CE 279 – Computational Geotechnics

Assignment – 1

Finite Difference Method

Due date: Feb 26th, 2020

1. Consider the following 1D wave equation on a 10 m domain.

$$\ddot{u} = c^2 u''$$

With Initial conditions: $u(x,0) = \frac{1}{1+x^2}$

$$\dot{u}(x,0) = 0$$

And with the following boundary conditions

Case 1: Fixed boundaries on both edges
Case 2: Fixed boundary on x = -5 m and stress-free boundary at x = 5 m
Case 3: Fixed boundary on x = -5 m and $u'(5,t) = -\dot{u}(5,t)$

Assume c = 1 m/s and total simulation time =20 s

a. Solve the above equation analytically for boundary condition cases 1 and 2. [20 pts]
b. Solve the above equation numerically using finite difference method using central difference for time and space. Please submit source code [20 pts]

(i) Derive the stability criterion for solving this equation using central difference method and choose time step accordingly [20 pts]

(ii) Plot a 3D surface plot to demonstrate wave propagation as a function of x and t for all three boundary conditions cases and comment on the wave reflection characteristics at the x = 5 m boundary. [20 pts]

(iii) Compare numerical solution with analytical solution for boundary condition cases 1 and 2. Calculate error as the L2 norm of [u_analytical(x,7) – u_numerical(x,7)]. Plot this error as a function of different grid sizes (different Δx) and comment on the convergence of the algorithm. [15 pts]

(iv) For any one of the boundary condition cases, plot the 3D surface plot for u(x,t) for an unstable time step. [5 pts]

Discussion question [0 pts – just food for thought] – responses are welcome but not mandatory

If you were to solve this problem in parallel by splitting the computational domain into two, then the -5 <= x <= 0 domain would lie on one computer (say Comp_1) and 0<=x<=5 would belong to another computer (say Comp_2). Comp_1 would not have the solution corresponding to any grid point from Comp_2 and vice-versa. One of the computationally expensive parts of a parallel solve is the transfer of data from one computer to the other. So, it is best to transfer just the minimum amount of data required for each computer to do its job. For solving this problem in parallel, please identify how many data transfers are required

at each time step and which data needs to be transferred from comp_1 to comp_2 and also from comp_2 to comp_1.

General comments:

(i) Collaboration on the assignment is encouraged. If you do collaborate with your classmates, please indicate their names on the front page of your assignment.

(ii) Please email me the completed assignment before Feb 27. If you need an extension for any legit reasons, please come and talk to me.

(iii) Please submit the source code along with the assignment. For the numerical solution, use any coding software/language that you are familiar with. C++ and Fortran are very powerful coding languages compared to MATLAB, and assignments are a great way to learn these languages, especially if you are working in a group.