EE5801 Tutorial 3

Q1

A sphere (of radius a) has a volume charge density of $\rho(0<r<a)=\rho_o r/a$ where ρ_o is a constant and r is the distance from the center of the sphere.

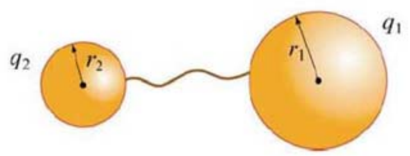
(a) Derive expressions for the electric field inside the sphere (where $r < a$) and outside the sphere (where $r > a$).

(b) The charged sphere is subsequently placed concentrically inside a metallic spherical shell (of inner radius b and outer radius c). Derive expressions for the electric field in the exterior region (where $r > c$) for the following cases:

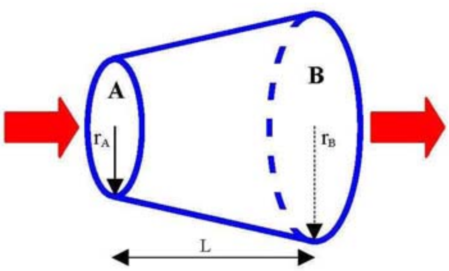
(i) when the spherical shell is left unearthed
(ii) after the spherical shell has been earthed.

Q2

(a) Two small spheres (carrying charges q_1 and q_2) are connected by a conducting wire as depicted in Figure 2(a). Show that $E_1/E_2 = r_2/r_1$ where E_k is the electric field normal to the surface of sphere k (with radius r_k) if the spheres' separation is much larger than both radii.



(b) Depicted in Figure 2(b) is a length L of truncated cone where r_A and r_B are the radii of the circular cross-sections at the end surfaces A and B. Show that the resistance for current flowing from A to B is given by $1/(\pi\sigma r_A r_B)$ where σ is the conductivity of the cone material.



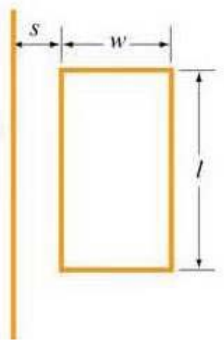
(c) The current distribution within a cylindrical wire becomes progressively non-uniform as the operating frequency is increased. The current density within the wire (of radius a) may then be expressed in the following manner:

$$J(0 < r < a) = J_o \exp\left(\frac{r^2 - a^2}{\delta^2}\right)$$

If the total current carried by the wire is denoted as I, show that the parameters J_o and δ in the current-density expression are related via $J_o = 1/(\pi\delta^2)$ when $\delta <<$ a.

Q3

Figure 3 depicts a rectangular wire loop (of length *l* and width *w*) which is placed in the vicinity of a long straight wire. Determine the mutual inductance between these two (with separation *s*).



Q4

Figure 4(a) depicts a circular wire loop (with radius r = 50 cm) which is connected to a resistor (with resistance R = 100Ω). The uniform magnetic field B in the vicinity varies with time in accordance with the plot given in Figure 4(b).

Sketch the variation of the current flowing in R as a function of time. B is in the +z direction (as denoted by the circles with enclosed dots) and the corresponding positive convention for the circular loop is given by the faint arrow arc.

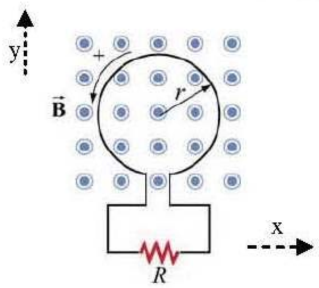


Figure 4(a)

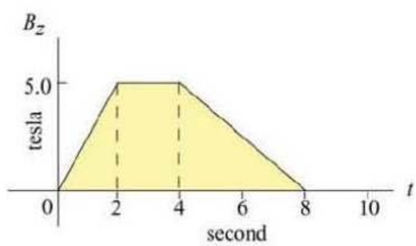


Figure 4(b)