Project 1

High-Frequency Financial Econometrics

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The purpose of this project is to get a better understanding of Jump-Diffusion processes. In this project You will learn how to simulate specific versions of a Jump-Diffusion process. You will simulate both the diffusion and jump parts of a Jump-Diffusion process, under constant volatility and under stochastic volatility.

This project is Due on September 6th by 11:59 PM. You must push your local repository with the `report.pdf` file back to GitHub before the deadline.

All plots in the report must be self-contained. Self-contained means that a reader who only sees your figure (image and caption, but not the surrounding text) can understand what you are plotting. This translates to all plots having axis titles, correct units on the axis, and a caption that summarizes what is plotted.

Exercise 1

The objective of this exercise is to simulate the Jump-Diffusion model with constant coefficients.

Let X denote the process for the log-price of an asset. Then, we want to simulate the model:

$$dX_t = \underbrace{\mu dt + \sqrt{c}\, dW_t}_{\text{diffusion}} + \underbrace{d\tilde{Y_t}}_{\text{jumps}}$$

Assume that:

$$\begin{aligned}
n &= 80 \\
T &= 1.25 \times 252 \\
\mu &= 0, \forall t \in [0, T] \\
\sqrt{c} \equiv \sigma &= 0.011, \forall t \in [0, T] \\
X_0 &= \log 75 \\
\lambda &= \tfrac{15}{252} \\
\sigma_{jump} &= 25\sqrt{\sigma^2/n}
\end{aligned}$$

A.

Simulate the continuous (diffusion) part of the model as explained in Lecture 2 - Simulation. No Euler discretization is required here.

Remember, to simulate just the diffusion part of the model you have to compute:

$$\tilde{X}_i = \tilde{X}_{i-1} + \mu\Delta_n + \sigma\sqrt{\Delta_n}Z_i \text{ for } i = 1, 2, \ldots, nT$$

Convert the simulated log-prices to prices:

$$P_i = \mathrm{e}^{X_i} \text{ for } i = 0, 1, 2, \ldots, nT$$

Make a plot of the simulated prices across time.

B.

Simulate the jump part of the model as explained in Lecture 2 - Simulation. No Euler discretization is required here.

Remember, to simulate the jump part of the model we compute:

$$\tilde{Y}_i = \sum_{k=1}^{N} 1_{\left\{U_k \leq \frac{i}{nT}\right\}} J_k \text{ for } i = 0, 1, 2, \ldots, nT$$

Make a plot of the simulated compound Poisson process across time.

C.

Combine the simulations from the diffusion and jump parts to obtain the simulations for the full Jump-Diffusion model. Convert the values from log-prices to prices. Plot the simulated prices across time and discuss the plot.

D.

Make a time series plot that has the simulated prices using only the diffusion part of the process, and the simulated prices using both the diffusion and jump parts. Discuss the differences.

Exercise 2

The objective of this exercise is to simulate the Jump-Diffusion model with stochastic volatility:

Let X denote the process for the log-price of an asset. Then, we want to simulate the model:

$$\begin{aligned} dX_t &= \sqrt{c_t}dW_t \\ dc_t &= \rho(\mu_c - c_t)dt + \sigma_c\sqrt{c_t}dW_t^c \end{aligned}$$

where W and W^c are independent.

Assume that:

$$
\begin{aligned}
n &= 80 \\
T &= 1.25 \times 252 \\
n_E &= 20 \times n \\
\rho &= 0.03 \\
\mu_c &= 1 \\
\sigma_c &= 0.40 \\
c_0 &= \mu_c \\
X_0 &= \log 75
\end{aligned}
$$

A.

Simulate the stochastic variance process c_t following Lecture 2 - Simulation. The use of Euler discretization is now necessary.

To simulate the process of c_t you have to compute:

$$c_j = c_{j-1} + \rho(\mu_c - c_{j-1})\delta + \sigma_c\sqrt{c_{j-1}}\sqrt{\delta}\tilde{Z}_j \text{ for } j = 1, 2, \ldots, n_E T$$

Here we will employ an additional rule. If the value of c_j for some j gets too small, we will reflect it to a bigger value. That is, if $c_j < 0.01$ for some j, make it equal to 0.01 instead.

Make a plot of the ultra high frequency values of c_j for $j = 0, 1, 2, \ldots, n_E T$.

B.

Generate the ultra high frequency log-prices $\tilde{X}_j$ for $j = 0, 1, 2, \ldots, n_E T$.

Remember, to simulate the log-prices when the variance is stochastic we compute:

$$\tilde{X}_j \equiv \tilde{X}_{j-1} + \sqrt{\tilde{c}_{j-1}}\sqrt{\delta}Z_j \text{ for } j = 1, 2, \ldots, n_E T$$

Use these prices to compute the log-returns $\Delta_j^{n_E} X$ for $j = 1, 2, \ldots, n_E T$. Make a plot of the ultra high frequency returns across time. Does the return volatility show a pattern?

C.

Generate the log-prices X_i sampled at the lower frequency. Compute the log-returns for $i = 1, 2, \ldots, nT$ and make a plot. Does the volatility pattern remain in the 5-min data?