

## MA3 – WEEK 6

TA: WANYING (KATE) HUANG

### 1. EXAMPLES

**Example 1.1.** Suppose  $X_1, X_2, X_3, X_4$  are uncorrelated random variables (i.e.,  $Cov(X_i, X_j) = 0$  for  $i \neq j$ ) where each  $X_i$  has an expectation of 0 and variance of 1. What is the correlation between  $X_1 + X_2$  and  $X_2 + X_3$ ? What about the correlation between  $X_1 + X_2$  and  $X_3 + X_4$ ?

**Example 1.2.** Let  $Y_1, Y_2, Y_3$  be i.i.d. Exponential with  $\lambda = 1$ . Let  $Y_{min} = \min\{Y_1, Y_2, Y_3\}$ . Compare  $f_{Y_{min}}(y)$  with  $f_{Y_1}(y)$  and compute  $\mathbb{P}[Y_1 < 1]$  and  $\mathbb{P}[Y_{min} < 1]$  (intuitively think about which one should be larger).

**Example 1.3.** Given the joint pdf

$$f_{X,Y}(x, y) = 2 \cdot e^{-(x+y)}, 0 \leq x \leq y, y \geq 0,$$

find

- (1)  $\mathbb{P}[Y < 1 | X < 1]$
- (2)  $\mathbb{P}[Y < 1 | X = 1]$
- (3)  $f_{Y|X}(y)$
- (4)  $\mathbb{E}[Y|X]$
- (5)  $Var[Y|X]$