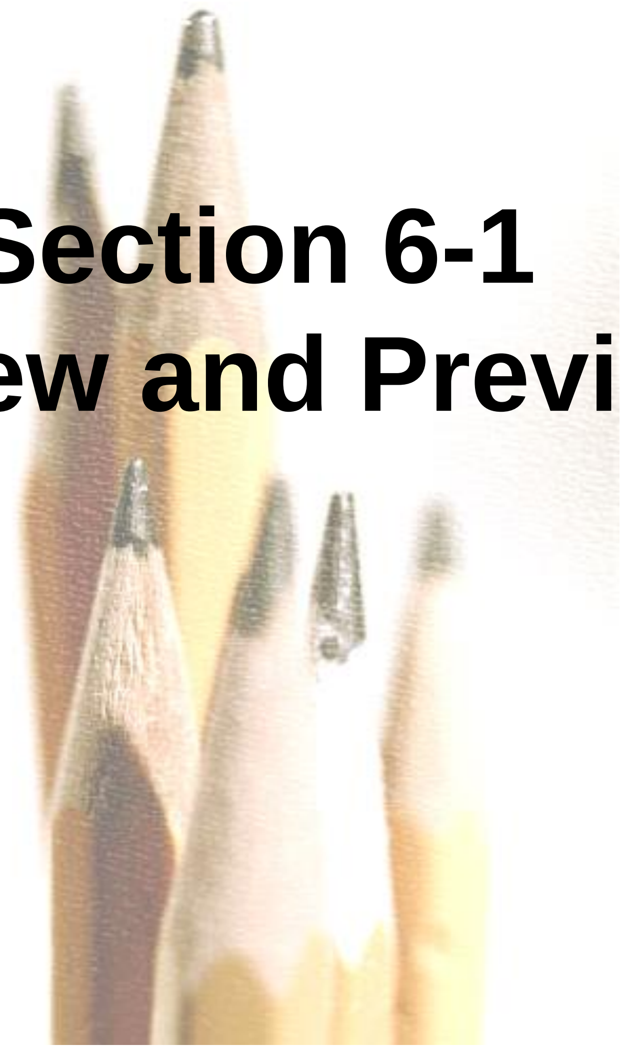
Chapter 6
Normal Probability Distributions

Section 6-1
Review and Preview



Review

- Chapter 2: Distribution of data
- Chapter 3: Measures of data sets, including measures of center and variation
- Chapter 4: Principles of probability
- Chapter 5: Discrete probability distributions

Preview

Chapter focus is on:

- **Continuous random variables**
- **Normal distributions**

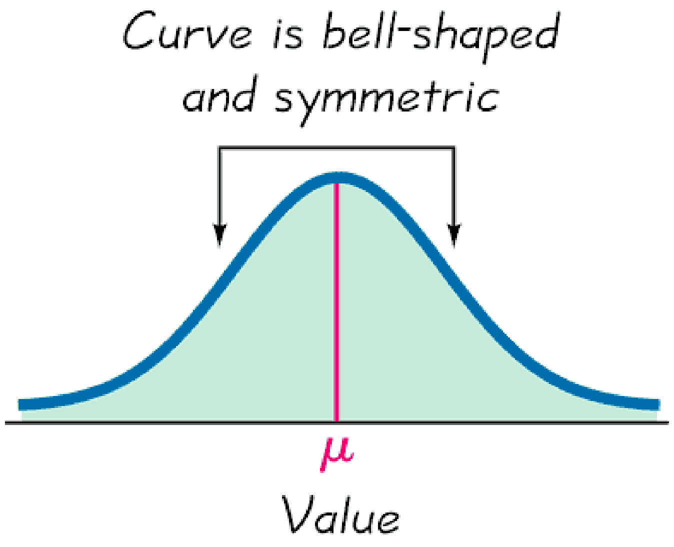


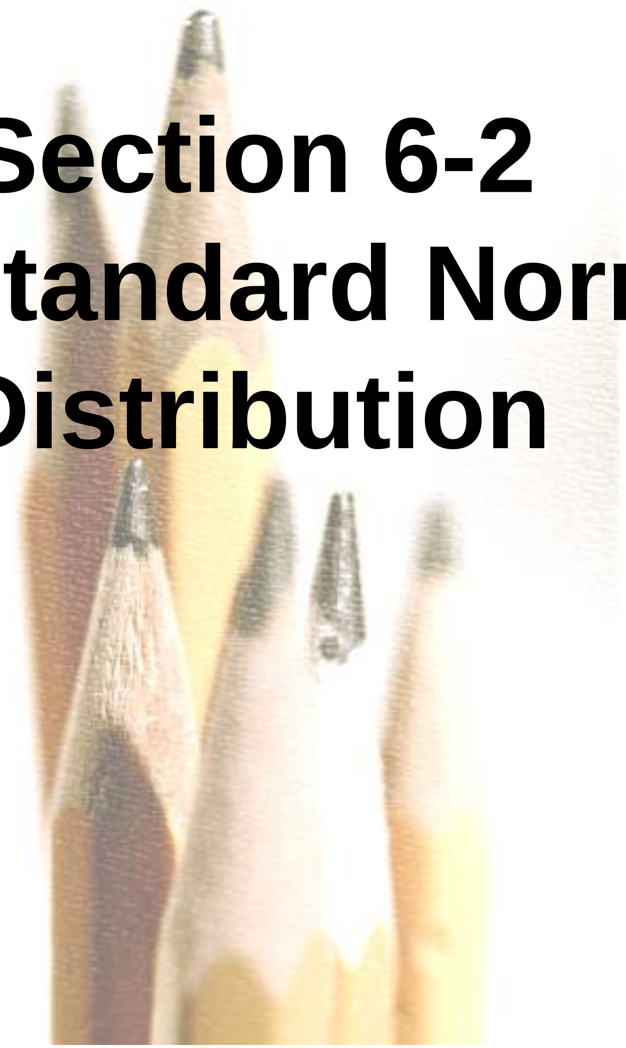
Figure 6-1

$$y = f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

Formula 6-1

Distribution determined by fixed values of mean and standard deviation. Put in 'x' to get 'y'. Plot looks like Fig 6-1

Section 6-2
The Standard Normal Distribution



Key Concept

This section presents the *standard normal distribution* which has three properties:

1. It's graph is bell-shaped.

2. It's mean is equal to 0 ($\mu = 0$).

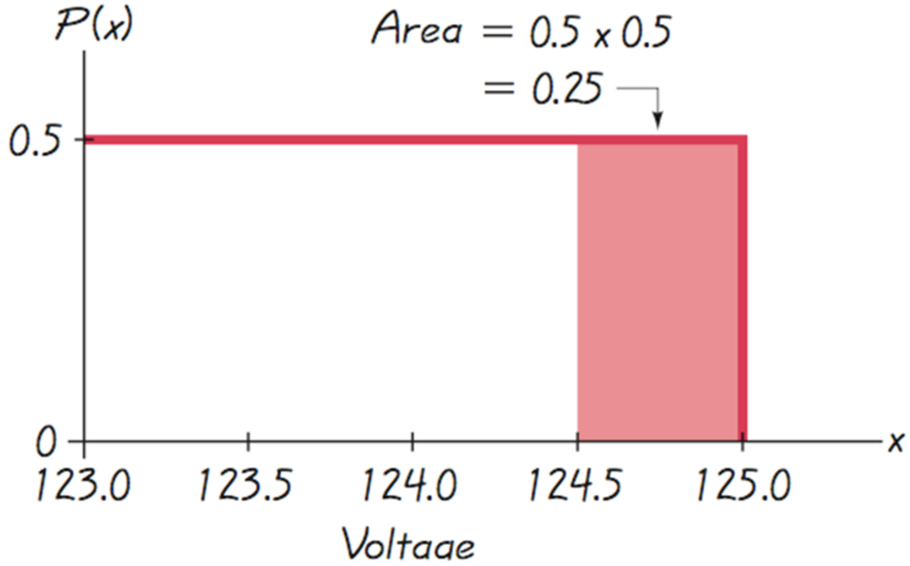
3. It's standard deviation is equal to 1 ($\sigma = 1$).

Develop the skill to find areas (or probabilities or relative frequencies) corresponding to various regions under the graph of the standard normal distribution. Find *z*-scores that correspond to area under the graph.

Uniform Distribution

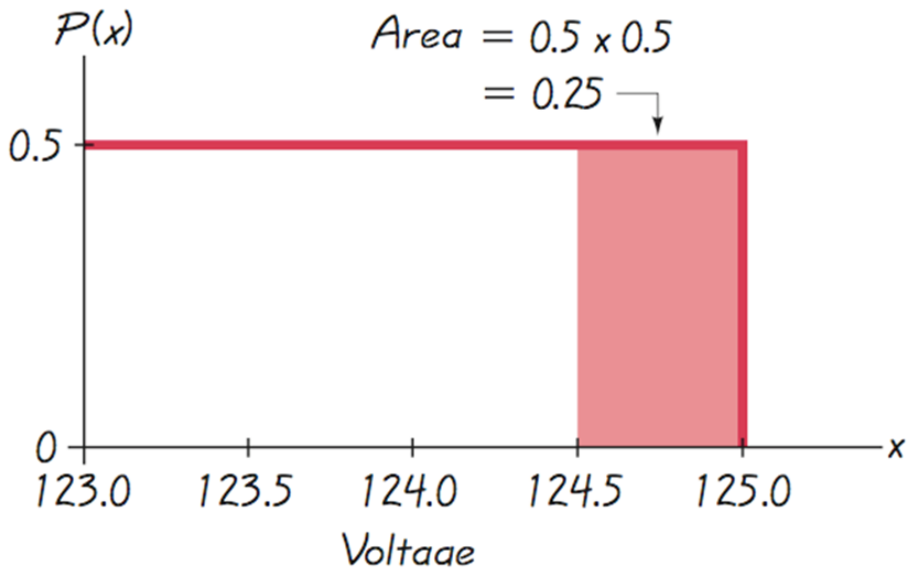
Before moving to normal distribution, lets study uniform distribution as it highlights few important concepts

A continuous random variable has a uniform distribution if its values are spread evenly over the range of probabilities. The graph of a uniform distribution results in a rectangular shape.



Uniform Distribution

- The area under the graph of a probability distribution is equal to 1
- There is a correspondence between area and probability (or relative frequency), so some probabilities can be found by identifying the corresponding areas.
- The graph of a continuous probability distribution, such as in Figure 6-2, is called a **density curve**



EXAMPLE 1

Home Power Supply The Newport Power and Light Company provides electricity with voltage levels that are uniformly distributed between 123.0 volts and 125.0 volts. That is, any voltage amount between 123.0 volts and 125.0 volts is possible, and all of the possible values are equally likely. If we randomly select one of the voltage levels and represent its value by the random variable *x*, then *x* has a distribution that can be graphed as in Figure 6-2.

Area under the graph of density curve is always 1

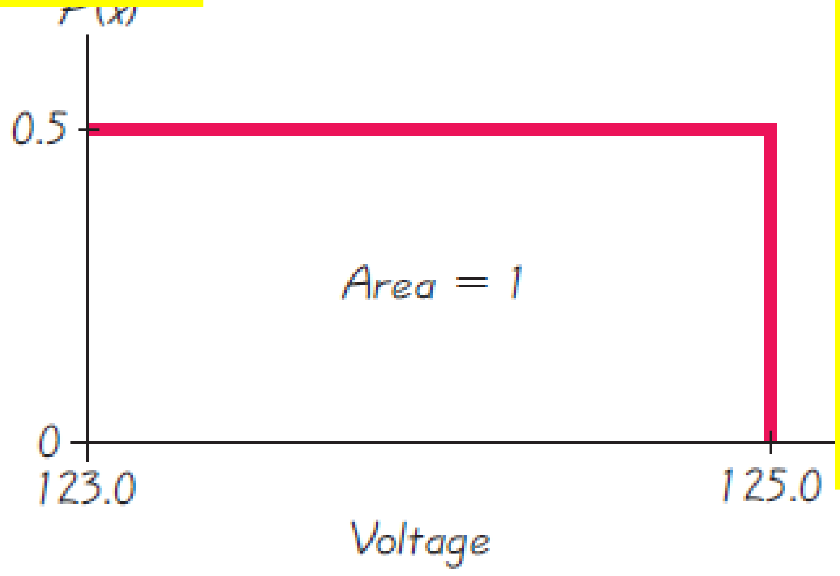


Figure 6-2 Uniform Distribution of Voltage Levels

With a continuous probability the probability of getting any single *exact* value is 0. We work out the probabilities of range of values

Density Curve

A density curve must satisfy the following properties:

1. **The total area under the curve must equal 1.**
2. **Every point on the curve must have a vertical height that is 0 or greater. (That is, the curve cannot fall below the *x*-axis.)**

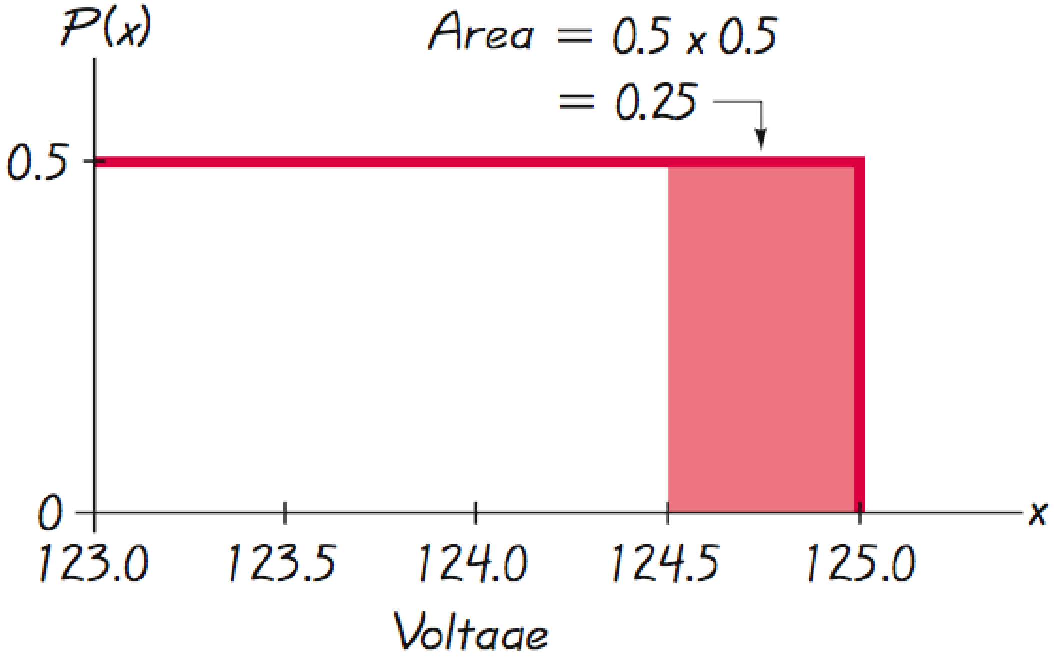
Area and Probability

Because the total area under the density curve is equal to 1, there is a correspondence between *area* and *probability*.

With a continuous probability the probability of getting any single *exact* value is 0.

Using Area to Find Probability

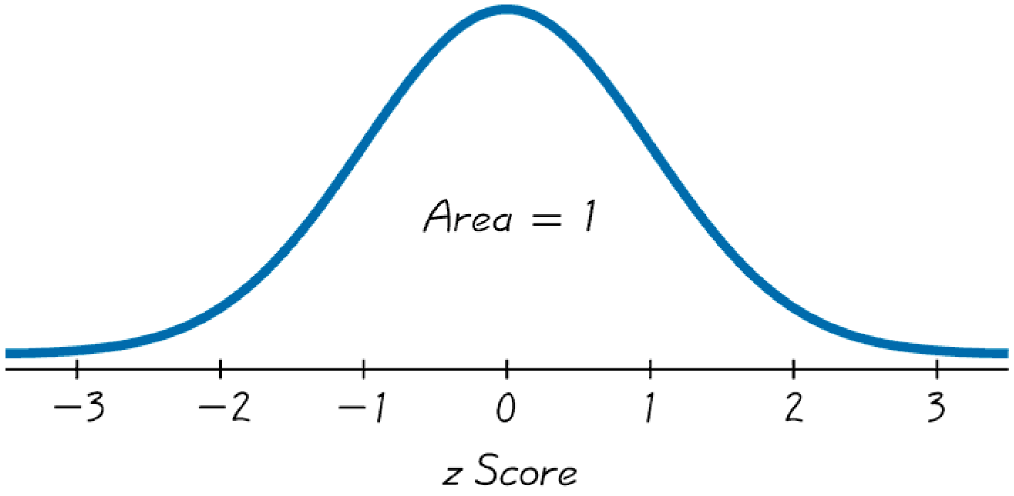
Given the uniform distribution illustrated, find the probability that a randomly selected voltage level is greater than 124.5 volts.



Shaded area represents voltage levels greater than 124.5 volts. Correspondence between area and probability: 0.25.

Standard Normal Distribution

The standard normal distribution is a normal probability distribution with $\mu = 0$ and $\sigma = 1$. The total area under its density curve is equal to 1.



$$y = f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

$$P(x) = \int \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}\,dx$$

It is not easy to find areas in Figure, so mathematicians have calculated many different areas under the curve, and those areas are presented in a form of Table

Finding Probabilities When Given *z*-scores

Various methods for calculating probabilities are available, we will be focusing on Table approach

- **Table A-2 (in Appendix A)**
- **Formulas and Tables insert card**
- **Find areas for many different regions**

Table A-2

TABLE A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	* .0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	* .0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

Table A-2

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599

Using Table A-2

1. **It is designed only for the *standard* normal distribution, which has a mean of 0 and a standard deviation of 1.**

2. **It is on two pages, with one page for *negative* *z*-scores and the other page for *positive* *z*-scores.**

3. **Each value in the body of the table is a *cumulative area from the left* up to a vertical boundary above a specific *z*-score.**

Using Table A-2

4. **When working with a graph, avoid confusion between *z*-scores and areas.**

   ***z* Score**

   Distance along horizontal scale of the standard normal distribution; refer to the leftmost column and top row of Table A-2.

   Area

   Region under the curve; refer to the values in the body of Table A-2.

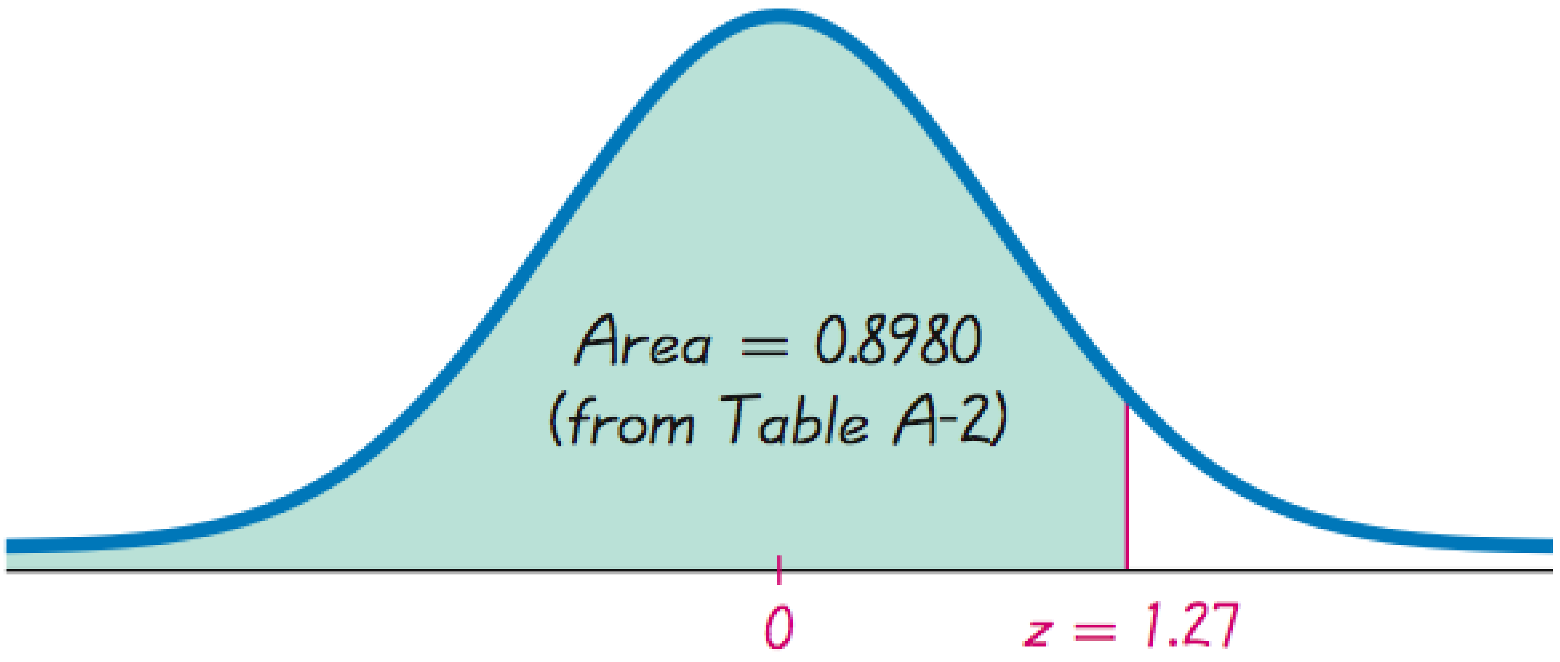
5. **The part of the z-score denoting hundredths is found across the top.**

Example - Thermometers

The Precision Scientific Instrument Company manufactures thermometers that are supposed to give readings of 0ºC at the freezing point of water. Tests on a large sample of these instruments reveal that at the freezing point of water, some thermometers give readings below 0º (denoted by negative numbers) and some give readings above 0º (denoted by positive numbers). Assume that the mean reading is 0ºC and the standard deviation of the readings is 1.00ºC. Also assume that the readings are normally distributed. If one thermometer is randomly selected, find the probability that, at the freezing point of water, the reading is less than 1.27º.

Example - (Continued)

$P(z < 1.27) =$



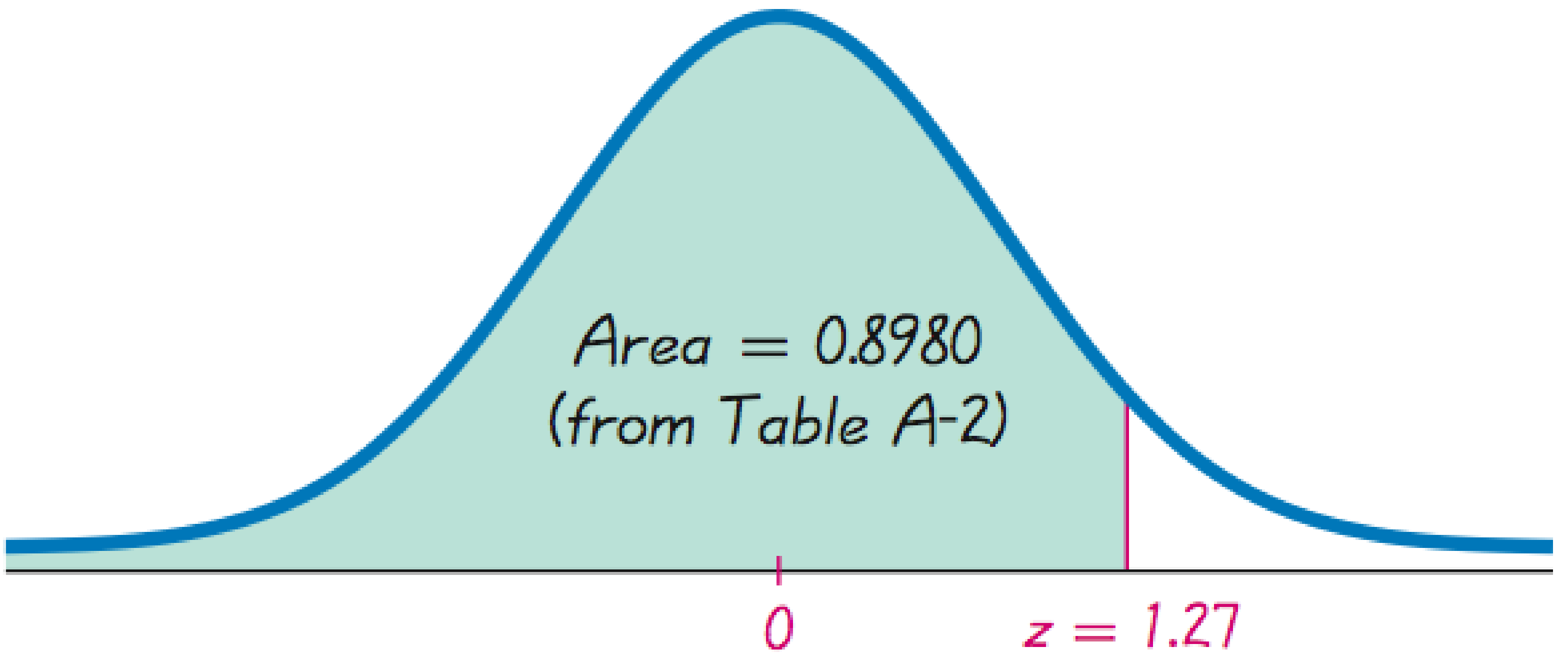
Look at Table A-2

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292

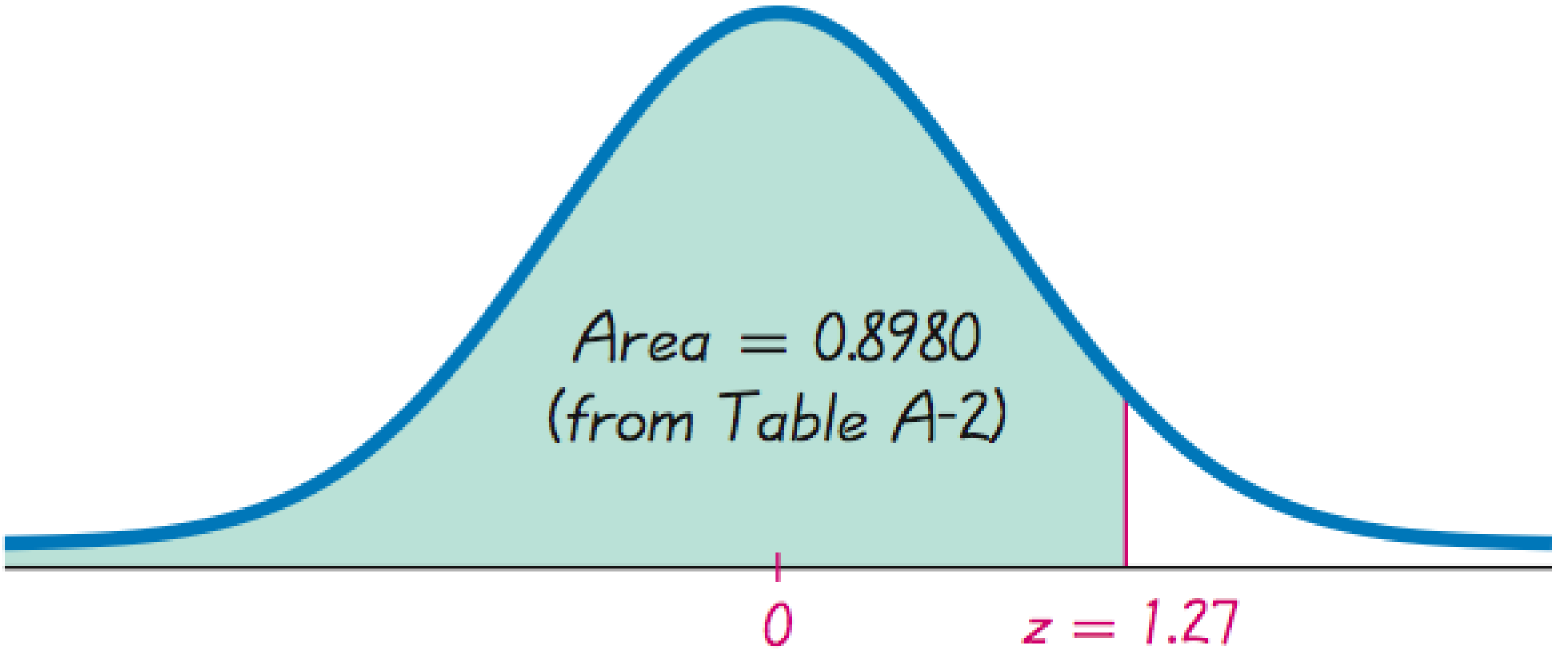
Example - cont

P (z < 1.27) = 0.8980



Example - cont

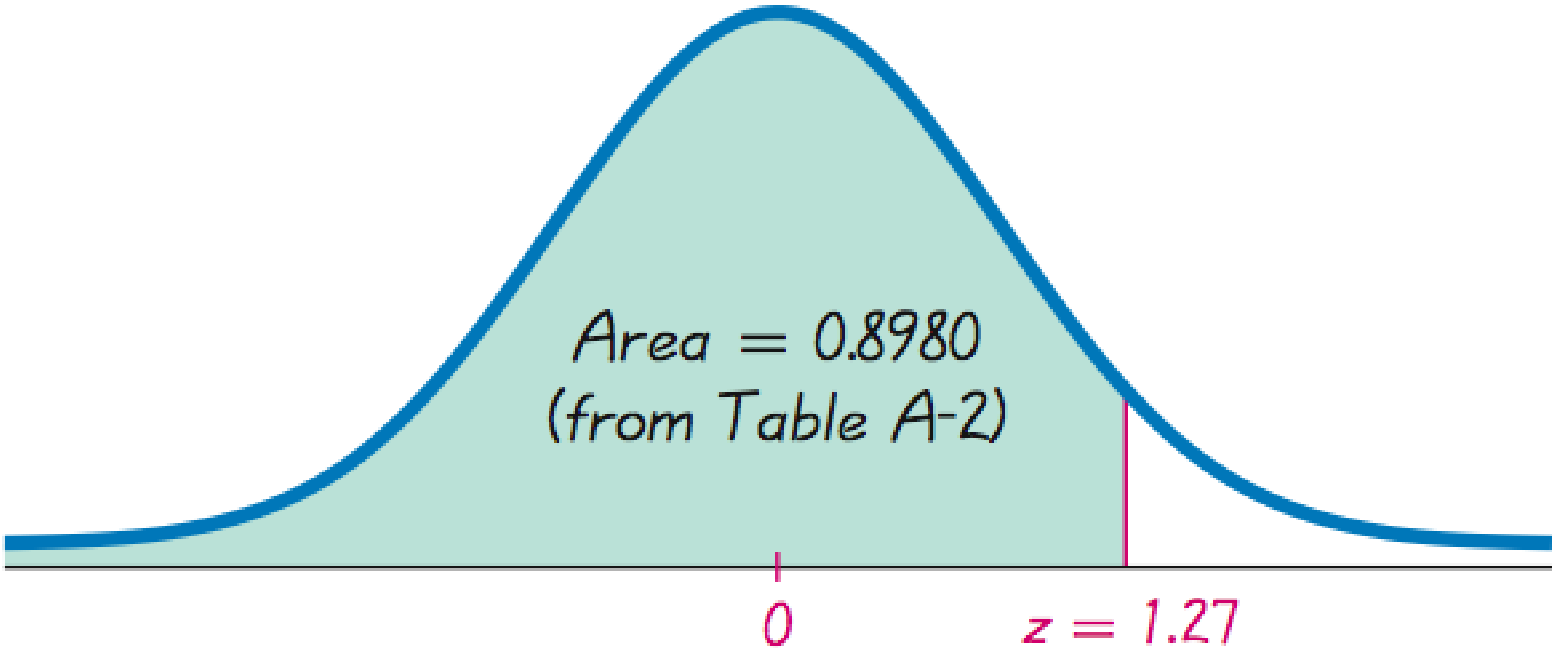
P (z < 1.27) = 0.8980



The *probability* of randomly selecting a thermometer with a reading less than 1.27º is 0.8980.

Example - cont

P (z < 1.27) = 0.8980



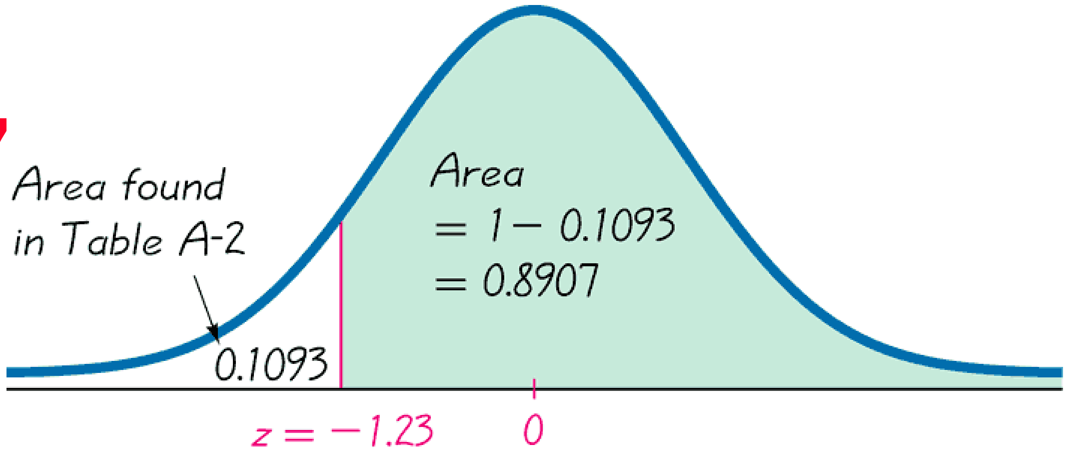
Or 89.80% will have readings below 1.27º.

Example - Thermometers Again

If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above –1.23 degrees.

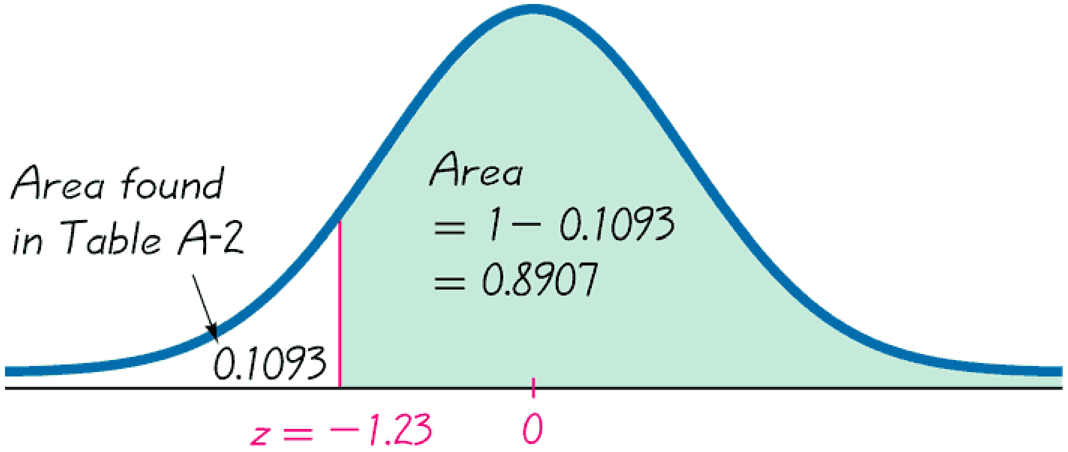
From table with negative z-score we find that the cumulative area from left up to z=-1.23 is .1093, hence
$P(z < -1.23) = .1093$
As total area under the curve is 1, this implies shaded area in the fig will be 1-.1093=.8907, so

$P(z > -1.23) = 0.8907$



Example - cont

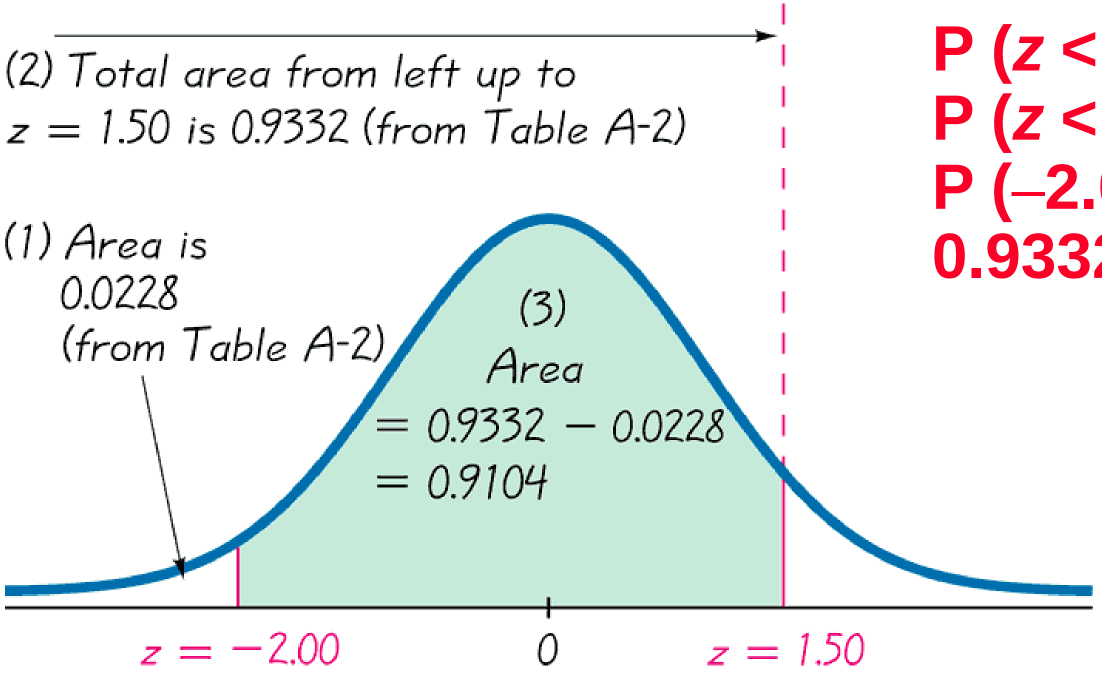
P (z > –1.23) = 0.8907



89.07% of the thermometers have readings above –1.23 degrees.

Example - Thermometers III

A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.

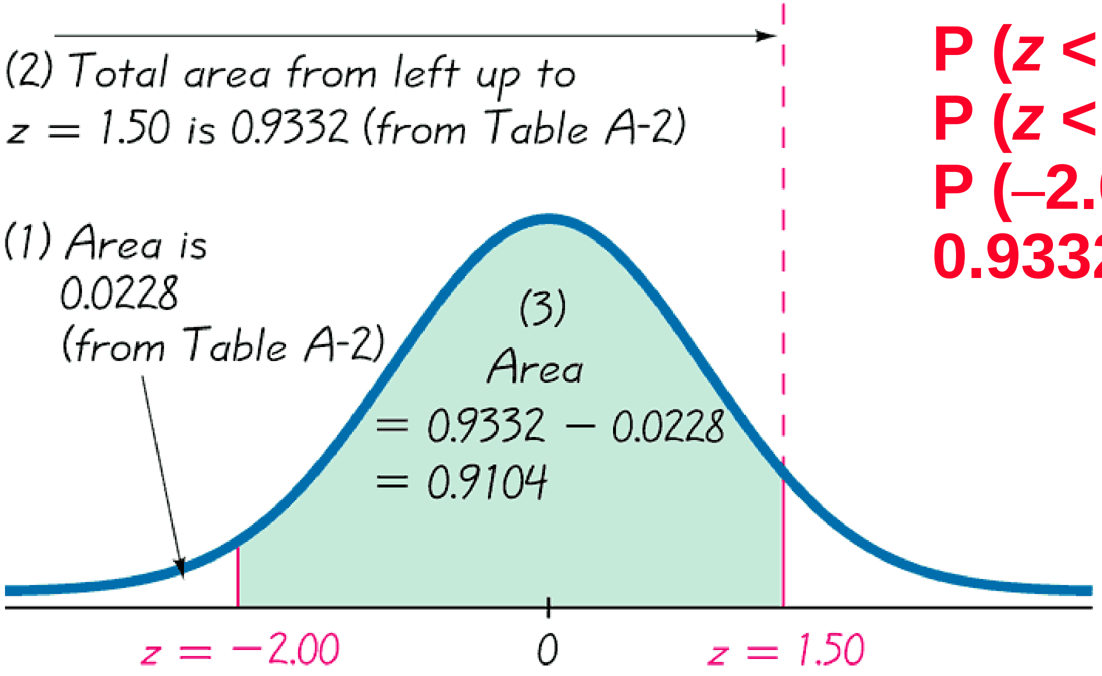


P (z < –2.00) = 0.0228
P (z < 1.50) = 0.9332
P (–2.00 < z < 1.50) =
0.9332 – 0.0228 = 0.9104

The probability that the chosen thermometer has a reading between – 2.00 and 1.50 degrees is 0.9104.

Example - cont

A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.



$P(z < -2.00) = 0.0228$
$P(z < 1.50) = 0.9332$
$P(-2.00 < z < 1.50) =$
$0.9332 - 0.0228 = 0.9104$

If many thermometers are selected and tested at the freezing point of water, then 91.04% of them will read between –2.00 and 1.50 degrees.

Notation

$$P(a < z < b)$$

denotes the probability that the *z* score is between *a* and *b*.

$$P(z > a)$$

denotes the probability that the *z* score is greater than *a*.

$$P(z < a)$$

denotes the probability that the *z* score is less than *a*.

With a continuous probability distribution such as the normal distribution, the probability of getting any single *exact* value is 0.