LFM推导

	item 1	item 2	item 3	item 4			class 1	class 2	class 3			item 1	item 2	item 3	item 4	
user 1	R11	R12	R13	R14		user 1	P11	P12	P13		class 1	Q11	Q12	Q13	Q14	
user 2	R21	R22	R23	R24	_	user 2	P21	P22	P23	X	class 2	Q21	Q22	Q23	Q24	
user 3	R31	R32	R33	R34		user 3	P31	P32	P33		class 3	Q31	Q32	Q33	Q34	
		R			,	P					Q					

$$L = \sum_{u=1}^{m} \sum_{i=1}^{n} (P_u^T Q_i - R_{ui})^2 + \sum_{u=1}^{m} \lambda |P_u|^2 + \sum_{i=1}^{n} \lambda |Q_i|^2$$

因为我们认为用户之间是相互独立的,也就是说可以求出P1的增量, P2的增量

再把这些Pu给拼起来,成为一个矩阵P。所以有

$$L_u = \sum_{i=1}^n \left(P_u^T Q_i - R_{ui}
ight)^2 + \lambda |P_u|^2 + \sum_{i=1}^n \lambda |Q_i|^2.$$

两个未知数怎么办? P 和O

思路: 先固定Q, 求P; 再反过来, 固定P, 求Q;

循环反复, 交替进行, 直到达到要求或者一定批次

想最小化损失函数? 求导数、负梯度方向走

$$P_u = P_u - \alpha \frac{\partial L_u}{\partial P_u}$$

$$egin{aligned} rac{\partial L_u}{\partial P_u} \ &= \sum_{i=1}^n 2(P_u^T Q_i - R_{ui})Q_i + 2\lambda P_u \ &= 2 \left[\sum_{i=1}^n (P_u^T Q_i - R_{ui})Q_i + \lambda P_u
ight] \ &= 2 \left[(P_u^T Q_1 - R_{u1}) \left[egin{aligned} Q_{11} \ Q_{12} \ \dots \ Q_{1K} \end{aligned}
ight] + (P_u^T Q_2 - R_{u2}) \left[egin{aligned} Q_{21} \ Q_{22} \ \dots \ Q_{2K} \end{aligned}
ight] + \dots + (P_u^T Q_n - R_{un}) \left[egin{aligned} Q_{n1} \ Q_{n2} \ \dots \ Q_{nK} \end{aligned}
ight] + \lambda \left[egin{aligned} P_{11} \ P_{12} \ \dots \ P_{1K} \end{aligned}
ight] \end{aligned}$$

所以,有

$$P_{u} = P_{u} - \alpha \frac{\partial L_{u}}{\partial P_{u}}$$

$$\begin{bmatrix} P_{11} \\ P_{12} \\ \dots \\ P_{1K} \end{bmatrix} - \alpha * 2 \begin{bmatrix} (P_{u}^{T}Q_{1} - R_{u1}) \begin{bmatrix} Q_{11} \\ Q_{12} \\ \dots \\ Q_{1K} \end{bmatrix} + (P_{u}^{T}Q_{2} - R_{u2}) \begin{bmatrix} Q_{21} \\ Q_{22} \\ \dots \\ Q_{2K} \end{bmatrix} + \dots + (P_{u}^{T}Q_{n} - R_{un}) \begin{bmatrix} Q_{n1} \\ Q_{n2} \\ \dots \\ Q_{nK} \end{bmatrix} + \lambda \begin{bmatrix} P_{11} \\ P_{12} \\ \dots \\ P_{1K} \end{bmatrix} \end{bmatrix}$$

所以

$$P_{uk} = P_{uk} - \alpha * \sum_{i=1}^{n} (P_u^T Q_i - R_{ui}) Q_{ik}$$

推出这条公式,我们就去写代码啦