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Probability and Statistics B.

1. The following 18 measurements are obtained of a pollutant in a body of water: 10.25, 10.37, 10.66, 10.47, 10.56, 10.22, 10.44, 10.38, 10.63, 10.40, 10.39, 10.26, 10.32, 10.35, 10.54, 10.33, 10.48, 10.68 milligrams per liter. Unfortunately, we don't have any previous experience with this type of experiment. Calculate a 95% lower one-sided bound confidence limit for the mean concentration in this body of water, assuming an approximately normal distribution

D1 : $\bar{x} = 10.42$ mg/l

$$s = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{N-1}}$$

$$s = \sqrt{\frac{(10,25 - 10,42)^2 + \ldots + (10.68 - 10,42)^2}{18-1}}$$

$$s = 0.139$$

$$\alpha = 0,05 \rightarrow t_{0.025} = 2,110 \text{ for } \vartheta = n-1 = 17$$

D2 : 95% confidence limit for lower one-sided bound

D3 : $\mu > \bar{x} - t_{\alpha/2} \frac{s}{\sqrt{n}}$

$$\mu > 10.42 - (2,110) \frac{0.139}{\sqrt{18}}$$

$$\mu > 10.35 \text{ mg/L}$$

So the lower bound of 95% confidence limit is not less than 10,35 mg/L

2. A study was conducted to determine whether treating cows with an experimental antibiotic influences a change in its body weight. To test the null hypothesis against the alternative hypothesis, we obtained the weight change measurements* from a random sample of 36 cows, which results in a mean of 1.29 kg with a standard deviation of 5.34 kg. Rewriting these into hypotheses statement:

   H_0: the antibiotic treatment has no effect on a cow body weight ($\mu = 0$ kg)

   H_1: the antibiotic treatment has an effect on a cow body weight ($\mu \neq 0$ kg)

   a. Calculate the probability of rejecting the null hypothesis if in fact it is true. Which type of error is this?
   b. Do you reject the null hypothesis? let's say if we choose the null hypothesis p-value = 0.05. Explain your answer! **

**) The measurements consist of both weight gain (positive weight change) and weight loss (negative weight change) obtained from the samples*

***) We reject the null hypothesis if α > p-value*

D_1 : $\bar{x} = 1.29$ kg gained $\sigma = 5.34$ kg

$H_0 = (\mu = 0 \text{ kg})$ $n = 36$ cows

$H_1 = (\mu \neq 0 \text{ kg})$

D_2 : a. Probability of rejecting H_0, what type of error?

b. Do you reject H_0? when p-Value = 0.05, explain!

D_3 : a. This error is a type I error (false positive)

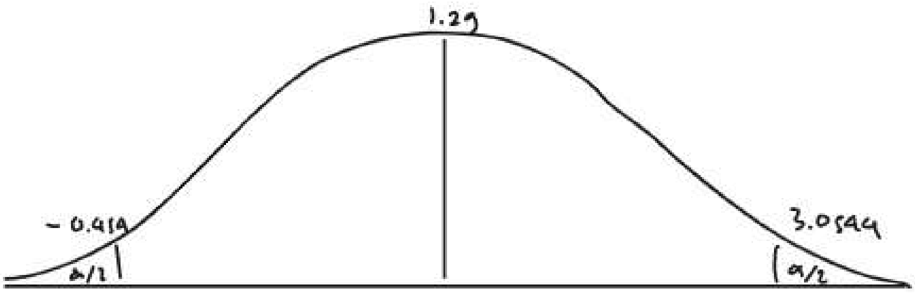
we have to search the critical region first, with 95% confidence interval

$$\alpha = 0.05 \rightarrow z_{\alpha/2} = z_{0.025} = 1.96$$

$$\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} > \mu > \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$1.29 - (1.96) \frac{5.34}{\sqrt{36}} > \mu > 1.29 + (1.96) \frac{5.34}{\sqrt{36}}$$

$$-0.4599 > \mu > 3.0599$$



Commit type I error → $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{1.29 - (-0.45)}{5.34/\sqrt{36}}$

$z_1 = 1.95$

$= \dfrac{1.29 - 3.03}{5.34/\sqrt{36}}$

$z_2 = -1.95$

$\alpha = P(z < -1.95) + (P > 1.95) = 0.0512$

b. We accept the null hypothesis, because the $\alpha \cong$ P-value → $0.0512 \cong 0.05$

3. A relay-specialized company developed a new contact material to increase the electrical life expectancy of their relay product. The company claims that there is an increase in the relay life expectancy by a mean of 5000 operations with a standard deviation of 120 operations. To test the hypothesis that $\mu = 5000$ against the alternative that $\mu < 5000$, a random sample of 50 pieces of relay is tested. The critical region is defined to be $\bar{x} < 4970$.
   a. Find the probability of committing a type I error when H_0 is true
   b. Evaluate β for the alternatives $\mu = 4970$ or $\mu = 4960$

D_1 : $\bar{x}$: 5000 operations $n = 50$ $H_0 : \mu = 5000$

σ : 120 operations $H_1 : \mu < 5000$

D_2 : a. Probability of commiting type I error when H_0 is true

b. Evaluate β for the alternatives $\mu = 4970$ or $\mu = 4960$

D_3 : a. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{4970 - 5000}{120/\sqrt{50}}$

$z = -1.76$

$P(z < -1.76) = 0.0392$

b. for $\mu = 4970$

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{4970 - 4970}{120 / \sqrt{50}}$$

$z = 0$

$P(z > 0) = 0.5 \rightarrow \beta = P(z > 0) = 0.5$

for $\mu = 4960$

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{4970 - 4960}{120 / \sqrt{50}}$$

$z = 0.58$

$P(z > 0.58) = 0.28$