Aljabar Boolean Laws

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FTI - ITS

NOT, AND and OR Laws

NOT Laws

X	$\bar{X}$
0	1
1	0

$$\bar{0} = 1$$
$$\bar{1} = 0$$
$$\bar{\bar{X}} = X$$

Involution

AND Laws

X	Y	$X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

$$X \cdot 0 = 0$$
$$X \cdot 1 = X$$
$$X \cdot X = X$$
$$X \cdot \bar{X} = 0$$

OR Laws

X	Y	$X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

$$X + 0 = X$$
$$X + 1 = 1$$
$$X + X = X$$
$$X + \bar{X} = 1$$

XOR and EQV Laws

XOR Laws

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

$$X \oplus 0 = X$$
$$X \oplus 1 = \bar{X}$$
$$X \oplus X = 0$$
$$X \oplus \bar{X} = 1$$

EQV Laws

X	Y	$X \odot Y$
0	0	1
0	1	0
1	0	0
1	1	1

$$X \odot 0 = \bar{X}$$
$$X \odot 1 = X$$
$$X \odot X = 1$$
$$X \odot \bar{X} = 0$$

DeMorgan Laws

$$\overline{X \cdot Y} = \bar{X} + \bar{Y}$$

$$\overline{X + Y} = \bar{X} \cdot \bar{Y}$$

$$\overline{X \odot Y} = \bar{X} \oplus \bar{Y} = X \oplus Y$$

$$\overline{X \oplus Y} = \bar{X} \odot \bar{Y} = X \odot Y$$

$$\bar{X} \oplus Y = X \oplus \bar{Y} = \overline{X \oplus Y} = X \odot Y$$

$$\bar{X} \odot Y = X \odot \bar{Y} = \overline{X \odot Y} = X \oplus Y$$

Associative Laws

$$(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z) = X \cdot Y \cdot Z$$

$$(X + Y) + Z = X + (Y + Z) = X + Y + Z$$

$$(X \odot Y) \odot Z = X \odot (Y \odot Z) = X \odot Y \odot Z$$

$$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$$

Commutative Laws

$$X \cdot Y \cdot Z = X \cdot Z \cdot Y = Y \cdot X \cdot Z = \cdots$$

$$X + Y + Z = X + Z + Y = Y + X + Z = \cdots$$

$$X \odot Y \odot Z = X \odot Z \odot Y = Y \odot X \odot Z$$

$$X \oplus Y \oplus Z = X \oplus Z \oplus Y = Y \oplus X \oplus Z$$

Factoring Laws

$$(X \cdot Y) + (X \cdot Z) = X \cdot (Y + Z)$$

$$(X \cdot Y) \oplus (X \cdot Z) = X \cdot (Y \oplus Z)$$

Absorptive Laws

$$\begin{aligned}[X \cdot (\bar{X} + Y)] &= (X \cdot \bar{X}) + (X \cdot Y) \\ &= 0 + (X \cdot Y) \\ &= X.Y\end{aligned}$$

$$[X \cdot (\bar{X} + Y)] = (X \cdot Y)$$

$$\begin{aligned}X + (\bar{X} \cdot Y) &= \overline{\overline{X + (\bar{X} \cdot Y)}} \\ &= \overline{\bar{X} \cdot \overline{(\bar{X} \cdot Y)}} \\ &= \overline{\bar{X} \cdot (\bar{\bar{X}} + \bar{Y})} \\ &= \overline{(\bar{X} \cdot X) + (\bar{X} \cdot \bar{Y})} \\ &= \overline{0 + (\bar{X} \cdot \bar{Y})} \\ &= \overline{\bar{X} \cdot \bar{Y}} \\ &= X + Y\end{aligned}$$

$$X + (\bar{X} \cdot Y) = X + Y$$

Absorptive Laws

$$
\begin{aligned}
[X \cdot (\bar{X} \oplus Y)] &= (X \cdot \bar{X}) \oplus (X \cdot Y) \\
&= 0 \oplus (X \cdot Y) \\
&= X.Y
\end{aligned}
$$

$$\boxed{[X \cdot (\bar{X} \oplus Y)] = (X \cdot Y)}$$

$$
\begin{aligned}
X + (\bar{X} \odot Y) &= \overline{\overline{X + (\bar{X} \odot Y)}} \\
&= \overline{\bar{X} \cdot \overline{\overline{(\bar{X} \odot Y)}}} \\
&= \overline{\bar{X} \ \cdot (\bar{\bar{X}} \oplus \bar{Y})} \\
&= \overline{(\bar{X} \cdot X) \oplus (\bar{X} \cdot \bar{Y})} \\
&= \overline{0 \oplus (\bar{X} \cdot \bar{Y})} \\
&= \overline{\bar{X} \cdot \bar{Y}} \\
&= X + Y
\end{aligned}
$$

$$\boxed{X + (\bar{X} \odot Y) = X + Y}$$

Distributive Laws

$$(X+Y)\cdot(X+Z) = X\cdot X + X\cdot Z + Y\cdot X + Y\cdot Z$$

$$= [X + (X\cdot Z) + (X\cdot Y)] + (Y\cdot Z)$$

$$= [X\cdot(1+Y+Z)] + (Y\cdot Z)$$

$$= (X\cdot 1) + (Y\cdot Z)$$

$$= X + (Y\cdot Z)$$

$$\boxed{(X+Y)\cdot(X+Z) = X + (Y\cdot Z)}$$

Distributive Laws

$$(X + Y) \odot (X + Z) = X + (Y \odot Z)$$

X	Y	Z	$X + Y$	$X + Z$	$Y \odot Z$	$(X + Y) \odot (X + Z)$	$X + (Y \odot Z)$
0	0	0	0	0	1	1	1
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1
1	0	1	1	1	0	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

Corollary

If two function α and β, never take the logic '1' simultaneously, then

The logic Operator (+) and ($\oplus$) are interchangeable

$$\propto \cdot \beta = 0 \qquad \therefore \propto + \beta = \propto \oplus \beta$$

If two function α and β, never take the logic '0' simultaneously, then

$$\propto + \beta = 1 \qquad \therefore \propto \cdot \beta = \propto \odot \beta$$

The logic Operator ($\cdot$) and ($\odot$) are interchangeable