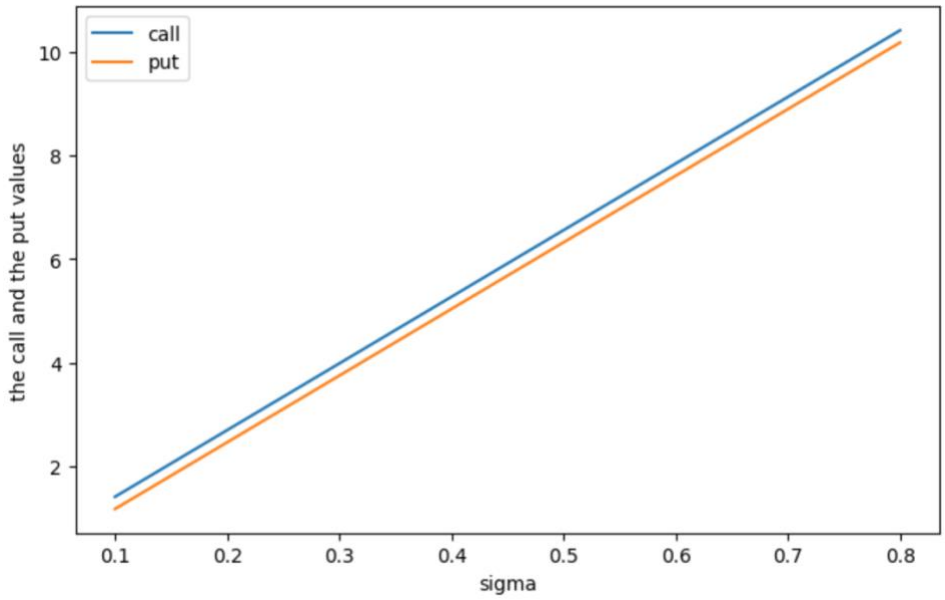
Problem 1

Assume you have a call and a put option with the following

- **Current Stock Price $165**
- **Current Date 03/03/2023**
- **Options Expiration Date 03/17/2023**
- **Risk Free Rate of 4.25%**
- **Continuously Compounding Coupon of 0.53%**

Calculate the time to maturity using calendar days (not trading days). For a range of implied volatilities between 10% and 80%, plot the value of the call and the put. Discuss these graphs. How does the supply and demand affect the implied volatility?

The problem is straightforward. In order to get the plot, we just need to calculate the value of the call and the put using Black-Scholes model for a range of implied volatilities between 10% and 80%. The result is as follows.



If we interpret the plot from the perspective of option supply and demand, we can tell that the demand is positively related with the implied volatility and the supply is negatively related with it. This is because as the demand increases and the supply decreases, the call and the put values increase. The implied volatility is positively related to the call and the put values, and therefore the higher the demand, the larger the implied volatility, and the higher the supply, the smaller the implied volatility.

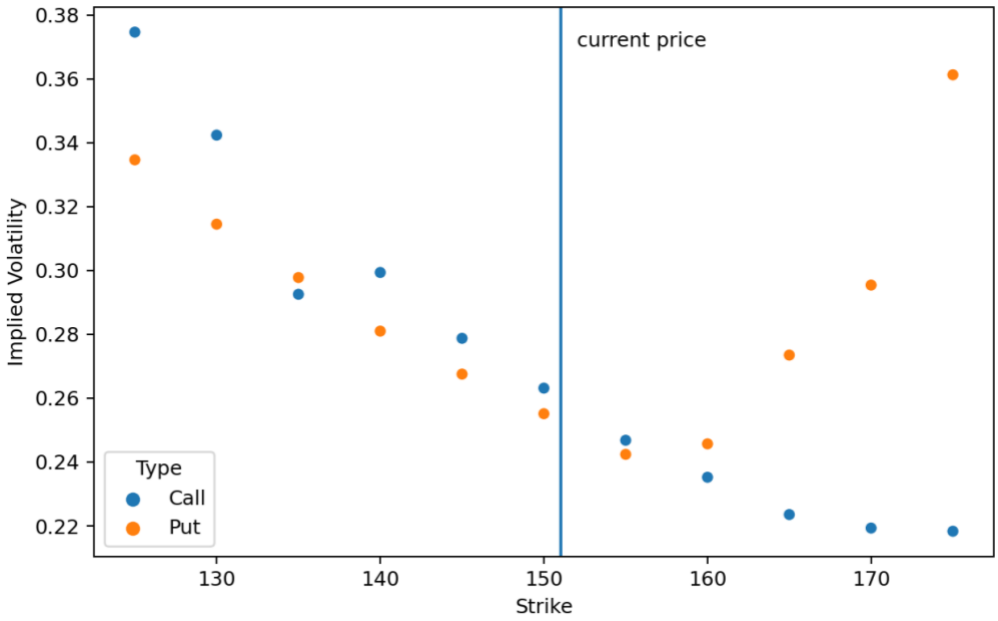
Problem 2

Use the options found in AAPL_Options.csv

- **Current AAPL price is 151.03**
- **Current Date, Risk Free Rate and Dividend Rate are the same as problem #1**

Calculate the implied volatility for each option. Plot the implied volatility vs the strike price for Puts and Calls. Discuss the shape of these graphs. What market dynamics could make these graphs? There are bonus points available on this question based on your discussion.

The problem is also straightforward. We can calculate the implied volatility using Black-Scholes and scipy.optimize.fsolve, and then plot the implied volatility vs. the given strike price. The result is as below.



As the strike price increases, the implied volatility of the call options generally decreases, whereas the implied volatility of the put options at first decreases and then starts to increase when the strike price reaches about $155 (the current price is $151.03).

The put options show a volatility smile – the options that are furthest in the money or out of the money have the highest implied volatility and the options near the money have the lowest implied volatility. This shows that in the money and out of the money options are more in demand than near the money options. The reason could be that investors expect extreme events to happen or the market to be volatile. This could be due to expectations of upcoming events, such as economic releases, geopolitical risks, or earnings announcements that could potentially cause significant price movements. The fact that the volatility smile has the lowest point greater than the current price suggests that the market expects a higher probability of downward moves in the underlying asset's price compared to upward moves.

The call options show a volatility reverse skew – the implied volatility is higher on lower strikes and out of the money calls are more in demand than in the money calls. This often occurs at times when investors have market concerns and buy calls to put a limit on potential losses for the perceived risks. This suggests that the market has a more bearish outlook.

Taken together, these observations suggest that the market is pricing in a higher likelihood of a significant downward move in the underlying asset's price, with a relatively lower probability of

a significant upward move. It is also worth noting that Black-Scholes assumes a constant volatility, but in reality, the implied volatility varies.

Problem3

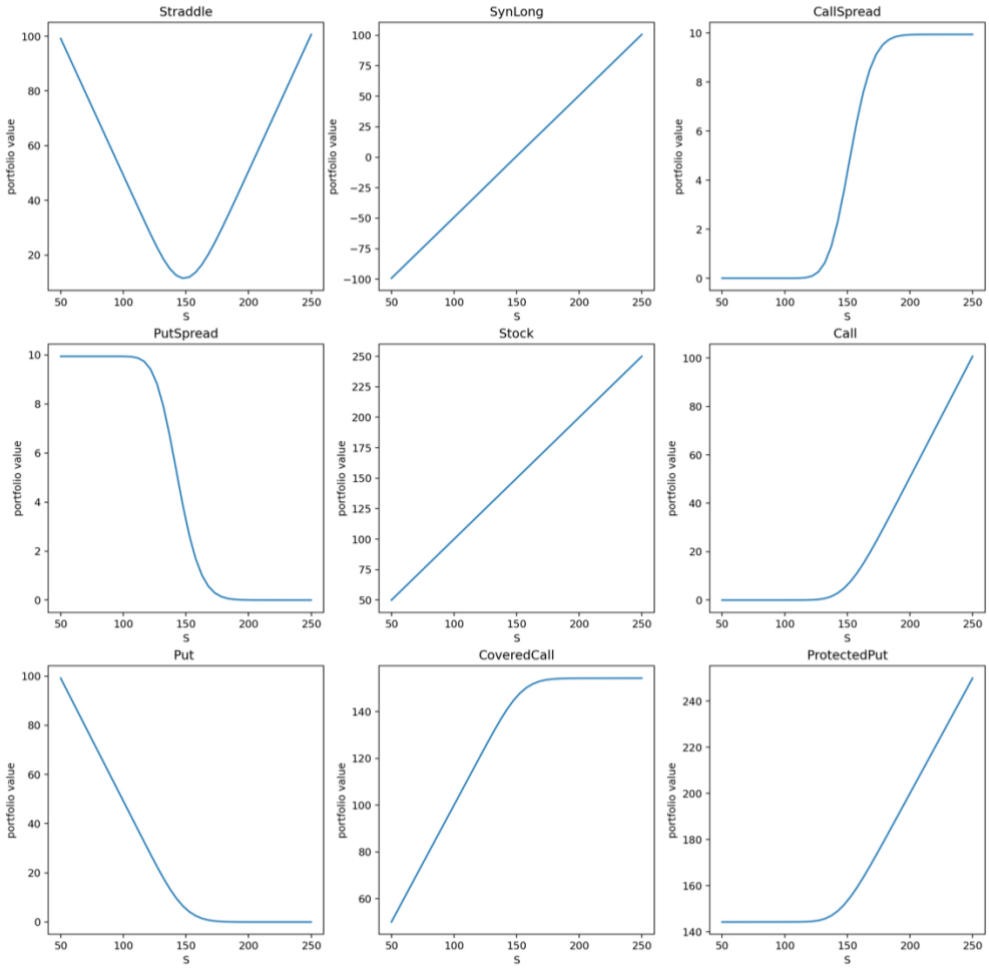
Use the portfolios found in problem3.csv

- **Current AAPL price is 151.03**
- **Current Date, Risk Free Rate and Dividend Rate are the same as problem #1**

For each of the portfolios, graph the portfolio value over a range of underlying values. Plot the portfolio values and discuss the shapes. Bonus points available for tying these graphs to other topics discussed in the lecture.

Using DailyPrices.csv. Calculate the log returns of AAPL. Demean the series. Fit an AR(1) model to AAPL returns. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Discuss.

To get options' portfolio value, we first calculate the implied volatility, which we then use to calculate the value of the call and the put. We multiply this value by the holding to get the portfolio value. For stocks, we just need to multiply the stock price by the holding. The plots for the portfolio value vs. underlying price of each portfolio is as follows



Straddle: The strategy involves buying both a put and a call with the same expiration date and strike price. Because of its nature, it is profitable when the underlying price either rises or falls from the strike price by more than the option price. The portfolio value increases more as the underlying price rises or falls more. The near "V" shape with the bottom at around the current underlying price also reflects these.

SynLong: The strategy involves longing a call while shorting a put of the same expiration date and strike price. When the underlying price rises, it profits from the call, but when the underlying falls, it suffers a loss because of the put shorted. This is consistent with the fact that the portfolio value increases/decreases linearly as the underlying price rises/falls.

CallSpread: The strategy involves buying a call and simultaneously selling a call of the same expiration date but a higher strike price. The portfolio value increases when the underlying price has a limited increase (when the increase is smaller than the difference between the two strike prices). When the underlying price rises beyond the range, the gains are capped; when the underlying price falls, the losses are limited (at the amount of buying a call minus selling a call). These are reflected by the shape.

PutSpread: The strategy involves buying a put and simultaneously selling a put of the same expiration date but a lower strike price. When the underlying price falls below the strike price of the put sold, the portfolio value is capped at the difference between the two strike prices. When the underlying price is between the strike prices, the portfolio value decreases as the underlying

price increases. When the underlying price rises above the put option bought, the losses are limited at the amount of buying a put minus selling a put. The shape is reflective of these.
Stock: The strategy just involves buying the stock. The portfolio value is a linear instrument of the underlying price.
Call: The strategy just involves buying a call option. The payoff is the max(0, the underlying price minus the strike). The downside risk is limited. As the underlying price rises, the portfolio value increases.
Put: The strategy just involves buying a put option. The payoff is the max(0, the strike minus the underlying price). The downside risk is limited as well. As the underlying price falls, the portfolio value increases.
CoveredCall: The strategy involves buying a stock and shorting a call of the same underlying asset. Theoretically, the maximum loss is equivalent to the purchase price of the underlying less the price of the call option sold. As the underlying price increases, the portfolio value increases until the underlying price rises above the strike price. The graph shows these.
ProtectedPut: The strategy involves buying the stock and longing a put of the same underlying asset. The portfolio value is capped at the strike price because of the put bought, and as the underlying price increases, the portfolio value increases because of the stock bought. The shape is illustrative of these.

After fitting an AR(1) model to AAPL returns and simulating AAPL returns 10 days ahead, we calculate the simulated prices 10 days later according to the simulated returns. We can then calculate the portfolio value for each simulation of each portfolio. By subtracting the current portfolio value from it, we get the profit/loss. Mean, VaR, and ES are obtained correspondingly. The results are as below.

	Mean	VaR	ES
Straddle	2.658813	0.138180	0.144531
SynLong	0.001118	17.706815	20.712603
CallSpread	0.032579	3.786905	4.045239
PutSpread	0.379652	2.513104	2.705215
Stock	0.061939	17.547137	20.515924
Call	1.329965	5.899570	6.200008
Put	1.328848	4.164096	4.435482
CoveredCall	-1.342873	13.808719	16.656662
ProtectedPut	1.244350	8.036230	8.535641

According to my simulations, the mean of the returns ranges from -1.343 to 2.659. Straddle has the largest mean while CoveredCall has the smallest and the only negative mean. VaR ranges from 0.138 to 17.707. SynLong and Stock have the largest VaRs while Straddle has the smallest. Likewise, SynLong and Stock have the largest ESs while Straddle has the smallest. ES ranges from 0.145 to 20.713. It seems that Straddle is the best strategy here and is probably the only strategy that is worth a try.