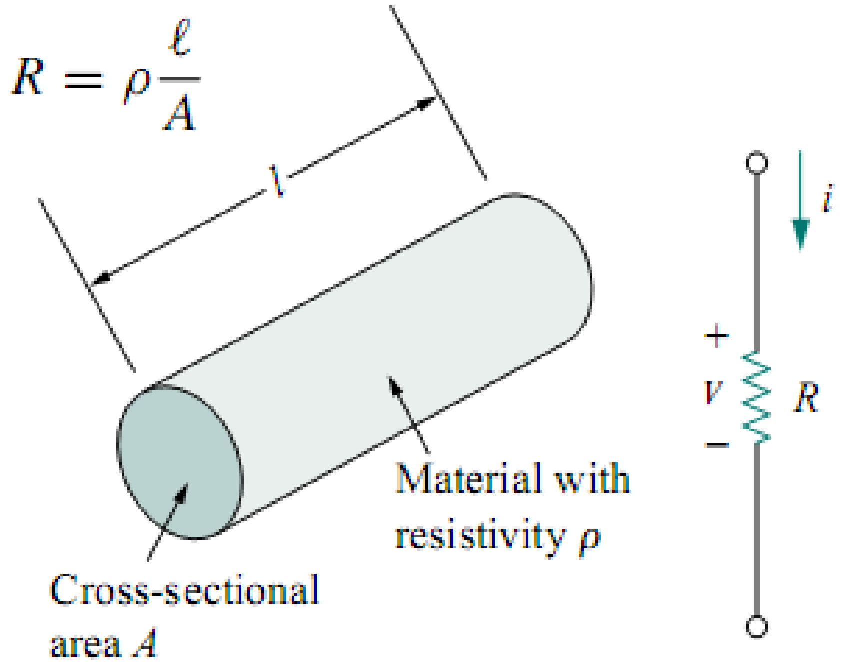
Basic Laws
(Part 1)

Georg Simon Ohm (1787–1854), a German physicist, in 1826 experimentally determined the most basic law relating voltage and current for a resistor.

Gustav Robert Kirchhoff (1824–1887), a German physicist, stated two basic laws in 1847 concerning the relationship between the currents and voltages in an electrical network. Kirchhoff’s laws, along with Ohm’s law, form the basis of circuit theory.

Ohm's Law

Materials in general have a characteristic behavior of resisting the flow of electric charge.This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol *R*.



Material	Resistivity ($\Omega \cdot m$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

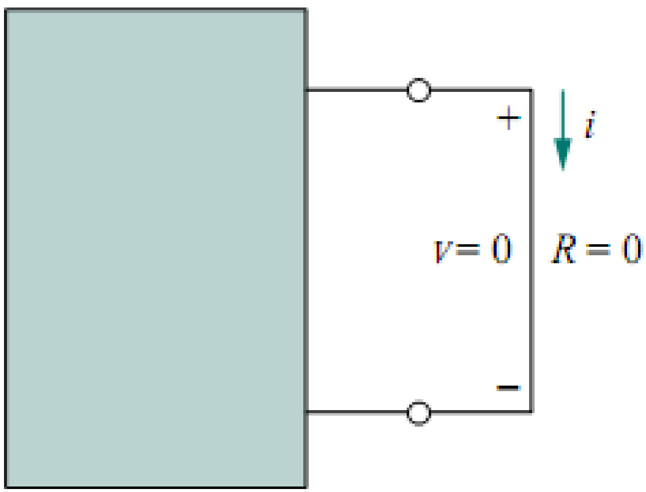
$$\boxed{v = iR}$$

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

$$R = \frac{v}{i}$$

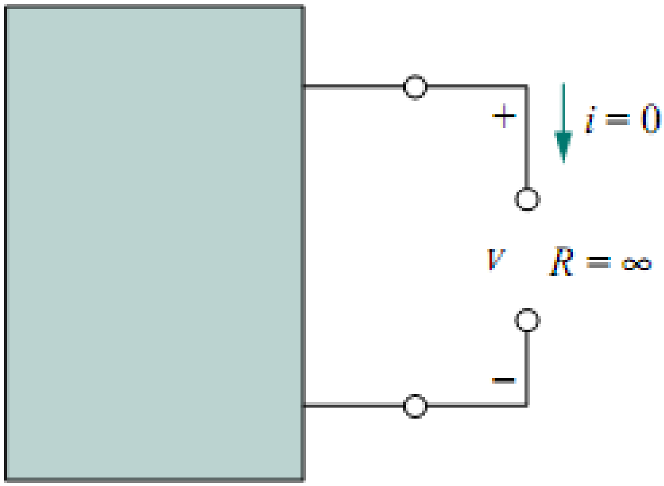
$$1\ \Omega = 1\ \mathrm{V/A}$$

A **short circuit** is a circuit element with resistance approaching zero.

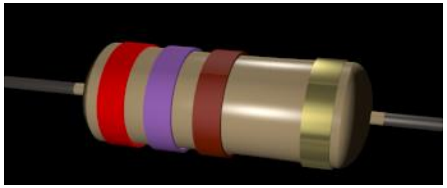


$$v = iR = 0$$

An open circuit is a circuit element with resistance approaching infinity.

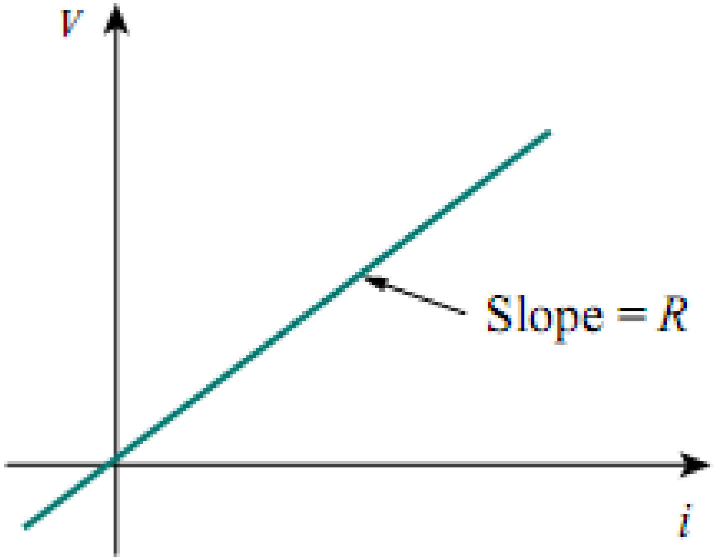


$$i = \lim_{R \to \infty} \frac{v}{R} = 0$$



Tolerance	Colour	
± 5%	Gold	
± 10%	Silver	
± 20%	No band	

Number	Colour	
0	Black	
1	Brown	
2	Red	
3	Orange	
4	Yellow	
5	Green	
6	Blue	
7	Violet	
8	Gray	
9	White	



(a)

A useful quantity in circuit analysis is the reciprocal of resistance *R*, known as *conductance* and denoted by *G*

$$\boxed{G = \frac{1}{R} = \frac{i}{v}}$$

Conductance is the ability of an element to conduct electric current; it is measured in mhos (℧) or siemens (S).

$$i = Gv$$

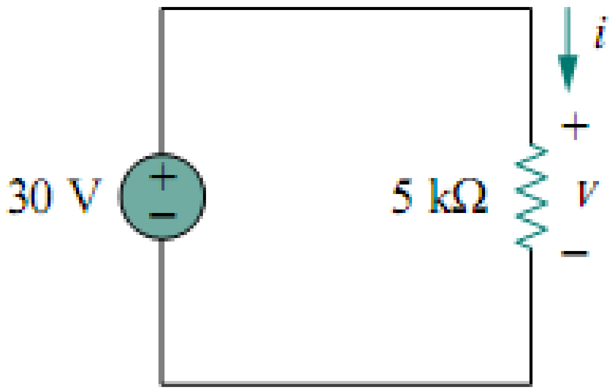
The power dissipated by a resistor can be expressed in terms of *R*

$$p = vi = i^2 R = \frac{v^2}{R}$$

The power dissipated by a resistor may also be expressed in terms of *G*

$$p = vi = v^2 G = \frac{i^2}{G}$$

Example



In the circuit, calculate the current *i*, the conductance *G*, and the power *p*.

Solution

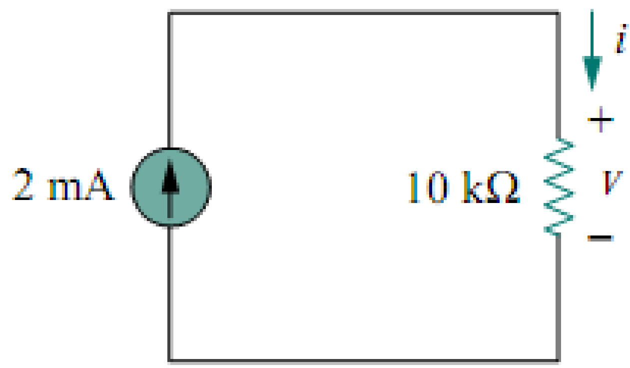
$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

Practice Problem

For the circuit below, calculate the voltage *v*, the conductance *G*, and the power *p*.



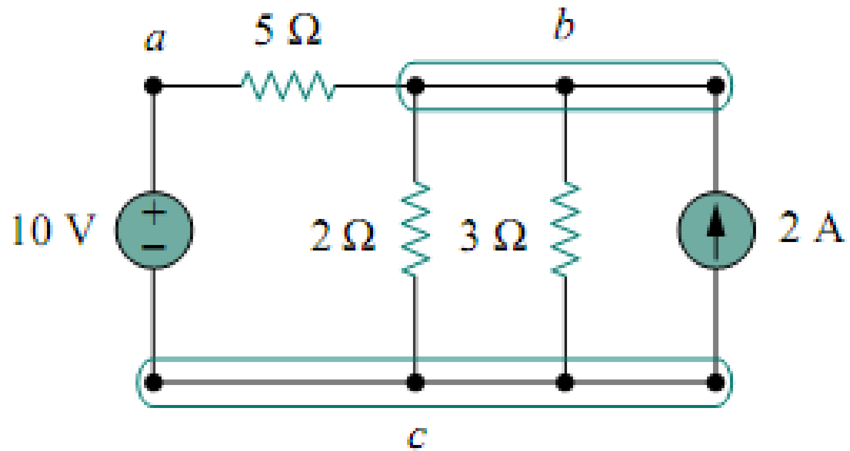
Nodes, Branches, and Loop

Since the elements of an electric circuit can be interconnected in several ways, we need to understand some basic concepts of network topology. In network topology, we study the properties relating to the placement of elements in the network and the geometric configuration of the network. Such elements include branches, nodes, and loops.

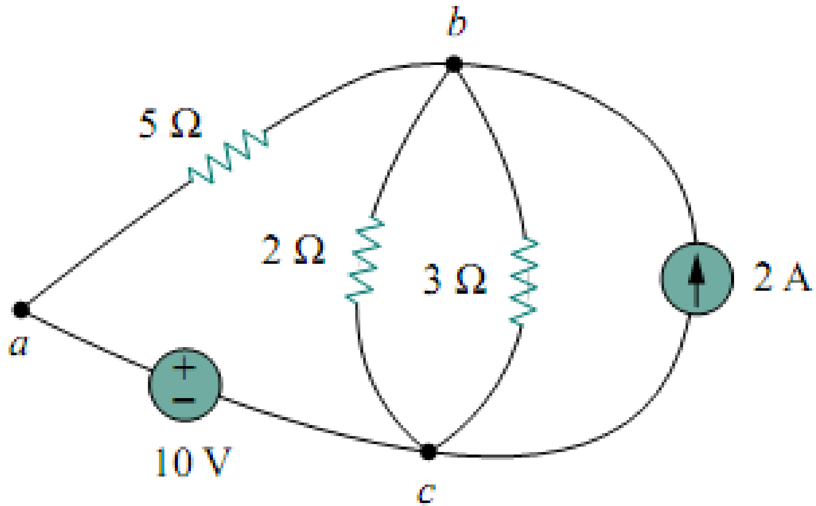
A **branch** represents a single element such as a voltage source or a resistor.

A **node** is the point of connection between two or more branches.

A **loop** is any closed path in a circuit.

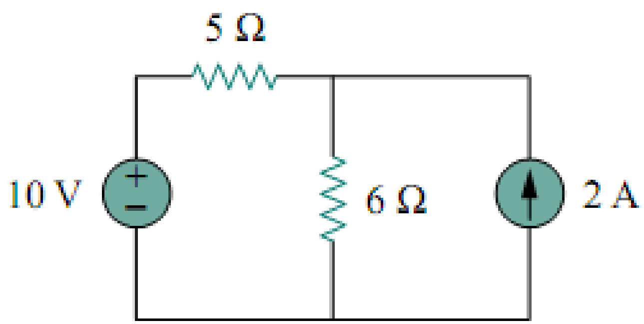


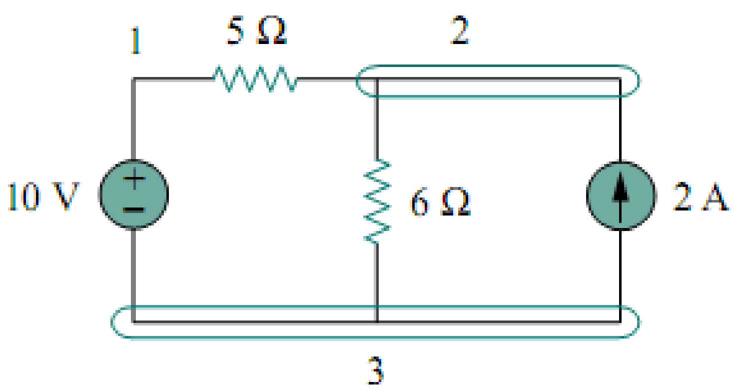
$$\boxed{b = l + n - 1}$$



Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.

Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

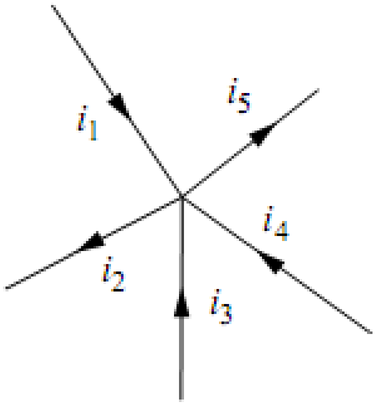




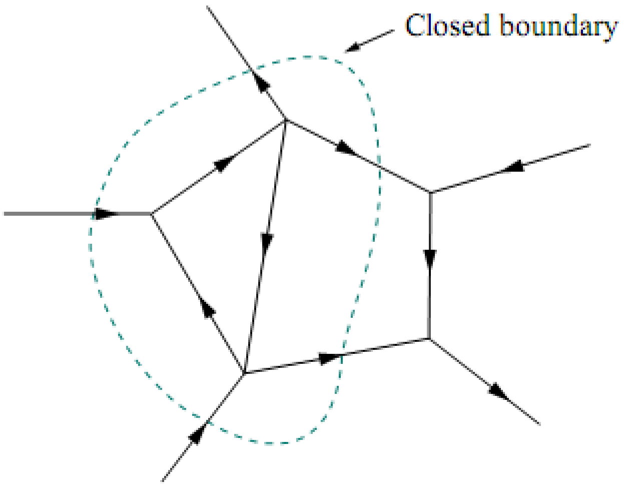
Kirchhoff's Laws

Kirchhoff's Current Law (KCL)

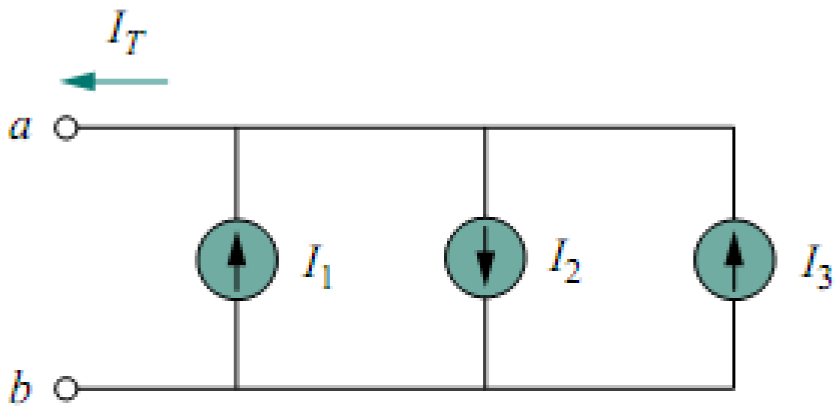
Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.



$$\boxed{\sum_{n=1}^{N} i_n = 0}$$



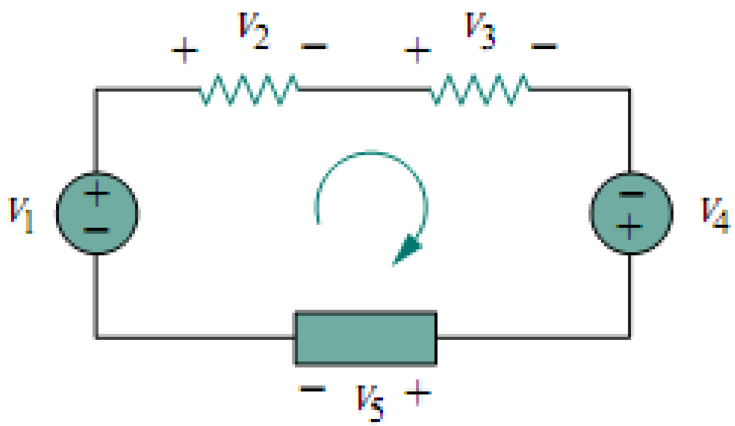
The sum of the currents entering a node is equal to the sum of the currents leaving the node.



$$I_T = I_1 - I_2 + I_3$$

Kirchhoff's Voltage Law (KVL)

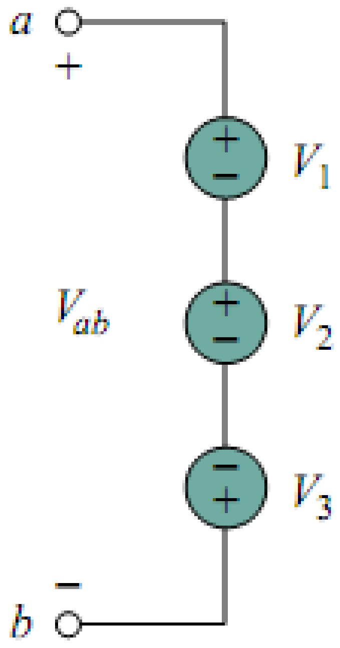
Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.



$$\sum_{m=1}^{M} v_m = 0$$

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

When voltage sources are connected in series, KVL can be applied to obtain the total voltage.

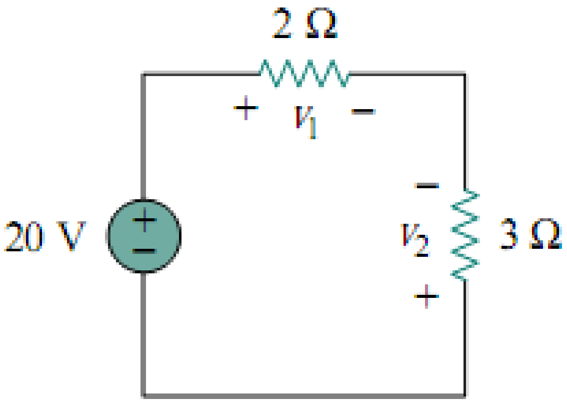


$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

$$V_{ab} = V_1 + V_2 - V_3$$

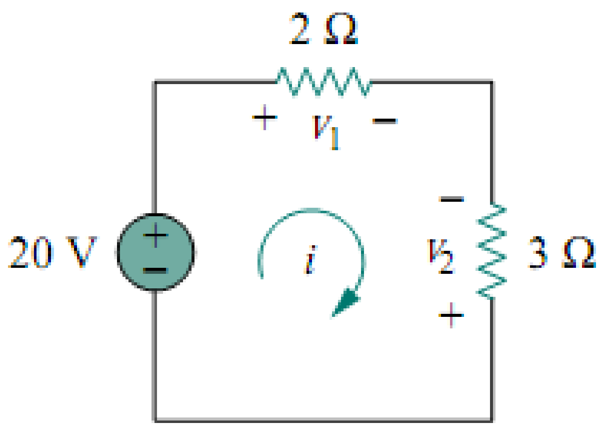
Examples

1. Find voltages v_1 and v_2.



Solution

To find v_1 and v_2, we apply Ohm's law and Kirchhoff's voltage law. Assume that current *i* flows through the loop.



$$v_1 = 2i, \qquad v_2 = -3i \tag{1}$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \qquad (2)$$

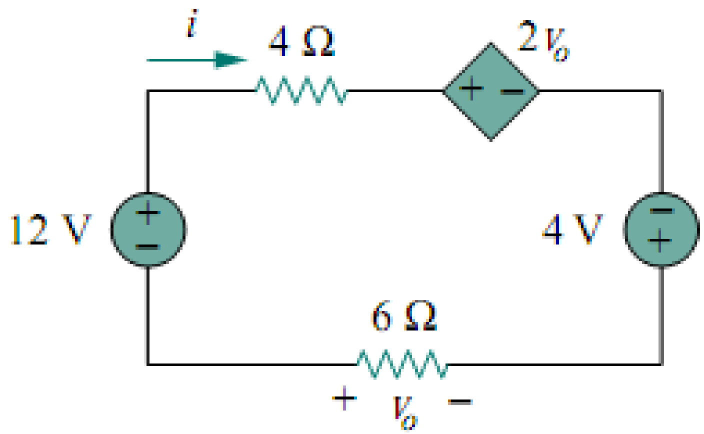
Substituting Eq.(1) into Eq.(2), we obtain

$$-20 + 2i + 3i = 0 \quad \text{or} \quad 5i = 20 \quad \Longrightarrow \quad i = 4\text{ A}$$

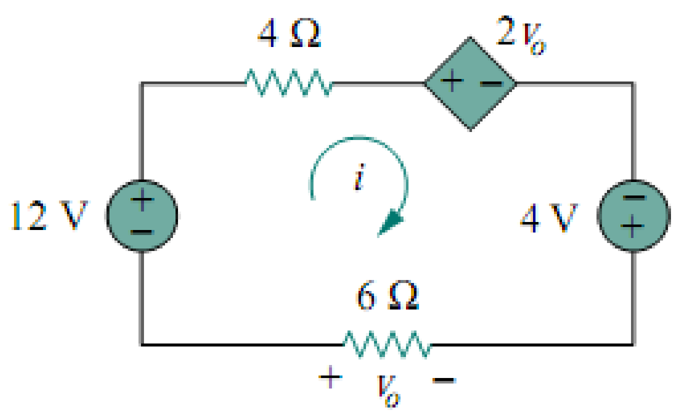
Substituting *i* in Eq.(1) finally gives

$$v_1 = 8\text{ V}, \qquad v_2 = -12\text{ V}$$

2. Find current *i* and voltage v_o.



Solution



$$-12 + 4i + 2v_o - 4 + 6i = 0 \quad (1)$$

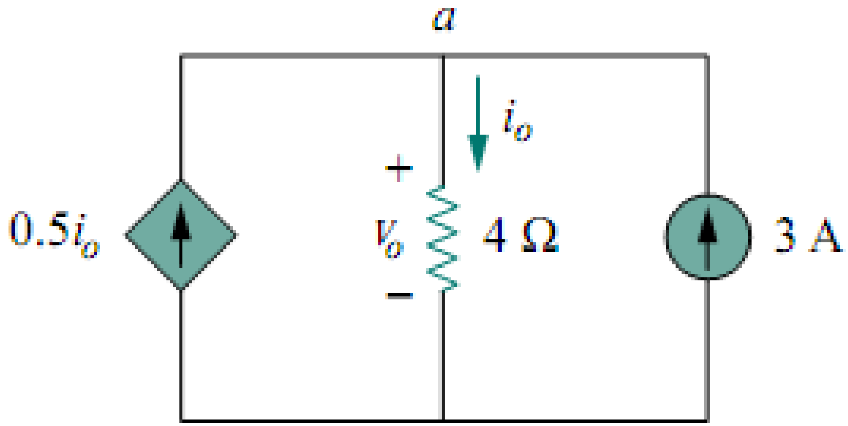
$$v_o = -6i \quad (2)$$

From Eq.(1) and Eq.(2),

$$-16 + 10i - 12i = 0 \quad \Longrightarrow \quad i = -8 \text{ A}$$

$$v_o = 48 \text{ V}.$$

3. Find current i_o and voltage v_o.



Solution

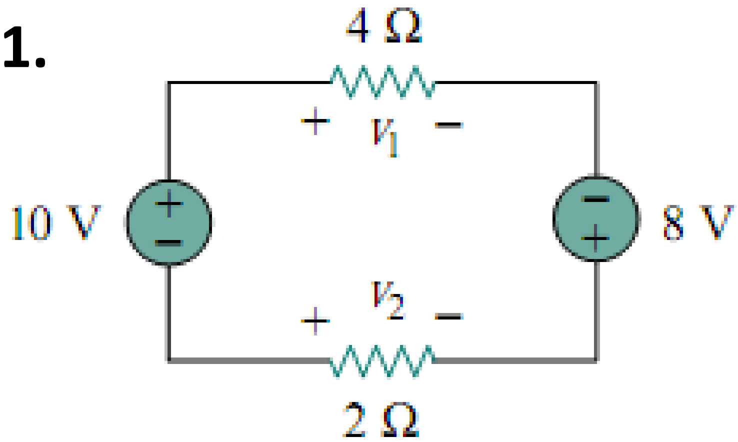
Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \quad\Longrightarrow\quad i_o = 6 \text{ A}$$

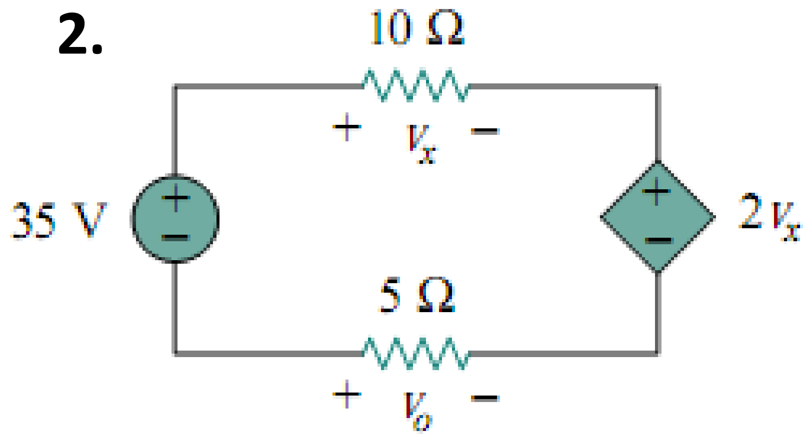
Practice Problems

Find voltages and currents in circuits below

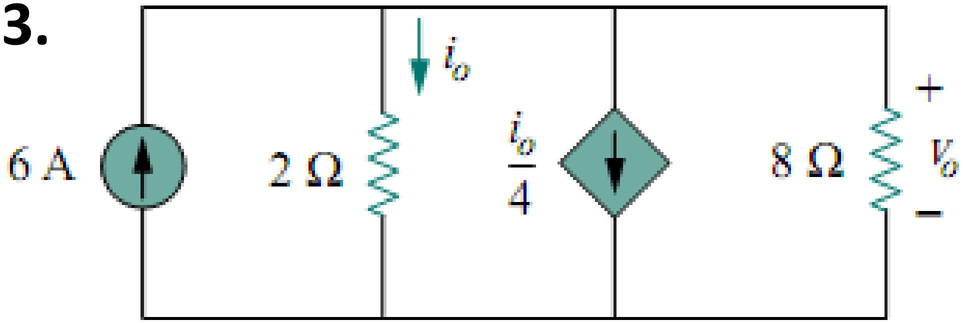
1.



2.



3.



Selected Problems

Determine the number of branches and nodes in the circuit in Fig. 2.71.

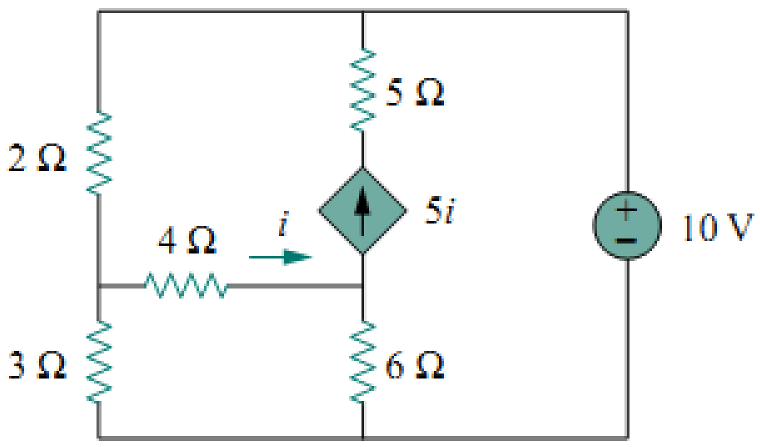


Figure 2.71 For Prob. 2.7.

Obtain v_1 through v_3 in the circuit of Fig. 2.78.

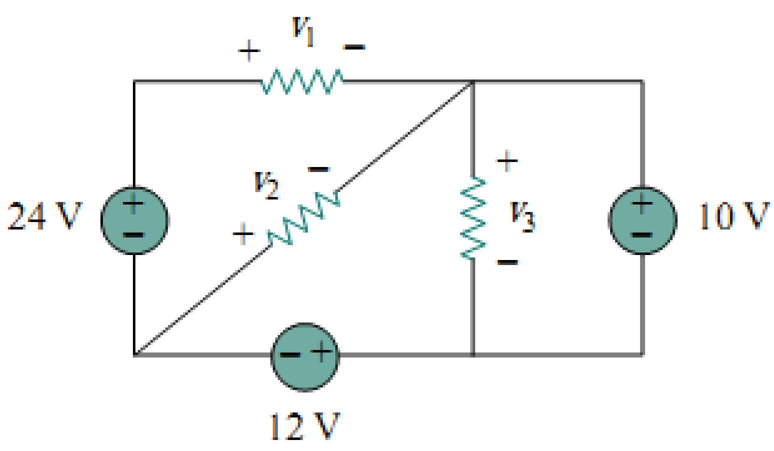


Figure 2.78 For Prob. 2.14.

Find I and V_{ab} in the circuit of Fig. 2.79.

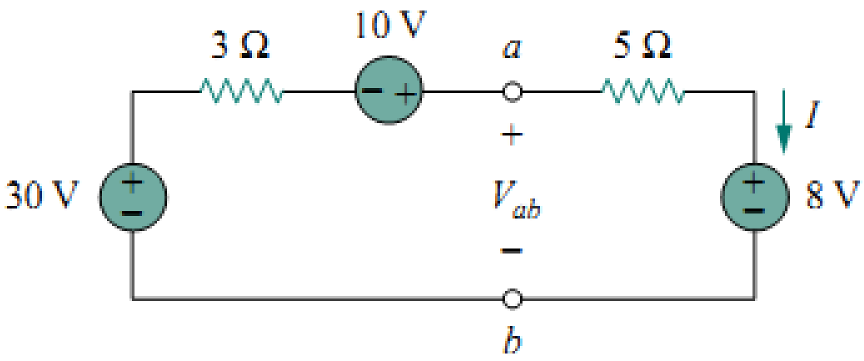


Figure 2.79 For Prob. 2.15.

Find V_o in the circuit in Fig. 2.83 and the power dissipated by the controlled source.

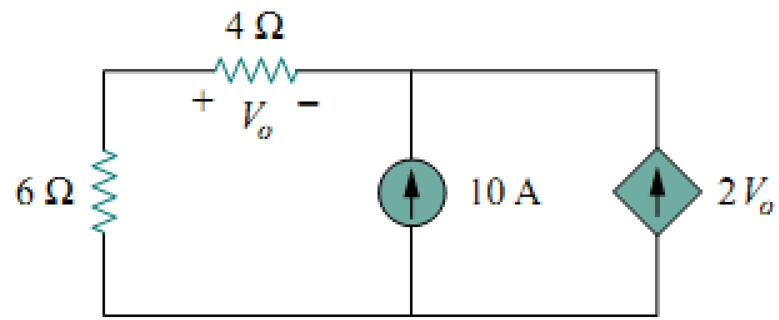


Figure 2.83 For Prob. 2.19.

2.21 For the network in Fig. 2.85, find the current, voltage, and power associated with the 20-kΩ resistor.

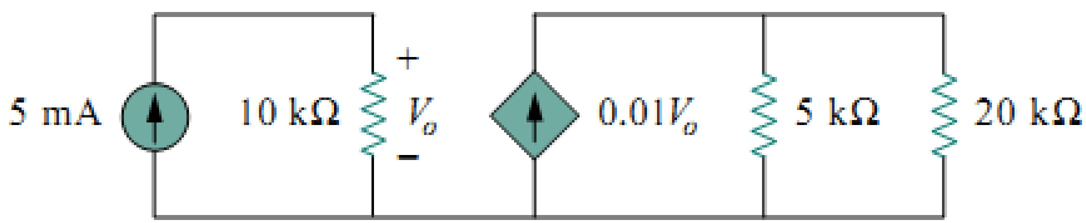


Figure 2.85 For Prob. 2.21.

For the circuit in Fig. 2.86, find i_1 and i_2.

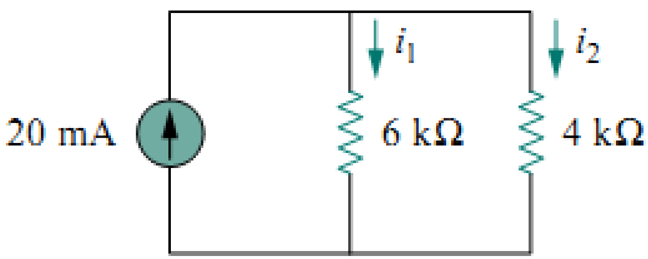


Figure 2.86 For Prob. 2.22.

Find v_1 and v_2 in the circuit in Fig. 2.87.

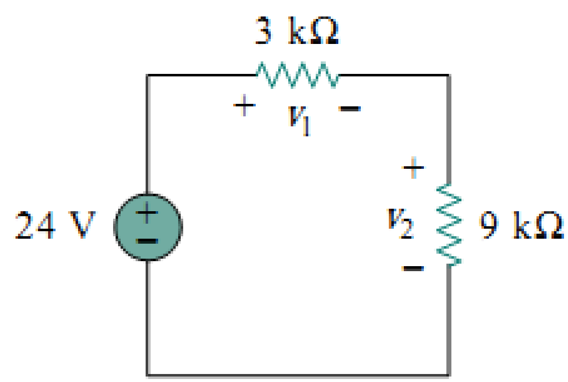


Figure 2.87 For Prob. 2.23.

Find V_o and I_o in the circuit of Fig. 2.96.

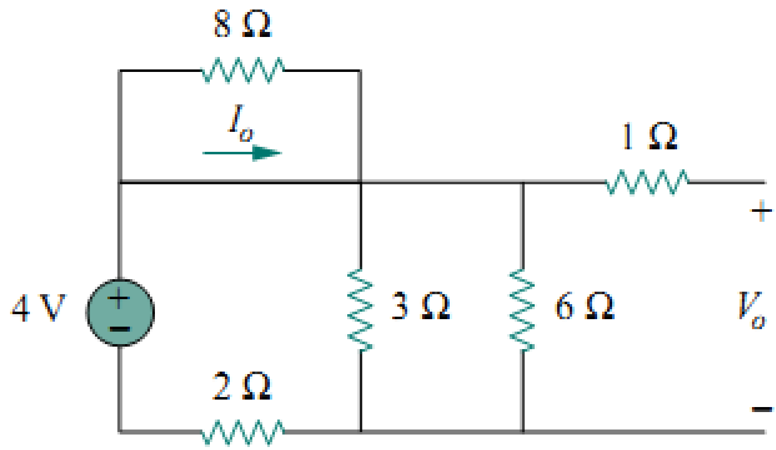


Figure 2.96 For Prob. 2.32.