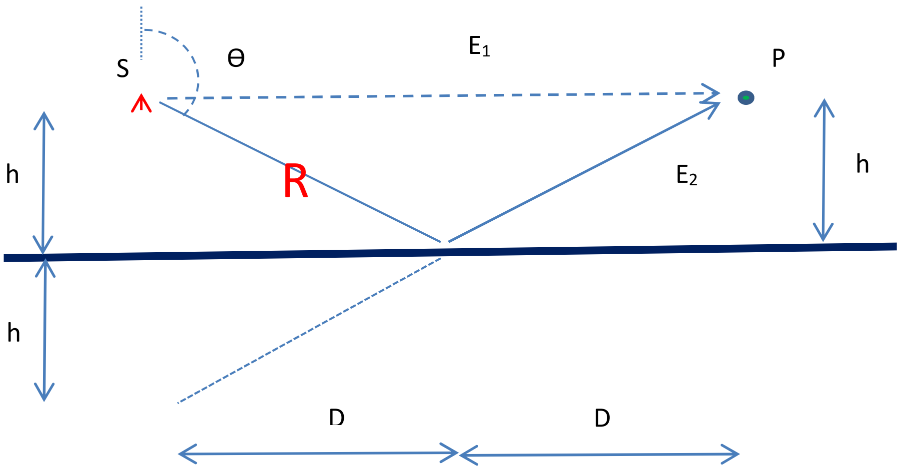
EE5801 Tutorial 2 Solutions

Q1

$$E_\theta = j\frac{\eta k I_0 l}{4\pi}\sin\theta\left(\frac{e^{-jkr}}{r}\right)$$

$$H_\phi = j\frac{k I_0 l}{4\pi}\sin\theta\left(\frac{e^{-jkr}}{r}\right)$$

$$\text{Note}: E_r = H_r = E_\phi = H_\theta = 0$$



$E_1 = E_{o1}\, e^{-jk(2D)}/(2D)$, where $E_{o1} = j\eta k I_o l \sin\Theta/(4\pi)$ with $\Theta = \pi/2$, or $E_{o1} = j\eta k I_o l/(4\pi)$

$E_2 = E_{o2}\, \Gamma e^{-jk(2R)}/(2R)$, where Γ is the ground reflection coefficient = 1 for vertical polarization, $E_{o2} = jk\eta I_o l \sin\Theta/(4\pi)$ with $\sin\Theta = \cos(\pi/2 - \Theta) = D/R = 1 - (1/2)(h/D)^2 \sim 1$ (for amplitude). Thus $E_{o2} = E_{o1}$.

To arrive at the above result, we have used $R^{-1} = (D^2 + h^2)^{-1/2} = D^{-1}\,[1 - (1/2)(h/D)^2]$ through Taylor Series expansion. Similarly, for R in the (amplitude) denominator of E_2, we can approximate it to be D.

On the other hand, the R in the phase term of E_2 cannot be approximated as D, we must use its Taylor expansion: $R = (D2 + h2)^{1/2} = D\,[1 + (1/2)(h/D)^2]$

Hence, finally, we have:

$$E_2 = E_{01} e^{-jk2D[1+\frac{1}{2}(\frac{h}{D})^2]}/(2D)$$

And $E_{total} = E_1 + E_2 = \dfrac{E_{01}}{2D} e^{-jk2D[1+\frac{1}{4}(\frac{h}{D})^2]} \{ e^{jk2D(\frac{1}{4})(\frac{h}{D})^2} + e^{-jk2D(\frac{1}{4})(\frac{h}{D})^2} \}$

Or $E_{total} = E_{01} \dfrac{e^{-jk2D}}{2D} \underline{e^{-jk2D(\frac{1}{4})(\frac{h}{D})^2}} \underline{2\cos\{k2D(\frac{1}{4})(\frac{h}{D})^2\}}$

Note the extra phase term and the extra cosine term which changes the amplitude of the received signal with respect to h (fixed D) or D (fixed h).

If the dipole is horizontally polarized, $\Gamma = -1$ (by boundary condition or image theorem), then

$$E_{total} = E_1 + E_2 = \frac{E_{01}}{2D} e^{-jk2D[1+\frac{1}{4}(\frac{h}{D})^2]} \{ e^{jk2D(\frac{1}{4})(\frac{h}{D})^2} - e^{-jk2D(\frac{1}{4})(\frac{h}{D})^2} \}$$

$$E_{total} = E_{01} \frac{e^{-jk2D}}{2D} e^{-jk2D(\frac{1}{4})(\frac{h}{D})^2} 2\sin\{k2D(\frac{1}{4})(\frac{h}{D})^2\}$$

Note the similarity (extra phase term and extra sinusoidal variation), and the difference (cosine for vertical dipole and sine for horizontal dipole).

Q2

Transmitting gain 6dBi = 3.98 (note that, $10*\log_{10}(x)$ dBi)

From Friis equation:

Power density = $P_tG_t/(4\pi R^2) = 100 \times 3.98/(4\pi \times (10\times10^3)^2) = 3.167\times10^{-7}$ W/m^2

Power density = $E^2/2\eta_o = E^2/(2\times120\pi) = 3.167\times10^{-7}$, thus E = 15.5 mV/m

Receiving gain = 20dBi = 100

10GHz, wavelength = 0.03m

Effective aperture = $G_r\lambda^2/(4\pi) = 100 \times 0.03^2/(4\pi) = 7.16\times10^{-3}$ m^2

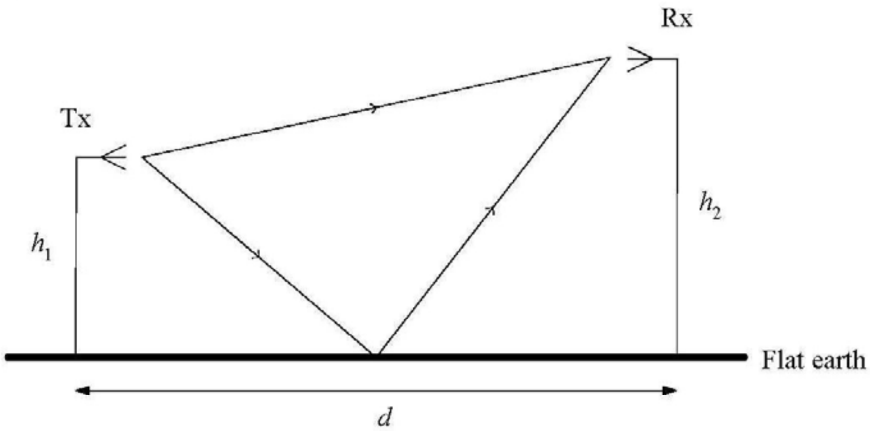
Thus, received power Pr = $3.167\times10^{-7} \times 7.16\times10^{-3} = 2.268$ nW

Q3

The total field at the receiver consists of a direct contribution and an indirect contribution due to reflection from the earth surface. The ratio of the total field strength at the receiver to the field strength of the direct contribution is given by

$$F = 1 + |\Gamma| \exp j\left(\psi - \frac{4\pi h_1 h_2}{\lambda d}\right),$$

where $\Gamma = |\Gamma| e^{j\psi}$ is the complex reflection coefficient.



Given that $\Gamma = 0.4\angle 0°$, $h_1 = h_2 = 40$ m, $d = 20\times10^3$ m and $\lambda = c/f = 0.075$ m, we find that

$F = 1.268 - j0.29$

$\rightarrow \quad |F| = 1.3$ or $|F| = 20\log(1.3) = 2.28$ dB

Varying the antenna height, the maximum field strength will occur when

$$\exp j\left(\psi - \frac{4\pi h_1 h_2}{\lambda d}\right) = +1, \text{ i.e. } \frac{4\pi h_1 h_2}{\lambda d} - \psi = 2n\pi, \quad n = 0, 1, \ldots,$$

giving $F_{\max} = 1 + |\Gamma|$.

The minimum field strength will occur when

$$\exp j\left(\psi - \frac{4\pi h_1 h_2}{\lambda d}\right) = -1, \text{ i.e. } \frac{4\pi h_1 h_2}{\lambda d} - \psi = (2n+1)\pi, \quad n = 0, 1, \ldots$$

giving $F_{\min} = 1 - |\Gamma|$.

In this case, we find that $\frac{F_{\max}}{F_{\min}} = \frac{1.4}{0.6} = 2.67$ or $\frac{F_{\max}}{F_{\min}} = 20\log(2.67) = 7.36$ dB.