Math 297 Discussion 1*

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HW setup: Complex numbers

1. Complex number, real part, imaginary part
A complex number takes the form $z = x + iy$ where x and y are real and i is an imaginary number that satisfies $i^2 = -1$. We call x and y the **real part** and the **imaginary part** of z, respectively, and we write $x = \text{Re}(z)$ and $y = \text{Im}(z)$.

2. Complex plane
We make the identification: the complex number $z = x + iy \in \mathbb{C}$ is identified with the point $(x, y) \in \mathbb{R}^2$. The x and y axis of $\mathbb{R}^2$ are called the **real axis** and **imaginary axis** because they correspond to the real and purely imaginary numbers.

3. Arithmetic rules

Question. *Think about the geometric meaning of the rules:*

- *What geometric transformation of vectors in the complex plane does addition of complex numbers correspond to?*
- *How about multiplication by i?*

4. Absolute value
Define $|z| = (x^2 + y^2)^{1/2}$, precisely the distance from the origin to the point (x, y).

5. Complex conjugate
$\bar{z} = x - iy$, obtained by a reflection across the real axis in the plane.

6. Polar form
$z = re^{i\theta}$, where $r \geq 0$, $\theta \in \mathbb{R}$ is called the **argument** of z (unique up to a multiple of 2π), often denoted by $\arg z$.
$e^{i\theta} = cos\theta + isin\theta$.
$r = |z|$ since $|e^{i\theta}| = 1$.
Note: If $z = re^{i\theta}$ and $w = se^{i\phi}$ then $zw = rse^{i(\theta+\phi)}$, so multiplication by a complex number corresponds to a homothety in $\mathbb{R}^2$ (i.e., a rotation composed with a dilation).

Fun topic: The pigeonhole principle

This is an interesting topic whose charm can only be conveyed through an array of examples and applications.

*Reference: Elias M. Stein, Rami Shakarchi. *Complex Analysis.* Princeton University Press, Princeton, 2003.