

Relation between Permutation and Combination

Mathematical Relation: $nCr = \frac{n!}{(n-r)!r!} \mid nPr = \frac{n!}{(n-r)!}$

① So, $nPr = nCr \times r!$

② Total count of characters in power set: power set = $(1+1)^n = 2^n$

$$2^n = nC_0 + nC_1 + nC_2 + nC_3 + \dots + nC_{n-3} + nC_{n-2} + nC_{n-1} + nC_n$$

Diagram showing groupings of terms in the binomial expansion:

- Group 1: $nC_0 + nC_1$ (labeled $n-char$)
- Group 2: $nC_2 + nC_3$ (labeled $n-char$)
- Group 3: $nC_{n-3} + nC_{n-2}$ (labeled $n-char$)
- Group 4: $nC_{n-1} + nC_n$ (labeled $n-char$)

Since there are 2^n terms, and 2 terms together gives 'n' char.

So, total number of chars in power set = $((2^n)/2) * n = 2^{n-1} * n$

Permutation and combination in terms of arrangement :

Permutation: Arranging 'r' distinct items at 'n' positions. Eg. Arranging 2 distinct items(a,b) at 3 positions. $3P_2 = 6$

Combination: Arranging 'r' identical items at 'n' positions. Eg. Arranging 2 (i,i) at 3 positions $\rightarrow 3C_2 = 3$

permutation of 2 identical items at 3 positions nCr

$$3C_2 = \begin{array}{ccc} i & i & \\ \underline{i} & \underline{\quad} & \underline{i} \\ \underline{\quad} & \underline{i} & \underline{i} \end{array}$$

permutation of 2 distinct items at 3 positions nPr

$$\left. \begin{array}{ccc|ccc} 1 & 2 & - & 2 & 1 & - \\ 1 & - & 2 & 2 & - & 1 \\ - & 1 & 2 & - & 2 & 1 \end{array} \right\} 3P_2 = 6$$

Since $r=2$, it means against each combination there will be $r!$ copies($2!=2$) of permutation.

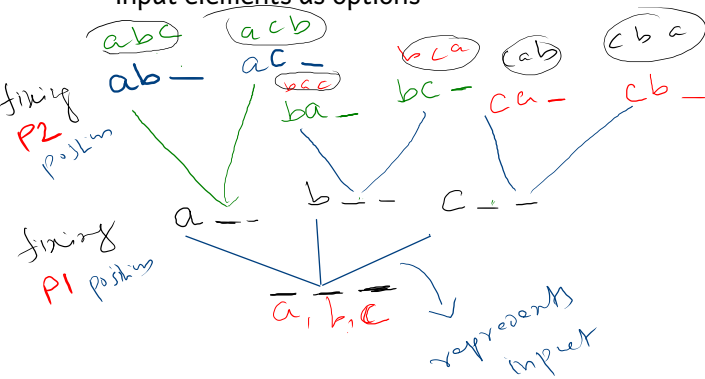
$$2^2 \times -$$

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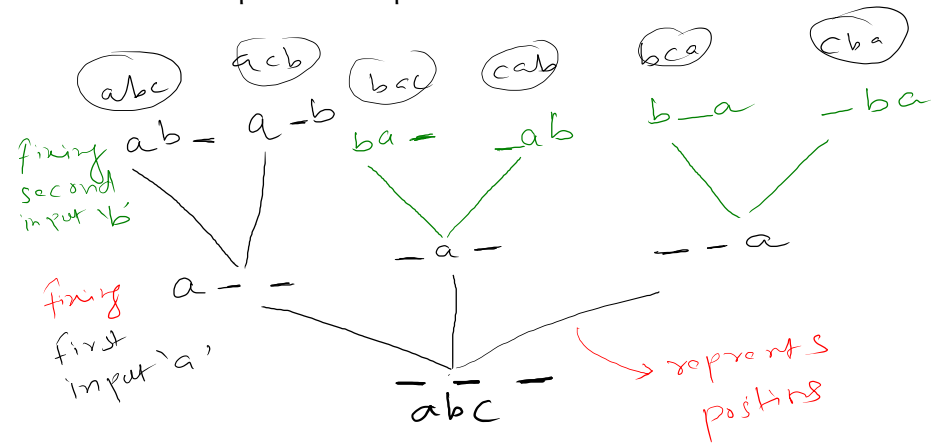
Default approach for permutation tree formation:

1. By fixing the position and taking input elements as options
2. By fixing the input element and taking positions as options

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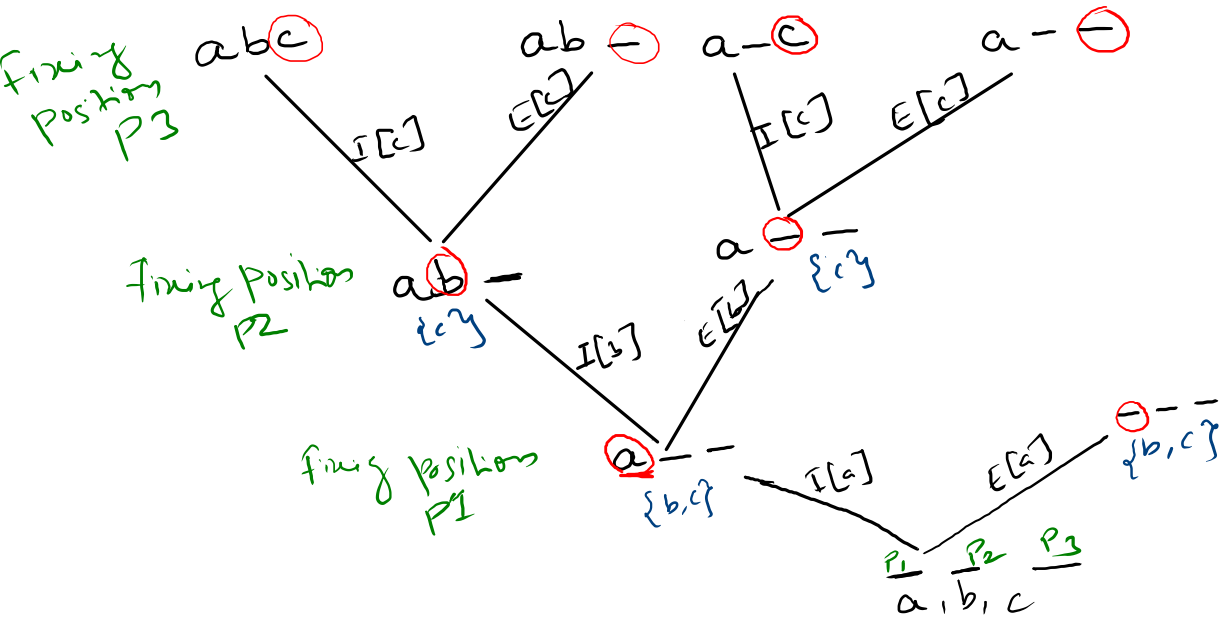


2. By fixing the input element and taking positions as options



Default approach for Combination tree formation:

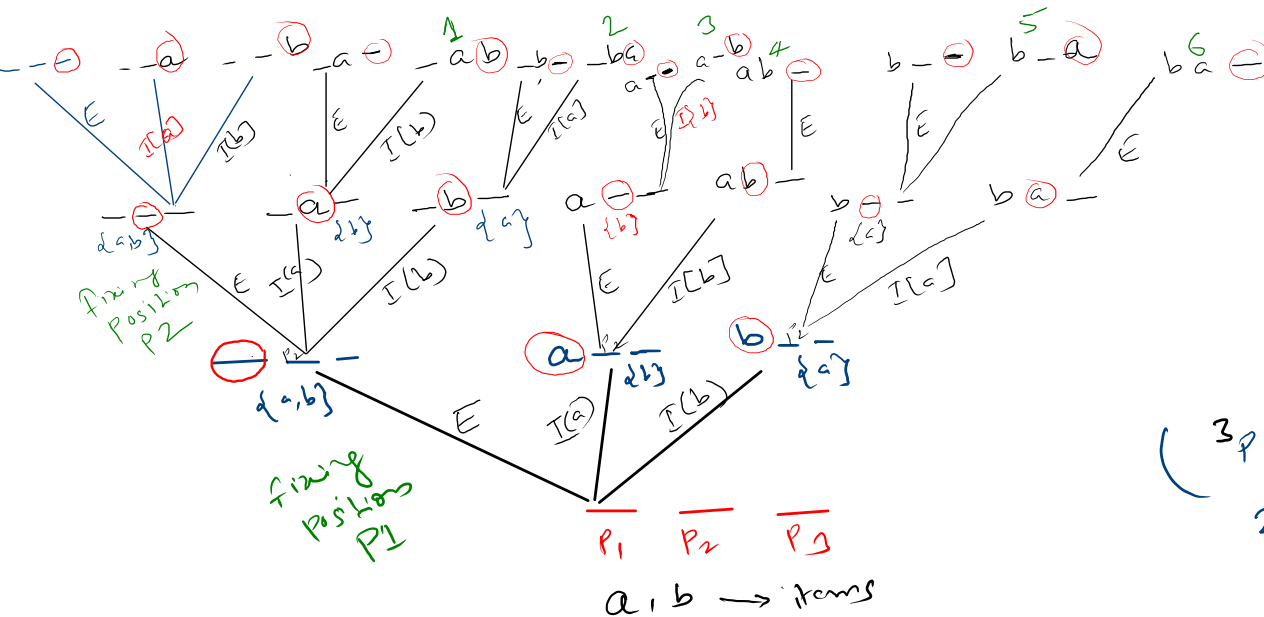
- ## Position fixing using include exclude tree



Relation between Permutation and Combination

Question 1: Print permutation using modified default combination strategy i.e. using modified Include Exclude strategy

target \rightarrow print nPr using nCr strategy.



-ab
-ba
a-b
b-a
ab=
ba-

${}^3P_2 = 6$
(3P_2 i.e., need to permute 2 items at 3 position)

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Question 2: Print combination using prermuation strategy of fixing input and taking position as options

Note: If we allow only to place the input in lexicographic order, then we will get combinations from permutation strategy.

