Master Theorem Dividing function

$T(n) = aT(n/b) + n^k \log^p n$

Case 2 $\log_b^a = K$

eg. $T(n) = 2T(n/2) + n\log n$

$\log_b^a = \log_2^2 = 1$; $k=1$; $p=1$

i.e $\boxed{\log_b^a = K}$

Note: when $K \geq \log_b^a$; $\Theta(n)$ depends on $f(n)$.

For Case 2 $\Theta(n) = n^K \times (\log n)^p$

This term is fixed as K has its existence because $K = \log_b^a$

now p will decide logn part

Case 2 A: $p > -1$ (0.5, 0, 1, 2)

$\boxed{\Theta(n) = f(n) \times \log n}$

$= n^k (\log n)^p \times \log n$

$= n^k (\log n)^{p+1}$

$T(n) = 2T(n/2) + n^1 \log^3 n$

$\log_b^a = \log_2^2 = 1 = K$

$\Theta(n) = n\log^3 n \times \log n = n\log^4 n$

p is +ve

Case 2 B: $p < -1$ (-1.1, -2, -3 -- ∞)

$f(n) = n^k \times \frac{1}{\log n^2}$

greater than 1

This term is too small so ignore it

Thus $\Theta(n) = n^K$

part of f(n)

eg. $T(n) = 2T(n/2) + n^1 \log^{-3} n)$

$\log_b^a = K$; $p = -3$

$\Theta(n) = n \log^{-3} n$

$= n$

not depend on this term

Case 2 C: $p = -1$ $f(n) = n^k \times \frac{1}{\log n}$

$\boxed{\Theta(n) = n^k \log(\log n)}$