Learning reduced-order LCS

$$\min_{\boldsymbol{\theta}} \ \mathcal{L}_{\text{vio}}(\boldsymbol{\theta}, \mathcal{D}_{\text{buffer}})$$

with the violation-based loss:

$$\mathcal{L}_{\text{vio}} = \min_{\boldsymbol{\lambda}_k \geq 0,\, \boldsymbol{\phi}_k \geq 0} \frac{1}{2} \left\| A\boldsymbol{x}_k^{\boldsymbol{f}} + B\boldsymbol{u}_k^{\boldsymbol{g}-\text{MPC}} + C\boldsymbol{\lambda}_k + \boldsymbol{d} - \boldsymbol{x}_{k+1}^{\boldsymbol{f}} \right\|^2 + \frac{1}{\epsilon} \left(\boldsymbol{\lambda}_k^{\text{T}} \boldsymbol{\phi}_k + \frac{1}{2\gamma} \left\| D\boldsymbol{x}_k^{\boldsymbol{f}} + E\boldsymbol{u}_k^{\boldsymbol{g}-\text{MPC}} + F\boldsymbol{\lambda}_k + \boldsymbol{c} - \boldsymbol{\phi}_k \right\|^2 \right)$$

LCS model

Trust-region $(\bar{\boldsymbol{u}}, \boldsymbol{\Delta})$

Trust-region LCS g-MPC controller

$$\min_{\boldsymbol{u}_{0:T-1}} \ \sum_{t=0}^{T-1} c_{\boldsymbol{\beta}}(\boldsymbol{x}_t, \boldsymbol{u}_t) + h_{\boldsymbol{\beta}}(\boldsymbol{x}_T)$$

s.t.

$$\boldsymbol{x}_{t+1} = A\boldsymbol{x}_t + B\boldsymbol{u}_t + C\boldsymbol{\lambda}_t + \boldsymbol{d}$$

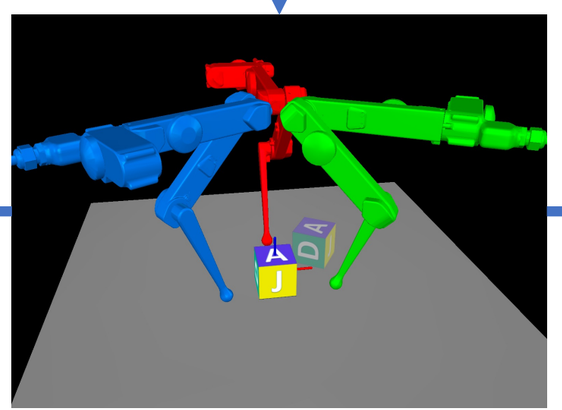
$$\boldsymbol{0} \leq \boldsymbol{\lambda}_t \perp D\boldsymbol{x}_t + E\boldsymbol{u}_t + F\boldsymbol{\lambda}_t + \boldsymbol{c} \geq \boldsymbol{0}$$

$$\boldsymbol{u}_t \in [\bar{\boldsymbol{u}} - \eta\, \boldsymbol{\Delta}, \bar{\boldsymbol{u}} + \eta\, \boldsymbol{\Delta}] \quad \text{(trust region)}$$

$$\boldsymbol{x}_0 = \boldsymbol{x}_k^{\boldsymbol{f}}$$

$\boldsymbol{u}_k^{\boldsymbol{g}-\text{MPC}}$

$\boldsymbol{x}_{k+1}^{\boldsymbol{f}}$



$$\boldsymbol{x}_{k+1}^{\boldsymbol{f}} = \boldsymbol{f}(\boldsymbol{x}_k^{\boldsymbol{f}}, \boldsymbol{u}_k^{\boldsymbol{g}-\text{MPC}})$$

Full-order hybrid dynamics

Rollout with LCS-MPC controller

$$\left(\boldsymbol{x}_k^{\boldsymbol{f}}, \boldsymbol{u}_k^{\boldsymbol{g}-\text{MPC}}, \boldsymbol{x}_{k+1}^{\boldsymbol{f}}\right)$$

$\mathcal{D}_{\text{buffer}}$

$$\mathcal{D}_{\text{buffer}} = \left\{ \left(\boldsymbol{x}_k^{\boldsymbol{f}}, \boldsymbol{u}_k^{\boldsymbol{g}-\text{MPC}}, \boldsymbol{x}_{k+1}^{\boldsymbol{f}} \right) \right\}$$

Trust-region calculation:

$$\bar{\boldsymbol{u}} = \text{mean}\,(\boldsymbol{u}_1^{\boldsymbol{g}-\text{MPC}}, \boldsymbol{u}_2^{\boldsymbol{g}-\text{MPC}}, \ldots, \boldsymbol{u}_k^{\boldsymbol{g}-\text{MPC}}, \ldots)$$

$$\boldsymbol{\Delta} = \text{std}\,(\boldsymbol{u}_1^{\boldsymbol{g}-\text{MPC}}, \boldsymbol{u}_2^{\boldsymbol{g}-\text{MPC}}, \ldots, \boldsymbol{u}_k^{\boldsymbol{g}-\text{MPC}}, \ldots)$$

Rollout Buffer