

MA3 – WEEK 1
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1. KEY CONCEPTS

- Multinomial coefficient:

$$\binom{n}{n_1, \dots, n_r} = \frac{n!}{n_1! \cdots n_r!},$$

where $\sum_{i=1}^r n_i = n$.

- distribute n objects into r distinctive groups of size n_1, \dots, n_r where the order between groups matters (distinctive groups)

- Binomial coefficient

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$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1) \cdots (n-k+1)}{k!}$$

- draw k objects from an urn of n objects where the order among these k objects does not matter.
- if the order matters, then multiply it by the number of possible combinations among these k objects which is $k!$.

- Probability of some event $A \subseteq \Omega$ where Ω denote the set of all possible outcomes:

$$\mathbb{P}[A] = \frac{\text{total number of ways that } A \text{ can happen}}{\text{total number of outcomes in } \Omega} = \frac{|A|}{|\Omega|}.$$

- Exclusion-inclusion principle:

$$\mathbb{P}[E_1 \cup E_2] = \mathbb{P}[E_1] + \mathbb{P}[E_2] - \mathbb{P}[E_1 \cap E_2].$$

$$\begin{aligned} \mathbb{P}[E_1 \cup E_2 \cup E_3] &= \sum_{i=1}^3 \mathbb{P}[E_i] - (\mathbb{P}[E_1 \cap E_2] + \mathbb{P}[E_1 \cap E_3] + \mathbb{P}[E_2 \cap E_3]) \\ &\quad + \mathbb{P}[E_1 \cap E_2 \cap E_3] \end{aligned}$$

- think of Venn diagram
- odd + , even -

- Complement of events:

$$\mathbb{P}[A^c] = \mathbb{P}[\Omega \setminus A] = 1 - \mathbb{P}[A]$$

- Conditional probability: for $\mathbb{P}[F] > 0$,

$$\mathbb{P}[E|F] = \frac{\mathbb{P}[E \cap F]}{\mathbb{P}[F]}.$$

- Total law of probability: $\mathbb{P}[A] = \mathbb{P}[A \cap B] + \mathbb{P}[A \cap B^c]$
- Bayes' rule: $\mathbb{P}[A|B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}$, $\mathbb{P}[B] > 0$.
- $\Rightarrow \mathbb{P}[A] = \mathbb{P}[A \cap B] + \mathbb{P}[A \cap B^c] = \mathbb{P}[A|B]\mathbb{P}[B] + \mathbb{P}[A|B^c]\mathbb{P}[B^c]$.

2. EXAMPLES

Example 2.1 (Permutation). How many different 7-place Californian license plates are possible? (1-number, 3 consecutive letters and 3 consecutive numbers)

Example 2.2 (Dividing into distinctive groups). (1) How many possible ways to arrange the word “Wollongong”?

(2) The game of bridge is played by 4 players, each of whom is dealt 13 cards. How many bridge deals are possible?

(3) What is the coefficient of $w^2x^3yz^3$ in the expansion of $(w + x + y + z)^9$?

Example 2.3 (“Balls in urns”). Nine students, five men and four women, interview for four summer internships sponsored by a city newspaper.

(1) In how many ways can the newspaper choose a set of four interns?

(2) In how many ways can the newspaper choose a set of four interns if it must include two men and two women in each set?

(3) What is the probability that out of 4 interns that have been chosen randomly, not everyone is of the same sex?

Example 2.4 (Placement problem). There are 5 seats in a row in a movie theater. You know that two of your friends just had a fight and they do not want to sit together for the duration of the movie. How many possible ways can you assign seats for your 5 friends?

Example 2.5 (Cards problems). In a standard deck of 52 cards (perfectly shuffled)

(1) what is the probability that the k -th card is an queen?

(2) what is the probability that the first queen appears in the k -th place, where $k \leq 52$?

(3) a bridge hand (thirteen cards) is dealt. Let A be the event that the hand contains four aces; let B be the event that the hand contains four kings. Find $P(A \cup B)$.

(4) two cards are distributed to each of three players. What is the probability that at most one player has one ace and one king? (hint: exclusion-inclusion principle)

Example 2.6 (Dice problem). (1) Five fair dice are rolled. What is the probability that the faces showing constitute a “full house”—that is, three faces show one number and two faces show a second number?

(2) Roll a fair dice until a 5 or 6 comes up. What is the probability that we see 5 before 6?