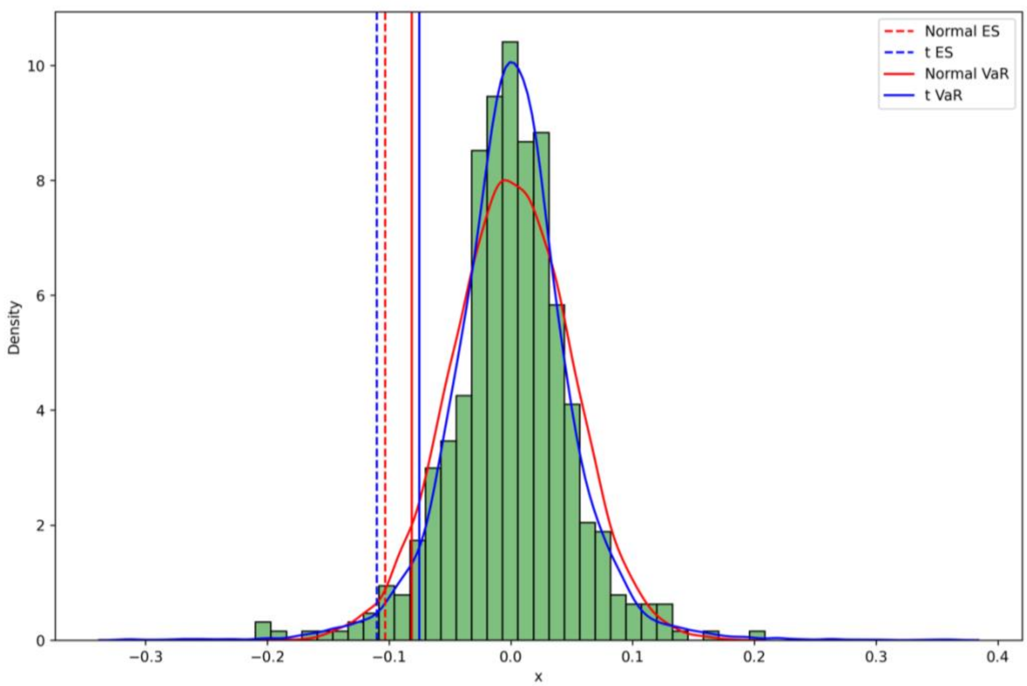
Xinyi (Michelle) Wan

Problem 1:
Use the data in problem1.csv. Fit a Normal Distribution and a Generalized T distribution to this data. Calculate the VaR and ES for both fitted distributions.
Overlay the graphs the distribution PDFs, VaR, and ES values. What do you notice? Explain the differences.

This problem is very straightforward, and I just used the functions I wrote for the previous assignment to fit a normal and a t-distribution and calculate the VaR values. The ES value is simply the conditional expectation of loss given that the loss is beyond the VaR value.

The results are as follows.




```
VaR of Normal distribution is  0.0816093938350451
VaR of t distribution is  0.07543967166231191
ES of Normal distribution is  0.10336079010489807
ES of t distribution is  0.11037516138718864
```

We can notice that the fitted t-distribution has fatter tails than the fitted normal distribution, which implies that extreme events are more likely to occur according to the t-distribution. Therefore, the expected shortfall value of the t-distribution is larger than its counterpart. However, fatter tails do not always mean a larger VaR, as indicated by the VaR values of the two distributions — the normal distribution actually has a larger VaR. Overall, the t-distribution seems to be a better fit of the data, given its fatter tails and higher peak (more probability mass concentrated around zero).

Problem 2:
In your main repository, create a Library for risk management. Create modules, classes, packages, etc as you see fit. Include all the functionality we have discussed so far in class. Make sure it includes

1. **Covariance estimation techniques.**
2. **Non-PSD fixes for correlation matrices**
3. **Simulation Methods**
4. **VaR calculation methods (all discussed)**
5. **ES calculation**

Create a test suite and show that each function performs as expected.

The functions are in library.py and the test cases are in week05.ipynb. All the functions perform as expected.

Problem 3:
Use your repository from #2.
Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0. This file contains the stock holdings of 3 portfolios. You own each of these portfolios.
Fit a Generalized T model to each stock and calculate the VaR and ES of each portfolio as well as your total VaR and ES. Compare the results from this to your VaR form Problem 3 from Week 4.

We can start by fitting a t-distribution to each stock through MLE and then transforming the observations into a uniform vector U using the CDF of the fitted t-distribution for each stock. This is part of fitting the copula, which is described in detail below.

1. For each variable $i \in 1 \ldots n$
    a. Transform the observation vector X_i in to a uniform vector U_i with $u_i \in [0, 1]$ using the CDF for x_i, $F_i(x_i)$.
    b. Transform the uniform vector U_i into a Standard Normal vector, Z_i using the normal quantile function.
2. Calculate the correlation matrix of Z

After fitting the copula, we can simulate draws from the Gaussian copula using the multivariate normal simulation function we wrote earlier and then go backwards through the previous transformation to obtain the simulated X values -

1. Simulation $NSim$ draws from the multivariate normal
2. For each variable $Z_i \in 1 \ldots n$.
    a. Transform Z_i into a uniform variable using the standard normal CDF, $\Phi(z_i) = U_i$.
    b. Transform U_i into the fitted distribution using the quantile of the fitted distribution, $F_i^{-1}(U_i) = X_i$

The results are as follows.

Portfolio	Portfolio Value	Delta Normal VaR	Normal Monte Carlo VaR	Historical VaR	Generalized t	
					VaR	ES
A	299950.059	5670.203	5699.183	6942.176	7949.042	10486.344
B	294385.591	4494.598	4553.183	6205.737	6671.064	8647.323
C	270042.831	3786.589	3784.857	5265.971	5560.313	7281.307
Total	864378.480	13577.075	13458.327	17929.478	19865.298	26356.298

We can tell that the Delta Normal VaR values and the Normal Monte Carlo VaR values are much smaller than the Generalized t VaR values. The Historical VaR values are also smaller than the Generalized t VaR values but are the closest among the three methods from last week.

As previously discussed, both the Delta Normal VaR and the Normal Monte Carlo VaR are less accurate than the historical VaR. We can tell from last week's plots that the historical method renders fatter tails, meaning that there are more extreme losses than the Delta Normal and the Monte Carlo methods account for. The generalized t simulation takes this into consideration with its fatter tails. It also gives a more comprehensive, flexible representation of portfolios' risk by considering the joint probabilities of the returns through copulas and allowing for simulations of a greater number of extreme scenarios that may not be present in the historical data.