

SVM for Classification of Spam Email Messages

EE5904 / ME5404 Project I

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Report due on 21 April 2023, 23:59 Singapore time

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Project Goal

- Implement a SVM to classify spam or not a spam for the **Spam Email Data Set**
- Spam Email Data Set:
 - 4061 samples of email metadata taken from UC Irvine Machine Learning Repository
 - 57 features per sample
 - Label: +1 (spam), -1 (non-spam) | 1 and 0 in original dataset
 - http://archive.ics.uci.edu/ml/datasets/spambase

Class Distribution:
Spam 1813 (39.4%)
Non-Spam 2788 (60.6%)



Project Goal

• The dataset is divided into 3 subset according to **Project Requirement**

• Training set: 2000

• Test set: 1536

• Eval set: 600 (will not provide)

	Value
⊞ train_data	57x2000 double
train_label	2000x1 double

Name	Value
	57x1536 double
test_label	1536x1 double

Name	Value
eval_data	57x600 double
eval_label	600x1 double



Train \longrightarrow Construction: For a given training set $S = \{(\mathbf{x}_1, d_1), \dots, (\mathbf{x}_N, d_N)\}$, find optimal hyperplane $(\mathbf{w}_{\circ}, b_{\circ})$ such that, for all $i \in \{1, 2, \dots, N\}$,

$$\mathbf{x}_i \qquad \mathbf{w}_{\circ}^T \mathbf{x}_i + b_{\circ} \qquad \mathbf{sgn}[g(\mathbf{x}_i)] \qquad \mathbf{y}_i = d_i$$

Test Testing: For a given test set $\bar{S} = \{(\bar{\mathbf{x}}_1, \bar{d}_1), \dots, (\bar{\mathbf{x}}_{\bar{N}}, \bar{d}_{\bar{N}})\}$, compute output \bar{y}_i of SVM (with \mathbf{w}_\circ and b_\circ) for all $i \in \{1, 2, \dots, \bar{N}\}$, and compare it against the known \bar{d}_i to evaluate performance of SVM

$$\mathbf{\bar{x}}_{i}$$
 $\mathbf{w}_{\circ}^{T}\bar{\mathbf{x}}_{i} + b_{\circ}$ $\mathbf{g}\left(\mathbf{\bar{x}}_{i}\right)$ $\operatorname{sgn}[g\left(\mathbf{\bar{x}}_{i}\right)]$

Evaluate \longrightarrow Application: Given a SVM with hyperplane $(\mathbf{w}_{\circ}, b_{\circ})$, classify a data point \mathbf{x}_{new} that is not in $\Sigma = S \cup \bar{S}$:

$$\mathbf{x}_{\text{new}}$$
 $\mathbf{w}_{\circ}^{T}\mathbf{x}_{\text{new}} + b_{\circ}$ $\mathbf{g}(\mathbf{x}_{\text{new}})$ $\mathbf{sgn}[g(\mathbf{x}_{\text{new}})]$



Task1: Training

Task1 – Data



Training set – 2000 samples

- Given 'train.mat'
 - Features (57 x 2000)
 - Label (2000 x 1)
- Features of a sample

Name	Value
train_data	57x2000 double
train_label	2000x1 double

Label: +1 (spam), -1 (non-spam)

Task1 – Training Set



Import the training set (i.e. train.mat)

- train_data (57 x 2000)
- train_label (2000 x 1)

Preprocess the 'data' (Various methods can be used including Sample scaling and Standardization [CHOOSE ONE METHOD])^{a, b}

- Scale the data Rescale the individual sample x such that ||x|| = 1
- Standardize the data Transform each <u>feature</u> by removing the <u>mean</u> value of each feature and then dividing by each <u>feature's standard</u> deviation

Please ensure the 'label' is mapped into the set of $\{-1, +1\}$

^a https://scikit-learn.org/stable/modules/preprocessing.html

b https://en.wikipedia.org/wiki/Feature_scaling

Task1 – Kernels



Hard-margin SVM with the linear kernel

$$K(x_1, x_2) = x_1^T x_2$$

Hard-margin SVM with a polynomial kernel

$$K(x_1, x_2) = (x_1^T x_2 + 1)^p$$

Soft-margin SVM with a polynomial kernel

$$K(x_1, x_2) = (x_1^T x_2 + 1)^p$$

Task1 – Hard and Soft Margins



Hard margin $0 \le \alpha_i$

• $C = + \infty$ (In theory)

$$0 \leq \alpha_i \leq C$$

C = Large value (In practice e.g. 10⁶)

Soft margin $0 \le \alpha_i \le C$

C = 0.1, 0.6, 1.1, 2.1



How to calculate α_i

Use quadprog function (Quadratic programming)

Description

Solver for quadratic objective functions with linear constraints.

quadprog finds a minimum for a problem specified by

$$\min_{x} \frac{1}{2} x^{T} H x + f^{T} x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

H, A, and Aeq are matrices, and f, b, beq, lb, ub, and x are vectors.

You can pass f, lb, and ub as vectors or matrices; see Matrix Arguments.

x = quadprog(H,f,A,b,Aeq,beq,lb,ub,x0,options) solves the preceding problem using the optimization options specified in options. Use optimoptions to create options. If you do not want to give an initial point, set x0 = [].





Maximize
$$Q(\alpha) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j d_i d_j K(\mathbf{x}_i, \mathbf{x}_j)$$

Subject to :
$$\sum_{i=1}^{N} \alpha_i d_i = 0$$
, $0 \le \alpha_i \le C$

Description

Solver for quadratic objective functions with linear constraints. quadprog finds a minimum for a problem specified by

$$\min_{x} \frac{1}{2} x^{T} H x + f^{T} x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

Convert the problem from 'Max' to 'Min'

• Max Q(α) \rightarrow Min - Q(α)

If f is to be maximized instead, such a maximization problem Slide 62 can be expressed as a minimization problem by the transformation

$$\max_{\mathbf{w}} f(\mathbf{w}) = -\min_{\mathbf{w}} \left[-f(\mathbf{w}) \right]$$





Maximize
$$Q(\alpha) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j d_i d_j K(\mathbf{x}_i, \mathbf{x}_j)$$
 Subject to :
$$\sum_{i=1}^{N} \alpha_i d_i = 0 , \ 0 \le \alpha_i \le C$$

Subject to :
$$\sum_{i=1}^{N} \alpha_i d_i = 0, \ 0 \le \alpha_i \le C$$

Description

Not used

Solver for quadratic objective functions with linear constraints. quadprog finds a minimum for a problem specified by

$$\min_{x} \frac{1}{2} x^{T} H x + f^{T} x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

$$\min_{x} \frac{1}{2} x^{T} H x + f^{T} x$$

$$H_{ij} = d_{i} d_{j} K(x_{i}, x_{j})$$

$$f = (-1, -1, \dots, -1)^{T}$$

$$A = []$$

$$b = []$$

$$Aeq \cdot x = beq,$$

$$Aeq = (d_{1}, d_{2}, \dots, d_{N})$$

$$beq = 0$$

$$lb \leq x \leq ub.$$

$$lb = (0, 0, \dots, 0)^{T}$$

$$ub = (C, C, \dots, C)^{T}$$

x = quadprog(H,f,A,b,Aeq,beq,lb,ub,x0,options)



Hard-margin SVM with the <u>linear kernel</u>

$$K(x_1, x_2) = x_1^T x_2$$

For illustration only

$$\min_{x} \frac{1}{2} x^{T} H x + f^{T} x \qquad H(i,j) = d_{i} d_{j} x_{i}^{T} x_{j}$$

$$f = -\text{ones}(2000,1)$$

$$A \cdot x \leq b, \qquad A = []$$

$$b = []$$

$$Aeq \cdot x = beq, \qquad Aeq = train_label'$$

$$beq = 0$$

$$lb \leq x \leq ub. \qquad lb = zeros(2000,1)$$

$$ub = \text{ones}(2000,1) * C$$

$$x0 = []$$

$$options = optimset('LargeScale','off','MaxIter',1000)$$



Hard-margin SVM with a polynomial kernel

$$K(x_1, x_2) = (x_1^T x_2 + 1)^p$$

For illustration only

$$\min_{x} \frac{1}{2} x^{T} H x + f^{T} x \qquad H(i,j) = d_{i} d_{j} (x_{1}^{T} x_{2} + 1)^{p}$$

$$f = -\operatorname{ones}(2000,1)$$

$$A \cdot x \leq b, \qquad A = []$$

$$b = []$$

$$Aeq \cdot x = beq, \qquad Aeq = train_label'$$

$$beq = 0$$

$$lb \leq x \leq ub. \qquad lb = zeros(2000,1)$$

$$ub = \operatorname{ones}(2000,1) * C$$

x0 = [] options = optimset('LargeScale','off','MaxIter', 1000)



Soft-margin SVM with a polynomial kernel

$$K(x_1, x_2) = (x_1^T x_2 + 1)^p$$

For illustration only

$$\min_{x} \frac{1}{2} x^{T} H x + f^{T} x \qquad H(i,j) = d_{i} d_{j} (x_{1}^{T} x_{2} + 1)^{p}$$

$$f = -\operatorname{ones}(2000,1)$$

$$A = []$$

$$b = []$$

$$Aeq \cdot x = beq,$$

$$deq \cdot x = beq,$$

$$deq = train_label'$$

$$beq = 0$$

$$lb \leq x \leq ub.$$

$$deq = train_label'$$

$$beq = 0$$

$$lb = zeros(2000,1)$$

$$ub = \operatorname{ones}(2000,1) * C$$

$$x0 = [] \qquad options = optimset('LargeScale','off','MaxIter',1000)$$

Task1 – Select support vectors



Based on KKT conditions

- For a support vector, $\alpha_i \neq 0$ (In theory, $\alpha_i > 0$)
- However, in practice, $\alpha_i > threshold$
- How to decide?
 - \circ Choose an appropriate threshold (e.g. 1e-4) to determine the corresponding α_i to the support vectors
 - Then make the other smaller α_i values which are less than the threshold zero

Task1 – Discriminant function



Hard Margin SVM with Linear Kernel

$$\begin{array}{ll} \text{Maximizing}: & Q(\pmb{\alpha}) = \sum_{i=1}^N \alpha_i - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \alpha_i \, \alpha_j \, d_i \, d_j \, \mathbf{x}_i^T \mathbf{x}_j \\ \\ \text{Subject to}: & \text{(1)} & \sum_{i=1}^N \alpha_i \, d_i = 0 \end{array}$$

Subject to : (1)
$$\sum_{i=1}^{N} \alpha_i d_i = 0$$

(2)
$$\alpha_i \geq 0$$

Discriminant function

$$g(\mathbf{x}) = \mathbf{w}_{\circ}^{T} \mathbf{x} + b_{\circ}$$

After $\alpha_{\circ,i}$ is obtained, we can calculate \mathbf{w}_{\circ} and b_{\circ} as follows:

$$\mathbf{w}_{\circ} = \sum_{i=1}^{N} \alpha_{\circ,i} \, d_i \, \mathbf{x}_i \,, \quad b_{\circ} = \frac{1}{d^{(s)}} - \mathbf{w}_{\circ}^T \mathbf{x}^{(s)}$$
 where $\mathbf{x}^{(s)}$ is a support vector with label $d^{(s)}$

Task1 – Discriminant function



Soft Margin SVM with Linear Kernel

$$\text{Maximize}: \quad Q(\boldsymbol{\alpha}) = \sum_{i=1}^N \alpha_i - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \alpha_i \alpha_j d_i d_j \mathbf{x}_i^T \mathbf{x}_j$$

Subject to : $\sum_{i=1}^N \alpha_i d_i = 0$ and $0 \le \alpha_i \le C$

Discriminant function

$$g(\mathbf{x}) = \mathbf{w}_{\circ}^{T} \mathbf{x} + b_{\circ}$$

After $\alpha_{\circ,i}$ is obtained, we can calculate \mathbf{w}_{\circ} as follows:

$$\mathbf{w}_{\circ} = \sum_{i=1}^{N} \alpha_{\circ,i} \, d_i \, \mathbf{x}_i$$

After \mathbf{w}_{\circ} is obtained, we can calculate b_{\circ} as follows:

2 Take b_{\circ} as the average of all such $b_{\circ,i}$

① For each example \mathbf{x}_i with $0 < \alpha_i \le C$,

$$b_{\circ,i} = \frac{1}{d_i} - \mathbf{w}_{\circ}^T \mathbf{x}_i$$

$$b_{\circ} = \frac{\sum_{i=1}^{m} b_{\circ,i}}{m}$$

where m is the total number of \mathbf{x}_i with $0 < \alpha_i \le C$.

Task1 – Discriminant function



Soft Margin SVM with Nonlinear Kernel

$$\begin{array}{lll} \text{Maximize}: & Q(\pmb{\alpha}) = \sum_{i=1}^N \alpha_i - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \alpha_i \alpha_j d_i d_j \pmb{\varphi}^T(\mathbf{x}_i) \pmb{\varphi}(\mathbf{x}_j) \\ \text{Subject to}: & \sum_{i=1}^N \alpha_i d_i = 0 \,, \, 0 \leq \alpha_i \leq C \end{array} \qquad \begin{array}{ll} \text{Discriminant function} \\ g(\mathbf{x}) & = \sum_{i=1}^N \alpha_{\circ,i} d_i K(\mathbf{x},\mathbf{x}_i) + b_{\circ} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots & \vdots & \vdots \\ i = 1 & \vdots$$

Subject to: $\sum_{i=1}^{N} \alpha_i d_i = 0, \ 0 \le \alpha_i \le C$

$$g(\mathbf{x}) = \sum_{i=1}^{N} \alpha_{\circ,i} d_i K(\mathbf{x}, \mathbf{x}_i) + b_{\circ}$$

Determine b_0 in Slide 123

$$g(\mathbf{x}) = \sum_{i=1}^{N} \alpha_{\circ,i} d_i K(\mathbf{x}, \mathbf{x}_i) + b_{\circ}$$

using the fact that for a support vector $\mathbf{x}^{(s)}$

$$g(\mathbf{x}^{(s)}) = \pm 1 = d^{(s)}$$

Take b_{\circ} as the average of all such $b_{\circ,i}$

$$b_{\circ} = \frac{\sum_{i=1}^{m} b_{\circ,i}}{m}$$

where m is the total number of \mathbf{x}_i with $0 < \alpha_i \le C$.

Task1 – Summary



Given a training set $S = \{(\mathbf{x}_i, d_i)\}, i = 1, \dots, N$

Slide 123

1 Find a suitable kernel

Choose expression then check Mercer's condition

Soft Margin -

- 2 Choose a value for C
- 3 Solve for $\alpha_{\circ,i}$

4 Determine b_{\circ} in

Kernel

$$g(\mathbf{x}) = \sum_{i=1}^{N} \alpha_{\circ,i} d_i K(\mathbf{x}, \mathbf{x}_i) + b_{\circ}$$

using the fact that for a support vector $\mathbf{x}^{(s)}$

$$g(\mathbf{x}^{(s)}) = \pm 1 = d^{(s)}$$

Quadratic - programming

$$\text{Maximize}: \quad Q(\alpha) = \sum_{i=1}^N \alpha_i - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \alpha_i \alpha_j d_i d_j K(\mathbf{x}_i, \mathbf{x}_j)$$

Subject to :
$$\sum_{i=1}^{N} \alpha_i d_i = 0, \ 0 \le \alpha_i \le C$$

Support vector machine: $\frac{\mathbf{x}}{\operatorname{sgn}[g(\mathbf{x})]}$



Task2: Testing

Task2 – Data



Test set – 1536 samples

- Given 'test.mat'
 - Features (57 x 1536)
 - Label (1536 x 1)
- Features of a sample

Label: +1 (spam), -1 (non-spam)

Task2 – Test set



Import the test set (i.e. test.mat)

- test_data (57 x 1536)
- test_label (1536 x 1)

Preprocess the 'data' (Various methods can be used including Sample scaling and Standardization [CHOOSE ONE USE for TRAINING])^{a, b}

- Scale the data Rescale the individual sample x such that ||x|| = 1
- Standardize the data Transform each <u>feature</u> in the same manner with the training data. Use the <u>mean and variance of each feature</u> from <u>your</u> <u>training set</u>.

Please ensure the 'label' is mapped into the set of {-1, +1}

^a https://scikit-learn.org/stable/modules/preprocessing.html

b https://en.wikipedia.org/wiki/Feature scaling

Task2 – Test set



Discriminant function

$$g(\mathbf{x}) = \mathbf{w}_{\circ}^{T} \boldsymbol{\varphi}(\mathbf{x}) + b_{\circ} = \sum_{i=1}^{N} \alpha_{\circ,i} d_{i} \underbrace{\boldsymbol{\varphi}^{T}(\mathbf{x}_{i}) \boldsymbol{\varphi}(\mathbf{x})}_{K(\mathbf{x}_{i},\mathbf{x})} + b_{\circ}$$

To classify a new data point x_{new}

$$d_{\mathsf{new}} = \mathsf{sgn}\left[g(\mathbf{x}_{\mathsf{new}})\right]$$

For illustrations only

$$g(x_{test}) = \sum_{i=1}^{N} \alpha_{o,i} d_i K(x_i, x_{test}) + b_0$$
 If $g(x_{test}) > 0$
$$x_{test_label = +1}$$
 Else
$$x_{test_label = -1}$$

Task2 – Test set



Discriminant function

$$g(\mathbf{x}) = \mathbf{w}_{\circ}^{T} \boldsymbol{\varphi}(\mathbf{x}) + b_{\circ} = \sum_{i=1}^{N} \alpha_{\circ,i} d_{i} \underbrace{\boldsymbol{\varphi}^{T}(\mathbf{x}_{i}) \boldsymbol{\varphi}(\mathbf{x})}_{K(\mathbf{x}_{i},\mathbf{x})} + b_{\circ}$$

To classify a new data point x_{new}

$$d_{\mathsf{new}} = \mathsf{sgn}\left[g(\mathbf{x}_{\mathsf{new}})\right]$$

Type of SVM	Training accuracy				Test accuracy			
Hard margin with								
Linear kernel	?				?			
Hard margin with	p=2	p=3	p = 4	p = 5	p=2	p=3	p=4	<i>p</i> = 5
polynomial kernel	?	?	?	?	?	?	?	?
Soft margin with								
polynomial kernel	C = 0.1	C = 0.6	C = 1.1	C = 2.1	C = 0.1	C = 0.6	C = 1.1	C = 2.1
p=1	?	?	?	?	?	?	?	?
p=2	?	?	?	?	?	?	?	?
p=3	?	?	?	?	?	?	?	?
p=4	?	?	?	?	?	?	?	?
p = 5	?	?	?	?	?	?	?	?



Task3: Evaluation

Task3 – Data



Evaluation set – 600 samples

- Not Given 'eval.mat'
 - eval_data (57 x 600)
 - eval_label (600 x 1)

Task3 – Evaluation



Design your own SVM

- Hard margin or Soft margin?
- Linear or Polynomial kernel?
- What are the values for p and C?



To produce the best performance

To classify the 600 samples in the evaluation set

Not Given – 'eval.mat' eval_data (57 x 600) eval_label (600 x 1)

Output: A column vector (600 x 1) named 'eval_predicted'





Preprocess your data – Choose one method

Sample scaling/ Mean normalization/standardization/Rescaling ...

Use the <u>training set statistics</u> to preprocess the other data sets

For training set, test set and eval set

Check Mercer Condition for Kernel suitability

Don't use inbuilt MATLAB functions like 'fitsvm', 'predict'



Mercer's condition

For training set $S = \{(\mathbf{x}_i, d_i)\}, i = 1, 2, ..., N$, the Gram matrix

$$\mathbf{K} = \begin{bmatrix} K(\mathbf{x}_1, \mathbf{x}_1) & \dots & K(\mathbf{x}_1, \mathbf{x}_N) \\ \vdots & \ddots & \vdots \\ K(\mathbf{x}_N, \mathbf{x}_1) & \dots & K(\mathbf{x}_N, \mathbf{x}_N) \end{bmatrix} \in R^{N \times N}$$

is positive semi-definite (i.e., its eigenvalues are nonnegative)

• In practice, the eigenvalues of the matrix K always contain some very small negative values. We need to set a very small negative value (-1e-4 or -1e-6) as the threshold. As long as there is no eigenvalue smaller than it, then we believe that Mercer's condition is established.



Procedure to build SVM

- Preprocess data
- Choose a suitable kernel
 - Linear/ Nonlinear ?
- Choose C
 - Hard margin/Soft margin
 - Hard margin $0 \le \alpha_i$
 - \circ C = +∞ (In theory)
 - C = Large value (In practice e.g. 10⁶)
- Solve for α_i
 - Quadratic programming
- Support vector selection
 - Choose an appropriate threshold (e.g. 1e-4) to determine the corresponding α_i to the support vectors and make smaller α_i values zero.
- Determine the discriminant function g(x)



Submit all your codes that you have implemented for the entire project

Submit all required files – even if it is 'train.mat'

Make sure your codes run without error

All codes should be executable with the given datasets in the workspace without any additional inputs



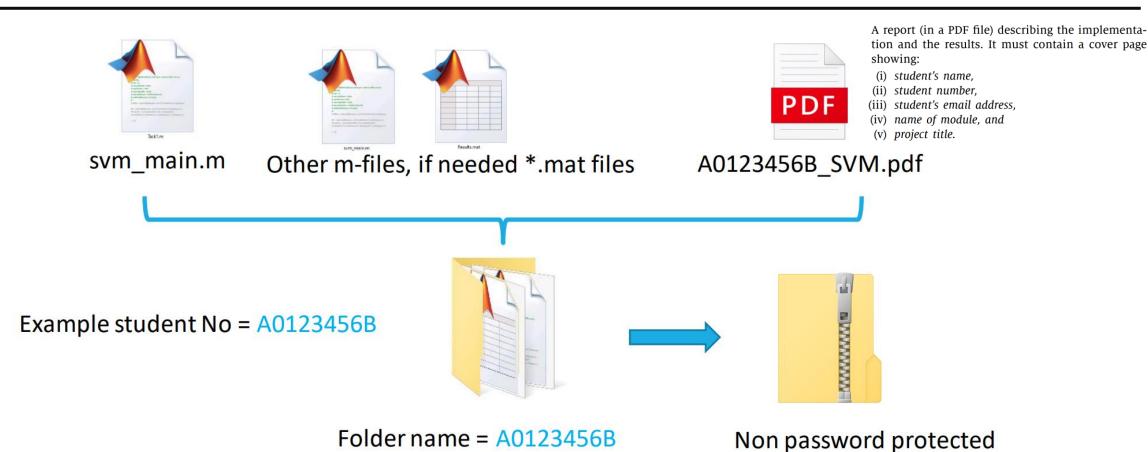
Report

- Details on Implementation
- Completed Table 1
- Discussion under following subheadings
 - Data pre-processing
 - Admissibility of the kernels
 - Existence of optimal hyperplanes
 - Table 1 Comments on results (with supporting arguments)
 - Task 3 Discussion on design decisions

Type of SVM	Training accuracy				Test accuracy			
Hard margin with								
Linear kernel	?				?			
Hard margin with	p=2	p=3	p=4	p = 5	p=2	p=3	p=4	p=5
polynomial kernel	?	?	?	?	?	?	?	?
Soft margin with								
polynomial kernel	C = 0.1	C = 0.6	C = 1.1	C = 2.1	C = 0.1	C = 0.6	C = 1.1	C = 2.1
p=1	?	?	?	?	?	?	?	?
p=2	?	?	?	?	?	?	?	?
p=3	?	?	?	?	?	?	?	?
p=4	?	?	?	?	?	?	?	?
p=5	?	?	?	?	?	?	?	?

TABLE I: Results of SVM classification.





Report due on 21 April 2023, 23:59 Singapore time

Thank you