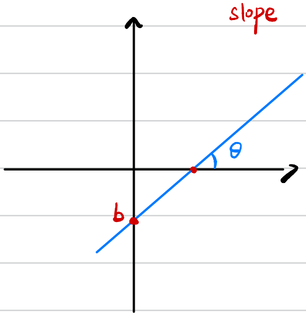
Lecture 1.

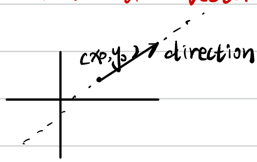
① Lines : $y = ax + b$. * goes accross $(0,b)$, $(-\frac{b}{a}, 0)$

a: slope, b: intercept



* uniformly determined by 2 elements (point, slop, intercept)

* direction vector : $(1,a)$ or $(m_1, m_2) \Rightarrow (1, \frac{m_2}{m_1})$



$$\left.\begin{array}{l} y = y_0 + a\cdot\Delta \\ x = x_0 + \Delta \end{array}\right\} \Rightarrow y = y_0 + a(x - x_0)$$

$$y = ax + y_0 - ax_0.$$

* parallel & perpendicular.

$a_1 = a_2$ $\qquad$ $a_1 \cdot a_2 = -1 \Rightarrow \begin{bmatrix} 1 \\ a_1 \end{bmatrix}^T \cdot \begin{bmatrix} 1 \\ a_2 \end{bmatrix} = 0$.

direction vectors are perpendicular.

② Representation of a function.

$y = f(x)$, $x \in D$. $\qquad$ D: domain.

$\{ f(x) \mid x \in D \}$: range

Find domains ★ (with function composition).

e.g.: If $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$, find the domain of

(a): $g \circ f$

(b) $g \circ g$.

solution: (a): $g \circ f = \sqrt{2 - \sqrt{x}}$. domain of g gives: $2 - \sqrt{x} \geq 0$. $\Rightarrow$ $x \in [0, 4]$.

domain of f gives: $x \geq 0$.

(b): $g \circ g = \sqrt{2-\sqrt{2-x}}$, domain of g (outer) gives $2-\sqrt{2-x} \geq 0$.

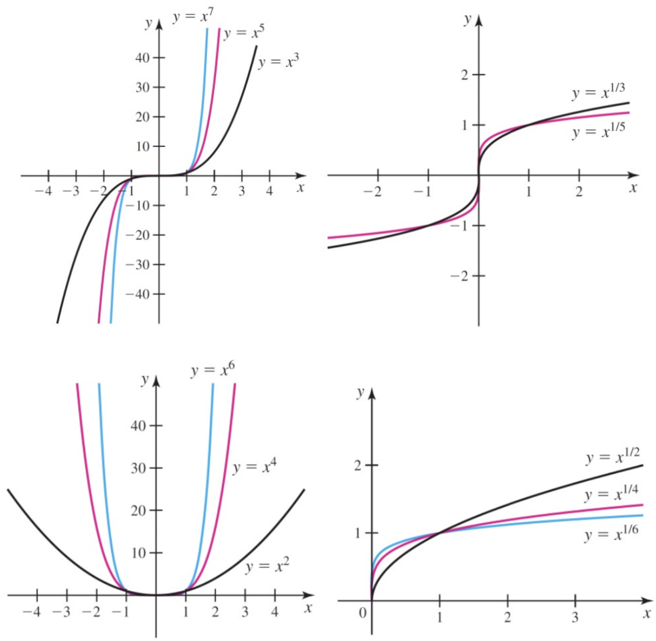
domain of g (inner) gives $\sqrt{2-x} \geq 0$.

$\Downarrow$

$x \in [-2, 2]$.

★ power function:

Graphs of Power Functions and Root Functions



Consider $y = x^\alpha$. Think: (a) what if $\alpha \to +\infty$ → indicator function.

(b) what if $\alpha \to 0$ → $y = 1$

Lecture 2.

function transformations.

① function shift:

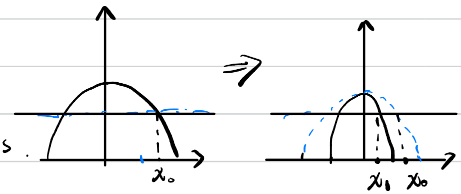
$y = f(x) + c$

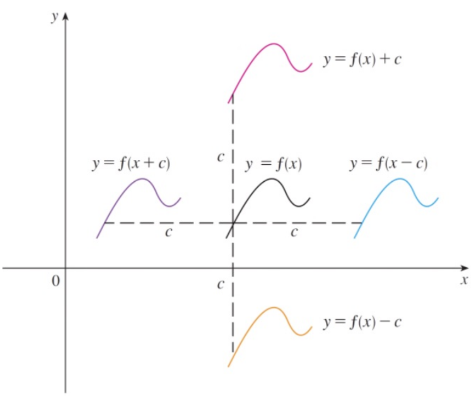
$y = f(x+c)$

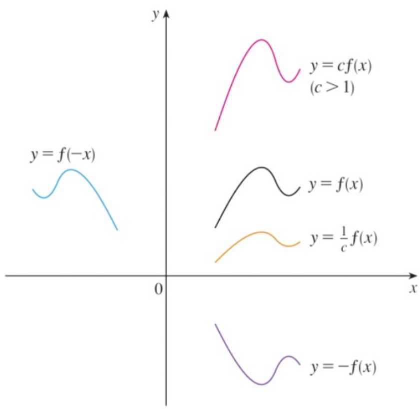
② function stretching and reflection:

$y = cf(x)$

$y = f(cx)$. $\rightarrow$ $c>1$, compress.







③ Composite function ☆.

$f \circ g(x) = f(g(x))$.

$f \circ g \neq g \circ f$.

e.g: Suppose $f(x) = x^2$, $g(x) = \sqrt{x}$. what is $f \circ g$, $g \circ f$?
Are they equivalent?

$f \circ g(x) = (\sqrt{x})^2 = x$, domain: $x \geq 0$.

not equivalent!

$g \circ f(x) = \sqrt{x^2} = |x|$, domain $\mathbb{R}$.

④. inverse of functions.

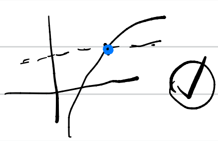
Only bijective function can be inversed.

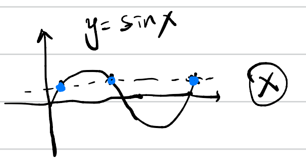
for any y in the range of $f(x)$, there exists Only one x. such that

$$f(x) = y.$$

or. if $f(x_1) = y$, and $f(x_2) = y$, then $x_1 = x_2$.

$y = \ln x$



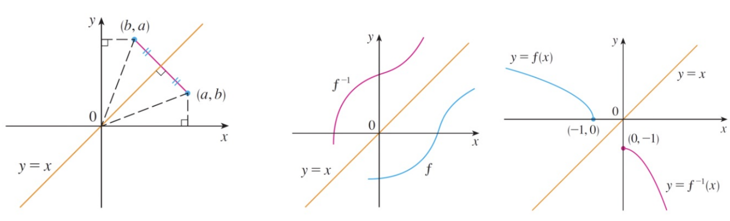


$y = \ln x \to x = e^y \Rightarrow g = e^x$

$y = x^2 \to x = \sqrt{y}$ (for $x \geq 0$). $g = x^{\frac{1}{2}}$.

(a) If a function $f(x)$ is bijective, the function $f(x)$ has an inverse function $f^{-1}(x)$. i.e. It's inverse function is well defined.

(b) The graph of $y = f^{-1}(x)$ is obtained by reflecting the graph of $y = f(x)$ about the line $= y$.



exponential : $e = 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \cdots = \lim_{n\to\infty} (1+\frac{1}{n})^n$.

What is $\lim_{n\to\infty} (1-\frac{1}{n})^n$? $\to \frac{1}{e}$.

$(1-\frac{1}{n})^n = (\frac{n-1}{n-1+1})^n = (\frac{1}{1+\frac{1}{n-1}})^n \to \frac{1}{e}$, since $(\frac{1}{1+\frac{1}{n-1}})^{n-1} \to \frac{1}{e}$

what is $1+2+3+\cdots+n+\cdots$? $n^2 \to +\infty$

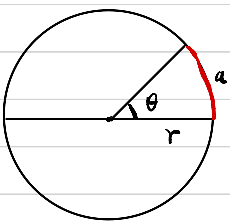
$1+\frac{1}{2}+\frac{1}{3}+\cdots+$? $\ln(n) \to +\infty$

$1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}+\cdots$? $\frac{\pi^2}{6}$

$1+\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots$? 2.

Lecture 3

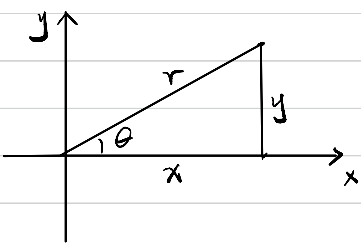
Trigonometric function.



The angle θ is measured by radians (or rad). $\theta \to \frac{a}{r}$.

π rad $= 180°$.

2π rad $= 360° = 1$ round.



$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$. $\cot\theta = \frac{1}{\tan\theta}$.

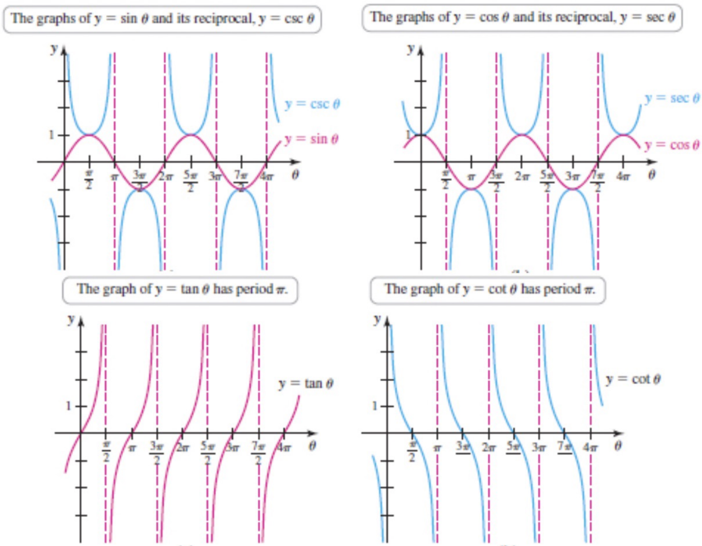
Important identities:

$\sin^2\theta + \cos^2\theta = 1 \to 1 + \cot^2\theta = \csc^2\theta$.

$\downarrow$

$1 + \tan^2\theta = \sec^2\theta$.

Period of $\sin\theta$, $\cos\theta$. $\tan\theta$.



$\sin(\frac{\pi}{2}-\theta) = \cos\theta$. $\quad \tan(\frac{\pi}{2}-\theta) = \cot\theta$

$\cos(\frac{\pi}{2}-\theta) = \sin\theta$

Addition formula:

$\sin(x+y) = \sin x\cos y + \cos x\sin y$.

$\cos(x+y) = \cos x\cos y - \sin x\sin y$.

$\tan(x+y) = \dfrac{\tan x + \tan y}{1-\tan x\tan y}$.

How about subtraction? Double-angle?

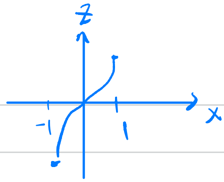
$y=-y$ $\quad$ $y=x$.

e.g. Show that $y = \cos(\sin^{-1}x)$ $x\in[-1,1]$ is equal to $\sqrt{1-x^2}$.

Denote $z = \sin^{-1}x$, $\quad \cos z^2 + \sin^2 z = 1$. $\Rightarrow$ $y^2 + [\sin(\sin^{-1}x)]^2 = 1$.

$y^2 + x^2 = 1$.

Notice that. z :



$z \in [-\pi, \pi]$.

Thus. $y = \cos(z) \geq 0$. , i.e, $y = \sqrt{1-x^2}$.