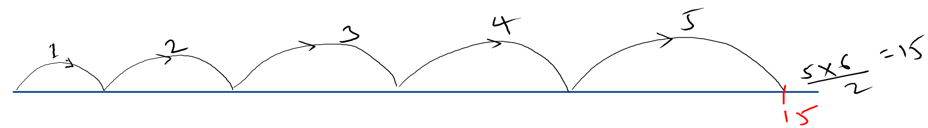
Minimum jumps to reach a point with +i or -i moves

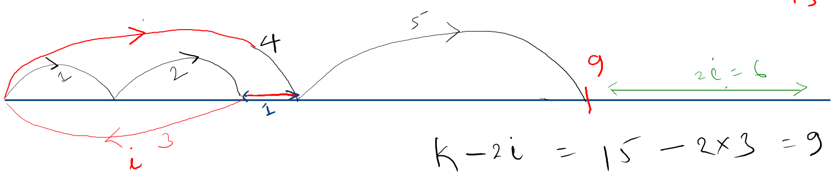
1. Given an integer X.
2. The task is to find the minimum number of jumps to reach a point X in the number line starting from zero.
3. The first jump made can be of length one unit and each successive jump will be exactly one unit longer than the previous jump in length.
4. It is allowed to go either left or right in each jump.

OBSERVATIONS:

1. $K = 0+1+2+3+4+..+n = n(n+1)/2$

If we reverse the direction of ith jump, the loss of distance is $2i$. So total distance covered is $K - 2i$





$K - 2i = 15 - 2\times3 = 9$

n	n×(n+1)/2	
0	0	
1	1	
2	3	
4	10	
5	15	odd
6	21	
7	28	even
8	36	
9	45	odd
10	55	

Observation 2: Distance covered in sequence follows the alternate pair of even and odd

Observation 3:

a. odd -/+ odd = even

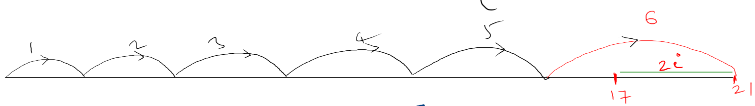
b. even -/+ even = even

c. odd - even = odd

d. even - odd = odd

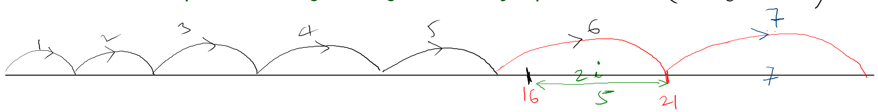
e. odd + even = odd/even

CASE 1 : 2i is even post traversing the target (target = 17)



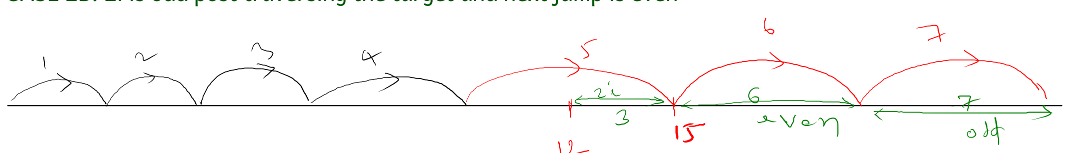
$2i = 21 - 17$
$= 4$
$i = 2 \rightarrow$ means we should reverse jump − 2

CASE 2A : 2i is odd post traversing the target and next jump is also odd (target = 16)



$2i = 5 + 7$ (odd + odd = even)
$= 12$
$i = 6$

CASE 2B: 2i is odd post traversing the target and next jump is even (target = 12)



odd + even + odd = even
as odd + odd = even
&
even + even = even

If jump-size to be reversed is greater than the last jump-size then we should pick group of jumps whose summation is equal to the jump-size to be reversed.

$2i = 3 + 6 + 7 = 16$
$i = 8$

Example:

Jump to be reversed = 8, which is greater than last jump i.e. 7.

Group of jumps to be reversed will be more than 1:

a. (7,1) , (6,2), (5,3), (5,2,1), (4,3,1)

In such a situation we can always reverse 1st and last jump.