EE5801 Tutorial 1 Solutions

Q1

$$3\delta_s = 1.2 \text{ km} = 1200 \text{ m}$$
$$\delta_s = 400 \text{ m}.$$

Hence,

$$\alpha = \frac{1}{\delta_s} = \frac{1}{400} = 2.5 \times 10^{-3} \quad \text{(Np/m)}.$$

Since $\varepsilon''/\varepsilon' \ll 1$, we can use low-loss approximation:

$$\alpha = \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{2\pi f \varepsilon_r'' \varepsilon_0}{2\sqrt{\varepsilon_r'}\sqrt{\varepsilon_0}}\sqrt{\mu_0} = \frac{\pi f \varepsilon_r''}{c\sqrt{\varepsilon_r'}} = \frac{\pi f \times 10^{-2}}{3 \times 10^8 \sqrt{3}} = 6f \times 10^{-11} \text{Np/m}.$$

For $\alpha = 2.5 \times 10^{-3} = 6f \times 10^{-11}$,

$$f = 41.6 \text{ MHz}.$$

Since α increases with increasing frequency, the useable frequency range is

$$f \leq 41.6 \text{ MHz}.$$

Q2

(i) The conduction and displacement current densities are given by $\mathbf{J}_c = \sigma\mathbf{E}$ and

$$\mathbf{J}_D = j\omega\varepsilon'\mathbf{E}.$$

Thus

$$\frac{|\mathbf{J}_c|}{|\mathbf{J}_D|} = \frac{\sigma}{\omega\varepsilon'} = \frac{\sigma}{\omega\varepsilon_r'\varepsilon_0} = 108 >> 1$$

Since $\frac{\sigma}{\omega\varepsilon} >> 1$ this is a good conductor.

(ii) $$\alpha \approx \sqrt{\frac{\omega\mu\sigma}{2}} = 0.435 \text{ Np/m}$$

The skin depth is $\delta = \frac{1}{\alpha} = 2.3 \text{ m}$.

Q3

(i) Since $f = 5 \times 10^6$ Hz, we obtain $\omega = 10^7 \pi$ rad/s.

Here $\frac{\sigma}{\omega\varepsilon'} \approx 200 >> 1$. We may therefore approximate seawater as a good conductor at this frequency.

$$\alpha = \beta = \sqrt{\pi f \mu_r \mu_0 \sigma} = 8.89 \text{ Np/m or rad/m}$$

$$\eta = (1+j)\sqrt{\frac{\pi f \mu_r \mu_0}{\sigma}} = \frac{\pi}{\sqrt{2}}(1+j) = \pi e^{j\pi/4} \quad \Omega$$

$$u_p = \frac{\omega}{\beta} = 3.53 \times 10^6 \text{ m/s}$$

$$\lambda = \frac{2\pi}{\beta} = 0.707 \text{ m}$$

$$\delta = 1/\alpha = 0.112 \text{ m}$$

(ii) $\exp(-2\alpha z_1) = \frac{10^{-4}}{1}$ $\quad z_1 = \frac{1}{2\alpha}\ln 10^4 = 0.518$ m

Q4

Medium 1 $\varepsilon_r = 100$, $\mu_r = 1$ and $\sigma = 0.4$ S/m;

$\sigma/(\omega\varepsilon) = 0.4/(2\pi x100x10^6 x(1/(36\pi))x10^{-9}x100) = 0.72$

$\eta_c = \sqrt{(\mu/\varepsilon)} = 120\pi/\sqrt{(100)} = 12\pi$

Medium 2 $\varepsilon_r = 2$, $\mu_r = 1$ and $\sigma = 100$ S/m.

$\sigma/(\omega\varepsilon) = 100/(2\pi x100x10^6 x(1/(36\pi))x10^{-9}x2) = 9000$

$\alpha = \sqrt{(\pi f\mu\sigma)} = \sqrt{(\pi x100x10^6 x4\pi x10^{-7}x100)} = 198.7$

$\eta_c = (\sqrt{2})(\alpha/\sigma)e^{j\pi/4} = 2.81\ e^{j\pi/4}$

reflection coefficient $\Gamma = (2.81\ e^{j\pi/4} - 12\pi)/(2.81\ e^{j\pi/4} + 12\pi) = 0.9\underline{/174^o}$

transmission coefficient $\tau = 2 \times 2.81\ e^{j\pi/4}/(2.81\ e^{j\pi/4} + 12\pi) = 0.14\underline{/42.1^o}$

check that $1 + \Gamma = \tau$ (within numerical rounding off errors)

$|E_{inc}| = 1$ V/m

$|H_{inc}|$ = 1/12π = 0.027 A/m

P_{inc} = ½ x 1 x 0.027 = 14 mW/m^2

$|E_{rfl}|$ = 0.9 V/m

$|H_{rfl}|$ = 0.9/12π = 0.024 A/m

P_{rfl} = ½ x 0.9 x 0.024 = 11 mW/m^2

$|E_{tran}|$ = 0.14 V/m

$|H_{tran}|$ = 0.14/2.81 = 0.05 A/m

P_{tran} = ½ x 0.14 x 0.05 = 4 mW/m^2

Thus $P_{inc} = P_{rfl} + P_{tran}$ within rounding off error

Another way to check is $|\Gamma|^2 + |\tau|^2\eta_1/\eta_2 = 1$ (why? You may like to derive this equation)