CEG5103 / EE5023 – Wireless Networks
Part 1: Tutorial 1 – Answers

1. Given: v = 80 kph = 80 x 10^3/3600 = 22.2 m/s
   f_c = 850 MHz ; $A^2/2\sigma^2$ = 0.01 ; $A/2\sigma$ = 0.1 ; c = 3 x 10^8 m/s

(a) $$N_A = \sqrt{2\pi} f_d \frac{A}{\sqrt{2}\sigma} \exp\left(-\frac{A^2}{2\sigma^2}\right)$$

Now $f_d = \frac{v}{\lambda} = v\frac{f_c}{c} = 22.2\frac{850\times10^6}{3\times10^8} = 62.96$ and substituting this into N_A, we have:

$$N_A = \sqrt{2\pi}(62.96)(0.1)\exp(-0.01) = 15.63 \quad \text{fades/s}$$

(b) $$\bar{t}_F = \frac{1}{\sqrt{2\pi}f_d}\frac{\sqrt{2}\sigma}{A}\left[\exp\left(\frac{A^2}{2\sigma^2}\right)-1\right] = \frac{1}{\sqrt{2\pi}(62.96)}(\overset{10}{\cancel{0.1}})\left[\exp(0.01)-1\right] = 0.000637 \quad \text{seconds}$$

(c) $$\bar{t}_{IF} = \frac{1}{\sqrt{2\pi}f_d}\frac{\sqrt{2}\sigma}{A} = \frac{1}{\sqrt{2\pi}(62.96)}\frac{1}{0.1} = 0.06336 \quad \text{seconds}$$

(d) Received carrier frequency = $f_c + f_d$ = 850 MHz + 62.96 Hz = <u>850,000,062.96 Hz</u>

(e) Bit rate = 200 kbps $\Rightarrow$ Symbol rate = 100 ksymbols/s
$\therefore$ Symbol duration = 1/(100 x 10^3) = 10^{-5} seconds

Since N_A = 15.63, $\therefore$ 1/N_A = 1/15.63 = 0.064 seconds
$\therefore$ On average, each fade occurs over many symbols and we have <u>slow fading</u>.

2. Given: N_A = 100 fades/s ; $A^2/2\sigma^2$ = 0.01 ; c = 3x10^8 m/s ; f_c = 1 GHz

(a) Using $N_A = \sqrt{2\pi} f_d \frac{A}{\sqrt{2}\sigma} \exp\left(-\frac{A^2}{2\sigma^2}\right)$, we have:

$$100 = \sqrt{2\pi} f_d (0.1) \exp(-0.01)$$

$$f_d = \frac{100}{\sqrt{2\pi}(0.1)\exp(-0.01)} = 402.95 \text{ Hz}$$

Since $f_d = \frac{v}{\lambda} = v\frac{f_c}{c}$

$$\therefore \quad v = \frac{f_d c}{f_c} = \frac{402.95 \times 3 \times 10^8}{10^9} = 120.89 \quad \text{m/s}$$

(b) $$\bar{t}_F = \frac{1}{\sqrt{2\pi} f_d} \frac{\sqrt{2}\sigma}{A}\left[\exp\left(\frac{A^2}{2\sigma^2}\right) - 1\right]$$

$$0.01 = \frac{1}{\sqrt{2\pi} f_d}\left(\frac{1}{0.1}\right)\left[\exp(0.01) - 1\right]$$

$$\therefore \quad f_d = \left(\frac{1}{0.01}\right)\frac{1}{\sqrt{2\pi}}\left(\frac{1}{0.1}\right)\left[\exp(0.01) - 1\right] = 4.0094$$

$$\therefore \quad v = f_d \frac{c}{f_c} = (4.0094)\frac{3 \times 10^8}{10^9} = 1.20283 \quad \text{m/s}$$

3. Given: v = 15 m/s ; f_c = 2 GHz ; c = 3x10^8 m/s ; $A^2/2\sigma^2$ = 0.01

(a) When the vehicle is at the boundary of coverage and is 30^o from the direction of motion that we have the maximum relative velocity at v.cos(30^o).

(b) Maximum relative velocity = v.cos(30^o) = 15cos(30^o) = (15$\sqrt{3}$)/2 m/s

$$f_d = v\frac{f_c}{c} = \frac{15\sqrt{3}}{2}\frac{2 \times 10^9}{3 \times 10^8} = 50\sqrt{3} \quad \text{Hz}$$

$$N_A = \sqrt{2\pi} f_d \frac{A}{\sqrt{2}\sigma}\exp\left(-\frac{A^2}{2\sigma^2}\right) = \sqrt{2\pi}(50\sqrt{3})(0.1)\exp(-0.01) = 21.495 \quad \text{fades/s}$$

4. Given: $\bar{\tau} = 1.5\mu s$

$$\therefore \quad \text{Max. symbol rate } = \frac{1}{\bar{\tau}} = \frac{1}{1.5 \times 10^{-6}} = 666.67 \quad \text{ksymbols/s}$$

5. Given: v = 20 m/s ; f_c = 2 GHz ; c = 3x10^8 m/s ; $A^2/2\sigma^2 = 0.01$

(a)
$$f_d = v\frac{f_c}{c} = 20\frac{2\times10^9}{3\times10^8} = \frac{400}{3} \quad \text{Hz}$$
$$N_A = \sqrt{2\pi}f_d\frac{A}{\sqrt{2}\sigma}\exp\left(-\frac{A^2}{2\sigma^2}\right) = \sqrt{2\pi}\left(\frac{400}{3}\right)(0.1)\exp(-0.01) = 33.089 \quad \text{fades/s}$$
$$\bar{t}_F = \frac{1}{\sqrt{2\pi}f_d}\frac{\sqrt{2}\sigma}{A}\left[\exp\left(\frac{A^2}{2\sigma^2}\right)-1\right] = \frac{1}{\sqrt{2\pi}}\left(\frac{3}{400}\right)\left(\frac{1}{0.1}\right)\left[\exp(0.01)-1\right] = 0.00030071$$
$\therefore$ Prob. of being in a fade $= \bar{t}_F.N_A = (0.00030071)(33.09) = 0.0099502$

(b) Note that the prob. of being in a fade is
$$\bar{t}_F.N_A = \left[\frac{1}{\sqrt{2\pi}f_d}\frac{\sqrt{2}\sigma}{A}\left\{\exp\left(\frac{A^2}{2\sigma^2}\right)-1\right\}\right]\left[\sqrt{2\pi}f_d\frac{A}{\sqrt{2}\sigma}\exp\left(-\frac{A^2}{2\sigma^2}\right)\right] = \left[\left\{\exp\left(\frac{A^2}{2\sigma^2}\right)-1\right\}\right]\left[\exp\left(-\frac{A^2}{2\sigma^2}\right)\right]$$

This is independent of the vehicular speed.
Hence the prob. of being in a fade will not change with speed.