DC Circuits

Methods of Analysis

Introduction

Having understood the fundamental laws of circuit theory (Ohm’s law and Kirchhoff’s laws), we are now prepared to apply these laws to develop two powerful techniques for circuit analysis: **nodal analysis**, which is based on a systematic application of Kirchhoff’s current law (KCL), and **mesh analysis**, which is based on a systematic application of Kirchhoff’s voltage law (KVL).

With the two techniques to be developed in this chapter, we can analyze almost any circuit by obtaining a set of simultaneous equations that are then solved to obtain the required values of current or voltage.

Nodal Analysis

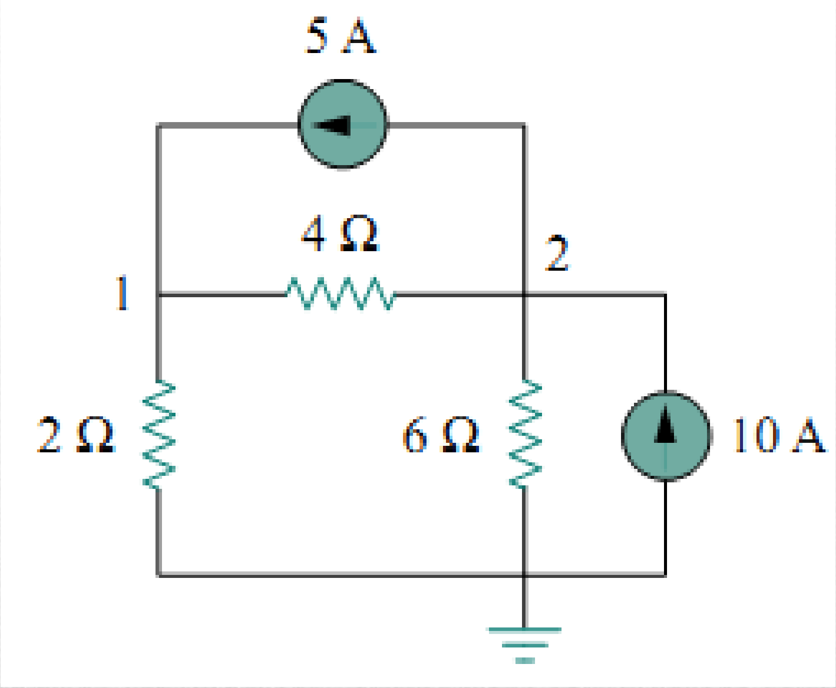
Nodal analysis provides a general procedure for analyzing circuits **using node voltages as the circuit variables**.

Steps to Determine Node Voltages:

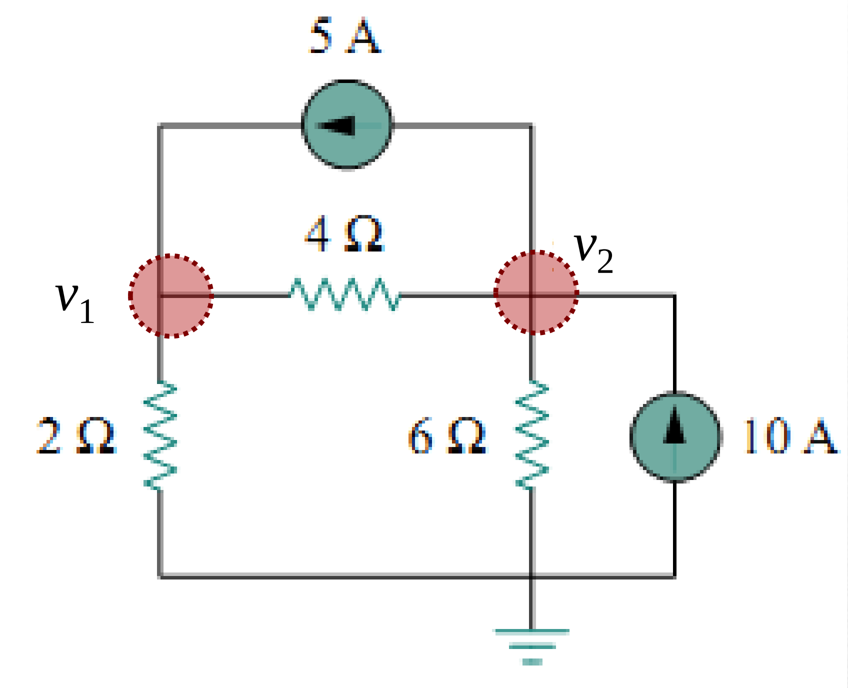
1. Select a node as the reference node. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Example

1. Calculate the node voltages in the circuit below.



Solution



At node 1

$$\frac{v_1}{2} + \frac{v_1 - v_2}{4} - 5 = 0$$

$$\left(\frac{1}{2} + \frac{1}{4}\right) v_1 - \frac{1}{4} v_2 = 5$$

$$\boxed{3v_1 - v_2 = 20} \quad (1)$$

At node 2

$$\frac{v_2}{6} + \frac{v_2 - v_1}{4} + 5 - 10 = 0$$

$$-\frac{1}{4} v_1 + \left(\frac{1}{6} + \frac{1}{4}\right) v_2 = 5$$

$$\boxed{-3v_1 + 5v_2 = 60} \quad (2)$$

Add Equation (1) and (2)

$$3v_1 - v_2 = 20$$
$$-3v_1 + 5v_2 = 60$$
$$+$$

$$4v_2 = 80$$

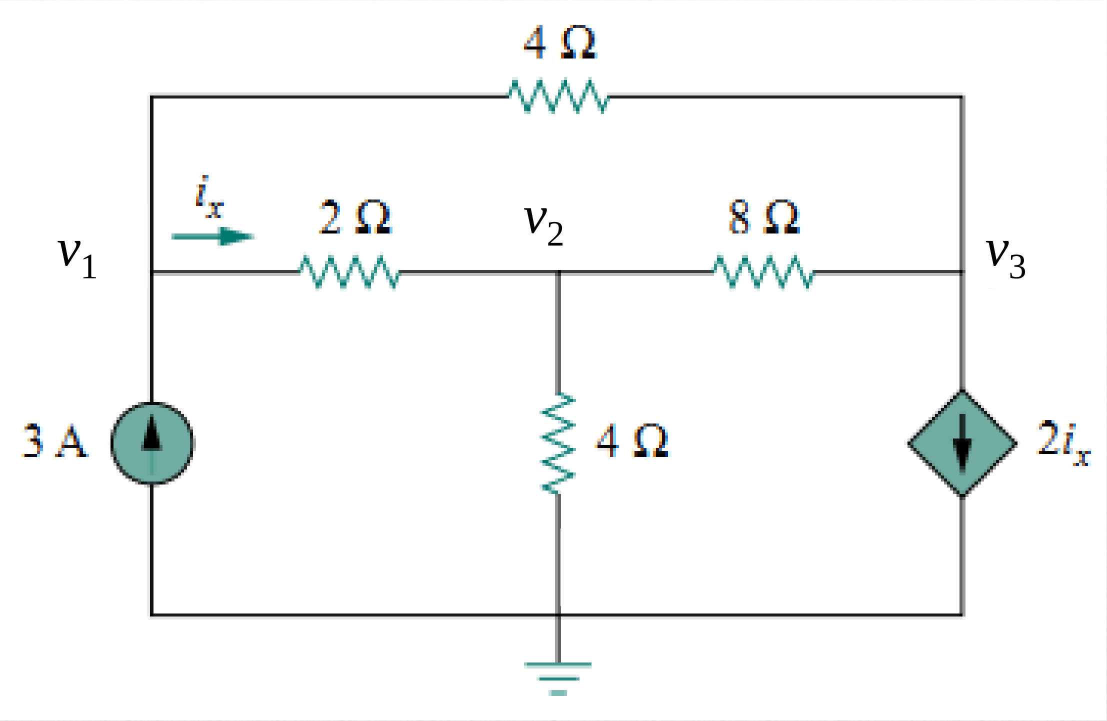
$$\boxed{v_2 = 20 \text{ V}}$$

Subtitute v_2 to Equation (1)

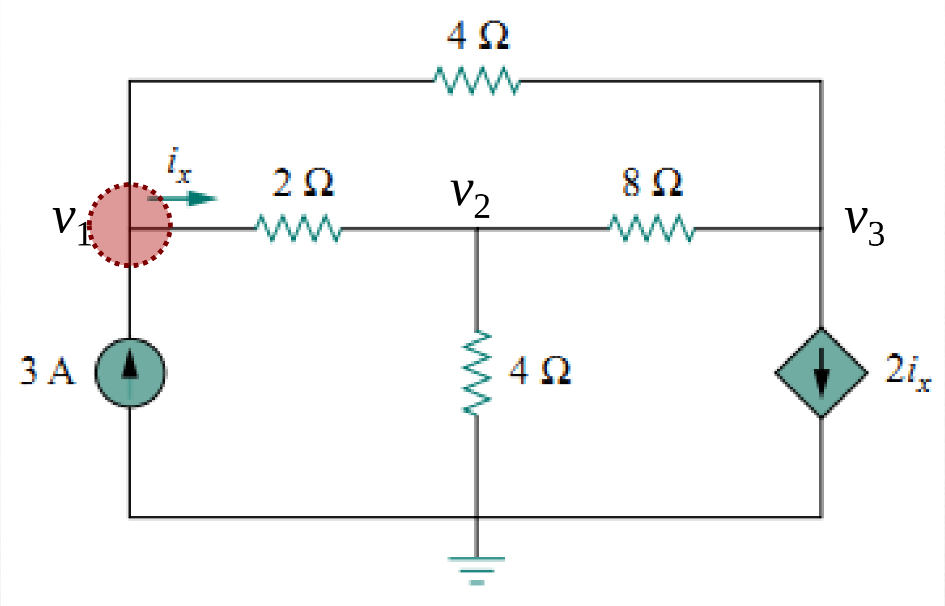
$$3v_1 - 20 = 20$$

$$\boxed{v_1 = 13{,}33 \text{ V}}$$

2. Find the simultaneous equations for node voltages in the circuit below.



Solution

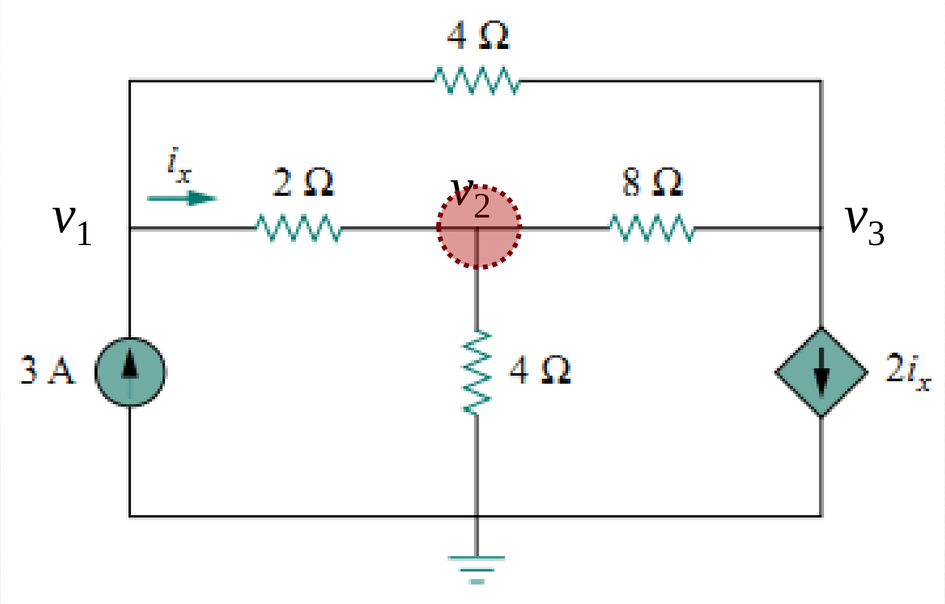


At node 1

$$\left(\frac{1}{2}+\frac{1}{4}\right)v_1-\frac{1}{2}v_2-\frac{1}{4}v_3-3=0$$

$$\frac{3}{4}v_1-\frac{1}{2}v_2-\frac{1}{4}v_3=3$$

$$\boxed{3v_1-2v_2-v_3=12}$$

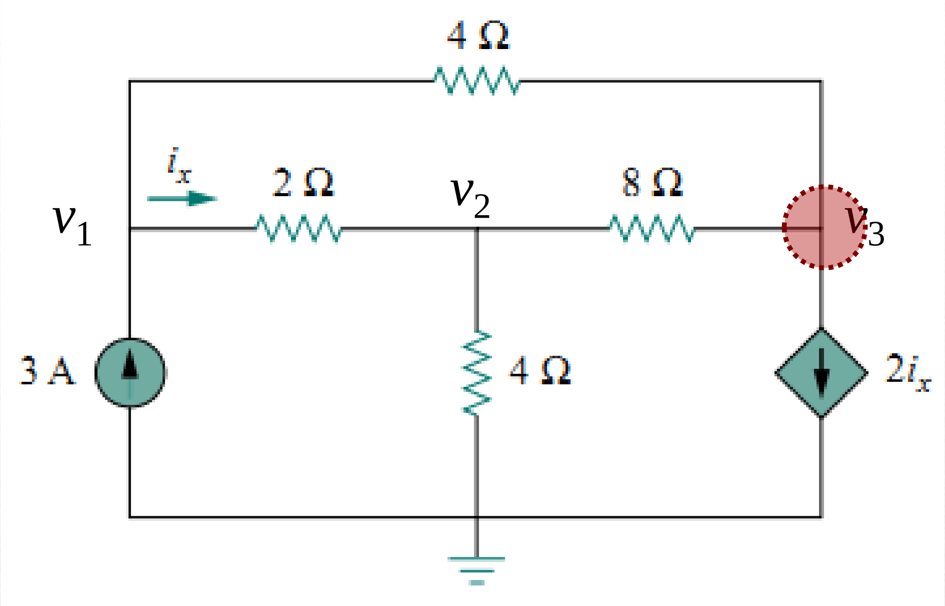


At node 2

$$-\frac{1}{2}v_1 + \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)v_2 - \frac{1}{8}v_3 = 0$$

$$-\frac{1}{2}v_1 + \frac{7}{8}v_2 - \frac{1}{8}v_3 = 0$$

$$\boxed{-4v_1 + 7v_2 - v_3 = 0}$$



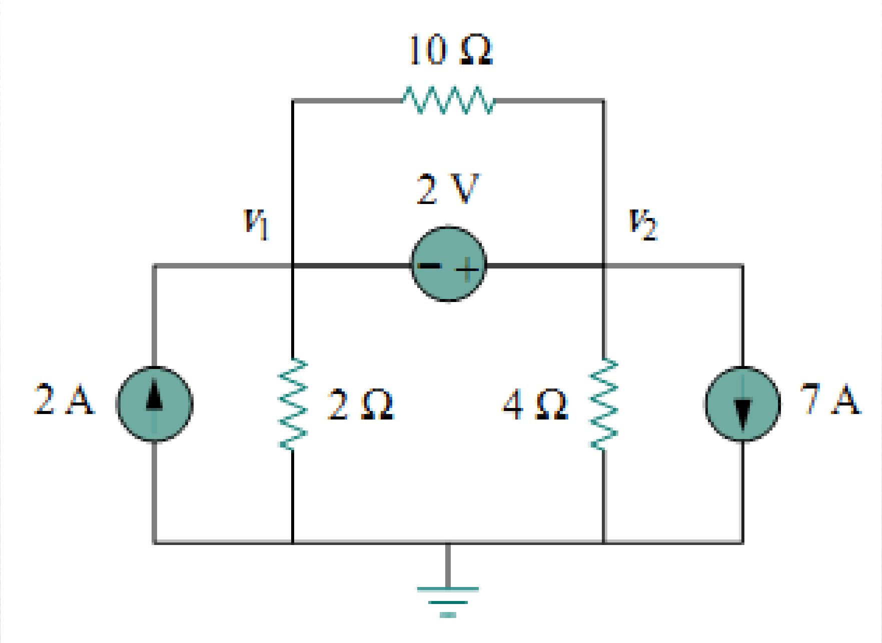
At node 3

$$-\frac{1}{4}v_1 - \frac{1}{8}v_2 + \left(\frac{1}{4} + \frac{1}{8}\right)v_3 + 2\left(\frac{v_1 - v_2}{2}\right) = 0$$

$$\frac{3}{4}v_1 - \frac{9}{8}v_2 + \frac{3}{8}v_3 = 0$$

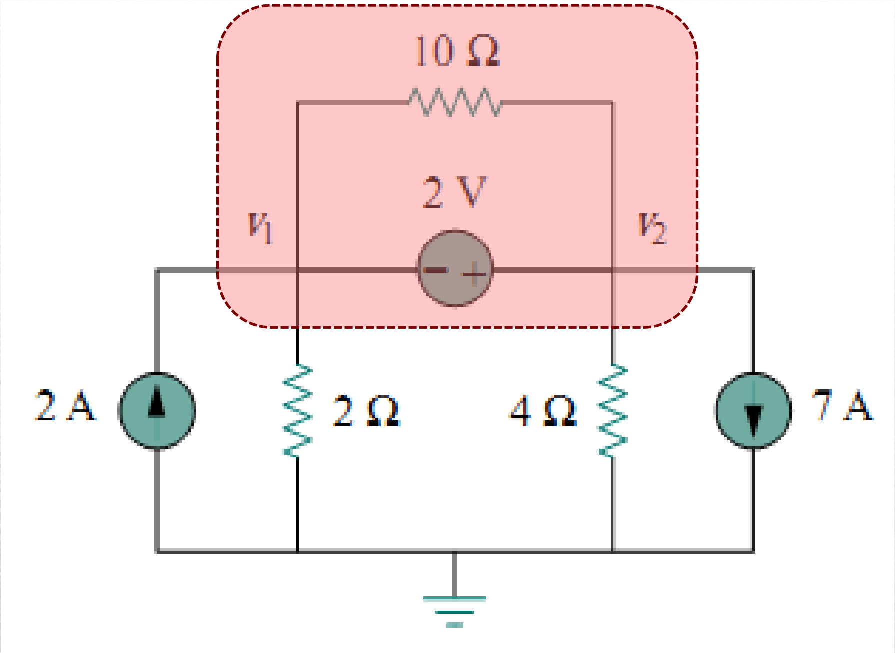
$$\boxed{2v_1 - 3v_2 + v_3 = 0}$$

3. For the circuit shown in figure below, find the node voltages.



A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

Solution



Voltage source in the supernode

$$\boxed{-v_1 + v_2 = 2} \qquad (1)$$

Applying KCL to the supernode

$$\frac{1}{2}v_1 + \frac{1}{4}v_2 - 2 + 7 = 0$$

$$\frac{1}{2}v_1 + \frac{1}{4}v_2 = -5$$

$$\boxed{2v_1 + v_2 = -20} \qquad (2)$$

Suctract Equation (2) from (1)

$$-v_1 + v_2 = 2$$
$$2v_1 + v_2 = -20 \quad -$$

$$-3v_1 = 22$$

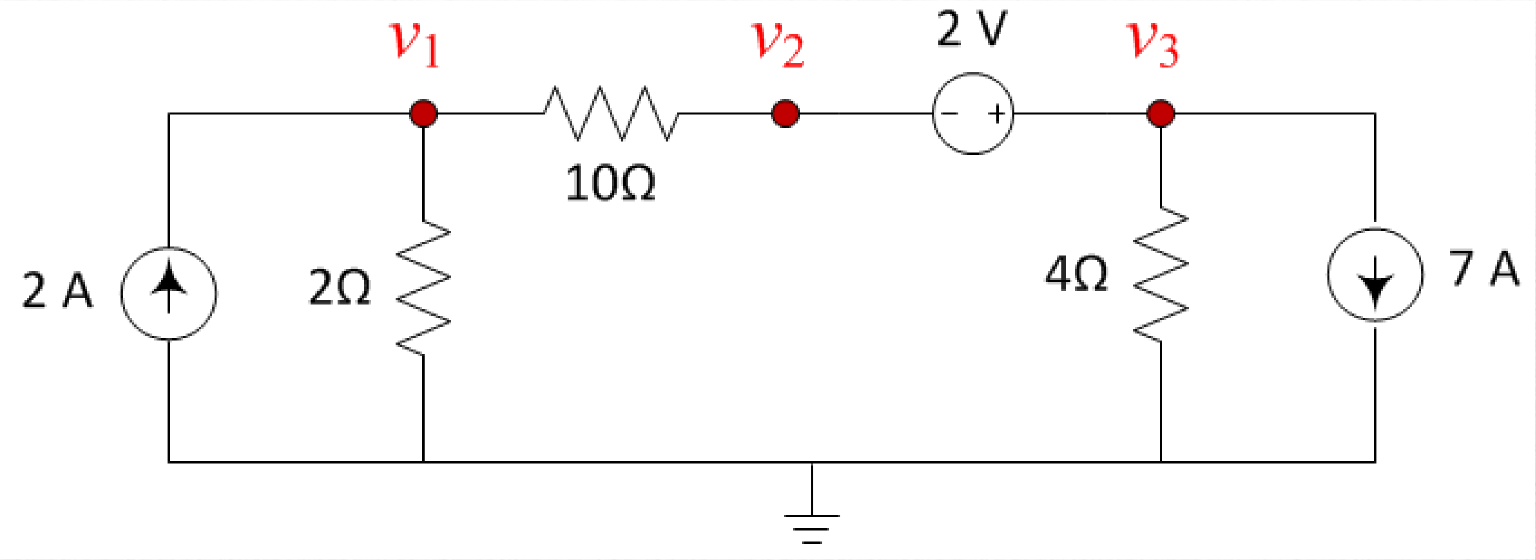
$$\boxed{v_1 = -7{,}33 \text{ V}}$$

Subtitute v_1 to Equation (1)

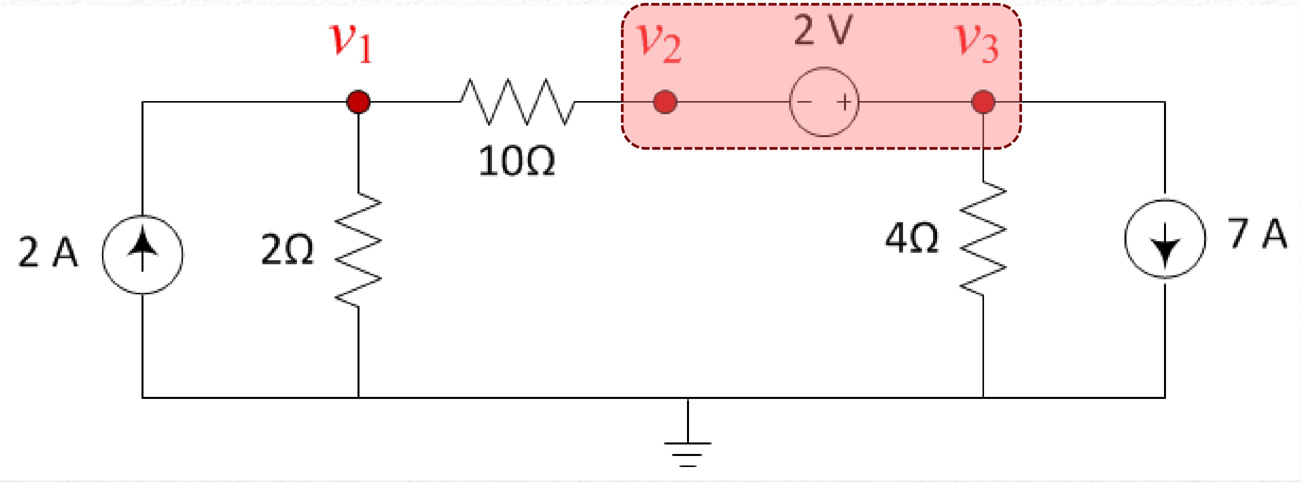
$$-(-7{,}33) + v_2 = 2$$

$$\boxed{v_2 = -5{,}33 \text{ V}}$$

4. For the circuit shown in figure below, find the simultaneous equation for the node voltages.



Solution



Voltage source

$$\boxed{-v_2 + v_3 = 2} \quad (1)$$

KCL at node 1

$$\left(\frac{1}{2} + \frac{1}{10}\right) v_1 - \frac{1}{10} v_2 - 2 = 0$$

$$\boxed{6v_1 - v_2 = 20} \quad (2)$$

KCL at supernode

$$-\frac{1}{10} v_1 + \frac{1}{10} v_2 + \frac{1}{4} v_3 + 7 = 0$$

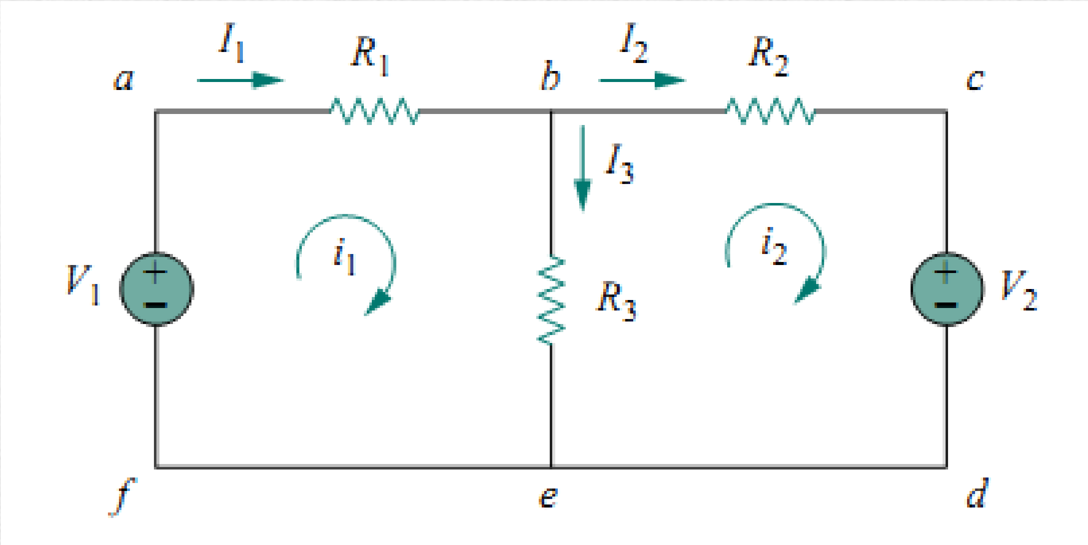
$$-\frac{2}{20} v_1 + \frac{2}{20} v_2 + \frac{5}{20} v_3 = -7$$

$$\boxed{-2v_1 + 2v_2 + 5v_3 = -140} \quad (3)$$

Mesh Analysis

Mesh analysis provides another general procedure for analyzing circuits, **using mesh currents as the circuit variables**.

A mesh is a loop which does not contain any other loops within it.

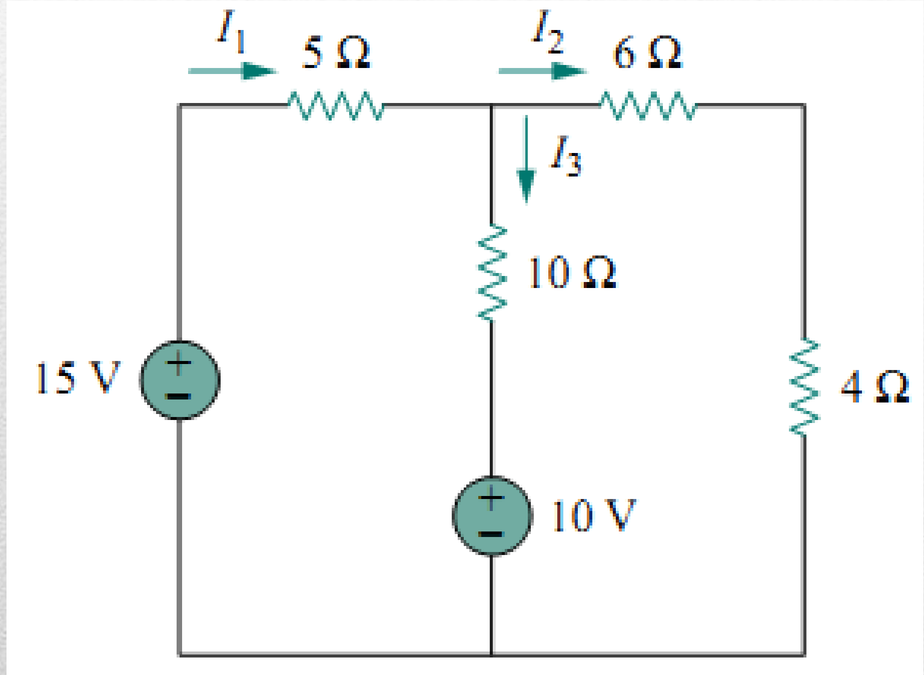


Steps to Determine Mesh Currents:

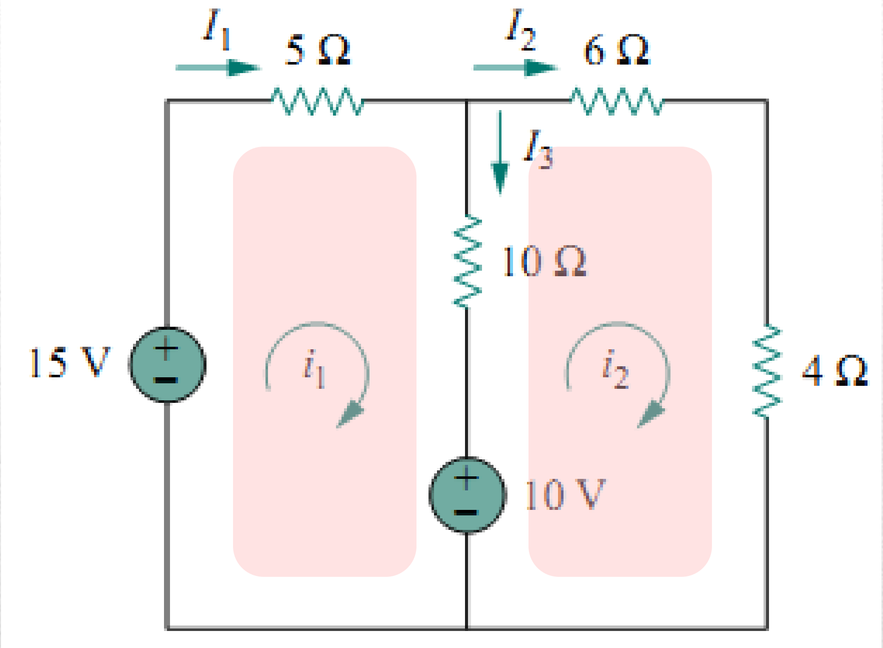
1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Example

1. For the circuit in Figure below, find the branch currents I_1, I_2, and I_3 using mesh analysis.



Solution



For mesh 1

$$(5+10)\ i_1 - 10\ i_2 + 10 - 15 = 0$$

$$15\ i_1 - 10\ i_2 = 5$$

$$\boxed{3\ i_1 - 2\ i_2 = 1} \qquad (1)$$

For mesh 2

$$-10\ i_1 + (10+6+4)\ i_2 - 10 = 0$$

$$-10\ i_1 + 20\ i_2 = 10$$

$$\boxed{-\ i_1 + 2\ i_2 = 1} \qquad (2)$$

Add Equation (1) and (2)

$$3\,i_1 - 2\,i_2 = 1$$

$$-\,i_1 + 2\,i_2 = 1$$

$+$

$$2\,i_1 = 2 \qquad i_1 = 1\ \text{A}$$

Subtitute i_1 to Equation (1)

$$3 - 2\,i_2 = 1$$

$$i_2 = 1\ \text{A}$$

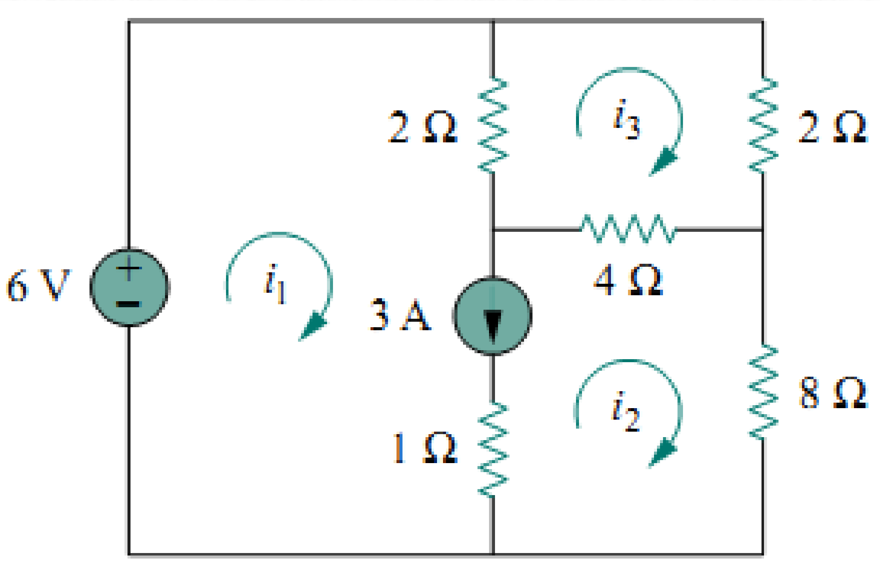
Calculate I_1, I_2, and I_3

$$I_1 = i_1 = 1\ \text{A}$$

$$I_2 = i_2 = 1\ \text{A}$$

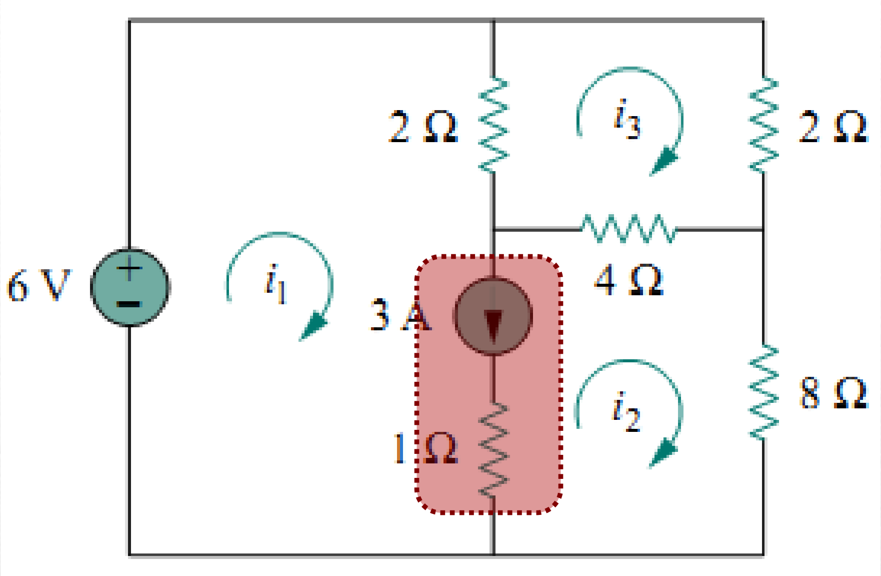
$$I_3 = i_1 - i_2 = 0\ \text{A}$$

2. Use mesh analysis to determine mesh current equations in circuit below



A supermesh results when two meshes have a (dependent or independent) current source in common.

Solution



Current at supermesh

$$\boxed{i_1 - i_2 = 3}$$

At mesh 1 and 2

$$2i_1 + (4+8)i_2 - (2+4)i_3 - 6 = 0$$

$$\boxed{2i_1 + 12i_2 - 6i_3 = 6}$$

At mesh 3

$$-2i_1 - 4i_2 + (2+4+2)i_3 = 0$$

$$\boxed{-2i_1 - 4i_2 + 8i_3 = 0}$$