Chapter 5
Probability Distributions

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Section 5-1
Review and Preview

Review and Preview

This chapter combines the methods of descriptive statistics presented in Chapter 2 and 3 and those of probability presented in Chapter 4 to describe and analyze

probability distributions.

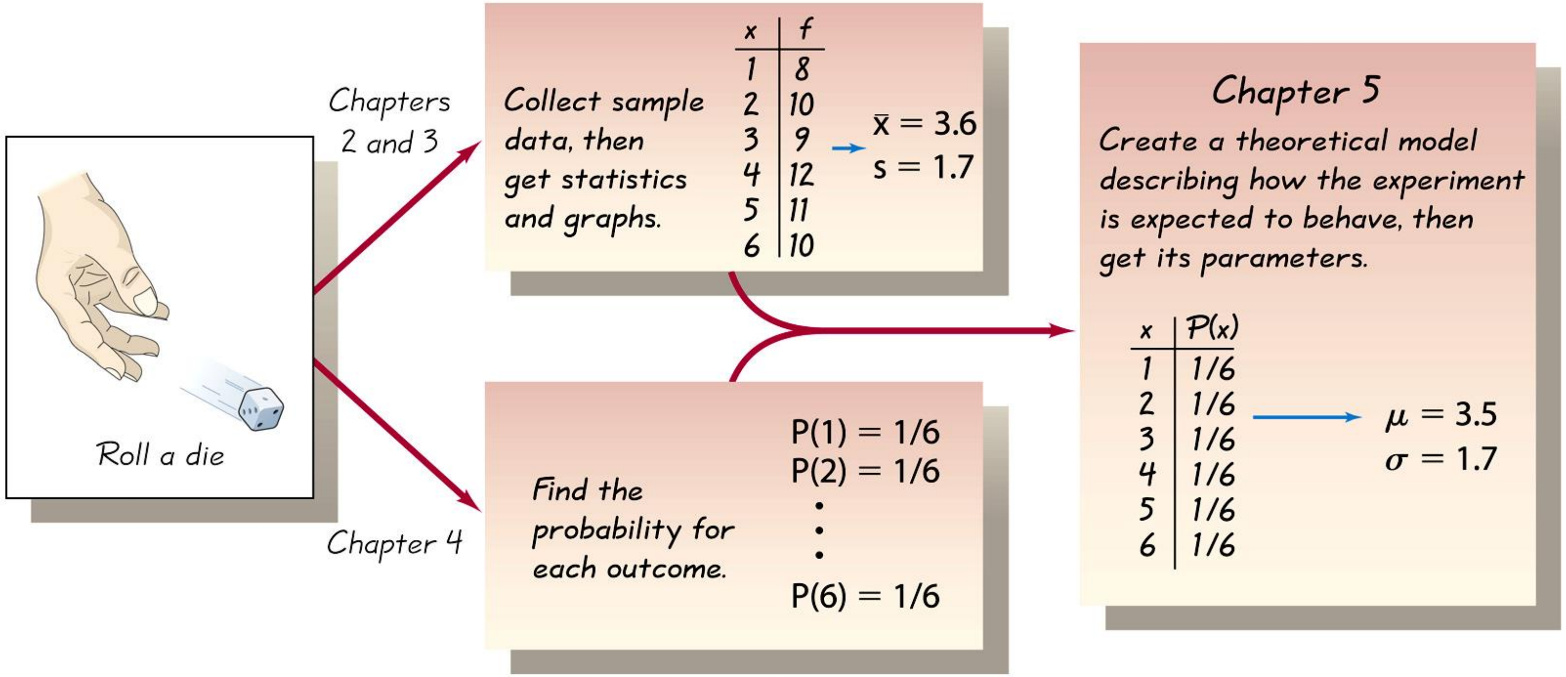
Probability Distributions describe what will probably happen instead of what actually did happen, and they are often given in the format of a graph, table, or formula.

Preview

In order to fully understand probability distributions, we must first understand the concept of a random variable, and be able to distinguish between discrete and continuous random variables. In this chapter we focus on discrete probability distributions. In particular, we discuss binomial and Poisson probability distributions.

Combining Descriptive Methods and Probabilities

In this chapter we will construct probability distributions by presenting possible outcomes along with the relative frequencies we expect.



Section 5-2
Random Variables

Key Concept

- The concept of random variables and how they relate to probability distributions
- Distinguish between discrete random variables and continuous random variables
- Develop formulas for finding the mean, variance, and standard deviation for a probability distribution
- Determine whether outcomes are likely to occur by chance or they are unusual (in the sense that they are not likely to occur by chance)

Random Variable Probability Distribution

- **Random variable**
  a variable (typically represented by x) that has a single numerical value, determined by chance, for each outcome of a procedure
- **Probability distribution**
  a description that gives the probability for each value of the random variable; often expressed in the format of a graph, table, or formula

Example

Each parent contributes one gene to an offspring and, depending on the combination of genes, that offspring could inherit the dominant trait or the recessive trait. Mendel conducted an experiment using pea plants. The pods of pea plants can be green or yellow. When one pea carrying a dominant green gene and a recessive yellow gene is crossed with another pea carrying the same green yellow genes, the offspring can inherit any one of four combinations of genes, as shown in the table below

Gene from Parent 1		Gene from Parent 2		Offspring Genes		Color of Offspring Pod
green	+	green	→	green/green	→	green
green	+	yellow	→	green/yellow	→	green
yellow	+	green	→	yellow/green	→	green
yellow	+	yellow	→	yellow/yellow	→	yellow

EXAMPLE 1 **Genetics** Consider the offspring of peas from parents both having the green/yellow combination of pod genes. Under these conditions, the probability that the offspring has a green pod is 3/4 or 0.75. That is, $P(\text{green}) = 0.75$. If five such offspring are obtained, and if we let

$$x = \text{number of peas with green pods among 5 offspring peas}$$

then x is a random variable because its value depends on chance. Table 5-1 is a probability distribution because it gives the probability for each value of the random variable x. (In Section 5-3 we will see how to find the probability values, such as those listed in Table 5-1.)

x (Number of Peas with Green Pods)	$P(x)$
0	0.001
1	0.015
2	0.088
3	0.264
4	0.396
5	0.237

- **If outcome (x) of some experiment is probabilistic, Then x is RV**
- **The distribution giving probability for each value of RV (x) is Probability Distribution**

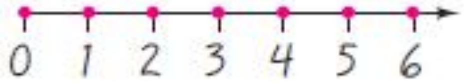
Discrete and Continuous Random Variables

- **Discrete random variable**
  either a finite number of values or countable number of values, where "countable" refers to the fact that there might be infinitely many values, but they result from a counting process, so that the number of values is 0 or 1 or 2 or 3, etc

- **Continuous random variable**
  infinitely many values, and those values can be associated with measurements on a continuous scale without gaps or interruptions

(a) Discrete Random Variable: Count of the number of movie patrons.

Graph of Discrete Values



(b) Continuous Random Variable: The measured voltage of a smoke detector battery.

Graph of Continuous Values

EXAMPLE 2 The following are examples of discrete and continuous random variables.

1. **Discrete** Let $x =$ the number of eggs that a hen lays in a day. This is a *discrete* random variable because its only possible values are 0, or 1, or 2, and so on. No hen can lay 2.343115 eggs, which would have been possible if the data had come from a continuous scale.

2. **Discrete** The count of the number of statistics students present in class on a given day is a whole number and is therefore a discrete random variable. The counting device shown in Figure 5-2(a) is capable of indicating only a finite number of values, so it is used to obtain values for a *discrete* random variable.

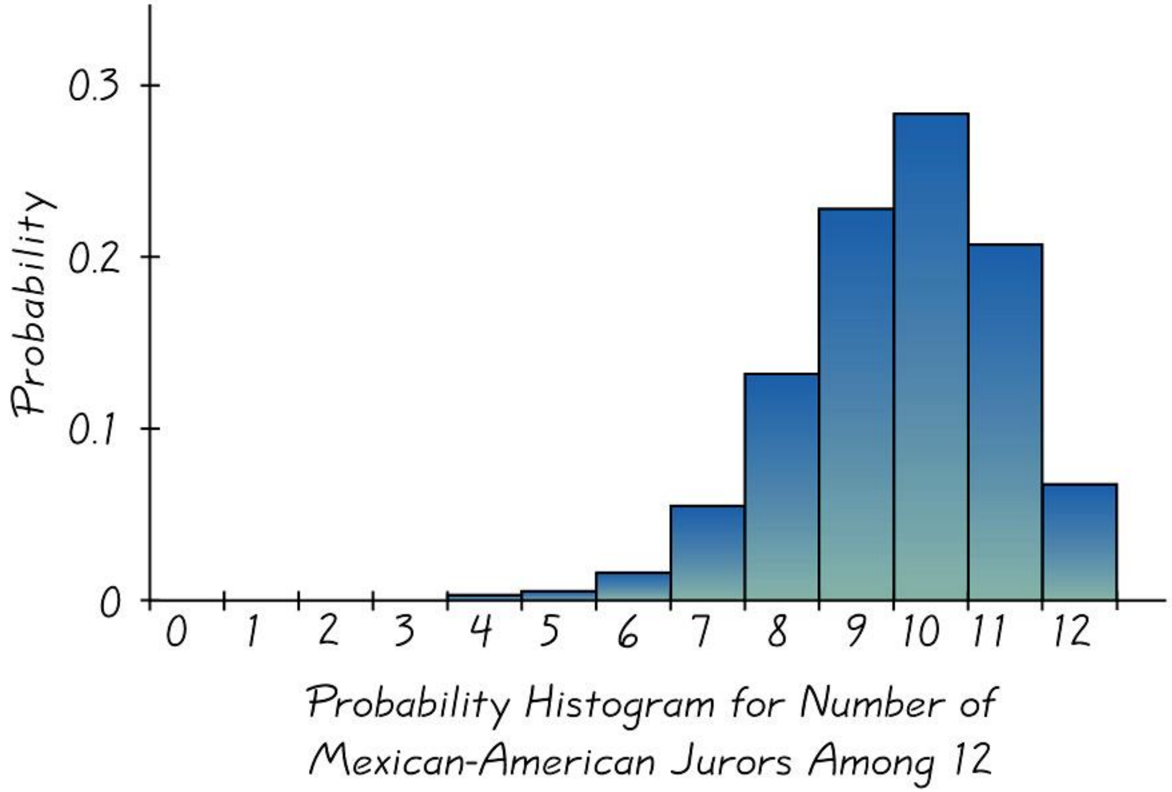
3. **Continuous** Let $x =$ the amount of milk a cow produces in one day. This is a *continuous* random variable because it can have any value over a continuous span. During a single day, a cow might yield an amount of milk that can be any value between 0 gallons and 5 gallons. It would be possible to get 4.123456 gallons, because the cow is not restricted to the discrete amounts of 0, 1, 2, 3, 4, or 5 gallons.

4. **Continuous** The measure of voltage for a particular smoke detector battery can be any value between 0 volts and 9 volts. It is therefore a continuous random variable. The voltmeter shown in Figure 5-2(b) is capable of indicating values on a continuous scale, so it can be used to obtain values for a *continuous* random variable.

Graphs

The probability histogram is very similar to a relative frequency histogram, but the vertical scale shows probabilities.

x (Number of Peas with Green Pods)	$P(x)$
0	0.001
1	0.015
2	0.088
3	0.264
4	0.396
5	0.237



Requirements for Probability Distribution

$$\sum P(x) = 1$$

where x assumes all possible values.

$$0 \le P(x) \le 1$$

for every individual value of x.

Table 5-2 Cell Phones per Household

x	$P(x)$
0	0.19
1	0.26
2	0.33
3	0.13

EXAMPLE 3

Cell Phones Based on a survey conducted by Frank N. Magid Associates, Table 5-2 lists the probabilities for the number of cell phones in use per household. Does Table 5-2 describe a probability distribution?

SOLUTION

To be a probability distribution, $P(x)$ must satisfy the preceding two requirements. But

$$\begin{aligned}\Sigma P(x) &= P(0) + P(1) + P(2) + P(3)\\ &= 0.19 + 0.26 + 0.33 + 0.13\\ &= 0.91 \quad [\text{showing that } \Sigma P(x) \neq 1]\end{aligned}$$

Because the first requirement is not satisfied, we conclude that Table 5-2 does *not* describe a probability distribution.

EXAMPLE 4 Does $P(x) = \frac{x}{10}$ (where x can be 0, 1, 2, 3, or 4) determine a probability distribution?

SOLUTION For the given formula we find that $P(0) = 0/10$, $P(1) = 1/10$, $P(2) = 2/10$, $P(3) = 3/10$, and $P(4) = 4/10$, so that

1. $\Sigma P(x) = \frac{0}{10} + \frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = \frac{10}{10} = 1$

2. Each of the $P(x)$ values is between 0 and 1.

Because both requirements are satisfied, the formula given in this example is a probability distribution.

Mean, Variance and Standard Deviation of a Probability Distribution

$$\mu = \Sigma\,[x \cdot P(x)]$$ Mean

$$\sigma^2 = \Sigma\,[(x - \mu)^2 \cdot P(x)]$$ Variance

$$\sigma^2 = \Sigma\,[x^2 \cdot P(x)] - \mu^2$$ Variance (shortcut)

$$\sigma = \sqrt{\Sigma\,[x^2 \cdot P(x)] - \mu^2}$$ Standard Deviation

Roundoff Rule for μ, σ, and σ^2

Round results by carrying one more decimal place than the number of decimal places used for the random variable x.

If the values of x are integers, round μ, σ, and σ^2 to one decimal place.

EXAMPLE 5 **Finding the Mean, Variance, and Standard Deviation**

Table 5-1 describes the probability distribution for the number of peas with green pods among 5 offspring peas obtained from parents both having the green/yellow pair of genes. Find the mean, variance, and standard deviation for the probability distribution described in Table 5-1 from Example 1.

SOLUTION In Table 5-3, the two columns at the left describe the probability distribution given earlier in Table 5-1, and we create the three columns at the right for the purposes of the calculations required.

Using Formulas 5-1 and 5-2 and the table results, we get

Mean: $\mu = \Sigma[x \cdot P(x)] = 3.752 = 3.8$

Variance: $\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)] = 0.940574 = 0.9$

The standard deviation is the square root of the variance, so

Standard deviation: $\sigma = \sqrt{0.940574} = 0.969832 = 1.0$

x	$P(x)$
0	0.001
1	0.015
2	0.088
3	0.264
4	0.396
5	0.237
Total	

Table 5-3 Calculating μ, σ, and σ^2 for a Probability Distribution

x	$P(x)$	$x \cdot P(x)$	$(x - \mu)^2 \cdot P(x)$
0	0.001	$0 \cdot 0.001 = 0.000$	$(0 - 3.752)^2 \cdot 0.001 = 0.014078$
1	0.015	$1 \cdot 0.015 = 0.015$	$(1 - 3.752)^2 \cdot 0.015 = 0.113603$
2	0.088	$2 \cdot 0.088 = 0.176$	$(2 - 3.752)^2 \cdot 0.088 = 0.270116$
3	0.264	$3 \cdot 0.264 = 0.792$	$(3 - 3.752)^2 \cdot 0.264 = 0.149293$
4	0.396	$4 \cdot 0.396 = 1.584$	$(4 - 3.752)^2 \cdot 0.396 = 0.024356$
5	0.237	$5 \cdot 0.237 = 1.185$	$(5 - 3.752)^2 \cdot 0.237 = 0.369128$
Total		3.752 ↑ $\mu = \Sigma[x \cdot P(x)]$	0.940574 ↑ $\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)]$

Identifying *Unusual* Results
Range Rule of Thumb

According to the **range rule of thumb**, most values should lie within 2 standard deviations of the mean.

We can therefore identify "unusual" values by determining if they lie outside these limits:

Maximum usual value = $\mu + 2\sigma$

Minimum usual value = $\mu - 2\sigma$

EXAMPLE 6 **Identifying Unusual Results with the Range Rule of Thumb**

In Example 5 we found that for groups of 5 offspring (generated from parents both having the green/yellow pair of genes), the mean number of peas with green pods is 3.8, and the standard deviation is 1.0. Use those results and the range rule of thumb to find the maximum and minimum usual values. Based on the results, determine whether it is unusual to generate 5 offspring peas and find that only 1 of them has a green pod.

SOLUTION Using the range rule of thumb, we can find the maximum and minimum usual values as follows:

$$\text{maximum usual value:} \quad \mu + 2\sigma = 3.8 + 2(1.0) = 5.8$$

$$\text{minimum usual value:} \quad \mu - 2\sigma = 3.8 - 2(1.0) = 1.8$$

Identifying Unusual Results Probabilities

Rare Event Rule for Inferential Statistics

If, under a given assumption (such as the assumption that a coin is fair), the probability of a particular observed event (such as 992 heads in 1000 tosses of a coin) is extremely small, we conclude that the assumption is probably not correct.

Identifying Unusual Results Probabilities

Using Probabilities to Determine When Results Are Unusual

- **Unusually high**: x successes among n trials is an **unusually high** number of successes if $P(x \text{ or more}) \le 0.05$.

- **Unusually low**: x successes among n trials is an **unusually low** number of successes if $P(x \text{ or fewer}) \le 0.05$.

EXAMPLE 7 **Identifying Unusual Results with Probabilities** Use *probabilities* to determine whether 1 is an unusually low number of peas with green pods when 5 offspring are generated from parents both having the green/yellow pair of genes.

SOLUTION To determine whether 1 is an unusually low number of peas with green pods (among 5 offspring), we need to find the probability of getting 1 or fewer peas with green pods. By referring to Table 5-1 on page 205 we can easily get the following results:

$$P(1 \text{ or fewer}) = P(1 \text{ or } 0) = 0.015 + 0.001 = 0.016.$$

INTERPRETATION Because the probability 0.016 is less than 0.05, we conclude that the result of 1 pea with a green pod is *unusually low*. There is a very small likelihood (0.016) of getting 1 or fewer peas with green pods.

(Number of Peas with Green Pods)	$P(x)$
0	0.001
1	0.015
2	0.088
3	0.264
4	0.396
5	0.237

Expected Value

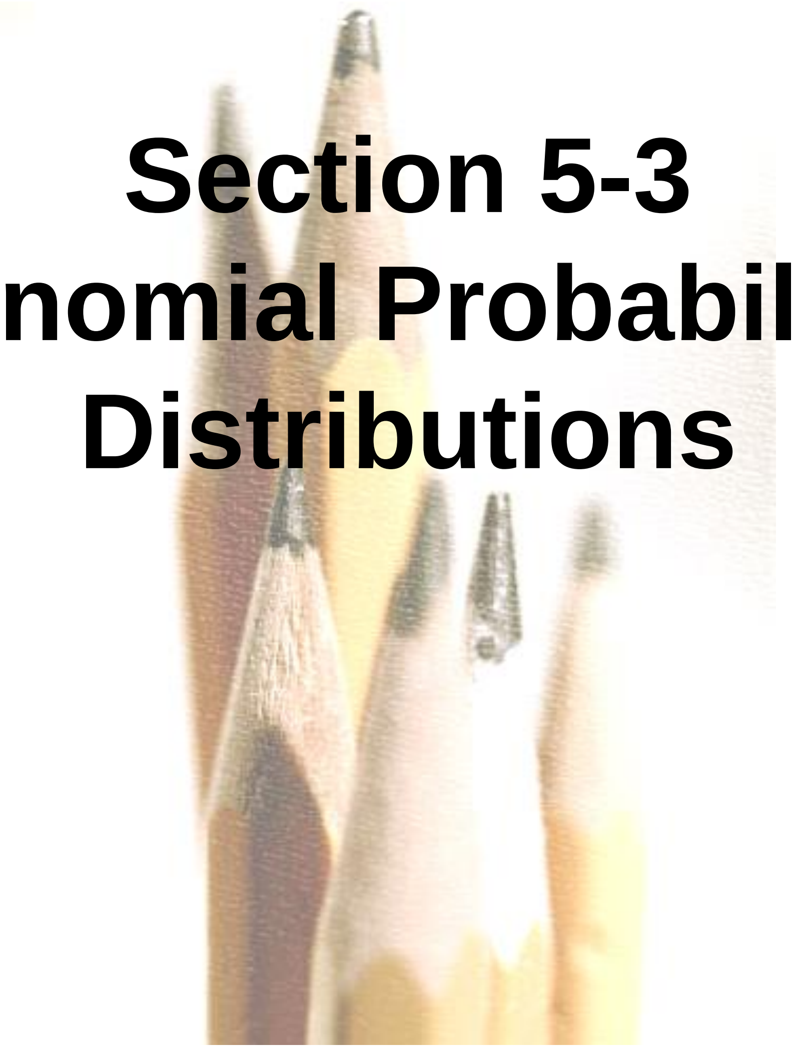
The **expected value** of a discrete random variable is denoted by E, and it represents the mean value of the outcomes. It is obtained by finding the value of $\Sigma\,[x \cdot P(x)]$.

$$E = \Sigma\,[x \cdot P(x)]$$

Recap

In this section we have discussed:

- **Combining methods of descriptive statistics with probability.**
- **Random variables and probability distributions.**
- **Probability histograms.**
- **Requirements for a probability distribution.**
- **Mean, variance and standard deviation of a probability distribution.**
- **Identifying unusual results.**
- **Expected value.**



Section 5-3
Binomial Probability Distributions

Key Concept

This section presents a basic definition of a binomial distribution along with notation, and methods for finding probability values.

Binomial probability distributions allow us to deal with circumstances in which the outcomes belong to two relevant categories such as acceptable/defective or survived/died.

Binomial Probability Distribution

A binomial probability distribution results from a procedure that meets all the following requirements:

1. **The procedure has a fixed number of trials.**
2. **The trials must be independent. (The outcome of any individual trial doesn't affect the probabilities in the other trials.)**
3. **Each trial must have all outcomes classified into two categories (commonly referred to as success and failure).**
4. **The probability of a success remains the same in all trials.**

Notation for Binomial Probability Distributions

S and F (success and failure) denote the two possible categories of all outcomes; p and q will denote the probabilities of S and F, respectively, so

$P(S) = p$ (p = probability of success)

$P(F) = 1 - p = q$ (q = probability of failure)

Notation (continued)

n denotes the fixed number of trials.

x denotes a specific number of successes in ***n*** trials, so ***x*** can be any whole number between **0** and ***n***, inclusive.

p denotes the probability of **success** in ***one*** of the ***n*** trials.

q denotes the probability of **failure** in **one** of the ***n*** trials.

P*(*x*)** denotes the probability of getting exactly ***x successes among the ***n*** trials.

Important Hints

The word *success* as used here is arbitrary and does not necessarily represent something good. Either of the two possible categories may be called the success S as long as its probability is identified as *p.* (The value of *q* can always be found by subtracting *p* from 1)

- **Be sure that *x* and *p* both refer to the same category being called a success.**

- **When sampling without replacement, consider events to be independent if $n < 0.05N$.**

EXAMPLE 1 **Genetics** Consider an experiment in which 5 offspring peas are generated from 2 parents each having the green/yellow combination of genes for pod color. Recall from the Chapter Problem that the probability an offspring pea will have a green pod is $^3/_4$ or 0.75. That is, $P(\text{green pod}) = 0.75$. Suppose we want to find the probability that exactly 3 of the 5 offspring peas have a green pod.

a. Does this procedure result in a binomial distribution?

b. If this procedure does result in a binomial distribution, identify the values of n, x, p, and q.

SOLUTION

a. This procedure does satisfy the requirements for a binomial distribution, as shown below.

1. The number of trials (5) is fixed.
2. The 5 trials are independent, because the probability of any offspring pea having a green pod is not affected by the outcome of any other offspring pea.
3. Each of the 5 trials has two categories of outcomes: The pea has a green pod or it does not.
4. For each offspring pea, the probability that it has a green pod is 3/4 or 0.75, and that probability remains the same for each of the 5 peas.

b. Having concluded that the given procedure does result in a binomial distribution, we now proceed to identify the values of n, x, p, and q.

1. With 5 offspring peas, we have $n = 5$.
2. We want the probability of exactly 3 peas with green pods, so $x = 3$.
3. The probability of success (getting a pea with a green pod) for one selection is 0.75, so $p = 0.75$.
4. The probability of failure (not getting a green pod) is 0.25, so $q = 0.25$.

Again, it is very important to be sure that x and p both refer to the same concept of "success." In this example, we use x to count the number of peas with green pods, so p must be the probability that a pea has a green pod. Therefore, x and p do use the same concept of success (green pod) here.

Methods for Finding Probabilities

We will now discuss the methods for finding the probabilities corresponding to the random variable x in a binomial distribution.

Probability: Using the Binomial Probability Formula

$$P(x) = \frac{n\,!}{(n - x\,)!x!} \cdot p^{x} \cdot q^{n-x}$$

for $x = 0, 1, 2, \ldots, n$

where

n = number of trials

x = number of successes among n trials

p = probability of success in any one trial

q = probability of failure in any one trial ($q = 1 - p$)

SC EXAMPLE 2 **Genetics** Assuming that the probability of a pea having a green pod is 0.75 (as in the Chapter Problem and Example 1), use the binomial probability formula to find the probability of getting exactly 3 peas with green pods when 5 offspring peas are generated. That is, find $P(3)$ given that $n = 5$, $x = 3$, $p = 0.75$, and $q = 0.25$.

SOLUTION Using the given values of n, x, p, and q in the binomial probability formula (Formula 5-5), we get

$$\begin{aligned} P(3) &= \frac{5!}{(5-3)!\,3!} \cdot 0.75^3 \cdot 0.25^{5-3} \\ &= \frac{5!}{2!3!} \cdot 0.421875 \cdot 0.0625 \\ &= (10)(0.421875)(0.0625) = 0.263671875 \end{aligned}$$

The probability of getting exactly 3 peas with green pods among 5 offspring peas is 0.264 (rounded to three significant digits).

SC EXAMPLE 3 **McDonald's Brand Recognition** The fast food chain McDonald's has a brand name recognition rate of 95% around the world (based on data from Retail Marketing Group). Assuming that we randomly select 5 people, use Table A-1 to find the following.

a. The probability that exactly 3 of the 5 people recognize McDonald's

b. The probability that the number of people who recognize McDonald's is 3 or fewer

SOLUTION

a. The displayed excerpt from Table A-1 on the top of the next page shows that when $n = 5$ and $p = 0.95$, the probability of $x = 3$ is given by $P(3) = 0.021$.

b. "3 or fewer" successes means that the number of successes is 3 or 2 or 1 or 0.

$$\begin{aligned} P(3 \text{ or fewer}) &= P(3 \text{ or } 2 \text{ or } 1 \text{ or } 0) \\ &= P(3) + P(2) + P(1) + P(0) \\ &= 0.021 + 0.001 + 0 + 0 \\ &= 0.022 \end{aligned}$$

Example from word file

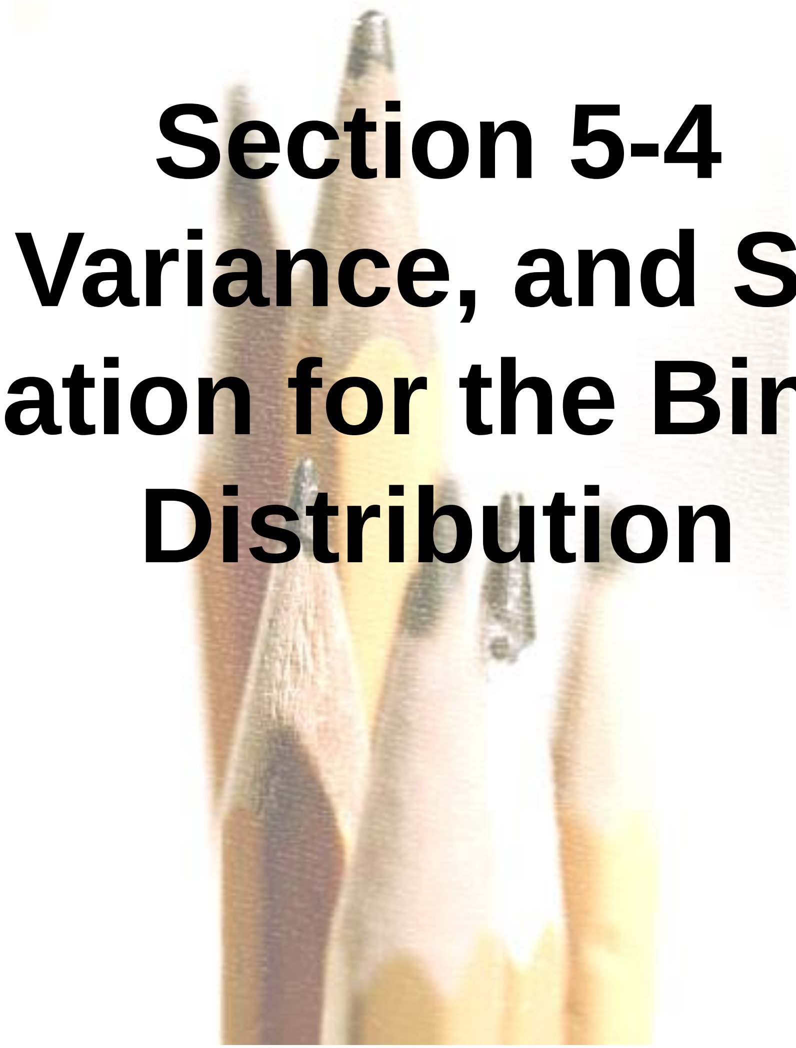
Binomial Distribution

Recap

In this section we have discussed:

- The definition of the binomial probability distribution.
- Notation.
- Important hints.
- Three computational methods.
- Rationale for the formula.

Section 5-4
Mean, Variance, and Standard Deviation for the Binomial Distribution



Key Concept

In this section we consider important characteristics of a binomial distribution including center, variation and distribution. That is, given a particular binomial probability distribution we can find its mean, variance and standard deviation.

A strong emphasis is placed on interpreting and understanding those values.

For Any Discrete Probability Distribution: Formulas

Mean $\mu = \Sigma[x \cdot P(x)]$

Variance $\sigma^2 = [\Sigma x^2 \cdot P(x)] - \mu^2$

Std. Dev $\sigma = \sqrt{[\Sigma x^2 \cdot P(x)] - \mu^2}$

Binomial Distribution: Formulas

Mean $\mu = n \bullet p$

Variance $\sigma^2 = n \bullet p \bullet q$

Std. Dev. $\sigma = \sqrt{n \bullet p \bullet q}$

Where

n **= number of fixed trials**

p **= probability of success in one of the** ***n*** **trials**

q **= probability of failure in one of the** ***n*** **trials**

Any of the method mentioned above or in previous slide may be used

EXAMPLE 1 **Genetics** Use Formulas 5-6 and 5-8 to find the mean and standard deviation for the numbers of peas with green pods when groups of 5 offspring peas are generated. Assume that there is a 0.75 probability that an offspring pea has a green pod (as described in the Chapter Problem).

SOLUTION Using the values $n = 5$, $p = 0.75$, and $q = 0.25$, Formulas 5-6 and 5-8 can be applied as follows:

$$\mu = np = (5)(0.75) = 3.8 \quad \text{(rounded)}$$
$$\sigma = \sqrt{npq} = \sqrt{(5)(0.75)(0.25)} = 1.0 \quad \text{(rounded)}$$

Interpretation of Results

It is especially important to interpret results. The **range rule of thumb** suggests that values are unusual if they lie outside of these limits:

$$\text{Maximum usual values} = \mu + 2\sigma$$

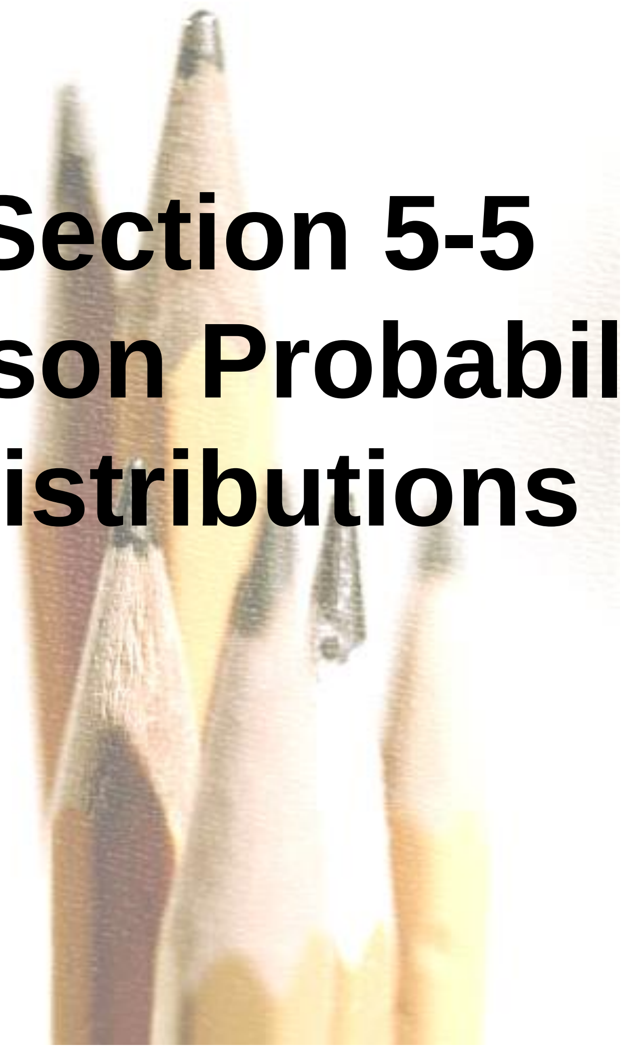
$$\text{Minimum usual values} = \mu - 2\sigma$$

Recap

In this section we have discussed:

- **Mean, variance and standard deviation formulas for any discrete probability distribution.**
- **Mean, variance and standard deviation formulas for the binomial probability distribution.**
- **Interpreting results.**

Section 5-5
Poisson Probability Distributions



Key Concept

The Poisson distribution is another discrete probability distribution which is important because it is often used for describing the behavior of rare events (with small probabilities).

Poisson Distribution

The Poisson distribution is a discrete probability distribution that applies to occurrences of some event over a specified interval. The random variable *x* is the number of occurrences of the event in an interval. The interval can be time, distance, area, volume, or some similar unit.

Formula

$$P(x) = \frac{\mu^{x} \cdot e^{-\mu}}{x!} \quad \text{where } e \approx 2.71828$$

Requirements of the Poisson Distribution

- The random variable x is the number of occurrences of an event over some interval.
- The occurrences must be random.
- The occurrences must be independent of each other.
- The occurrences must be uniformly distributed over the interval being used.

Parameters

- The mean is μ.
- The standard deviation is $\sigma = \sqrt{\mu}$.

Parameters of the Poisson Distribution

- **The mean is μ.**

- **The standard deviation is $\sigma = \sqrt{\mu}$.**

SC EXAMPLE 1

Earthquakes For a recent period of 100 years, there were 93 major earthquakes (measuring at least 6.0 on the Richter scale) in the world (based on data from the *World Almanac and Book of Facts*). Assume that the Poisson distribution is a suitable model.

a. Find the mean number of major earthquakes per year.

b. If $P(x)$ is the probability of x earthquakes in a randomly selected year, find $P(0)$, $P(1)$, $P(2)$, $P(3)$, $P(4)$, $P(5)$, $P(6)$, and $P(7)$.

c. The actual results are as follows: 47 years (0 major earthquakes); 31 years (1 major earthquake); 13 years (2 major earthquakes); 5 years (3 major earthquakes); 2 years (4 major earthquakes); 0 years (5 major earthquakes); 1 year (6 major earthquakes); 1 year (7 major earthquakes). How do these actual results compare to the probabilities found in part (b)? Does the Poisson distribution appear to be a good model in this case?

SOLUTION

a. The Poisson distribution applies because we are dealing with the occurrences of a event (earthquakes) over some interval (a year). The mean number of earthquake per year is

$$\mu = \frac{\text{number of earthquakes}}{\text{number of years}} = \frac{93}{100} = 0.93$$

b. Using Formula 5-9, the calculation for $x = 2$ earthquakes in a year is as follows (with μ replaced by 0.93 and e replaced by 2.71828):

$$P(2) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \frac{0.93^2 \cdot 2.71828^{-0.93}}{2!} = \frac{0.8649 \cdot 0.394554}{2} = 0.171$$

The probability of exactly 2 earthquakes in a year is $P(2) = 0.171$. Using the same procedure to find the other probabilities, we get these results: $P(0) = 0.395$, $P(1) = 0.367$, $P(2) = 0.171$, $P(3) = 0.0529$, $P(4) = 0.0123$, $P(5) = 0.00229$, $P(6) = 0.000355$, and $P(7) = 0.0000471$.

c. The probability of $P(0) = 0.395$ from part (b) is the likelihood of getting 0 earthquakes in one year. So in 100 years, the expected number of years with 0 earthquakes is $100 \times 0.395 = 39.5$ years. Using the probabilities from part (b), here are all of the expected frequencies: 39.5, 36.7, 17.1, 5.29, 1.23, 0.229, 0.0355, and 0.00471. These expected frequencies compare reasonably well with the actual frequencies of 47, 31, 13, 5, 2, 0, 1, and 1. Because the expected frequencies agree reasonably well with the actual frequencies, the Poisson distribution is a good model in this case.

Difference from a Binomial Distribution

The Poisson distribution differs from the binomial distribution in these fundamental ways:

- **The binomial distribution is affected by the sample size n and the probability p, whereas the Poisson distribution is affected only by the mean μ.**
- **In a binomial distribution the possible values of the random variable x are 0, 1, . . . n, but a Poisson distribution has possible x values of 0, 1, 2, . . . , with no upper limit.**

Poisson as an Approximation to the Binomial Distribution

The Poisson distribution is sometimes used to approximate the binomial distribution when n is large and p is small.

Rule of Thumb

- $n \geq 100$
- $np \leq 10$

Poisson as an Approximation to the Binomial Distribution - μ

If both of the following requirements are met,

- $n \geq 100$
- $np \leq 10$

then use the following formula to calculate μ,

Value for μ

$$\mu = n \cdot p$$

EXAMPLE 2

Illinois Pick 3 In the Illinois Pick 3 game, you pay 50¢ to select a sequence of three digits, such as 729. If you play this game once every day, find the probability of winning exactly once in 365 days.

SOLUTION

Because the time interval is 365 days, $n = 365$. Because there is one winning set of numbers among the 1000 that are possible (from 000 to 999), $p = 1/1000$. With $n = 365$ and $p = 1/1000$, the conditions $n \geq 100$ and $np \leq 10$ are both satisfied, so we can use the Poisson distribution as an approximation to the binomial distribution. We first need the value of μ, which is found as follows:

$$\mu = np = 365 \cdot \frac{1}{1000} = 0.365$$

Having found the value of μ, we can now find $P(1)$ by using $x = 1$, $\mu = 0.365$, and $e = 2.71828$, as shown here:

$$P(1) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \frac{0.365^1 \cdot 2.71828^{-0.365}}{1!} = \frac{0.253}{1} = 0.253$$

Recap

In this section we have discussed:

- **Definition of the Poisson distribution.**
- **Requirements for the Poisson distribution.**
- **Difference between a Poisson distribution and a binomial distribution.**
- **Poisson approximation to the binomial.**