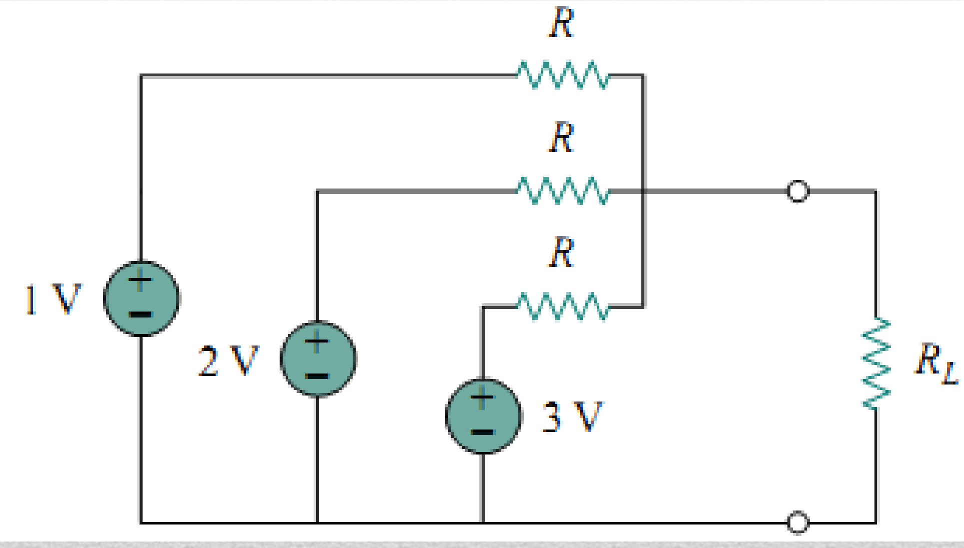
DC Circuits

Circuit Theorems

Linearity Property

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

$$f(x_1 + x_2 + \cdots + x_n) = f(x_1) + f(x_2) + \cdots + f(x_n)$$



Superposition Theorem

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Steps to Apply Superposition Principle:

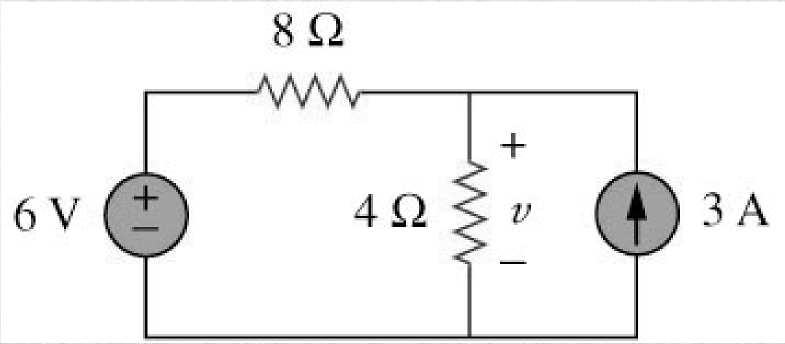
1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Two things have to be keep in mind:

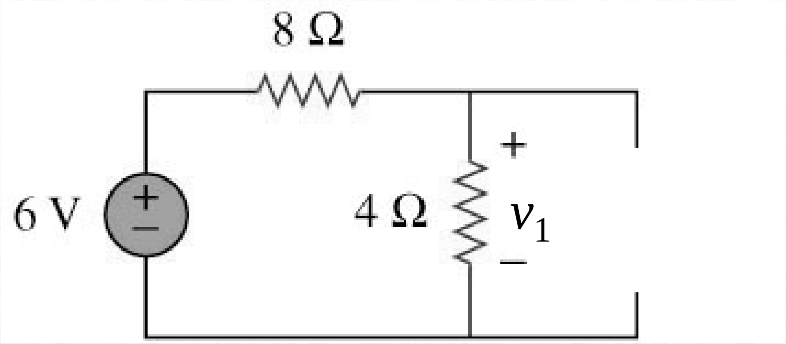
1. When we say turn off all other independent sources:
   - Independent voltage sources are replaced by 0 V (short circuit)
   - Independent current sources are replaced by 0 A (open circuit).
2. Dependent sources are left intact because they are controlled by circuit variables.

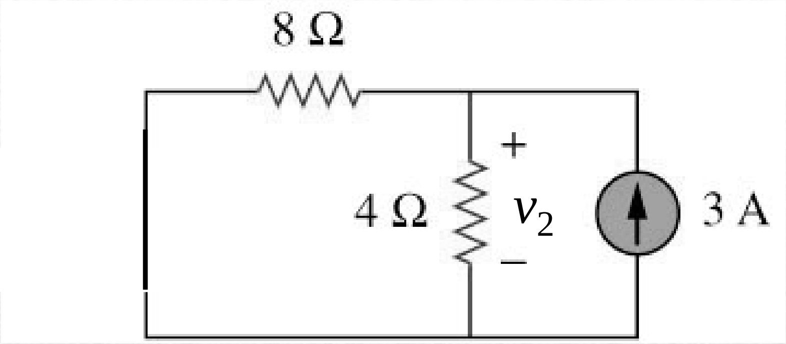
Example

Use the superposition theorem to find *v* in the circuit shown below.



Solution





$$v = v_1 + v_2$$

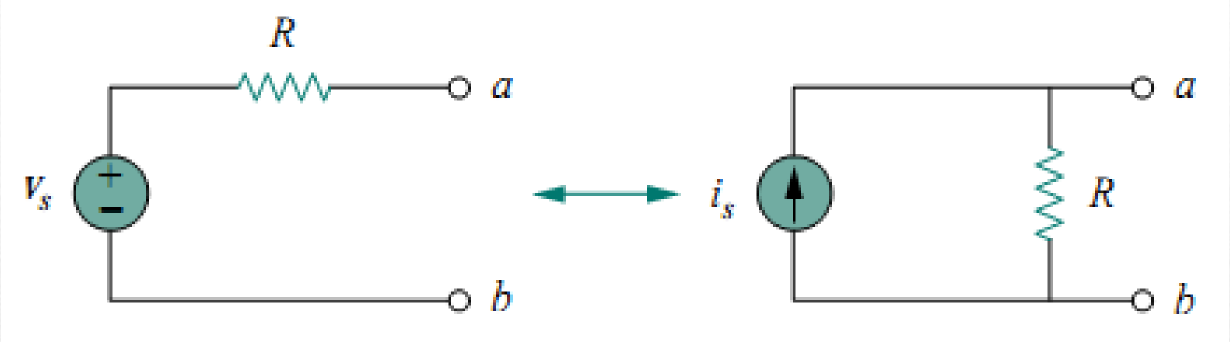
$$v = \frac{4}{4+8} \times 6 + (4 \parallel 8) \times 3$$

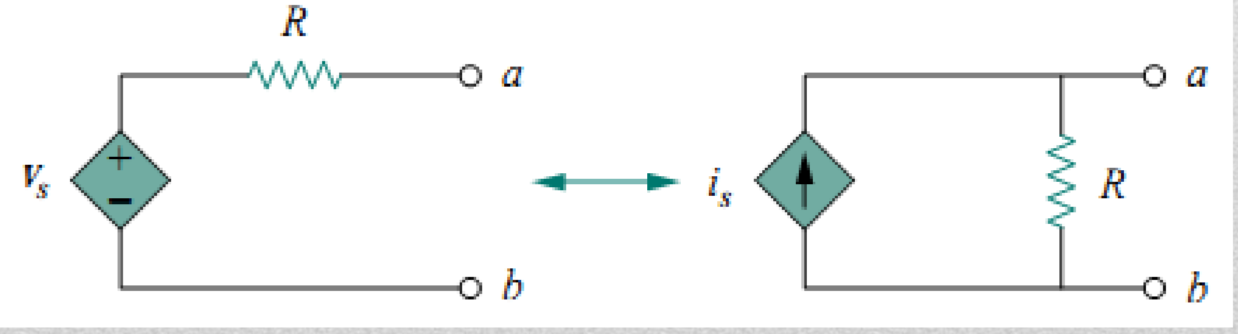
$$v = \frac{1}{3} \times 6 + \left(\frac{4 \times 8}{4+8}\right) \times 3$$

$$\boxed{v = 10\ \mathrm{V}}$$

Source Transformation

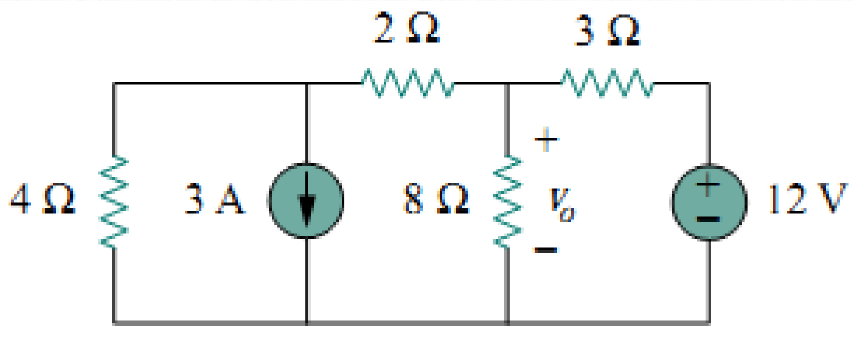
A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.



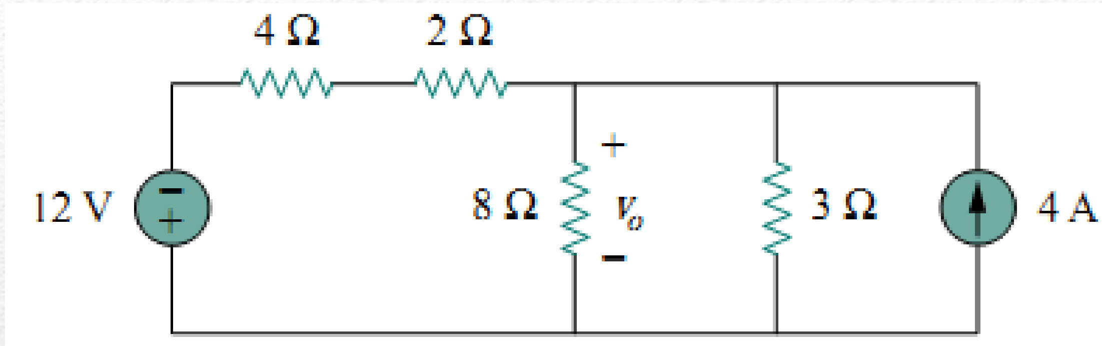


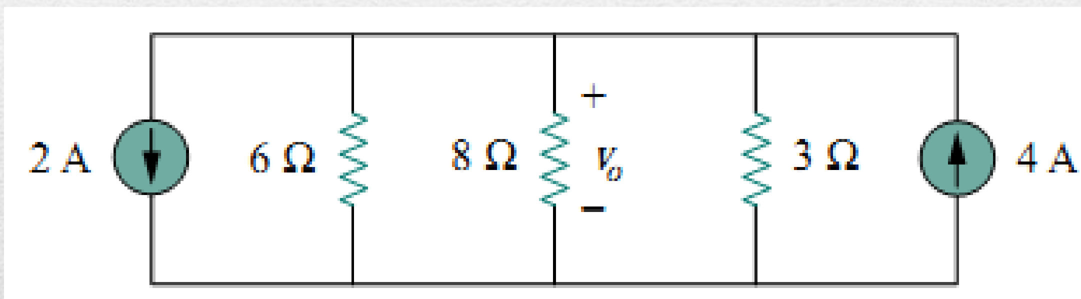
Example

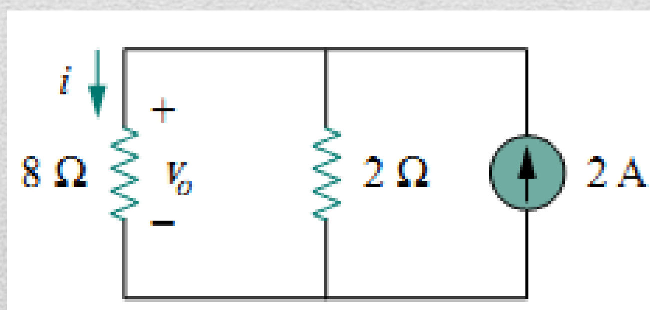
Use source transformation to find v_o in the circuit below



Solution







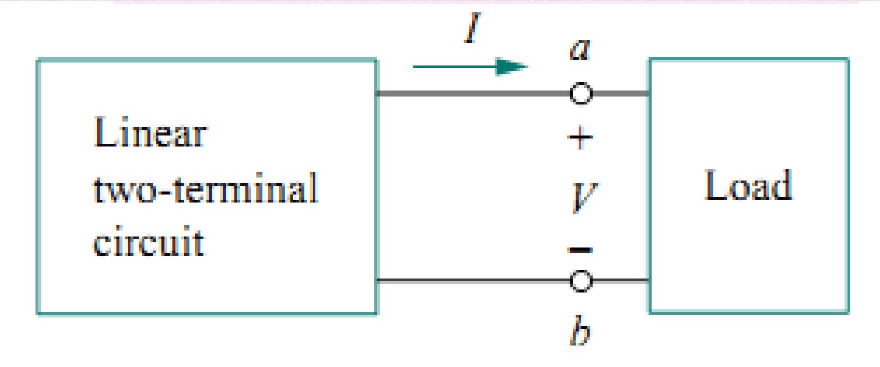
$$v_o = 2 \times (2 \parallel 8)$$

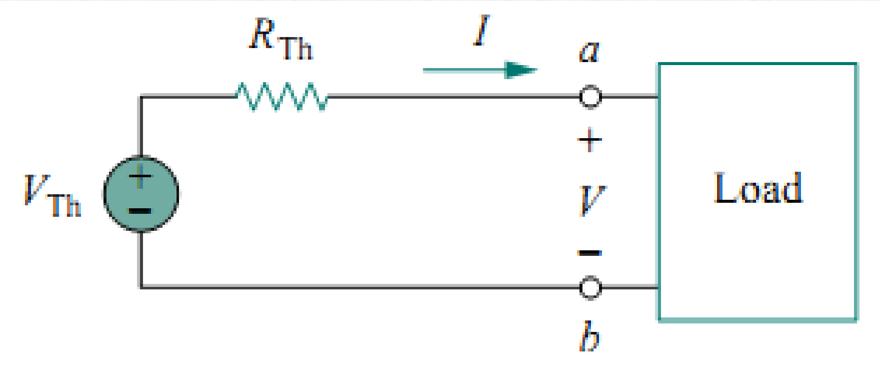
$$v_o = 2 \times 1{,}6$$

$$\boxed{v_o = 3{,}2 \text{ V}}$$

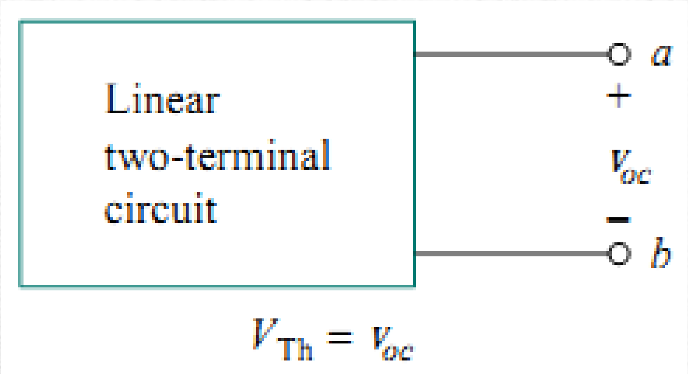
Thevenin's Theorem

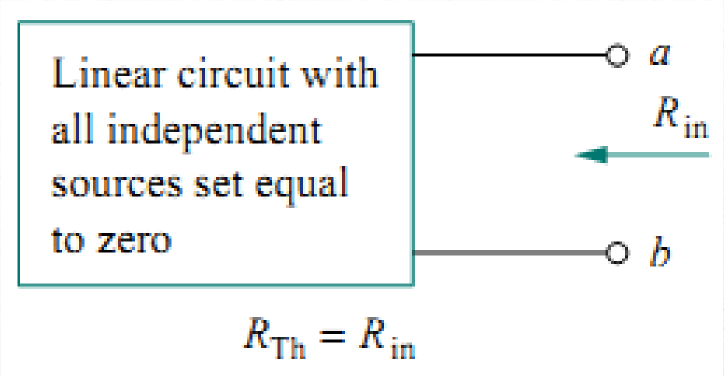
Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



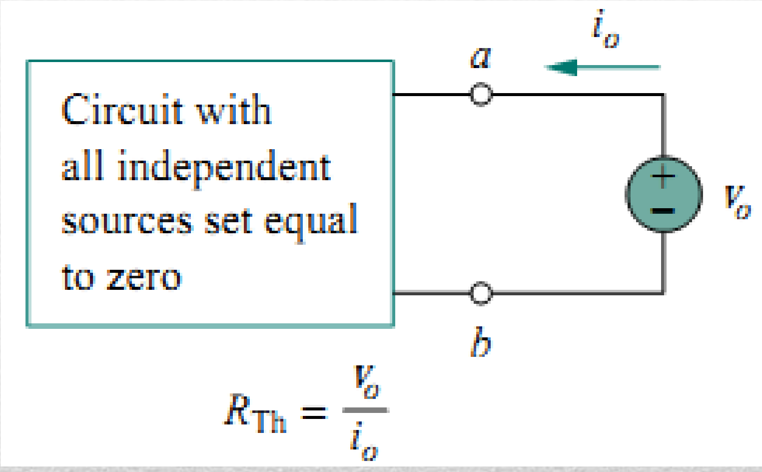


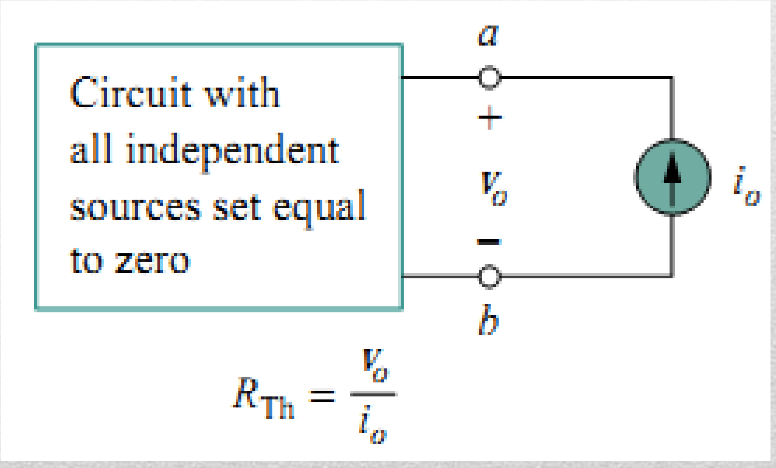
Finding V_{Th} and R_{Th}.





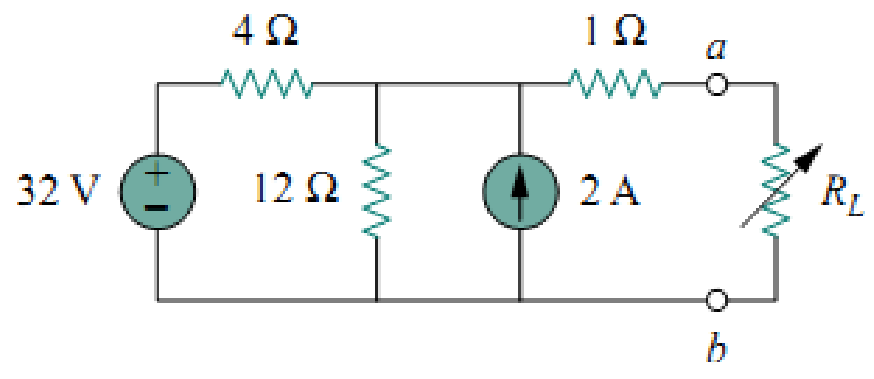
Finding R_{Th} when circuit has dependent sources.



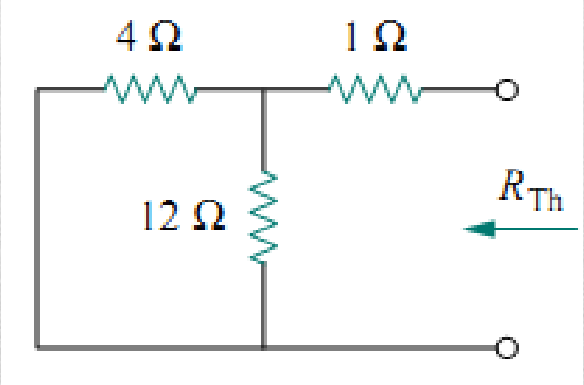


Example

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below.



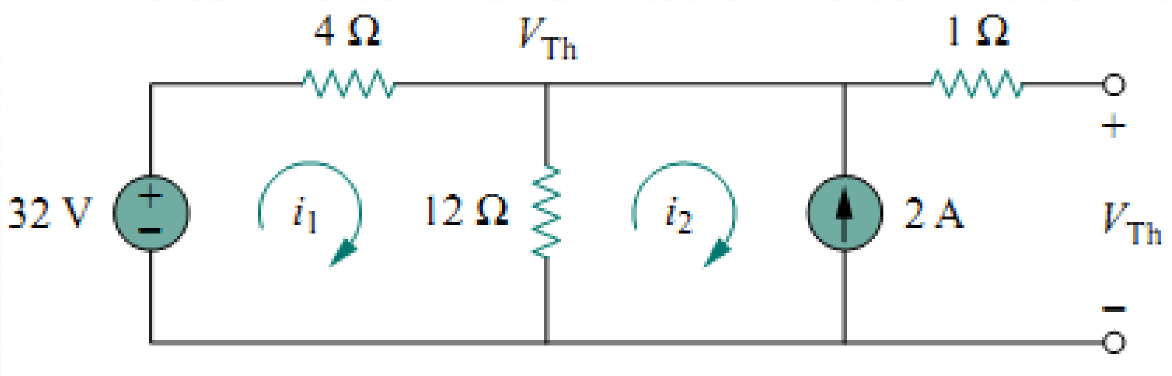
Solution



$$R_{\mathrm{Th}} = (4 \parallel 12) + 1$$

$$R_{\mathrm{Th}} = \frac{4 \times 12}{16} + 1$$

$$\boxed{R_{\mathrm{Th}} = 4\ \Omega}$$

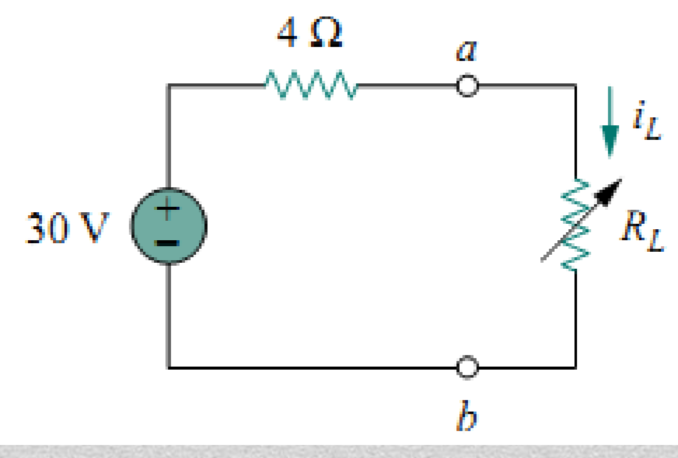


Using Superposition theorem,

$$V_{Th} = (4 \parallel 12) \times 2 + \frac{12}{4+12} \times 32$$

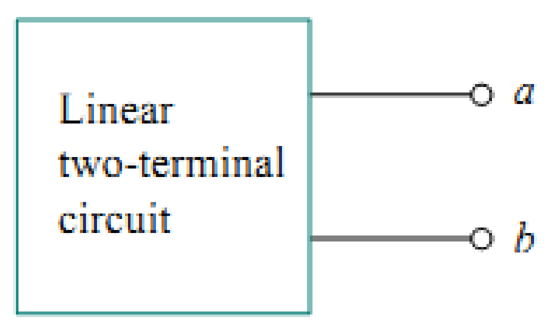
$$V_{Th} = 3 \times 2 + \frac{3}{4} \times 32 = 6 + 24$$

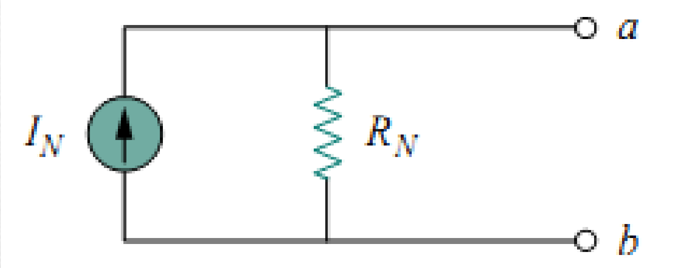
$$\boxed{V_{Th} = 30\ \text{V}}$$



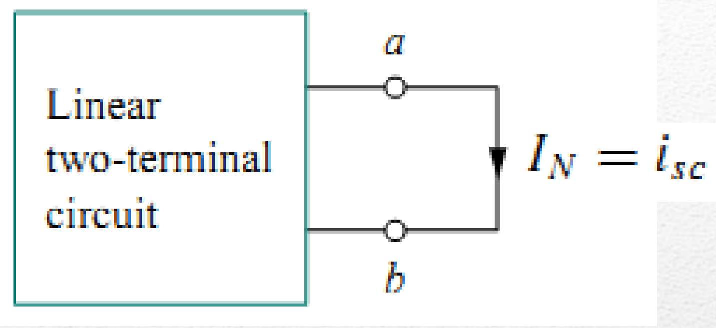
Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.





$$\boxed{R_N = R_{Th}}$$



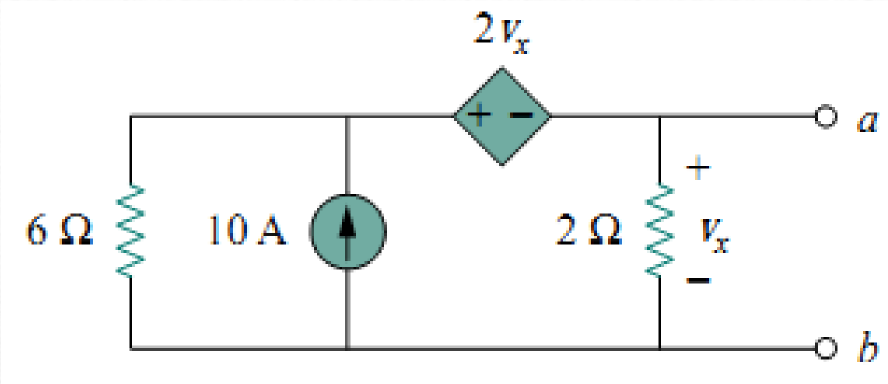
$$V_{\mathrm{Th}} = v_{oc}$$

$$I_N = i_{sc}$$

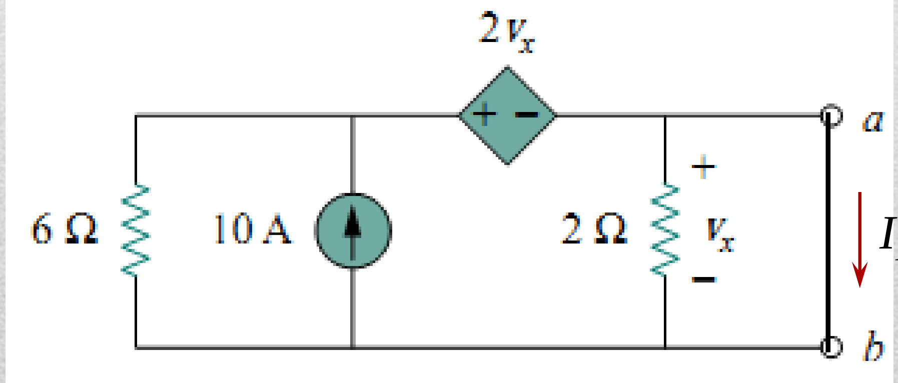
$$R_{\mathrm{Th}} = \frac{v_{oc}}{i_{sc}} = R_N$$

Example

Find the Norton equivalent circuit of the circuit in figure below



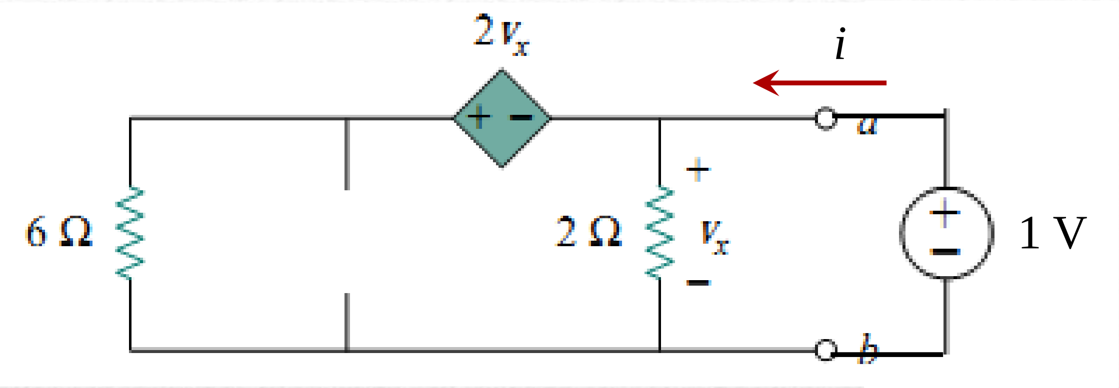
Solution



$V_x = 0$

$\boxed{I_N = 10 \text{ A}}$

Calculate R_{TH} (Method 1)

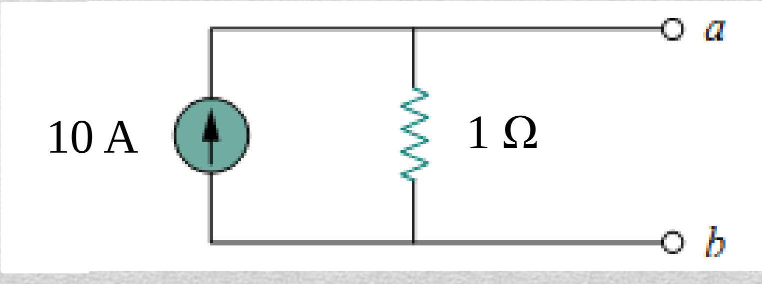


$$i = \frac{V_x}{2} + \frac{2V_x + V_x}{6} \quad (1)$$

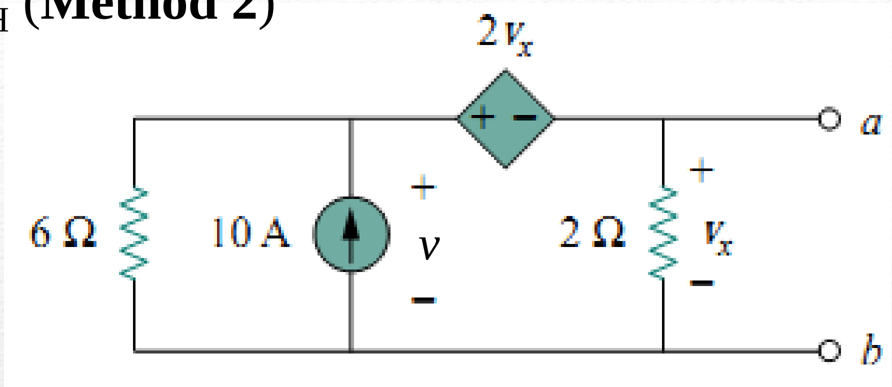
$$V_x = 1 \quad (2)$$

From Eq. (1) and (2) : $i = 1\ \text{A}$

$$R_{\text{Th}} = \frac{1}{i} = \frac{1}{1} = 1\ \Omega$$



Calculate R_{TH} (Method 2)



$$\frac{v}{6} - 10 + \frac{V_x}{2} = 0$$

$$v = 2V_x + V_x$$

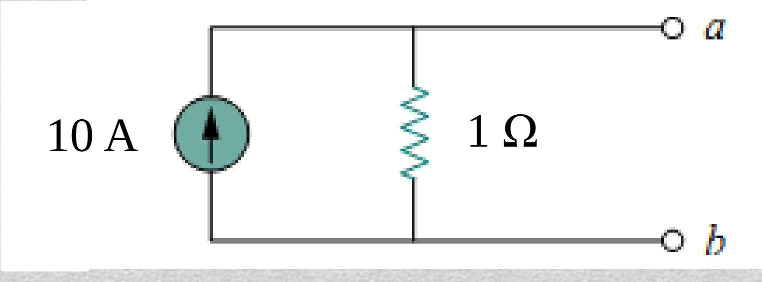
$$\boxed{v + 3V_x = 60} \quad \text{(1)}$$

$$\boxed{-v + 3V_x = 0} \quad \text{(2)}$$

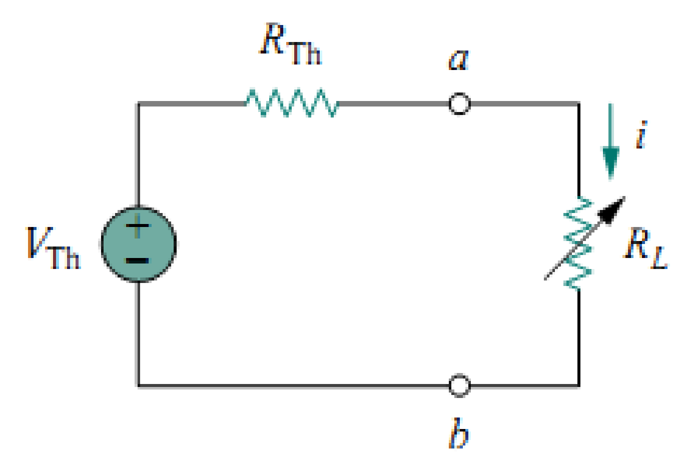
From Eq. (1) and (2) : $6V_x = 60 \implies V_x=10$

$$V_{\mathrm{Th}} = V_x = 10\ \mathrm{V}$$

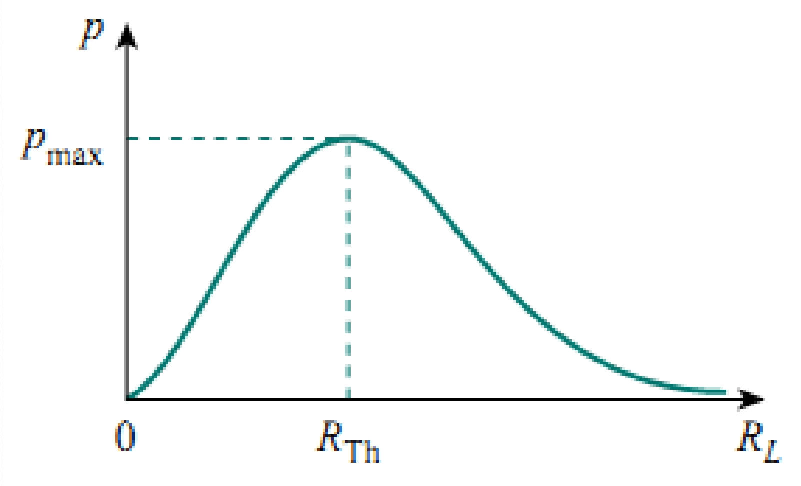
$$R_{\mathrm{Th}} = \frac{V_{\mathrm{Th}}}{I_N} = \frac{10}{10} = 1\ \Omega$$



Maximum Power Transfer



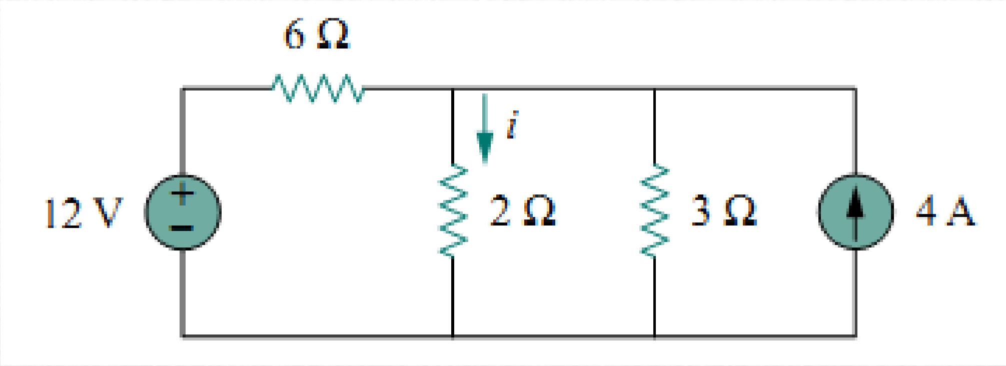
$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$



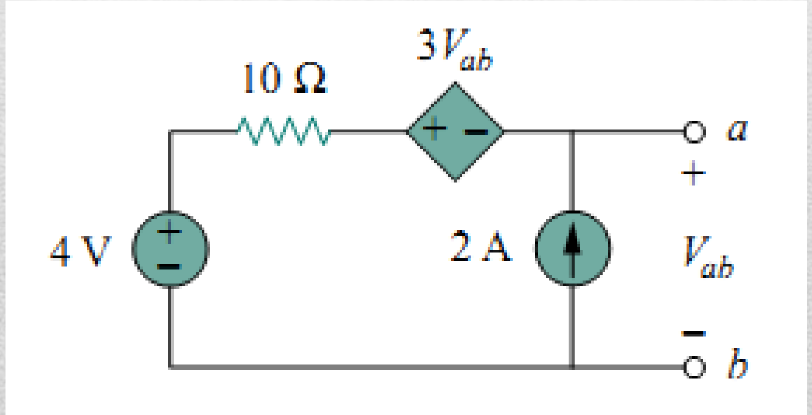
Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

Homework

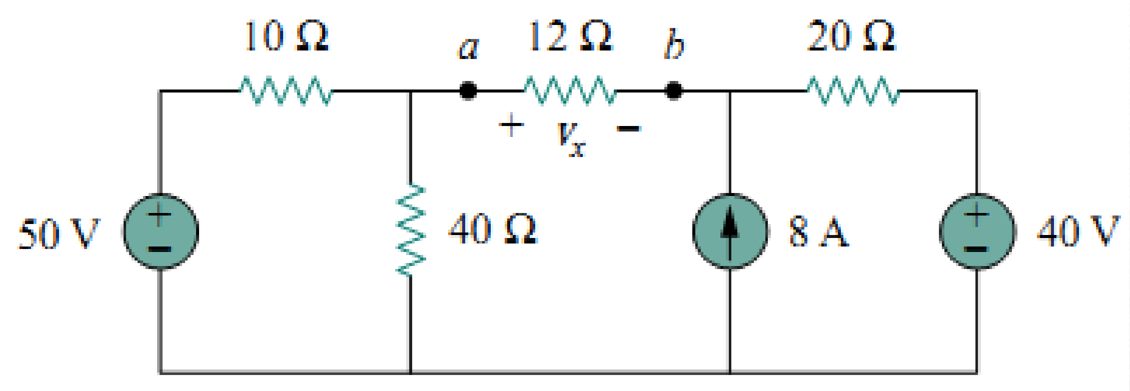
1 Use superposition principle to find i



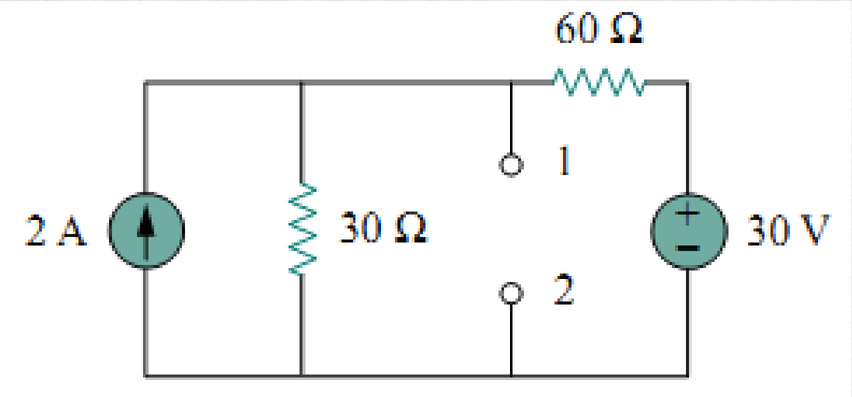
2 For the circuit in figure below, find the terminal voltage V_{ab} using superposition.



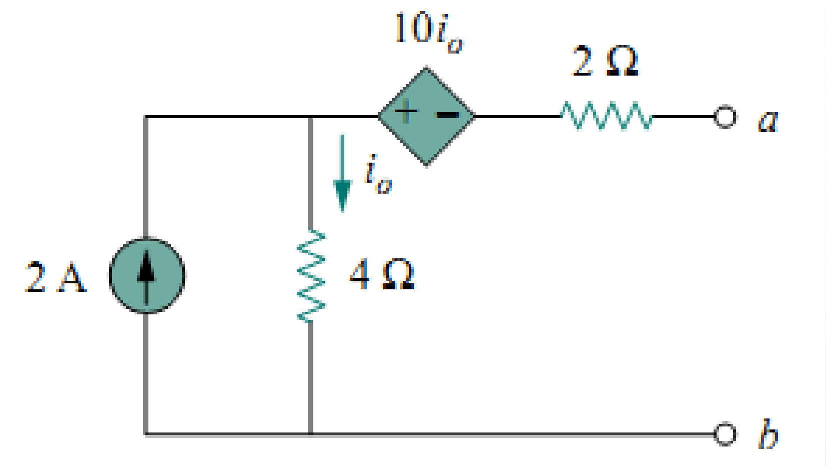
3 Apply source transformation to find V_x in the circuit



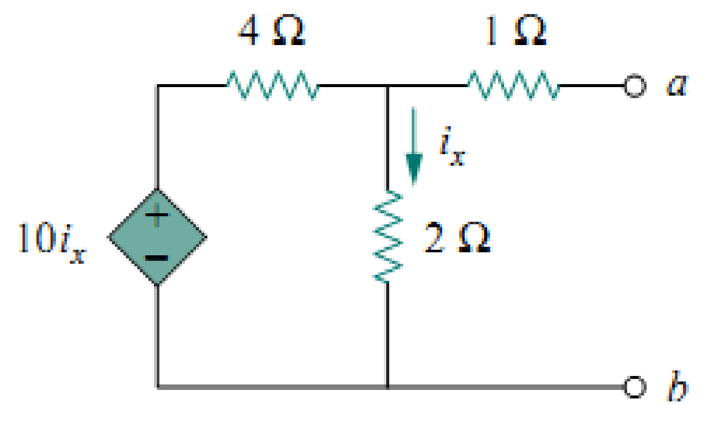
4 Determine R_{Th} and V_{Th} at terminals 1-2 of the circuit



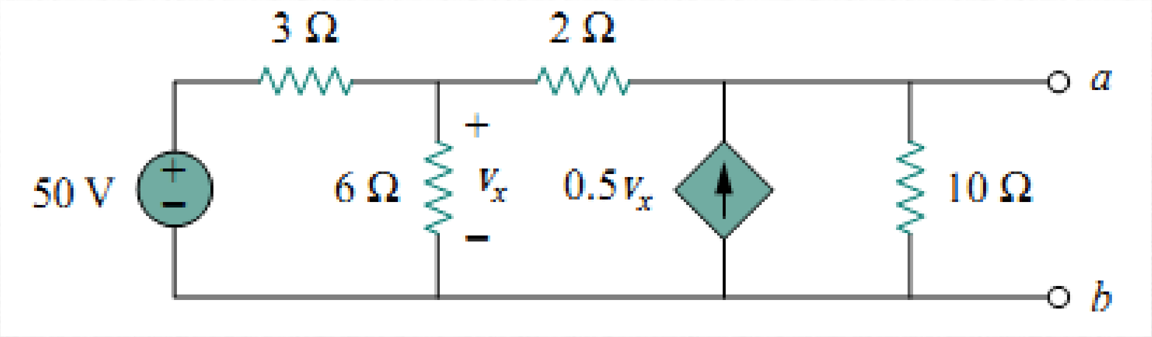
5 Determine the Norton equivalent at terminals *a*-*b* for the circuit



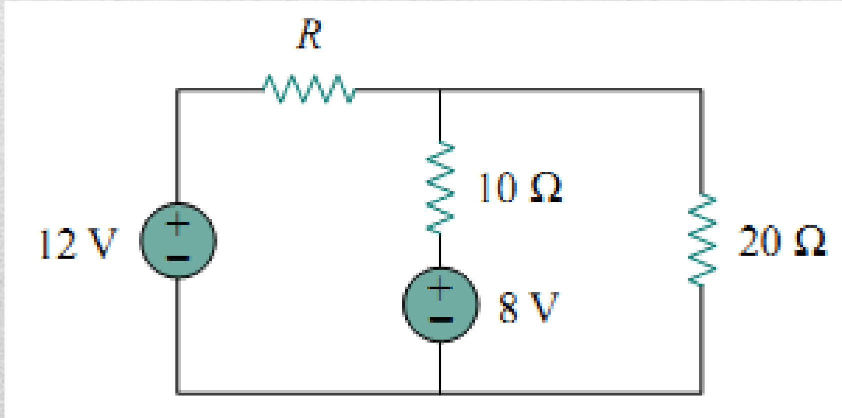
6 Obtain the Thevenin equivalent seen at terminals *a*-*b* of the circuit



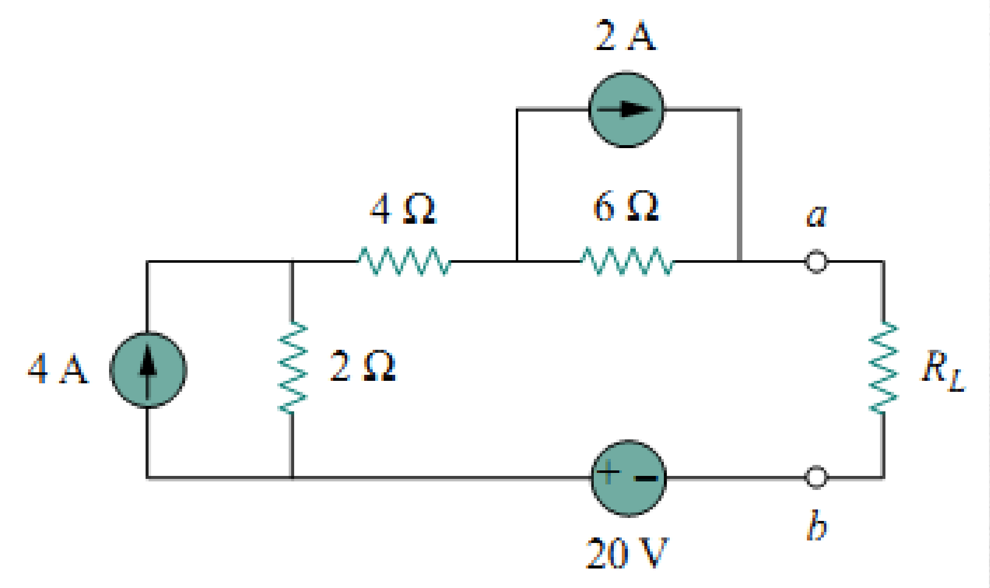
7 Obtain the Thevenin and Norton equivalent circuits at the terminals *a-b* for the circuit



8 Compute the value of *R* that results in maximum power transfer to the 10-Ω resistor. Find the maximum power.



9 Find R_L for maximum power deliverable to R_L, and determine that maximum power.



10 For the circuit in figure below, determine the value of *R* such that the maximum power delivered to the load is 3 mW.