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Constructor University Bremen  
Natural Science Laboratory  
General Electrical Engineering II  
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## Lab Experiment 5- Filter

Author: Wanzia Nambule

Instructor: Pagel Uwe

Experiment conducted by: Wanzia Nambule, Dahyun Ko

Place of execution: Teaching Lab EE

Rotation II, Bench 12

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# 1.INTRODUCTION

The objective of this experiment was to demonstrate the behavior of basic passive RLC Networks using sinusoidal signals of varying frequencies as input signals. Additionally, learning how to measure the attributes of these circuits as well as how to evaluate and display the results was also another objective of the experiment.

## 2.THEORY

### FILTER

A filter is a network used to select a frequency or a range of frequencies of an input signal, while rejecting all other frequencies. There are several ways to construct these filters. One way is to use active components like transistors or operational amplifiers together with networks of resistors, capacitors, and inductors. Another way is to use digital signal processors together with analog to digital and digital to analog convertors. The simplest way is to use a passive network of resistors, capacitors, or inductors. There are four general types of filters. High Pass, Low Pass, Band Pass, and Notch filters.

#### Filter Properties:

- **Frequency response-** The frequency response is the measure of the filter's response at the output to a signal of varying frequency but constant amplitude at its input. The frequency response is typically characterized by the magnitude of the system's response, measured in dB, and the phase shift relative to the input signal, measured in radians, versus frequency.

The so-called 'Order' of a filter describes the general behavior on how good it damps the unwanted frequencies. Simple filters like the ones from our experiment are of '1. Order'.

- **Cutoff frequency** - The cutoff frequency or corner frequency is the frequency either above which or below which the power output of the filter is half the power of the passband, and since voltage is proportional to power P,  $V_{out}$  is  $\sqrt{\frac{1}{2}}$  of the  $V_{out}$  in the passband. This happens to be close to -3 decibels, and the cutoff frequency is referred to as the -3dB point.
- **Center frequency** - A bandpass circuit and a notch filter has two cutoff frequencies. Their geometric mean is the center frequency. The geometric mean of two numbers is:

$$f_{bw} = \sqrt{f_1 * f_2}$$

- **Bandwidth** - The bandwidth for a bandpass or notch filter is the difference between the upper and lower cutoff frequencies.
- **Time constant** - In an RC circuit, the value of the time constant (in seconds) is equal to the product of the circuit resistance (in Ohms) and the circuit capacitance (in Farads), i.e.  $\tau = RC$ . It is

the time required to charge the capacitor, through the resistor, to 63.2% (~ 63%) percent of full charge; or to discharge it to 36.8% (~ 37%) of its initial voltage. These values are derived from the mathematical constant  $e$ , specifically  $1 - e^{-1}$  and  $e^{-1}$  respectively.  $e$  is the base of the natural logarithm.

- **Angular frequency  $\omega$**  - Is a scalar measure of rotation rate. One revolution is equal to  $2\pi$ , hence

$$\omega = \frac{2\pi}{T} = 2\pi f$$

The unit of omega is radians, i.e. one full cycle is  $2\pi$ , which is about 6.28. You will also find that it is given in degrees, where one full cycle corresponds to  $360^\circ$ . So,  $2\pi$  correspond to  $360^\circ$ , or one radian is about  $57^\circ$ , a useful number to know by heart.

## High Pass

A High Pass is a circuit which transfers signals with high frequencies nearly unchanged. With low frequencies the signal is attenuated and the phase shift of the output signal is in advance to the input signal, i.e. the phase shift is positive. To the right passive RL and RC circuits are shown. For both types we get the amplitude ratio  $\underline{A}(j\omega)$  and the phase shift  $\varphi$  from the voltage divider formula. Since we have AC we use the complex form (to follow the calculations, you have to apply your knowledge about complex numbers).

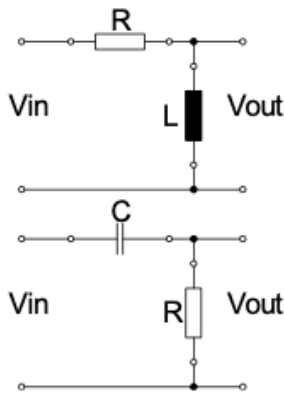


Figure 1

### RL combination:

$$\underline{A}(j\omega) = \frac{1}{1 - j(\frac{R}{\omega L})}, \text{ with } \omega = 2\pi f \dots \dots \dots (1)$$

### RC combination:

$$\underline{A}(j\omega) = \frac{1}{1 + j(\frac{1}{\omega RC})} \dots \dots \dots (2)$$

Using formula i and ii we get terms for magnitude and phase shift.

$$|\underline{A}| = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}} \text{ and } \varphi = \arctan\left(\frac{R}{\omega L}\right) \dots \dots \dots (3)$$

$$|\underline{A}| = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}} \text{ and } \varphi = \arctan\left(\frac{1}{\omega RC}\right) \dots \dots \dots (4)$$

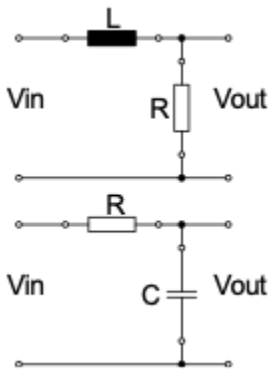
These are the formulas for magnitude and phase shift. To get the cutoff frequency we set  $|\underline{A}| = \frac{1}{\sqrt{2}}$  or  $-3dB$  and the phase  $+45^\circ$   $\omega RC$  or  $\frac{R}{\omega L}$  becomes 1 at this point.

$$f_{-3dB} = \frac{1}{2\pi RC} \text{ or } f_{-3dB} = \frac{R}{2\pi L} \dots \dots \dots (5)$$

With the formulas for the magnitude and phase you can verify the result of the experiment. Formula (5) already gives you an estimation about the general behavior without any calculations or measurements.

### Low Pass

A Low Pass is a circuit which transfers signals with low frequencies nearly unchanged. With high frequencies the signal is attenuated and the phase shift of the output signal follows the input signal, i.e. the phase shift is negative. To the right passive RL and RC circuits are shown. For both types we get the amplitude ratio  $A(j\omega)$  and the phase shift  $\varphi$  from the voltage divider formula. Since we have AC we use the complex form (the calculations are in complete analogy to the previous section).



### RL combination:

$$\underline{A}(j\omega) = \frac{V_{out}}{V_{in}} = \frac{R}{R + j\omega L} = \frac{1}{1 + \frac{j\omega L}{R}} \dots \dots \dots (6)$$

### RC combination:

$$\underline{A}(j\omega) = \frac{V_{out}}{V_{in}} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC} \dots \dots \dots (7)$$

Using formula (6) and (7) we get the magnitude and phase shift.

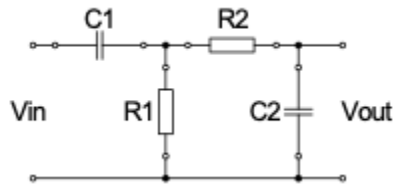
$$|A| = \frac{1}{\sqrt{1+(\omega L/R)^2}} \text{ and } \varphi = -\tan^{-1}\left(\frac{\omega L}{R}\right) \dots \dots \dots (8)$$

$$|A| = \frac{1}{\sqrt{1+(\omega RC)^2}} \text{ and } \varphi = -\tan^{-1}(\omega RC) \dots \dots \dots (9)$$

The formulas for the cutoff frequency are the same as for the High Pass. The magnitude is again  $|A| = \frac{1}{\sqrt{2}}$  or -3dB. Since the phase is negative the shift is  $-45^\circ$  at this point, i.e. the voltage of the output lags by  $-45^\circ$  behind the input voltage! With the formulas for the magnitude and phase you can verify the result of the experiment. Formula (5) already gives you an estimation about the general behavior without any calculations or measurements.

### Band Pass

A band-pass filter is a device that passes frequencies within a certain range and attenuates frequencies outside that range. A RLC combination may be used to create the circuit. The simplest way to build a band-pass is to combine a high and a low pass.



To calculate magnitude and frequency response we combine the formulas for high and low pass.

$$A_{high}(j\omega) = \frac{V_{out(high)}}{V_{in}} \dots \dots \dots (10)$$

$$A_{low}(j\omega) = \frac{V_{out(low)}}{V_{out(high)}} \dots \dots \dots (11)$$

Formula (11) inserted in formula (10) gives

$$A_{high}(j\omega) * A_{low}(j\omega) = \frac{V_{out(low)}}{V_{in}} \dots \dots \dots (12)$$

Written in a different way

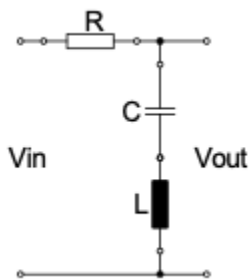
$$|A|_{hi} * e^{j\omega_{hi}} * |A|_{lo} * e^{j\omega_{lo}} = |A|_{hi} * |A|_{lo} * e^{j(\omega_{hi} + \omega_{lo})} = \frac{V_{out,lo}}{V_{in}} \dots\dots\dots(13)$$

we can see that we can use the already derived formulas to calculate magnitude and phase shift for the band-pass. The magnitude for high and low pass have to be multiplied and the phases have to be added. The formula for the cutoff frequency is also the same as for high and low pass (5). The only difference is that we have to apply it for each stage of the filter!

### Notch Filter

A notch filter (or band-stop filter or band-rejection filter) is a filter that passes most frequencies unaltered, but attenuates those in a specific range to very low levels. It is the opposite of a band-pass filter. A RLC combination is used to create the circuit. It is shown to the left.

We get the magnitude  $\underline{A}(j\omega)$  and the phase shift  $\varphi$  from the voltage divider formula. Since the design (in this particular example) is different from the band pass filter, we have to do new calculations. It is also possible to design a band pass filter from this circuit. All you have to do is to use the voltage drop over the resistor as the output.



$$\underline{A}(j\omega) = \frac{V_{out}}{V_{in}} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})} = \frac{1}{1 - j(\frac{R}{\omega L - \frac{1}{\omega C}})} \dots\dots\dots(14)$$

From this formula a term of the magnitude and a term for the phase shift can be derived

$$|A| = \frac{1}{\sqrt{1 + (\frac{R}{\omega L - \frac{1}{\omega C}})^2}} \text{ and } \varphi = \tan^{-1}(\frac{R}{\omega L - \frac{1}{\omega C}}) \dots\dots\dots(15)$$

The center frequency of the band stop filter is at the point where  $|A|$  is close to 0.

This is the case if  $\omega L - \frac{1}{\omega C} = 0$ . Solved for  $f$  we get

$$f = \frac{1}{2\pi\sqrt{LC}} \dots\dots\dots(16)$$

The cutoff frequency is defined when the magnitude  $|A| = \frac{1}{\sqrt{2}}$  and the phase shift is  $\pm 45^\circ$ . Formula (15) is solved for  $\omega$  at these points. There are four solutions!

$$\omega_{cut-Low} = \frac{-RC \pm \sqrt{(RC)^2 + 4LC}}{2LC} \dots \dots \dots (17)$$

$$\omega_{cut-high} = \frac{RC \pm \sqrt{(RC)^2 + 4LC}}{2LC} \dots \dots \dots (18)$$

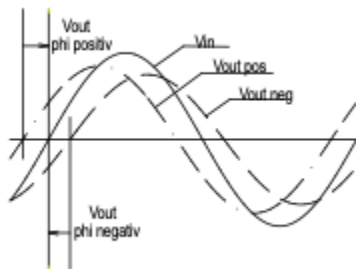
And only two of the 4 solutions are practically relevant.  $f$  can be found from the relation  $\omega = 2\pi f$ .

### How to determine the output characteristic of a filter

There are several ways to describe the properties of these kinds of networks like time response or frequency response. In our experiments we analyze the frequency response, meaning what happens at the output when the frequency at the input changes. The amplitude is measured with frequencies over several decades and the ratio is calculated. The unit of the result is decibels, or dB. dB is a logarithmic value often used for such kind of ratios. It is defined as

$$A = 20 \log_{10} \frac{V_{out}}{V_{in}} \dots \dots \dots (19)$$

The second quantity measured is the phase shift  $\phi$  between input and output signal. It is taken relatively to the input signal. If the output signal is ahead the input signal  $\phi$  is positive else it is negative.

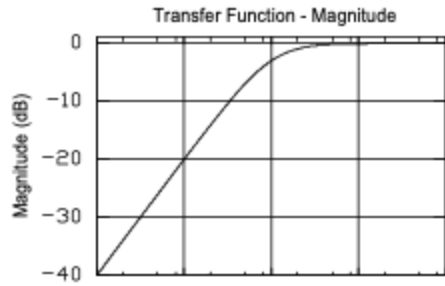


This information is usually used in conjunction with the magnitude, to evaluate how much an output signal is phase-shifted against an input signal. Like the magnitude the phase shift  $\phi$  is generally a function of frequency.

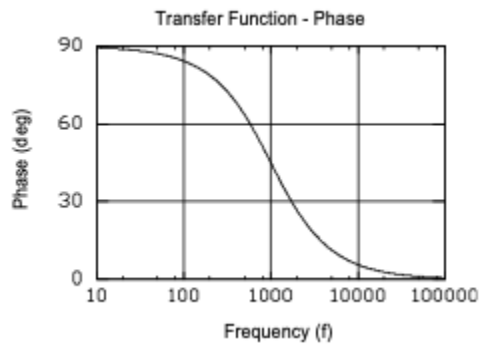
The usual way to present the result is the Bode Plot. The Bode plot describes the output response of a frequency-dependent system for a normalized input. It is often used in signal processing to show the transfer function of a system. It consists of two graphs. The magnitude and the phase plot.

### Bode Plot

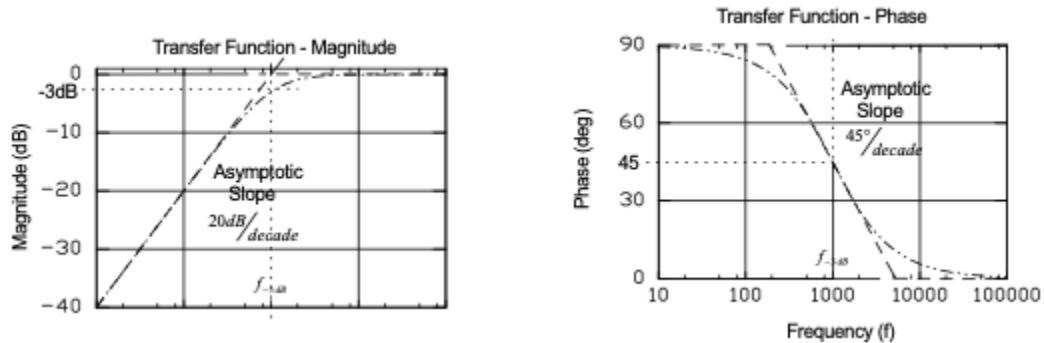
A Bode magnitude plot is a graph of magnitude in dB (this is a log scale, see above!) against frequency on a logarithmic scale.



A Bode phase plot is a graph of phase against frequency on a logarithmic scale



The magnitude-frequency plot can often be approximated to straight lines in a Bode plot (Asymptotic Bode Plot).

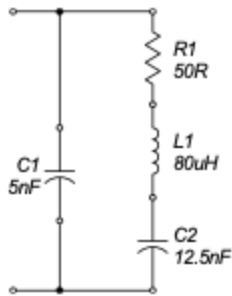


## Nyquist Plot

A Nyquist plot is a parametric plot of a frequency response. In Cartesian coordinates, the real part of the transfer function is plotted on the X axis. The imaginary part is plotted on the Y axis. The frequency is swept as a parameter, resulting in a plot per frequency.

Given is the following circuit:



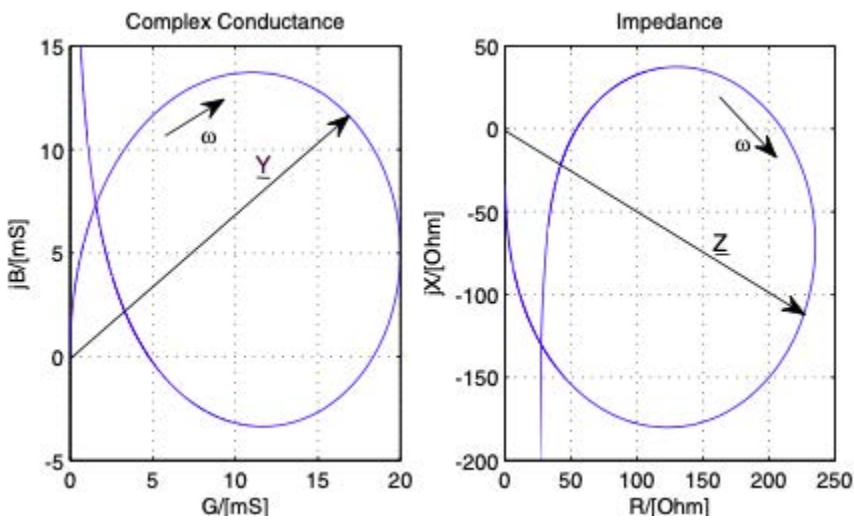


How is the change of the the complex conductance and the impedance when  $\omega$  changes?

$Z_s = R_1 + j(\omega L_1 - \frac{1}{\omega C_2})$  is the RLC series circuit.

Added the parallel capacitor  $Y_{all} = j\omega C_1 + \frac{1}{Z_s}$

With  $\omega$  varied the left Nyquist plot is generated. The right plot is  $\frac{1}{Y_{all}}$ .



summarize in one words...

### 3. EXECUTION

#### Experiment Setup

Workbench No.12

Used tools and instruments:

Breadboard, Tools box from workbench, Multimeters TENMA and ELABO, Generator, Oscilloscope, BNC Cable, BNC T-connector, BNC-Banana

#### Experiment Part 1 – Setup

##### Lo-Pass

##### Objective

The aim of this segment of the experiment was to ascertain the attributes of a Low-Pass filter and its behavior across various frequencies. The anticipated outcome was to present the results in the form of a Bode plot.

##### Preparation 1

The generator was linked through the BNC-to-Kleps cable. Ch1 of the oscilloscope was designated for the input signal, while Ch2 was allocated for the output signal. Voltage probes were employed for both channels, and an attenuation of 16 was configured to refine the graphs. The depicted circuit was assembled according to the provided instructions. The circuit in figure 1 below was built on the breadboard.

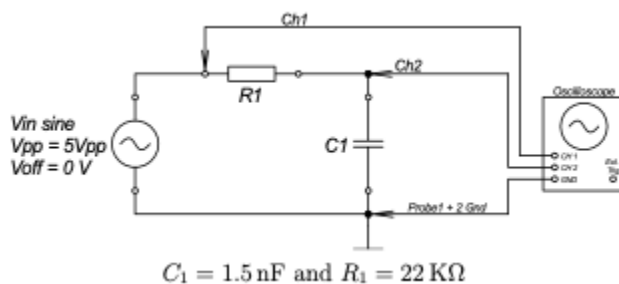


Figure 1: showing the circuit setup

##### Execution and Results 1

The generator's frequency was varied from 50 Hz to 100 KHz, with the table documenting the amplitude and phase shift observed between the input and output signals. The hardcopies provide a visual representation of how the data was displayed on the oscilloscope. The hard copies were taken at the beginning and end, to show a visual change in the amplitude and phase shift.

	input		output	
frequency(Hz)	Vpp(V)	Vpp(V)	Vpp(V)	Phase shift (°)
50	10.2	10.1	10.1	-7.20E-01
100	10.2	10.1	10.1	-1.44
200	10.2	10.1	10.1	-1.73
500	10.2	10	10	-5.76
1k	10.2	9.92	9.92	-13.6
2k	10.3	9.28	9.28	-23
5k	10.3	6.8	6.8	-47.5
10k	10.3	4.24	4.24	-64.8
20k	10.2	2.24	2.24	-75.4
50k	11.2	0.98	0.98	-85
100k	11.2	0.549	0.549	-84.3

Table 1: Showing the frequency, input and output peak-to-peak voltage and phase shift of low Pass

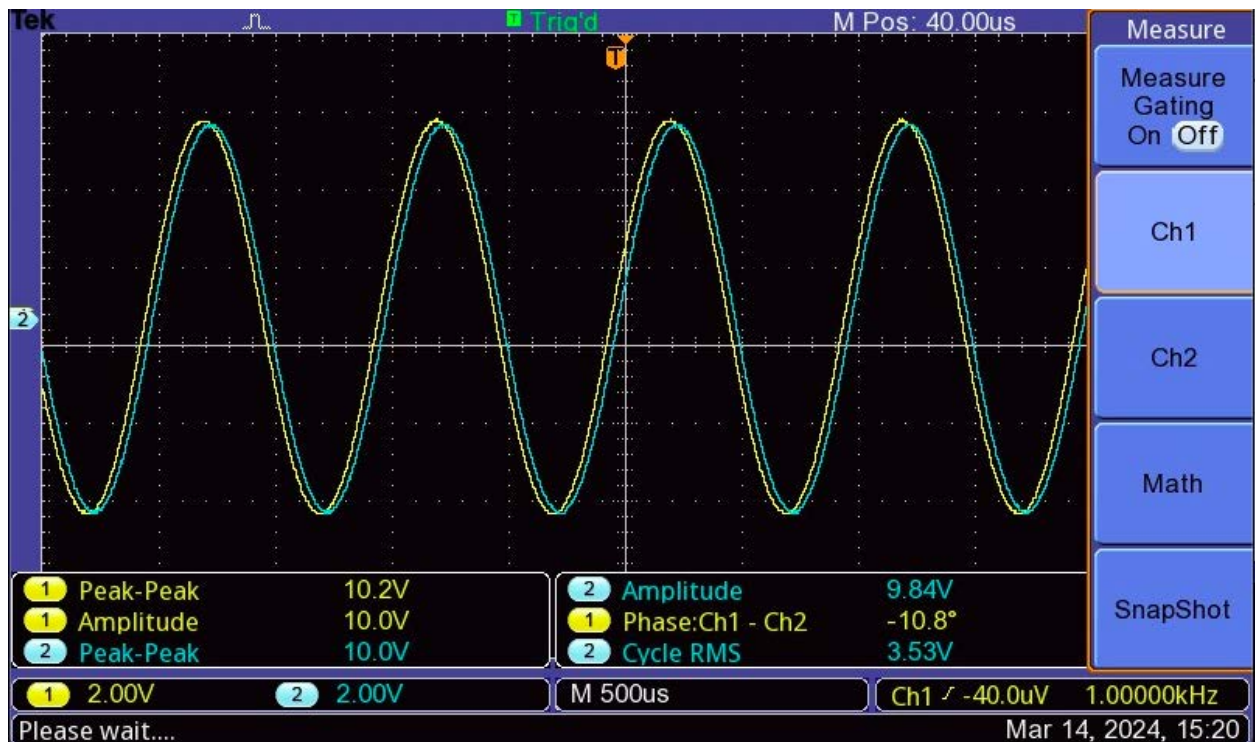


Figure 2: Showing the input and output peak-to-peak voltage and phase shift of low Pass at 1KHz.

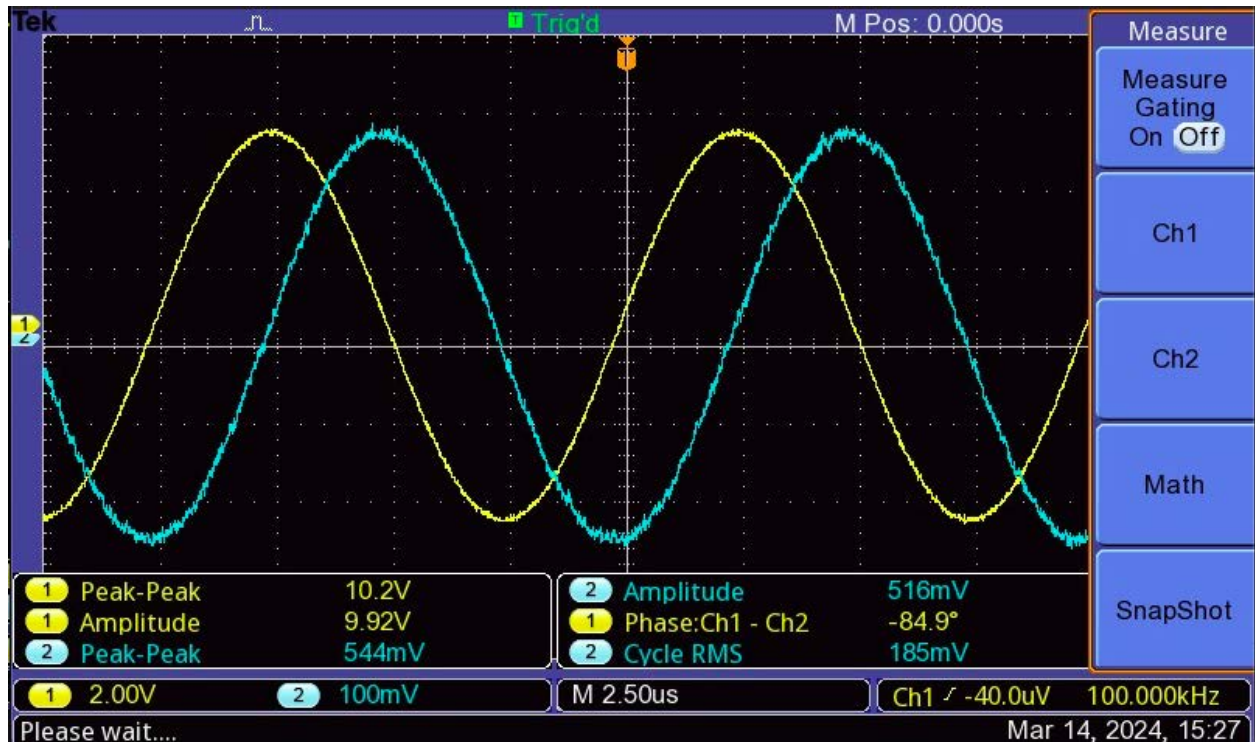


Figure 3: Showing the input and output peak-to-peak voltage and phase shift of low Pass at 100KHz.

## Experiment Part 2 – Setup

### RC-Band-Pass

#### Objective

The aim of this phase of the experiment was to ascertain the properties of a Band-Pass filter and its characteristics across a range of frequencies. The outcomes were intended to be depicted through both Bode and Nyquist plots.

#### Preparation 2

Utilizing a 1.5 nF capacitor and an 8.2 KΩ resistor, alongside a combination of a 100 nF capacitor and a 10.0 KΩ RC resistor, a bandpass filter was constructed. To distinguish between high-pass and low-pass configurations, cutoff frequencies were computed. The signal generator was linked to the input, designated as CH-1, via a BNC-to-Kleps cable. Input and output signals were measured using the oscilloscope. The cutoff frequencies of the RC combinations were determined as outlined below.

For the first combination we get:

$$f_{-3dB} = \frac{1}{2\pi RC}$$

$$f_{-3dB} = \frac{1}{2\pi \times 10^{-9} \times 100 \times 10000}$$

$$f_{-3dB} = 159.155Hz$$

**For the second combination we get:**

$$f_{-3dB} = \frac{1}{2\pi RC}$$

$$f_{-3dB} = \frac{1}{2\pi \times 10^{-9} \times 1.5 \times 8200}$$

$$f_{-3dB} = 12939.43Hz$$

Based on the above calculations, the first combination (100nF and 10kΩ) is high pass and the second combination (1.5nF and 8.2KΩ) is low pass. The circuit below was built on the breadboard, incorporating the necessary resistors and capacitors as illustrated in the figure below.

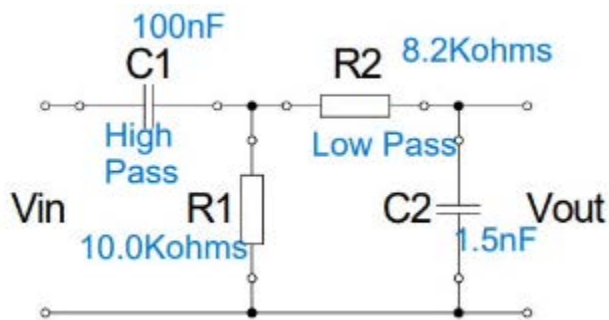


Figure 4

## Execution and Results 2

Similar to part 1, the generator's frequency was varied from 50 Hz to 100 KHz, with the table documenting the amplitude and phase shift observed between the input and output signals, but this time using the Sine signal with 5Vpp amplitude with no offset, and indeed the circuit behaved like expected from the calculation of the cutoff frequencies. The hardcopies provide a visual representation of how the data was displayed on the oscilloscope. The hard copies were taken at the beginning and end, to show a visual change in the amplitude and phase shift. In short the process was followed according to the provided instructions to assess the behavior of the circuit under different frequency conditions and record relevant data.

	input		output	
frequency(Hz)	Vpp(V)	Vpp(V)	Phase Shift(°)	
50	10.8	2.92	73.40	
100	10.6	5.2	57	
200	10.6	7.66	36.5	
500	10.6	9.44	14	
1k	10.6	9.76	2.16	
2k	10.8	10.4	-2.58	
5k	11.2	9.8	-22.9	
10k	11.2	8.6	-39.4	
20k	11	6	-57.4	
50k	10.8	3.2	-60.9	
100k	11	2	-82.6	

this is the high pass end!!

Table 2: Showing the frequency, input and output peak-to-peak voltage and phase shift of low Pass

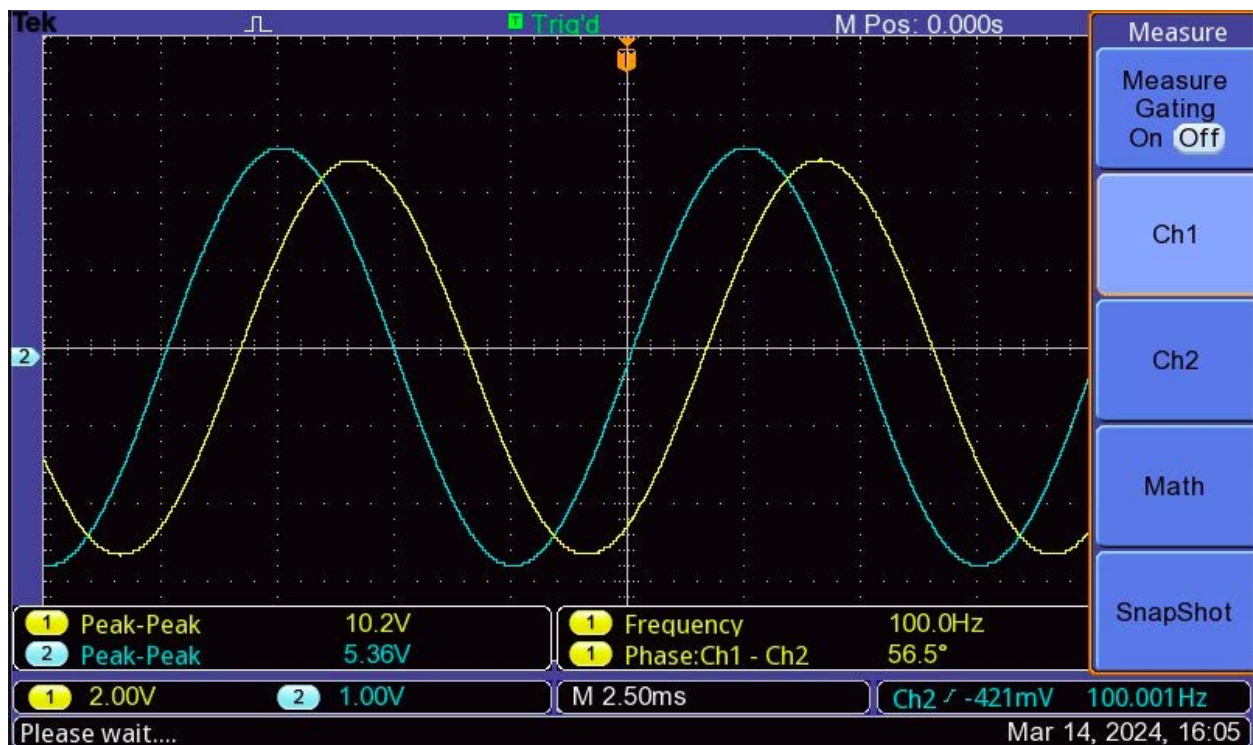


Figure 5: Showing the input and output peak-to-peak voltage and phase shift at 100Hz



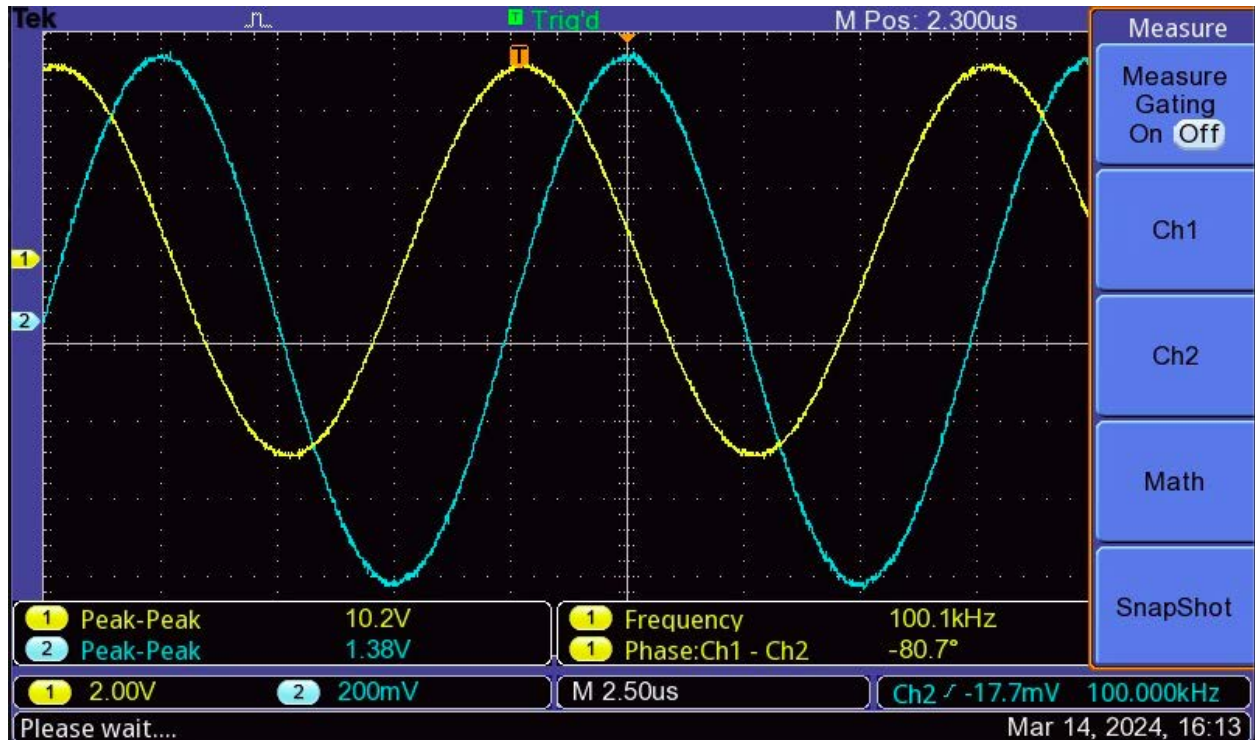


Figure 6: Showing the input and output peak-to-peak voltage and phase shift at 100KHz

## 4. EVALUATION

### Part 1: Lo-Pass

Calculating for magnitude using one of the measurements

Output voltage=10.1, Input Voltage=10.2

$$\text{Magnitude} = 20 \log_{10} \left( \frac{V_{out}}{V_{in}} \right)$$

$$\text{Magnitude} = 20 \log_{10} \left( \frac{10.1}{10.2} \right)$$

$$\text{Magnitude} = -0.0856 \text{ dB}$$

Using the formula above to calculate the magnitude at different measured frequencies in excel, we get:

	input		output		
frequency(Hz)	Vpp(V)	Vpp(V)	Phase shift(°)	magnitude(dB)	
50	10.2	10.1	-7.20E-01	-0.08557595959	
100	10.2	10.1	-1.44	-0.08557595959	
200	10.2	10.1	-1.73	-0.08557595959	
500	10.2	10	-5.76	-0.1720034352	
1k	10.2	9.92	-13.6	-0.2417699922	
2k	10.3	9.28	-23	-0.9057849697	
5k	10.3	6.8	-47.5	-3.60656624	
10k	10.3	4.24	-64.8	-7.709427362	
20k	10.2	2.24	-75.4	-13.16704307	
50k	11.2	0.98	-85	-21.15983894	
100k	11.2	0.549	-84.3	-26.19291356	

Table 3 :showing input and output voltages, phase shift and magnitude of low pass at different frequencies

Bode Magnitude Plot

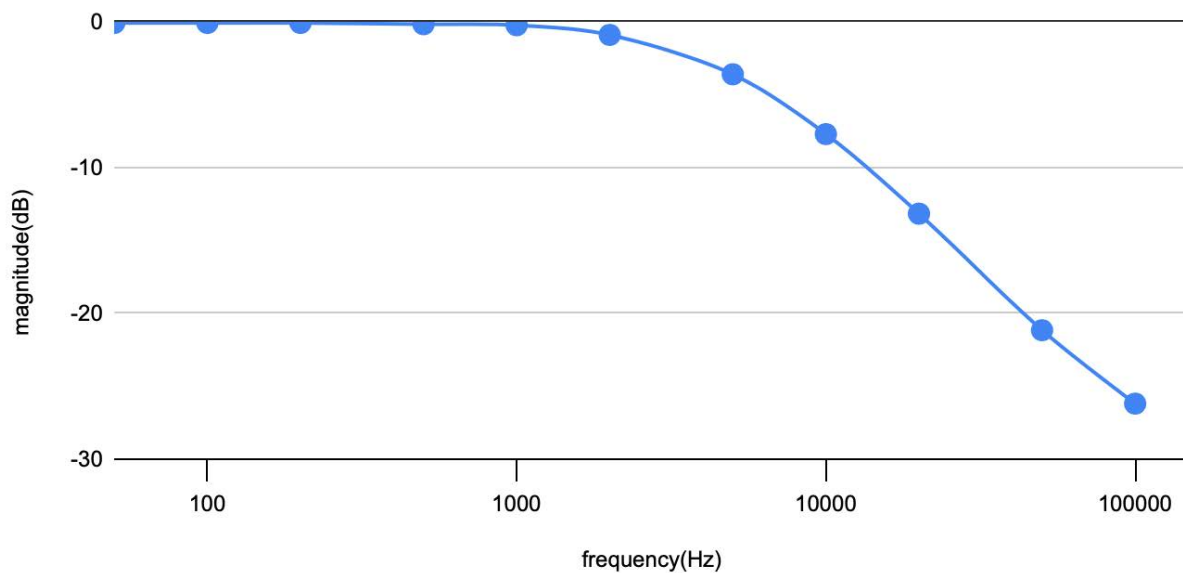


Figure 7 : Showing Bode Magnitude Plot from measured values



## Bode Phase Plot

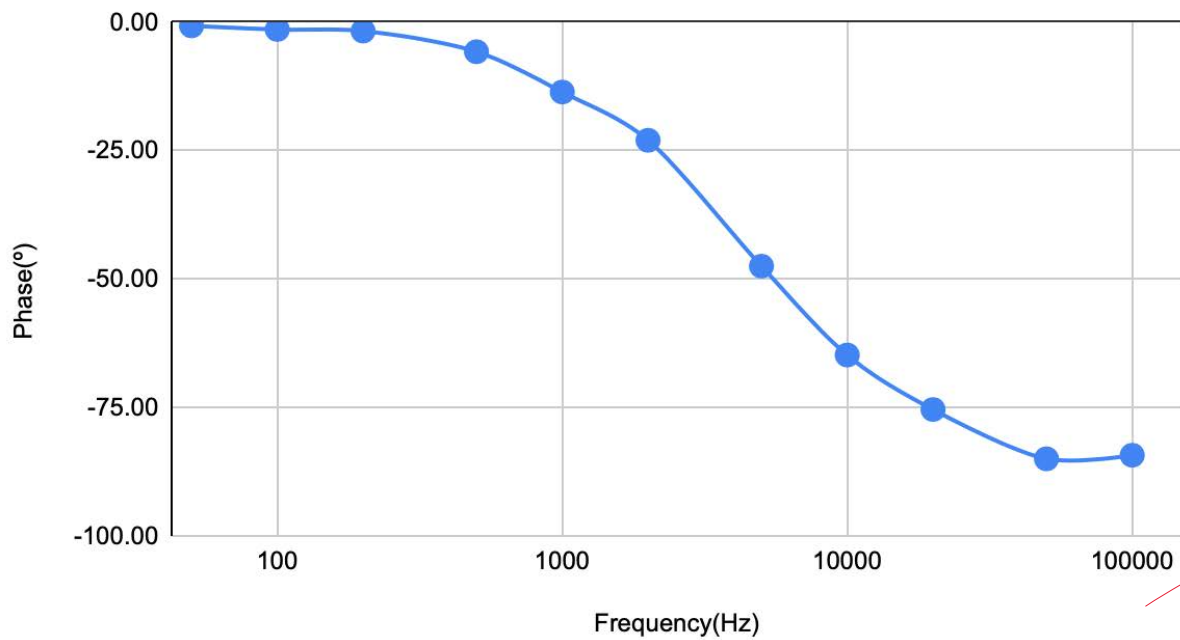


Figure 8: Showing Bode Phase Plot from measured values

Calculating the theoretical magnitude and phase using the formulas provided in the theory we get

$$f = 50\text{Hz}, \omega = 2\pi f = 2\pi \times 50 = 314.159\text{rads/s}$$

$$R = 22000\Omega \text{ and } C = 1.5\text{nF}$$

$$\text{Magnitude} = 20\log_{10} \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\text{Magnitude} = 20\log_{10} \frac{1}{\sqrt{1+(314.159 \times 22000 \times 1.5 \times 10^{-9})^2}}$$

$$\text{Magnitude} = -4.668 \times 10^{-4} \text{ dB}$$

$$\varphi = -\arctan(\omega RC)$$

$$\varphi = -\arctan(314.159 \times 22000 \times 1.5 \times 10^{-9})$$

$$\varphi = -0.59^\circ$$

Using the same methods to calculate for other frequency values in excel, we get the values in the table below

frequency(Hz)	w (rads/s)	phase Shift(°)	Magnitude(dB)
50	314.1592654	-0.5939787203	-0.0004667545912
100	628.3185307	-1.187829796	-0.001866717456
200	1256.637061	-2.374639417	-0.007462060445
500	3141.592654	-5.918855154	-0.04642890307
1000	6283.185307	-11.71401278	-0.1828097595
2000	12566.37061	-22.52330752	-0.6891574886
5000	31415.92654	-46.03302698	-3.169762224
10000	62831.85307	-64.25255815	-7.242102843
20000	125663.7061	-76.44231426	-12.59994991
50000	314159.2654	-84.4904349	-20.35349644
100000	628318.5307	-87.2388345	-26.34396619

Table 4: showing calculated phase shift and magnitude at different frequencies

Drawing the Bode magnitude and phase plot from the theoretical values in table above we get

### Bode Magnitude Plot (Theoretical)

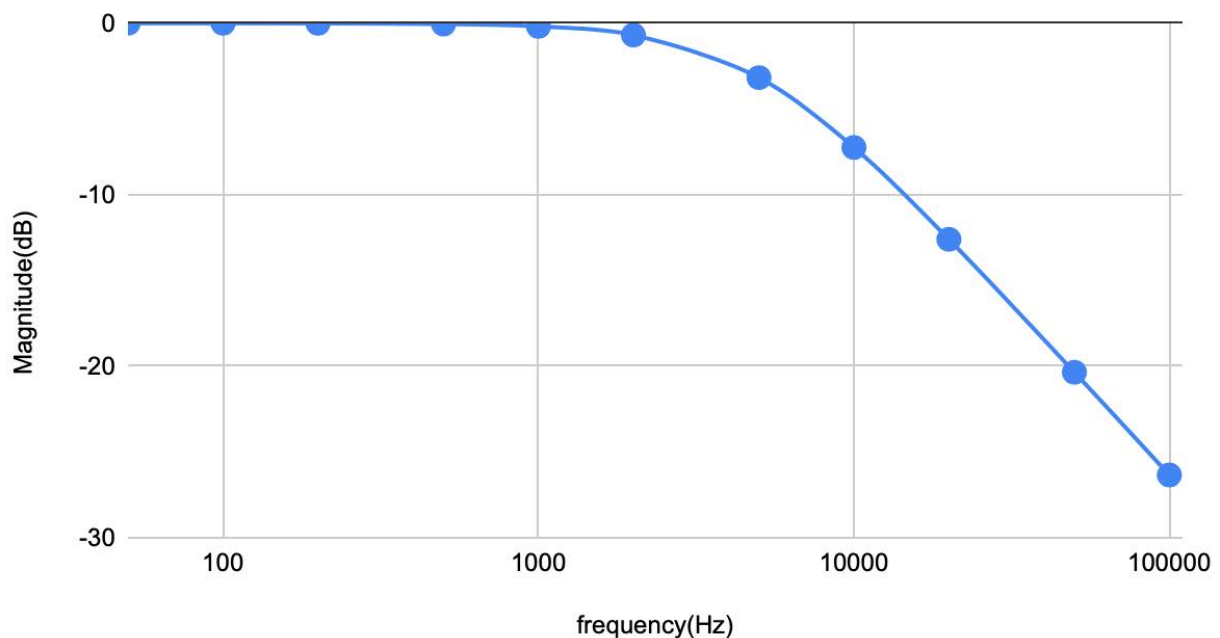


Figure 9: Showing Bode Magnitude Plot from calculated values

### Bode Phase Plot (Theoretical)

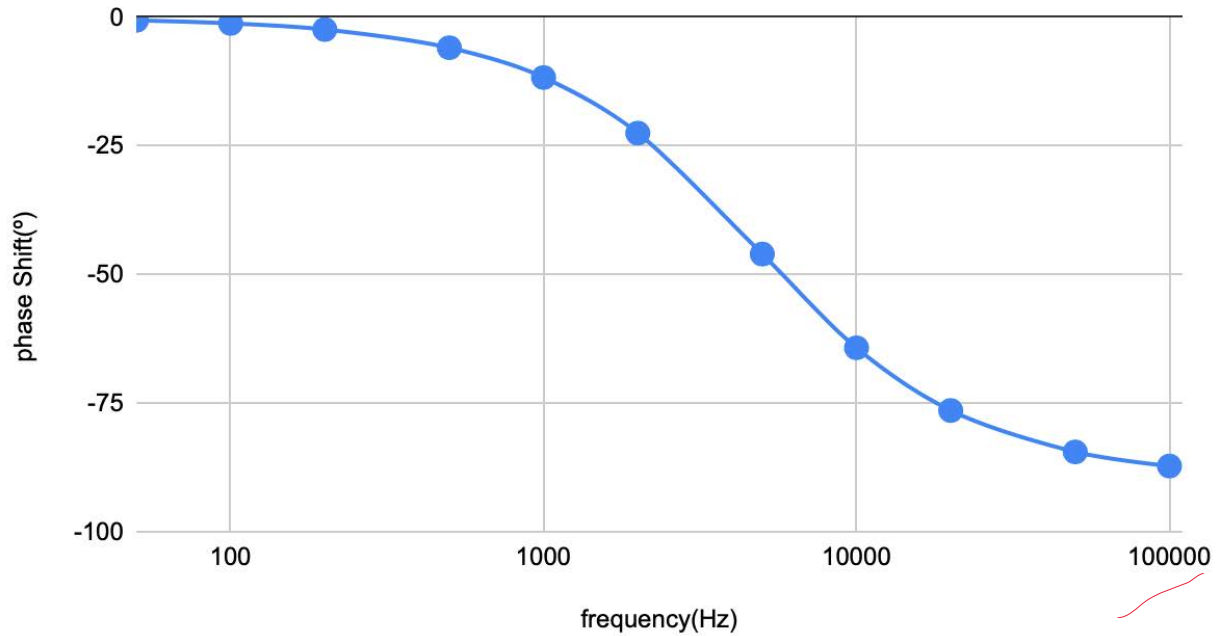


Figure 10: Showing Bode Phase Plot from calculated values

Comparing the measured and calculated plots we get

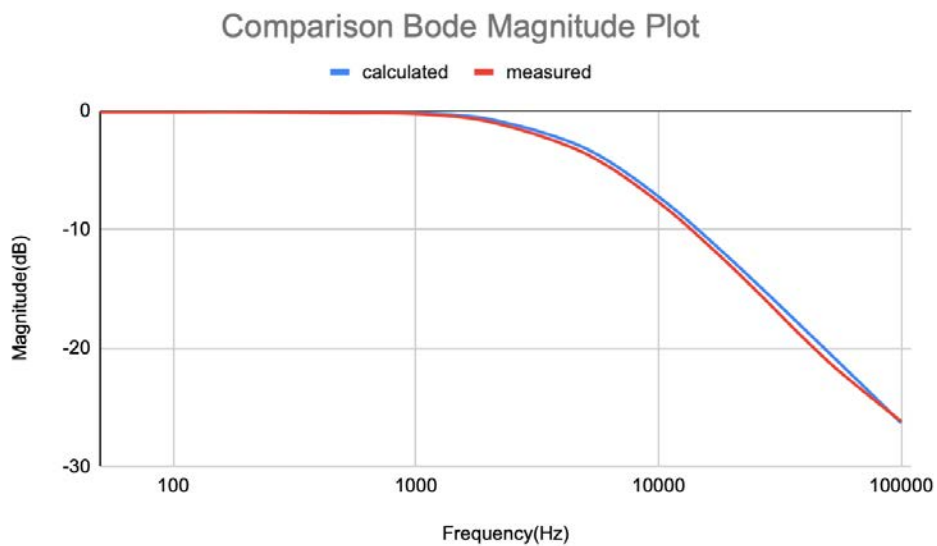


Figure 11: Showing a comparison Bode Magnitude Plot of calculated and measured values

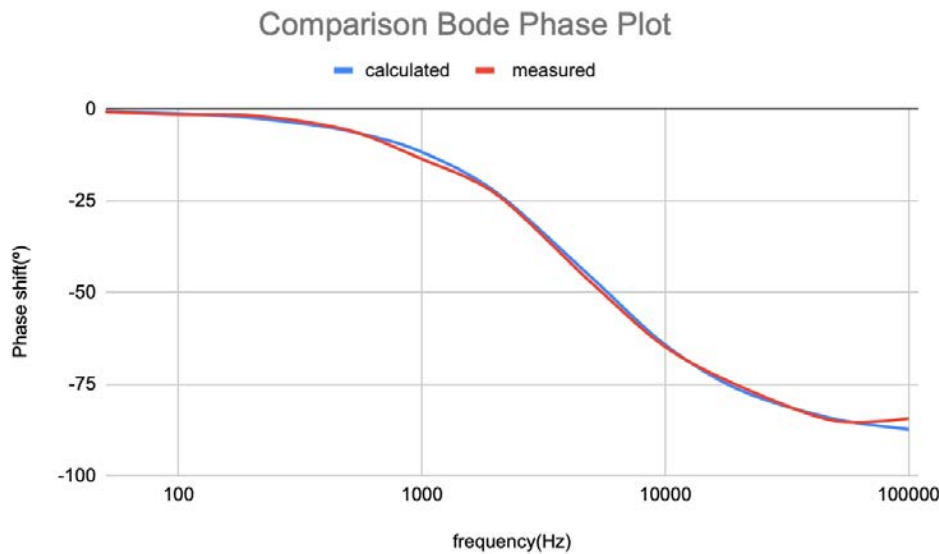


Figure 12: Showing a comparison Bode Phase Plot of calculated and measured values

On the graph, both the measured and calculated values are displayed. The results indicate that, generally, the values are quite similar, especially at higher frequencies. However, there are slight discrepancies which can be attributed to flaws in the oscilloscope's instruments and limitations in the resistors, capacitors, or other components utilized during the experiment. Additionally, external factors like noise or interference within the circuit may have impacted the accuracy of the measured values.

Calculating the -3dB frequency from the components given we get

$$f_{-3dB} = \frac{1}{2\pi RC}$$

$$f_{-3dB} = \frac{1}{2\pi \times 22000 \times 1.5 \times 10^{-9}}$$

$$f_{-3dB} = 4822.88Hz$$

Reading it from figure with the measured values we get

$$f_{-3dB} = 4890Hz$$

Comparing the calculated and read value of  $f_{-3dB}$ , we see that they are almost the same and that they have a very small difference, which may have been caused by slight discrepancies attributed to flaws in the oscilloscope's instruments and limitations in the resistors, capacitors, or other components utilized during the experiment.

Calculating the phase shift at  $\omega_{-3dB}$  from the components given we get

$$\varphi = -\arctan(\omega RC)$$

$$\varphi = -\arctan(2\pi \times 4822.88 \times 22000 \times 1.5 \times 10^{-9})$$

$$\varphi = -45.0^\circ$$

Reading it from figure with the measured values we get

$$\varphi = -45.5^\circ$$

Comparing the calculated and read value of the phase shift at  $\omega_{-3dB}$ , we see that they are almost the same and that they have a very small difference, which may have been caused by slight discrepancies attributed to flaws in the oscilloscope's instruments and limitations in the resistors, capacitors, or other components utilized during the experiment.

Finding the gradient  $|A|$  per decade (unit=dB/decade), we get any two points on the plot that slant perfectly and find their difference in magnitude.

$$\text{gradient of } |A| = -26.19291356 - (-7.709427362)$$

$$\text{gradient of } |A| = -18.4834862 \text{ dB/decade}$$

The presence of the negative sign indicates a decrease in decibels per decade as frequency rises, confirming the signal's attenuation behavior.

Calculating the limits of the amplitude ratio in dB and the phase of the Lo Pass for each question below we get:

$$\text{a) } f \ll f_{-3dB}$$

Calculating the limit of the magnitude we get:

$$\lim_{f \rightarrow 0} \frac{1}{\sqrt{1+(2\pi fRC)^2}}$$

$$\lim_{f \rightarrow 0} \frac{1}{\sqrt{1+(0)^2}}$$

$$\lim_{f \rightarrow 0} = 1$$

Calculating the limit of the phase we get:

$$\lim_{f \rightarrow 0} -\tan^{-1}(2\pi fRC) = 0^\circ$$

$$\text{b) } f \gg f_{-3dB}$$

Calculating the limit of the magnitude we get:

$$\lim_{f \rightarrow \infty} \frac{1}{\sqrt{1+(2\pi fRC)^2}}$$

$$\lim_{f \rightarrow \infty} \frac{1}{\sqrt{1+(\infty)^2}} = 0$$

Calculating the limit of the phase we get:

$$\lim_{f \rightarrow \infty} -\tan^{-1}(2\pi fRC)$$

$$\lim_{f \rightarrow \infty} -\tan^{-1}(\infty) = -90^\circ$$

c)  $f = f_{-3dB}$

Calculating the limit of the magnitude we get:

$$\lim_{f \rightarrow \infty} \frac{1}{\sqrt{1+(\frac{1}{2\pi RC} \times 2\pi RC)^2}} = \frac{1}{\sqrt{2}}$$

Calculating the limit of the phase we get:

$$\lim_{f \rightarrow \infty} -\tan^{-1}\left(\frac{1}{2\pi RC} \times 2\pi RC\right)$$

$$\lim_{f \rightarrow \infty} -\tan^{-1}(1) = -45^\circ$$

## Part 2: Band Pass

### Calculating for magnitude using one of the measurements

Output voltage=2.92, Input Voltage=10.8

$$\text{Magnitude} = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right)$$

$$\text{Magnitude} = 20\log_{10}\left(\frac{2.92}{10.8}\right)$$

$$\text{Magnitude} = -11.361dB$$

Using the formula above to calculate the magnitude at different measured frequencies in excel, we get:

	input	output		
frequency(Hz)	Vpp(V)	Vpp(V)	Phase shift(°)	magnitude(dB)
50	10.8	2.92	73.40	-11.36081808
100	10.6	5.2	57	-6.186050433
200	10.6	7.66	36.5	-2.821541913
500	10.6	9.44	14	-1.006677419
1k	10.6	9.76	2.16	-0.717120952
2k	10.8	10.4	-2.58	-0.3278083238
5k	11.2	9.8	-22.9	-1.15983894
10k	11.2	8.6	-39.4	-2.294391429
20k	11	6	-57.4	-5.264828695
50k	10.8	3.2	-60.9	-10.56547554
100k	11	2	-82.6	-14.80725379

Table 5: showing input and output voltages, phase shift and magnitude of high pass at different frequencies

Bode Magnitude Plot

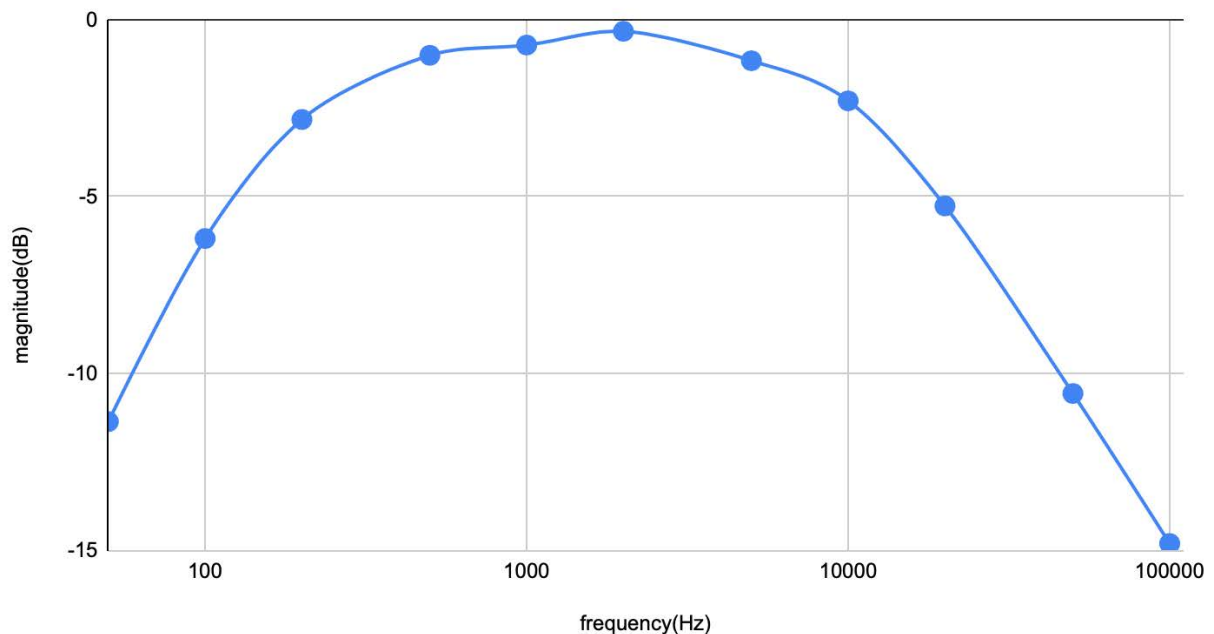


Figure 13: Showing Bode Magnitude Plot from measured values

## Bode Phase Plot

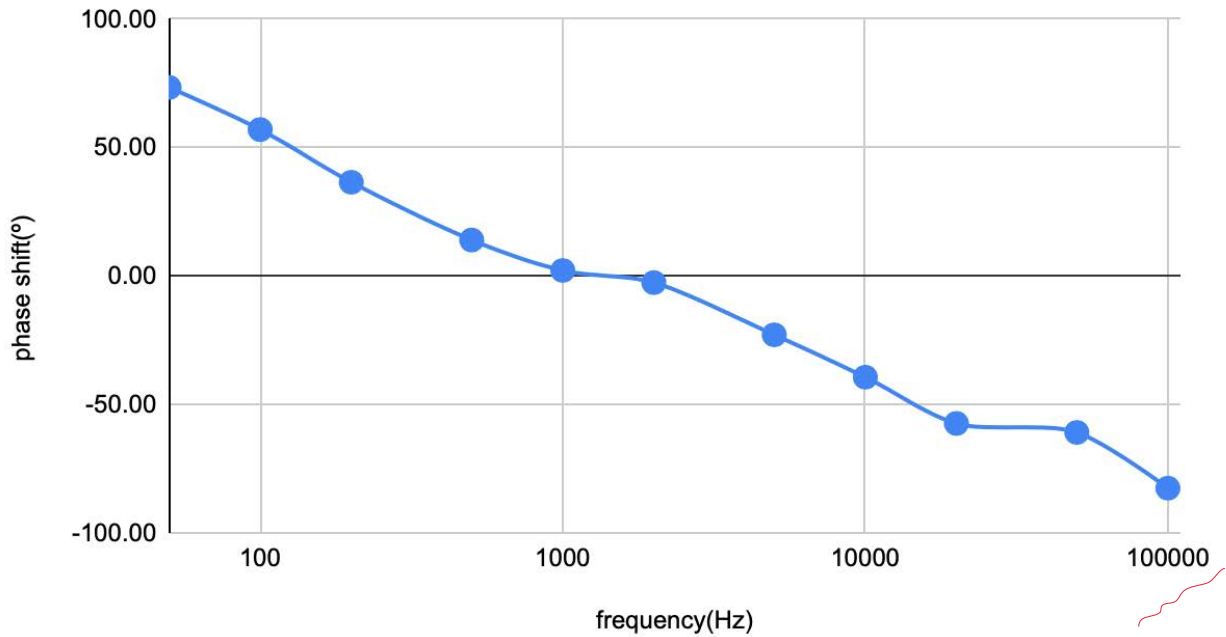


Figure 14: Showing Bode Phase Plot from measured values

Calculating the theoretical magnitude and phase using the nominal values of the elements and the measured values, we get

$$f = 50\text{Hz}, \omega = 2\pi f = 2\pi \times 50 = 314.159\text{rads/s}$$

$$|A|_{high} = \frac{1}{\sqrt{1+(\frac{1}{\omega RC})^2}}, \text{ when } R = 10000\Omega \text{ and } C = 100\text{nF}$$

$$|A|_{high} = \frac{1}{\sqrt{1+(\frac{1}{314.159 \times 10000 \times 100 \times 10^{-9}})^2}}$$

$$|A|_{high} = 0.300\text{dB}$$

$$|A|_{low} = \frac{1}{\sqrt{1+(\omega RC)^2}}, \text{ when } R = 8200\Omega \text{ and } C = 1.5\text{nF}$$

$$|A|_{low} = \frac{1}{\sqrt{1+(314.159 \times 8200 \times 1.5 \times 10^{-9})^2}}$$

$$|A|_{low} = 1.000\text{dB}$$

$$|A|_{total} = 20\log_{10}(|A|_{high} \times |A|_{low})$$

$$|A|_{total} = 20\log_{10}(0.300 \times 1.000)$$



$$|A|_{total} = -10.458 \text{ dB}$$

Calculating for the phase:

$$\varphi_{high} = \arctan\left(\frac{1}{\omega RC}\right), \text{ when } R = 10000\Omega \text{ and } C = 100\text{nF}$$

$$\varphi_{high} = \arctan\left(\frac{1}{314.159 \times 10000 \times 100 \times 10^{-9}}\right)$$

$$\varphi_{high} = 72.560^\circ$$

$$\varphi_{low} = -\arctan(\omega RC), \text{ when } R = 8200\Omega \text{ and } C = 1.5\text{nF}$$

$$\varphi_{low} = -\arctan(314.159 \times 8200 \times 1.5 \times 10^{-9})$$

$$\varphi_{low} = -0.221^\circ$$

$$\varphi_{total} = \varphi_{high} + \varphi_{low}$$

$$\varphi_{total} = 72.560 + (-0.221)$$

$$\varphi_{total} = 72.339^\circ$$

Using the same methods above to calculate for other frequency values in excel, we get the values in the table below

frequency(Hz)	w (rads/s)	phase Shift(°)	Magnitude(dB)
50	314.1592654	72.33800661	-10.465843
100	628.3185307	57.41530118	-5.481732132
200	1256.637061	37.62635777	-2.131583778
500	3141.592654	15.44388813	-0.4256198574
1000	6283.185307	4.623845311	-0.1344998049
2000	12566.37061	-4.236603177	-0.1299515402
5000	31415.92654	-19.30415874	-0.6087973268
10000	62831.85307	-36.78617624	-2.034880516
20000	125663.7061	-56.64235575	-5.301087989
50000	314159.2654	-75.30844267	-12.02267188
100000	628318.5307	-82.53603093	-17.83382197

Table 6: showing calculated phase shift and magnitude of high pass at different frequencies

### Bode Magnitude Plot (Theoretical)

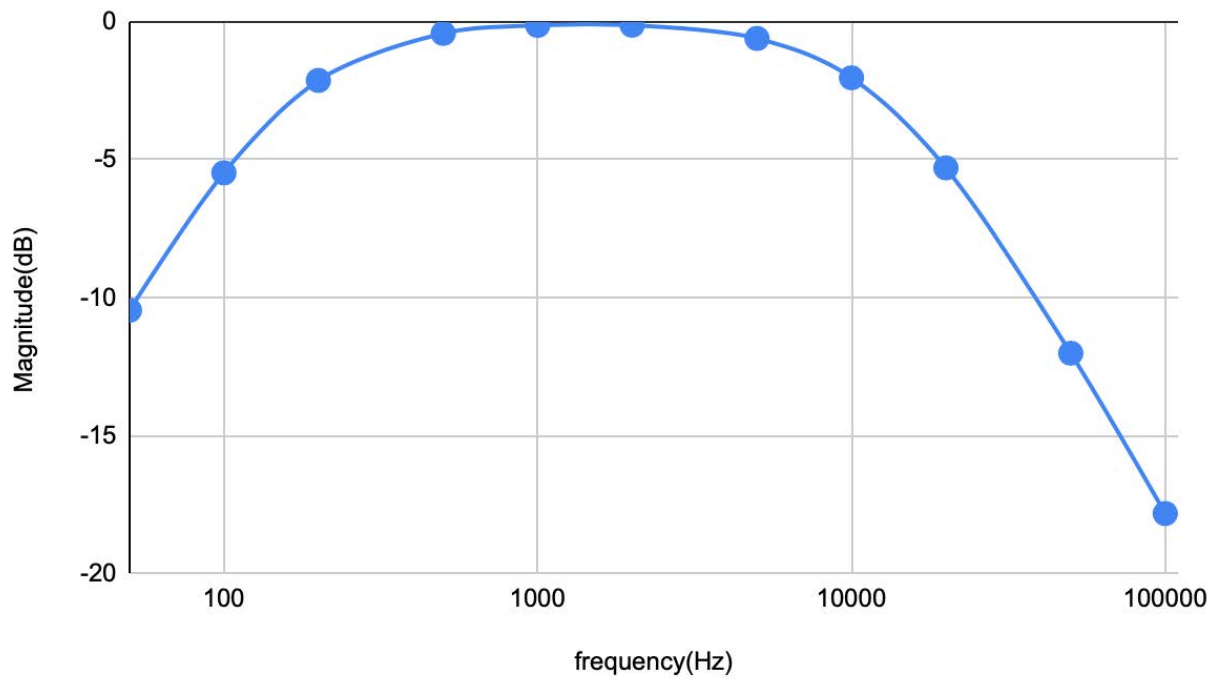


Figure 15: Showing Bode Magnitude Plot from calculated values

### Bode Phase Plot (Theoretical)

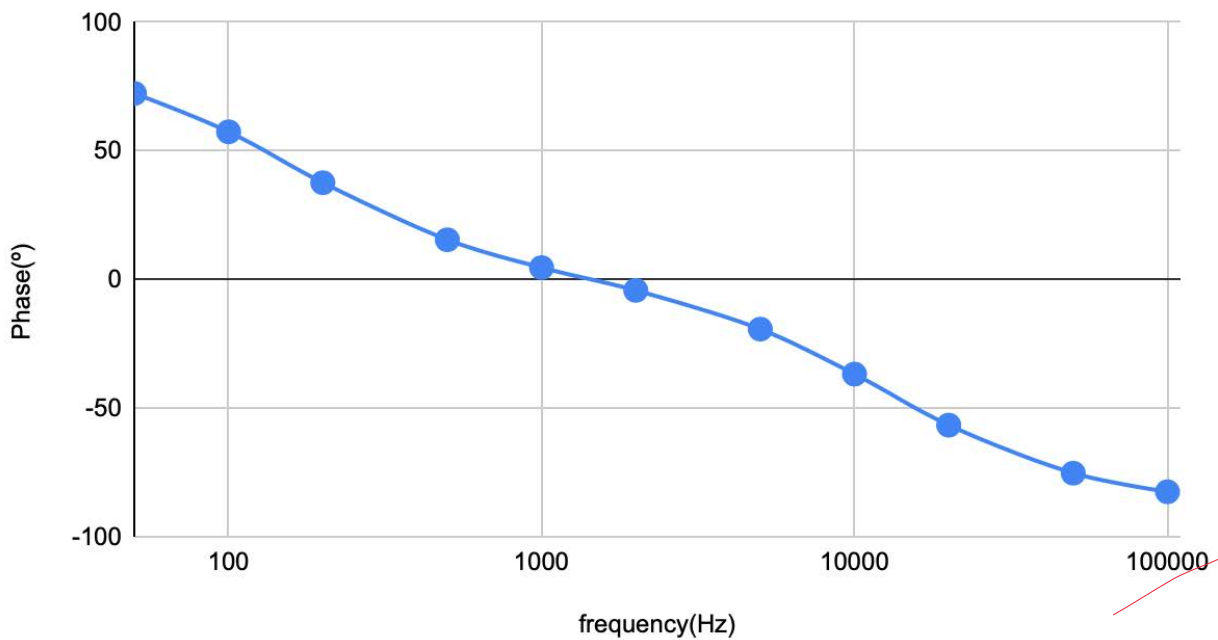


Figure 16: Showing Bode Phase Plot from calculated values

plot measured and theoretical together...

Calculating the cutoff frequencies using the given components:

For High pass frequency, we get:

$$f_{-3dB} = \frac{1}{2\pi RC}, \text{ when } R = 10000\Omega \text{ and } C = 100nF$$

$$f_{-3dB} = \frac{1}{2\pi \times 10000 \times 100 \times 10^{-9}}$$

$$f_{-3dB} = 159.155Hz$$

For Low pass frequency, we get:

$$f_{-3dB} = \frac{1}{2\pi RC}, \text{ when } R = 8200\Omega \text{ and } C = 1.5nF$$

$$f_{-3dB} = \frac{1}{2\pi \times 8200 \times 1.5 \times 10^{-9}}$$

$$f_{-3dB} = 12939.43Hz$$

Calculating the center-frequency using the given components, we get:

$$\text{center - frequency} = \sqrt{\text{High pass frequency} \times \text{Low pass frequency}}$$

$$\text{center - frequency} = \sqrt{159.155 \times 12939.43}$$

$$\text{center - frequency} = 1435.05Hz$$

Calculating the bandwidth using the given components, we get:

$$\text{bandwidth} = \text{Low pass frequency} - \text{High pass frequency}$$

$$\text{bandwidth} = 12939.43 - 159.155$$

$$\text{bandwidth} = 12780.28Hz$$

Calculating the phase shift at the cutoff frequencies, we get:

For High Phase, we get:

$$\varphi_{high} = \arctan\left(\frac{1}{2\pi fRC}\right), \text{ when } R = 10000\Omega \text{ and } C = 100nF$$

$$\varphi_{high} = \arctan\left(\frac{1}{2\pi \times 159.155 \times 10000 \times 100 \times 10^{-9}}\right)$$

$$\varphi_{high} = 45.000^\circ$$

For Low Phase, we get:

$$\varphi_{low} = -\arctan(2\pi fRC), \text{ when } R = 8200\Omega \text{ and } C = 1.5nF$$

$$\varphi_{low} = -\arctan(2\pi \times 12939.43 \times 8200 \times 1.5 \times 10^{-9})$$

$$\varphi_{low} = 45.000^\circ$$

$$\varphi_{total} = |\varphi_{high}| - |\varphi_{low}|$$

$$\varphi_{total} = |45.000| - |45.000|$$

$$\varphi_{total} = 0^\circ$$

Comparing the calculated and measured values, we see that the theoretical calculations closely align with the actual measurements across most frequencies, indicating a successful and reliable experiment with minimal errors. However, discrepancies were noted at higher frequencies, potentially attributed to the components used in the experiment, such as capacitors and inductors, which may not have behaved as expected according to Ohm's law. Furthermore, the oscilloscope's ~~limited bandwidth~~ might have contributed to measurement inaccuracies when assessing the output voltage. These experimental apparatuses and components likely contributed to the disparities between the theoretical predictions and the measured values.

not the bandwidth... accuracy!!

Drawing the Nyquist plot  $UR=f(f)$  for the used frequency values:

Calculating the Nyquist plot values used in plotting the graph:

Calculating for the imaginary part for one of the frequencies:

Calculating for the real part for one of the frequencies:

$$real\ part = V_{out} \times \cos(\varphi)$$

$$real\ part = 2.92 \times \cos(73.40)$$

$$real\ part = 0.834$$

$$imaginary\ part = V_{out} \times \sin(\varphi)$$

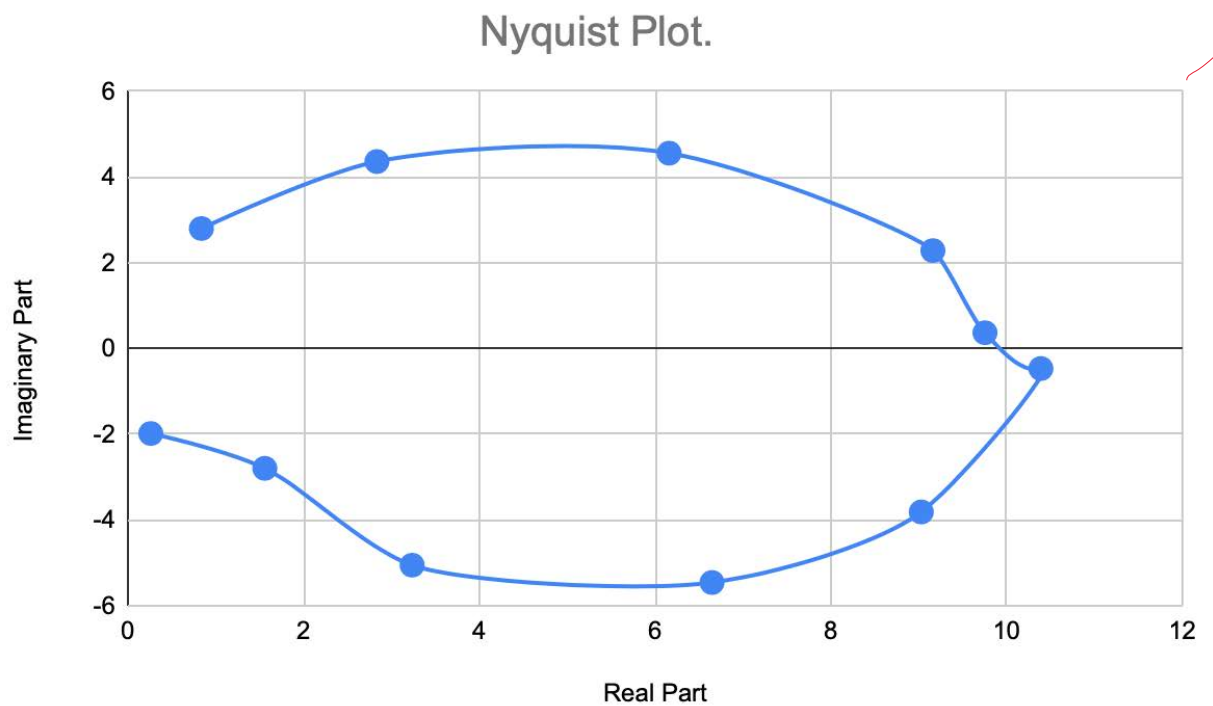
$$imaginary\ part = 2.92 \times \sin(73.40)$$

$$imaginary\ part = 2.798$$

Using the same methods above to calculate for other frequency values in excel, we get the values in the table below

	input	output			
frequency (Hz)	Vpp(V)	Vpp(V)	Phase Shift(°)	Real Part	Imaginary Part
50	10.8	2.92	73.40	0.834	2.798
100	10.6	5.2	57	2.832	4.361
200	10.6	7.66	36.5	6.158	4.556
500	10.6	9.44	14	9.16	2.284
1000	10.6	9.76	2.16	9.753	0.368
2000	10.8	10.4	-2.58	10.389	-0.468
5000	11.2	9.8	-22.9	9.028	-3.813
10000	11.2	8.6	-39.4	6.646	-5.459
20000	11	6	-57.4	3.233	-5.055
50000	10.8	3.2	-60.9	1.556	-2.796
100000	11	2	-82.6	0.258	-1.983

Table 7: Showing calculated real and imaginary parts.



labeled axis.. what is the unit here??

Figure 17: Showing a Nyquist Plot

## **5. CONCLUSION**

The initial phase of the experiment involved examining the characteristics of the Low-Pass filter within a specified frequency range. This included measuring both input and output voltage, as well as determining and adjusting phase shifts to facilitate the Bode plotting of theoretical and actual magnitude and phase graphs. While there were minor disparities between the calculated and theoretical values, the experiment was still a success and showed accuracy. The primary source of error, particularly evident at higher frequencies, was attributed to the oscilloscope.

Similarly, the Band-Pass circuit consisted of resistor and capacitor pairs, measurements of input and output voltages, along with phase shifts. To verify the accuracy of the gathered data, a Nyquist plot was generated. As observed in the low pass filter experiment, the oscilloscope emerged as the principal error source, especially noticeable with increasing frequency.

## **6. REFERENCE**

1. Pagel Uwe, General Electrical Engineering II Lab Manual (2024). Constructor University

## **7. APPENDIX**

### **Experiment 6: Operational Amplifier**

Part 1			
R1=100k $\Omega$			
	input	output	
frequency(Hz)	Vpp(mV)	Vpp(mV)	Phase
1.00E+03	504	492	177
2.00E+03	508	476	176
5.00E+03	508	464	177
10k	508	460	177
20k	508	456	176
50k	508	452	171
100k	508	448	164
200k	508	424	148
500k	508	280	103
1M	508	140	65.5
2M	504	56	20.8
5M	500	16	-62.7

R1= 22k $\Omega$			
	input	output	
frequency(Hz)	Vpp(mV)	Vpp(mV)	Phase
1k	520	240	178
2k	516	240	179
5k	528	228	175
10k	528	232	178
20k	532	228	175
50k	524	228	176
100k	524	224	168
200k	528	224	158
500k	528	200	121
1M	524	128	75.8
2M	524	64	44.1
5M	512	36	12.6



R1=1k $\Omega$			
	input	output	
frequency(Hz)	Vpp(mV)	Vpp(V)	Phase
1k	516	4.84	176
2k	516	4.68	174
5k	516	4.56	174
10k	516	4.48	170
20k	520	4.36	162
50k	524	3.36	133
100k	528	1.92	112
200k	524	1.04	92.3
500k	528	480mV	76.2
1M	520	192mV	56.7
2M	524	92mV	33.7
5M	528	42mV	30.1

Part 2			
	input	output	
frequency(Hz)	Vpp(V)	Vpp(V)	Phase
10	1.01	16.4	100
20	1.01	8.4	95
50	1.01	3.6	93.1
100	1	1.66	90.3
200	1	832mV	90.7
500	1.01	328mV	88.5
1.00E+03	1.01	164mV	88.5

Part 3				
Ammeter	Ammeter			
Current(mA)	Voltage	V-	V+	Vout
96.96	0.2479	5.089	4.842	-2.4751
200mA range				