EE5801 Tutorial 4 Solutions

Q1

$\varepsilon_r = 2.9$

$Z_0 = 75\ \Omega$

$Z_L = 300\ \Omega$

$l = 38\text{ mm} = 0.038\text{ m}$

since $u_p = \dfrac{\omega}{k} = \dfrac{c}{\sqrt{\varepsilon_r}}$, we have

$$k = \frac{\omega}{c}\sqrt{\varepsilon_r} = 71.33\ \text{ rad/m}$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(kl)}{Z_0 + jZ_L \tan(kl)} = 82.90 + j118.01\ \ \Omega.$$

Q2

From page 5 of lecture notes: characteristic impedance = √(L/C) = (h/w)120π for air-spaced parallel plate. Thus, Zo = (1cm/2cm)120π = 60π

From page 8 of lecture note, assuming lossless (although there is a resistance due to skin depth R = L/(σA) = 20cm/(σwδ), where skin depth δ=1/√(πfμσ)=2.3μm and thus R = 20cm/(σx2cmx2.3μm) = 0.073Ω or 0.30 Ω/m

When the ground is a perfect short (0Ω), the input impedance is $Z_{in} = -jZ_o\tan(kz)$

Thus, Zin = -j60π tan(16.755x0.2) = -j60πx0.212 = -j40Ω

If the ground impedance is a typical 5Ω, then the input impedance is

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(kz)}{Z_o + jZ_L \tan(kz)} = 60\pi \frac{5 + j60\pi x0.212}{60\pi + j5x0.212} = 60\pi \frac{5 + j40}{60\pi + j1.06}$$

$$Z_{in} = 60\pi \frac{40.31e^{82.87^0}}{188.5e^{0.32^o}} = 40.3e^{82.55^0} = 5.23 + j40$$

Q3

$$Z_{\text{in}2} = Z_{02}\frac{Z_{L2} + jZ_{02}\tan(2\pi\, 3/8)}{Z_{02} + jZ_{L2}\tan(2\pi\, 3/8)} = 118.042 - j92.553\ \Omega$$

$$Z_{\text{in}3} = Z_{03}\frac{Z_{L3} + jZ_{03}\tan(2\pi\, 3/8)}{Z_{03} + jZ_{L3}\tan(2\pi\, 3/8)} = 95.244 - j30.472\ \Omega$$

Total impedance as seen from BB':

$$Z_{BB'} = Z_{\text{in}2} \,//\, Z_{\text{in}3} = 54.818 - j26.575\ \Omega$$

Total impedance as seen from AA':

$$Z_{AA'} = Z_{01}\frac{Z_{BB'} + jZ_{01}\tan(2\pi\, 3/8)}{Z_{01} + jZ_{BB'}\tan(2\pi\, 3/8)} = 126.884 - j332.878\ \Omega$$

Total current at AA': $I_{AA'} = \dfrac{100\angle 0°}{300 + Z_{AA'}} = 0.1456 + j0.1136 = 0.1847\angle 0.6623\text{A}$

Average power supplied to transmission line at AA':

$$P_{AA'} = \tfrac{1}{2}\left|I_{AA'}\right|^2 \operatorname{Re}\left[Z_{AA'}\right] = \tfrac{1}{2}(0.1847)^2(126.884) = 2.1643\ \text{W}\,.$$

Since transmission line 1 is lossless, the average power supplied to the parallel transmission lines at BB' is $P_{BB'} = P_{AA'} = 2.1643$ W . The total average power supplied to the two antennas must therefore be equal to $P_{BB'}$, since transmission lines 2 and 3 are lossless.

But

$$P_{BB'} = \tfrac{1}{2}\operatorname{Re}\left[V_{BB'}\, I_{BB'}^*\right] = \tfrac{1}{2}\operatorname{Re}\left[V_{BB'}\,(V_{BB'}^* / Z_{BB'}^*)\right] = \tfrac{1}{2}\left|V_{BB'}\right|^2 \operatorname{Re}\left[1/Z_{BB'}^*\right] = \tfrac{1}{2}\left|V_{BB'}\right|^2 (0.01477)$$

$$\rightarrow\ \left|V_{BB'}\right|^2 = 293.067$$

$$P_{L2} = \tfrac{1}{2}\left|V_{BB'}\right|^2 \operatorname{Re}\left[1/Z_{\text{in}2}^*\right] = 0.7688\ \text{W}$$

$$P_{L3} = \tfrac{1}{2}\left|V_{BB'}\right|^2 \operatorname{Re}\left[1/Z_{\text{in}3}^*\right] = 1.3956\ \text{W}$$

Note that $P_{L2} + P_{L3} = P_{BB'} = 2.1643$ W

Q4

Input impedance of the antenna $Z_{in} = 35 + j10\ \Omega$

Normalized input impedance is $z_{in} = \dfrac{Z_{in}}{Z_0} = \dfrac{35 + j10}{50} = 0.7 + j0.2$

Normalized input admittance is $y_{in} = \dfrac{1}{z_{in}} = 1.32\text{-}j0.38$

<u>Steps:</u>

1. Enter y_{in} into the Smith chart as shown. Note that y_{in} is diagonally opposite z_{in}.
2. Turn clockwise (towards generator) to point A which gives the first solution. Point A's normalized admittance is 1.0-j0.44. The distance traveled from y_{in} to point A measuring on the Smith chart is d_1 which is

$$d_1 = (0.3585 - 0.3045)\lambda = 0.054\lambda$$

   This is the position of the stub from the load.
3. To cancel the reactive part of -j0.44, an open-circuit parallel stub, whose normalized admittance at the open end is at point S on the Smith chart, must have

   a length ℓ_1 measuring (towards generator) from point S to point S_A on the Smith chart in order to give a normalized admittance of j0.44. That is,

$$\ell_1 = 0.0660\lambda$$

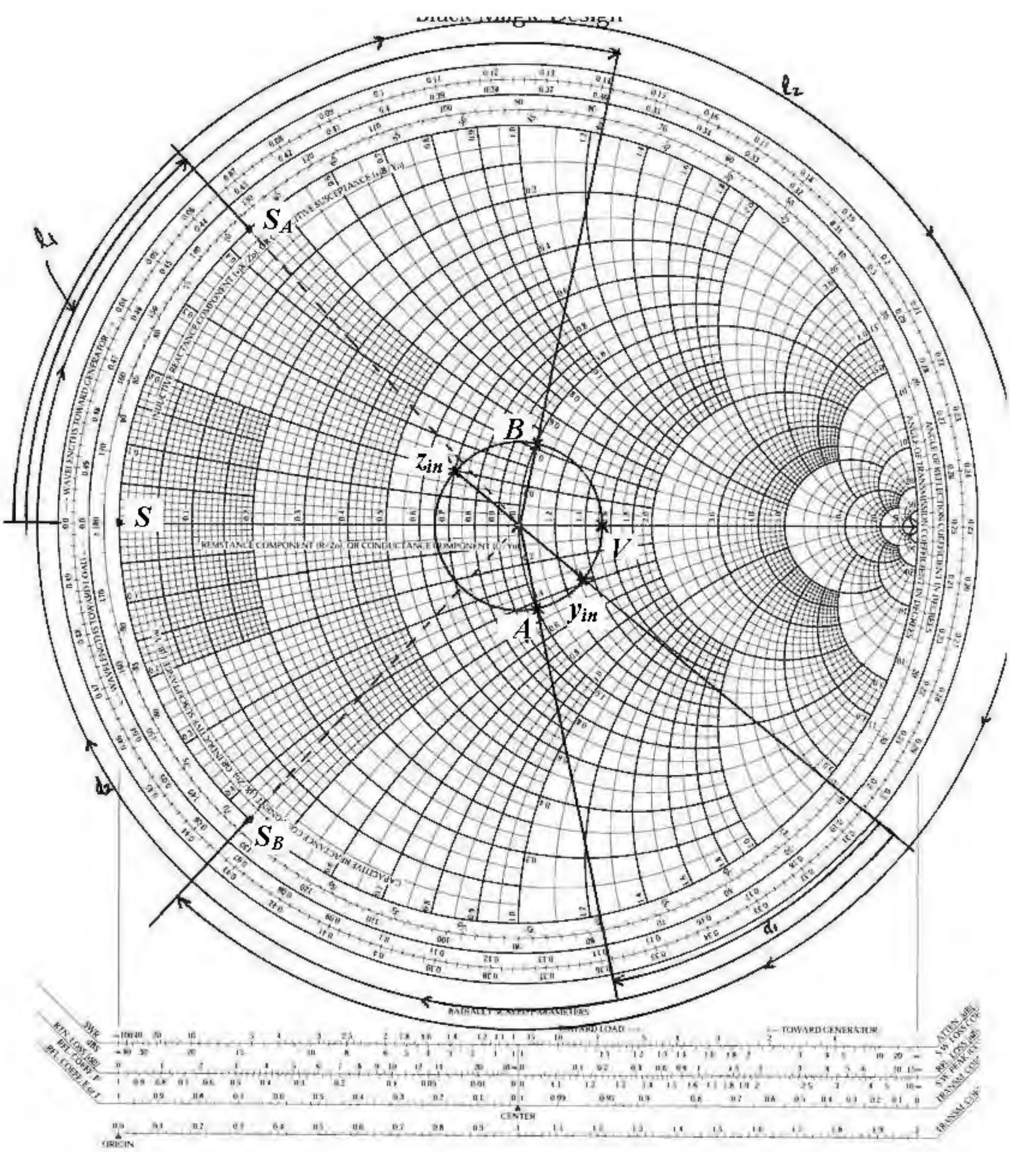
   This is the length of the stub.
4. There is another possible solution. This is achieved by turning (towards generator) from y_{in} to point B on the Smith chart. Point B's normalized admittance is 1.0+j0.44. The distance traveled from y_{in} to point B measuring on the Smith chart is d_2 which is

$$d_2 = (0.500 - 0.3045)\lambda + 0.1415\lambda = 0.337\lambda$$

   This is the position of the stub from the load for the second solution.
5. To cancel the reactive part of j0.44, an open-circuit parallel stub, whose normalized admittance at the open end is at point S on the Smith chart, must have a length ℓ_2 measuring (towards generator) from point S to point S_B on the Smith chart in order to give a normalized admittance of -j0.44. That is,

$$\ell_2 = 0.434\lambda$$

   This is the length of the stub for the second solution.


S_A
l_1
l_2
B
z_{in}
S
V
y_{in}
A
d_2
S_B
d_1
CENTER
ORIGIN