

Financial Engineering

Homework 2

Due at 07:00 pm (Korea Standard Time) on Saturday, February 18.

Submit one file: written solutions with executable Python code

Problem 1. There are three bonds in the market as follows:

1. A bond with 4% coupon rate (paid annually), 10 years to maturity, and \$1,000 face value
2. A bond with 4% plus current (short) rate (paid annually), 10 years to maturity, and \$1,000 face value
3. A bond with 8% minus current (short) rate (paid annually), 10 years to maturity, and \$1,000 face value

The prices of the bonds are \$950, \$1,100, and \$900, respectively.

- (a) Derive the price of a zero-coupon bond with 10 years to maturity and \$1,000 face value.
- (b) Derive the price of a floating-rate bond (coupon paid annually) with 10 years to maturity and \$1,000 face value.

Problem 2. Under continuous compounding the Macaulay duration becomes

$$D = \frac{\sum_{k=0}^n t_k e^{-\lambda t_k} c_k}{P}$$

Where λ is the yield and

$$P = \sum_{k=0}^n e^{-\lambda t_k} c_k$$

Find $dP/d\lambda$ in terms of D and P .

Problem 3. Mr A. Gaylord manages a pension fund and believes that the his stock selection ability is excellent. However, he is worried because the market could go down. He considers entering an equity swap where each quarter i , up to quarter M , he pays counterparty B the previous quarters total rate of return r_i on the S&P 500 index times some notional principal and receives payments at a fixed rate r on the same principal. The total rate of return includes dividends. Specifically, $1 + r_i = (S_i + d_i)/S_{i-1}$, where S_i and d_i are the values of the index at i and the dividends received from $i - 1$ to i , respectively. Derive the value of such a swap by the following steps:

- (a) Let $V_{i-1}(S_i + d_i)$ denote the value at time $i - 1$ of receiving $S_i + d_i$ at time i . Argue that $V_{i-1}(S_i + d_i) = S_{i-1}$ and find $V_{i-1}(r_i)$
- (b) Find $V_0(r_i)$
- (c) Find $\sum_{i=1}^M V_0(r_i)$
- (d) Find the value of the swap

Problem 4. Farmer D. Jones has a crop of grapefruit juice that will be ready for harvest and sale as 150, 000 pounds of grapefruit juice in 3 months. Jones is worried about possible price changes, so he is considering hedging. There is no futures contract for grapefruit juice, but there is a futures contract for orange juice. His son, Gavin, recently studied minimum-variance hedging and suggests it as a possible approach. Currently the spot prices are \$1.20 per pound for orange juice and \$1.50 per pound for grapefruit juice. The standard deviation of the prices of orange juice and grapefruit juice is about 20% per year, and the correlation coefficient between them is about .7. What is the minimum variance hedge for farmer Jones, and how effective is this hedge as compared to no hedge?

Problem 5. A rabbit sits at the bottom of a staircase with n stairs. The rabbit can hop up only one or two or three stairs at a time. How many different combinations are there for the rabbit to ascend to the top of the stairs? Try to solve this problem with Python using recursive function.