CPSC 538G Assignment #2

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1 Question 1

1.1 a

(20 points) Show, by induction on the structure of the graph, that for each vertex c, $B \to q(c)$

First we define a truth table for $B \to q(c)$

B	$q(c)$	$B \to q(c)$
T	T	T
T	F	F
F	T	T
F	F	T

(i) $c \in$ **root** A

By definition $q(c) = true$. In the case where $B = true$ we have $true \to true$, in the case where B is false we have $false \to true$ which holds. Therefore, $B \to q(c)$ if $c \in$ root A.

(ii) $c \in$ **root** B

By definition $q(c) = c$. If $B = true$, then $B \to q(c)$ becomes $true \to q(c)$ where $c \in B$ which reduces to $true \to true$. If $B = false$, $q(c) \in [true, false]$, so $false \to [true, false]$ which holds. Therefore $B \to q(c)$ if $c \in$ root B.

(iii) $q(c) = q(c_1) \wedge q(c_2)$

If $c_1, c_2 \in A$, $q(c) == true$, see **(i)**. If $c_1, c_2 \in B$, $q(c) = c_1 \wedge c_2$ and $B \to c_1 \wedge c_2$, see **(ii)**. If $c_1 \in A \wedge c_2 \in B$ then $B \to true \wedge c_2$. Which reduces to $B \to c_2$, see **(ii)**

1.2 b

(20 points) Show, by induction on the structure of the graph, that for each vertex c, $(p(c) \wedge q(c) \to c$

First we define a truth table for $(p(c) \wedge q(c) \to c$; taking note that only the second case leads to a contradition.

$p(c)$	$q(c)$	c	$(p(c) \wedge q(c) \to c$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

(i) $c \in$ **root** A

If $c \in$ root A, $q(c) = T$ and $p(c) = g(c)$. This simplifies the formulat to $T \wedge g(c) \rightarrow c$. Where $g(c) \in c$, $g(c) = true \rightarrow c$. If $g(c) = false$, $c \in [true, false]$ so $false \rightarrow [true, false]$ which holds. Therefore $q(c) \wedge p(c) \rightarrow c$ if $c \in$ root A.

(ii) $c \in$ **root** B

If $c \in$ root B then $p(c) = true$ and $q(c) = c$. This reduces to $true \wedge c \rightarrow c$, in tern $true \rightarrow true$, $false \rightarrow false$. Therefore $q(c) \wedge p(c) \rightarrow c$ if $c \in$ root B.

(iii) c **has parents** c_1,c_2 **on variable** v **which is local to** A

If v is local to A then $p(c) = p(c_1) \vee p(c_2)$ and $q(c) = q(c_1) \wedge q(c_2)$, so we have $(p(c_1) \vee p(c_2)) \wedge (q(c_1) \wedge q(c_2)) \rightarrow c$. Reducing $p(c)$ to it's globals we get $(g(c_1) \vee g(c_2)) \wedge (q(c_1) \wedge q(c_2)) \rightarrow c$. If $q(c_1)$,$q(c_2) \in A$ they resolve to true, and from **a iii** we know .