

Relation between Permutation and Combination

Mathematical Relation: $nCr = \frac{n!}{(n-r)!r!} \mid nPr = \frac{n!}{(n-r)!}$

① So, $nPr = nCr \times r!$

② Total count of characters in power set: power set = $(1+1)^n = 2^n$

$$2^n = nC_0 + nC_1 + nC_2 + nC_3 + \dots + nC_{n-3} + nC_{n-2} + nC_{n-1} + nC_n$$

Diagram showing groupings of terms in the binomial expansion:

- Group 1: $nC_0 + nC_1$ (labeled $n-char$)
- Group 2: $nC_2 + nC_3$ (labeled $n-char$)
- Group 3: $nC_{n-3} + nC_{n-2}$ (labeled $n-char$)
- Group 4: $nC_{n-1} + nC_n$ (labeled $n-char$)

Since there are 2^n terms, and 2 terms together gives 'n' char.

So, total number of chars in power set = $((2^n)/2) * n = 2^{n-1} * n$

Permutation and combination in terms of arrangement :

Permutation: Arranging 'r' distinct items at 'n' positions. Eg. Arranging 2 distinct items(a,b) at 3 positions. $3P_2 = 6$

Combination: Arranging 'r' identical items at 'n' positions. Eg. Arranging 2 (i,i) at 3 positions $\rightarrow 3C_2 = 3$

permutation of 2 identical items at 3 positions nCr

$$3C_2 = \begin{array}{ccc} i & i & \\ \underline{i} & \underline{\quad} & \underline{i} \\ \underline{\quad} & \underline{i} & \underline{i} \end{array}$$

permutation of 2 distinct items at 3 positions nPr

$$\left. \begin{array}{ccc|ccc} 1 & 2 & - & 2 & 1 & - \\ 1 & - & 2 & 2 & - & 1 \\ - & 1 & 2 & - & 2 & 1 \end{array} \right\} 3P_2 = 6$$

Since $r=2$, it means against each combination there will be $r!$ copies($2!=2$) of permutation.

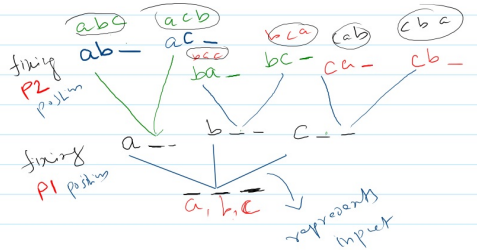
$$2^2 \times -$$

Approach for permutation tree formation:

1. By fixing the position and taking input elements as options
2. By fixing the input element and taking positions as options

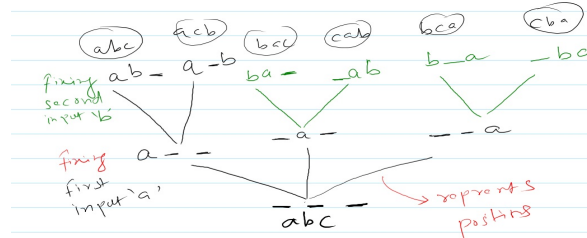
1A. By fixing the position and taking input elements as

options where $\text{input_count} == \text{position_count}$

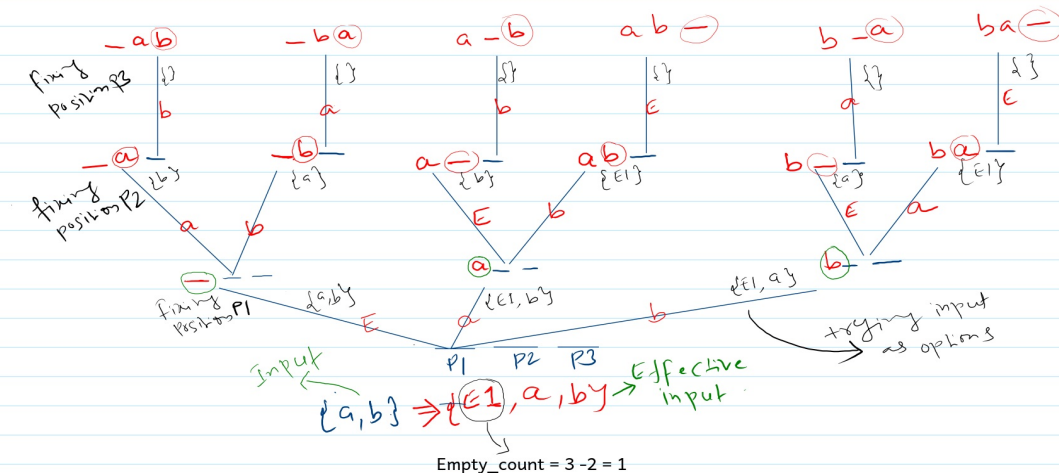


2. By fixing the input element and taking positions as

options where $\text{input_count} \leq \text{position_count}$



1B. By fixing the position and taking input elements as options where $\text{input_count} < \text{position_count}$



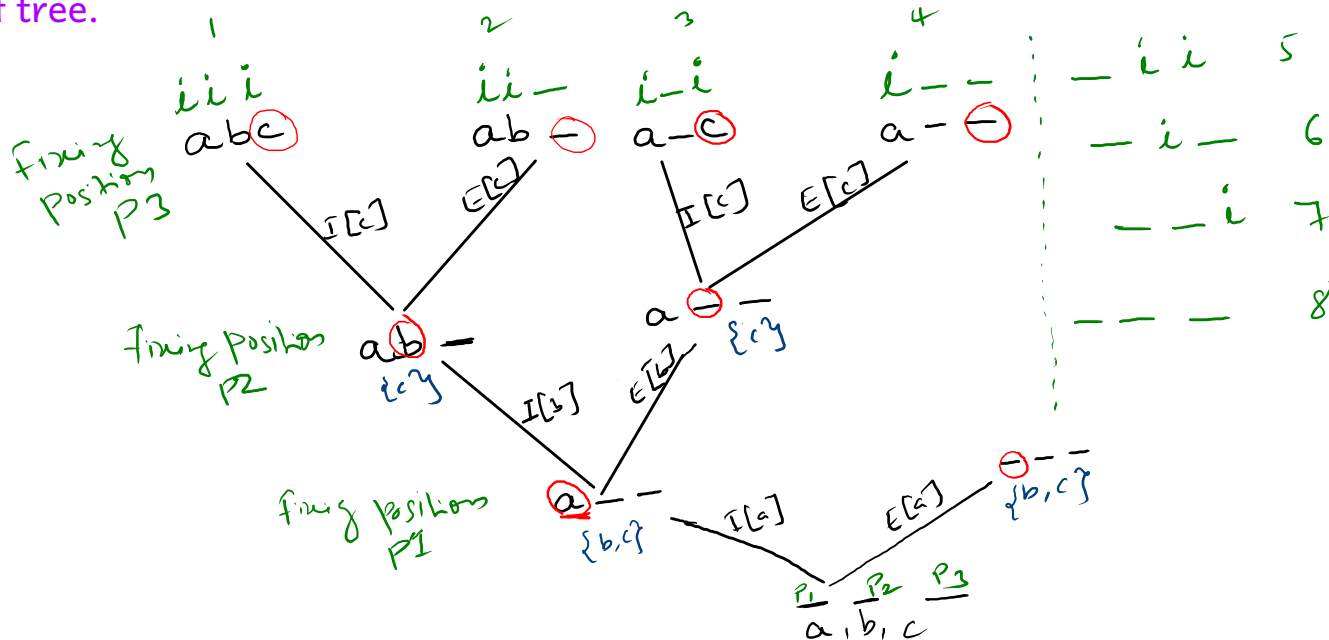
$$\text{EMPTY_COUNT} = \text{POSITION_COUNT} - \text{INPUT_COUNT}$$

Approaches for Combination tree formation:

1. Pascal_Identity based Include_Exclude_Tree tree by fixing position
2. Pascal_Identity_Expansion based Include_Tree by fixing position

1. Pascal_Identity based Include_Exclude_Tree tree by fixing position where
 $\text{input_count} \leq \text{postion_count}$

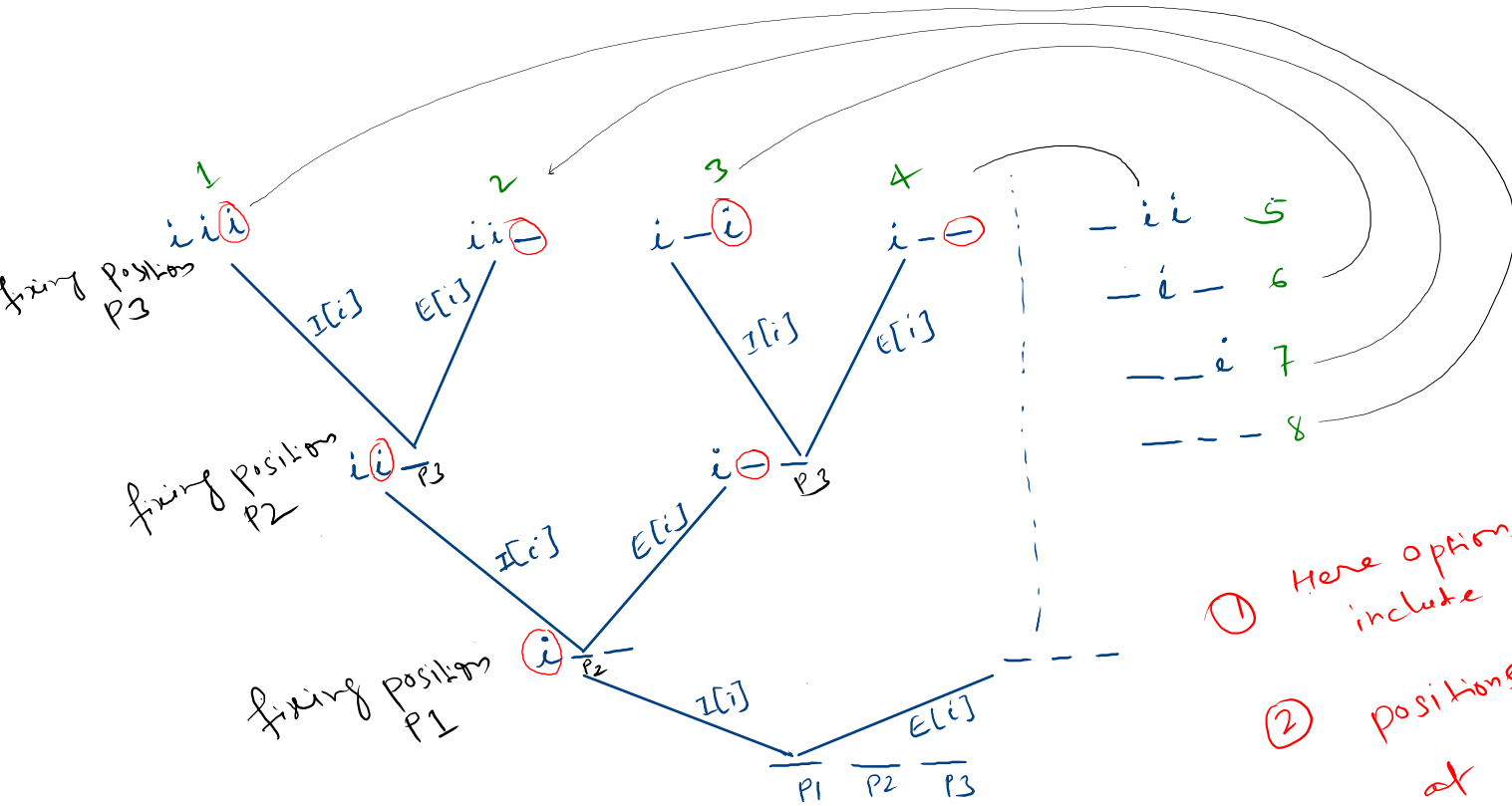
Note: Position is fixed at each level, and include(i) & exclude(i) are taken as options i.e. branches of tree.



1. Pascal_Identity based Include_Exclude_Tree tree by fixing position where

$\text{input_count} \leq \text{position_count}$

power set by placing 'i' on n given position

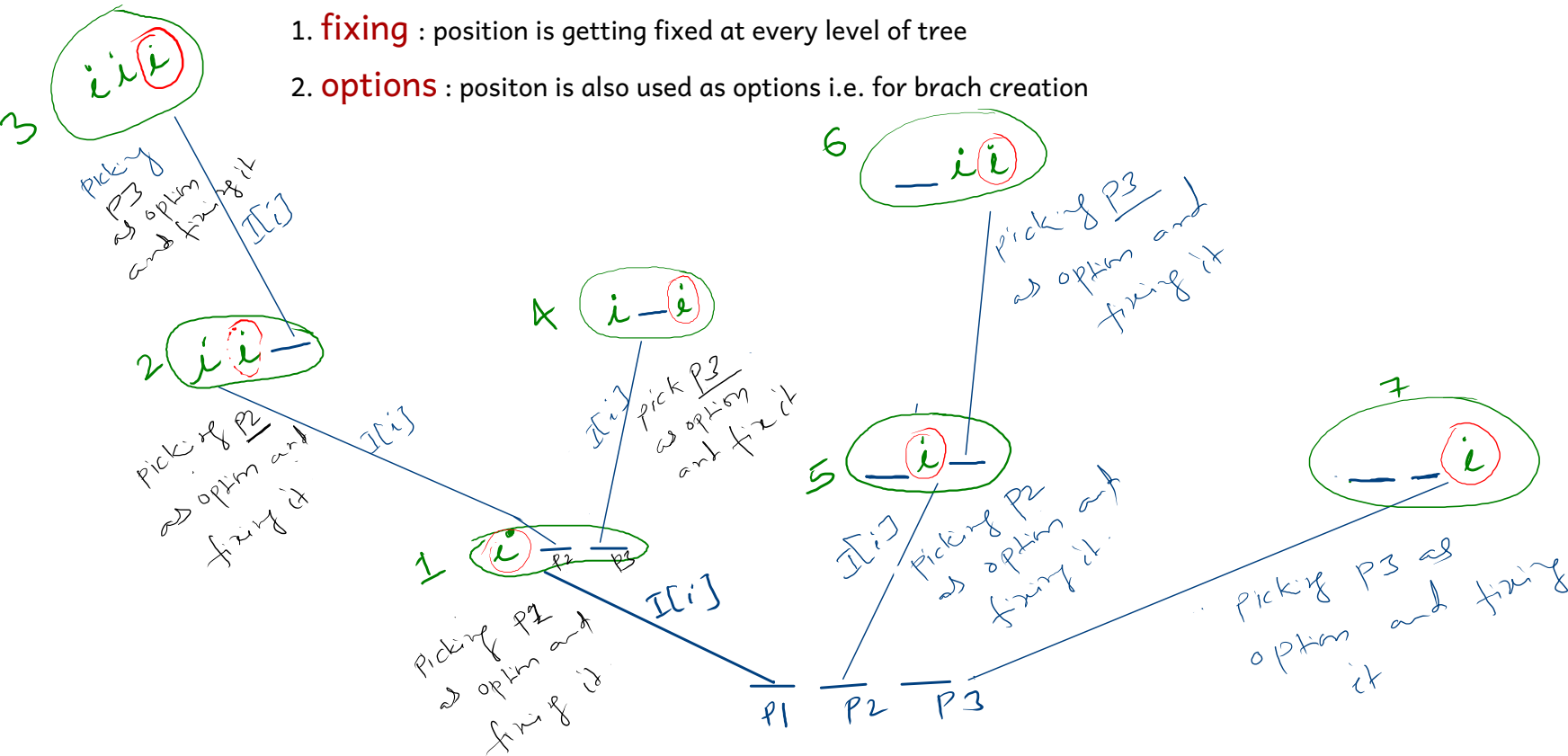


2. Pascal_Identity_Expansion based Include_Tree by fixing position where $\text{input_count} \leq \text{position_count}$

Power set by placing 'i' at 'n' given positions

In pascal identity Expansion strategy position is used for both :

1. **fixing** : position is getting fixed at every level of tree
2. **options** : position is also used as options i.e. for brach creation



Relation between Permutation and Combination

Question : Print combination using prermuation strategy of fixing input and taking position as options

Note: If we allow only to place the input in lexicographic order, then we will get combinations from permutation strategy.

