Josephus solution for k = 2

observations:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$f(n)$	1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1

2^1 2^2 2^3 2^4

Safe point

increasing odd sequence

Josephus safepoint for k=2 is an increasing odd sequence that restarts with 1 whenever the index n is a power of 2. Therefore, if we choose M and L so that n=2^M +L, then safepoint is Lth odd sequence point that is (2*L + 1)

Q. What is the 5th odd and 5th even number?

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

1st odd, 2nd odd, 3rd odd, 5th odd

$$\boxed{n^{th} \text{ odd} = 2n-1 \text{ or } 2n+1}$$

$$\boxed{n^{th} \text{ even} = 2n}$$

Understanding m & L

Let's understand $k = 2$ & $n = 13$.

$m+1 = 4$

$\lceil \log_2 13 \rceil$

$m = 3$

$\lfloor \log_2 13 \rfloor$

$8 \quad 13 \quad 16$

$8 \rightarrow 2^3 \qquad 16 \rightarrow 2^4$

$\log_2 8 = 3 \qquad \log_2 16 = 4$

$n = 13, \quad n = 2^m + L$

$= 2^3 + 5$

So, for $n = 13$, $m = 3$ & $L = 5$

for $n = 13$, $\boxed{n = 2^m + L}$

So, $\boxed{L = n - 2^m}$

Safe point is L^{th} odd sequence point i.e $(2L+1)$

$\Rightarrow 2L+1 = 2(n-2) + 1$

$= 2(n - 2^{\lfloor \log_2 n \rfloor}) + 1$

$\Longrightarrow$

$\lfloor \log_2 n \rfloor = m$

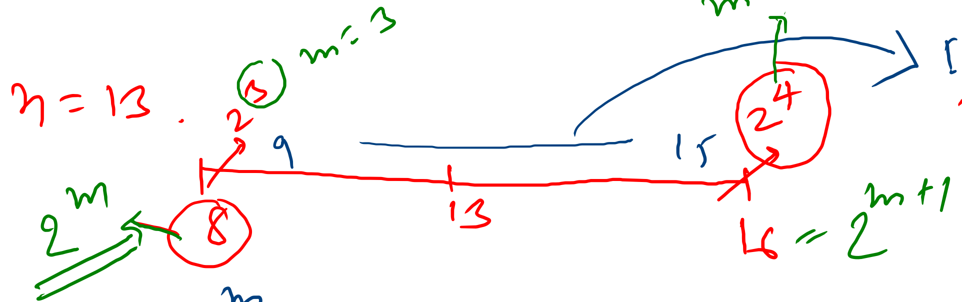
$\lfloor \log_2 13 \rfloor = 3$

So, $\lfloor \log_2 n \rfloor = m$

$\lceil \log_2 n \rceil = m+1$

Calculating m & $m+1$ using bitwise

Finding safe point using bitwise operation for $k = 2$.



If we multiply any number from this range will be greater than $\underline{2^{m+1}} = 2^4 = 16$

$n = 2^m + L$; $n = 8$ to $n = 15$ can be represented as $\underline{2^3 + L}$, using $L = 0$ to 7

$8 = 2^3 + 0$

$9 = 2^3 + 1$

$10 = 2^3 + 2$

$11 = 2^3 + 3$

$\vdots$

$15 = 2^3 + 7$

$$\boxed{\begin{aligned} 2^m &= \text{Integer.highestOneBit}(n) \\ 2 \times 2^m &= \underline{2^{m+1}} = \text{Integer.highestOneBit}(2 \times n) \end{aligned}}$$

Multiply by 2 using bitwise operator

$$(n<<1) == 2 \times n$$

13 =

2^3	2^2	2^1	2^0
1	1	0	1

2×2^3 $\quad$ 2×2^1

(13<<1) =

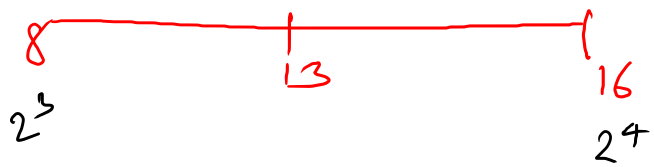
2^4	2^3	2^2	2^1	2^0
1	1	0	1	0

→ effectively we are multiplying each bit position by 2.

Divide by 2 using bitwise operation

$(n>>1)$ → it divides each bit position by 2.

$n = 13,\ k = 2,$



$$n = 2^m + L$$

$$2n = 2 \times 2^m + 2L \quad \text{— eq } (1)$$

Safe point $= 2L + 1$

In eq 1, if we add <u>1</u> and remove <u>2×2^m</u> term will give Safe point.

Strategy to get Safepoint using bitwise operator:

$$n = 2^m + L$$

<u>Step1</u> (i) multiply the 'n' by 2

$$2n = 2 \times 2^m + 2L$$

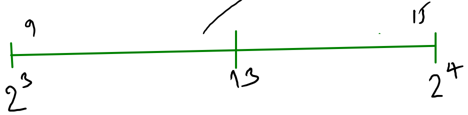
Step2 (ii) add 1 to the output of step 1

$$2n + 1 = 2 \times 2^m + \underbrace{(2L + 1)}_{\to \text{ safe point}}$$

Step(iii) make 2×2^m term as zero in step ii

$n = 13, \quad k = 2$

If we multiply any number by 2 from this range will be greater than or equal to 2^4 i.e. 2^{m+1}



Step1 = multiply the n by 2

$$2n = 2 \times 2^m + 2L$$

bitwise multiply by 2

$(n << 1)$

$$n << 1 = (2^m << 1) + (L << 1)$$

Step2 add 1 to step1

$((n << 1) \mid 1)$

$$(n << 1) \mid 1 = 2^m << 1 + ((L << 1) \mid 1)$$

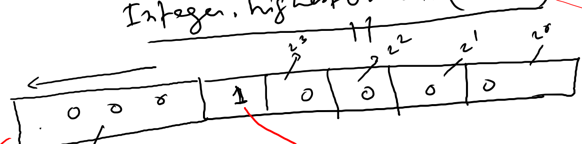
step 3 remove $2^m << 1$ term.

$2^m << 1 = 2^{m+1}$ → We will create an 'and map' having $(m+1)^{th}$ bit position as zero and other bits as 1.

$$\text{Integer.highestOneBit}(2 \times n) = 2^{m+1} = 2^{3+1}$$

13

$2 \times n = 2 \times 13 = 26$ greater than $2^{3+1} = 2^4$



all zero in remaining 27 bits

2^{m+1}

If we take complement of this will give desired mask containing all 1's except at $(m+1)^{th}$ bit position.