

An Introduction to MATH 1013

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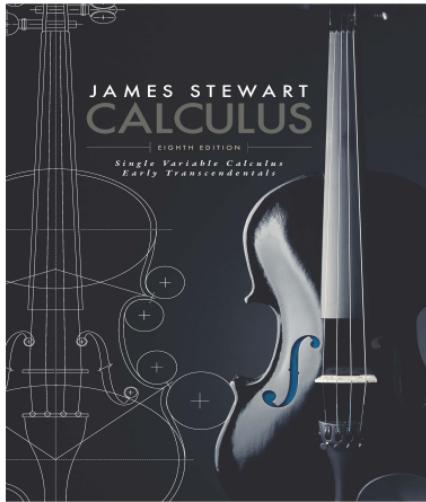
The Coverage of MATH 1013 and Textbook

Coverage:

- basic functions, limit, derivative, application, and integration

Textbook:

- Stewart's Single Variable Calculus: Early Transcendentals, 8th Edition



Textbook: Stewart's Single Variable Calculus

- Only Chapters 2, 3, 4, and 5 are required

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Textbook: Stewart's Single Variable Calculus

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- It is highly advised to read the book and do exercises!

About Tutorial T7-B and T7-C

Arrangement:

- T7-B: Tu 17:30 - 18:20, LT-F
- T7-C: Fr 17:30 - 18:20, Room 6602

Contact: wmaah@connect.ust.hk

- All questions **regarding the knowledge of this course** are welcome.
- Please don't ask questions about **homework and grading**.
- Generally, I will reply ASAP.

Tutorial Notes:

- Handwritten notes + printed notes (just for reference)
- Will be uploaded to Canvas and my personal website

Some Important Resources

MIT OpenCourseWare - Calculus by Professor Gilbert Strang:

MIT OPEN COURSEWARE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

RES.18-001 | Spring 2005 | Undergraduate

Calculus Online Textbook

Textbook

Instructor's Manual

Study Guide

Related Resources

Textbook

Published in 1991 by [Wellesley-Cambridge Press](#), the book is a useful resource for educators and self-learners alike. It is well organized, covers single variable and multivariable calculus in depth, and is rich with applications. There is also an online [Instructor's Manual](#) and a student [Study Guide](#).

The complete textbook is also available as a single file. ([PDF - 38.5MB](#))



Highlights of Calculus

MIT Professor Gilbert Strang has created [a series of videos](#) to show ways in which calculus is important in our lives. The videos, which include real-life examples to illustrate the concepts, are ideal for high school students, college students, and anyone interested in learning the basics of calculus.

• [Watch the videos](#)

Textbook Components

- Table of Contents ([PDF](#))
- Answers to Odd-Numbered Problems ([PDF - 2.4MB](#))
- Equations ([PDF](#))

Course Info

INSTRUCTOR
Prof. Gilbert Strang

DEPARTMENTS
Supplemental Resources

AS TAUGHT IN
Spring 2005

LEVEL
Undergraduate

TOPICS

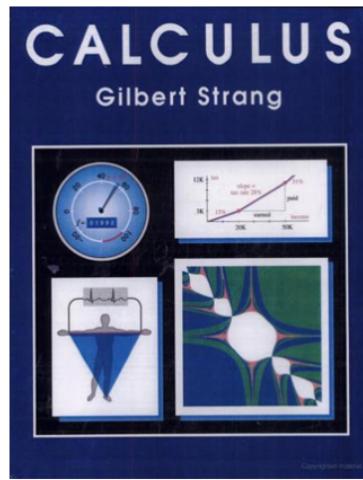
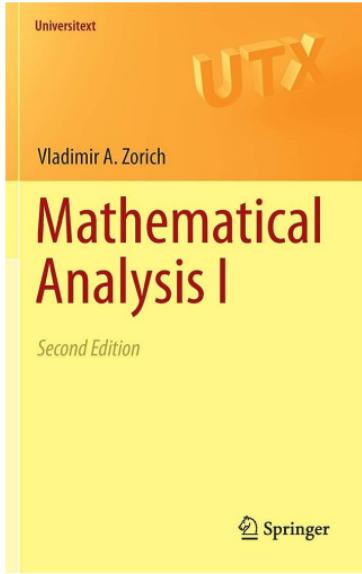
Mathematics

- Calculus
- Differential Equations

<https://ocw.mit.edu/courses/res-18-001-calculus-online-textbook-spring-2005/pages/textbook/>

Some Important Resources

Other books:



with abundant exercises for you!

Some Important Resources

GPT

 Polish Paragraph
GPT-4  Share

what is the limit of $\sin x/x$ when x is approaching 0

 GPT-4 

The limit of the function $\frac{\sin(x)}{x}$ as x approaches 0 is 1.

This is a well-known limit in calculus and can be proven using L'Hopital's rule or the squeeze theorem. The limit is often written as:

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

This limit is important in many areas of mathematics, including calculus and trigonometry.

 Share  Like  Dislike

Tell me more. →

Can you explain how L'Hopital's rule is used to prove the limit of $\sin x/x$ as x approaches 0? →

What is the squeeze theorem and how is it used to prove the limit of $\sin x/x$ as x approaches 0? →

Some Important Resources

GPT

GPT-4 Poe

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Tell me how to prove it

GPT-4 Poe

Sure, here's a sketch of the proof, using the squeeze theorem. This theorem states that if we have three functions, $f(x)$, $g(x)$, and $h(x)$, and for a certain range of x , we know that $f(x) \leq g(x) \leq h(x)$, then if the limit as x approaches a certain value (let's call it ' c ') of $f(x)$ and $h(x)$ are both L , then the limit of $g(x)$ as x approaches ' c ' must also be L .

1.

First, consider a unit circle (a circle with a radius of 1) and an angle x (in radians) subtended at the center of the circle.

Some Important Resources

Wolfram Alpha

9:47 PM Sun 10 Sept

Examples

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WolframAlpha

limit x to 0, sin(x)/x

Limit

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

Step-by-step solution

Plot

(x from -9.4 to 9.4)

Series expansion at x = 0

$$1 - \frac{x^2}{6} + \frac{x^4}{120} + O(x^6)$$

(Taylor series)

[More terms](#)

Related Wolfram|Alpha queries

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Wolfram Alpha

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Examples

WolframAlpha

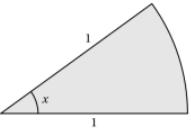
Basic Step-by-Step Solution X

Limit

Find the following limit:

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$$

Draw a sector S with a central angle of x radians and radius 1. This will be used to establish inequalities that can be used in the squeeze theorem:



The area of a sector with a central angle of θ radians and radius r is $\frac{\theta r^2}{2}$. This means the area of S is $A = \frac{x}{2}$:

$$Area S = \frac{x}{2}$$

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Examples

WolframAlpha

Basic Step-by-Step Solution

$\sin(x)$

$\cos(x)$

$\tan(x)$

$\sin^2(x)$

$\cos^2(x)$

$\tan^2(x)$

$\frac{\sin(x)}{x}$

$\frac{\cos(x)}{x}$

$\frac{\tan(x)}{x}$

$\frac{\sin^2(x)}{x}$

$\frac{\cos^2(x)}{x}$

$\frac{\tan^2(x)}{x}$

Mathematics

Statistics

Chemistry

Physics

Space & Astronomy

Engineering

Materials

Life Sciences

Computational Science

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Music

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Area $T = \frac{\sin(x)}{2}$

Area $s = \frac{x \cos^2(x)}{2}$

By construction Area $s \leq \text{Area } T \leq \text{Area } S$, which gives:

$$\frac{x \cos^2(x)}{2} \leq \frac{\sin(x)}{2} \leq \frac{x}{2}$$

Multiply the established inequality through by $\frac{2}{x}$:

$$\cos^2(x) \leq \frac{\sin(x)}{x} \leq 1$$

Since $\cos^2(x) \leq \frac{\sin(x)}{x} \leq 1$ and $\lim_{x \rightarrow 0} \cos^2(x) = 1$, the squeeze theorem implies:

Answer:

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

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Several Tips

- For freshmen, get used to the self-learning
- Exploit all the resources to empower yourself
- Understanding math is the key, but practice makes perfect

Thanks

Thank you for listening!