ITS
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Probability and Statistics

Expected Values

Expected Values

- The Expected Value of x is the sum of the products of the values of x and their corresponding probabilities

$$E(x) = \sum_{n=1}^{N} x_n \, P(x_n)$$

- In probability theory, the expected value (also called expectation, expectancy, mathematical expectation, mean, average, or first moment) is a generalization of the weighted average

Average

- Average is a special scenario of expected value in which the probabilities are equal

$$\bar{x} = \mu = \frac{\sum x}{n}$$

- For example, the average of the numbers 2, 3, 4, 7, and 9 (summing to 25) is 5
- Depending on the context, an average might be another statistic such as the median, or mode

Average vs Expected Value: Same weight

- Example of students who scored a certain value on their test

$$\{76,81,100,92\}$$

- The average will be:

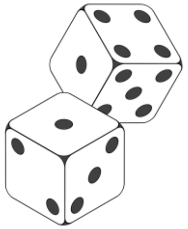
$$\bar{x} = \frac{\sum x}{n} = \frac{76 + 81 + 100 + 92}{4} = 87.25$$

- Whereas the expected value will be:

$$E(x) = \sum_{n=1}^{N} x_n \, P(x_n) = \left(\frac{1}{4}\right) 76 + \left(\frac{1}{4}\right) 81 + \left(\frac{1}{4}\right) 100 + \left(\frac{1}{4}\right) 92 = 87.25$$

Average vs Expected Value: Different Weight

- Dice Example:



x	P(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

Equal Chance

All equal 16.67%

x	P(x)
1	3/20
2	3/20
3	3/20
4	3/20
5	1/5
6	1/5

5 and 6 are more likely than others

5 and 6: 20% each, others: 15%

Dice Example

- Average can be only used for equal dice:

$$\bar{x} = \frac{1+2+3+4+5+6}{6} = 3.5$$

- However, Expected value can be used for both **Equal** and **Unequal Dice Example**

Equal Dice:

$$E(x) = \left(\frac{1}{6}\right)1 + \left(\frac{1}{6}\right)2 + \left(\frac{1}{6}\right)3 + \left(\frac{1}{6}\right)4 + \left(\frac{1}{6}\right)5 + \left(\frac{1}{6}\right)6 = 3.5$$

Unequal Dice:

$$E(x) = \left(\frac{3}{20}\right)1 + \left(\frac{3}{20}\right)2 + \left(\frac{3}{20}\right)3 + \left(\frac{3}{20}\right)4 + \left(\frac{1}{5}\right)5 + \left(\frac{1}{5}\right)6 = 3.7$$

Job Example:

- Supposed we have two different works in a week:
  - Job 1: \$20 / hour, work 8 hours/week
  - Job 2: \$12 / hour, work 16 hours/week
- This can not be calculated using **Average**
- Instead, we will use Weighted Average a.k.a **Expected Value**

$$E(x) = \$20 \,.\, 8\, hrs/w + \$12 \,.\, 16 hrs/w = \$352/week$$

- Using Average will give you false results of \$384 / week

Purpose for Expected Values

1) Assists in making mathematically sound decisions for future events.

2) Used when making investments, determining a price for numerous services, prioritizing events, and in calculating return on investment.

Example 1:

A third grade class was surveyed regarding the number of hours that they played electronic games each day. The probability distribution is given in the table below:

# of Hours (x)	**Probability P(x)**
0	0.3
1	0.4
2	0.2
3	0.1

Calculate the Expected Value of the quantity of time that a third grader spends each day playing electronic games.

$$E(x) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + \dots + x_n P(x_n)$$

# of Hours (x)	Probability P(x)
0	0.3
1	0.4
2	0.2
3	0.1

Expected value, $E(x) = 0\,(0.3) + 1\,(0.4) + 2\,(0.2) + 3\,(0.1)$

Expected value, $E(x) = 0 + 0.4 + 0.4 + 0.3$

Expected value, $E(x) = 1.1\ hours$

Conclusion: Third graders spend 1.1 hrs playing video games each day.

Example 2:

Find the expected number of boys for a three-child family. Assume girls and boys are equally likely. Key: b=Boy; g = Girl

8 Combos
bbb
bbg
bgb
bgg
gbb
gbg
ggb
ggg

# of Boys	Probability	Product
x	P(x)	x P(x)
0	1/8	0
1	3/8	3/8
2	3/8	6/8
3	1/8	3/8
	Expected Value: E(x)	=0 + 3/8 + 6/8 + 3/8 = 12/8 or 1.5 boys

Concl: The expected # of boys for a 3-child family is 1.5 boys.

Example 3:

Finding Expected Winnings

A player pays $3 to play the following game:

Win $7 by rolling a 6 on a single die,
Win $1 by rolling any other number.

What are the expected net winnings for the game?

Number	**Payoff**	**Net**	**P(x)**	**x P(x)**
1, 2, 3, 4, 5				
6				

Finding Expected Winnings

A player pays \$3 to play the following game:

Win \$7 by rolling a 6 on a single die,
Win \$1 by rolling any other number.

What are the expected net winnings for the game?

Number	**Payoff**	**Net**	**P(x)**	**x P(x)**
1, 2, 3, 4, 5	\$1	\$1-\$3 = -\$2	5/6	$-\$2\ \frac{5}{6} = -\$1.67$
6	\$7	\$7-\$3 = \$4	1/6	$\$4\ \frac{1}{6} = \$0.67$
			Expected Value	$-\$1.67 + \$0.67 = -\$1$

ANS: The player will not have an expected net winning for the game, since his Expected Value is a **loss of \$1.00.**

Fair Games/Expected Value

- The <u>expected value</u> of a game is the amount, <u>on average</u>, of money you win per game. The expected value (in terms of a game) is calculated as follows:

- $(x) = (\$\, paid\ if\ you\ win) * (P(winning))$

- A game is a <u>fair game</u> when the cost of each game equals the expected value (what you put in, you get out).

Deciding if a Game is <u>Fair</u>, <u>Favors the House</u>, <u>Favors the Player</u>

A **fair game** is one in which the net winnings are zero.

An unfair game against the player has a **<u>negative</u>** expected winnings.

An unfair game in favor of the player has a **<u>positive</u>** expected winnings.

Example 1: Two dice are rolled

A player gets $5 if the two dice show the same number,
or if the numbers on the dice are different then the player pays $1.

Number	Payoff	P(x)	x P(x)
Same for both dice	$5	6/36 or 1/6	$5 x 1/6 = $5/6
Different number	-$1	30/36 or 5/6	-$1 x 5/6 = -$5/6

a. What is the probability of winning $5?

ANS: 6/36 or 1/6 probability of winning $5

b. What is the probability of paying a $1?

ANS: 30/36 or 5/6 probability of losing $1.

Example 1:

A player gets $5 if the two dice show the same number, or if the numbers on the dice are different then the player pays $1.

Number	Payoff	P(x)	x P(x)
Same for both dice	$5	6/36 or 1/6	$5 x 1/6 = $5/6
Different number	-$1	30/36 or 5/6	-$1 x 5/6 = -$5/6

c. What is the expected value of this game?

$$E(x) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + \ldots + x_n P(x_n)$$

$$E(x) = \$5(1/6) + (-\$1)(5/6) = \$5/6 - \$5/6 = 0$$

d. The Expected Value is $0. This would be a fair game, neither the House or Player is favored.

END OF LECTURE