1) $2a_{n-1} + 2^n = 2(n - 1) \times 2^{n-1} + 2^n = n2^n = a_n$

2) **A)** Let a_n be the number of ways to climb n stairs. For someone to climb n stairs, they can take one step at a time and then climb n-1 stairs, they can take two steps at a time and then climb n-2 stairs, or they can take three steps at a time and then climb n-3 stairs.
From this, we can create a recurrence relation for all $n \leq 3 : a_n = a_{n-1} + a_{n-2} + a_{n-3}$

B) Because there is only one way to do nothing (ie. climb no stairs) and there is only one way to climb one stair, the initial conditions are $a_0 = 1$ and $a_1 = 1.$

C) To calculate how many ways a person can climb a flight of eight stairs, we can find the value of each term, which is the sum of the previous two terms.

$a_0 = 1,\ a_1 = 1,\ \therefore\ a_2 = 2$

$a_3 = a_1 + a_2 = 1 + 2 = 3$

$a_4 = a_2 + a_3 = 5$

$a_5 = a_3 + a_4 = 8$

$a_6 = a_4 + a_5 = 13$

$a_7 = a_5 + a_6 = 21$

$a_8 = a_6 + a_7 = 34$

Answer: 34 ways to climb a flight of eight stairs.

3) $a_n = 8a_{n-1} - 16a_{n-2}$

↳ $a_n - 8a_{n-1} + 16a_{n-2} = 0$

↳ $a_2 - 8a_{2-1} + 16a_{2-2} = 0$

↳ $a_2 - 8a + 16 = 0$

↳ $(a - 4)_2 = 0$

$\therefore\ a = 4$

$a_n = 8a_{n-1} - 16a_{n-2}$
$\therefore a_n = A4^n + Bn4^n$

Option C must be the solution, where $a_n = n4^n$.
↳ $a_{n-1} = (n - 1) \times 4^{n-1}$ and $a_{n-2} = (n - 2) \times 4^{n-2}$
$\therefore a_n = n \times 4^n$
$= 16n \times 4^{n-2}$
$= 4^{n-2} \times (32n - 16n)$
$= 4^{n-2} \times (32(n-1) - 16(n-2))$
$= 8 \times 4(n-1) \times 4^{n-2} - 16 \times (n-2) \times 4^{n-2}$
$= 8a_{n-1} - 16a_{n-2}$

4) A) $a_n = a_{n-1} + n, a_0 = 1$
$a_n = a_{n-1} + n$
$= (a_{n-2} + n - 1) + n = a_{n-2} + 2n - 1$
$= (a_{n-3} + n - 2) + 2n - 1 = a_{n-3} + 3n - (1 + 2)$
...
$= a_{n-n} + n^2 - (1 + 2 + 3 + ... + (n-1)$
$= a_0 + n^2 - (n - 1)(1 + n - 2) \div 2$
$= 1 + n^2 - \frac{n^2 - n}{2}$
$= 1 + \frac{n^2 + n}{2}$

B) $a_n = na_{n-1}, a_0 = 5$
$a_n = na_{n-1}$
$= n((n - 1) \times a_{n-2})$
$= n(n - 1)(n - 2)a_{n-3}$
...
$= n(n - 1)(n - 2) \times ... \times a_{n-n}$
$= n! \times a_0$
$= n! \times 5$

5) A) Let a_n represent the number of bees in a colony after n days. Since we know that the number of bees tripes every day, we can assume that the number of bees is the number of bees yesterday multiplied by 3. This gives us the recurrence relation of $a_n = 3a_{n-1}$.

B) Given: $a_n = 3a_{n-1}$, $a_0 = 100$, where a_0 represents 100 bees and n = 10, where n represents the number of days.

$a_n = 3a_{n-1} = 3^1 a_{n-1}$
$= 3^1(3a^{n-2}) = 3^2 a_{n-2}$
$= 3^2(3a^{n-3}) = 3^3 a_{n-3}$
...
$= 3^n a_{n-n}$
$= 3^n a_0$
$= 3^{10} \times 100$
$= 5{,}904{,}900$ bees after 100 days.