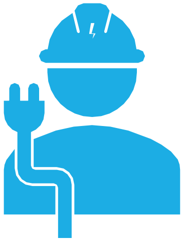
Capacitors and Inductors



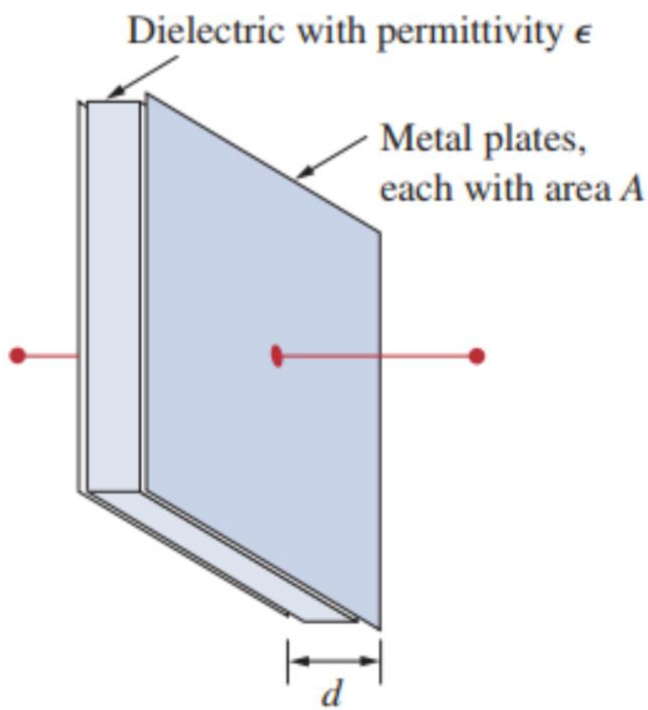
Introduction

So far we have limited our study to resistive circuits. In this chapter, we shall introduce two new and important passive linear circuit elements: the **capacitor** and the **inductor**. Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called **storage elements**.

The application of resistive circuits is quite limited. With the introduction of capacitors and inductors, we will be able to analyze more important and practical circuits. Be assured that the circuit analysis techniques covered in previous weeks are equally applicable to circuits with capacitors and inductors.

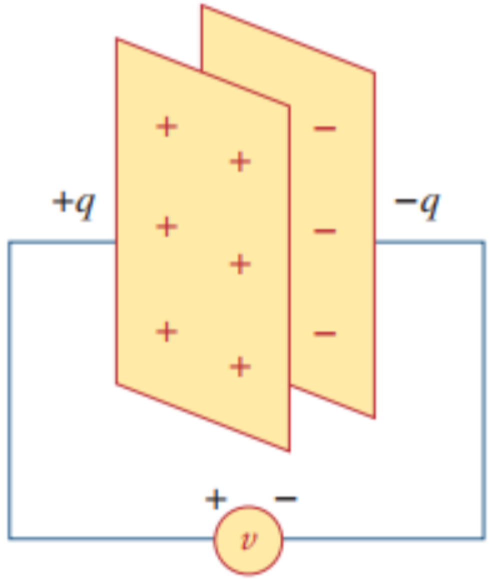
Capacitors

A **capacitor** is a passive element designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems.



$$C = \frac{\epsilon A}{d}$$

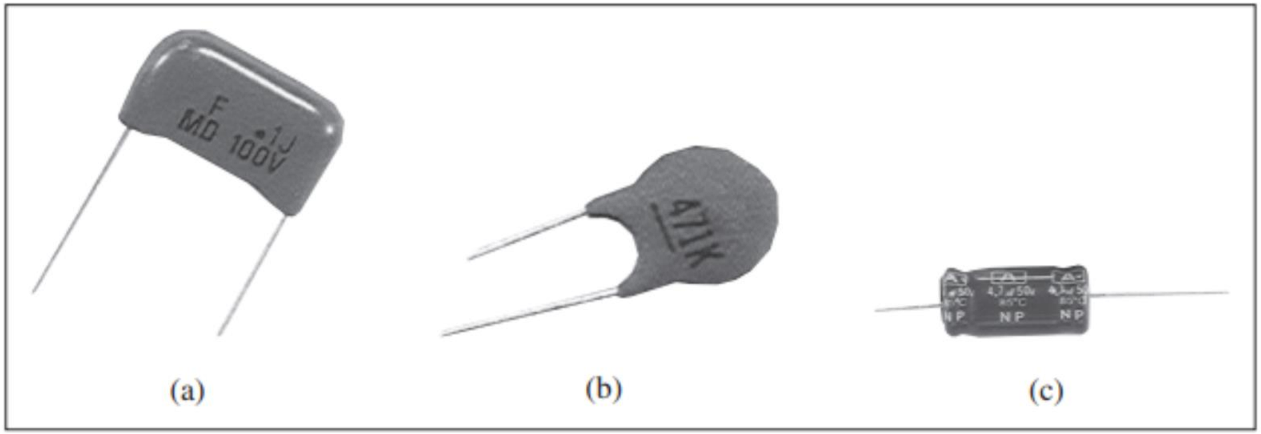
A capacitor consists of two conducting plates separated by an insulator (or dielectric).



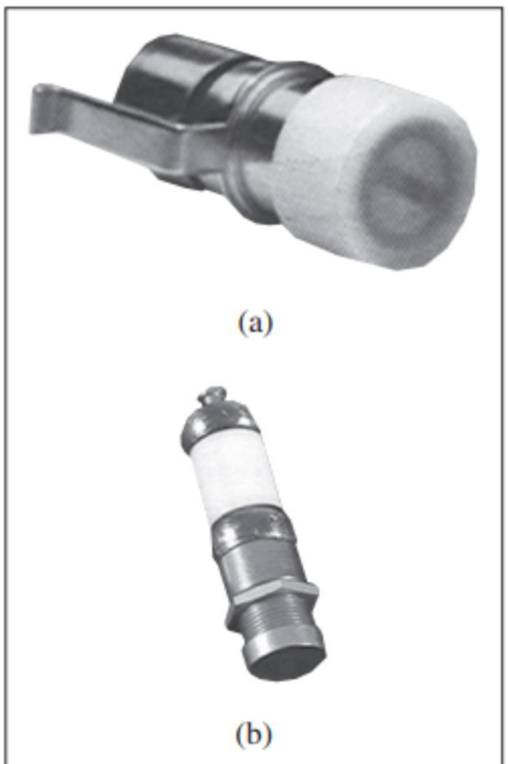
When a voltage source v is connected to the capacitor, the source deposits a positive charge q on one plate and a negative charge $-q$ on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that

$$\boxed{q = Cv}$$

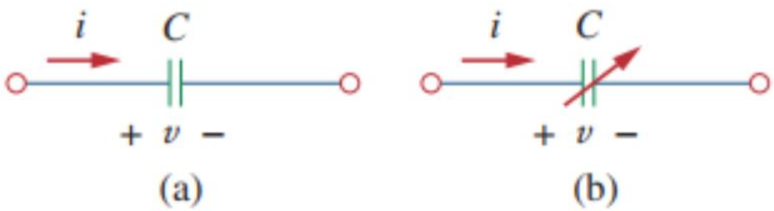
Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).



Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.

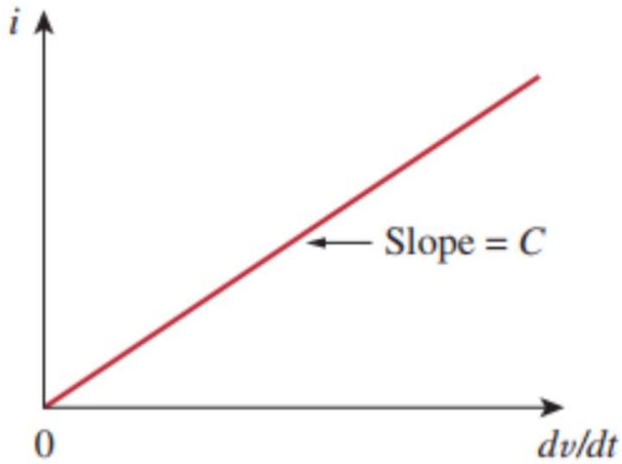


Variable capacitors: (a) trimmer capacitor, (b) filmtrim capacitor.



Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

The current-voltage relationship of the capacitor



$$\boxed{i = C\frac{dv}{dt}}$$

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)}$$

The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt}$$

The energy stored in the capacitor is therefore

$$w = \int_{-\infty}^{t} p(\tau)\,d\tau = C\int_{-\infty}^{t} v\frac{dv}{d\tau}\,d\tau = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

We note that $v(-\infty) = 0$, because the capacitor was uncharged at $t = -\infty$, Thus,

$$\boxed{w = \frac{1}{2}Cv^2}$$

Important properties of a capacitor

(1) When the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero.

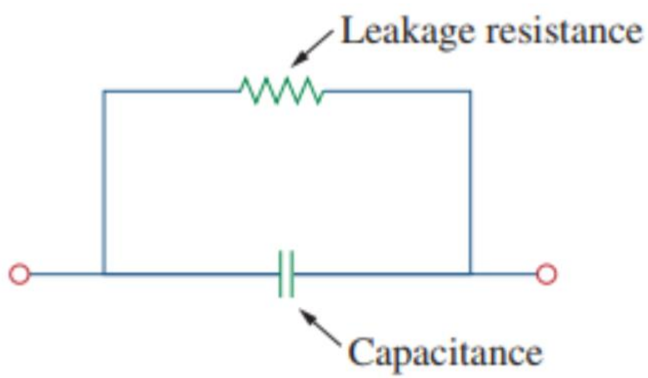
A capacitor is an open circuit to dc.

(2) The voltage on the capacitor must be continuous.

The voltage on a capacitor cannot change abruptly.

(3) The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

(4) A real, nonideal capacitor has a parallel-model leakage resistance



Example 1

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:

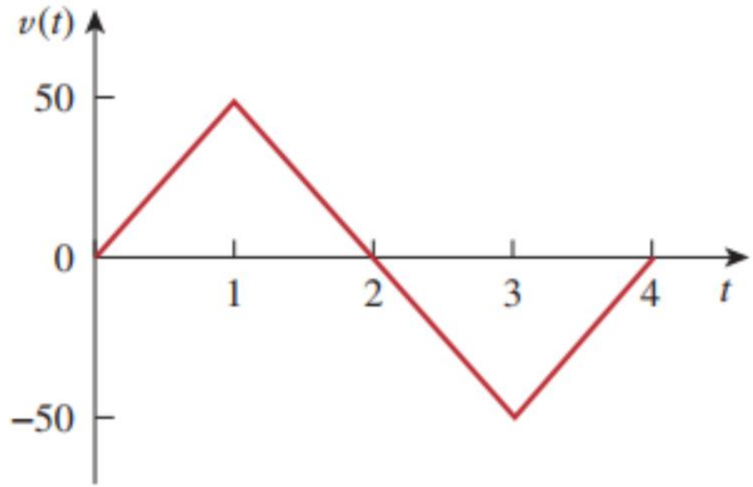
By definition, the current is

$$i(t) = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(10 \cos 6000t)$$

$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A}$$

Example 2

Determine the current through a 200-μF capacitor whose voltage is



Solution:

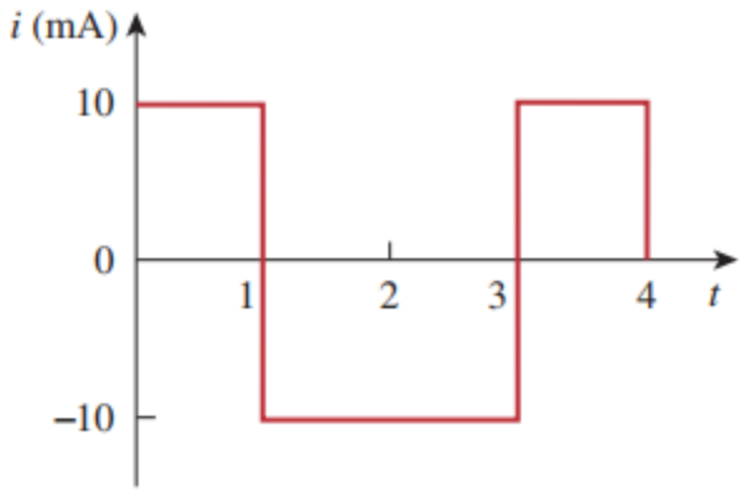
The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Since $i = C\,dv/dt$ and $C = 200\ \mu\text{F}$, we take the derivative of v to obtain

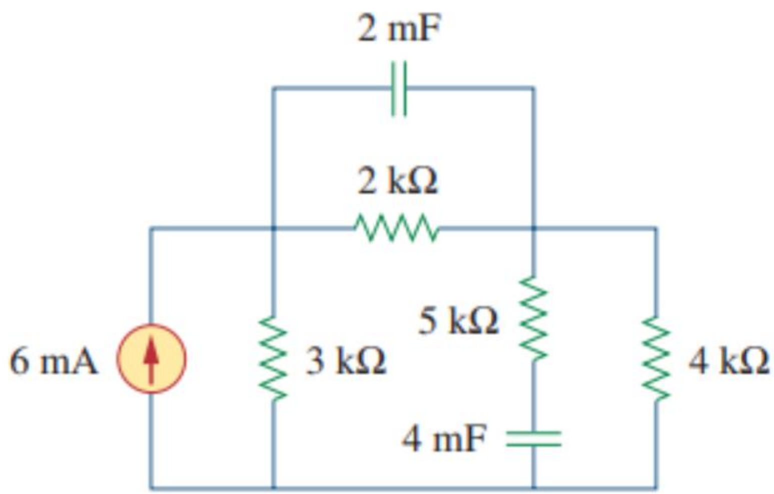
$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10\ \text{mA} & 0 < t < 1 \\ -10\ \text{mA} & 1 < t < 3 \\ 10\ \text{mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$



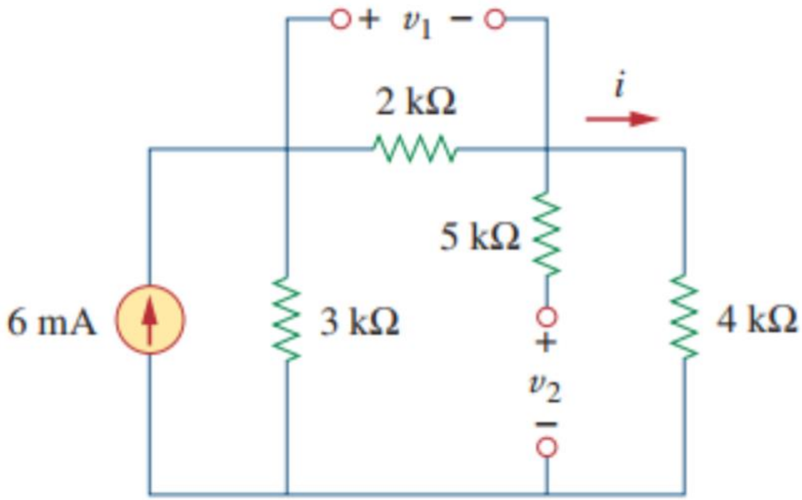
Example 3

Obtain the energy stored in each capacitor



Solution:

Under dc conditions, we replace each capacitor with an open circuit.



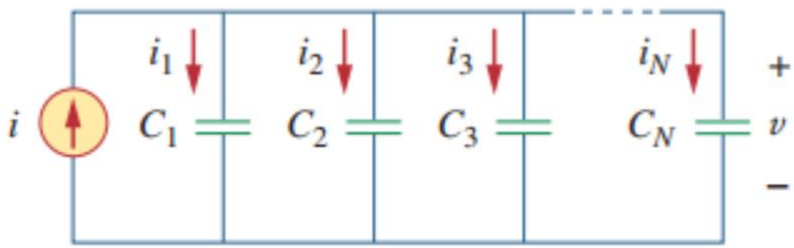
$$i = \frac{3}{3+2+4}(6\text{ mA}) = 2\text{ mA}$$

$$v_1 = 2000i = 4\text{ V} \qquad v_2 = 4000i = 8\text{ V}$$

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2\times 10^{-3})(4)^2 = 16\text{ mJ}$$

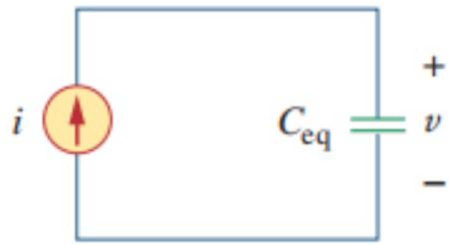
$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4\times 10^{-3})(8)^2 = 128\text{ mJ}$$

Series and Parallel Capacitors



(a)

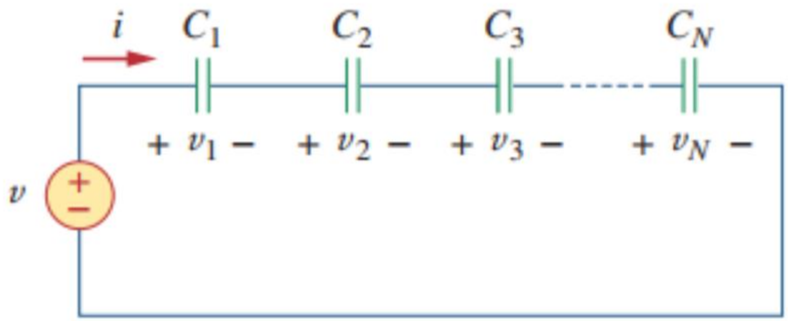
(a) Parallel-connected N capacitors, (b) equivalent circuit for the parallel capacitors.



(b)

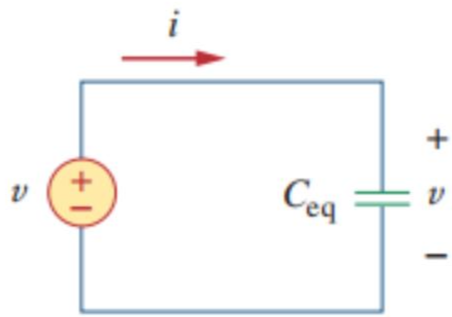
$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$$

The **equivalent capacitance** of N parallel-connected capacitors is the sum of the individual capacitances.



(a)

(a) Series-connected N capacitors, (b) equivalent circuit for the series capacitor.



(b)

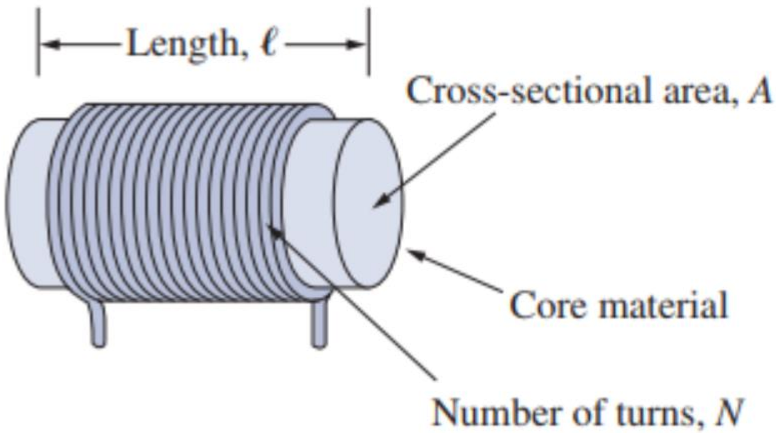
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

The **equivalent capacitance** of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

Inductors

An **inductor** is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors.

Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire



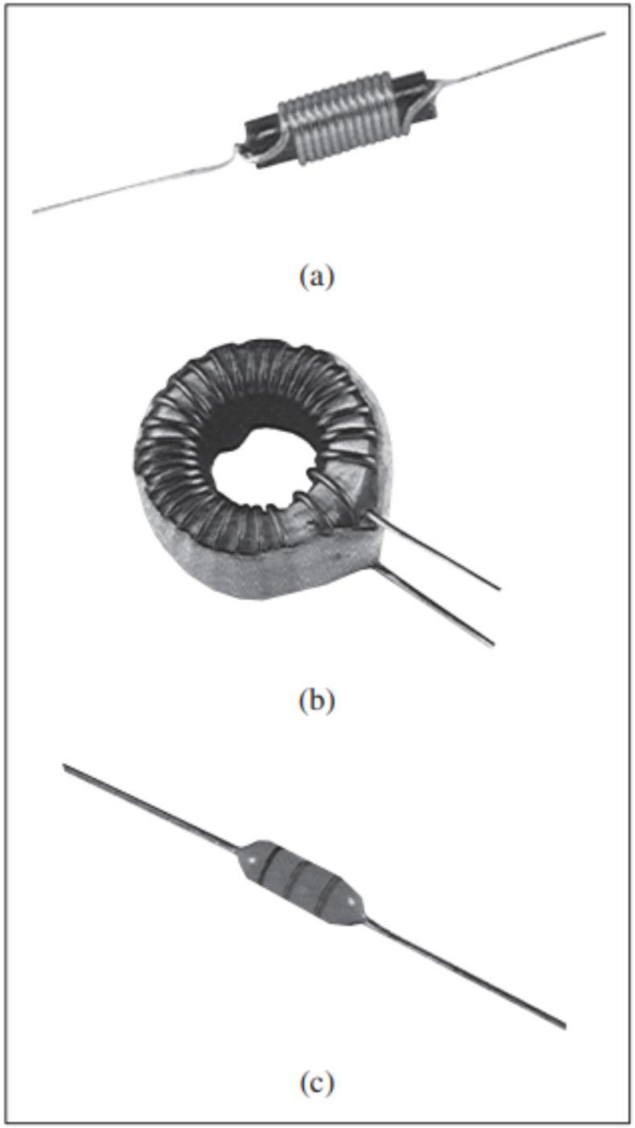
$$L = \frac{N^2 \mu A}{\ell}$$

An **inductor** consists of a coil of conducting wire.

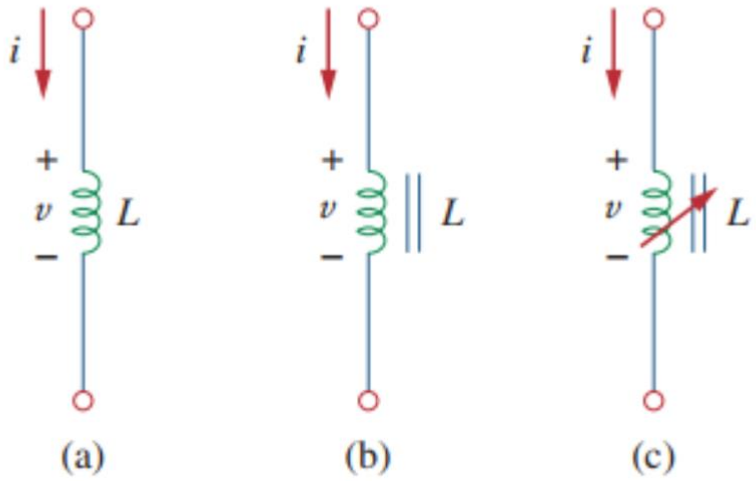
If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current. Using the passive sign convention,

$$\boxed{v = L\frac{di}{dt}}$$

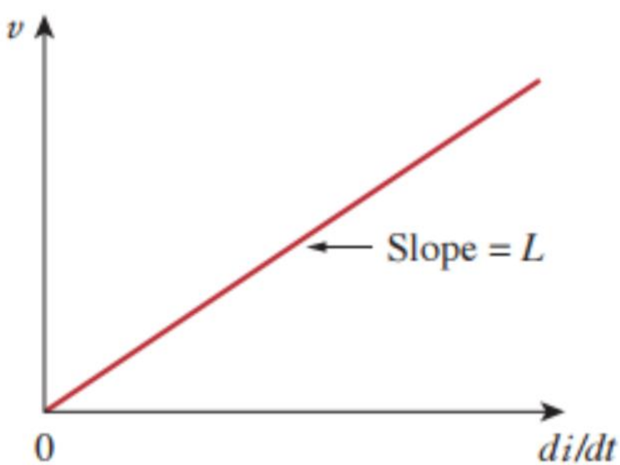
Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).



Various types of inductors: (a) solenoidal wound inductor, (b) toroidal inductor, (c) chip inductor.



Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.



$$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$$

The inductor is designed to store energy in its magnetic field. The power delivered to the inductor is

$$p = vi = \left(L\frac{di}{dt}\right)i$$

The energy stored is

$$w = \int_{-\infty}^{t} p(\tau)\,d\tau = L\int_{-\infty}^{t} \frac{di}{d\tau}\,i d\tau$$
$$= L\int_{-\infty}^{t} i\,di = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

Since $i(-\infty) = 0$

$$\boxed{w = \frac{1}{2}Li^2}$$

Important properties of an inductor

(1) The voltage across an inductor is zero when the current is constant.

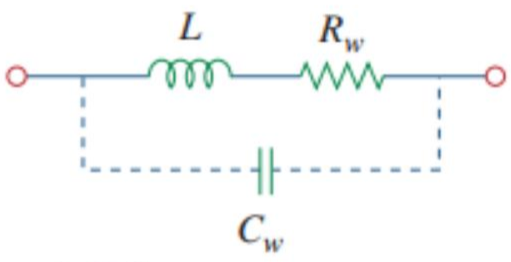
An inductor acts like a short circuit to dc.

(2) An important property of the inductor is its opposition to the change in current flowing through it.

The current through an inductor cannot change instantaneously.

(3) The ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.

(4) A practical, nonideal inductor has a significant resistive component.



Example 1

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:

Since $v = L\,di/dt$ and $L = 0.1$ H,

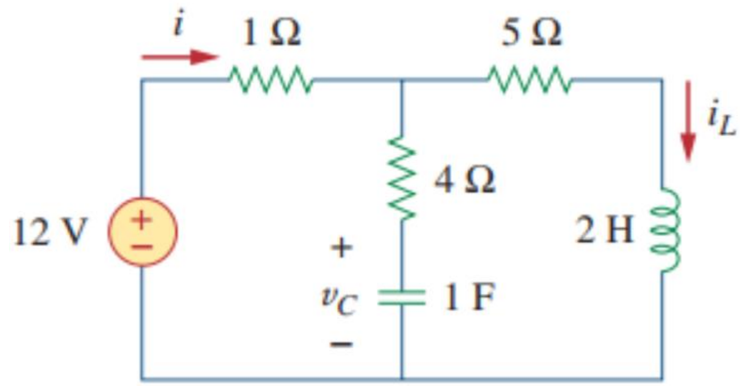
$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t) \text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

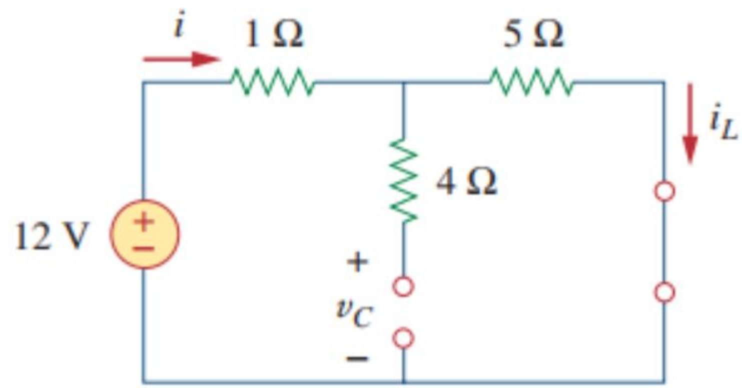
Example 2

Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.



Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit



$$i = i_L = \frac{12}{1 + 5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

$$v_C = 5i = 10 \text{ V}$$

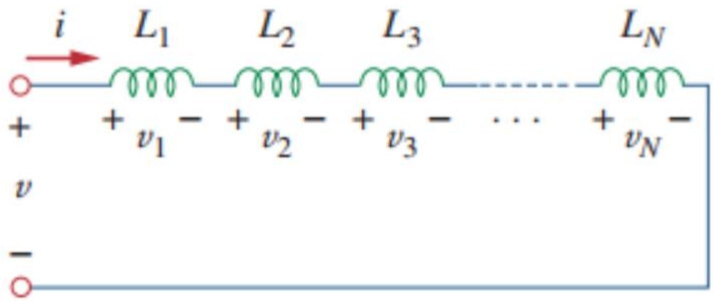
(b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

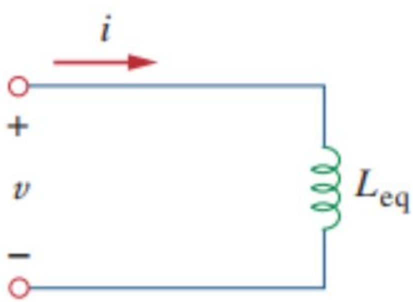
and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

Series and Parallel Inductors



(a)

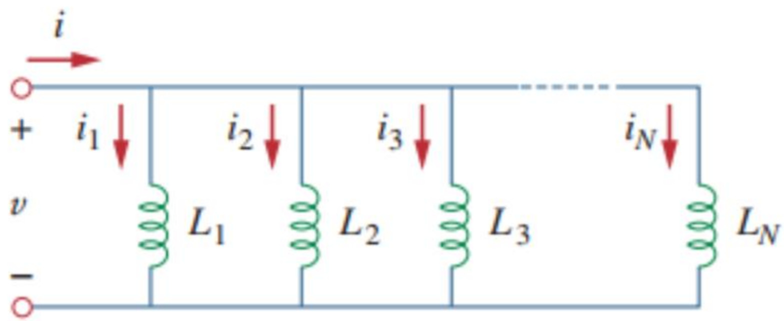


(b)

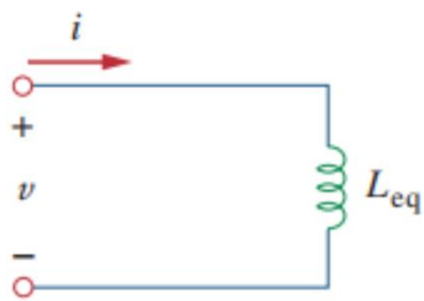
(a) A series connection of N inductors, (b) equivalent circuit for the series inductors.

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

The **equivalent inductance** of series-connected inductors is the sum of the individual inductances.



(a)



(b)

(a) A parallel connection of N inductors, (b) equivalent circuit for the parallel inductors.

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

The **equivalent inductance** of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

Summary

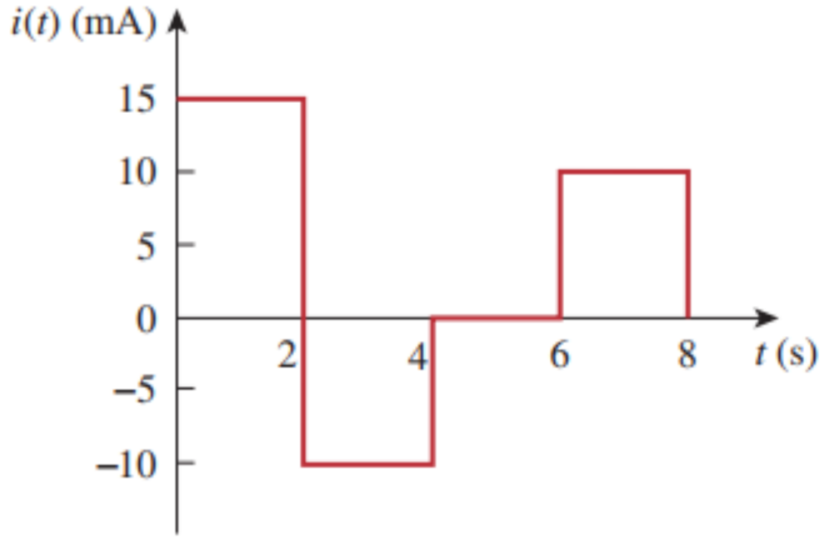
Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

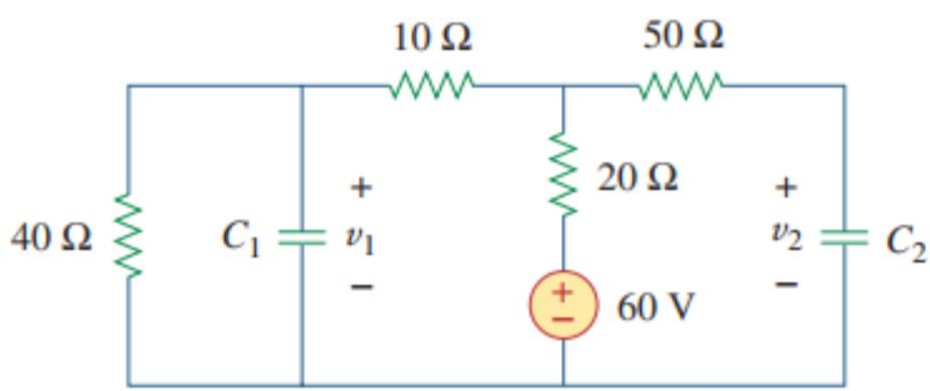
† *Passive sign convention is assumed.*

Problems

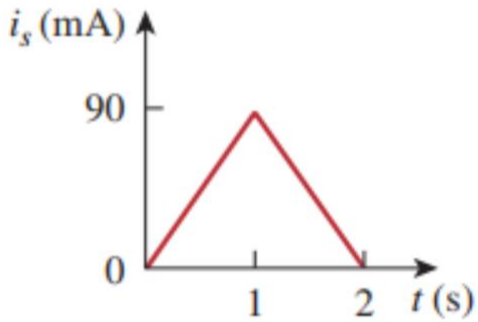
[1] A 4-mF capacitor has the current waveform shown below. Assuming that $v(0) = 10\text{ V}$, sketch the voltage waveform $v(t)$.

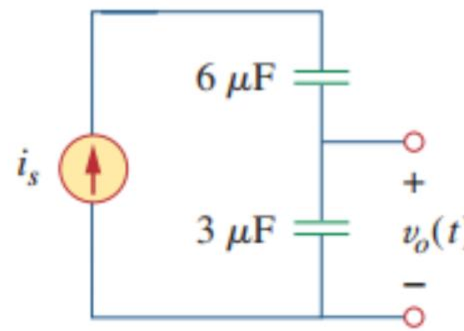


[2] Find the voltage across the capacitors in the circuit under dc conditions.

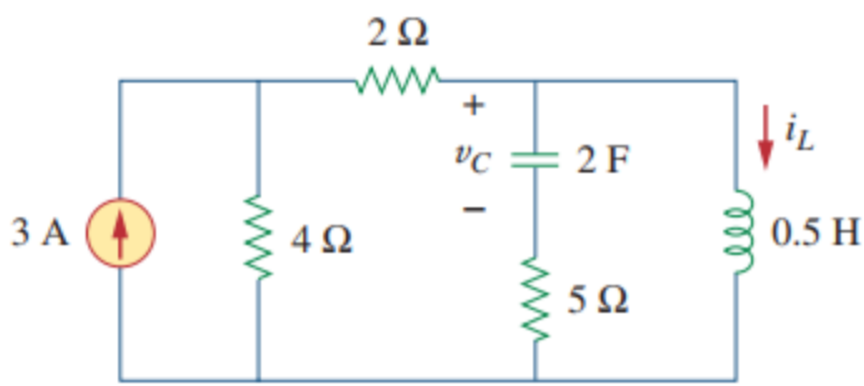


[3] Assuming that the capacitors are initially uncharged, find $v_o(t)$

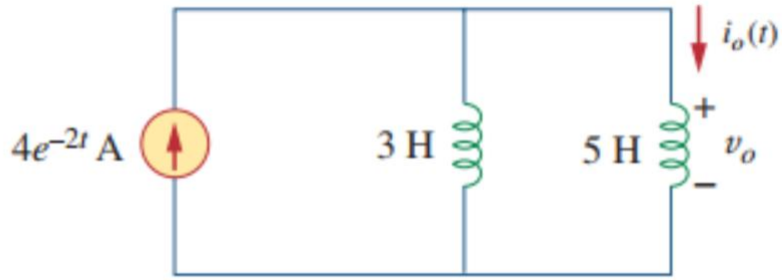




[4] Find v_C, i_L, and the energy stored in the capacitor and inductor under dc conditions.



[5] In the circuit, $i_o(0) = 2$ A. Determine $i_o(t)$ and $v_o(t)$ for $t > 0$.



[6] For the circuit below, sketch v_o