Math 297 Discussion 6*

Annie Xu

Feburary 20, 2019

Recall examples of topologies we've seen.

More Topological Concepts

Recall the definition of continuity of functions defined on topological spaces.

Definition 0.1. *Let $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ be topological spaces. The map $f : X \to Y$ is a **homeomorphism** if f is a continuous bijective map with continuous inverse. If there's such f between X and Y, we say X and Y are homeomorphic.*

From the above definition, if f is a continuous bijective open(closed) map, then f is a homeomorphism. (Why?)

Definition 0.2. *A topological space X is said to be **disconnected** if it is the union of two disjoint nonempty open sets. Otherwise, X is said to be connected.*

An alternate definition of connectedness is the following:
A space X is connected if and only if the only subsets of X that are both open and closed in X are the empty set and X itself. (Why?)

Examples
1. Let X denote a two-point space in the indiscrete topology. Is X connected?
2. Let Y denote the subspace $[-1, 0) \cup (0, 1]$ of the real line $\mathbb{R}$.
3. Let X be the subspace $[-1, 1]$ of the real line.
4. The rationals $\mathbb{Q}$.

Theorem 1. *The image of a connected space under a continuous map is connected.*

This is a very useful theorem, often called *connected image theorem.*

Question. *Let $\{A_n\}$ be a sequence of connected subspaces of X, such that $A_n \cap A_{n+1} \neq \emptyset$ for all n. Show that $\bigcup A_n$ is connected.*

Question. *A space is totally disconnected if its only connected subspaces are one-point sets. Show that if X has the discrete topology, then X is totally disconnected. Does the converse hold?*

*Reference: James Munkres. *Topology.* Prentice Hall, NJ, 2000.