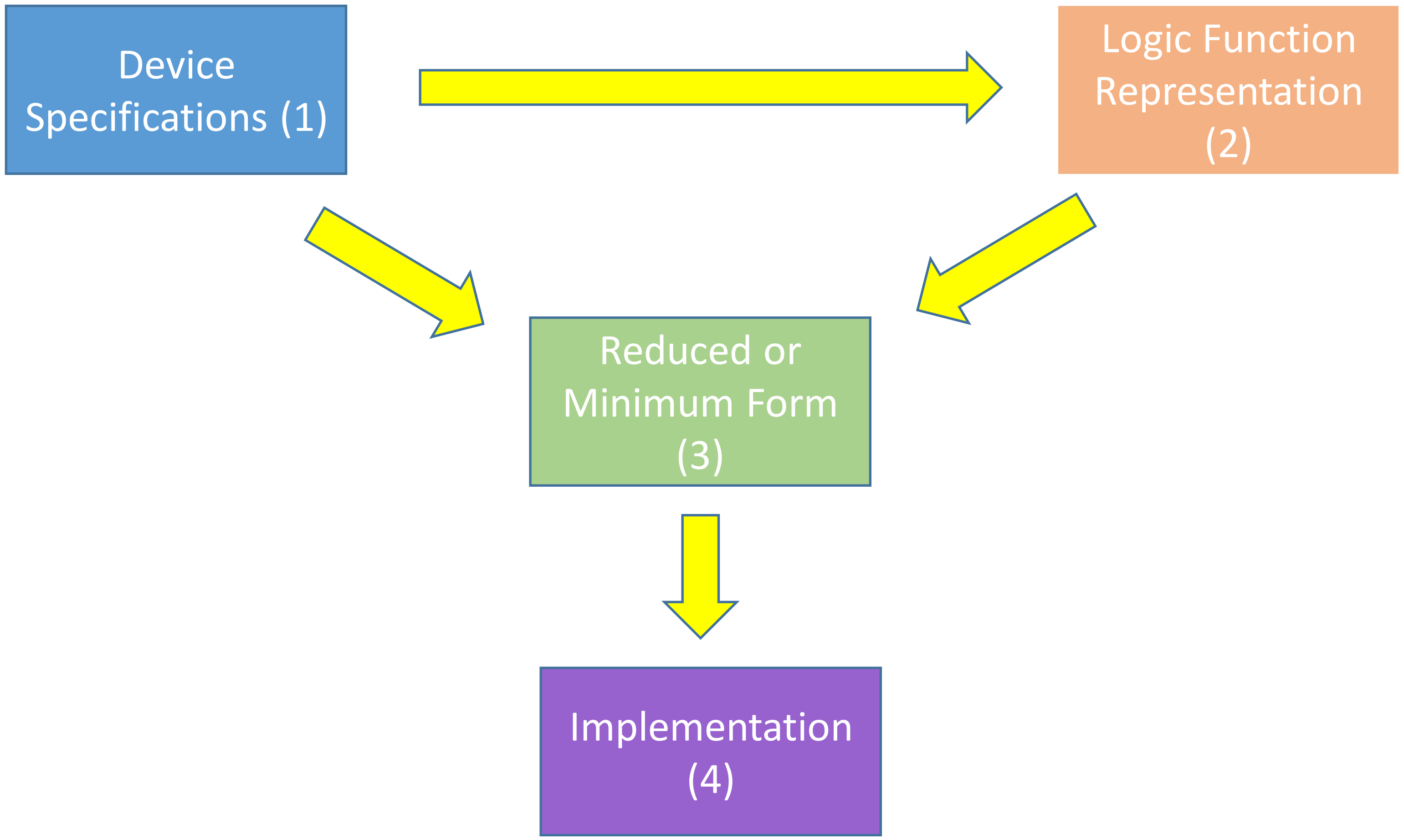
Function Representation and Reduction

Oleh : Ahmad Zaini
EC.184405 Rangkaian Digital dan Lab
Teknik Komputer
FTEIC - ITS

The Sequence Of Logic Circuit Design



Sum of Product Terms (SOP)

Or-ing of AND-ed terms $f(A,B,C) = A + B'C + A'BC$

Minterm : " ***Any ANDed term containing all the variables of a function In complemented or uncomplemented form"***

Representation

$$m_i = m_i(A, B, C, \ldots)$$

Complemented variable = Logic '0'
Uncomplemented variable = Logic '1'

Canonical SOP

"A function composed completely of a logical sum of MINTERM"

$$f(x,y,z) = x'y'z + x'yz' + x'yz + xyz$$

$$f(x,y,z) = 001 + 010 + 011 + 111$$

$$f(x,y,z) = m_1 + m_2 + m_3 + m_7$$

$$f(x,y,z) = \sum m(1,2,3,7)$$

Row	xyz	Minterm
0	000	x'y'z'
1	001	x'y'z
2	010	x'yz'
3	011	x'yz
4	100	xy'z'
5	101	xy'z
6	110	xyz'
7	111	xyz

Product of Sum Terms (POS)

AND-ing of OR-ed terms $f(A,B,C) = (A + B')(A' + B + C)(B + C)$

Maxterm : **" *Any ORed term containing all the variables of a function In complemented or uncomplemented form*"**

Representation

$$M_i = M_i(A, B, C, \dots)$$

Complemented variable = Logic '1'
Uncomplemented variable = Logic '0'

Canonical POS

"A function composed completely of a logical Product of MAXTERM"

Row	xyz	Maxterm	F
0	000	x+y+z	0
1	001	x+y+z'	1
2	010	x+y'+z	1
3	011	x+y'+z'	1
4	100	x'+y+z	0
5	101	x'+y+z'	0
6	110	x'+y'+z	0
7	111	x'+y'+z'	1

$$F(x,y,z) = (x+y+z)\cdot(x'+y+z)\cdot(x'+y+z')\cdot(x'+y'+z)$$

$$F(x,y,z) = 000 \cdot 100 \cdot 101 \cdot 110$$

$$F(x,y,z) = M_0 \cdot M_4 \cdot M_5 \cdot M_6$$

$$F(x,y,z) = \prod M_i(0,4,5,6)$$

Logic Function Graphics

First Order

A	
0	A’
1	A

Second Order

A \ B	0	1
0	A’B’	A’B
1	AB’	AB

Third Order

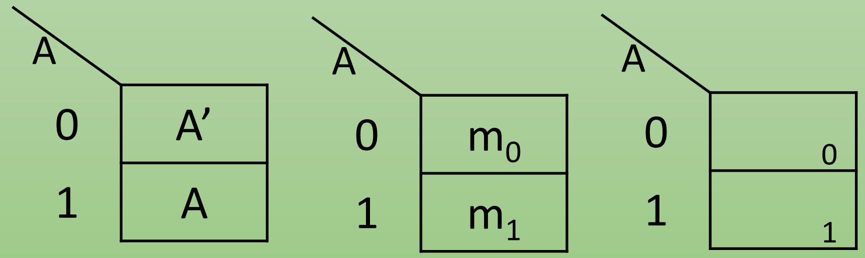
	B			
A	0	0	1	1
0	A’B’C’	A’B’C	A’BC	A’BC’
1	AB’C’	AB’C	ABC	ABC’
C	0	1	1	0

Fourth Order

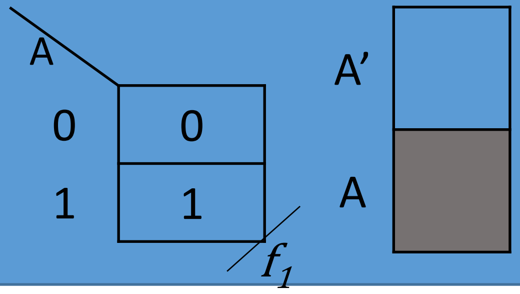
	C				
A	0	0	1	1	B
0	A’B’C’D’	A’B’C’D	A’B’CD	A’B’CD’	0
0	A’BC’D’	A’BC’D	A’BCD	A’BCD’	1
1	ABC’D’	ABC’D	ABCD	ABCD’	1
1	AB’C’D’	AB’C’D	AB’CD	AB’CD’	0
D	0	1	1	0	

First Order K-maps

A	m_i
0	$A' = m_0$
1	$A = m_1$

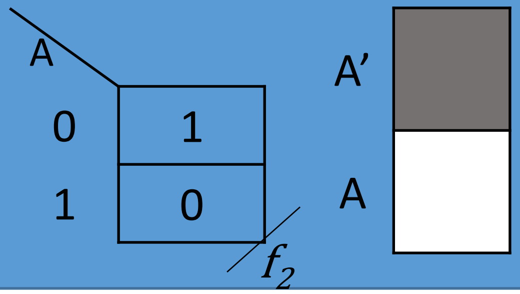


A	f_1
0	0
1	1



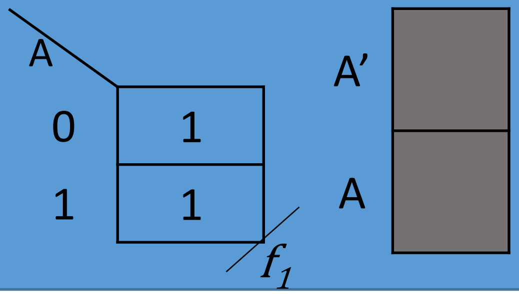
$f_1 = A$

A	f_2
0	1
1	0



$f_2 = A'$

A	f_3
0	1
1	1



$f_3 = 1$

Second Order K-maps

A	B	m_i
0	0	$A' \cdot B' = m_0$
0	1	$A' \cdot B = m_1$
1	0	$A \cdot B' = m_2$
1	1	$A \cdot B = m_3$

A \ B	0	1
0	A'B'	A'B
1	AB'	AB

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

A \ B	0	1
0	0	1
1	2	3

B \ A	0	1
0	0	2
1	1	3

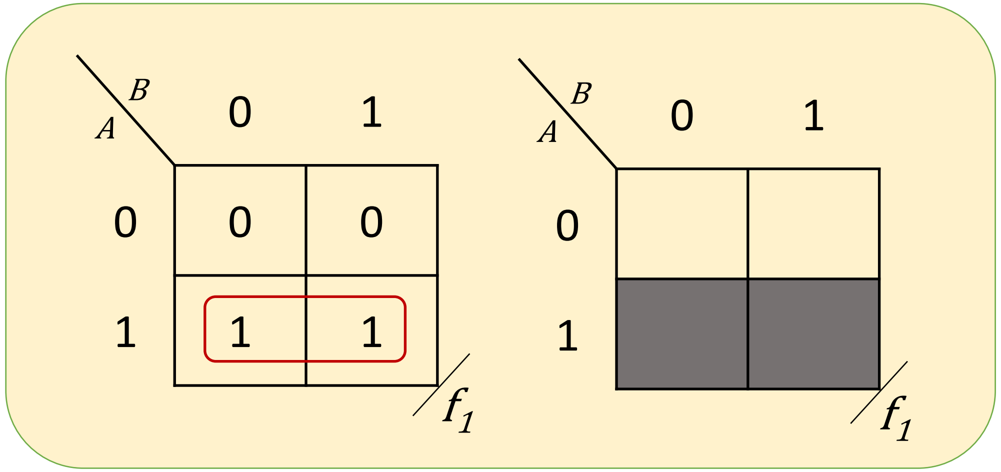
Second Order K-maps

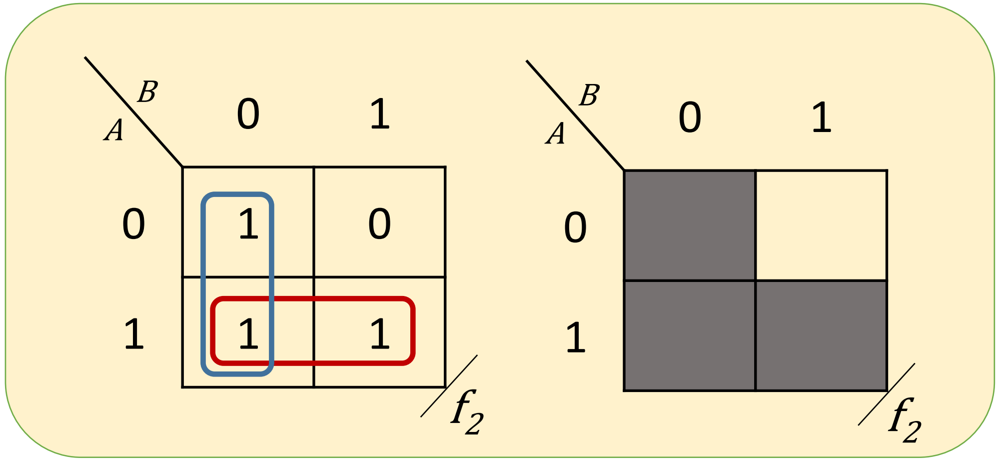
A	B	f_1
0	0	0
0	1	0
1	0	1
1	1	1

A	B	f_2
0	0	1
0	1	0
1	0	1
1	1	1

$f_1(A,B) = \sum m\,(2,3) = AB' + AB$ ← SOP → $f_2(A,B) = \sum m\,(0,2,3) = A'B' + AB' + AB$

$F_1(A,B) = \prod M\,(0,1) = (A+B)\cdot(A+B')$ ← POS → $F_2(A,B) = \prod M\,(1) = (A+B')$





Third Order K-maps

A	B	C	m_i
0	0	0	$A'\cdot B'\cdot C' = m_0$
0	0	1	$A'\cdot B'\cdot C = m_1$
0	1	0	$A'\cdot B\cdot C' = m_2$
0	1	1	$A'\cdot B\cdot C = m_3$
1	0	0	$A\cdot B'\cdot C' = m_4$
1	0	1	$A\cdot B'\cdot C = m_5$
1	1	0	$A\cdot B\cdot C' = m_6$
1	1	1	$A\cdot B\cdot C = m_7$

		B'		B	
	A \ BC	0	0	1	1
A'	0	A'B'C'	A'B'C	A'BC	A'BC'
A	1	AB'C'	AB'C	ABC	ABC'
		0	1	1	0
		C'	C		C'

		B'		B	
	A \ BC	0	0	1	1
A'	0	m_0	m_1	m_3	m_2
A	1	m_4	m_5	m_7	m_6
		0	1	1	0
		C'	C		C'

		B'		B	
	A \ BC	0	0	1	1
A'	0	0	1	3	2
A	1	4	5	7	6
		0	1	1	0
		C'	C		C'

Fourth Order K-maps

A	B	C	D	m_i
0	0	0	0	$A' \cdot B' \cdot C' \cdot D' = m_0$
0	0	0	1	$A' \cdot B' \cdot C' \cdot D = m_1$
0	0	1	0	$A' \cdot B' \cdot C \cdot D' = m_2$
0	0	1	1	$A' \cdot B' \cdot C \cdot D = m_3$
0	1	0	0	$A' \cdot B \cdot C' \cdot D' = m_4$
0	1	0	1	$A' \cdot B \cdot C' \cdot D = m_5$
0	1	1	0	$A' \cdot B \cdot C \cdot D' = m_6$
0	1	1	1	$A' \cdot B \cdot C \cdot D = m_7$

A	B	C	D	m_i
1	0	0	0	$A \cdot B' \cdot C' \cdot D' = m_8$
1	0	0	1	$A \cdot B' \cdot C' \cdot D = m_9$
1	0	1	0	$A \cdot B' \cdot C \cdot D' = m_{10}$
1	0	1	1	$A \cdot B' \cdot C \cdot D = m_{11}$
1	1	0	0	$A \cdot B \cdot C' \cdot D' = m_{12}$
1	1	0	1	$A \cdot B \cdot C' \cdot D = m_{13}$
1	1	1	0	$A \cdot B \cdot C \cdot D' = m_{14}$
1	1	1	1	$A \cdot B \cdot C \cdot D = m_{15}$

Fourth Order K-maps

		C'		C			
	CD / *AB*	0	0	1	1		
A'	0	A'B'C'D'	A'B'C'D	A'B'CD	A'B'CD'	0	B'
	0	A'BC'D'	A'BC'D	A'BCD	A'BCD'	1	B
A	1	ABC'D'	ABC'D	ABCD	ABCD'	1	
	1	AB'C'D'	AB'C'D	AB'CD	AB'CD'	0	B'
		0	1	1	0		
		D'	D		D'		

		C'		C			
	CD / *AB*	0	0	1	1		
A'	0	m_0	m_1	m_3	m_2	0	B'
	0	m_4	m_5	m_7	m_6	1	B
A	1	m_{12}	m_{13}	m_{15}	m_{14}	1	
	1	m_8	m_9	m_{11}	m_{10}	0	B'
		0	1	1	0		
		D'	D		D'		

Loop Protocol

- ***Monads*** (n = 0), Single Minterms or Maxterms which have no logic adjacencies should be looped out first.

- ***Diads*** (n = 1), Group 0f two logically adjcent Minterms or Maxterms which cannot be grouped in any other way to form larger 2^n groups should be looped out following the monads. A reduction of one variable for each diad will result.

- ***Quads*** (n = 2), Group 0f four logically adjcent Minterms or Maxterms which cannot be grouped in any other way to form larger 2^n groups should be looped out following the diads. A reduction of two variables for each quad will result.

- ***Octads*** (n = 3), Group 0f eight logically adjcent Minterms or Maxterms which cannot be further combined to form a hexadecad (sixteen adjacencies) should be looped out next. A reduction of three variables for per octad will result.

Loop Protocol

		C'	C'	C	C		
	AB \ CD	00	01	11	10		
A'	00	0	0	1	0	0	B'
A'	01	1	1	1	0	1	B
A	11	0	1	1	1	1	B
A	10	0	1	0	0	0	B'
		0	1	1	0		
		D'	D	D	D'		

		C'	C'	C	C		
	AB \ CD	00	01	11	10		
A'	00	1	1	0	0	0	B'
A'	01	1	0	0	0	1	B
A	11	1	0	1	1	1	B
A	10	1	1	0	0	0	B'
		0	1	1	0		
		D'	D	D	D'		

Prime Implicants (PIs)

“ Any Single or Groups of 2^n adjacent Minterms or Maxterms that they can’t be combined with other 2^n adjacent groups in any way to produce term of fewer variables ”

ESSENTIAL PRIME IMPLICANTs (EPIs) ; Single way PIs, which must be used to achieve minimum cover

OPTIONAL PRIME IMPLICANTs (OPIs) ; Optional way PIs, which are used for alternative minimum cover

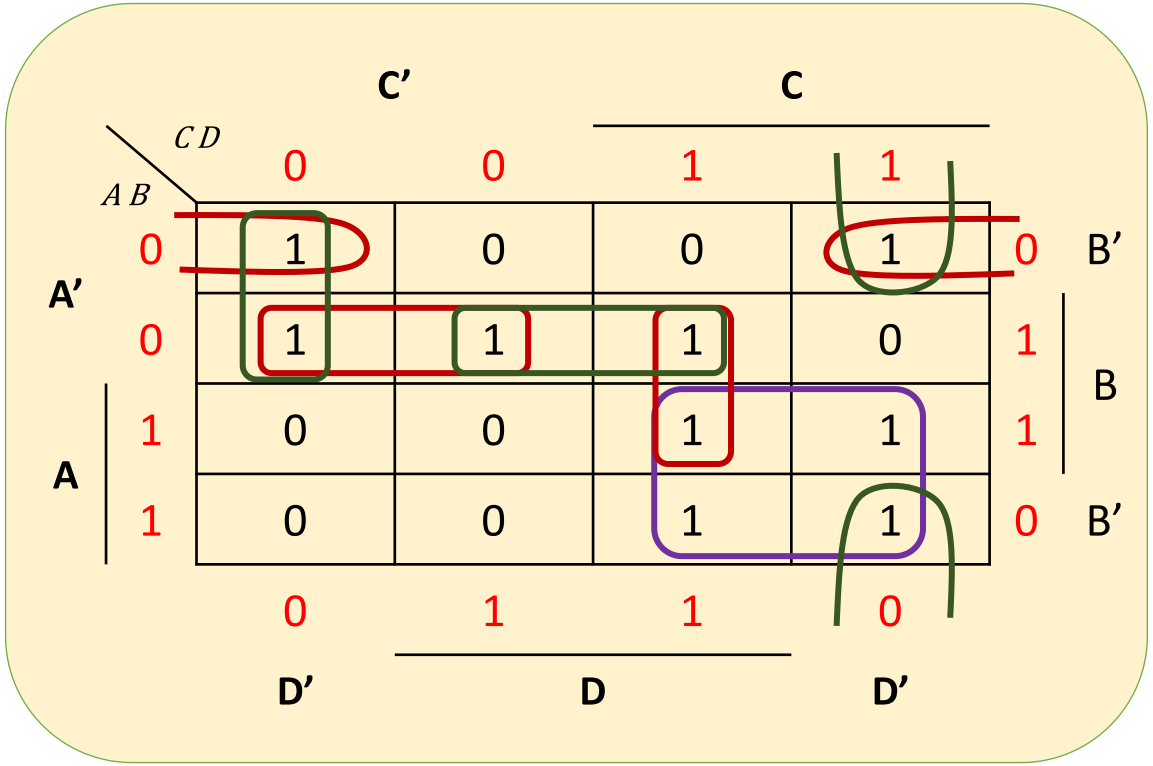
REDUNDANT PRIME IMPLICANTs (RPIs) ; Superfluous PIs, which cannot be used if minimum cover is to result

$$f(A,B,C,D) = \sum m(0,2,4,5,7,10,11,14,15)$$

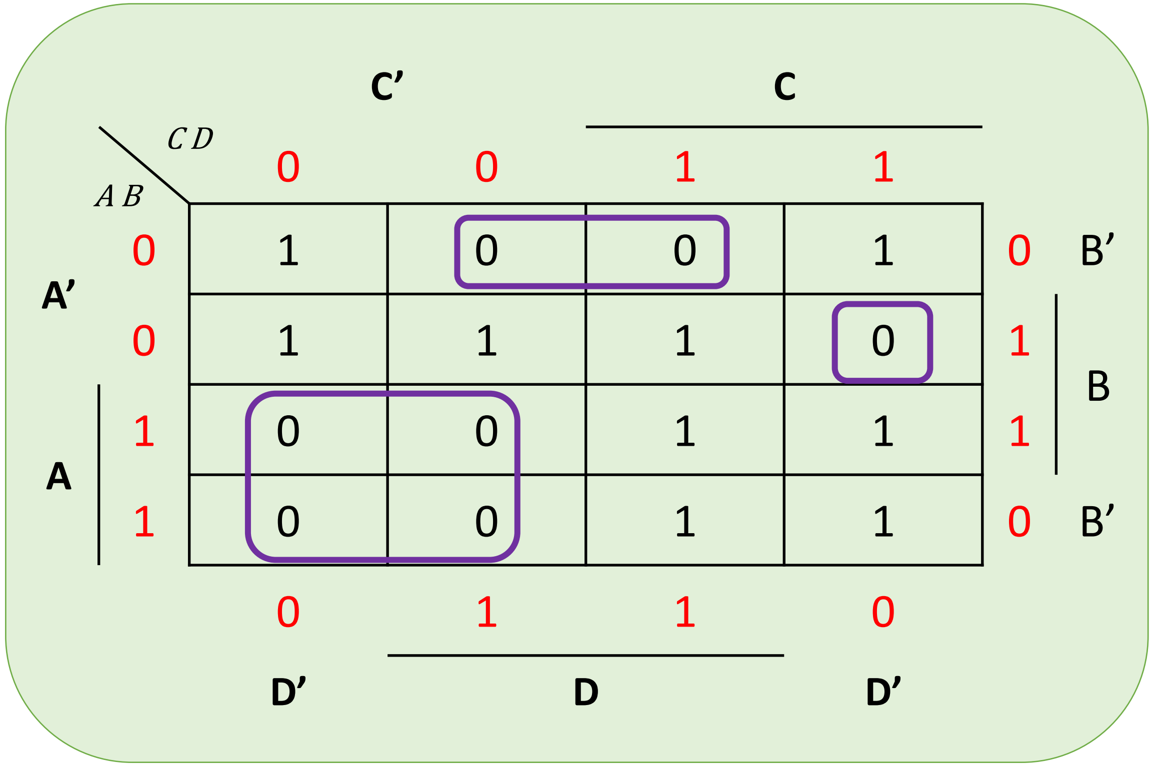
$$f = \bar{A}\bar{B}\bar{D} + \bar{A}B\bar{C} + BCD + AC$$

$$f = \bar{A}\bar{C}\bar{D} + \bar{A}BD + \bar{B}C\bar{D} + AC$$

$$F = (A + \bar{B} + \bar{C} + D)(A + B + \bar{D})(\bar{A} + C)$$



SOP



POS

Don’t Cares “∅” (Non-essetial State)

- Very often, the specification of a function is incomplete
- Output state is unimportant for that particular set of inputs or input state never occurs
- Any input combination whose state is unimportant is a “don’t care” state (*d* in SOP and *D* in POS)
- Useful feature for minimization of states
- Example, with minterms AB’C (101) and ABC’(110) are don’t cares
  - Minterm – $F(A,B,C) = \Sigma m(0,1,2) + \Sigma d(5,6)$
  - Maxterm – $F(A,B,C) = \Pi M(3,4,7) \, . \, \Pi D(5,6)$

$$F(A,B,C,D) = \prod M\,(0,1,4,6,8,14,15)\cdot \emptyset(2,3,9)$$

Essential Maxterms — Nonessential Maxterms

$$F = (\bar{A}+\bar{B}+\bar{C})(A+D)(B+C)$$

$$f = \bar{A}BD + AB\bar{C} + \bar{B}C$$

AB \ CD	00 (C'D')	01 (C'D)	11 (CD)	10 (CD')
00 (A'B')	0	0	∅	∅
01 (A'B)	0	1	1	0
11 (AB)	1	1	0	0
10 (AB')	0	∅	1	1

POS

AB \ CD	00 (C'D')	01 (C'D)	11 (CD)	10 (CD')
00 (A'B')	0	0	∅	∅
01 (A'B)	0	1	1	0
11 (AB)	1	1	0	0
10 (AB')	0	∅	1	1

SOP

Entered Variable (EV)

Compressed form of N Variables K-maps into a K-maps of order n < N, then (N-n) variables must be as Entered Variable

$$Map\ Key = 2^{N-n}$$

1. Loop out all Evs following the loop out protocol

2. Loop Out the 1's for SOP representation or the 0's for POS representation as a "clean up" operation, also following the loop out protocol

Decimal		A	B	C	f
0	0	0	0	0	0
	1	0	0	1	0
1	2	0	1	0	1
	3	0	1	1	0
2	4	1	0	0	0
	5	1	0	1	1
3	6	1	1	0	1
	7	1	1	1	1

C		
0	0	0
1	0	

C		
0	1	$\bar{C}$
1	0	

C		
0	0	C
1	1	

C		
0	1	1
1	1	

		B'		B	
$A \backslash BC$		0	0	1	1
A'	0	0	0	0	1
A	1	**0**	1	1	1
		0	1	1	0
		C'	C		C'

A	B	f
0	0	0
0	1	C'
1	0	C
1	1	1

$A \backslash B$	0	1
0	0	C'
1	C	1

$$Map\ Key = 2^{N-n}$$

$$Map\ Key = 2^{3-2}$$

$$Map\ Key = 2$$

$$f = AC + B\bar{C}$$

$$F = (A + \bar{C})(B + C)$$

Decimal		*A*	*B*	*C*	*f*
0	0	0	0	0	0
	1	0	0	1	0
1	2	0	1	0	1
	3	0	1	1	0
2	4	1	0	0	0
	5	1	0	1	1
3	6	1	1	0	1
	7	1	1	1	1

C		
0	0	0
1	0	

C		
0	1	$\bar{C}$
1	0	

C		
0	0	C
1	1	

C		
0	1	1
1	1	

		B'		B	
	$A \backslash BC$	0 0	0 1	1 1	1 0
A'	0	0	0	0	1
A	1	**0**	1	1	1
		0	1	1	0
		C'	C	C	C'

A	*B*	*f*
0	0	0
0	1	C'
1	0	C
1	1	1

$A \backslash B$	0	1
0	0	C'
1	C	1

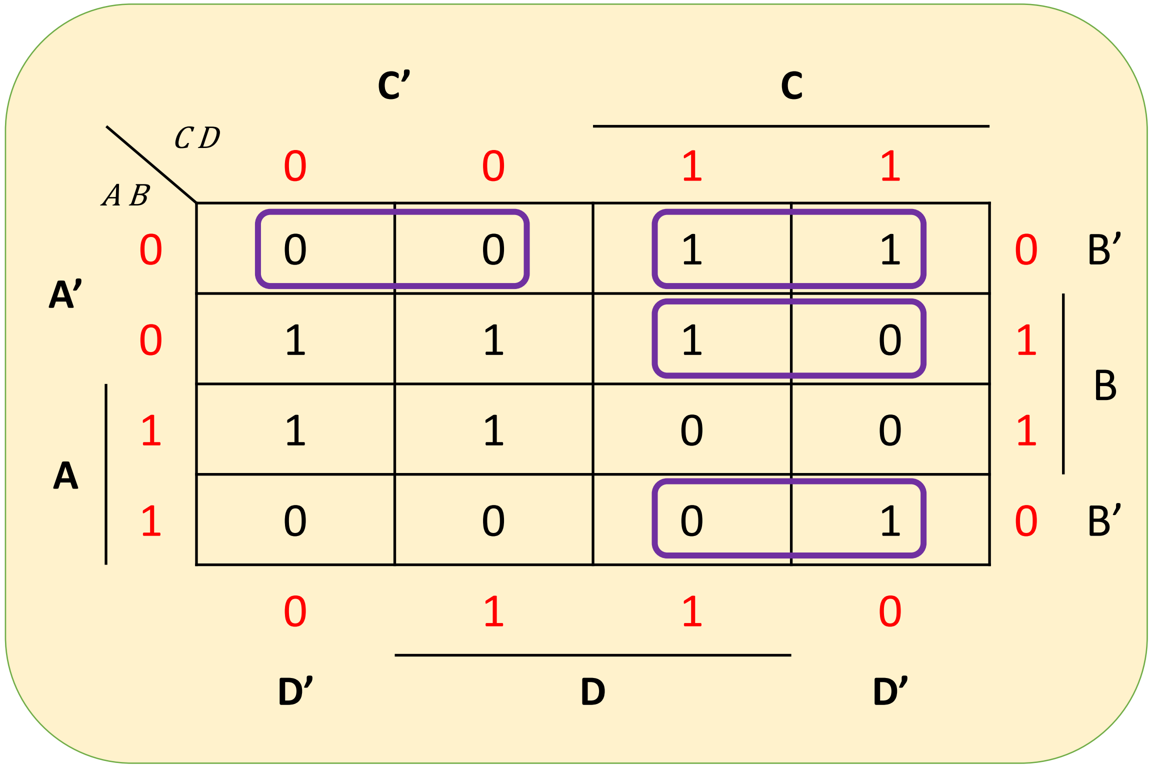
$$Map\ Key = 2^{N-n}$$

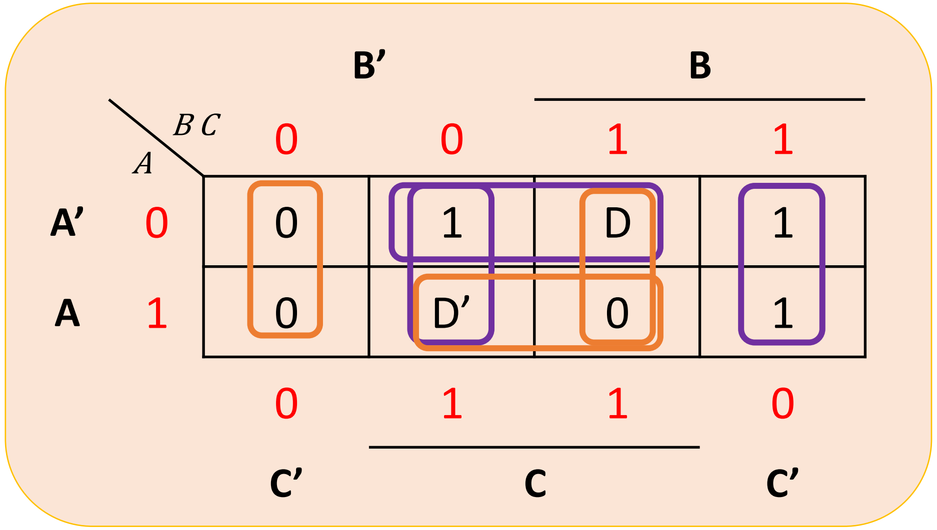
$$Map\ Key = 2^{3-2}$$

$$Map\ Key = 2$$

$$f = AC + B\bar{C}$$

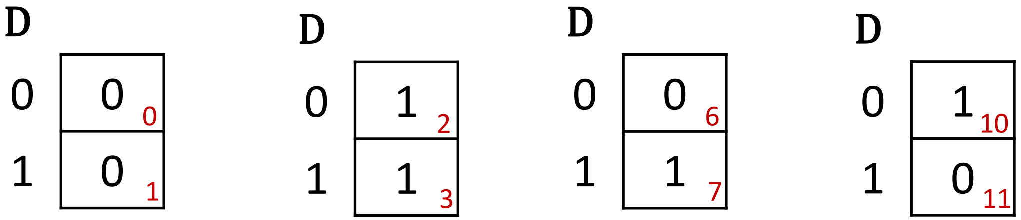
$$F = (A + \bar{C})(B + C)$$





$$f = \bar{A}CD + \bar{B}C\bar{D} + B\bar{C}$$

$$F = (\bar{A} + \bar{C} + \bar{D})\,(\bar{B} + \bar{C} + D)\,(B + C)$$



		C′	C′	C	C		
$AB \backslash CD$		0	0	1	1		
A′	0	0	0	1	1	0	B′
A′	0	1	1	1	0	1	B
A	1	1	1	0	0	1	B
A	1	0	0	0	1	0	B′
		0	1	1	0		
		D′	D	D	D′		