MA3 – Week 1

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1. Key Concepts

- Multinominal coefficient:
$$\binom{n}{n_1, \dots, n_r} = \frac{n!}{n_1! \cdots n_r!},$$
where $\sum_{i=1}^{r} n_i = n$.
  - distribute n objects into r distinctive groups of size $n_1, \dots, n_r$ where the order between groups matters (distinctive groups)
- Binominal coefficient
  - $$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$
  - draw k objects from an urn of n objects where the order among these k objects does not matter.
  - if the order matters, then multiply it by the number of possible combinations among these k objects which is $k!$.

- Probability of some event $A \subseteq \Omega$ where Ω denote the set of all possible outcomes:
$$\mathbb{P}[A] = \frac{\text{total number of ways that } A \text{ can happen}}{\text{total number of outcomes in } \Omega} = \frac{|A|}{|\Omega|}.$$
- Exclusion-inclusion principle:
$$\mathbb{P}[E_1 \cup E_2] = \mathbb{P}[E_1] + \mathbb{P}[E_2] - \mathbb{P}[E_1 \cap E_2].$$
$$\mathbb{P}[E_1 \cup E_2 \cup E_3] = \sum_{i=1}^{3} \mathbb{P}[E_i] - (\mathbb{P}[E_1 \cap E_2] + \mathbb{P}[E_1 \cap E_3] + \mathbb{P}[E_2 \cap E_3]) + \mathbb{P}[E_1 \cap E_2 \cap E_3]$$
  - think of Venn diagram
  - odd + , even -
- Complement of events:
$$\mathbb{P}[A^c] = \mathbb{P}[\Omega \setminus A] = 1 - \mathbb{P}[A]$$
- Conditional probability: for $\mathbb{P}[F] > 0$,
$$\mathbb{P}[E|F] = \frac{\mathbb{P}[E \cap F]}{\mathbb{P}[F]}.$$
- Total law of probability: $\mathbb{P}[A] = \mathbb{P}[A \cap B] + \mathbb{P}[A \cap B^c]$
- Bayes' rule: $\mathbb{P}[A|B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}$, $\mathbb{P}[B] > 0$.
- $\Rightarrow \mathbb{P}[A] = \mathbb{P}[A] = \mathbb{P}[A|B]\mathbb{P}[B] + \mathbb{P}[A|B^c]\mathbb{P}[B^c]$.

2. Examples

Example 2.1 (Permutation)**.** How many different 7-place Californian license plates are possible? (1-number, 3 consecutive letters and 3 consecutive numbers)

Example 2.2 (Dividing into distinctive groups)**.**

1. How many possible ways to arrange the word "Wollongong"?
2. The game of bridge is played by 4 players, each of whom is dealt 13 cards. How many bridge deals are possible?
3. What is the coefficient of $w^2x^3yz^3$ in the expansion of $(w+x+y+z)^9$?

Example 2.3 ("Balls in urns")**.** Nine students, five men and four women, interview for four summer internships sponsored by a city newspaper.

1. In how many ways can the newspaper choose a set of four interns?
2. In how many ways can the newspaper choose a set of four interns if it must include two men and two women in each set?
3. What is the probability that out of 4 interns that have been chosen randomly, not everyone is of the same sex?

Example 2.4 (Placement problem)**.** There are 5 seats in a row in a movie theater. You know that two of your friends just had a fight and they do not want to sit together for the duration of the movie. How many possible ways can you assign seats for your 5 friends?

Example 2.5 (Cards problems)**.** In a standard deck of 52 cards (perfectly shuffled)

1. what is the probability that the k-th card is an queen?
2. what is the probability that the first queen appears in the k-th place, where $k \leq 52$?
3. a bridge hand (thirteen cards) is dealt. Let A be the event that the hand contains four aces; let B be the event that the hand contains four kings. Find $P(A \cup B)$.
4. two cards are distributed to each of three players. What is the probability that at most one player has one ace and one king? (hint: exclusion-inclusion principle)

Example 2.6 (Dice problem)**.**

1. Five fair dice are rolled. What is the probability that the faces showing constitute a "full house"—that is, three faces show one number and two faces show a second number?
2. Roll a fair dice until a 5 or 6 comes up. What is the probability that we see 5 before 6?