FBA | QUANTITATIVE FINANCE RESEARCH GROUP

Financial Engineering

Homework 4

Due at 07:00 pm (Korea Standard Time) on Saturday, March 11.

Submit one file: written solutions with executable Python code

Problem 1. Derive the below forward equations.

$$
\begin{aligned}
P^e_{k+1,s} &= \frac{P^e_{k,s-1}}{2(1+r_{k,s-1})} + \frac{P^e_{k,s}}{2(1+r_{k,s})}, \quad 0 < s < k+1 \\
P^e_{k+1,0} &= \frac{1}{2}\frac{P^e_{k,0}}{(1+r_{k,0})} \\
P^e_{k+1,k+1} &= \frac{1}{2}\frac{P^e_{k,k}}{(1+r_{k,k})}.
\end{aligned}
$$

with $P^e_{0,0} = 1$.

Problem 2. Use the forward equations to compute the state prices for $t = 0, \cdots, 6$ in the short-rate lattice below. Now answer the following questions:

Short Rate Lattice					
					0.183
				0.146	0.132
			0.117	0.105	0.095
		0.094	0.084	0.076	0.068
	0.075	0.068	0.061	0.055	0.049
0.060	0.054	0.049	0.044	0.039	0.035
t=0	t=1	t=2	t=3	t=4	t=5

(a) Compute the price of a 4-period zero-coupon bond with face value 100 that expires at t = 4.

(b) Compute the price of a European call option on the zero-coupon bond of (a) that expires at t=2 and has strike $84

(c) Compute the price of a forward contract for delivery at t = 4 of a 2-year 10% coupon-bearing bond where we assume that delivery takes place just after a coupon has been paid.

(d) Compute the price of a caplet that expires at t = 6 with strike = 2%.

Problem 3. You are given an incomplete specification of the term structure, as specified by the spot rates and forward rates noted next. You also know that the price of a 6-year bond with coupon rate 10% is \$145.749 and the price of a 6-year bond with coupon rate 5% is \$100.315. For all bonds, the face value is \$100, and the coupons are paid annually. Assuming continuous compounding, find the missing rates.

$$s_1 = ?, s_2 = 6.9\%, s_3 = 7.5\%, s_4 = ?, s_5 = 8.4\%, s_6 = ?$$

$$f_{1,2} = 7.8\%, f_{2,3} = 8.7\%, f_{5,6} = ?, f_{1,3} = 8.25\%, f_{2,4} = 11.55\%$$

Problem 4. If X follows standard normal distribution ($X \sim N(0,1)$), what is $\mathrm{E}[X^n]$ for $n = 1,2,3,4$?

Problem 5. Solve the corresponding leetcode problem below and register the solution on GitHub.

https://leetcode.com/problems/intersection-of-two-arrays-ii/

https://github.com/fbaquant/leetcode-challenge/issues

Problem 6. Solve the corresponding leetcode problem below and register the solution on GitHub.

https://leetcode.com/problems/valid-anagram/

https://github.com/fbaquant/leetcode-challenge/issues