How to compute the GRS test statistic -- Recipe for a solution in Excel:

1) Run a time-series regression for each of the test assets. Use the L given portfolio returns as explanatory variables. Note: use excess returns, $r_{it} - r_{ft}$.

2) Compute the time-series regression residuals:

$$\hat{e}_{ti} = r_{it} - r_{ft} - \hat{a}_i - \hat{b}_{1i} F_{1t} - \ldots - \hat{b}_{Li} F_{Lt} \tag{0.1}$$

3) Compute the estimated covariance matrix of the time-series regression residuals:

$$\hat{\Sigma} = \begin{bmatrix} \hat{\Sigma}_{11} & \cdots & \hat{\Sigma}_{1N} \\ \vdots & \ddots & \vdots \\ \hat{\Sigma}_{N1} & \cdots & \hat{\Sigma}_{NN} \end{bmatrix}, \text{ where } \hat{\Sigma}_{ij} = \sum_{t=1}^{T} \frac{\hat{e}_{it}\,\hat{e}_{jt}}{T\text{-}L\text{-}1} \text{ for } i, j = 1, \ldots, N . \tag{0.2}$$

You should use an unbiased covariance matrix. For instance, Excel's Data Analysis add-in package divides the sum of products by T, so no adjustment for the estimated covariance matrix is required.

4) Compute the sample means, $\hat{m}$, and sample covariance matrix , $\hat{\Omega}$, of the L factor returns:

$$\hat{m} = \begin{bmatrix} \hat{\bar{F}}_1 \\ \vdots \\ \hat{\bar{F}}_L \end{bmatrix}, \text{ and} \tag{0.3}$$

$$\hat{\Omega} = \begin{bmatrix} \hat{\Omega}_{11} & \cdots & \hat{\Omega}_{1L} \\ \vdots & \ddots & \vdots \\ \hat{\Omega}_{L1} & \cdots & \hat{\Omega}_{LL} \end{bmatrix}, \text{ where } \hat{\Omega}_{ij} = \frac{1}{T} \sum_{t=1}^{T} \left(F_{it} - \bar{F}_i \right) \left(F_{jt} - \bar{F}_j \right) \text{ for } i, j = 1, \ldots, L . \tag{0.4}$$

5) Use MINVERSE and MMULT to compute:

$$\hat{W} = \frac{\hat{a}' \hat{\Sigma}^{-1} \hat{a}}{1 + \hat{m}' \hat{\Omega}^{-1} \hat{m}} \tag{0.5}$$

As a simple example, consider the CAPM. The excess market return is the only factor (i.e., L = 1). In this case, $\hat{m}$ and $\hat{\Omega}$ are scalars, so

$$\hat{W} = \frac{\hat{a}'\hat{\Sigma}^{-1}\hat{a}}{1 + \hat{m}_m^2 / \hat{s}_m^2}, \tag{0.6}$$

where $\hat{m}_m^2$ and $\hat{s}_m^2$ are the estimated average excess market return and variance, respectively.

Under the null hypothesis, $W = 0$. Note that if all a 's are close to zero, the numerator of (0.5) is also close to zero, and W is close to zero.

6) Normalize W to follow a convenient F-distribution:

$$W_{Normalized} = \left[\frac{T}{N}\right]\left[\frac{T - N - L}{T - L - 1}\right] W \longrightarrow F\left(N, T - N - L\right) \tag{0.7}$$

7) Find the p-value:

$$p = \text{FDIST}\left(\hat{W}_{Normalized}, N, T - N - L\right) \tag{0.8}$$

As an example, consider Fama-French's three-factor model. What does a low p-value in a GRS-test mean? It mean that we reject that $\left(r_M - r_f\right)$, *HML*, and *SMB* are priced to be mean-variance efficient (actually minimum-variance).