berry curvature and Berry curvature-like term of spin Hall conductivity along a piecewise linear path in the BZ (see examples 17, 18 and 29 of the tutorial).
- kslice: Calculation of k-space quantities on a planar slice of the BZ (see examples 17, 18 and 29 of the tutorial).
- berry: Calculation of properties related to the BZ integral of the Berry curvature, Berry connection and Berry curvature-like term of spin Hall conductivity, including anomalous Hall conductivity, orbital magnetisation, optical conductivity, nonlinear shift current and spin Hall conductivity (see Chap. 12 and examples 18, 19, 25, 29 and 30 of the tutorial).
- gyrotropic: Calculation of gyrotropic properties, including natural and current0induced optical rotation, and the current-induced magnetization (see Chap. 13 and examples of the tutorial).
- BoltzWann: Calculation of electronic transport properties for bulk materials using the semiclassical Boltzmann transport equation (see Chap. 14 and example 16 of the tutorial).
- geninterp (Generic Band Interpolation): Calculation band energies (and band derivatives) on a generic list of k points (see Chap. 15).

11.5 Keyword List

On the next pages the list of available postw90 input keywords is reported. In particular, Table 11.1 reports keywords that affect the generic behavior of all modules of postw90. Often, these are "global" variables that can be overridden by module-specific keywords (as for instance the kmesh flag). The subsequent tables describe the input parameters for each specific module.

A description of the behaviour of the global flags is described Sec. 11.6; the description of the flags specific to the modules can be found in the following sections.

Keyword	Type	Description	
Globa	Global Parameters of postw90		
KMESH	I	Dimensions of the uniform interpo-	
		lation k -mesh (one or three integers)	
KMESH_SPACING	R	Minimum spacing between k points	
		\mid in Å ⁻¹	
ADPT_SMR	L	Use adaptive smearing	
ADPT_SMR_FAC	R	Adaptive smearing prefactor	
ADPT_SMR_MAX	P	Maximum allowed value for the	
		adaptive energy smearing (eV)	
SMR_TYPE	S	Analytical form used for the broad-	
		ened delta function	
SMR_FIXED_EN_WIDTH	P	Energy smearing (if non-adaptive)	
NUM_ELEC_PER_STATE	I	Number of electrons per state	
SCISSORS_SHIFT	P	Scissors shift applied to the conduc-	
		tion bands (eV) (deprecated)	
NUM_VALENCE_BANDS	I	Number of valence bands	
SPIN_DECOMP	L	Decompose various properties into	
		up-spin, down-spin, and possibly	
		spin-flip parts	
SPIN_AXIS_POLAR	P	Polar angle of the spin quantization	
		axis (deg)	
SPIN_AXIS_AZIMUTH	P	Azimuthal angle of the spin quanti-	
		zation axis (deg)	
SPIN_MOMENT*	L	Determines whether to evaluate the	
		spin magnetic moment per cell	
uHu_formatted	L	Read a formatted seedname.uHu file	
SPN_FORMATTED	L	Read a formatted seedname.spn file	
BERRY_CURV_UNIT	S	Unit of Berry curvature	

Table 11.1: seedname.win file keywords controlling the general behaviour of the modules in postw90. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

The keyword spin_moment does not affect the behavior of the modules in postw90, and does not really belong to any of them. It is listed here for lack of a better place.

Keyword	Type	Description
berry]	Paramet	ers
BERRY	L	Calculate Berry-type quantities
BERRY_TASK	S	List of properties to compute
[BERRY_]KMESH	I	Dimensions of the uniform interpo-
		lation k -mesh (one or three integers)
[BERRY_]KMESH_SPACING	R	Minimum spacing between k points in $\mathring{\mathbf{A}}^{-1}$
BERRY_CURV_ADPT_KMESH	I	Linear dimension of the adaptively
		refined k -mesh used to compute the
		anomalous/spin Hall conductivity

BERRY_CURV_ADPT_KMESH_THRESH	Р	Threshold magnitude of the Berry curvature for adaptive refinement
KUBO_FREQ_MIN	Р	Lower limit of the frequency range for optical spectra, JDOS, shift cur- rent and spin Hall conductivity (eV)
KUBO_FREQ_MAX	Р	Upper limit of the frequency range for optical spectra, JDOS, shift current and spin Hall conductivity (eV)
KUBO_FREQ_STEP	R	Step for increasing the optical frequency in the specified range
KUBO_EIGVAL_MAX	Р	Maximum energy eigenvalue included when evaluating the Kubo-Greenwood conductivity, JDOS, shift current and spin Hall conductivity
[KUBO_]ADPT_SMR	L	Use adaptive energy smearing for the optical conductivity, JDOS, shift current and spin Hall conductivity
[KUBO_]ADPT_SMR_FAC	R	Adaptive smearing prefactor
KUBO ADPT SMR MAX	Р	Maximum allowed value for the
1 _1		adaptive energy smearing (eV)
[KUBO_]SMR_TYPE	S	Analytical form used for the broad-
[KUBO_]SMR_FIXED_EN_WIDTH	P	ened delta function when computing the optical conductivity, JDOS, shift current and spin Hall conductivity Energy smearing (if non-adaptive)
		for the optical conductivity, JDOS, shift current and spin Hall conductivity (eV)
SC_ETA	R	Energy broadening of energy differences in the sum over virtual states when computing shift current
SC_PHASE_CONV	I	Convention for phase factor of Bloch states when computing shift current
SC_W_THR	R	Frequency threshold for speeding up delta function integration when computing shift current
SHC_FREQ_SCAN	L	Calculate Fermi energy scan or frequency scan of spin Hall conductivity
SHC_METHOD	S	How to obtain the spin current matrix elements for SHC
SHC_ALPHA	I	The spin current direction of spin Hall conductivity
SHC_BETA	I	The direction of applied electrical field of spin Hall conductivity
SHC_GAMMA	I	The spin direction of the spin current of spin Hall conductivity
	l	Tells of spin Hair conductivity

$SHC_BANDSHIFT$	L	Rigid bandshift of the conduction
		bands
SHC_BANDSHIFT_FIRSTBAND	I	Index of the first band to shift
SHC_BANDSHIFT_ENERGYSHIFT	Р	Energy shift of the conduction bands
		(eV)

Table 11.5: seedname.win file keywords controlling the berry module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description
do	s Paran	neters
DOS	L	Calculate the density of states and
		related properties
DOS_TASK	S	List of properties to compute
DOS_ENERGY_MIN	P	Lower limit of the energy range for
		computing the DOS (eV)
DOS_ENERGY_MAX	P	Upper limit of the energy range for
		computing the DOS (eV)
DOS_ENERGY_STEP	R	Step for increasing the energy in the
		specified range (eV)
DOS_PROJECT	I	List of WFs onto which the DOS is
		projected
[DOS_]KMESH	I	Dimensions of the uniform interpo-
		lation k -mesh (one or three integers)
[DOS_]KMESH_SPACING	R	Minimum spacing between k points
		in $Å^{-1}$
[DOS_]ADPT_SMR	L	Use adaptive smearing for the DOS
[DOS_]ADPT_SMR_FAC	R	Adaptive smearing prefactor
[DOS_]ADPT_SMR_MAX	P	Maximum allowed value for the
		adaptive energy smearing (eV)
[DOS_]SMR_FIXED_EN_WIDTH	P	Energy smearing (if non-adaptive)
		for the DOS (eV)
[DOS]SMR TYPE	S	Analytical form used for the broad-
, _		ened delta function when computing
		the DOS.

Table 11.2: seedname.win file keywords controlling the dos module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description
kpath Parameters		
КРАТН	L	Calculate properties along a piece-
		wise linear path in the BZ
KPATH_TASK	L	List of properties to evaluate
KPATH_NUM_POINTS	I	Number of points in the first kpath
		segment
KPATH_BANDS_COLOUR	S	Property used to colour the energy
		bands along the path

Table 11.3: seedname.win file keywords controlling the kpath module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description
ksli	ce Para	ameters
KSLICE	L	Calculate properties on a slice in the
		BZ
KSLICE_TASK	S	List of properties to evaluate
KSLICE_CORNER	R	Position of the corner of the slice
KSLICE_B1	R	First vector defining the slice
KSLICE_B2	R	Second vector defining the slice
KSLICE_2DKMESH	I	Dimensions of the uniform interpo-
		lation k -mesh on the slice (one or
		two integers)
KSLICE_FERMI_LEVEL	P	This parameter is not used anymore.
		Use FERMI_ENERGY instead.
KSLICE_FERMI_LINES_COLOUR	S	Property used to colour the Fermi
		lines

Table 11.4: seedname.win file keywords controlling the kslice module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description
berry P	aramete	ers
GYROTROPIC	L	Calculate gyrotropic quantities
GYROTROPIC_TASK	L	List of properties to compute
[GYROTROPIC_]KMESH	I	Dimensions of the uniform interpo-
		lation k -mesh (one or three integers)
[GYROTROPIC_]KMESH_SPACING	R	Minimum spacing between k points
		$\int \ln \mathring{A}^{-1}$
GYROTROPIC_FREQ_MIN	P	Lower limit of the frequency range
		for optical rotation (eV)
GYROTROPIC_FREQ_MAX	Р	Upper limit of the frequency range
		for optical rotation (eV)
GYROTROPIC_FREQ_STEP	Р	Step for increasing the optical fre-
		quency in the specified range
GYROTROPIC_EIGVAL_MAX	Р	Maximum energy eigenvalue in-
		cluded when evaluating the inter-
		band natural optical activity
GYROTROPIC_DEGEN_THRESH	Р	threshold to exclude degenerate
		bands from the calculation
[GYROTROPIC_]SMR_TYPE	S	Analytical form used for the broad-
		ened delta function
[GYROTROPIC_]SMR_FIXED_EN_WIDTH	Р	Energy smearing (eV)
[GYROTROPIC_]BAND_LIST	I	list of bands used in the calculation
GYROTROPIC_BOX_CENTER	R	The center and three basis vectors,
GYROTROPIC_BOX_B1	R	defining the box for integration (in
GYROTROPIC_BOX_B2	R	reduced coordinates, three real
GYROTROPIC_BOX_B3	R	numbers for each vector)

Table 11.6: seedname.win file keywords controlling the gyrotropic module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description		
BoltzWann Parameters				
BOLTZWANN	L	Calculate Boltzmann transport co-		
		efficients		
[BOLTZ_]KMESH	I	Dimensions of the uniform interpo-		
		lation k -mesh (one or three integers)		
[BOLTZ_]KMESH_SPACING	R	Minimum spacing between k points		
, _		$\int \ln \mathring{A}^{-1}$		
BOLTZ 2D DIR	\mathbf{S}	Non-periodic direction (for 2D sys-		
		tems only)		
BOLTZ_RELAX_TIME	Р	Relaxation time in fs		
BOLTZ_MU_MIN	Р	Minimum value of the chemical po-		
		tential μ in eV		
BOLTZ_MU_MAX	Р	Maximum value of the chemical po-		
		tential μ in eV		
BOLTZ_MU_STEP	R	Step for μ in eV		
BOLTZ_TEMP_MIN	Р	Minimum value of the tempera-		
		ture T in Kelvin		
BOLTZ TEMP MAX	Р	Maximum value of the tempera-		
		ture T in Kelvin		
BOLTZ_TEMP_STEP	R	Step for T in Kelvin		
BOLTZ_TDF_ENERGY_STEP	R	Energy step for the TDF (eV)		
BOLTZ_TDF_SMR_FIXED_EN_WIDTH	Р	Energy smearing for the TDF (eV)		
BOLTZ_TDF_SMR_TYPE	\mathbf{S}	Smearing type for the TDF		
BOLTZ_CALC_ALSO_DOS	L	Calculate also DOS while calculat-		
		ing the TDF		
BOLTZ_DOS_ENERGY_MIN	Р	Minimum value of the energy for the		
		DOS in eV		
BOLTZ DOS ENERGY MAX	Р	Maximum value of the energy for the		
		DOS in eV		
BOLTZ DOS ENERGY STEP	R	Step for the DOS in eV		
BOLTZ DOS SMR TYPE	\mathbf{S}	Smearing type for the DOS		
BOLTZ DOS ADPT SMR	L	Use adaptive smearing for the DOS		
[BOLTZ_DOS_]ADPT_SMR_FAC	R	Adaptive smearing prefactor		
BOLTZ_DOS_ADPT_SMR_MAX	Р	Maximum allowed value for the		
,		adaptive energy smearing (eV)		
BOLTZ DOS SMR FIXED EN WIDTH	Р	Energy smearing (if non-adaptive)		
		for the DOS (eV)		
BOLTZ_BANDSHIFT	L	Rigid bandshift of the conduction		
_		bands		
BOLTZ_BANDSHIFT_FIRSTBAND	I	Index of the first band to shift		
BOLTZ_BANDSHIFT_ENERGYSHIFT	Р	Energy shift of the conduction bands		
		(eV)		

Table 11.7: seedname.win file keywords controlling the BoltzWann module (calculation of the Boltzmann transport coefficients in the Wannier basis). Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description
gen	interp	Parameters
GENINTERP	L	Calculate bands for given set of k
		points
GENINTERP_ALSOFIRSTDER	L	Calculate also first derivatives
GENINTERP_SINGLE_FILE	L	Write a single file or one for each
		process

Table 11.8: seedname.win file keywords controlling the Generic Band Interpolation (geninterp) module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

11.6 Global variables

11.6.1 integer :: kmesh(:)

Dimensions of the interpolation grid used in postw90.x.

Not to be confused with the mp_grid input flag, which instead specifies the Monkhorst-Pack grid used in the ab-initio calculation!

If three integers l m n are given, the reciprocal-space cell subtended by the three primitive translations is sampled on a uniform $l \times m \times n$ grid (including Γ). If only one integer m is given, an $m \times m \times m$ grid is used.

If you use a module which needs a k-mesh, either kmesh_spacing or kmesh must be defined.

11.6.2 real(kind=dp) :: kmesh_spacing

An alternative way of specifying the interpolation grid. This flag defines the minimum distance for neighboring k points along each of the three directions in k space.

The units are $Å^{-1}$.

If you use a module which needs a k-mesh, either kmesh_spacing or kmesh must be defined.

11.6.3 logical :: adpt_smr

Determines whether to use an adaptive scheme for broadening the DOS and similar quantities defined on the energy axis. If true, the values for the smearing widths are controlled by the flag adpt_smr_fac.

The default value is true.

11.6.4 real(kind=dp) :: adpt_smr_fac

The width $\eta_{n\mathbf{k}}$ of the broadened delta function used to determine the contribution to the spectral property (DOS, ...) from band n at point \mathbf{k} is calculated as

$$\eta_{n\mathbf{k}} = \alpha |\nabla_{\mathbf{k}} \varepsilon_{n\mathbf{k}}| \Delta k,$$

where $\varepsilon_{n\mathbf{k}}$ is the energy eigenvalue and the dimensionless factor α is given by adpt_smr_fac. Δk is taken to be the largest of the mesh spacings along the three reciprocal lattice vectors $\mathbf{b_1}$, $\mathbf{b_2}$, and $\mathbf{b_3}$. If the calculated value of $\eta_{n\mathbf{k}}$ exceeds adpt_smr_max, the latter value is used.

The default value is $\sqrt{2}$.

11.6.5 real(kind=dp) :: adpt_smr_max

See description given immediately above.

The units are eV. The default value is 1.0.

11.6.6 character(len=120) :: smr_type

Defines the analytical form used for the broadened delta function in the computation of the DOS and similar quantities defined on the energy axis.

- gauss: Gaussian smearing
- m-pN: derivative of the N-th order Methfessel-Paxton function $(N \ge 0)$. Example: m-p2 for the second-order Methfessel-Paxton function. If only m-p is provided, the first-order function is used, i.e., it is equivalent to m-p1.
- m-v or cold: derivative of the Marzari-Vanderbilt cold-smearing function
- f-d: derivative of the Fermi-Dirac distribution function

The default value is gauss.

11.6.7 logical :: smr_fixed_en_width

Energy width for the smearing function for the DOS. Used only if adpt_smr is false.

The units are eV. The default value is 0 eV. Note that if the width is smaller than twice the energy step (e.g. dos_energy_step for the dos module), the DOS will be unsmeared (thus the default is to have an unsmeared properties when adpt_smr is set to false.).

11.6.8 integer :: num_elec_per_state

Number of electrons per state. It can only take the values one or two.

The default value is 1 if spinors=true, 2 otherwise.

11.6.9 real(kind=dp) :: scissors_shift

Scissors shift applied to the conduction bands.

Note! This variable is deprecated and will be removed in future versions of the code. This applies the scissors shift only to the Hamiltonian, but also other matrices might need to be updated if a scissors shift is applied. If you are using BoltzWann, consider using boltz_bandshift instead. If you are calculating spin Hall conductivity, consider using shc_bandshift instead.

The units are eV. The default value is 0 eV (i.e., no scissors shift applied).

11.6.10 integer :: num_valence_bands

Number of valence bands of the system. Used in different modules and for the scissors shift.

No default value.

11.6.11 logical :: spin_decomp

If true, extra columns are added to some output files (such as seedname-dos.dat for the dos module, and analogously for the berry and BoltzWann modules).

For the dos and BoltzWann modules, two further columns are generated, which contain the decomposition of the required property (e.g., total or orbital-projected DOS) of a spinor calculation into up-spin and down-spin parts (relative to the quantization axis defined by the input variables $spin_axis_polar$ and $spin_axis_azimuth$). For the berry module with $berry_task = kubo$, three extra columns are added to $seedname_jdos.dat$, containing the decomposition of the JDOS into $up \rightarrow up$, $down \rightarrow down$, and $spin_flip$ transitions. In the same way, six extra columns are added to the data files $seedname_kubo*.dat$ where the complex optical conductivity is stored.

The file seedname.spn must be present at input. Furthermore, if this variable is set to true it requires num_elec_per_state = 1.

The default value is false.

11.6.12 real(kind=dp) :: spin_axis_polar

Polar angle of the spin quantization axis.

The units are degrees. The default value is 0.

11.6.13 real(kind=dp) :: spin_axis_azimuth

Azimuthal angle of the spin quantization axis.

The units are degrees. The default value is 0.

11.6.14 logical :: spin_moment

Determines whether to evaluate the spin moment.

The default value is false.

11.6.15 logical :: uHu_formatted

If uHu_formatted=true, then the uHu matrix elements will be read from disk as formatted (ie ASCII) files; otherwise they will be read as unformatted files.

The default value of this parameter is false.

11.6.16 logical :: spn_formatted

If spn_formatted=true, then the spin matrix elements will be read from disk as formatted (ie ASCII) files; otherwise they will be read as unformatted files. Unformatted is generally preferable as the files will take less disk space and I/O is significantly faster. However such files will not be transferable

between all machine architectures and formatted files should be used if transferability is required (i.e., for test cases).

The default value is false.

11.6.17 character(len=20) :: berry_curv_unit

Unit in which the Berry curvature is specified at input (in berry_curv_adpt_kmesh_thresh) or written to file (when kpath_task=curv or kpath_task=shc or kslice_task=curv or kslice_task=shc).

- ang2: Angstrom²
- bohr2: Bohr² (atomic units)

The default value is ang2.

11.6.18 real(kind=dp) :: sc_eta

The width η used to broaden energy differences in denominators of the form

$$\frac{1}{\varepsilon_{n\mathbf{k}} - \varepsilon_{m\mathbf{k}}} \to \operatorname{Re} \frac{1}{\varepsilon_{n\mathbf{k}} - \varepsilon_{m\mathbf{k}} + i\eta}.$$

The above is needed in shift-current calculations in order to avoid numerical problems caused by near-degeneracies in the sum over virtual states.

The units are eV. The default value is 0.4.

11.6.19 integer :: sc_phase_conv

Convention for the expansion of the Bloch states in shift-current calculations. It can only take the values one or two. We follow the convention of Ref. [11]:

• 1: Include Wannier centre $\tau_n = \langle w_{n0} | \mathbf{r} | w_{n0} \rangle$ in the phase factor (so-called tight-binding convention):

$$|u_{n\mathbf{k}}\rangle = \sum_{\mathbf{R}} e^{-i\mathbf{k}(\mathbf{r} - \mathbf{R} - \boldsymbol{\tau}_n)} |w_{n\mathbf{R}}\rangle$$

• 2: Do not include Wannier centre in the phase factor (usual Wannier90 convention):

$$|u_{n\mathbf{k}}\rangle = \sum_{\mathbf{R}} e^{-i\mathbf{k}(\mathbf{r}-\mathbf{R})} |w_{n\mathbf{R}}\rangle$$

The convention does not affect the full shift-current matrix element, but it does affect the weights of the internal components that compose it (see Ref. [12]).

The default value is 1.

11.6.20 real(kind=dp) :: sc_w_thr

Parameter α_t for speeding up the frequency integration in shift-current calculations. It settles the frequency threshold $\omega_t = \alpha_t \eta_{n\mathbf{k}}$ (a factor times the broadening) beyond which the delta functions are taken as zero.

The default value is 5.0.

11.7 DOS

Note that the behavior of the dos module is also influenced by the value of some global flags (listed in Table 11.1), as spin_decomp, spin_axis_polar, spin_axis_azimuth, scissors_shift, etc. Some of the global flags can be possibly overridden by local flags of the DOS module, listed below, which have the same name of the global flag but are prefixed by dos_.

11.7.1 logical :: dos

Determines whether to enter the DOS routines.

The default value is false.

11.7.2 character(len=20) :: dos_task

The quantity to compute when dos=true

The valid options for this parameter are:

- dos_plot Density of states. An output data file seedname-dos.dat is created, containing the energy values in eV in the first column, and the total DOS per unit cell and unit energy range (in eV^{-1}) in the second. Two additional columns are present if spin_decomp=true

The default value is dos_plot.

11.7.3 real(kind=dp) :: dos_energy_min

Lower limit of the energy range for computing the DOS. Units are eV.

The default value is the minimum value of the energy eigenvalues stored in **seedname.eig**, minus 0.6667.

11.7.4 real(kind=dp) :: dos_energy_max

Upper limit of the energy range for computing the DOS. Units are eV.

If an inner energy window was specified, the default value is the upper bound of the innter energy window, plus 0.6667. Otherwise it is the maximum value of the energy eigenvalues stored in seedname.eig, plus 0.6667.

11.7.5 real(kind=dp) :: dos_energy_step

Energy step for the grid of energies used to plot the dos. Units are eV.

The default value is 0.01 eV.

11.7.6 integer :: dos_project(:)

If present postw90 computes, instead of the total DOS, the partial DOS projected onto the WFs listed. The WFs are numbered according to the file seedname.wout.

For example, to project onto WFs 2, 6, 7, 8, and 12:

dos_project : 2, 6-8, 12

The DOS projected onto a set $\mathcal S$ of orbitals is calculated as

$$\rho_{\mathcal{S}}(E) = \frac{1}{N_k} \sum_{\mathbf{k}} \sum_{n} \langle \psi_{n\mathbf{k}}^{(H)} | \hat{P}_{\mathbf{k}}(\mathcal{S}) | \psi_{n\mathbf{k}}^{(H)} \rangle \delta(\varepsilon_{n\mathbf{k}} - E)$$
(11.1)

$$\hat{P}_{\mathbf{k}}(\mathcal{S}) = \sum_{m \in \mathcal{S}} |\psi_{n\mathbf{k}}^{(W)}\rangle \langle \psi_{n\mathbf{k}}^{(W)}|, \tag{11.2}$$

where N_k is the number of mesh points used to sample the BZ, and the superscript (H) and (W) refer to *Hamiltonian gauge* and *Wannier gauge* [13].

11.7.7 integer :: dos_kmesh(:)

Overrides the kmesh global variable (see Sec. 11.6).

11.7.8 real(kind=dp) :: dos_kmesh_spacing

Overrides the kmesh_spacing global variable (see Sec. 11.6).

11.7.9 logical :: dos_adpt_smr

Overrides the adpt_smr global variable (see Sec. 11.6).

11.7.10 real(kind=dp) :: dos_adpt_smr_fac

Overrides the adpt_smr_fac global variable (see Sec. 11.6).

11.7.11 real(kind=dp) :: dos_adpt_smr_max

Overrides the adpt_smr_max global variable (see Sec. 11.6).

11.7.12 logical :: dos_smr_fixed_en_width

Overrides the smr_fixed_en_width global variable (see Sec. 11.6).

Note that if the width is smaller than twice the energy step dos_energy_step, the DOS will be unsmeared (thus the default is to have an unsmeared DOS).

11.7.13 character(len=20) :: dos_smr_type

Overrides the ${\tt smr_type}$ global variable (see Sec. 11.6).

11.8 kpath

11.8.1 logical :: kpath

Determines whether to enter the kpath routines.

The default value is false.

11.8.2 character(len=20) :: kpath_task

The quantities to plot when kpath=true

The valid options for this parameter are:

- bands Energy bands, in eV. The following files are created:
 - · seedname-bands.dat (data file)
 - · seedname-bands.gnu (gnuplot script)
 - · seedname-bands.py (python script)
 - · seedname-path.kpt (list of k-points along the path, written in the pwscf format)
- curv Minus the Berry curvature given by Eq. (12.18) of Ch. 12, in units of berry_curv_unit. The following files are created:
 - · seedname-curv.dat (data file)
 - seedname-curv_{x,y,z}.gnu (gnuplot scripts)
 - seedname-curv_{x,y,z}.py (python scripts)
- morb The integrand of the k-space orbital magnetization formula [Eq. (12.20) of Ch. 12] in $eV \cdot Å^2$. Four output files are created:
 - · seedname-morb.dat (data file)
 - seedname-morb_{x,y,z}.gnu (gnuplot scripts)
 - seedname-morb_{x,y,z}.py (python scripts)
- shc The band-projected Berry curvature-like term of spin Hall conductivity given by Eq. (12.23) of Ch. 12, in units of berry_curv_unit. The following files are created:
 - · seedname-shc.dat (data file)
 - · seedname-shc.gnu (gnuplot scripts)
 - · seedname-shc.py (python scripts)
- Any combination of the above. The following combinations are of special interest

```
kpath_task = bands+curv
kpath_task = bands+morb
```

kpath_task = bands+shc

They generate the following files:

- · seedname-bands.dat (data file)
- · seedname-{curv,morb,shc}.dat (data file)
- · seedname-bands+{curv,morb}_{x,y,z}.py or seedname-bands+shc.py (python scripts)

Two-panel figures are produced, with the energy bands within ± 0.65 eV of the Fermi level in the top panel, and the Berry curvature (or k-space orbital magnetization, or k-resolved Berry curvature-like term of spin Hall conductivity) in the bottom panel.

The default value is bands.

11.8.3 integer :: kpath_num_points

If kpath = true, then the number of points along the first section of the bandstructure plot given by $kpoint_path$. Other sections will have the same density of k-points.

The default value is 100.

11.8.4 character(len=20) :: kpath_bands_colour

When kpath_task=bands, colour code the energy bands according to the specified quantity.

The valid options for this parameter are:

- spin Spin projection (in units of $\hbar/2$) along the quantization axis defined by the variables spin_axis_polar and spin_axis_azimuth, for a spinor calculation
- shc Band-projected Berry curvature-like term of spin Hall conductivity (in units of berry_curv_unit) defined by the variables shc_alpha, shc_beta and shc_gamma, for a spinor calculation
- none no colour coding

The default value is none.

11.9 kslice

11.9.1 logical :: kslice

Determines whether to enter the kslice routines.

The default value is false.

11.9.2 character(len=20) :: kslice_task

The quantity to plot when kslice=true

The valid options for this parameter are:

- fermi_lines Lines of intersection between constant-energy surfaces and the slice. The energy level is specified by the keyword fermi_energy. Output files:
 - · seedname-kslice-fermi-spn.dat (data file when kslice_fermi_lines_colour = spin)
 - · seedname-bnd_n.dat (gnuplot data files when kslice_fermi_lines_colour = none)
 - · seedname-kslice-coord.dat (python data files when kslice_fermi_lines_colour = none)
 - · seedname-kslice-bands.dat (python data file when kslice_fermi_lines_colour = none)
 - · seedname-kslice-fermi_lines.gnu (gnuplot script)
 - seedname-kslice-fermi_lines.py (python script)
- curv[+fermi_lines] Heatmap of the Berry curvature of the occupied states [together with the constant-energy contours]. The unit of Berry curvature is berry_curv_unit.

Output files:

- · seedname-kslice-coord.dat (data files)
- · seedname-kslice-curv.dat (data file)
- · [seedname-kslice-bands.dat] (data file)
- seedname-kslice-curv_{x,y,z}[+fermi_lines].py (python scripts)
- morb[+fermi_lines] Heatmap of the k-space orbital magnetization in eV·Å² [together with the constant-energy contours]. Output files:
 - · seedname-kslice-coord.dat (data files)
 - · seedname-kslice-morb.dat (data file)
 - · [seedname-kslice-bands.dat] (data file)
 - seedname-kslice-morb_{x,y,z}[+fermi_lines].py (python scripts)
- shc[+fermi_lines] Heatmap of the Berry curvature-like term of the occupied states [together with the constant-energy contours]. The unit of Berry curvature-like term is berry_curv_unit.
 Output files:
 - · seedname-kslice-coord.dat (data files)

- · seedname-kslice-shc.dat (data file)
- · [seedname-kslice-bands.dat] (data file)
- · seedname-kslice-shc[+fermi_lines].py (python scripts)

The default value is fermi_lines.

Note: When kslice_fermi_lines_colour = none the gnuplot scripts draw the k-slices with a square shape, even when kslice_b1 and kslice_b2 below are not at right angles, or do not have equal lengths. (The python scripts draw the slices with the correct parallelogram shape.)

11.9.3 real(kind=dp) :: kslice_corner(3)

Reduced coordinates of the lower-left corner of the slice in k-space.

The default value is (0.0, 0.0, 0.0)

11.9.4 real(kind=dp) :: kslice_b1(3)

Reduced coordinates of the first reciprocal-space vector defining the slice.

The default value is (1.0, 0.0, 0.0).

11.9.5 real(kind=dp) :: kslice_b2(3)

Reduced coordinates of the second reciprocal-space vector defining the slice.

The default value is (0.0, 1.0, 0.0).

11.9.6 integer :: kslice_2dkmesh(2)

Dimensions of the k-point grid covering the slice. If two integers m n are given, the slice is sampled on a uniform $m \times n$ grid. If only one integer m is given, an $m \times m$ grid is used.

The default value for kslice_kmesh is 50.

11.9.7 character(len=20) :: kslice_fermi_lines_colour

When kslice_task=fermi_lines (but not when combined with curv or morb), colour code the Fermi lines according to the specified quantity.

The valid options for this parameter are:

- spin Spin projection (in units of $\hbar/2$) along the quantization axis defined by the variables spin_axis_polar and spin_axis_azimuth, for a spinor calculation
- none no colour coding

The default value is none.

11.10 berry

11.10.1 logical :: berry

Determines whether to enter the berry routines.

The default value is false.

11.10.2 character(len=120) :: berry_task

The quantity to compute when berry=true

The valid options for this parameter are:

- kubo Complex optical conductivity and joint density of states. Output files:
 - · seedname-kubo-S_{xx,yy,zz,xy,xz,yz}.dat (data files). First column: optical frequency $\hbar\omega$ in eV. Second and third columns: real and imaginary parts of the symmetric conductivity $\sigma_{\alpha\beta}^{S}(\hbar\omega) = \sigma_{\beta\alpha}^{S}(\hbar\omega)$ in S/cm. Six additional columns are present if spin_decomp = true.
 - · seedname-kubo-A_{yz,zx,xy}.dat (data files). First column: optical frequency $\hbar\omega$ in eV. Second and third columns: real and imaginary parts of the antisymmetric conductivity $\sigma_{\alpha\beta}^{A}(\hbar\omega) = -\sigma_{\beta\alpha}^{A}(\hbar\omega)$ in S/cm. Six additional columns are present if spin_decomp = true.
 - · seedname-jdos.dat (data file). First column: energy difference $\hbar\omega$ in eV between conduction (c) and valence (v) states with the same crystal momentum **k**. Second column: joint density of states $\rho_{cv}(\hbar\omega)$ (number of states per unit cell per unit energy range, in eV⁻¹). Three additional columns are present if spin_decomp = true.
- ahc Anomalous Hall conductivity, in S/cm. The three independent components $\sigma_x = \sigma_{yz}$, $\sigma_y = \sigma_{zx}$, and $\sigma_z = \sigma_{xy}$ are computed. Output files:
 - seedname-ahc-fermiscan.dat (data file). The first column contains the Fermi level ε_F in eV, and the following three column the values of $\sigma_{x,y,z}(\varepsilon_F)$. This file is written if a range of Fermi energies is specified via fermi_energy_min and fermi_energy_max. If a single Fermi energy is given, the AHC is printed in seedname.wpout only.
- morb Orbital magnetisation, in bohr magnetons per cell.
 Output files:
 - seedname-morb-fermiscan.dat (data file). The first column contains the Fermi level ε_F in eV, and the following three column the values of $M_{x,y,z}^{\text{orb}}(\varepsilon_F)$. This file is written if a range of Fermi energies is specified via fermi_energy_min and fermi_energy_max. If a single Fermi energy is given, \mathbf{M}^{orb} is printed in seedname.wpout only.
- shc Spin Hall conductivity (SHC), in (\hbar/e) S/cm. Output files:
 - seedname-shc-fermiscan.dat (data file). The first column is the number of entries in the list, the second column contains the Fermi level ε_F in eV, and the last column contains the values of $\sigma_{\alpha\beta}^{\rm spin\gamma}(\varepsilon_F)$. This file is written if a range of Fermi energies is specified via fermi_energy_min and fermi_energy_max. If a single Fermi energy is given, the file will contain SHC at this specific energy.

· seedname-shc-freqscan.dat (data file). The first column is the number of the entry in the list, the second column contains the frequency $\hbar\omega$ in eV, and the following two columns contain the values of the real part $\Re[\sigma_{\alpha\beta}^{\mathrm{spin}\gamma}(\omega)]$ and imaginary part $\Im[\sigma_{\alpha\beta}^{\mathrm{spin}\gamma}(\omega)]$ of ac SHC. This file is written if a range of frequencies is specified via kubo_freq_min and kubo_freq_max.

There is no default value.

```
11.10.3 integer :: berry_kmesh(:)
```

Overrides the kmesh global variable (see Sec. 11.6).

```
11.10.4 real(kind=dp) :: berry_kmesh_spacing
```

Overrides the kmesh_spacing global variable (see Sec. 11.6).

```
11.10.5 integer :: berry_curv_adpt_kmesh
```

If a positive integer n is given and berry_task=ahc[or berry_task=shc], an $n \times n \times n$ mesh is placed around points on the uniform mesh (defined by either berry_kmesh or berry_kmesh_spacing) where the magnitude of the k-space Berry curvature[k-space Berry curvature-like term of SHC] exceeds the threshold value specified in berry_curv_adpt_kmesh_thresh. This can be used to densify the BZ integration mesh around spikes in the Berry curvature[Berry curvature-like term of SHC].

The default value is 1.

```
11.10.6 real(kind=dp) :: berry_curv_adpt_kmesh_thresh
```

Magnitude of the Berry curvature[Berry curvature-like term of SHC] (in units of berry_curv_unit) that triggers adaptive mesh refinement when berry_task=ahc[berry_task=shc].

The default value is 100.0.

```
11.10.7 real(kind=dp) :: kubo_freq_min
```

Lower limit of the frequency range for computing the optical conductivity, JDOS and ac SHC. Units are eV.

The default value 0.0.

11.10.8 real(kind=dp) :: kubo_freq_max

Upper limit of the frequency range for computing the optical conductivity, JDOS and ac SHC. Units are eV.

If an inner energy window was specified, the default value is dis_froz_max-fermi_energy+0.6667. Otherwise it is the difference between the maximum and the minimum energy eigenvalue stored in seedname.eig, plus 0.6667.

11.10.9 real(kind=dp) :: kubo_freq_step

Difference between consecutive values of the optical frequency between kubo_freq_min and kubo_freq_max. Units are eV.

The default value is 0.01.

11.10.10 real(kind=dp) :: kubo_eigval_max

Maximum energy eigenvalue of the eigenstates to be included in the evaluation of the optical conductivity, JDOS and ac SHC. Units are eV.

If an inner energy window was specified, the default value is the upper bound of the inner energy window plus 0.6667. Otherwise it is the maximum energy eigenvalue stored in seedname.eig plus 0.6667.

11.10.11 logical :: kubo_adpt_smr

Overrides the adpt_smr global variable (see Sec. 11.6).

11.10.12 real(kind=dp) :: kubo_adpt_smr_fac

Overrides the adpt_smr_fac global variable (see Sec. 11.6).

11.10.13 real(kind=dp) :: kubo_adpt_smr_max

Overrides the adpt_smr_max global variable (see Sec. 11.6).

```
11.10.14 logical :: kubo_smr_fixed_en_width
```

Overrides the smr_fixed_en_width global variable (see Sec. 11.6).

11.10.15 character(len=120) :: kubo_smr_type

Overrides the smr_type global variable (see Sec. 11.6).

11.10.16 logical :: shc_freq_scan

Determines whether to calculate the frequency scan (i.e. the ac SHC) or the Fermi energy scan (i.e. the dc SHC) of the spin Hall conductivity.

The default value is false, which means dc SHC is calculated.

11.10.17 character(len=120) :: shc_method

If it is qiao/ryoo, the ac or dc SHC is calculated using Junfeng Qiao's/Jihoon Ryoo's method. To calculate the Kubo formula, the spin current matrix elements are required, and the two methods differ in the degree of approximation. For details, see the section 12.5.

11.10.18 integer :: shc_alpha

The α index of spin Hall conductivity $\sigma_{\alpha\beta}^{\text{spin}\gamma}$, i.e. the direction of spin current. Possible values are 1, 2 and 3, representing the x, y and z directions respectively.

The default value is 1.

11.10.19 integer :: shc_beta

The β index of spin Hall conductivity $\sigma_{\alpha\beta}^{\text{spin}\gamma}$, i.e. the direction of applied electric field. Possible values are 1, 2 and 3, representing the x, y and z directions respectively.

The default value is 2.

11.10.20 integer :: shc_gamma

The γ index of spin Hall conductivity $\sigma_{\alpha\beta}^{\rm spin\gamma}$, i.e. the spin direction of spin current. Possible values are 1, 2 and 3, representing the x, y and z directions respectively.

The default value is 3.

If all the shc_alpha, shc_beta and shc_gamma are set as default values, the $\sigma_{xy}^{\text{spin}z}$ is computed.

11.10.21 logical :: shc_bandshift

Shift all conduction bands by a given amount (defined by shc_bandshift_energyshift).

Note: this flag slightly differs from the global scissors_shift flag: with shc_bandshift, an exact rigid shift is applied after interpolation; scissors_shift applies instead the shift before interpolation. As a consequence, results may slightly differ (and this is why we provide both possibilities). Note also that with scissors_shift you have to provide the number of valence bands num_valence_bands, while with shc_bandshift you should provide the first band to shift shc_bandshift_firstband = num_valence_bands+1.

The default value is false.

11.10.22 integer :: shc_bandshift_firstband

Index of the first conduction band to shift.

That means that all bands with index $i \ge \mathtt{shc_bandshift_firstband}$ will be shifted by $\mathtt{shc_bandshift_energyshift}$, if $\mathtt{shc_bandshift}$ is true.

The units are eV. No default value; if shc_bandshift is true, this flag must be provided.

$11.10.23 \quad \texttt{real(kind=dp)} \; :: \; \; \texttt{shc_bandshift_energyshift}$

Energy shift of the conduction bands.

The units are eV. No default value; if shc_bandshift is true, this flag must be provided.

11.11 Gyrotropic

11.11.1 logical :: gyrotropic

Determines whether to enter the gyrotropic routines.

The default value is false.

11.11.2 character(len=120) :: gyrotropic_task

The quantity to compute when gyrotropic=true

May contain one or more of the following valid options (note that each option starts with a '-'):

- -D0 The Berry-curvature dipole tensor Eq. (13.1) (dimensionless)

 Output file: seedname-gyrotropic-D.dat (see Sec. 11.11.3 for file format description)
- -Dw The finite-frequency Berry-curvature dipole tensor Eq. (13.2) (dimensionless)

 Output file: seedname-gyrotropic-tildeD.dat (see Sec. 11.11.3 for file format description)
- -C The ohmic conductivity tensor Eq. (13.4) (Ampere/cm)
 Output file: seedname-gyrotropic-C.dat (see Sec. 11.11.3 for file format description)
- -K The orbital contribution to the kME tensor Eq. (13.5) (Ampere)
 Output file: seedname-gyrotropic-K_orb.dat (see Sec. 11.11.3 for file format description)
 - -spin: if this task is present, compute also the spin contribution.
 Output file: seedname-gyrotropic-K_spin.dat
- -NOA The orbital contribution to the NOA Eq. (13.5) (Å)
 Output file: seedname-gyrotropic-NOA_orb.dat (see Sec. 11.11.3 for file format description)
 - -spin: if this task is present, compute also the spin contribution.
 Output file: seedname-gyrotropic-NOA_spin.dat
- -dos the density of states Output file: seedname-gyrotropic-DOS.dat. First column energy (eV), second column DOS $(1/(eV \times ^3))$

There is no default value.

11.11.3 output data format

The calculated tensors are written as functions of Fermi level E_F (first column) and frequency ω (second column). If the tensor does not denend on ω , the second column is filled by zeros. Data is grouped in blocks of the same ω separated by two blank lines. In case of natural optical activity the columns 3 to 11 contain the independent components of γ_{abc} (antisymmetric in ab): yzx, zxy, xyz, yzy, yzz, zxz, xyy, yzz and zxx. For tensors C_{ab} , D_{ab} ,

11.11.4 integer :: gyrotropic_kmesh(:)

Overrides the kmesh global variable (see Sec. 11.6).

11.11.5 real(kind=dp) :: gyrotropic_kmesh_spacing

Overrides the kmesh_spacing global variable (see Sec. 11.6).

11.11.6 real(kind=dp) :: gyrotropic_freq_min

Lower limit of the frequency range for computing the optical activity.

Units are eV. The default value 0.0.

11.11.7 real(kind=dp) :: gyrotropic_freq_max

Upper limit of the frequency range for computing the optical activity. Units are eV.

If an inner energy window was specified, the default value is dis_froz_max-fermi_energy+0.6667. Otherwise it is the difference between the maximum and the minimum energy eigenvalue stored in seedname.eig, plus 0.6667.

11.11.8 real(kind=dp) :: gyrotropic_freq_step

Difference between consecutive values of the optical frequency between <code>gyrotropic_freq_min</code> and <code>gyrotropic_freq_max</code>.

Units are eV. The default value is 0.01.

11.11.9 real(kind=dp) :: gyrotropic_eigval_max

Maximum energy eigenvalue of the eigenstates to be included in the evaluation of the Natural optical activity. Units are eV.

If an inner energy window was specified, the default value is the upper bound of the inner energy window plus 0.6667. Otherwise it is the maximum energy eigenvalue stored in seedname.eig plus 0.6667.

11.11.10 logical :: gyrotropic_smr_fixed_en_width

Overrides the smr_fixed_en_width global variable (see Sec. 11.6).

11.11.11 character(len=120) :: gyrotropic_smr_type

Overrides the smr_type global variable (see Sec. 11.6).

11.11.12 character(len=120) :: gyrotropic_degen_thresh

The threshould to eliminate degenerate bands from the calculation in order to avoid divergences.

Units are eV. The dfault value is 0.

```
11.11.13 character(len=120) :: gyrotropic_box_center
```

- three real numbers. Optionally the integration may be restricted to a parallelogram, centered at gyrotropic_box_center and defined by vectors gyrotropic_box_b{1,2,3}

In reduced coordinates. Default value is 0.5 0.5 0.5

```
11.11.14 character(len=120) :: gyrotropic_box_b1
```

- three real numbers. In reduced coordinates. Default value is $1.0\ 0.0\ 0.0$

```
11.11.15 character(len=120) :: gyrotropic_box_b2
```

- three real numbers. In reduced coordinates. Default value is $0.0\ 1.0\ 0.0$

11.11.16 character(len=120) :: gyrotropic_box_b3

- three real numbers. In reduced coordinates. Default value is $0.0\ 0.0\ 1.0$

11.12 BoltzWann

11.12.1 logical :: boltzwann

Determines whether to enter the BoltzWann routines.

The default value is false.

11.12.2 integer :: boltz_kmesh(:)

It determines the interpolation k mesh used to calculate the TDF (from which the transport coefficient are calculated). If boltz_calc_also_dos is true, the same k mesh is used also for the DOS. Overrides the kmesh global variable (see Sec. 11.6).

11.12.3 real(kind=dp) :: boltz_kmesh_spacing

Overrides the kmesh_spacing global variable (see Sec. 11.6).

11.12.4 character(len=4) :: boltz_2d_dir

For two-dimensional systems, the direction along which the system is non-periodic. It can assume the following values: \mathbf{x} for a 2D system on the yz plane, \mathbf{y} for a 2D system on the xz plane, \mathbf{z} for a 2D system on the xy plane, or no for a 3D system with periodicity along all three directions.

This value is used when calculating the Seebeck coefficient, where the electrical conductivity tensor needs to be inverted. If the value is different from zero, only the relevant 2×2 sub-block of the electrical conductivity is inverted.

The default value is no.

11.12.5 real(kind=dp) :: boltz_relax_time

The relaxation time to be used for the calculation of the TDF and the transport coefficients.

The units are fs. The default value is 10 fs.

11.12.6 real(kind=dp) :: boltz_mu_min

Minimum value for the chemical potential μ for which we want to calculate the transport coefficients. The units are eV. No default value.

11.12.7 real(kind=dp) :: boltz_mu_max

Maximum value for the chemical potential μ for which we want to calculate the transport coefficients. The units are eV. No default value.

11.12.8 real(kind=dp) :: boltz_mu_step

Energy step for the grid of chemical potentials μ for which we want to calculate the transport coefficients.

The units are eV. No default value.

11.12.9 real(kind=dp) :: boltz_temp_min

Minimum value for the temperature T for which we want to calculate the transport coefficients.

The units are K. No default value.

11.12.10 real(kind=dp) :: boltz_temp_max

Maximum value for the temperature T for which we want to calculate the transport coefficients.

The units are K. No default value.

11.12.11 real(kind=dp) :: boltz_temp_step

Energy step for the grid of temperatures T for which we want to calculate the transport coefficients.

The units are K. No default value.

11.12.12 real(kind=dp) :: boltz_tdf_energy_step

Energy step for the grid of energies for the TDF.

The units are eV. The default value is 0.001 eV.

11.12.13 character(len=120) :: boltz_tdf_smr_type

The type of smearing function to be used for the TDF. The available strings are the same of the global smr_type input flag.

The default value is the one given via the smr_type input flag (if defined).

11.12.14 real(kind=dp) :: boltz_tdf_smr_fixed_en_width

Energy width for the smearing function. Note that for the TDF, a standard (non-adaptive) smearing scheme is used.

The units are eV. The default value is 0 eV. Note that if the width is smaller than twice the energy step boltz_tdf_energy_step, the TDF will be unsmeared (thus the default is to have an unsmeared TDF).

11.12.15 logical :: boltz_calc_also_dos

Whether to calculate also the DOS while calculating the TDF.

If one needs also the DOS, it is faster to calculate the DOS using this flag instead of using independently the routines of the dos module, since in this way the interpolation on the k points will be performed only once.

The default value is false.

```
11.12.16 real(kind=dp) :: boltz_dos_energy_min
```

The minimum value for the energy grid for the calculation of the DOS.

The units are eV. The default value is minval(eigval)-0.6667, where minval(eigval) is the minimum eigenvalue returned by the ab-initio code on the ab-initio q me sh.

```
11.12.17 real(kind=dp) :: boltz_dos_energy_max
```

The maximum value for the energy grid for the calculation of the DOS.

The units are eV. The default value is maxval(eigval)+0.6667, where maxval(eigval) is the maximum eigenvalue returned by the ab-initio code on the ab-initio q me sh.

```
11.12.18 real(kind=dp) :: boltz_dos_energy_step
```

Energy step for the grid of energies for the DOS.

The units are eV. The default value is 0.001 eV.

```
11.12.19 character(len=120) :: boltz_dos_smr_type
```

Overrides the smr_type global variable (see Sec. 11.6).

```
11.12.20 logical :: boltz_dos_adpt_smr
```

Overrides the adpt_smr global variable (see Sec. 11.6).

```
11.12.21 real(kind=dp) :: boltz_dos_adpt_smr_fac
```

Overrides the adpt_smr_fac global variable (see Sec. 11.6).

```
11.12.22 real(kind=dp) :: boltz_dos_adpt_smr_max
```

Overrides the adpt_smr_max global variable (see Sec. 11.6).

11.12.23 logical :: boltz_dos_smr_fixed_en_width

Overrides the smr_fixed_en_width global variable (see Sec. 11.6).

11.12.24 logical :: boltz_bandshift

Shift all conduction bands by a given amount (defined by boltz_bandshift_energyshift).

Note: this flag slightly differs from the global scissors_shift flag: with boltz_bandshift, an exact rigid shift is applied after interpolation; scissors_shift applies instead the shift before interpolation. As a consequence, results may slightly differ (and this is why we provide both possibilities). Note also that with scissors_shift you have to provide the number of valence bands num_valence_bands, while with boltz_bandshift you should provide the first band to shift boltz_bandshift_firstband = num_valence_bands+1.

The default value is false.

11.12.25 integer :: boltz_bandshift_firstband

Index of the first conduction band to shift.

That means that all bands with index $i \ge \texttt{boltz_bandshift_firstband}$ will be shifted by $\texttt{boltz_bandshift_ener}$ if $\texttt{boltz_bandshift}$ is true.

The units are eV. No default value; if boltz_bandshift is true, this flag must be provided.

11.12.26 real(kind=dp) :: boltz_bandshift_energyshift

Energy shift of the conduction bands.

The units are eV. No default value; if boltz_bandshift is true, this flag must be provided.

11.13 Generic Band Interpolation

11.13.1 logical :: geninterp

Determines whether to enter the Generic Band Interpolation routines.

The default value is false.

11.13.2 logical :: geninterp_alsofirstder

Whether to calculate also the first derivatives of the bands at the given k points.

The default value is false.

11.13.3 logical :: geninterp_single_file

Whether to write a single seedname_geninterp.dat file (all I/O is done by the root node); or instead multiple files (one for each node) with names seedname_geninterp_NNNNN.dat, where NNNNN is the node number. See also the discussion in Sec. 15.1.2 on how to use this flag.

The default value is true.

Chapter 12

Overview of the berry module

The berry module of postw90 is called by setting berry = true and choosing one or more of the available options for berry_task. The routines in the berry module which compute the k-space Berry curvature, orbital magnetization and spin Hall conductivity are also called when kpath = true and kpath_task = {curv,morb,shc}, or when kslice = true and kslice_task = {curv,morb,shc}.

12.1 Background: Berry connection and curvature

The Berry connection is defined in terms of the cell-periodic Bloch states $|u_{n\mathbf{k}}\rangle = e^{-i\mathbf{k}\cdot\mathbf{r}}|\psi_{n\mathbf{k}}\rangle$ as

$$\mathbf{A}_n(\mathbf{k}) = \langle u_{n\mathbf{k}} | i \nabla_{\mathbf{k}} | u_{n\mathbf{k}} \rangle, \tag{12.1}$$

and the Berry curvature is the curl of the connection,

$$\Omega_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}) = -\operatorname{Im}\langle \nabla_{\mathbf{k}} u_{n\mathbf{k}} | \times | \nabla_{\mathbf{k}} u_{n\mathbf{k}} \rangle. \tag{12.2}$$

These two quantities play a central role in the description of several electronic properties of crystals [14]. In the following we will work with a matrix generalization of the Berry connection,

$$\mathbf{A}_{nm}(\mathbf{k}) = \langle u_{n\mathbf{k}} | i \nabla_{\mathbf{k}} | u_{m\mathbf{k}} \rangle = \mathbf{A}_{mn}^{*}(\mathbf{k}), \tag{12.3}$$

and write the curvature as an antisymmetric tensor,

$$\Omega_{n,\alpha\beta}(\mathbf{k}) = \epsilon_{\alpha\beta\gamma}\Omega_{n,\gamma}(\mathbf{k}) = -2\operatorname{Im}\langle\nabla_{k_{\alpha}}u_{n\mathbf{k}}|\nabla_{k_{\beta}}u_{n\mathbf{k}}\rangle. \tag{12.4}$$

12.2 berry_task=kubo: optical conductivity and joint density of states

The Kubo-Greenwood formula for the optical conductivity of a crystal in the independent-particle approximation reads

$$\sigma_{\alpha\beta}(\hbar\omega) = \frac{ie^2\hbar}{N_k\Omega_c} \sum_{\mathbf{k}} \sum_{n,m} \frac{f_{m\mathbf{k}} - f_{n\mathbf{k}}}{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}}} \frac{\langle \psi_{n\mathbf{k}} | v_{\alpha} | \psi_{m\mathbf{k}} \rangle \langle \psi_{m\mathbf{k}} | v_{\beta} | \psi_{n\mathbf{k}} \rangle}{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}} - (\hbar\omega + i\eta)}.$$
 (12.5)

Indices α, β denote Cartesian directions, Ω_c is the cell volume, N_k is the number of k-points used for sampling the Brillouin zone, and $f_{n\mathbf{k}} = f(\varepsilon_{n\mathbf{k}})$ is the Fermi-Dirac distribution function. $\hbar\omega$ is the optical frequency, and $\eta > 0$ is an adjustable smearing parameter with units of energy.

The off-diagonal velocity matrix elements can be expressed in terms of the connection matrix [15],

$$\langle \psi_{n\mathbf{k}} | \mathbf{v} | \psi_{m\mathbf{k}} \rangle = -\frac{i}{\hbar} (\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}}) \mathbf{A}_{nm}(\mathbf{k}) \quad (m \neq n).$$
 (12.6)

The conductivity becomes

$$\sigma_{\alpha\beta}(\hbar\omega) = \frac{1}{N_k} \sum_{\mathbf{k}} \sigma_{\mathbf{k},\alpha\beta}(\hbar\omega) \tag{12.7}$$

$$\sigma_{\mathbf{k},\alpha\beta}(\hbar\omega) = \frac{ie^2}{\hbar\Omega_c} \sum_{n,m} (f_{m\mathbf{k}} - f_{n\mathbf{k}}) \frac{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}}}{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}} - (\hbar\omega + i\eta)} A_{nm,\alpha}(\mathbf{k}) A_{mn,\beta}(\mathbf{k}). \tag{12.8}$$

Let us decompose it into Hermitian (dissipative) and anti-Hermitean (reactive) parts. Note that

$$\overline{\delta}(\varepsilon) = \frac{1}{\pi} \operatorname{Im} \left[\frac{1}{\varepsilon - i\eta} \right], \tag{12.9}$$

where $\bar{\delta}$ denotes a "broadended" delta-function. Using this identity we find for the Hermitean part

$$\sigma_{\mathbf{k},\alpha\beta}^{\mathrm{H}}(\hbar\omega) = -\frac{\pi e^2}{\hbar\Omega_c} \sum_{n,m} (f_{m\mathbf{k}} - f_{n\mathbf{k}})(\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}}) A_{nm,\alpha}(\mathbf{k}) A_{mn,\beta}(\mathbf{k}) \overline{\delta}(\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}} - \hbar\omega). \tag{12.10}$$

Improved numerical accuracy can be achieved by replacing the Lorentzian (12.9) with a Gaussian, or other shapes. The analytical form of $\bar{\delta}(\varepsilon)$ is controlled by the keyword [kubo_]smr_type.

The anti-Hermitean part of Eq. (12.8) is given by

$$\sigma_{\mathbf{k},\alpha\beta}^{\mathrm{AH}}(\hbar\omega) = \frac{ie^2}{\hbar\Omega_c} \sum_{n,m} (f_{m\mathbf{k}} - f_{n\mathbf{k}}) \mathrm{Re} \left[\frac{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}}}{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}} - (\hbar\omega + i\eta)} \right] A_{nm,\alpha}(\mathbf{k}) A_{mn,\beta}(\mathbf{k}). \tag{12.11}$$

Finally the joint density of states is

$$\rho_{cv}(\hbar\omega) = \frac{1}{N_k} \sum_{\mathbf{k}} \sum_{n,m} f_{n\mathbf{k}} (1 - f_{m\mathbf{k}}) \overline{\delta} (\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}} - \hbar\omega). \tag{12.12}$$

Equations (12.9–12.12) contain the parameter η , whose value can be chosen using the keyword [kubo_]smr_fixed_en_width. Better results can often be achieved by adjusting the value of η for each pair of states, i.e., $\eta \to \eta_{nmk}$. This is done as follows (see description of the keyword adpt_smr_fac)

$$\eta_{nm\mathbf{k}} = \alpha |\nabla_{\mathbf{k}}(\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}})|\Delta k.$$
(12.13)

The energy eigenvalues $\varepsilon_{n\mathbf{k}}$, band velocities $\nabla_{\mathbf{k}}\varepsilon_{n\mathbf{k}}$, and off-diagonal Berry connection $\mathbf{A}_{nm}(\mathbf{k})$ entering the previous four equations are evaluated over a k-point grid by Wannier interpolation, as described in Refs. [13, 16]. After averaging over the Brillouin zone, the Hermitean and anti-Hermitean parts of the conductivity are assembled into the symmetric and antisymmetric tensors

$$\sigma_{\alpha\beta}^{S} = \text{Re}\sigma_{\alpha\beta}^{H} + i\text{Im}\sigma_{\alpha\beta}^{AH}$$
(12.14)

$$\sigma_{\alpha\beta}^{A} = \text{Re}\sigma_{\alpha\beta}^{AH} + i\text{Im}\sigma_{\alpha\beta}^{H}, \qquad (12.15)$$

whose independent components are written as a function of frequency onto nine separate files.

12.3 berry_task=ahc: anomalous Hall conductivity

The antisymmetric tensor $\sigma_{\alpha\beta}^{A}$ is odd under time reversal, and therefore vanishes in non-magnetic systems, while in ferromagnets with spin-orbit coupling it is generally nonzero. The imaginary part $\text{Im}\sigma_{\alpha\beta}^{H}$ describes magnetic circular dichroism, and vanishes as $\omega \to 0$. The real part $\text{Re}\sigma_{\alpha\beta}^{AH}$ describes the anomalous Hall conductivity (AHC), and remains finite in the static limit.

The intrinsic dc AHC is obtained by setting $\eta = 0$ and $\omega = 0$ in Eq. (12.11). The contribution from point **k** in the Brillouin zone is

$$\sigma_{\mathbf{k},\alpha\beta}^{\mathrm{AH}}(0) = \frac{2e^2}{\hbar\Omega_c} \sum_{n,m} f_{n\mathbf{k}} (1 - f_{m\mathbf{k}}) \mathrm{Im} \langle \nabla_{k_{\alpha}} u_{n\mathbf{k}} | u_{m\mathbf{k}} \rangle \langle u_{m\mathbf{k}} | \nabla_{k_{\beta}} u_{n\mathbf{k}} \rangle, \tag{12.16}$$

where we replaced $f_{n\mathbf{k}} - f_{m\mathbf{k}}$ with $f_{n\mathbf{k}}(1 - f_{m\mathbf{k}}) - f_{m\mathbf{k}}(1 - f_{n\mathbf{k}})$.

This expression is not the most convenient for *ab initio* calculations, as the sums run over the complete set of occupied and empty states. In practice the sum over empty states can be truncated, but a relatively large number should be retained to obtain accurate results. Using the resolution of the identity $1 = \sum_{m} |u_{m\mathbf{k}}\rangle\langle u_{m\mathbf{k}}|$ and noting that the term $\sum_{n,m} f_{n\mathbf{k}} f_{m\mathbf{k}}(\ldots)$ vanishes identically, we arrive at the celebrated formula for the intrinsic AHC in terms of the Berry curvature,

$$\sigma_{\alpha\beta}^{\text{AH}}(0) = \frac{e^2}{\hbar} \frac{1}{N_k \Omega_c} \sum_{\mathbf{k}} (-1) \Omega_{\alpha\beta}(\mathbf{k}), \qquad (12.17)$$

$$\Omega_{\alpha\beta}(\mathbf{k}) = \sum_{n} f_{n\mathbf{k}} \Omega_{n,\alpha\beta}(\mathbf{k}). \tag{12.18}$$

Note that only *occupied* states enter this expression. Once we have a set of Wannier functions spanning the valence bands (together with a few low-lying conduction bands, typically) Eq. (12.17) can be evaluated by Wannier interpolation as described in Refs. [13, 17], with no truncation involved.

12.4 berry_task=morb: orbital magnetization

The ground-state orbital magnetization of a crystal is given by [14, 18]

$$\mathbf{M}^{\text{orb}} = \frac{e}{\hbar} \frac{1}{N_k \Omega_c} \sum_{\mathbf{k}} \mathbf{M}^{\text{orb}}(\mathbf{k}), \tag{12.19}$$

$$\mathbf{M}^{\mathrm{orb}}(\mathbf{k}) = \sum_{n} \frac{1}{2} f_{n\mathbf{k}} \operatorname{Im} \langle \nabla_{\mathbf{k}} u_{n\mathbf{k}} | \times (H_{\mathbf{k}} + \varepsilon_{n\mathbf{k}} - 2\varepsilon_{F}) | \nabla_{\mathbf{k}} u_{n\mathbf{k}} \rangle, \tag{12.20}$$

where ε_F is the Fermi energy. The Wannier-interpolation calculation is described in Ref. [17]. Note that the definition of $\mathbf{M}^{\text{orb}}(\mathbf{k})$ used here differs by a factor of -1/2 from the one in Eq. (97) and Fig. 2 of that work.

12.5 berry_task=shc: spin Hall conductivity

The Kubo-Greenwood formula for the intrinsic spin Hall conductivity (SHC) of a crystal in the independent-particle approximation reads [19–21]

$$\sigma_{\alpha\beta}^{\text{spin}\gamma}(\omega) = \frac{\hbar}{\Omega_c N_k} \sum_{\mathbf{k}} \sum_{n} f_{n\mathbf{k}} \sum_{m \neq n} \frac{2 \operatorname{Im}[\langle n\mathbf{k} | \hat{j}_{\alpha}^{\gamma} | m\mathbf{k} \rangle \langle m\mathbf{k} | - e\hat{v}_{\beta} | n\mathbf{k} \rangle]}{(\epsilon_{n\mathbf{k}} - \epsilon_{m\mathbf{k}})^2 - (\hbar\omega + i\eta)^2}.$$
 (12.21)

The spin current operator $\hat{j}_{\alpha}^{\gamma} = \frac{1}{2} \{\hat{s}_{\gamma}, \hat{v}_{\alpha}\}$ where the spin operator $\hat{s}_{\gamma} = \frac{\hbar}{2} \hat{\sigma}_{\gamma}$. Indices α, β denote Cartesian directions, γ denotes the direction of spin, commonly $\alpha = x, \beta = y, \gamma = z$. Ω_c is the cell volume, N_k is the number of k-points used for sampling the Brillouin zone, and $f_{n\mathbf{k}} = f(\varepsilon_{n\mathbf{k}})$ is the Fermi-Dirac distribution function. $\hbar\omega$ is the optical frequency, and $\eta > 0$ is an adjustable smearing parameter with unit of energy.

The velocity matrix element in the numerator is the same as Eq. (12.6), so the only unknown quantity is the spin current matrix $\langle n\mathbf{k}|\hat{j}_{\alpha}^{\gamma}|m\mathbf{k}\rangle$. We can use Wannier interpolation technique to efficiently calculate this matrix, and there are two derivation according to the degree of approximation. A noteworthy difference is the way in which two *ab-initio* matrix elements are evaluated,

$$\langle u_{n\mathbf{k}} | \sigma_{\gamma} H_{\mathbf{k}} | u_{m\mathbf{k}+\mathbf{b}} \rangle, \langle u_{n\mathbf{k}} | \sigma_{\gamma} | u_{m\mathbf{k}+\mathbf{b}} \rangle, \gamma = x, y, z$$
 (12.22)

These are evaluated by pw2wannier90 using Ryoo's method. In contrast, Qiao's method does not require pw2wannier90, but it assumes an approximation $1 \approx \sum_{l \in ab-initio \text{ bands}} |u_{lk}\rangle\langle u_{lk}|$. You can choose which method to evaluate this value with shc_method in the input file. For a full derivation please refer to Ref. [19] or Ref. [20].

The Eq. (12.21) can be further separated into band-projected Berry curvature-like term

$$\Omega_{n,\alpha\beta}^{\text{spin}\gamma}(\mathbf{k}) = \hbar^2 \sum_{m \neq n} \frac{-2 \operatorname{Im}[\langle n\mathbf{k} | \frac{1}{2} \{ \hat{\sigma}_{\gamma}, \hat{v}_{\alpha} \} | m\mathbf{k} \rangle \langle m\mathbf{k} | \hat{v}_{\beta} | n\mathbf{k} \rangle]}{(\epsilon_{n\mathbf{k}} - \epsilon_{m\mathbf{k}})^2 - (\hbar\omega + i\eta)^2},$$
(12.23)

k-resolved term which sums over occupied bands

$$\Omega_{\alpha\beta}^{\text{spin}\gamma}(\mathbf{k}) = \sum_{n} f_{n\mathbf{k}} \Omega_{n,\alpha\beta}^{\text{spin}\gamma}(\mathbf{k}), \qquad (12.24)$$

and the SHC is

$$\sigma_{\alpha\beta}^{\text{spin}\gamma}(\omega) = -\frac{e^2}{\hbar} \frac{1}{\Omega_c N_k} \sum_{\mathbf{k}} \Omega_{\alpha\beta}^{\text{spin}\gamma}(\mathbf{k}). \tag{12.25}$$

The unit of the $\Omega_{n,\alpha\beta}^{\mathrm{spin}\gamma}(\boldsymbol{k})$ is length² (Angstrom² or Bohr², depending on your choice of berry_curv_unit in the input file), and the unit of $\sigma_{\alpha\beta}^{\mathrm{spin}\gamma}$ is $(\hbar/e)\mathrm{S/cm}$ (the unit is written in the header of the output file). The case of $\omega=0$ corresponds to direct current (dc) SHC while that of $\omega\neq0$ corresponds to alternating current (ac) SHC or frequency-dependent SHC. Note in some papers Eq. (12.23) is called as spin Berry curvature. However, it was pointed out by Ref. [22] that this name is misleading, so we use a somewhat awkward name "Berry curvature-like term" to refer to Eq. (12.23). The k-resolved term Eq. (12.24) can be used to draw kslice plot, and the band-projected Berry curvature-like term Eq. (12.23) can be used to color the kpath plot.

Same as the case of optical conductivity, the parameter η contained in the Eq. (12.23) can be chosen using the keyword [kubo_]smr_fixed_en_width. Also, adaptive smearing can be employed by the keyword [kubo_]adpt_smr (see Examples 29 and 30 in the Tutorial).

Please cite the following paper [19] or [20] when publishing SHC results obtained using this method:

Junfeng Qiao, Jiaqi Zhou, Zhe Yuan, and Weisheng Zhao, Calculation of intrinsic spin Hall conductivity by Wannier interpolation, Phys. Rev. B. 98, 214402 (2018), DOI:10.1103/PhysRevB.98.214402.

Ji Hoon Ryoo, Cheol-hwan Park, and Ivo Souza,

Computation of intrinsic spin Hall conductivities from first principles using maximally localized Wannier functions,

Phys. Rev. B. 99, 235113 (2019), DOI:10.1103/PhysRevB.99.235113.

12.6 Needed matrix elements

All the quantities entering the formulas for the optical conductivity and AHC can be calculated by Wannier interpolation once the Hamiltonian and position matrix elements $\langle \mathbf{0}n|H|\mathbf{R}m\rangle$ and $\langle \mathbf{0}n|\mathbf{r}|\mathbf{R}m\rangle$ are known [13, 16]. Those matrix elements are readily available at the end of a standard MLWF calculation with wannier90. In particular, $\langle \mathbf{0}n|\mathbf{r}|\mathbf{R}m\rangle$ can be calculated by Fourier transforming the overlap matrices in Eq. (1.7),

$$\langle u_{n\mathbf{k}}|u_{m\mathbf{k}+\mathbf{b}}\rangle$$
.

Further Wannier matrix elements are needed for the orbital magnetization [17]. In order to calculate them using Fourier transforms, one more piece of information must be taken from the k-space ab-initio calculation, namely, the matrices

$$\langle u_{n\mathbf{k}+\mathbf{b}_1}|H_{\mathbf{k}}|u_{m\mathbf{k}+\mathbf{b}_2}\rangle$$

over the *ab-initio* k-point mesh [17]. These are evaluated by pw2wannier90, the interface routine between pwscf and wannier90, by adding to the input file seedname.pw2wan the line

write
$$uHu = .true$$
.

The calculation of spin Hall conductivity needs the spin matrix elements

$$\langle u_{n\mathbf{k}}|\sigma_{\gamma}|u_{m\mathbf{k}}\rangle, \gamma=x,y,z$$

from the ab-initio k-point mesh. These are also evaluated by pw2wannier90 by adding to the input file seedname.pw2wan the line

$$write_spn = .true.$$

If one uses Ryoo's method to calculate spin Hall conductivity, the further matrix elements are needed:

$$\langle u_{n\mathbf{k}}|\sigma_{\gamma}H_{\mathbf{k}}|u_{m\mathbf{k}+\mathbf{b}}\rangle, \langle u_{n\mathbf{k}}|\sigma_{\gamma}|u_{m\mathbf{k}+\mathbf{b}}\rangle, \gamma = x, y, z$$

and these are evaluated by adding to the input file seedname.pw2wan the lines

write
$$sHu = .true$$
.

$$write_sIu = .true.$$

Chapter 13

Overview of the gyrotropic module

The gyrotropic module of postw90 is called by setting gyrotropic = true and choosing one or more of the available options for gyrotropic_task. The module computes the quantities, studied in [23], where more details may be found.

13.1 gyrotropic_task=-d0: the Berry curvature dipole

The traceless dimensionless tensor

$$D_{ab} = \int [d\mathbf{k}] \sum_{n} \frac{\partial E_{n}}{\partial k_{a}} \Omega_{n}^{b} \left(-\frac{\partial f_{0}}{\partial E} \right)_{E=E_{n}}, \tag{13.1}$$

13.2 gyrotropic_task=-dw: the finite-frequency generalization of the Berry curvature dipole

$$\widetilde{D}_{ab}(\omega) = \int [d\mathbf{k}] \sum_{n} \frac{\partial E_{n}}{\partial k_{a}} \widetilde{\Omega}_{n}^{b}(\omega) \left(-\frac{\partial f_{0}}{\partial E} \right)_{E=E_{n}}, \tag{13.2}$$

where $\widetilde{\Omega}_{\boldsymbol{k}n}(\omega)$ is a finite-frequency generalization of the Berry curvature:

$$\widetilde{\Omega}_{kn}(\omega) = -\sum_{m} \frac{\omega_{kmn}^2}{\omega_{kmn}^2 - \omega^2} \operatorname{Im} \left(\mathbf{A}_{knm} \times \mathbf{A}_{kmn} \right)$$
(13.3)

Contrary to the Berry curvature, the divergence of $\tilde{\Omega}_{kn}(\omega)$ is generally nonzero. As a result, $\tilde{D}(\omega)$ can have a nonzero trace at finite frequencies, $\tilde{D}_{\parallel} \neq -2\tilde{D}_{\perp}$ in Te.

13.3 gyrotropic_task=-C: the ohmic conductivity

In the constant relaxation-time approximation the ohmic conductivity is expressed as $\sigma_{ab} = (2\pi e \tau/\hbar)C_{ab}$, with

$$C_{ab} = \frac{e}{h} \int [d\mathbf{k}] \sum_{n} \frac{\partial E_{n}}{\partial k_{a}} \frac{\partial E_{n}}{\partial k_{b}} \left(-\frac{\partial f_{0}}{\partial E} \right)_{E=E_{n}}$$
(13.4)

a positive quantity with units of surface current density (A/cm).

13.4 gyrotropic_task=-K: the kinetic magnetoelectric effect (kME)

A microscopic theory of the intrinsic kME effect in bulk crystals was recently developed [24, 25].

The response is described by

$$K_{ab} = \int [d\mathbf{k}] \sum_{n} \frac{\partial E_{n}}{\partial k_{a}} m_{n}^{b} \left(-\frac{\partial f_{0}}{\partial E} \right)_{E=E_{n}}, \tag{13.5}$$

which has the same form as Eq. (13.1), but with the Berry curvature replaced by the intrinsic magnetic moment m_{kn} of the Bloch electrons, which has the spin and orbital components given by [14]

$$m_{\mathbf{k}n}^{\text{spin}} = -\frac{1}{2} g_s \mu_{\text{B}} \langle \psi_{\mathbf{k}n} | \sigma | \psi_{\mathbf{k}n} \rangle$$
 (13.6)

$$m_{kn}^{\text{orb}} = \frac{e}{2\hbar} \text{Im} \langle \partial_k u_{kn} | \times (H_k - E_{kn}) | \partial_k u_{kn} \rangle,$$
 (13.7)

where $g_s \approx 2$ and we chose e > 0.

13.5 gyrotropic_task=-dos: the density of states

The density of states is calculated with the same width and type of smearing, as the other properties of the gyrotropic module

13.6 gyrotropic_task=-noa: the interband contribution to the natural optical activity

Natural optical rotatory power is given by [26]

$$\rho_0(\omega) = \frac{\omega^2}{2c^2} \operatorname{Re} \gamma_{xyz}(\omega). \tag{13.8}$$

for light propagating ling the main symmetry axis of a crystal z. Here $\gamma_{xyz}(\omega)$ is an anti-symmetric (in xy) tensor with units of length, which has both inter- and intraband contributions.

Following Ref. [27] for the interband contribution we write write, with $\partial_c \equiv \partial/\partial k_c$,

$$\operatorname{Re} \gamma_{abc}^{\text{inter}}(\omega) = \frac{e^2}{\varepsilon_0 \hbar^2} \int [d\mathbf{k}] \sum_{n,l}^{o,e} \left[\frac{1}{\omega_{ln}^2 - \omega^2} \operatorname{Re} \left(A_{ln}^b B_{nl}^{ac} - A_{ln}^a B_{nl}^{bc} \right) - \frac{3\omega_{ln}^2 - \omega^2}{(\omega_{ln}^2 - \omega^2)^2} \partial_c (E_l + E_n) \operatorname{Im} \left(A_{nl}^a A_{ln}^b \right) \right]. \quad (13.9)$$

The summations over n and l span the occupied (o) and empty (e) bands respectively, $\omega_{ln} = (E_l - E_n)/\hbar$, and $\mathbf{A}_{ln}(\mathbf{k})$ is given by (12.3) Finally, the matrix B_{nl}^{ac} has both orbital and spin contributions given by

$$B_{nl}^{ac\,\text{(orb)}} = \langle u_n | (\partial_a H) | \partial_c u_l \rangle - \langle \partial_c u_n | (\partial_a H) | u_l \rangle \tag{13.10}$$

and

$$B_{nl}^{ac \, (\text{spin})} = -\frac{i\hbar^2}{m_e} \epsilon_{abc} \langle u_n | \sigma_b | u_l \rangle. \tag{13.11}$$

The spin matrix elements contribute less than 0.5% of the total ρ_0^{inter} of Te. Expanding $H = \sum_m |u_m\rangle E_m\langle u_m|$ we obtain for the orbital matrix elements

$$B_{nl}^{ac \, (\text{orb})} = -i\partial_a (E_n + E_l) A_{nl}^c \sum_m \left\{ (E_n - E_m) A_{nm}^a A_{ml}^c - (E_l - E_m) A_{nm}^c A_{ml}^a \right\}.$$
 (13.12)

This reduces the calculation of $B^{\text{(orb)}}$ to the evaluation of band gradients and off-diagonal elements of the Berry connection matrix. Both operations can be carried out efficiently in a Wannier-function basis following Ref. [16].

13.7 gyrotropic_task=-spin: compute also the spin component of NOA and KME

Unless this task is specified, only the orbital contributions are calcuated in NOA and KME, thus contributions from Eqs. (13.6) and (13.11) are omitted.

Chapter 14

Electronic transport calculations with the BoltzWann module

By setting boltzwann = TRUE, postw90 will call the Boltzwann routines to calculate some transport coefficients using the Boltzmann transport equation in the relaxation time approximation.

In particular, the transport coefficients that are calculated are: the electrical conductivity σ , the Seebeck coefficient **S** and the coefficient **K** (defined below; it is the main ingredient of the thermal conductivity).

The list of parameters of the BoltzWann module are summarized in Table 11.7. An example of a Boltzmann transport calculation can be found in the wannier90 Tutorial.

Note: By default, the code assumes to be working with a 3D bulk material, with periodicity along all three spatial directions. If you are interested in studying 2D systems, set the correct value for the boltz_2d_dir variable (see Sec. 11.12.4 for the documentation). This is important for the evaluation of the Seebeck coefficient.

Please cite the following paper [28] when publishing results obtained using the BoltzWann module:

G. Pizzi, D. Volja, B. Kozinsky, M. Fornari, and N. Marzari,

BoltzWann: A code for the evaluation of thermoelectric and electronic transport properties with a maximally-localized Wannier functions basis,

Comp. Phys. Comm. 185, 422 (2014), DOI:10.1016/j.cpc.2013.09.015.

14.1 Theory

The theory of the electronic transport using the Boltzmann transport equations can be found for instance in Refs. [29–31]. Here we briefly summarize only the main results.

The current density \mathbf{J} and the heat current (or energy flux density) \mathbf{J}_Q can be written, respectively, as

$$\mathbf{J} = \boldsymbol{\sigma}(\mathbf{E} - \mathbf{S}\boldsymbol{\nabla}T) \tag{14.1}$$

$$\mathbf{J}_O = T\boldsymbol{\sigma}\mathbf{S}\mathbf{E} - \mathbf{K}\boldsymbol{\nabla}T,\tag{14.2}$$

where the electrical conductivity σ , the Seebeck coefficient S and K are 3×3 tensors, in general.

Note: the thermal conductivity κ (actually, the electronic part of the thermal conductivity), which is defined as the heat current per unit of temperature gradient in open-circuit experiments (i.e., with $\mathbf{J} = 0$) is not precisely \mathbf{K} , but $\kappa = \mathbf{K} - \mathbf{S}\boldsymbol{\sigma}\mathbf{S}T$ (see for instance Eq. (7.89) of Ref. [29] or Eq. (XI-57b) of Ref. [30]). The thermal conductivity κ can be then calculated from the $\boldsymbol{\sigma}$, \mathbf{S} and \mathbf{K} tensors output by the code.

These quantities depend on the value of the chemical potential μ and on the temperature T, and can be calculated as follows:

$$[\boldsymbol{\sigma}]_{ij}(\mu, T) = e^2 \int_{-\infty}^{+\infty} d\varepsilon \left(-\frac{\partial f(\varepsilon, \mu, T)}{\partial \varepsilon} \right) \Sigma_{ij}(\varepsilon), \tag{14.3}$$

$$[\boldsymbol{\sigma}\mathbf{S}]_{ij}(\mu, T) = \frac{e}{T} \int_{-\infty}^{+\infty} d\varepsilon \left(-\frac{\partial f(\varepsilon, \mu, T)}{\partial \varepsilon} \right) (\varepsilon - \mu) \Sigma_{ij}(\varepsilon), \tag{14.4}$$

$$[\mathbf{K}]_{ij}(\mu, T) = \frac{1}{T} \int_{-\infty}^{+\infty} d\varepsilon \left(-\frac{\partial f(\varepsilon, \mu, T)}{\partial \varepsilon} \right) (\varepsilon - \mu)^2 \Sigma_{ij}(\varepsilon), \tag{14.5}$$

where $[\boldsymbol{\sigma}\mathbf{S}]$ denotes the product of the two tensors $\boldsymbol{\sigma}$ and \mathbf{S} , $f(\varepsilon, \mu, T)$ is the usual Fermi–Dirac distribution function

$$f(\varepsilon, \mu, T) = \frac{1}{e^{(\varepsilon - \mu)/K_B T} + 1}$$

and $\Sigma_{ij}(\varepsilon)$ is the Transport Distribution Function (TDF) tensor, defined as

$$\Sigma_{ij}(\varepsilon) = \frac{1}{V} \sum_{n,\mathbf{k}} v_i(n,\mathbf{k}) v_j(n,\mathbf{k}) \tau(n,\mathbf{k}) \delta(\varepsilon - E_{n,k}).$$

In the above formula, the sum is over all bands n and all states \mathbf{k} (including spin, even if the spin index is not explicitly written here). $E_{n,\mathbf{k}}$ is the energy of the n-th band at \mathbf{k} , $v_i(n,\mathbf{k})$ is the i-th component of the band velocity at (n,\mathbf{k}) , δ is the Dirac's delta function, $V=N_k\Omega_c$ is the total volume of the system (N_k and Ω_c being the number of k-points used to sample the Brillouin zone and the unit cell volume, respectively), and finally τ is the relaxation time. In the relaxation-time approximation adopted here, τ is assumed as a constant, i.e., it is independent of n and \mathbf{k} and its value (in fs) is read from the input variable boltz_relax_time.

14.2 Files

14.2.1 seedname_boltzdos.dat

OUTPUT. Written by postw90 if boltz_calc_also_dos is true. Note that even if there are other general routines in postw90 which specifically calculate the DOS, it may be convenient to use the routines in BoltzWann setting boltz_calc_also_dos = true if one must also calculate the transport coefficients. In this way, the (time-demanding) band interpolation on the k mesh is performed only once, resulting in a much shorter execution time.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each energy ε on the grid, containing a number of columns. The first column is the energy ε . The following is the DOS at the given energy ε . The DOS can either be calculated using the adaptive smearing scheme¹ if boltz_dos_adpt_smr is true, or using a "standard" fixed smearing, whose

¹Note that in BoltzWann the adaptive (energy) smearing scheme also implements a simple adaptive k-mesh scheme: if at any given k point one of the band gradients is zero, then that k point is replaced by 8 neighboring k points. Thus, the final results for the DOS may be slightly different with respect to that given by the dos module.

type and value are defined by boltz_dos_smr_type and boltz_dos_smr_fixed_en_width, respectively. If spin decomposition is required (input flag spin_decomp), further columns are printed, with the spin-up projection of the DOS, followed by spin-down projection.

14.2.2 seedname_tdf.dat

OUTPUT. This file contains the Transport Distribution Function (TDF) tensor Σ on a grid of energies.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each energy ε on the grid, containing a number of columns. The first is the energy ε , the followings are the components if $\Sigma(\varepsilon)$ in the following order: Σ_{xx} , Σ_{xy} , Σ_{yy} , Σ_{xz} , Σ_{yz} , Σ_{zz} . If spin decomposition is required (input flag spin_decomp), 12 further columns are provided, with the 6 components of Σ for the spin up, followed by those for the spin down.

The energy ε is in eV, while Σ is in $\frac{1}{\hbar^2} \cdot \frac{eV \cdot fs}{\mathring{A}}$.

14.2.3 seedname_elcond.dat

OUTPUT. This file contains the electrical conductivity tensor σ on the grid of T and μ points.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each (μ, T) pair, containing 8 columns, which are respectively: μ , T, σ_{xx} , σ_{xy} , σ_{yy} , σ_{xz} , σ_{yz} , σ_{zz} . (The tensor is symmetric).

The chemical potential is in eV, the temperature is in K, and the components of the electrical conductivity tensor ar in SI units, i.e. in $1/\Omega/m$.

14.2.4 seedname_sigmas.dat

OUTPUT. This file contains the tensor σS , i.e. the product of the electrical conductivity tensor and of the Seebeck coefficient as defined by Eq. (14.4), on the grid of T and μ points.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each (μ, T) pair, containing 8 columns, which are respectively: μ , T, $(\sigma S)_{xx}$, $(\sigma S)_{xy}$, $(\sigma S)_{yy}$, $(\sigma S)_{xz}$, $(\sigma S)_{yz}$, $(\sigma S)_{zz}$. (The tensor is symmetric).

The chemical potential is in eV, the temperature is in K, and the components of the tensor ar in SI units, i.e. in A/m/K.

14.2.5 seedname_seebeck.dat

OUTPUT. This file contains the Seebeck tensor **S** on the grid of T and μ points.

Note that in the code the Seebeck coefficient is defined as zero when the determinant of the electrical conductivity σ is zero. If there is at least one (μ, T) pair for which det $\sigma = 0$, a warning is issued on the output file.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each (μ, T) pair, containing 11 columns, which are respectively: μ , T, S_{xx} , S_{xy} , S_{xz} , S_{yx} , S_{yy} , S_{yz} , S_{zx} , S_{zx} , S_{zz} , S_{zz} .

NOTE: therefore, the format of the columns of this file is different from the other three files (elcond, sigmas and kappa)!

The chemical potential is in eV, the temperature is in K, and the components of the Seebeck tensor ar in SI units, i.e. in V/K.

14.2.6 seedname_kappa.dat

OUTPUT. This file contains the tensor **K** defined in Sec. 14.1 on the grid of T and μ points.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each (μ, T) pair, containing 8 columns, which are respectively: μ , T, K_{xx} , K_{yy} , K_{xz} , K_{yz} , K_{zz} . (The tensor is symmetric).

The chemical potential is in eV, the temperature is in K, and the components of the **K** tensor are the SI units for the thermal conductivity, i.e. in W/m/K.

Chapter 15

Generic Band interpolation

By setting geninterp = TRUE, postw90 will calculate the band energies (and possibly the band derivatives, if also geninterp_alsofirstder is set to TRUE) on a generic list of k points provided by the user.

The list of parameters of the Generic Band Interpolation module are summarized in Table 11.8. The list of input k points for which the band have to be calculated is read from the file named seedname_geninterp.kpt. The format of this file is described below.

15.1 Files

15.1.1 seedname_geninterp.kpt

INPUT. Read by postw90 if geninterp is true.

The first line is a comment (its maximum allowed length is 500 characters).

The second line must contain crystal (or frac) if the k-point coordinates are given in crystallographic units, i.e., in fractional units with respect to the primitive reciprocal lattice vectors. Otherwise, it must contain cart (or abs) if instead the k-point coordinates are given in absolute coordinates (in units of 1/Å) along the k_x , k_y and k_z axes.

Note on units: In the case of absolute coordinates, if a_{lat} is the lattice constant expressed in angstrom, and you want to represent for instance the point $X = \frac{2\pi}{a_{lat}}[0.5, 0, 0]$, then you have to input for its x coordinate $k_x = 0.5 * 2 * \pi/a_{lat}$. As a practical example, if $a_{lat} = 4\text{Å}$, then $k_x = 0.78539816339745$ in absolute coordinates in units of 1/Å.

The third line must contain the number n of following k points.

The following n lines must contain the list of k points in the format

kpointidx k1 k2 k3

where $\mathtt{kpointidx}$ is an integer identifying the given k point, and $\mathtt{k1}$, $\mathtt{k2}$ and $\mathtt{k3}$ are the three coordinates of the k points in the chosen units.

15.1.2 seedname_geninterp.dat or seedname_geninterp_NNNNN.dat

OUTPUT. This file/these files contain the interpolated band energies (and also the band velocities if the input flag geninterp_alsofirstder is true).

If the flag geninterp_single_file is true, then a single file seedname_geninterp.dat is written by the code at the end of the calculation. If instead one sets geninterp_single_file to false, each process writes its own output file, named seedname_geninterp_00000.dat, seedname_geninterp_00001.dat,

This flag is useful when one wants to parallelize the calculation on many nodes, and it should be used especially for systems with a small number of Wannier functions, when one wants to compute the bands on a large number of k points (if the flag geninterp_single_file is true, instead, all the I/O is made by the root node, which is a significant bottleneck).

Important! The files are not deleted before the start of a calculation, but only the relevant files are overwritten. Therefore, if one first performs a calculation and then a second one with a smaller number of processors, care is needed to avoid to mix the results of the older calculations with those of the new one. In case of doubt, either check the date stamp in the first line of the seedname_geninterp_*.dat files, or simply delete the seedname_geninterp_*.dat files before starting the new calculation.

To join the files, on can simply use the following command:

```
cat seedname_geninterp_*.dat > seedname_geninterp.dat
```

or, if one wants to remove the comment lines:

```
rm seedname_geninterp.dat
for i in seedname_geninterp_*.dat ; do grep -v \# "$i" >> \
seedname_geninterp.dat ; done
```

The first few lines of each files are comments (starting with #), containing a datestamp, the comment line as it is read from the input file, and a header. The following lines contain the band energies (and derivatives) for each band and k point (the energy index runs faster than the k-point index). For each of these lines, the first four columns contain the k-point index as provided in the input, and the k coordinates (always in absolute coordinates, in units of 1/Å). The fifth column contains the band energy.

If geninterp_alsofirstder is true, three further columns are printed, containing the three first derivatives of the bands along the k_x , k_y and k_z directions (in units of eV·Å).

The k point coordinates are in units of 1/Å, the band energy is in eV.

Part IV

Appendices

Appendix A

Utilities

The wannier90 code is shipped with a few utility programs that may be useful in some occasions. In this chapter, we describe their use.

A.1 kmesh.pl

The wannier90 code requires the definition of a full Monkhorst-Pack grid of k points. In the input file the size of this mesh is given by means of the mp_grid variable. E.g., setting

 $mp_grid = 4 4 4$

tells wannier90 that we want to use a $4 \times 4 \times 4$ grid.

One has then to specify (inside the kpoints block in the the seedname.win file) the list of k points of the grid. Here, the kmesh.pl Perl script becomes useful, being able to generate the required list.

The script can be be found in the utility directory of the wannier90 distribution. To use it, simply type:

./kmesh.pl nx ny nz

where nx, ny and nz define the size of the Monkhorst-Pack grid that we want to use (for instance, in the above example of the $4 \times 4 \times 4$ grid, nx=ny=nz=4).

This produces on output the list of k points in Quantum Espresso format, where (apart from a header) the first three columns of each line are the k coordinates, and the fourth column is the weight of each k point. This list can be used to create the input file for the ab-initio \mathtt{nscf} calculation.

If one wants instead to generate the list of the k coordinates without the weight (in order to copy and paste the output inside the **seedname.win** file), one simply has to provide a fourth argument on the command line. For instance, for a $4 \times 4 \times 4$ k grid, use

./kmesh.pl 4 4 4 wannier

and then copy the output inside the in the kpoints block in the seedname.win file.

We suggest to always use this utility to generate the k grids. This allows to provide the k point coordinates with the accuracy required by wannier90, and moreover it makes sure that the k grid used in the ab-initio code and in wannier90 are the same.

A.2 w90chk2chk.x

During the calculation of the Wannier functions, wannier90 produces a .chk file that contains some information to restart the calculation.

This file is also required by the postw90 code. In particular, the postw90 code requires at least the .chk file, the .win input file, and (almost always) the .eig file. Specific modules may require further files: see the documentation of each module.

However, the .chk file is written in a machine-dependent format. If one wants to run wannier90 on a machine, and then continue the calculation with postw90 on a different machine (or with postw90 compiled with a different compiler), the file has to be converted first in a machine-independent "formatted" format on the first machine, and then converted back on the second machine.

To this aim, use the w90chk2chk.x executable. Note that this executable is not compiled by default: you can obtain it by executing

make w90chk2chk

in the main wannier90 directory.

A typical use is the following:

- 1. Calculate the Wannier functions with wannier90
- 2. At the end of the calculation you will find a seedname.chk file. Run (in the folder with this file) the command

```
w90chk2chk.x -export seedname
```

or equivalently

w90chk2chk.x -u2f seedname

(replacing seedname with the seedname of your calculation).

This command reads the seedname.chk file and creates a formatted file seedname.chk.fmt that is safe to be transferred between different machines.

- 3. Copy the seedname.chk.fmt file (together with the seedname.win and seedname.eig files) on the machine on which you want to run postw90.
- 4. On this second machine (after having compiled w90chk2chk.x) run

```
w90chk2chk.x -import seedname
```

or equivalently

w90chk2chk.x -f2u seedname

This command reads the seedname.chk.fmt file and creates an unformatted file seedname.chk ready to be used by postw90.

5. Run the postw90 code.

A.3 PL_assessment

The function of this utility is to assess the length of a principal layer (in the context of a Landauer-Buttiker quantum conductance calculation) of a periodic system using a calculation on a single unit cell with a dense k-point mesh.

The utility requires the real-space Hamiltonian in the MLWF basis, seedname_hr.dat.

The seedname_hr.dat file should be copied to a directory containing executable for the utility. Within that directory, run:

\\$> ./PL_assess.x nk1 nk2 nk3 num_wann

where:

nk1 is the number of k-points in x-direction nk2 is the number of k-points in y-direction nk3 is the number of k-points in z-direction num_wann is the number of wannier functions of your system

e.g.,

\\$> ./PL_assess.x 1 1 20 16

Note that the current implementation only allows for a single k-point in the direction transverse to the transport direction.

When prompted, enter the seedname.

The programme will return an output file seedname_pl.dat, containing four columns

- 1. Unit cell number, R
- 2. Average 'on-site' matrix element between MLWFs in the home unit cell, and the unit cell R lattice vectors away
- 3. Standard devaition of the quantity in (2)
- 4. Maximum absolute value in (2)

A.4 w90vdw

This utility provides an implementation of a method for calculating van der Waals energies based on the idea of density decomposition via MLWFs.

For theoretical details, please see the following publication and references therein:

Lampros Andrinopoulos, Nicholas D. M. Hine and Arash A. Mostofi, "Calculating dispersion interactions using maximally localized Wannier functions", *J. Chem. Phys.* **135**, 154105 (2011).

For further details of this program, please see the documentation in utility/w90vdw/doc/ and the related examples in utility/w90vdw/examples/.

A.5 w90pov

An utility to create Pov files (to render the Wannier functions using the PovRay utility) is provided inside utility/w90pov.

Please refer to the documentation inside utility/w90pov/doc for more information.

A.6 k mapper.py

The wannier90 code requires the definition of a full Monkhorst–Pack grid of k-vectors, which can be obtained by means of the kmesh.pl utility. In order to perform a GW calculation with the Yambo code, you need to perform a nscf calculation on a grid in the irreducible BZ. Moreover, you may need a finer grid to converge the GW calculation than what you need to interpolate the band structure. The k_mapper.py tools helps in finding the k-vectors indexes of a full grid needed for interpolation into the reduced grid needed for the GW calculation with Yambo.

Usage:

```
path/k_mapper.py nx ny nz QE_nscf_output
```

where path is the path of utility folder, QE_nscf_output is the path of the QE nscf output file given to Yambo.

A.7 gw2wannier90.py

This utility allows to sort in energy the input data of wannier90 (e.g. overlap matrices and energy eigenvalues). gw2wannier90.py allows to use wannier90 at the G_0W_0 level, where quasi-particle corrections can change the energy ordering of eigenvalues (Some wannier90 modules require states to be ordered in energy).

Usage:

path/gw2wannier90.py seedname options

where path is the path of utility folder.

Available options are:

```
mmn, amn, spn, unk, uhu, uiu,
spn_formatted, unk_formatted, uhu_formatted, uiu_formatted,
write_formatted
```

If no options are specified, all the files (mmn, amn, spn, UNK, uHu, uIu) are considered.

Binary (unformatted Fortran) files are supported, though not reccommended, since they are compiler-dependent. A safer choice is to use (bigger) formatted files, with options:

```
spn_formatted, uiu_formatted, uhu_formatted, unk_formatted
```

In default, the output format is the same as the input format. To generate formatted files with unformatted input, use option: write_formatted

A.8 w90spn2spn.x

The interface between ab-initio code and wannier90 (e.g. pw2wannier90.x) can produce a .spn file that is used by postw90 to calculate some spin related quantities.

The .spn file can be written in a machine-dependent or a machine-independent format depending on the input parameter spn_formatted (the default is false which means the .spn file is machine-dependent) of the pw2wannier90.x. If a .spn file has been generated on a machine with machine-dependent format, and then one wants to continue the calculation with postw90 on a different machine (or with postw90 compiled with a different compiler), the file has to be converted first in a machine-independent "formatted" format on the first machine.

To this aim, use the w90spn2spn.x executable. Note that this executable is not compiled by default: you can obtain it by executing

```
make w90spn2spn
```

in the main wannier90 directory.

A typical use is the following:

- 1. Calculate the .spn file, e.g. by pw2wannier90.x
- 2. At the end of the calculation you will find a seedname.spn file. If the file is "unformatted", run (in the folder with this file) the command

```
w90spn2spn.x -export seedname
```

or equivalently

 $\verb|w90spn2spn.x -u2f| seedname|$

(replacing seedname with the seedname of your calculation).

This command reads the seedname.spn file and creates a formatted file seedname.spn.fmt that is safe to be transferred between different machines.

3. Copy the seedname.spn.fmt file on the machine on which you want to run postw90.

4. On this second machine (after having compiled w90spn2spn.x) run

```
w90spn2spn.x -import seedname
or equivalently
w90spn2spn.x -f2u seedname
```

This command reads the seedname.spn.fmt file and creates an unformatted file seedname.spn ready to be used by postw90.

5. Run the postw90 code.

Note if spn_formatted is set to true in both pw2wannier90.x and postw90 input files, then the .spn file will be written and read as "formatted", so w90spn2spn.x is not needed. However, if an "unformatted" seedname.spn has been created and you do not want to rerun pw2wannier90.x, then w90spn2spn.x can be useful. Also, once a "formatted" seedname.spn has been generated, the step 4 can be skipped if spn_formatted is set to true in postw90 input file seedname.win.

Appendix B

Frequently Asked Questions

B.1 General Questions

B.1.1 What is wannier 90?

wannier90 is a computer package, written in Fortran90, for obtaining maximally-localised Wannier functions, using them to calculate bandstructures, Fermi surfaces, dielectric properties, sparse Hamiltonians and many things besides.

B.1.2 Where can I get wannier90?

The most recent release of wannier90 is always available on our website http://www.wannier.org.

B.1.3 Where can I get the most recent information about wannier90?

The latest news about wannier90 can be followed on our website http://www.wannier.org.

B.1.4 Is wannier90 free?

Yes! wannier90 is available for use free-of-charge under the GNU General Public Licence. See the file LICENSE that comes with the wannier90 distribution or the GNU hopepage at http://www.gnu.org.

B.2 Getting Help

B.2.1 Is there a Tutorial available for wannier90?

Yes! The examples directory of the wannier90 distribution contains input files for a number of tutorial calculations. The doc directory contains the accompanying tutorial handout.

B.2.2 Where do I get support for wannier90?

There are a number of options:

1. The wannier90 User Guide, available in the doc directory of the distribution, and from the webpage (http://www.wannier.org/user_guide.html)

- 2. The wannier90 webpage for the most recent announcements (http://www.wannier.org)
- 3. The wannier90 mailing list (see http://www.wannier.org/forum.html)

B.2.3 Is there a mailing list for wannier90?

Yes! You need to register: go to http://www.wannier.org/forum.html and follow the instructions.

B.3 Providing Help: Finding and Reporting Bugs

B.3.1 I think I found a bug. How do I report it?

- Check and double-check. Make sure it's a bug.
- Check that it is a bug in wannier90 and not a bug in the software interfaced to wannier90.
- Check that you're using the latest version of wannier90.
- Send us an email. Make sure to describe the problem and to attach all input and output files relating to the problem that you have found.

B.3.2 I have got an idea! How do I report a wish?

We're always happy to listen to suggestions. Email your idea to the wannier90 developers.

B.3.3 I want to help! How can I contribute to wannier90?

Great! There's always plenty of functionality to add. Email us to let us know about the functionality you'd like to contribute.

B.3.4 I like wannier90! Should I donate anything to its authors?

Our Swiss bank account number is... just kidding! There is no need to donate anything, please just cite our paper in any publications that arise from your use of wannier90:

[ref] G. Pizzi, V. Vitale, R. Arita, S. Blügel, F. Freimuth, G. Géranton, M. Gibertini, D. Gresch, C. Johnson, T. Koretsune, J. Ibañez-Azpiroz, H. Lee, J.M. Lihm, D. Marchand, A. Marrazzo, Y. Mokrousov, J.I. Mustafa, Y. Nohara, Y. Nomura, L. Paulatto, S. Poncé, T. Ponweiser, J. Qiao, F. Thöle, S.S. Tsirkin, M. Wierzbowska, N. Marzari, D. Vanderbilt, I.

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Souza, A.A. Mostofi, J.R. Yates,
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Wannier90 as a community code: new features and applications, J. Phys. Cond. Matt. 32, 165902 (2020)

https://doi.org/10.1088/1361-648X/ab51ff

If you are using versions 2.x of the code, cite instead:

[ref] A. A. Mostofi, J. R. Yates, G. Pizzi, Y.-S. Lee, I. Souza, D. Vanderbilt and N. Marzari, An updated version of wannier90: A Tool for Obtaining Maximally-Localised Wannier Functions, *Comput. Phys. Commun.* **185**, 2309 (2014) http://doi.org/10.1016/j.cpc.2014.05.003

B.4 Installation

B.4.1 How do I install wannier 90?

Follow the instructions in the file README.install in the main directory of the wannier90 distribution.

B.4.2 Are there wannier90 binaries available?

Not at present.

B.4.3 Is there anything else I need?

Yes. wannier90 works on top of an electronic structure calculation.

At the time of writing there are public, fully functioning, interfaces between wannier90 and PWSCF, ABINIT (http://www.abinit.org), SIESTA (http://www.icmab.es/siesta/), VASP (https://www.vasp.at), WIEN2K (http://www.wien2k.at), FLEUR (http://www.fleur.de), OPENMX (http://www.openmx-square.org/), GPAW (https://wiki.fysik.dtu.dk/gpaw/).

To use wannier90 in combination with PWSCF code (a plane-wave, pseudopotential, density-functional theory code, which is part of the quantum-espresso package) you will need to download PWSCF from the webpage http://www.quantum-espresso.org. Then compile PWSCF and the wannier90 interface program pw2wannier90. For instructions, please refer to the documentation that comes with the quantum-espresso distribution.

For examples of how to use PWSCF and wannier90 in conjunction with each other, see the wannier90 Tutorial.

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