

MA3 – WEEK 4

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1. EXAMPLES

Example 1.1. The number of miles a Tesla can run before dying out its battery is given an exponential distribution. The average number of miles is about 300,000. What is the probability that you can drive for 150,000 without replacing its battery?

Example 1.2. Let $\Phi(\cdot)$ be the CDF of $X \sim \mathcal{N}(0, 1)$ (standard normal distribution). What is the expected value and variance of $X = \Phi^{-1}(U)$ where $U \sim \mathcal{U}[0, 1]$ (uniform on the unit interval)?

Example 1.3. Suppose that two fair dice are tossed one time. Let X denote the number of 2's that appear, and Y the number of 3's. Write the matrix giving the joint probability density function for X and Y . Suppose a third random variable, Z , is defined, where $Z = X + Y$. Use $p_{X,Y}(x, y)$ to find $p_Z(z)$, and use $p_Z(z)$ to find F_Z .

Example 1.4. Consider two random variables X and Y , which are joint uniformly distributed on the unit square (i.e., $f_{X,Y}(x, y) = 1$) on $x, y \in [0, 1]$. Compute the following:

- (1) $\mathbb{E}[X]$
- (2) $\mathbb{E}[X|Y]$
- (3) $\mathbb{E}[X|Y > X]$