NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF ENGINEERING

EXAMINATION FOR

(Semester II: 2021/2022)

EE5904/ME5404 – NEURAL NETWORKS

April/May 2022 – Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. Please write only your Student Number. Do not write your name.
2. This paper contains **FOUR** (4) questions and comprises **FIVE** (5) printed pages.
3. Answer all **FOUR** (4) questions.
4. All questions carry **EQUAL** marks. The **TOTAL** marks are 100.
5. This is a **CLOSED BOOK** examination. But the candidate is permitted to bring into the examination hall a single A4 size *help sheet*. The candidate may refer to this sheet during the examination.
6. Calculators can be used in the examination, but no programmable calculator is allowed.

Q.1 Consider the following two-class pattern classification problem shown in Fig. 1. All the points inside the triangle (including the points on the three edges) belong to one class, while all the points outside the triangle belong to another class. The equations for the three lines are also shown in Fig. 1.

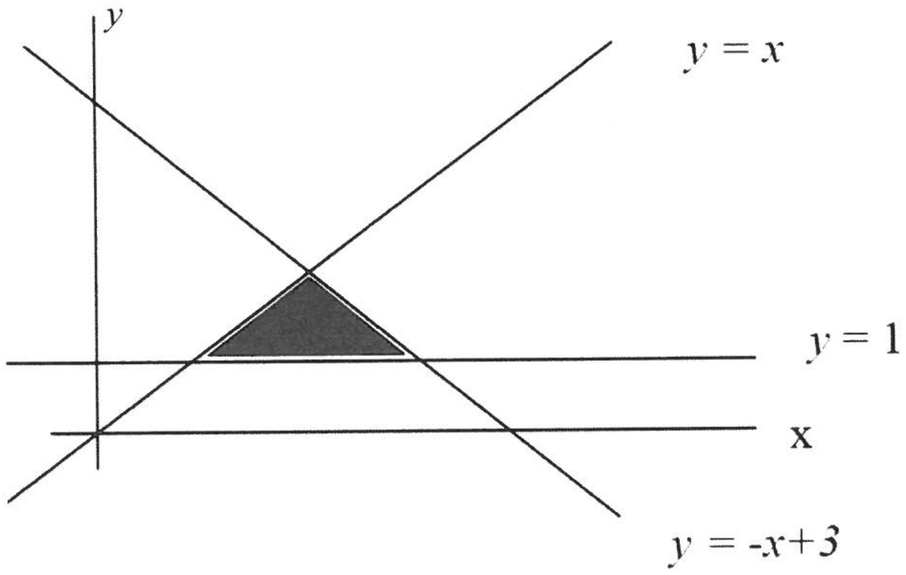

Fig. 1 A two-class pattern classification problem.

(a) Is this two-class pattern classification problem linearly separable or nonlinearly separable? Please supply a rigorous mathematical proof for your answer.

(10 marks)

(b) Design a multi-layer perceptron (MLP) with one hidden layer to solve this two-class pattern classification problem. Assume that the activation function of all the neurons is hard-limiter, i.e. step function. Design the weights and biases for all the neurons carefully such that the MLP can solve this pattern classification problem without any error. Justify your design.

(15 marks)

Q.2 Consider a two-dimensional pattern classification problem as shown in Fig. 2. There are four classes. The samples of the four classes are clustered around (0,0), (0,1), (1,0) and (1,1) as shown in Fig. 2.

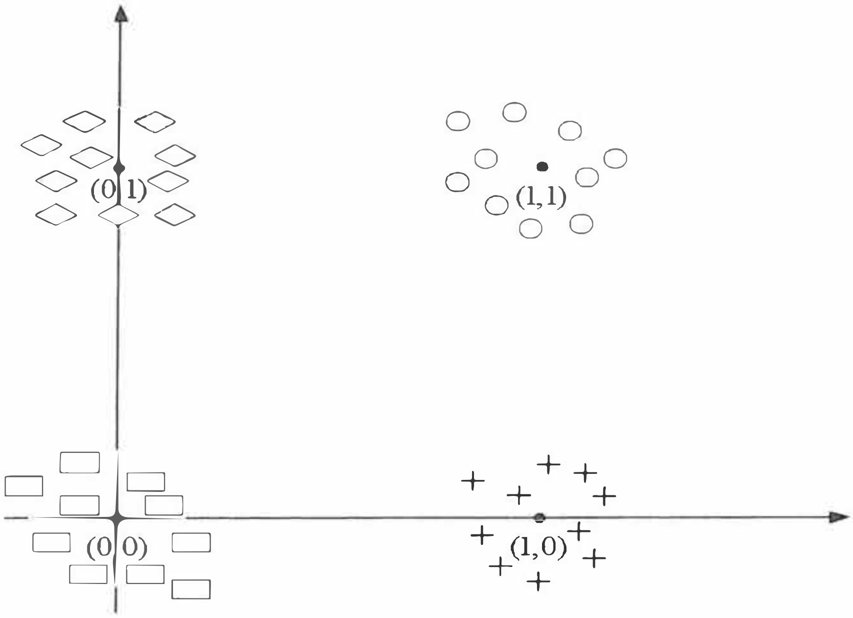

Fig. 2 The training samples for the pattern classification problem of Q2.

(a) Design a radial-basis function network (RBFN) to separate the samples of the four classes completely. You need to clearly specify all the design parameters including the number of hidden neurons, parameters associated with each center (the hidden neurons), the number of output neurons and the weights in the output layer.

(15 marks)

(b) Is it possible to use single-layer perceptron to solve this pattern classification problem? Justify your answer.

(5 marks)

(c) In self-organizing map (SOM), the concept of topological neighborhood is very important to assure that the weights of both the winner and its neighbors are adjusted at the same time with different rate. Give a simple example to demonstrate the potential problem if the simple "winner-takes-all" strategy is utilized in SOM. "Winner-takes-all" means that only the weights of the winner are adjusted while all others keep the same in each step.

(5 marks)

Q.3 Consider the training dataset in Table 3.1.

i	$\mathbf{x}_i = [x_1, x_2]^T$	d_i
1	$[0.8, -3]^T$	-1
2	$[1, 1]^T$	-1
3	$[0.8, 2]^T$	$+1$

Table 3.1

(a) Given a hyperplane that passes through two points $(0, -1)$ and $(1, 2)$, determine the geometric margin of the training dataset with respect to this hyperplane.

(10 marks)

(b) Suppose that a support vector machine is constructed using the training dataset. It is found that the value of one of the Lagrange multipliers of the dual problem is 1.92. For a given input vector $\mathbf{x} = [0, x_2]^T$, determine the value(s) of x_2 such that the output of the support vector machine is $+1$.

(10 marks)

(c) A new example with $\mathbf{x}_4 = [1, 0]^T$ and $d_4 = +1$ is added to the training dataset given in Table 3.1. Suppose that the kernel $K(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i^T\mathbf{x}_j + 1)^2$ is selected as a candidate for constructing a support vector machine using the expanded training dataset. Determine the Gram matrix.

(5 marks)

Q.4 Consider the grid environment with six states (numbered from 1 to 6) as shown in Figure 4.1, with the thick border indicating walls. Suppose that at each state the agent can move up (denoted by a_1), right (a_2), down (a_3), or left (a_4). When the agent moves from state s to state s', it receives a reward of 1 if $s > s'$ and 0 otherwise. For example, the agent receives a reward of 1 when moving from state 4 to state 3. Assume that state 1 and state 5 are goal (i.e., terminal) states. Let the discount rate γ be 0.7.

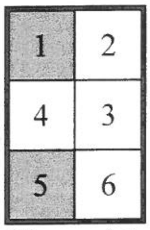

Figure 4.1

(a) Suppose that the agent, after taking an action at a state, enters the adjacent state in the direction of the intended action or remains in the same state in the case of a wall-collision. For example, at state 2, if the agent takes action a_1, it will remain in state 2; but if it takes action a_3 it will enter state 3. Determine the values of the Q-function for the policy: $\pi(2) = a_3$, $\pi(3) = a_4$, $\pi(4) = a_1$, and $\pi(6) = a_1$.

(10 marks)

(b) Suppose that the state transitions are now probabilistic, with the probabilities specified as follows. At a non-terminal state, after taking an action the agent ends up in the adjacent state in the direction of the intended action (or remains in the same state in the case of a wall-collision) with a probability of 0.8, and ends up in an adjacent state in the direction perpendicular to the intended direction (or remains in the same state in the case of a wall-collision) with a probability of 0.1. For example, if the agent takes action a_1 at state 4, it will end up in state 1 with a probability of 0.8, or end up at state 3 or state 4 with a probability of 0.1 for each case. Given the initial values of the Q-function as shown in Table 4.1, apply the value iteration algorithm for one iteration that starts with $Q(3, a_1)$, to calculate the values of $Q(3, a_1)$, $Q(3, a_2)$, $Q(3, a_3)$, and $Q(3, a_4)$, in the order as they are listed. (*Note: You can omit the rest of the iteration once you have calculated these four values.*)

State	**Action**			
	a_1	a_2	a_3	a_4
2	0.0	0.1	0.1	0.5
3	0.2	0.0	0.0	0.1
4	0.4	0.3	0.1	0.1
6	0.2	0.1	0.2	0.3

Table 4.1

(10 marks)

(c) Consider the state transition sequence $6 \xrightarrow{a_1} 3 \xrightarrow{a_4} 4 \xrightarrow{a_1} 1$, where the number to the left of an arrow indicates the state while the symbol above the arrow indicates the action taken at that state. Suppose that this sequence is executed in a trial using Q-learning (with $\alpha_k = 1/k$, and $\alpha_0 = 1$). Assume that the current values of the Q-function are all zero. Determine the values of the Q-function at the completion of this state transition sequence.

(5 marks)

END OF PAPER