Probability and Statistics

Statistical Inference (Hypothesis Testing)

Learning Outcome

By the end of this lecture you should be able to:

- Perform a hypothesis test with one sample
- Understand the concept of Null and Alternative Hypotheses
- Reviewing the decision outcomes and finding whether there are Type I or Type II errors on the test

What is Statistical Hypotheses?

In making an inference, rather than presenting a data based decision, we may produce a conclusion about some specific system.

“A **statistical hypothesis** is an assertion or conjecture concerning one or more populations”

A statistical hypothesis can be tested by using procedures which includes **acceptances** or **rejections**

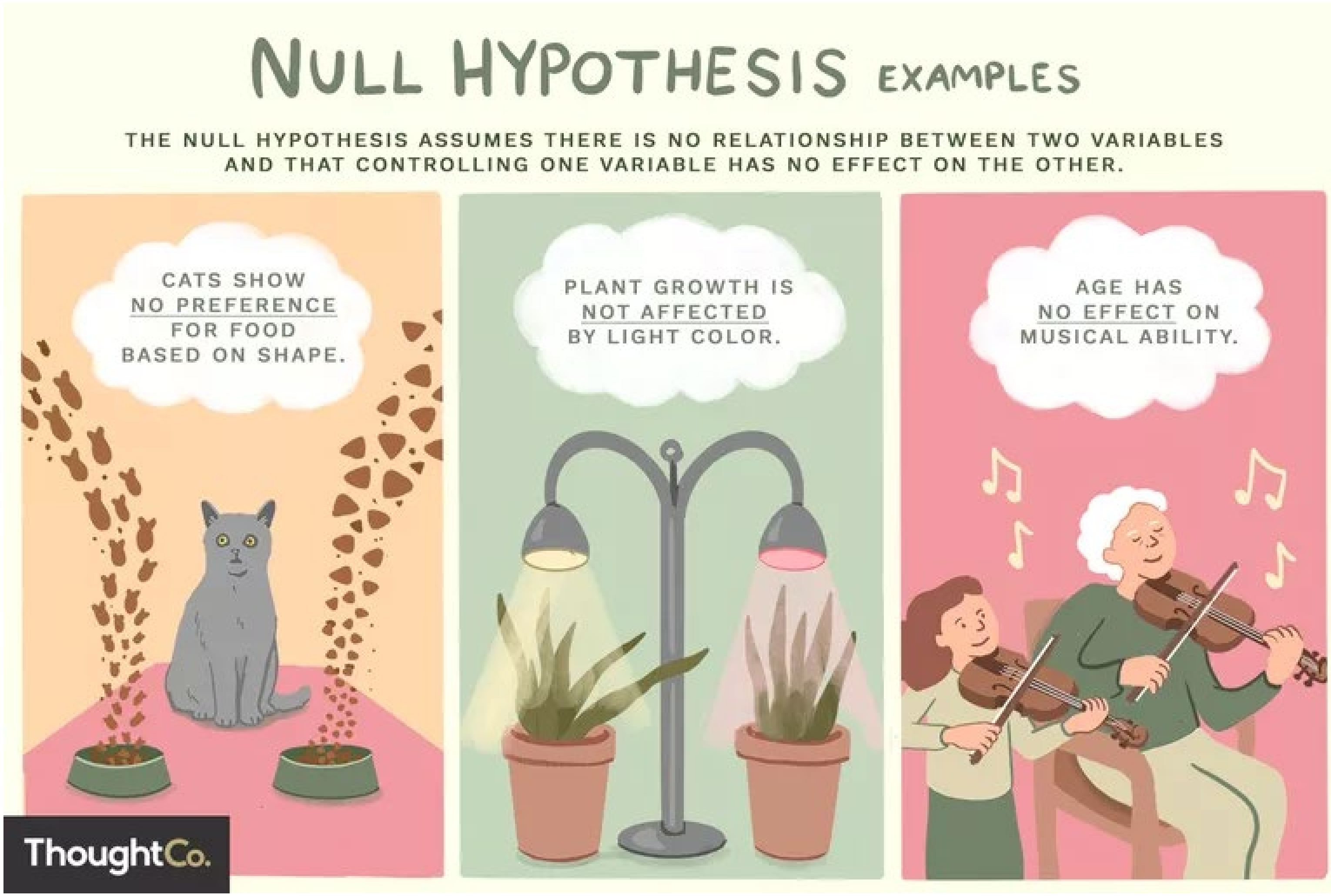
Null and Alternative Hypotheses

The null hypothesis:

Assumes that **there is no meaningful** relationship between two variables

The alternate hypothesis:

Assumes that **there is** a relationship between two variables


NULL HYPOTHESIS EXAMPLES
THE NULL HYPOTHESIS ASSUMES THERE IS NO RELATIONSHIP BETWEEN TWO VARIABLES AND THAT CONTROLLING ONE VARIABLE HAS NO EFFECT ON THE OTHER.
CATS SHOW NO PREFERENCE FOR FOOD BASED ON SHAPE.
PLANT GROWTH IS NOT AFFECTED BY LIGHT COLOR.
AGE HAS NO EFFECT ON MUSICAL ABILITY.
ThoughtCo.

Null Hypothesis Example

Question	**Null Hypothesis**
Are teens better at math than adults?	Age has no effect on mathematical ability.
Does taking aspirin every day reduce the chance of having a heart attack?	Taking aspirin daily does not affect heart attack risk.
Do teens use cell phones to access the internet more than adults?	Age has no effect on how cell phones are used for internet access.
Do cats care about the color of their food?	Cats express no food preference based on color.
Does chewing willow bark relieve pain?	There is no difference in pain relief after chewing willow bark versus taking a placebo.

Null and Alternative Hypotheses (Con't)

H_0 and H_1 are contradictory

One of good illustration of null and alternative hypotheses is in a Jury Trial:

H_0: defendant is innocent

H_1: defendant is guilty

Test Statistics

In many common situations the test statistic is stated as in the following:

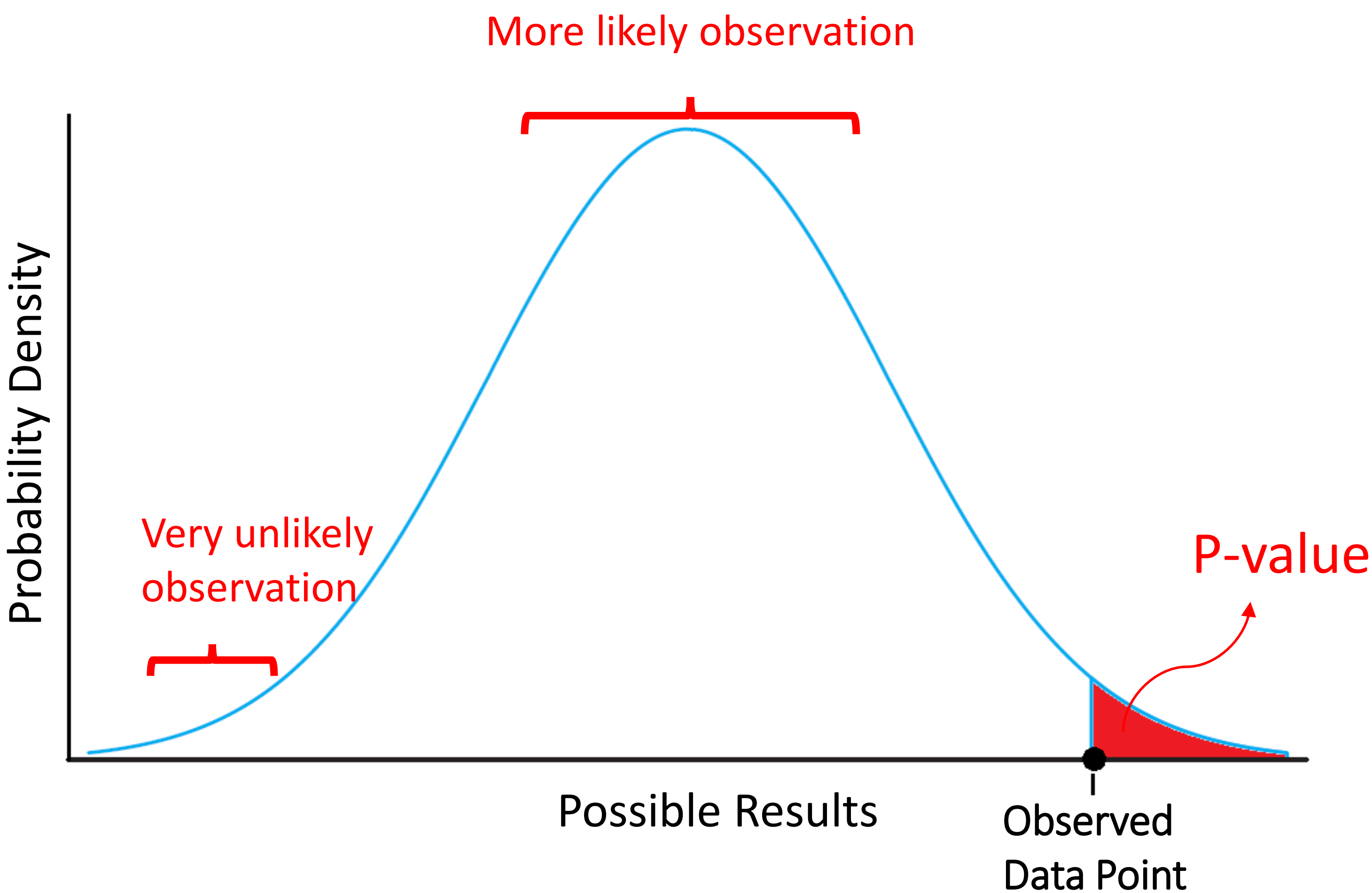
$$z = \frac{estimate\ - hypothesized\ value}{standard\ deviation\ of\ the\ estimate}$$

Let’s put this into a mathematical form:

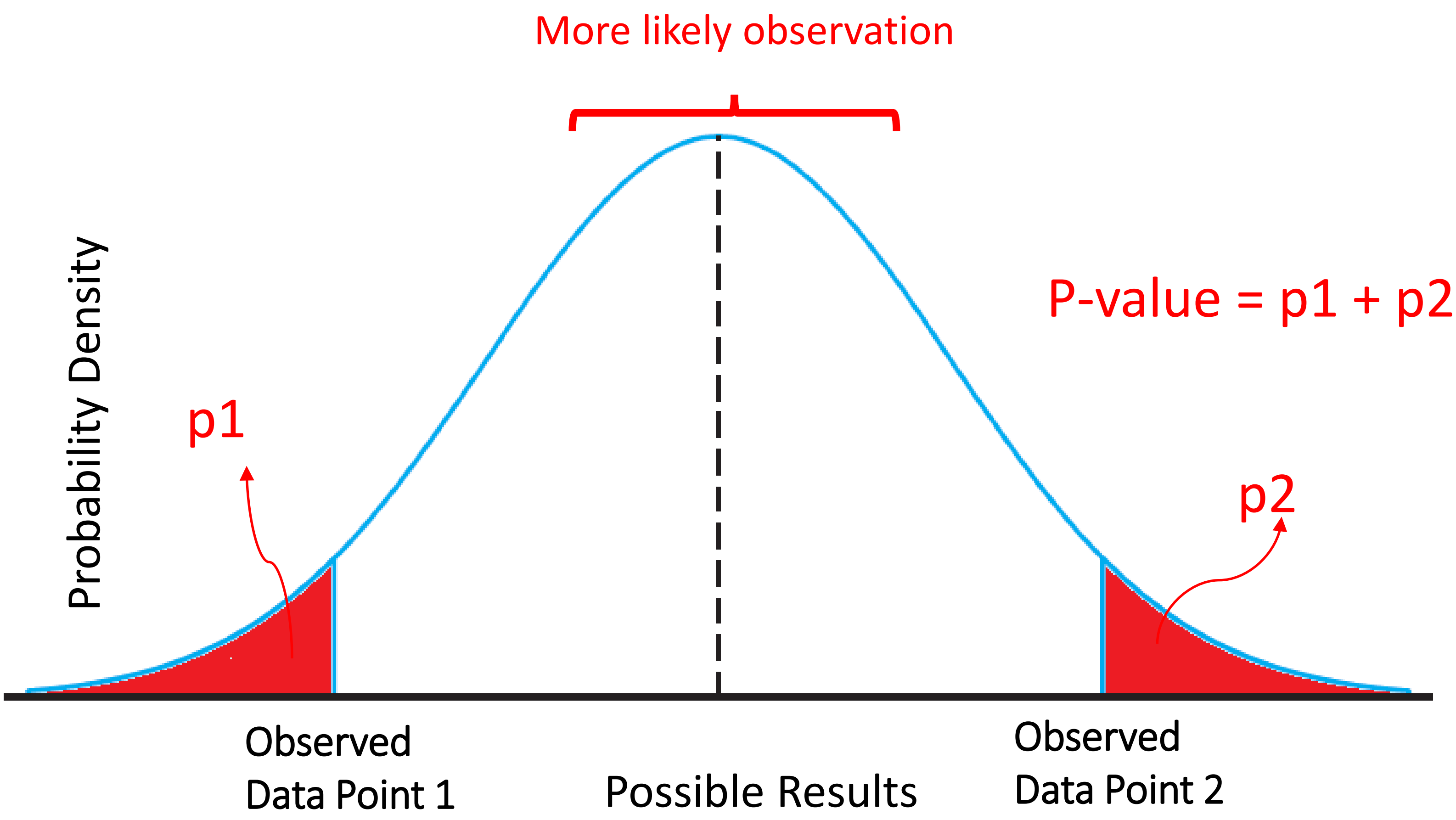
$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

P-Value Concept - One-tailed Test

P-value is the probability assuming H_0 is true



P-Value Concept - Two-tailed Test



Testing a Statistical Hypothesis

Table. Relations between correctness of the null hypothesis

	H_0 True	H_0 False
Do Not Reject H_0	Correct Inference	Type II Error
Reject H_0	Type I Error	Correct Inference

α = probability of a Type I error

β = probability of a Type II error

Probabilities of errors α and β should be as small as possible

The Power of the Test is 1 –β.

Example 2.2

The current cold vaccine is known to be only 25% effective (or less) after 2 years. A study is conducted to determine the effectiveness of a new cold vaccine for a longer period of time. 20 people are chosen at random, If more than 8 of those receiving the new vaccine surpass the 2-year period without contracting the virus, the new vaccine will be considered superior to the one presently in use.

H_0: The new vaccine is equally or less effective after a period of 2 years ($p \leq 0.25$)

H_1: The new vaccine is in fact superior than the current vaccine ($p > 0.25$)

Example 2.2 - continued

However, there are two common errors when making a conclusion from the hypothesis above:

Type I Error:

More than 8 people surpass the 2-year period but the new vaccine **MAY NOT** be better than the old vaccine

Type II Error:

Less than 8 People surpass the 2-year old period, but we are **UNABLE TO CONCLUDE** that the vaccine is better

Example 2.3

A medical device manufacturer has developed a new blood pressure meter, that the company claims has a mean time before failures (MTBF) of 15 years with a standard deviation of 0.5 year. To test if the claim is true, a random sample of 50 items will be tested. The critical region is defined to be $\bar{x} < 14.9$.

a. State the null and alternative hypotheses

b. Find the probability of committing a type I error when H_0 is true

c. Evaluate β for the alternatives μ = 14.8 and μ =14.9 years

Solution 2.3

a.) H_0: μ < 15

H_1: μ >= 15

b.)

$\sigma = 0.5 \qquad n = 50$

$$z = \frac{14.9\ - 15}{0.5/\sqrt{50}} = -1.41$$

$\alpha = P(Z < -1.41) = 0.0793$

c.) If $\mu = 14.8$, $z = \frac{14.9 - 14.8}{0.5/\sqrt{50}} = 1.41$.

So $\beta = P(Z > 1.41) = 0.0793$

If $\mu = 14.9$, $z = \frac{14.9 - 14.9}{0.5/\sqrt{50}} = 0$.

So $\beta = P(Z > 0) = 0.5$

Example 2.4

Consider the null hypothesis that the average weight of male students in a certain college is 68 kilograms against the alternative hypothesis that it is unequal to 68. A critical region for the test statistic is chosen to be $67 < \bar{x} < 69$. Assume that the standard deviation of the population weight to be 3.6 kilograms, based on a random sample of size 36 male students.

a. State the null and alternative hypotheses

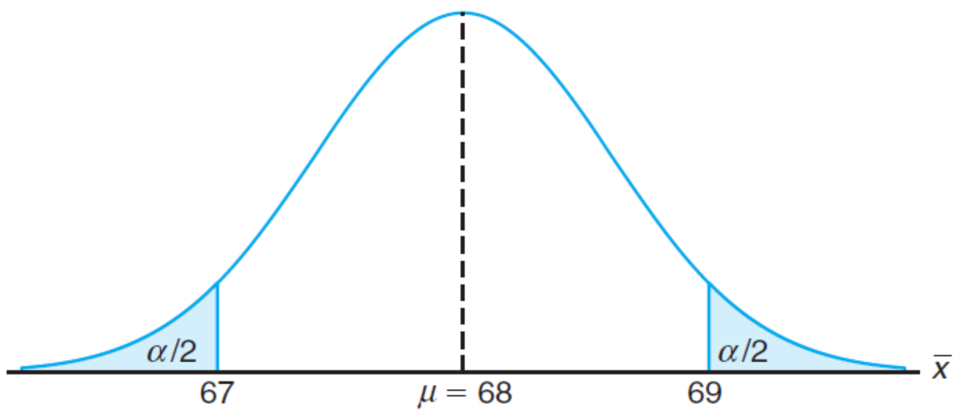
b. Find the probability of committing a type I error when H_0 is true

c. Evaluate β for the alternatives μ = 70 or μ = 66 kg

Solution 2.4

a.) H_0: μ = 68

H_1: μ ≠ 68



b.) $\sigma = 3.6 \qquad n = 36$

$$z_1 = \frac{67 - 68}{3.6/\sqrt{36}} = -1.67$$

$$z_2 = \frac{69 - 68}{3.6/\sqrt{36}} = 1.67$$

The probability of committing type I error:

$\alpha = P(Z < -1.67) + P(Z > 1.67) = 0.095$

Solution 2.4

c.)

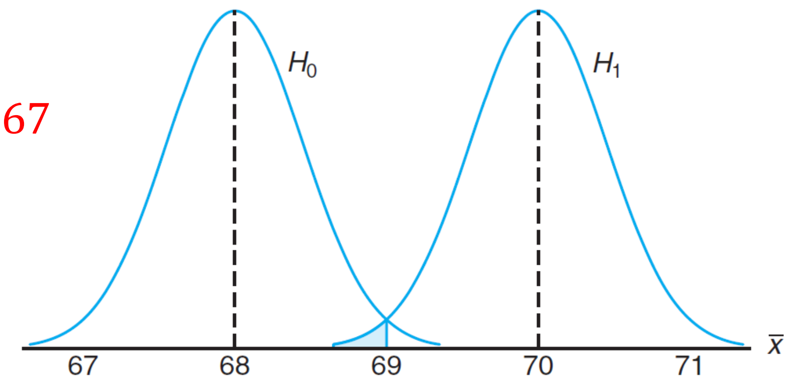
- $\boldsymbol{\beta = P(67 \leq Z \leq 69 \; when \; \mu = 70)}$

$$z_1 = \frac{67 - 70}{3.6/\sqrt{36}} = -5, \qquad z_2 = \frac{69 - 70}{3.6/\sqrt{36}} = -1.67$$

$$\beta = P(-5 < Z < -1.67)$$

$$= P(Z < -1.67) - P(Z < -5)$$

$$= 0.0485 - 0 = 0.0475$$



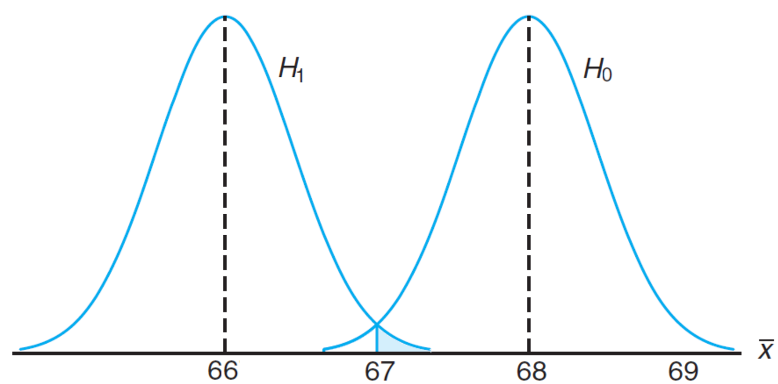
- $\boldsymbol{\beta = P(67 \leq Z \leq 69 \; when \; \mu = 66)}$

$$z_1 = \frac{67 - 66}{3.6/\sqrt{36}} = 1.67, \qquad z_2 = \frac{69 - 66}{3.6/\sqrt{36}} = 5$$

$$\beta = P(1.67 < Z < 5)$$

$$= P(Z < 5) - P(Z > 1.67)$$

$$= 1 - 0.9525 = 0.0475$$



References

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- “Introduction to the Practice of Statistics”, Sixth Edition, by David S.Moore, George P. McCabe, and Bruce A. Craig
- “Probability and Statistics for Engineering and The Sciences” by Devore, Jay L.