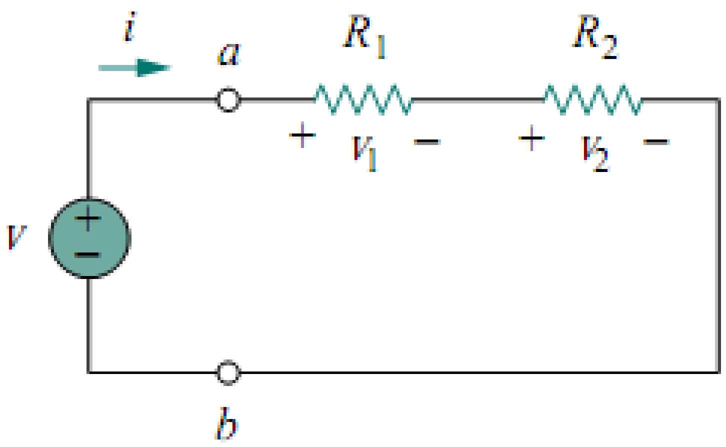
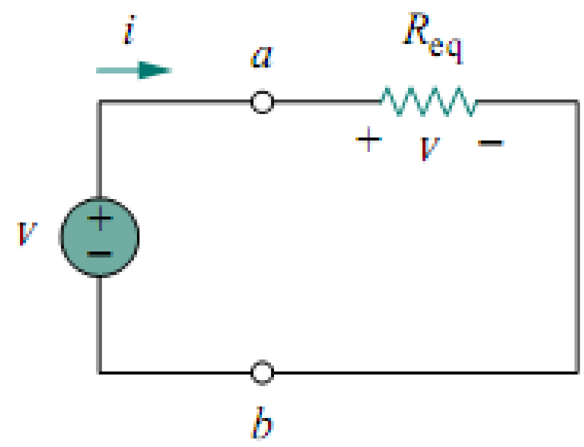
Basic Laws (2)

Series Resistors and Voltage Division





$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$v = i R_{eq}$$

$$R_{eq} = R_1 + R_2$$

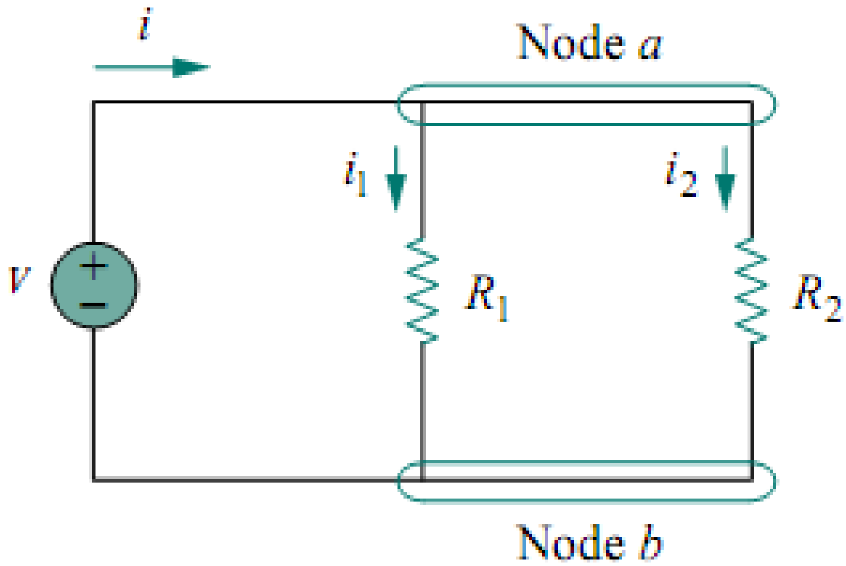
$$\boxed{v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v}$$

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

$$\boxed{R_{\text{eq}} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n}$$

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Parallel Resistors and Current Division



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

$$v = i\,R_{\mathrm{eq}} = \frac{i\,R_1\,R_2}{R_1 + R_2}$$

$$\boxed{i_1 = \frac{R_2\,i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\,i}{R_1 + R_2}}$$

$$i_1 = \frac{G_1}{G_1 + G_2} i$$

$$i_2 = \frac{G_2}{G_1 + G_2} i$$

The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

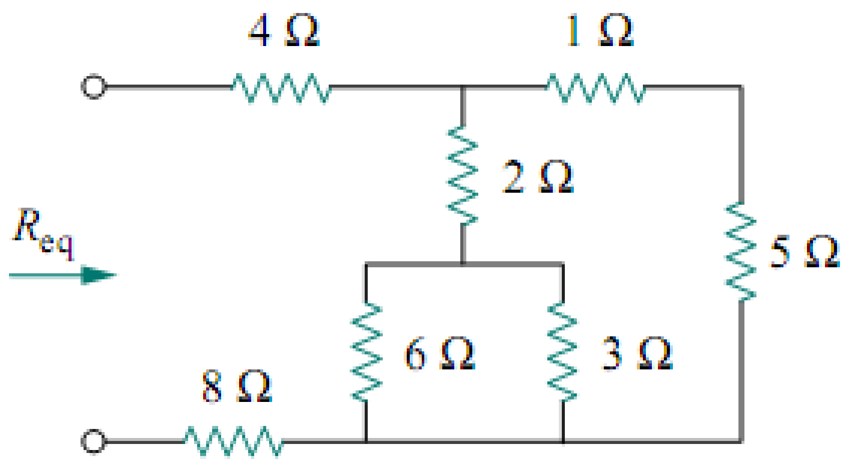
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

$$\boxed{G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N}$$

$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i$$

Example

Find R_{eq} for the circuit.



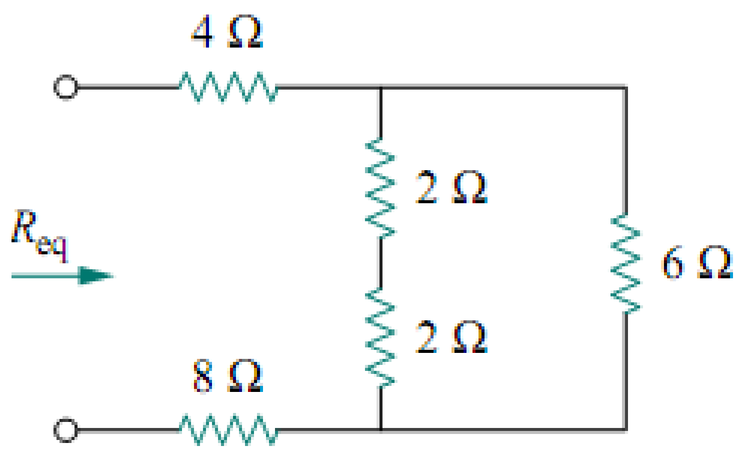
Solution

The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

Also,the 1-Ω and 5-Ω resistors are in series, hence their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

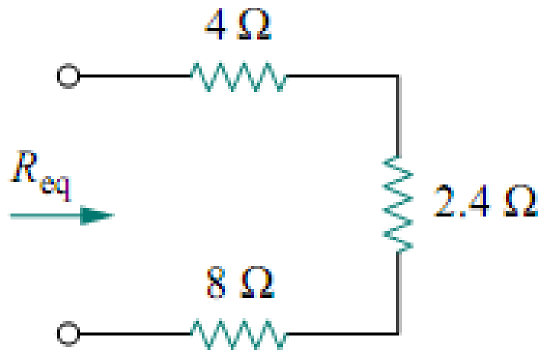


In Fig. above, we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

This 4-Ω resistor is now in parallel with the 6-Ω resistor; their equivalent resistance is

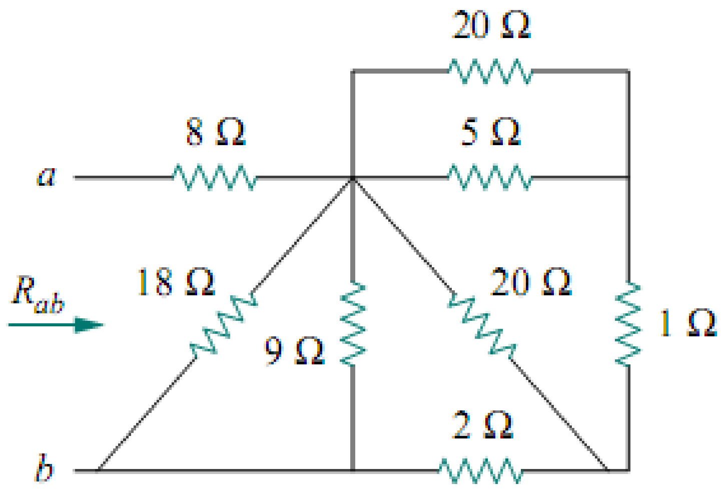
$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$



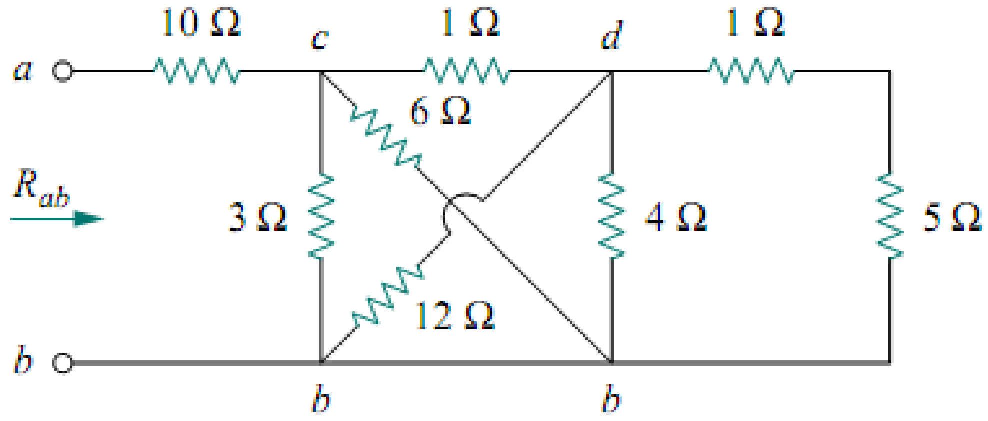
$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

Practice Problems (A)

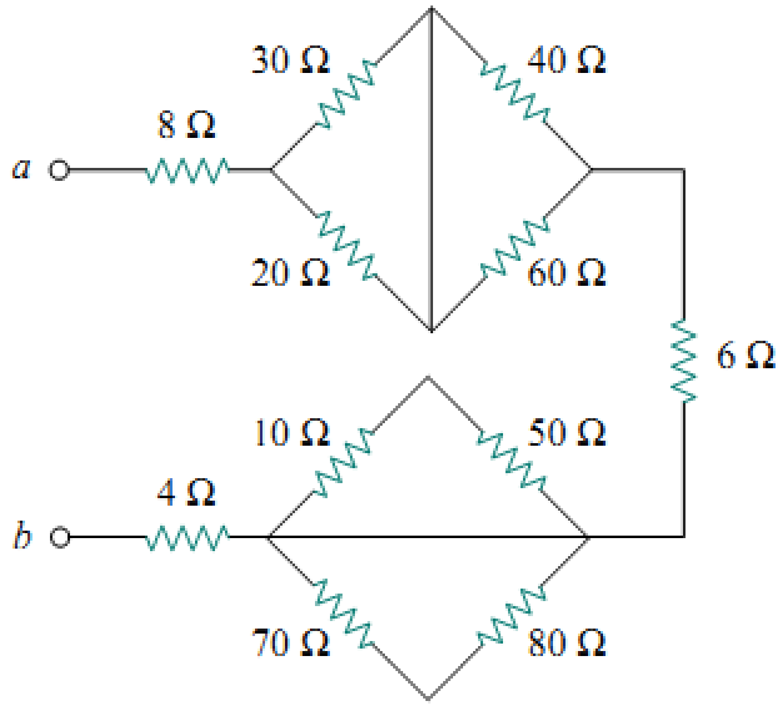
1. Find R_{ab}



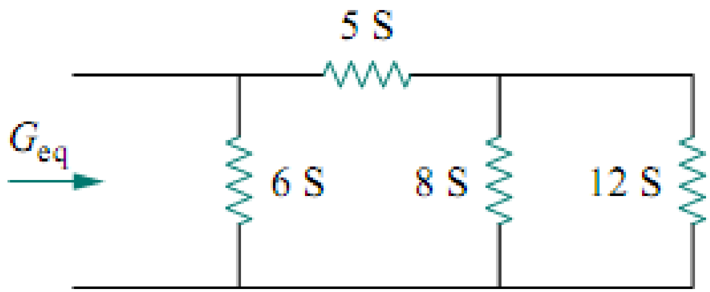
2. Find R_{ab}



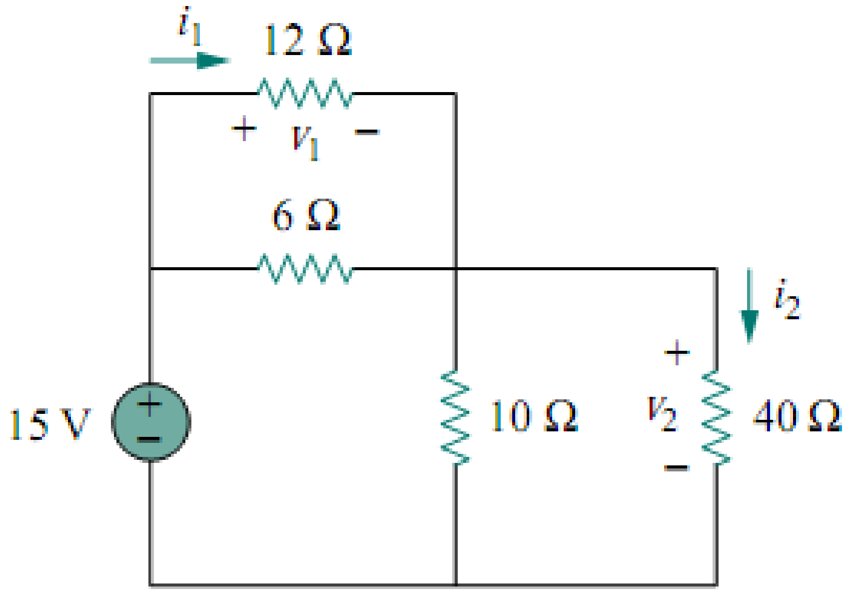
3. Find R_{eq} at terminals a-b



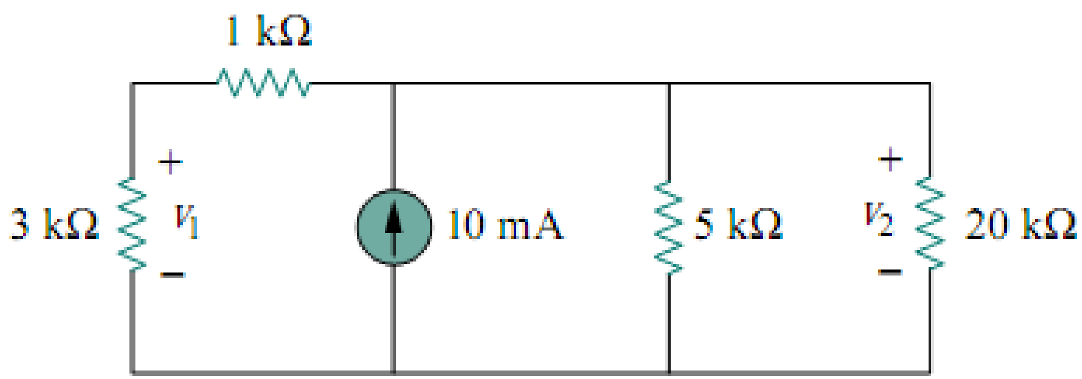
4. Find G_{eq}



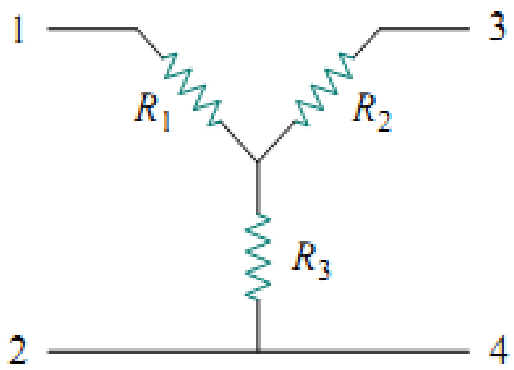
5. Find v_1 and v_2 in the circuit, and calculate the power dissipated in 12-Ω and 40-Ω resistors



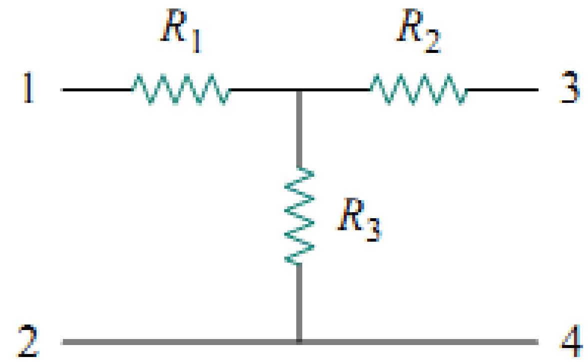
6. For the circuit, find: (a) v_1 and v_2, (b) the power dissipated in the 3-kΩ and 20-kΩ resistors, and (c) the power supplied by the current source.



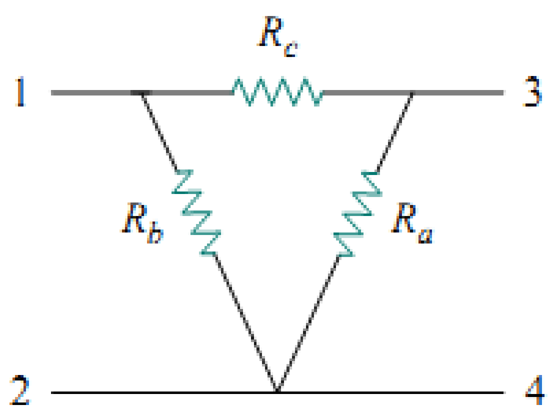
Wye-Delta Transformations



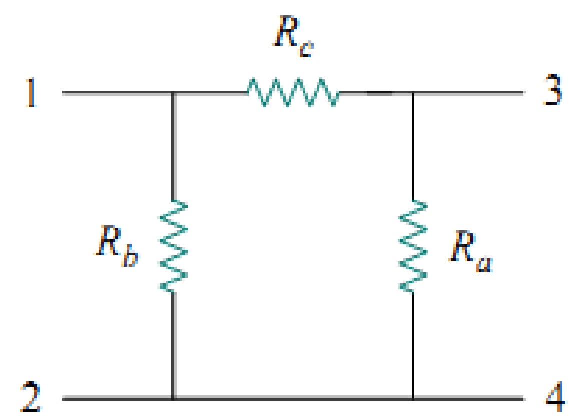
Y - network



T - network



Δ - network



Π - network

$$R_{12}(\mathrm{Y}) = R_1 + R_3$$
$$R_{12}(\Delta) = R_b \parallel (R_a + R_c)$$

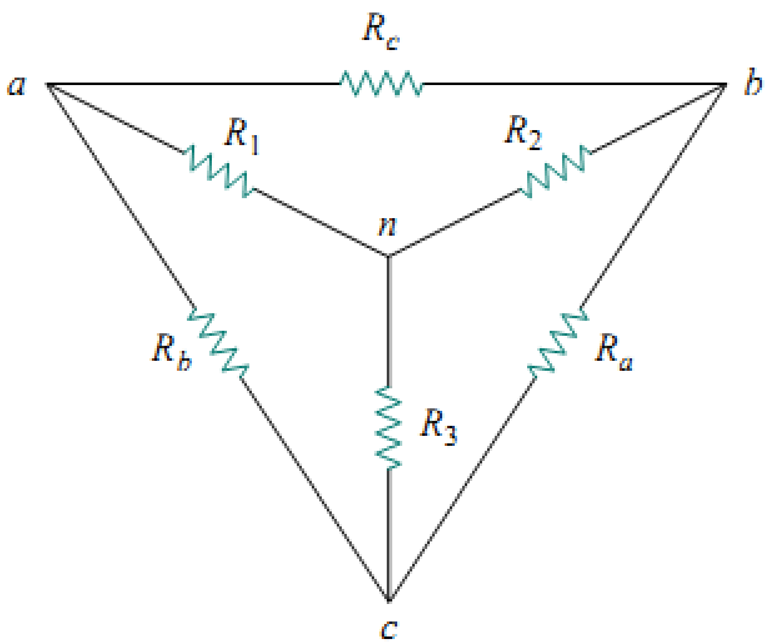
Setting $R_{12}(\mathrm{Y}) = R_{12}(\Delta)$ gives $R_{12} = R_1 + R_3 = \dfrac{R_b(R_a + R_c)}{R_a + R_b + R_c}$ A

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$ B

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$ C

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c}$$ D=A–C

Delta to Wye Conversion



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

E=D+B

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

F=D–B

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

G=A–E

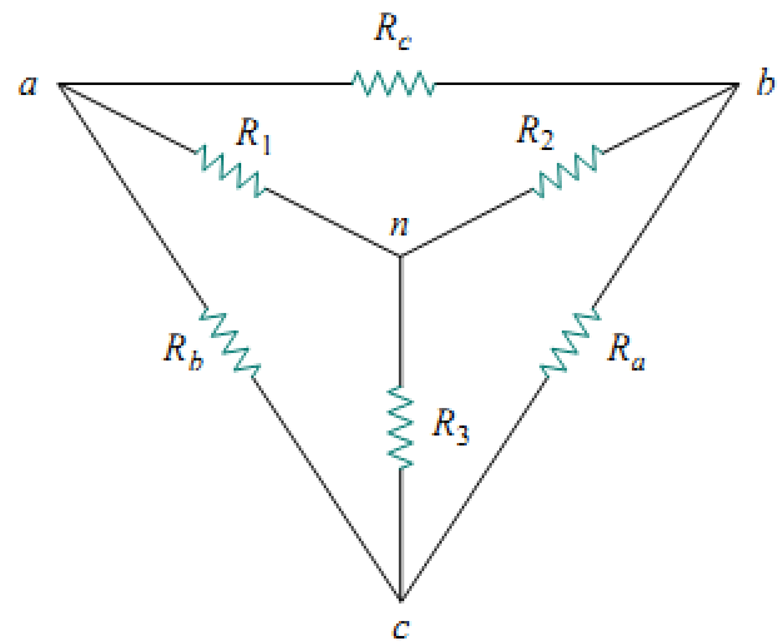
Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

Wye to Delta Conversion

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

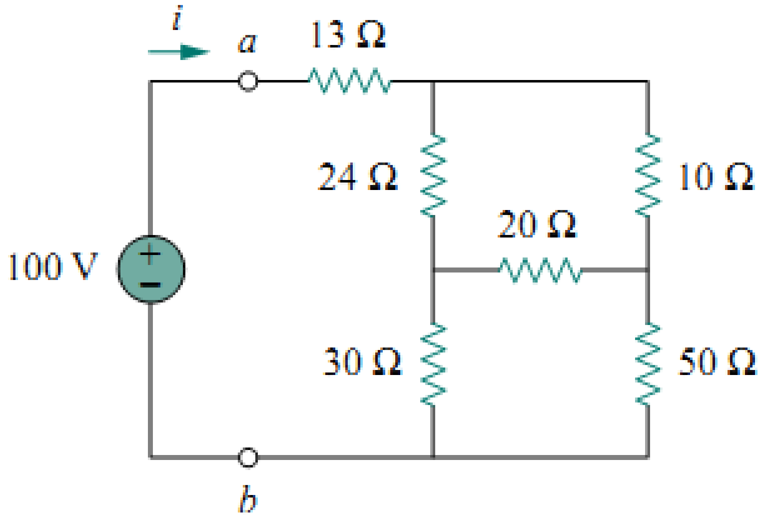
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$



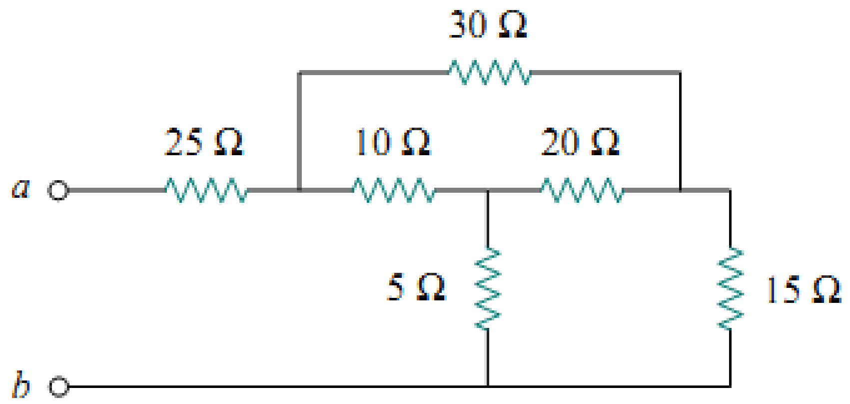
Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

Practice Problems (B)

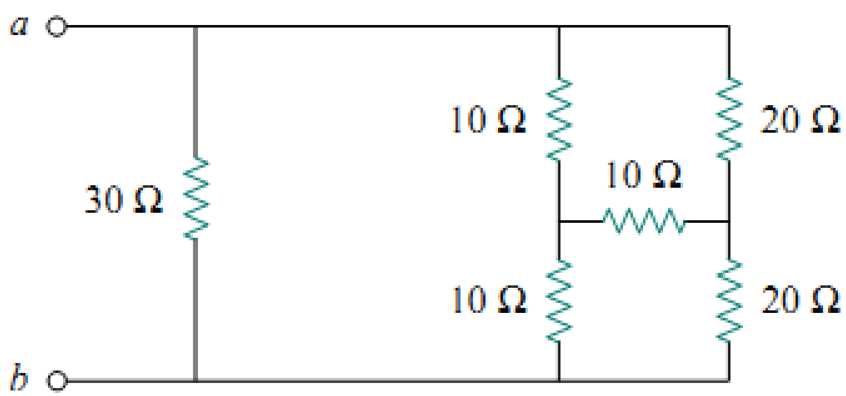
1. Find R_{ab} and i for bridge network below



2. Find R_{eq} at terminals a-b



3. Find R_{eq} at terminals a-b



Selected Problems

2.32 Find V_o and I_o in the circuit of Fig. 2.96.

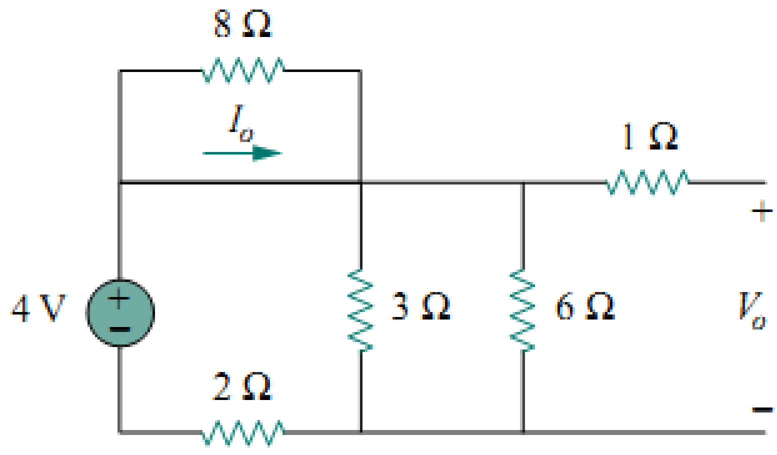


Figure 2.96 For Prob. 2.32.

2.36 For the ladder network in Fig. 2.100, find I and R_{eq}.

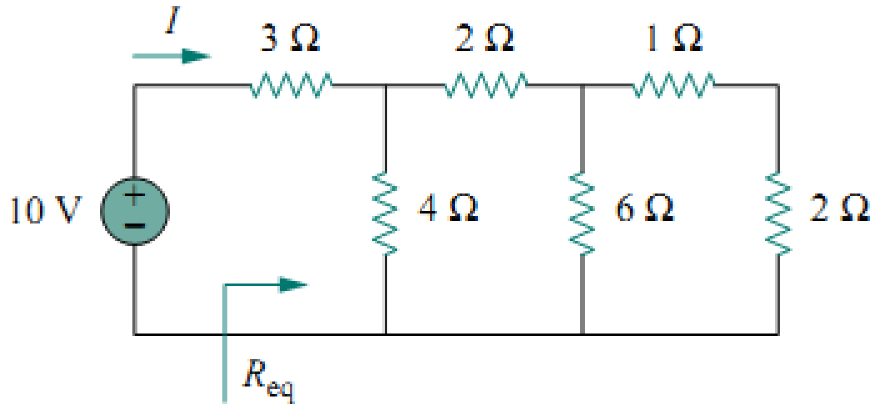


Figure 2.100 For Prob. 2.36.

2.42 Find the equivalent resistance R_{ab} in the circuit of Fig. 2.106.

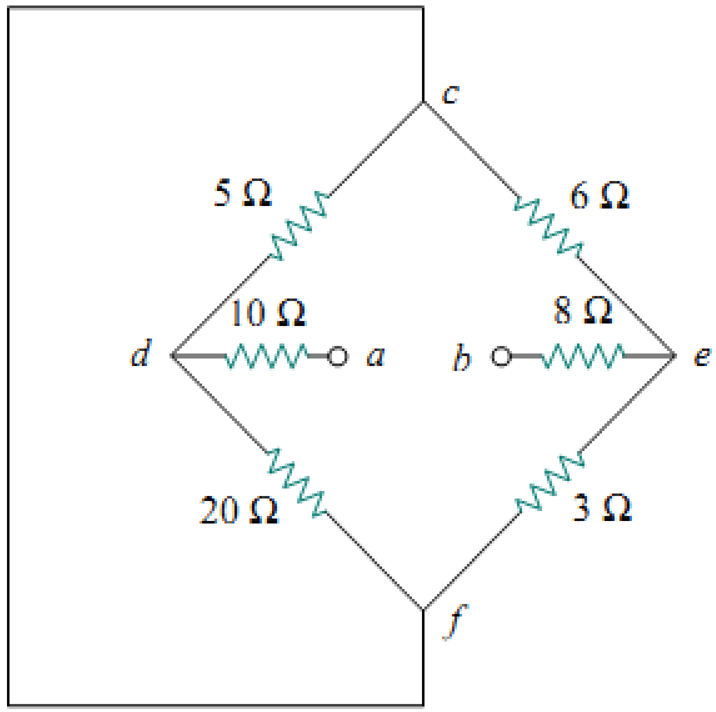


Figure 2.106 For Prob. 2.42.