MA3 – Week 1

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## 1. Key Concepts

- Multinominal coefficient:
$$\binom{n}{n_1, \ldots, n_r} = \frac{n!}{n_1! \cdots n_r!},$$
where $\sum_{i=1}^r n_i = n$.
  - distribute $n$ objects into $r$ distinctive groups of size $n_1, \ldots, n_r$ where the order between groups matters (distinctive groups)
  - Ex:
- Binominal coefficient
  - $$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$
  - draw $k$ objects from an urn of $n$ objects where the order among these $k$ objects does not matter.
  - if the order matters, then multiply it by the number of possible combinations among these $k$ objects which is $k!$.
- Probability of some event $A \subseteq \Omega$ where $\Omega$ denote the set of all possible outcomes:
$$\mathbb{P}[A] = \frac{\text{total number of ways that } A \text{ can happen}}{\text{total number of outcomes in } \Omega} = \frac{|A|}{|\Omega|}.$$
- Exclusion-inclusion principle:
$$\mathbb{P}[E_1 \cup E_2] = \mathbb{P}[E_1] + \mathbb{P}[E_2] - \mathbb{P}[E_1 \cap E_2].$$
$$\begin{aligned}\mathbb{P}[E_1 \cup E_2 \cup E_3] &= \sum_{i=1}^{3} \mathbb{P}[E_i] - (\mathbb{P}[E_1 \cap E_2] + \mathbb{P}[E_1 \cap E_3] + \mathbb{P}[E_2 \cap E_3]) \\ &\quad + \mathbb{P}[E_1 \cap E_2 \cap E_3]\end{aligned}$$
  - think of Venn diagram
  - odd + , even -
- Complement of events:
$$\mathbb{P}[A^c] = \mathbb{P}[\Omega \setminus A] = 1 - \mathbb{P}[A]$$
- Conditional probability: for $\mathbb{P}[F] > 0$,
$$\mathbb{P}[E|F] = \frac{\mathbb{P}[E \cap F]}{\mathbb{P}[F]}.$$
- Total law of probability: $\mathbb{P}[A] = \mathbb{P}[A \cap B] + \mathbb{P}[A \cap B^c]$
- Bayes' rule: $\mathbb{P}[A|B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}$, $\mathbb{P}[B] > 0$.
- $\Rightarrow \mathbb{P}[A] = \mathbb{P}[A] = \mathbb{P}[A|B]\mathbb{P}[B] + \mathbb{P}[A|B^c]\mathbb{P}[B^c]$.

## 2. Examples

**Example 2.1** (Permutation)**.** How many different 7-place Californian license plates are possible? (1-number, 3 consecutive letters and 3 consecutive numbers)

**Example 2.2** (Dividing into distinctive groups)**.**
(1) How many possible ways to arrange the word "Wollongong"?
(2) The game of bridge is played by 4 players, each of whom is dealt 13 cards. How many bridge deals are possible?
(3) What is the coefficient of $w^2x^3yz^3$ in the expansion of $(w+x+y+z)^9$?

**Example 2.3** ("Balls in urns")**.** Nine students, five men and four women, interview for four summer internships sponsored by a city newspaper.

(1) In how many ways can the newspaper choose a set of four interns?
(2) In how many ways can the newspaper choose a set of four interns if it must include two men and two women in each set?
(3) What is the probability that out of 4 interns that have been chosen randomly, not everyone is of the same sex?

**Example 2.4** (Placement problem)**.** There are 5 seats in a row in a movie theater. You know that two of your friends just had a fight and they do not want to sit together for the duration of the movie. How many possible ways can you assign seats for your 5 friends?

**Example 2.5** (Cards problems)**.** In a standard deck of 52 cards (perfectly shuffled)

(1) what is the probability that the $k$-th card is an queen?
(2) what is the probability that the first queen appears in the $k$-th place, where $k \leq 52$?
(3) a bridge hand (thirteen cards) is dealt. Let $A$ be the event that the hand contains four aces; let $B$ be the event that the hand contains four kings. Find $P(A \cup B)$.
(4) two cards are distributed to each of three players. What is the probability that at most one player has one ace and one king? (hint: exclusion-inclusion principle)

**Example 2.6** (Dice problem)**.**
(1) Five fair dice are rolled. What is the probability that the faces showing constitute a "full house"—that is, three faces show one number and two faces show a second number?
(2) Roll a fair dice until a 5 or 6 comes up. What is the probability that we see 5 before 6?