**Problem 1**

- Current Stock Price $165
- Strike Price $165
- Current Date 03/13/2022
- Options Expiration Date 04/15/2022
- Risk Free Rate of 4.25%
- Continuously Compounding Coupon of 0.53%

Implement the closed form Greeks for GBSM. Implement a finite difference derivative calculation. Compare the values between the two methods for both a call and a put.

Implement the binomial tree valuation for American options with and without discrete dividends. Assume the stock above:

- Pays dividend on 4/11/2022 of $0.88

Calculate the value of the call and the put. Calculate the Greeks of each. What is the sensitivity of the put and call to a change in the dividend amount?

The implementations of the closed form Greeks for GBSM and a finite difference derivative calculation are straightforward as detailed by the notes. The Black Scholes function we wrote for the previous assignment comes in handy when implementing the finite difference derivative approach. The results are as follows.

| Greeks | Call | | Put | |
|---|---|---|---|---|
| | Closed form | Finite difference derivative | Closed form | Finite difference derivative |
| Delta | 0.5340 | 0.5340 | -0.4655 | -0.4655 |
| Gamma | 0.0400 | 0.0400 | 0.0400 | 0.0400 |
| Vega | 19.7102 | 19.7099 | 19.7102 | 19.7099 |
| Theta | -24.8985 | -24.9911 | -18.7870 | -18.8796 |
| Rho | 7.5836 | 7.5835 | -7.2770 | -7.2771 |
| Carry Rho | 7.9662 | 7.9663 | -6.9444 | -6.9444 |

The Greeks calculated through the two methods are about the same, given △=0.0001*underlying price. One thing worth noting is that for the closed form Rho, we assume rf=b (no dividend payments), and thus it is not as accurate as the one calculated through the finite difference derivative method.

In terms of the implementations of the binomial tree valuation for both no dividend and with dividend cases, we use the recursive approach taught in class. We then calculate the Greeks and the sensitivity to dividend for both the call and the put through the finite difference derivative method as we did previously, but using the binomial tree valuation functions. The results are as below.

| | Call | Put |
|---|---|---|
| With dividend | 4.1200 | 4.1100 |
| Without dividend | 4.2275 | 3.7143 |
| **Greeks** | | |
| Delta | 0.5386 | -0.4931 |
| Gamma | -6.5247 | -1.3049 |
| Vega | 19.5148 | 19.8245 |
| Theta | -24.8868 | -18.6248 |
| Rho | 6.8297 | -7.2219 |
| Sensitivity to change in dividend amounts | -0.0936 | 0.5126 |

**Problem 2**

Using the options portfolios from Problem3 last week (named problem2.csv in this week's repo) and assuming:

- American Options
- Current Date 03/03/2023
- Current AAPL price is 151.03
- Risk Free Rate of 4.25%
- Dividend Payment of $1.00 on 3/15/2023

Using DailyPrices.csv. Fit a Normal distribution to AAPL returns – assume 0 mean return. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES.

Calculate VaR and ES using Delta-Normal.

Present all VaR and ES values a $ loss, not percentages. Compare these results to last week's results.

The problem is similar to one of last week's problems. Likewise, we can start by writing functions for calculating implied volatilities and portfolio values. We can then simulate returns using normal distribution for the next ten days and calculate the portfolio values ten days later. The mean, VaR, and ES obtained are as follows.

| | Mean | VaR | ES |
|---|---|---|---|
| **Straddle** | 2.934361 | -0.041406 | -0.021459 |
| **SynLong** | 0.099347 | 17.599745 | 21.093383 |
| **CallSpread** | 0.148110 | 3.546573 | 3.887153 |
| **PutSpread** | 0.439253 | 2.620574 | 2.767577 |
| **Stock** | 0.380886 | 16.671101 | 19.902287 |
| **Call** | 1.516854 | 5.631834 | 6.028156 |
| **Put** | 1.417507 | 4.312130 | 4.519575 |
| **CoveredCall** | -1.142948 | 13.042462 | 16.098311 |
| **ProtectedPut** | 1.613749 | 7.177510 | 7.603287 |

To calculate VaR and ES using Delta Normal, I used the below formula learned earlier this semester and got the following results (the one on the left). I also put last week's results on the right for reference.

$$VaR(\alpha) = -\, PV * F_X^{-1}(\alpha) * \sqrt{\nabla R^T \Sigma \, \nabla R}$$

| | VaR | ES |
|---|---|---|
| **Straddle** | 3.117919 | 3.909997 |
| **SynLong** | 17.652508 | 22.136957 |
| **CallSpread** | 5.663956 | 7.102830 |
| **PutSpread** | 4.784100 | 5.999454 |
| **Stock** | 17.581895 | 22.048405 |
| **Call** | 10.385214 | 13.023477 |
| **Put** | 7.267294 | 9.113480 |
| **CoveredCall** | 11.098402 | 13.917844 |
| **ProtectedPut** | 12.222024 | 15.326912 |

| | Mean | VaR | ES |
|---|---|---|---|
| **Straddle** | 2.658813 | 0.138180 | 0.144531 |
| **SynLong** | 0.001118 | 17.706815 | 20.712603 |
| **CallSpread** | 0.032579 | 3.786905 | 4.045239 |
| **PutSpread** | 0.379652 | 2.513104 | 2.705215 |
| **Stock** | 0.061939 | 17.547137 | 20.515924 |
| **Call** | 1.329965 | 5.899570 | 6.200008 |
| **Put** | 1.328848 | 4.164096 | 4.435482 |
| **CoveredCall** | -1.342873 | 13.808719 | 16.656662 |
| **ProtectedPut** | 1.244350 | 8.036230 | 8.535641 |

While the VaR and ES calculated using Monte Carlo through the normal distribution are really close to last week's results, the means are slightly larger. The VaR and ES calculated using Delta Normal are greater in multiple strategies such as CallSpread, PutSpread, and ProtectedPut, compared to last week's results.

**Problem 3**
Use the Fama French 3 factor return time series (F-F_Research_Data_Factors_daily.CSV) as well as the Carhart Momentum time series (F-F_Momentum_Factor_daily.CSV) to fit a 4-factor model to the following stocks.

| | | | |
|---|---|---|---|
| AAPL | FB | UNH | MA |
| MSFT | NVDA | HD | PFE |
| AMZN | BRK-B | PG | XOM |
| TSLA | JPM | V | DIS |
| GOOGL | JNJ | BAC | CSCO |

Fama stores values as percentages, you will need to divide by 100 (or multiply the stock returns by 100) to get like units.
Based on the past 10 years of factor returns, find the expected **annual** return of each stock.
Construct an annual covariance matrix for the 20 stocks.
Assume the risk-free rate is 0.0425. Find the super efficient portfolio.

We can start by obtaining alphas and betas through fitting a linear regression between returns of a stock and the factors. Then we can calculate $r_s$ according to the below formula. Subsequently, we get to obtain cumulative returns and then the expected annual returns of each stock, which are as follows.

$$r_s - r_{rf} = \alpha + \beta_{mkt}\left(r_{mkt} - r_{rf}\right) + \beta_{SMB}SMB + \beta_{HML}HML + \beta_{UMD}UMD + \epsilon_s$$

| Stock | Return | Stock | Return | Stock | Return | Stock | Return |
|---|---|---|---|---|---|---|---|
| AAPL | 0.1571 | NVDA | 0.2797 | PG | 0.0815 | DIS | -0.1554 |
| META | 0.0179 | HD | 0.1206 | XOM | 0.5218 | GOOGL | -0.0171 |
| UNH | 0.2538 | PFE | 0.0770 | TSLA | -0.0333 | JNJ | 0.1242 |
| MA | 0.2223 | AMZN | -0.0429 | JPM | 0.0983 | BAC | -0.1123 |
| MSFT | 0.1559 | BRK-B | 0.1299 | V | 0.2411 | CSCO | 0.1478 |

Due to the space restriction here, please see Cell 33 in week7.ipynb for the annual covariance matrix for the 20 stocks.

To obtain the super efficient portfolio, we need to find the weights that maximize the Sharpe ratio. Each weight has to be in between 0 and 1 and all together sums to 1. After solving this optimization problem, we can then calculate the Sharpe ratio.

The Sharpe ratio I got is 1.4707 and the weights are as follows.

| Stock | Weight | Stock | Weight | Stock | Weight | Stock | Weight |
|---|---|---|---|---|---|---|---|
| AAPL | 0 | NVDA | 0 | PG | 0 | DIS | 0 |
| META | 0 | HD | 0 | XOM | **0.5744** | GOOGL | 0 |
| UNH | **0.2257** | PFE | 0 | TSLA | 0 | JNJ | **0.0705** |
| MA | 0 | AMZN | 0 | JPM | 0 | BAC | 0 |
| MSFT | 0 | BRK-B | 0 | V | **0.1293** | CSCO | 0 |