Week 1 Summary Sheet
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Before you start, make sure you understand the basic operation between sets including the Union ($\cup$), Intersection ($\cap$), Difference ($-$) and Complement ($\bar{A}$)

Basic definitions

1. Alphabets
   - finite set of symbols
   - denoted as Σ, read as sigma
   - *examples*

     the binary alphabet, $\Sigma = \{0,1\}$

     a set of all lower-case letters, $\Sigma = \{a, b, c, d, e, \ldots\ldots. x, y, z\}$
2. Strings
   - finite sequence of symbols
   - w, x, y, z
   - *examples*

     given an alphabet $\Sigma = \{0,1\}$, define a string as $w = 010110$

     given an alphabet $\Sigma = \{a, b, c\}$, define a string as $x = aabbcc$

     all symbols must taken from a given alphabet
3. Empty strings
   - string with zero symbols, means nothing in the string
   - denoted as ε, read as epsilon
4. Length of a string
   - number of symbols in the string
   - length of w is $|w|$, read as the modulus of w
   - *examples*

     a given alphabet $\Sigma = \{0,1\}$, a string $w = 010110$, then $|w| = 6$

     for empty string, $|\varepsilon| = 0$
5. Power of an alphabet
   - Σ^* is a set of all strings over an alphabet Σ, also known as kleene start operation
   - $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \ldots\ldots \cup \Sigma^{infinite}$
   - more specifically, Σ^k is a set of all strings of length k chosen from Σ
   - read as Σ to the power of k
   - $\Sigma^0 = \{\varepsilon\}$
   - given $\Sigma = \{0,1\}$, then we have $\Sigma^1 = \{0,1\}$ and $\Sigma^2 = \{00,01,10,11\}$
   - Σ^+ is a set of nonempty strings from Σ
   - $\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \ldots\ldots \cup \Sigma^{infinite}$
   - $\Sigma^* = \Sigma^+ \cup \Sigma^0 = \Sigma^+ \cup \{\varepsilon\}$
   - ***powerset***

     a set of all possible subsets of the original set, including ε ($\emptyset$) and the set itself
6. Concatenation of strings
   - $x = 1010$, and $y = 0111$

- $xy = 10100111$
- guess what the concatenation of yx is
- $w\varepsilon = \varepsilon w = w$

7. Language
   - a set of strings chosen from Σ^*
   - denoted by L
   - *examples*

     7.1 language of all strings consisting of n 0s followed by n 1s, for $n \geq 0$

     it can be denoted as $L = \{\varepsilon, 01, 0011, 000111 ...\}$

     7.2 a set of strings of 0s and 1s with equal number of each

     it can be denoted as $L = \{\varepsilon, 01, 10, 0011, 1100, 000111, 111000 ...\}$

     7.3 a set of <u>binary numbers</u> whose value is a <u>prime*</u>

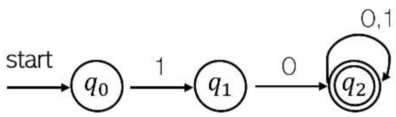
     It can be denoted as $L = \{10, 11, 101, 111, 1011 ....\}$

     * *A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself. Number like 2, 3, 5, 7, 11, 13*
   - empty language, denoted by φ, read as *phi, means no strings*

<u>**Finite Automata (FA)**</u>

1. introduction
   - a mathematical model, and no memory
   - consists of a set of states, transitions among states based on input symbols
   - deterministic finite automate (DFA)
   - non-deterministic finite automata (NFA)
2. applications
   - digital circuits, lexical analyzer, pattern matching (searching for a webpage)
3. design a DFA to accept the strings start with 10

   3.1 transition diagram



   3.2 list of transition functions

   a DFA $D = \{Q, \Sigma, \delta, q_0, F\}$

   a set of states are denoted as $Q = \{q_0, q_1, q_2\}$

   an alphabet is $\Sigma = \{0,1\}$

   transition functions are as follows, δ, read as delta

   - first transition is $\delta(q_0, 1) = q_1$, then $\delta(q_1, 0) = q_2$, then $\delta(q_2, 0) = q_2$, and $\delta(q_2, 1) = q_2$
   - start state is q_0
   - the final/ accept state is q_2

   3.3 transition table

States	Input symbols	
	0	1
→ q_0	X	q_1
q_1	q_2	X
* q_2	q_2	q_2