MA3 – WEEK 4 AND 5

TA: WANYING (KATE) HUANG

1. Examples

Example 1.1. The number of miles a Tesla can run before dying out its battery is given an exponential distribution. The average number of miles is about $300,000$. What is the probability that you can drive for $150,000$ without replacing its battery?

Example 1.2. Let $\Phi(\cdot)$ be the CDF of $X \sim \mathcal{N}(0,1)$ (standard normal distribution). What is the expected value and variance of $X = \Phi^{-1}(U)$ where $U \sim \mathcal{U}[0,1]$ (uniform on the unit interval)?

Example 1.3. Suppose that two fair dice are tossed one time. Let X denote the number of 2's that appear, and Y the number of 3's. Write the matrix giving the joint probability density function for X and Y . Suppose a third random variable, Z, is defined, where $Z = X + Y$. Use $p_{X,Y}(x, y)$ to find $p_Z(z)$, and use $p_Z(z)$ to find F_Z.

Example 1.4. Let X and Y be two continuous and independent random variable with marginal distributions $f_X(x) = x, 0 \leq x \leq 1$ and $f_Y(y) = 1, 0 \leq y \leq 1$. Find $\mathbb{P}[\frac{Y}{X} > 2]$.

Example 1.5. Let Y be a non-negative, continuous random variable. Show that $W = Y^2$ has pdf $f_W(w) = \frac{1}{2\sqrt{w}} f_Y(\sqrt{w})$.

Example 1.6. Consider two random variables X and Y, which are joint uniformly distributed on the unit square (i.e., $f_{X,Y}(x, y) = 1$) on $x, y \in [0, 1]$. Compute the following:

(1) $\mathbb{E}[X]$
(2) $\mathbb{E}[X|Y]$
(3) $\mathbb{E}[X|Y > X]$