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Constructor University Bremen
Natural Science Laboratory
General Electrical Engineering II
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Lab Experiment 4- Two-port Networks

Author: Wanzia Nambule

Instructor: Pagel Uwe

Experiment conducted by: Wanzia Nambule, Dahyun Ko
Place of execution: Teaching Lab EE
Rotation II, Bench 12

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1.INTRODUCTION

In this experiment we were focused on discovering the behaviors and properties of a selected Two-port Network.

2.THEORY

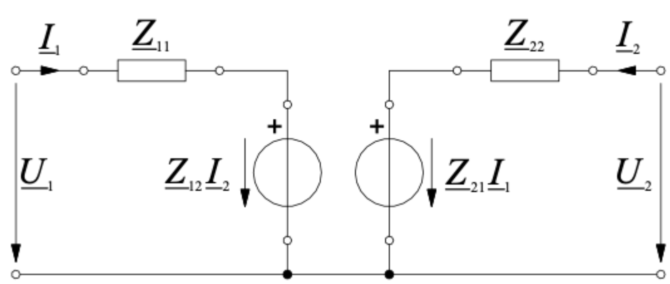
A two-port network refers to an electrical circuit featuring two sets of terminals, designated for input and output respectively. It is characterized by a pair of equations for both its input and output terminals, outlining how the network converts electrical signals between the two terminal pairs.

Two-port impedance network

A two-port impedance network is a specific type of two-port network that stands out due to its input and output impedances. When discussing a two-port impedance network, the input impedance refers to the impedance observed at its input terminals upon applying a signal, while the output impedance refers to the impedance observed at its output terminals.

To represent the input and output impedances of a two-port impedance network, the Z-parameter matrix is commonly employed. These parameters establish the relationship between the voltage and current at both the input and output ports of the network, providing a means to analyze its behavior.

Two-port Impedance Network



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

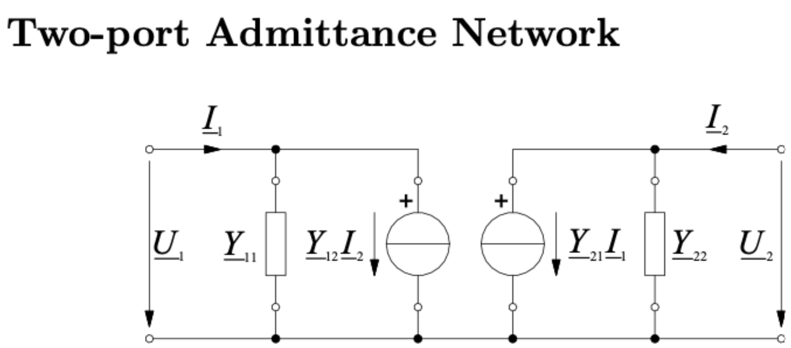
$$\begin{aligned} V1 &= Z_{11}I_1 + Z_{12}I_2 \\ V2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned} \qquad \text{or} \qquad \begin{vmatrix} V1 \\ V2 \end{vmatrix} = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} * \begin{vmatrix} I1 \\ I2 \end{vmatrix}$$

Figure 1:Two-port Impedance Network

Two-port Admittance Network

A two-port admittance network is a specific configuration within the realm of two-port networks, notable for its input and output admittance properties. In this context, the input admittance characterizes the admittance perceived at the input terminals of the network upon signal application, while the output admittance pertains to the admittance observed at the network's output terminals. It is also referred to as the inverse of the impedance.

To represent the input and output admittances of a two-port admittance network, the Y-parameter matrix is commonly employed. These parameters establish the relationship between the current and voltage at both the input and output ports of the network, facilitating the analysis of its behavior.



$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$\begin{aligned} I1 &= Y_{11}V_1 + Y_{12}V_2 \\ I2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned} \qquad \text{or} \qquad \begin{vmatrix} I1 \\ I2 \end{vmatrix} = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} * \begin{vmatrix} V1 \\ V2 \end{vmatrix}$$

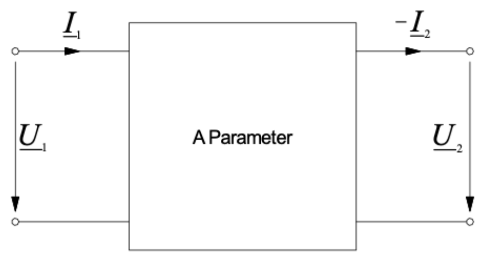
Figure 2:Two-port Admittance Network

Two-port Transmission Network

A two-port transmission network stands out among other two-port networks due to its transmission characteristics, elucidating how signals are converted from the input port to the output port. These transmission characteristics of a two-port transmission network are encapsulated by its ABCD-parameter matrix.

The ABCD-parameters establish a connection between the voltage and current at both the input and output ports of the network, offering insight into its operational behavior. They define how the network manipulates voltage and current signals, thereby enabling the determination of crucial parameters such as the reflection coefficient, transmission coefficient, and impedance transformation ratio.

Two-port Transmission ABCD-parameter Network



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

$$\begin{aligned} V1 &= A * V_2 + B * -I_2 \\ I1 &= C * V_2 + D * -I_2 \end{aligned} \qquad \text{or} \qquad \begin{vmatrix} V1 \\ I1 \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} * \begin{vmatrix} V2 \\ -I2 \end{vmatrix}$$

Figure 3:Two-port Transmission ABCD-parameter Network

Series-parallel combinations were employed to interconnect two two-port transmission networks. This involved connecting the output of network A to the input of network B, effectively linking the two networks in series. Consequently, the merged network obtained will feature two ports, and its parameters will be determined through a series of equations derived from the relationships that connect the individual networks' parameters to those of the combined network.

3. EXECUTION

Experiment Setup
Workbench No.12

Used tools and instruments:
Breadboard, Tools box from workbench, Multimeters TENMA and ELABO, Generator, Oscilloscope, BNC Cable, BNC T-connector, BNC-Banana

Experiment Part 1 – Setup
Two-port Z / Y Network

Objective
Our aim in this part of the experiment was to determine the Z and Y parameters.

Preparation 1
The two-port impedance network consisting of circuit 1 and circuit 2 circuits was constructed on a breadboard. This setup involved connecting specific wires on the breadboard and integrating resistors with values of 100, 330, and 270 ohms. The Tenma multimeter was utilized to accurately measure the voltage across these resistors.The circuit diagram below illustrates the configuration of the two-port impedance network composed of Z and Y components. Instead of directly measuring the current, which may lead to inaccuracies, the voltage was precisely measured across the relevant resistors. For instance, V1 was measured across R1 and R4. Ohm's law was then applied to determine the respective impedances. To measure other impedances, the voltage was measured across their respective resistors while setting certain currents to 0, ensuring accurate and reliable results.

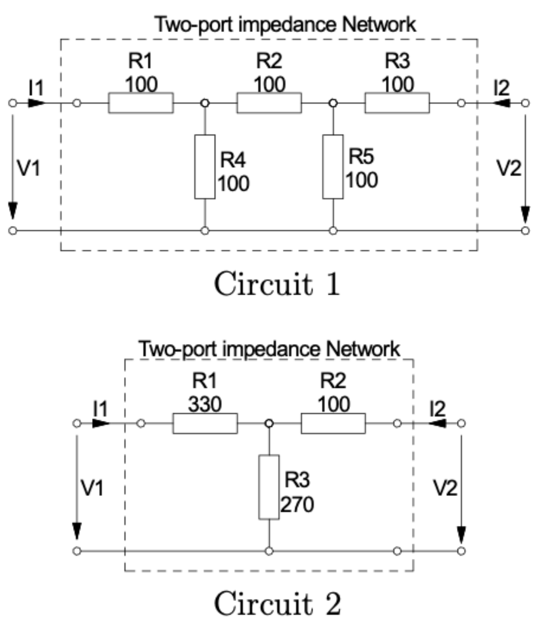


Figure 4:Two-port impedance Network

Execution and Results 1

To ensure accuracy and obtain better results, the decision was made to measure voltage rather than current. Ohm's law was then applied to calculate the respective impedances.

Measuring the Z parameters of Circuit 1 using suitable tools and methods

For Z_{11}, we just measured the resistance between $R_1 and\, R_4$ and for Z_{22}, we just measured the resistance between $R_5\ and\ R_3$, using the TENMA. For Z_{12}, we open circuit $I_1$and then measured the $V_1 and\, I_2$ and then used ohm's law.

To get I_2, we just use the measured voltage at R_3 and since we already have the resistance we go ahead and use ohms law

$I_2 = \frac{V_{R3}}{R_3}$, when $I_1$is equal to zero.

$I_2 = \frac{3.038}{100}$

$I_2 = 3.04 * 10^{-2}A$

And $V_1$is just the measured voltage between $R_1 and\, R_4$ when $I_1 = 0$

$Z_{12} = \frac{V_1}{I_2} = \frac{1.0066}{3.04*10^{-2}} = 33.13\Omega$

Circuit 1	
I1=0	
V1(V)	1.0066
I2(A)	3.04E-02

Table 1: Measured values

To get Z_{21}, we open the circuit to make $I_2 = 0$ and then measured the $V_2 and\, I_1$ and then used ohm's law.

To get I_1, we just use the measured voltage at R_1 and since we already have the resistance we go ahead and use ohms law

$I_1 = \frac{V_{R1}}{R_1}$, when $I_1$is equal to zero.

$I_1 = \frac{3.0299}{100}$

$I_1 = 3.03 * 10^{-2}A$

And $V_2$is just the measured voltage between $R_3 and\, R_5$ when $I_1 = 0$

$Z_{21} = \frac{V_2}{I_1} = \frac{1.0123}{3.03*10^{-2}} = 33.41\Omega$

I2=0	
V2(V)	1.0123
I1(A)	3.03E-02

Table 2: Measured values

All the Z parameters in circuit 1 can be summarized in table 3 below

Z11	166.55Ω
Z12	33.13Ω
Z21	33.41Ω
Z22	167.35Ω

Table 3: Z parameters in circuit 1

Measuring the Y parameters of Circuit 2 using suitable tools and methods

To get our Y_{11}, we just measure the resistance between $R_1 and\ R_3$ and then get its inverse. We do the same with Y_{22} , we just measure the resistance between $R_2 and\ R_3$ and then get its inverse

For Y_{12}, we short circuit V_1 and then measured the $V_2 and\ I_1$ and then used the inverse of ohm's law. To get I_1, we just use the measured voltage at R_1 and since we already have the resistance we go ahead and use ohms law

$V_{R1} =- 2.8859$

$I_1 = \frac{V_{R1}}{R_1}$, when V_1 is equal to zero.

$I_1 = \frac{-2.8859}{330}$

$I_1 =- 8.75 * 10^{-3} A$

And V_2 is just the measured voltage between $R_2 and\ R_3$

$V_2 = 5.058$

We then use the inverse of ohms law to get Y_{12}

$Y_{12} = \frac{I_1}{V_2} = \frac{-8.75*10^{-3}}{5.058} =- 1.73 * 10^{-3}\Omega$

For Y_{21}, we short circuit V_2 and then measured the $V_1 and\ I_2$ and then used the inverse of ohm's law. To get I_2, we just use the measured voltage at R_2 and since we already have the resistance we go ahead and use ohms law

$V_{R2} =- 0.8648$

$I_2 = \frac{V_{R2}}{R_2}$, when V_2 is equal to zero.

$I_2 = \frac{-0.8648}{100}$

$I_2 =- 8.65 * 10^{-3} A$

And V_1 is just the measured voltage between $R_1 and\ R_3$

$V_1 = 5.058$

We then use the inverse of ohms law to get Y_{12}

$$Y_{12} = \frac{I_1}{V_2} = \frac{-8.65*10^{-3}}{5.058} =- 1.71 * 10^{-3}\Omega$$

All the Y parameters in circuit 2 can be summarized in table 4 below

Y11	2.49E-03Ω
Y12	-1.73E-03Ω
Y21	-1.71E-03Ω
Y22	4.31E-03Ω

Table 4: Z parameters in circuit 2

We later connected Circuit 1 and Circuit 2 to a voltage supply with V1=5V and used a load resistor of 1KΩ at the outputs and measured V1, V2, I1, I2 for both circuits.

Circuit 1

To get V1, we just measured the voltage at R1 and R4

To get I1, we measured the voltage at R1 and since we already had the value of R1, we then used ohms

$$I_1 = \frac{V_{R1}}{R_1}$$

$$I_1 = \frac{3.0578}{100} = 0.030578A$$

To get V2, we just measured the voltage at RL

To get I2, we just used ohms law on the RL values, keeping in mind that it was flowing in the opposite direction

$$I_2 = \frac{V_{RL}}{RL} = \frac{-0.8722}{1000} =- 0.000879A$$

The voltage and current values of circuit 1 when the input is 5V and the output connected to 1kΩ, can be summarized in the table below

V1(V)	5.055
V2(V)	0.8722
I1(A)	0.030578
I2(A)	-0.000879

Table 6: current and voltage values of circuit 1

Circuit 2

To get V1, we just measured the voltage at R1 and R3

To get I1, we measured the voltage at R1 and since we already had the value of R1, we then used ohms

$$I_1 = \frac{V_{R1}}{R_1}$$

$I_1 = \frac{3.264}{330} = 9.89 * 10^{-3}A$

To get V2, we just measured the voltage at RL

To get I2, we just used ohms law on the RL values, keeping in mind that it was flowing in the opposite direction

$I_2 = \frac{V_{RL}}{RL} = \frac{-1.6342}{1000} =- 1.63 * 10^{-3}A$

The voltage and current values of circuit 2 when the input is 5V and the output connected to 1kΩ, can be summarized in the table below

V1(V)	5.059
V2(V)	1.6342
I1(A)	9.89E-03
I2(A)	-1.63E-03

Table 7: current and voltage values of circuit 2

Experiment Part 2 – Setup
Interconnection of Two-port Network

Objective

Our objective in this part of the experiment was to demonstrate the parallel connection of two two-port networks.

Preparation 2

Using the same circuits 1 and 2 from our previous experiment, we now connected them in parallel as shown in figure 5

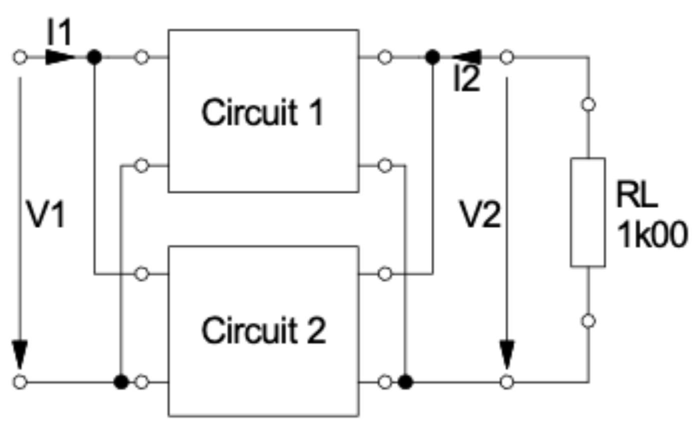


Figure 5: circuits 1 and 2 connected in parallel

Execution and Results 2

To ensure accuracy and obtain better results, the decision was made to measure voltage rather than current. Ohm's law was then applied to calculate the respective impedances.

Measuring the Z parameters of the resulting two-port circuit using suitable tools and methods

To get Z_{11}, we measured the resistance at R1(circuit 1) and R3(circuit 2) and to get Z_{22}, we measured the resistance at R3(circuit 1) and R3(circuit 2)

For Z_{12}, we open circuit I_1 and then measured the V_1 and I_2 and then used ohm's law.

To get I_2, we can clearly see that we have a current divider so we first calculate I_2 for circuit 1 and the I_2 for circuit 2 and then add the two currents

For circuit 1

$$I_{2.1} = \frac{V_{R3}}{R_3} = \frac{2.9491}{100} = 2.95 * 10^{-2} A$$

For circuit 2

$$I_{2.2} = \frac{V_{R2}}{R_2} = \frac{1.8792}{100} = 1.88 * 10^{-2} A$$

$$I_2 = I_{2.1} + I_{2.2} = 2.95 * 10^{-2} + 1.88 * 10^{-2} = 4.83 * 10^{-2} A$$

$V_1 = 1.7194V$, measured voltage at $R_1(circuit\ 1) and\ R_3(circuit\ 2)$ when $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2} = \frac{1.7194}{4.83*10^{-2}} = 35.61\Omega$$

For Z_{21}, we open circuit I_2 and then measured the V_2 and I_1 and then used ohm's law.

To get I_1, we can clearly see that we have a current divider so we first calculate I_1 for circuit 1 and the I_1 for circuit 2 and then add the two currents

For circuit 1

$$I_{1.1} = \frac{V_{R1}}{R_1} = \frac{2.9912}{100} = 2.99 * 10^{-2} A$$

For circuit 2

$$I_{1.2} = \frac{V_{R1}}{R_1} = \frac{3.4464}{330} = 1.04 * 10^{-2} A$$

$$I_1 = I_{1.1} + I_{1.2} = 2.99 * 10^{-2} + 1.04 * 10^{-2} = 4.04 * 10^{-2} A$$

$V_2 = 1.4211V$, measured voltage at $R_3(circuit\ 1) and\ R_3(circuit\ 2)$ when $I_2 = 0$

$$Z_{21} = \frac{V_2}{I_1} = \frac{1.4211}{4.99*10^{-2}} = 35.21\Omega$$

All the Z parameters in figure 5 can be summarized in table 8 below

Z11	125.03
Z12	35.61
Z21	35.21
Z22	94.95

Table 8:Z parameters of circuits 1 and 2 connected in parallel

We then connected the circuit in figure 5 to a voltage supply with $V_1 = 5$ V. Used a load of $R_L = 1k\Omega$ at the output, and then Measured V_1, V_2, I_1, I_2.
To get V1, we just measured the voltage between R1(circuit 1) and R3 (circuit 2) and I1, we measured the voltage at R1(circuit 1) and R3 (circuit 2) and then used ohms law to get the currents and later got sum of the two calculated currents.

For circuit 1

$$I_{1.1} = \frac{V_{R1}}{R_1} = \frac{2.9912}{100} = 2.99 * 10^{-2} A$$

For circuit 2

$$I_{1.2} = \frac{V_{R1}}{R_1} = \frac{3.4464}{330} = 1.05 * 10^{-2} A$$

$$I_1 = I_{1.1} + I_{1.2} = 2.99 * 10^{-2} + 1.05 * 10^{-2} = 4.04 * 10^{-2} A$$

And to get V2, we measured the voltage at RL and for I2 we used the ohms law on the RL parameters, keeping in mind that the current was flowing in the opposite direction

$$I_2 = \frac{-V_{RL}}{RL} = \frac{-1.2958}{1000} = 1.30 * 10^{-3} A$$

V1	5.052V
V2	1.2958V
I1	4.04E-02A
I2	-1.30E-03A

Table 9: measured voltages and currents at the input and output when the input is connected to 5V and RL=1KΩ at the output

Experiment Part 3 – Setup

Complex Two-port Networks / Cascading

Objective

Our aim in this part of the experiment was to demonstrate cascading of complex two-port networks.

Preparation 3

We built the circuits in figure 6 below on the breadboard and then cascaded both circuits as shown in figure 7. We then connected the resulting Two-port to a voltage supply with $\widehat{v}_s = 5V_{pp} @ 1000Hz$ at the generator. The 100Ω resistor was used to determine I1.

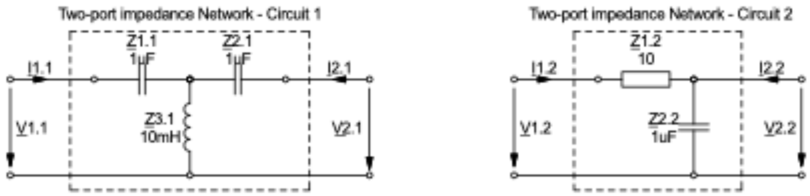


Figure 7: The two port impedance networks of circuit 1 and 2 before they were cascaded.

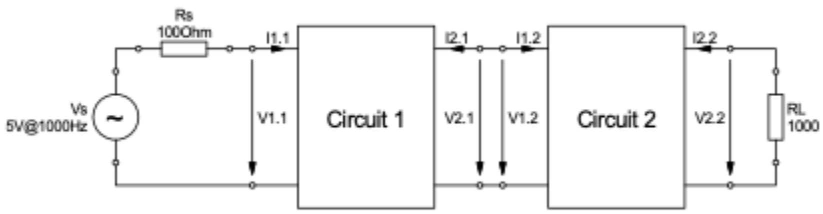


Figure 8:The two port impedance networks of circuit 1 and 2 after they were cascaded.

Execution and Results 3

We measured $\hat{v}_s$, $\hat{v}_{1.1}$, $\hat{i}_{1.1}$ at circuit 1, and $\hat{v}_{2.2}$, $\hat{i}_{2.2}$ at circuit 2.

Keeping in mind that $\hat{v}_{1.1}$ is the reference for the measurement and also paying attention to the polarity of $\hat{i}_{2.2}$.

Circuit 1		
V^s(V)	*7.36*	
Vs(V)	*2.59*	*46.8°*
V^1.1(V)	*4*	
V1.1(V)	*1.41*	*0°*
I^1.1(A)	*0.0336*	*46.8°*

write these values in phasor form...

Table 10: measured currents and voltages with phases of circuit 1

Circuit 2		
V^2.2(V)	*2.28*	
V2.2(mV)	*7.97E-01*	*163°*
I^2.2(A)	*-0.00228*	*163°*

Table 11: measured currents and voltages with phases of circuit 2

Impedance of all components used

Name	Impedance(Ω)
$Z_{1.1}$	0.586-152.03j
$Z_{1.2}$	10+0j
$Z_{2.1}$	0.586-152.03j
$Z_{2.2}$	0.586-152.03j
$Z_{3.1}$	4.4913+64.262j

Table 12: measured Impedance for all components used

4. EVALUATION

Part 1: Two-port Z / Y Network

Calculating the Z parameters of circuit 1 and 2 from the given resistor values, we do the following

For circuit 1:

Calculating for $Z_{11}, I_2 = 0$

We first calculate the equivalent resistance when I2=0**,** keeping in mind that R3 is inactive because there is no current flowing through it.

$$Rs = ((R2 + R5)||R4) + R1$$

$$Rs = \frac{(100+100)(100)}{100+100+100} + 100$$

$$Rs = 166.67\Omega$$

$$V_1 = RsI_1 = 166.67I_1$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{166.67I_1}{I_1} = 166.67\Omega$$

Calculating for $Z_{12}, I_1 = 0$

$$V1 = \left[\frac{R_4 \times R_5}{R_2+R_4+R_5}\right] \times I_2 = \frac{100\times100}{100+100+100}I_2 = 33.333I_2$$

$$Z_{12} = \frac{V1}{I_2} = \frac{33.333I_2}{I_2} = 33.33\Omega$$

Calculating for $Z_{21}, I_2 = 0$

$$V2 = \left[\frac{R_4 \times R_5}{R_2+R_4+R_5}\right] \times I_1 = \frac{100\times100}{100+100+100}I_1 = 33.333I_1$$

$Z_{21} = \frac{V2}{I_1} = \frac{33.333I_2}{I_1} = 33.33\Omega$

Calculating for $Z_{22}, I_1 = 0$

$V2 = (((R_2 + R_4)||R_5) + R_3)I_2 = \left[(\frac{200\times100}{200+100}) + 100\right]I_2 = 166.67I_2$

$Z_{22} = \frac{V_2}{I_2} = \frac{166.67I_2}{I_2} = 166.67\Omega$

All calculated Z parameters for circuit 1 can be summarized in the table below

Z Parameters	Value(Ω)
Z_{11}	166.67
Z_{12}	33.33
Z_{21}	33.33
Z_{22}	166.7

Table 13: Calculated Z parameters for circuit 1

For circuit 2

Calculating for $Z_{11}, I_2 = 0$

$V1 = (R_1 + R_3)I_1 = (330 + 270)I_1 = 600I_1$

$Z_{11} = \frac{V_1}{I_1} = \frac{600I_1}{I_1} = 600\Omega$

Calculating for $Z_{12}, I_1 = 0$

$Z_{12} = R_3 = 270\Omega$

Calculating for $Z_{21}, I_2 = 0$

$Z_{21} = R_3 = 270\Omega$

Calculating for $Z_{22}, I_1 = 0$

$V2 = (R_2 + R_3)I_2 = (100 + 270)I_2 = 370I_2$

$Z_{22} = \frac{V2}{I_2} = \frac{370I_2}{I_2} = 370\Omega$

All calculated Z parameters for circuit 2 can be summarized in the table 14 below

Z Parameters	Value(Ω)
Z_{11}	600
Z_{12}	270
Z_{21}	270
Z_{22}	370

Table 14: Calculated Z parameters for circuit 2

To calculate the Y parameters for circuit 1 and 2 using the given resistor values, we just find the inverse of the already calculated Z parameters in matrix form, since it has already been described in the theory part that Y is the inverse of Z.

For circuit 1:

$$Y = Z^{-1}$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$Z = \begin{bmatrix} 166.67 & 33.33 \\ 33.33 & 166.67 \end{bmatrix}$$

$$Z^{-1} = \frac{1}{(166.67\times166.67)-(33.33\times33.33)} \begin{bmatrix} 166.67 & -33.33 \\ -33.33 & 166.67 \end{bmatrix}$$

$$Z^{-1} = \frac{1}{26668} \begin{bmatrix} 166.67 & -33.33 \\ -33.33 & 166.67 \end{bmatrix}$$

$$Z^{-1} = \begin{bmatrix} 6.25\times10^{-3} & -1.25\times10^{-3} \\ -1.25\times10^{-3} & 6.25\times10^{-3} \end{bmatrix}$$

All calculated Y parameters for circuit 1 can be summarized in the table 15 below

Y parameters	Values(Ω)
Y_{11}	6.25×10^{-3}
Y_{12}	-1.25×10^{-3}
Y_{21}	-1.25×10^{-3}
Y_{22}	6.25×10^{-3}

Table 15: Calculated Y parameters for circuit 1

For circuit 2:

$$Y = Z^{-1}$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$Z = \begin{bmatrix} 600 & 270 \\ 270 & 370 \end{bmatrix}$$

$$Z^{-1} = \frac{1}{(600\times370)-(270\times270)}\begin{bmatrix} 370 & -270 \\ -270 & 600 \end{bmatrix}$$

$$Z^{-1} = \frac{1}{149100}\begin{bmatrix} 370 & -270 \\ -270 & 600 \end{bmatrix}$$

$$Z^{-1} = \begin{bmatrix} 2.48\times10^{-3} & -1.81\times10^{-3} \\ -1.81\times10^{-3} & 4.024\times10^{-3} \end{bmatrix}$$

All calculated Y parameters for circuit 2 can be summarized in the table 15 below

Y parameters	Values(Ω)
Y_{11}	2.48×10^{-3}
Y_{12}	-1.81$\times 10^{-3}$
Y_{21}	-1.81$\times 10^{-3}$
Y_{22}	4.024×10^{-3}

Table 16: Calculated Y parameters for circuit 2

Comparing the measured and calculated Z and Y parameters:

For circuit 1

Z parameters	Measured(Ω)	Calculated (Ω)
Z_{11}	166.67	166.55
Z_{12}	33.33	33.13
Z_{21}	33.33	33.41
Z_{22}	166.7	167.35

Table 17:measured and calculated Z parameters for circuit 1

For circuit 2

Y parameters	Measured(Ω)	Calculated (Ω)
Y_{11}	2.49×10^{-3}	2.48×10^{-3}
Y_{12}	-1.73×10^{-3}	-1.81×10^{-3}
Y_{21}	-1.71×10^{-3}	-1.81×10^{-3}
Y_{22}	4.31×10^{-3}	4.024×10^{-3}

Table 18:measured and calculated Y parameters for circuit 2

The values above show that the measured and calculated values are almost identical, despite some substantial differences in some pairs of values. These variations are expected and caused by instrumental mistakes when data was being measured. Some characteristics and all resistances were measured using the Tenma Multimeter in Ohm Range, which has better accuracy, but could still cause variations in the data.The voltage levels from the experiment were collected using the Elabo Multimeter was used to collect the voltage levels hence its precision impacted the findings. Due to truncation and error propagation, mathematical mistakes are unavoidable for calculated values and components throughout both the execution and evaluation processes.

Using the measured Z parameters from circuit 1 to verify the measured V1 and V2:

$$\begin{vmatrix} V1 \\ V2 \end{vmatrix} = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} * \begin{vmatrix} I1 \\ I2 \end{vmatrix}$$

$$\begin{bmatrix} V1 \\ V2 \end{bmatrix} = \begin{bmatrix} 166.55 & 33.13 \\ 33.41 & 167.35 \end{bmatrix} * \begin{bmatrix} 3.0304\times10^{-2} \\ -8.7100\times10^{-4} \end{bmatrix}$$

$$\begin{bmatrix} V1 \\ V2 \end{bmatrix} = \begin{bmatrix} 5.022 \\ 0.865 \end{bmatrix} V$$

Using the measured Y parameters from circuit 2 to verify the measured I1 and I2:

$$\begin{vmatrix} I1 \\ I2 \end{vmatrix} = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} * \begin{vmatrix} V1 \\ V2 \end{vmatrix}$$

$$\begin{bmatrix} I1 \\ I2 \end{bmatrix} = \begin{bmatrix} 2.49\times10^{-3} & -1.71\times10^{-3} \\ -1.73\times10^{-3} & 4.31\times10^{-3} \end{bmatrix} * \begin{bmatrix} 5.034 \\ 1.8013 \end{bmatrix}$$

$$\begin{bmatrix} I1 \\ I2 \end{bmatrix} = \begin{bmatrix} 9.45\times10^{-3} \\ -9.452\times10^{-4} \end{bmatrix} A$$

We can verify that the measured values of V1,V2,I1 and I2 and the calculated values are very close to being the same!

Part 2:Interconnection of Two-port Networks

Calculating the Z parameters of the parallel connected circuits by combining the measured Z and Y parameters from part 1 (8.3.3, Problem 1. and 2.) of the experiment.

Since we already calculated the Y parameters of both circuits in part 1 of the evaluation, we can just go ahead and find the sum and then find the inverse to make it Z

$Y = Y_1 + Y_2$, keeping in mind that Y_1 is the already calculated inverse of Z in circuit 1 and Y_2 is the already calculated Y in circuit 2.

$$Y = Y_1 + Y_2$$

$$Y = \begin{bmatrix} 6.25\times10^{-3} & -1.25\times10^{-3} \\ -1.25\times10^{-3} & 6.25\times10^{-3} \end{bmatrix} + \begin{bmatrix} 2.48\times10^{-3} & -1.81\times10^{-3} \\ -1.81\times10^{-3} & 4.024\times10^{-3} \end{bmatrix}$$

$$Y = \begin{bmatrix} 8.73\times10^{-3} & -3.06\times10^{-3} \\ -3.06\times10^{-3} & 10.27\times10^{-3} \end{bmatrix}$$,we then find the inverse to get Z

$$Y^{-1} = Z$$

$$Y^{-1} = \frac{1}{(8.73\times10^{-3}*10.27\times10^{-3})-(-3.06\times10^{-3}*-3.06\times10^{-3})}\begin{bmatrix} 10.27\times10^{-3} & 3.06\times10^{-3} \\ 3.06\times10^{-3} & 8.73\times10^{-3} \end{bmatrix}$$

$$Y^{-1} = \begin{bmatrix} 127.91 & 38.11 \\ 38.11 & 108.73 \end{bmatrix}$$

The comparison of the measured and calculated Z parameters of the interconnected Two-port Network can be summarized in the table below

Z parameters	Measured(Ω)	Calculated (Ω)
Z_{11}	125.03	127.91
Z_{12}	35.61	38.11
Z_{21}	35.21	38.11
Z_{22}	94.95	108,73

Table 19:measured and calculated Z parameters for the parallel interconnected *Two-port Network*

The calculated values almost match the measured values. The variations were caused by the instrumental accuracy of the Elabo and Tenma multimeters. Most of the numbers were derived using Ohm's Law, which might have caused inaccuracies in computation.When calculating the derived combined Z parameters from the individual values, error propagation and mathematical mistakes were unavoidable.

Using the measured Z or Y parameters to verify the measured V_1, V_2 or I_1, I_2 values from 8.4.3, Problem 3.

Using the measured Z parameters to verify the measured V1 and V2:

$$\begin{vmatrix} V1 \\ V2 \end{vmatrix} = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} * \begin{vmatrix} I1 \\ I2 \end{vmatrix}$$

$$\begin{bmatrix} V1 \\ V2 \end{bmatrix} = \begin{bmatrix} 127.91 & 38.11 \\ 38.11 & 108.73 \end{bmatrix} * \begin{bmatrix} 0.03982 \\ -0.001377 \end{bmatrix}$$

$$\begin{bmatrix} V1 \\ V2 \end{bmatrix} = \begin{bmatrix} 5.0409 \\ 1.3678 \end{bmatrix} V$$

Using the measured Y parameters to verify the measured I1 and I2:

$$\begin{vmatrix} I1 \\ I2 \end{vmatrix} = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} * \begin{vmatrix} V1 \\ V2 \end{vmatrix}$$

$$\begin{bmatrix} I1 \\ I2 \end{bmatrix} = \begin{bmatrix} 8.73\times10^{-3} & -3.06\times10^{-3} \\ -3.06\times10^{-3} & 10.27\times10^{-3} \end{bmatrix} * \begin{bmatrix} 5.038 \\ 1.3644 \end{bmatrix}$$

$$\begin{bmatrix} I1 \\ I2 \end{bmatrix} = \begin{bmatrix} 0.0398 \\ -1.4039\times10^{-3} \end{bmatrix} A$$

We can verify that the measured values of V1,V2,I1 and I2 and the calculated values are very close to being the same!

For a series connection of two-ports, is it always possible to determine z by the combination of two two-ports? Yes, for a series connection of two two-port networks, it is always possible to determine the impedance (Z) of the combination by analyzing the individual two-port networks.When two two-port networks are connected in series, the output of the first network is connected to the input of the second network, forming a series connection. In this configuration, you can determine the overall impedance of the series connection by adding the individual impedance matrices of the two two-port networks.If the individual two-port networks are described by their impedance parameters, you can use the sum of the impedance matrices to find the impedance of the series connection. This principle holds for any linear passive network. Here's a general formula for finding the impedance matrix of a series connection of two two-port networks:

$Z_{series} = Z_1 + Z_2$

Where:

- Z_{series} is the impedance matrix of the series connection.

- Z_1 is the impedance matrix of the first two-port network.

- Z_2 is the impedance matrix of the second two-port network.

Is this possible in our case? No, because a series connection between the two networks can possibly change the individual circuit, a short circuit is created and no current will pass through the two resistors at the wire of R4 and R5 in circuit 1

Part 3 : Complex Two-port Networks / Cascading

Determining the Z parameters of circuit 1

Calculating for $Z_{11}, I_2 = 0$

$V_1 = (C_1 + L)I_1 = ((0.586 - j152.03) + (4.4913 + j64.262))I_1 = (5.0773 - j87.768)I_1$

$Z_{11} = \frac{V_1}{I_1} = \frac{(5.0773 - j87.768)I_1}{I_1} = (5.0773 - j87.768)\Omega$

$Z_{11} = 87.915\angle - 86.69^o$

Calculating for $Z_{12}, I_1 = 0$

$V_1 = I_2 \times L$

$Z_{12} = \frac{V_!}{I_2} = \frac{I_2 \times L}{I_2} = L = (4.4913 + j64.262)\Omega$

$Z_{12} = 64.42\angle 86^o$

Calculating for $Z_{21}, I_2 = 0$

$V_2 = I_1 \times L$

$Z_{21} = \frac{V_2}{I_1} = \frac{I_1 \times L}{I_1} = L = (4.4913 + j64.262)\Omega$

$Z_{21} = 64.42\angle 86^o$

Calculating for $Z_{22}, I_1 = 0$

$V_2 = (C_2 + L)I_2 = ((0.586 - j152.03) + (4.4913 + j64.262))I_2 = (5.0773 - j87.768)I_2$

$Z_{22} = \frac{V_2}{I_2} = \frac{(5.0773 - j87.768)I_2}{I_2} = (5.0773 - j87.768)\Omega$

$Z_{22} = 87.915\angle - 86.69^o$

Determining the Z parameters of circuit 2

Calculating for $Z_{11}, I_2 = 0$

$V_1 = (C \ + R)I_1 = ((0.586 - j152.03) + (10 + j0))I_1 = (10.586 - j152.03)I_1$

$Z_{11} = \frac{V_1}{I_1} = \frac{(10.586 - j152.03)I_1}{I_1} = (10.586 - j152.03)\Omega$

$Z_{11} = 152.40\angle - 86.02^o$

Calculating for $Z_{12}, I_1 = 0$

$V_1 = I_2 \times C$

$Z_{12} = \frac{V_!}{I_2} = \frac{I_2 \times C}{I_2} = C = (0.586 - j152.03)\Omega$

$Z_{12} = 152.03\angle - 89.78^o$

Calculating for $Z_{21}, I_2 = 0$

$V_2 = I_1 \times C$

$Z_{21} = \frac{V_2}{I_1} = \frac{I_1 \times C}{I_1} = (0.586 - j152.03)\Omega$

$Z_{21} = 152.03\angle - 89.78^o$

Calculating for $Z_{22}, I_1 = 0$

$V_2 = I_2 \times C$

$Z_{22} = \frac{V_2}{I_2} = \frac{I_2 \times C}{I_2} = C = (0.586 - j152.03)\Omega$

$Z_{22} = 152.03\angle - 89.78^o$

Below are two matrices of the Z parameters calculated for circuits 1 and 2 in Phasor form

$$Z_1 = \begin{bmatrix} 87.915\angle{-86.69}^o & 64.42\angle 86^o \\ 64.42\angle 86^o & 87.915\angle{-86.69}^o \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} 152.40\angle{-86.02}^o & 152.03\angle{-89.78}^o \\ 152.03\angle{-89.78}^o & 152.03\angle{-89.78}^o \end{bmatrix}$$

Using the determined Z parameters to calculate the resulting cascaded ABCD parameters

$V_1 = Z_{11}I_1 + Z_{12}I_2$.........(i)

$V_2 = Z_{21}I_1 + Z_{22}I_2$.........(ii)

$V_1 = AV_2 - BI_2$...........(iii)

$I_1 = CV_2 - DI_2$...........(iv)

With the provided equations we get the i and iii and equate them to each other because they are equal and then manipulate ii and iv. Doing all this helps us make each required letter the subject of the formula. Following the mentioned steps gives us the equations below

$A = \frac{Z_{11}}{Z_{21}}$...........(1)

$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$......(2)

$C = \frac{1}{Z_{21}}$.........(3)

$D = \frac{Z_{22}}{Z_{21}}$...........(4)

Calculating ABC and D for circuit 1

$A = \frac{Z_{11}}{Z_{21}}$

$A = \frac{87.915\angle -86.69^o}{64.42\angle 86^o}$

$A = 1.36\angle - 172.69^o$

$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$

$B = \frac{(87.915\angle -86.69^o)(87.915\angle -86.69^o) - (64.42\angle 86^o)(64.42\angle 86^o)}{64.42\angle 86^o}$

this values seems to be wrong / recalc...

$B = 59.89\angle - 63.63^o$

$C = \frac{1}{Z_{21}}$

$C = \frac{1}{64.42\angle 86^o}$

$C = 0.0155\angle - 86^o$

$D = \frac{Z_{22}}{Z_{21}}$

$D = \frac{87.915\angle -86.69^o}{64.42\angle 86^o}$

$D = 1.365\angle - 172.69^o$

Calculating ABC and D for circuit 2

$A = \frac{Z_{11}}{Z_{21}}$

$A = \frac{152.40\angle -86.02^o}{152.03\angle -89.78^o}$

$A = 1.0024\angle 3.76^o$

$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$

$B = \frac{(152.40\angle -86.02^o)(152.03\angle -89.78^o) - (152.03\angle -89.78^o)(152.03\angle -89.78^o)}{152.03\angle -89.78^o}$

$B = 10\angle 0^o$

$C = \frac{1}{Z_{21}}$

$C = \frac{1}{152.03\angle -89.78^o}$

$C = 6.58 \times 10^{-3}\angle 89.78^o$

$D = \frac{Z_{22}}{Z_{21}}$

$$D = \frac{152.03\angle -89.78^o}{152.03\angle -89.78^o}$$

$$D = 1\angle 0^o$$

Using the calculated ABCD values calculated from circuit 1 and 2, we can now find the resulting cascaded ABCD parameters by simply multiplying the ABCD matrices 1 and 2.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \times \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.36\angle -172.69^o & \cancel{59.89\angle -63.63^o} \\ 0.0155\angle -86^o & 1.365\angle -172.69^o \end{bmatrix} \times \begin{bmatrix} 1.0024\angle 3.76^o & 10\angle 0^o \\ 6.58\times 10^{-3}\angle 89.78^o & 1\angle 0^o \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.988\angle -174.89^o & 56.92\angle -76.68^o \\ 0.025\angle -82.49^o & 1.38\angle -166.26^o \end{bmatrix}$$

Verifying the measured V1,I2 using the determined ABCD and the measured V2I2 using the formula below

$$\begin{vmatrix} V1 \\ I1 \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \times \begin{vmatrix} V2 \\ -I2 \end{vmatrix}$$

$$\begin{vmatrix} V1 \\ I1 \end{vmatrix} = \begin{vmatrix} 0.988\angle -174.89^o & 56.92\angle -76.68^o \\ 0.025\angle -82.49^o & 1.38\angle -166.26^o \end{vmatrix} \times \begin{vmatrix} 2.28\angle 0^o \\ -0.00228\angle 163^o \end{vmatrix}$$

$$\begin{bmatrix} V1 \\ I1 \end{bmatrix} = \begin{bmatrix} 2.236\angle -178.18^o \\ 0.058\angle -79.42^o \end{bmatrix}\begin{bmatrix} V \\ A \end{bmatrix}$$

ok, because of B from circuit 1 the calculation is wrong...

rest is fine!!!

Comparing the calculated values to the measurements we see that there is a large difference between the calculated values and the measured values of the matrix found. Most of the errors were due to the errors of rounding off values. Lastly some errors may have occurred during the conversion of the results from complex form to phasor form.

5. CONCLUSION

In summary, it's evident that impedance, admittance, and transmission parameters streamline the analysis of DC and AC two-port networks, whether individual or interconnected. The correlation among different factors has been demonstrated and confirmed, facilitating seamless circuit analysis. Nevertheless, errors often arise during data processing, particularly with measured data due to instrument inaccuracies. In the case of calculated values, mathematical errors can lead to minor or significant discrepancies. These errors become apparent when verifying voltages and currents using predicted two-port parameters. The estimated matrices don't perfectly align with the experimental data.

6. REFERENCE

1. Pagel Uwe, General Electrical Engineering II Lab Manual (2024). Constructor University
2. https://circuitbee.com/complex-number-calculator

7.APPENDIX

Experiment 3:Wheatstone Bridge

Part 1		
R1	22.03	Elabo range: 20
R3(kΩ)	2.201	Elabo range: 20
R4	8.219	Elabo range: 200
Vab(mV)	0	
R from Decade(kΩ)	81.89	

R from Elabo(kΩ)	82.23	Elabo range: 200

Part 2		
Vs(V)	1.017	Elabo range: 20
Vout(mV)	-0.01989	
Vs(V)	10.011	Elabo range: 20
Vout(mV)	-0.24288	

Part 3		
Z4(kΩ)	0.8179	Elabo range: 2
Z1(kΩ)	0.5594	Elabo range: 2
Z3		
R	39.869	
X	64.317	

Theoretical		
Radj(Ω)	3197.199	
Cadj(H)	-3.09E-08	
Vab(V)	2.8	Elabo range: 0.2
Radj(Ω)	3197	
Cadj(nF)	3.10E+01	
Z2		
R	3.2064k	