Xinyi (Michelle) Wan

Problem 1

Calculate and compare the expected value and standard deviation of price at time t (P_t), given each of the 3 types of price returns, assuming $r_t \sim N(0, \sigma^2)$. Simulate each return equation using $r_t \sim N(0, \sigma^2)$ and show the mean and standard deviation match your expectations.

We are going to simulate classical Brownian motion (1), arithmetic return system (2), and geometric Brownian motion (3), using $r_t \sim N(0, \sigma^2)$. We can start by specifying a price at t-1 and simulate the price at t for many times according to the formulas below. We then calculate the expected value and standard deviation for the numerous simulated prices at t.

$$P_t = P_{t-1} + r_t \quad (1)$$

$$P_t = P_{t-1}(1 + r_t) \quad (2)$$

$$P_t = P_{t-1}e^{r_t} \quad (3)$$

Sigma is set to be 0.2, and the price at t-1 is set to be 100. The results are as follows.

Type	Mean		Standard Deviation	
	Expectation	Simulation	Expectation	Simulation
Classical Brownian	$P_{t-1} = 100$	100.000	$\sigma = 0.2$	0.201
Arithmetic	$P_{t-1} = 100$	100.019	$P_{t-1} \cdot \sigma = 20$	20.092
Geometric Brownian	$P_{t-1} \cdot e^{\mu+\sigma^2/2}$ = 102.020	102.058	$P_{t-1} \cdot e^{\sigma^2/2} \cdot \sqrt{e^{\sigma^2} - 1}$ = 20.610	20.714

The means and standard deviations from simulations are consistent with the expected values.

Problem 2

Implement a function similar to the "return_calculate()" in this week's code. Allow the user to specify the method of return calculation.

Use DailyPrices.csv. Calculate the arithmetic returns for all prices.

Remove the mean from the series so that the mean(META)=0

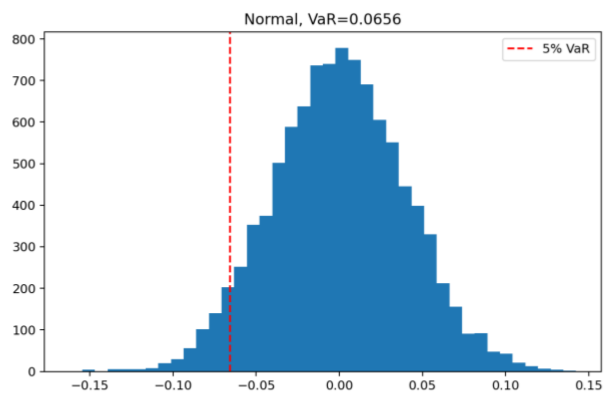
Calculate VaR

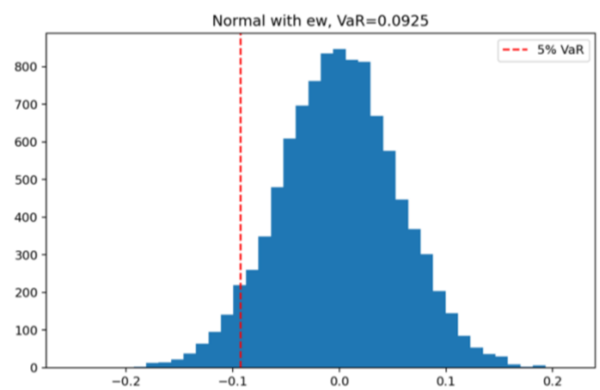
1. **Using a normal distribution.**
2. **Using a normal distribution with an Exponentially Weighted variance (λ = 0. 94)**
3. **Using a MLE fitted T distribution.**
4. **Using a fitted AR(1) model.**
5. **Using a Historic Simulation.**

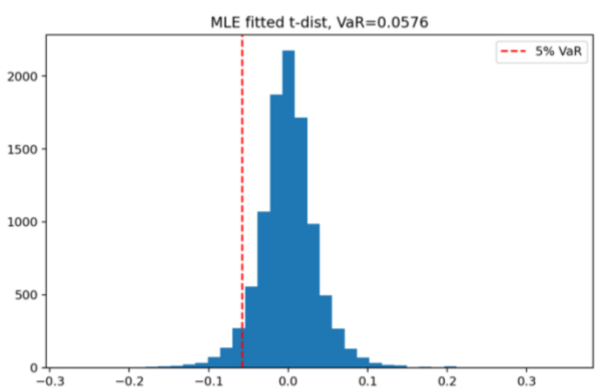
Compare the 5 values.

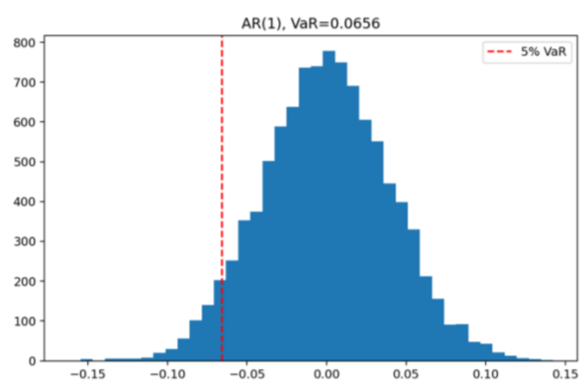
The implementation of return_calculate() and the calculation of the arithmetic returns for all prices are straightforward. To calculate VaR using a normal distribution, we just simulate returns directly from a normal distribution whose mean/std equals the mean/std of the observed returns. To calculate VaR using normal distribution with an exponentially weighted variance, we need to set the standard deviation of the normal distribution from which we simulate returns to the standard deviation that we obtain from the exponentially weighted covariance matrix. The matrix can be calculated using the function we previously wrote. To calculate VaR using a MLE fitted T distribution, we need to solve for the parameters of the t-distribution from which we simulate returns through maximizing the log likelihood function. To calculate VaR using a fitted AR(1) model, we obtain the parameters by fitting the model to the observed returns. Then we simulate returns at t+1 using the return at t (the most recent observed value) through $y_{t+1} = \alpha * y_t + u_{t+1}$, where $u_{t+1} \sim N(0, \sigma)$. α and σ are what we obtain. Using a Historic Simulation is fairly straightforward.

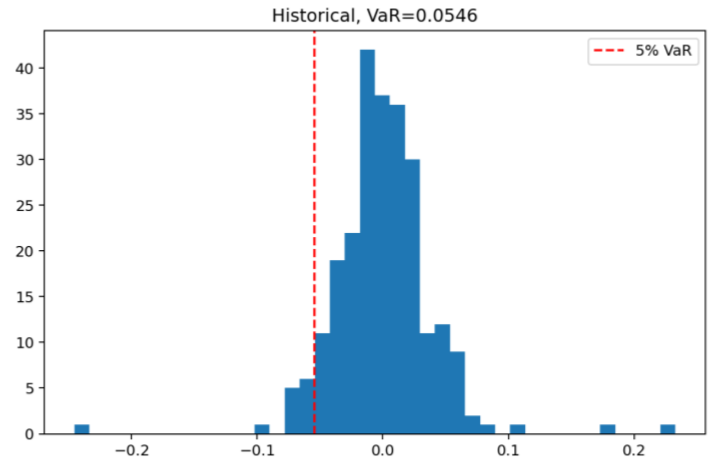
The results are as follows.











Simulation	VaR
normal distribution	0.06565
normal distribution with EW	0.09248
t-distribution	0.05763
AR(1)	0.06565
Historical	0.05462

The VaR we get from using a normal distribution and the one using AR(1) process are relatively close. The VaR we get from using a t-distribution and the one using a historical simulation are relatively close. The VaR obtained from using a normal distribution with EW is much higher than the rest, which may indicate that the recent observations are more volatile.

Problem 3
Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0. This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Using an exponentially weighted covariance with lambda = 0.94, calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings). Express VaR as a $. Discuss your methods and your results.
Choose a different model for returns and calculate VaR again. Why did you choose that model? How did the model change affect the results?

I calculated the VaR of each portfolio and the total VaR using all three methods discussed in class (Delta Normal VaR, Normal Monte Carlo VaR, and Historical VaR). Because our portfolios only contain stocks, which are linear instruments, the Delta Normal VaR calculation becomes much easier. When we solve for the gradient of the portfolio returns with respect to the underlying price returns,

$$\frac{dR}{dr_i} = \frac{P_i}{PV} \sum_{j=1}^{m} h_j \delta_j$$

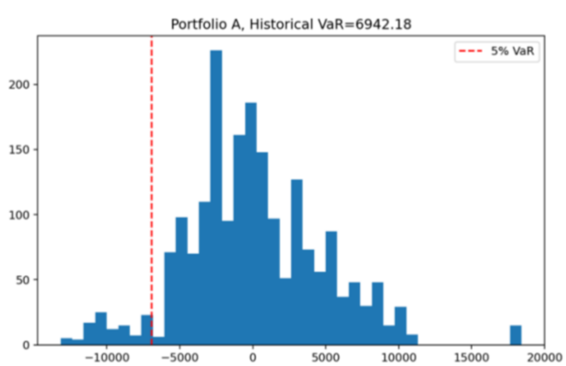
We should recognize that $\delta = \frac{dA_i}{dP_i}$, the first derivative of the asset with respect to the current price, can just be considered as an identity matrix. In addition, we should also take note of the fact that in

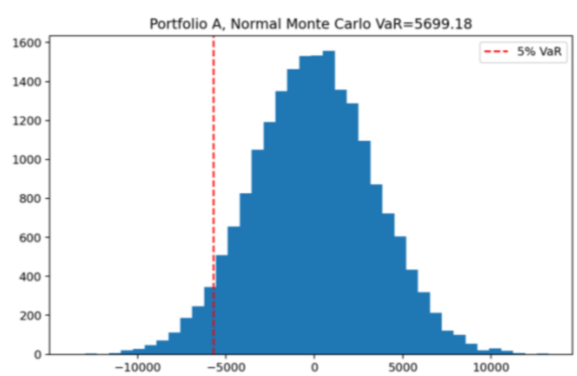
$$VaR(\alpha) = -PV * F_X^{-1}(\alpha) * \sqrt{\nabla R^T \Sigma \nabla R}$$

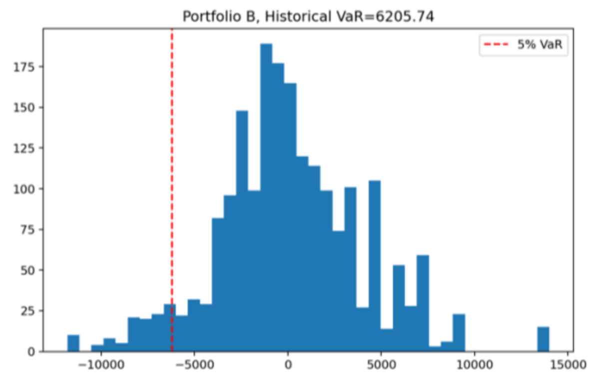
Σ is an exponentially weighted covariance matrix with lambda = 0.94.

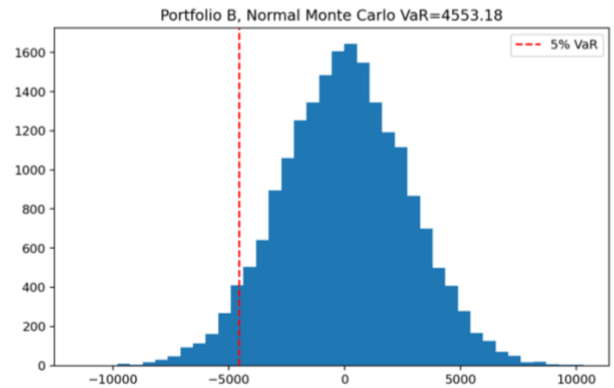
For the Monte Carlo method, we can take advantage of the multivariate normal simulation function we wrote previously to simulate returns. For the historical method, we simulate returns using the past observed values. The results are as follows.

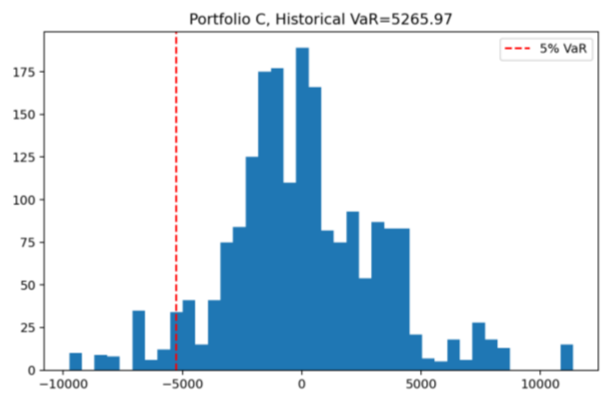
Portfolio	Portfolio Value	Delta Normal VaR	Normal Monte Carlo VaR	Historical VaR
A	299950.059	5670.203	5699.183	6942.176
B	294385.591	4494.598	4553.183	6205.737
C	270042.831	3786.589	3784.857	5265.971
Total	864378.480	13577.075	13458.327	17929.478

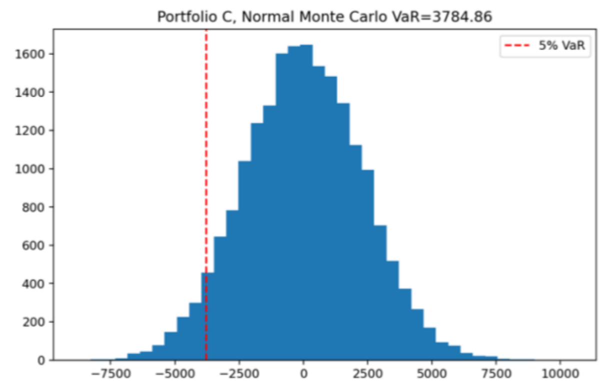


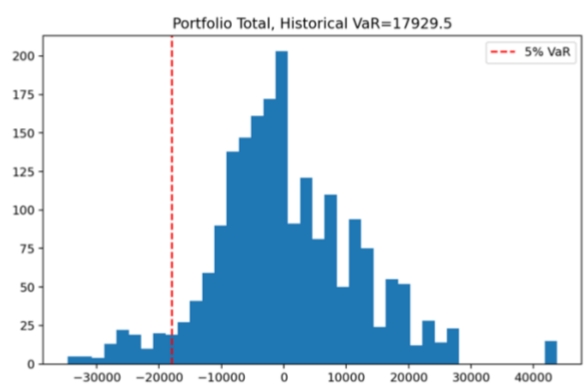


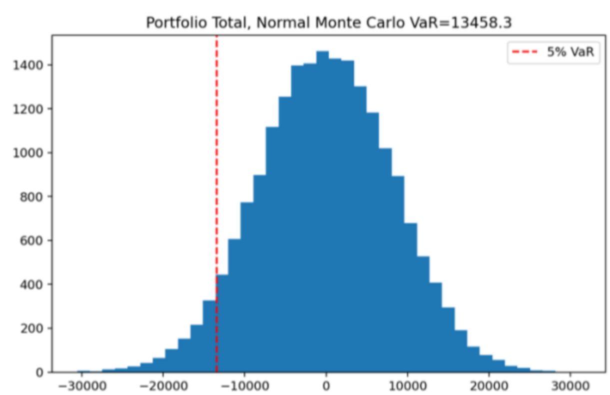












Both Delta Normal VaR and Normal Monte Carlo VaR assume that returns are multivariate normal. But the kurtoses of returns suggest that our data is not in line with the assumption. Therefore, the Delta Normal and the Monte Carlo methods are likely less accurate than the historical method. We can tell from the plots that the historical method renders fatter tails, meaning that there are more extreme losses than the Delta Normal and the Monte Carlo methods account for. Moreover, both the Delta Normal and the Monte Carlo methods weigh recent observations more heavily, due to the exponentially weighted covariance with lambda = 0.94. This may suggest that the stock is doing relatively well or that it is less volatile “recently”, since the Delta Normal and the Monte Carlo methods likely underestimate VaR. Even though I did three approaches, I had chosen the historical method, given its non-parametric nature. The method also turned out to be more appropriate in this scenario compared to the other two.