Statistics for Describing, Exploring, and Comparing Data

1 Measures of Center or Central Tendency

2 Measures of Variation or Dispersion

Review

- **Chapter 1**

  Distinguish between population and sample, parameter and statistic

  Good sampling methods: *simple random sample*, collect in appropriate ways

Preview

- **Important Statistics**

  Mean, median, standard deviation, variance

- **Understanding and Interpreting**

  these important statistics

Preview

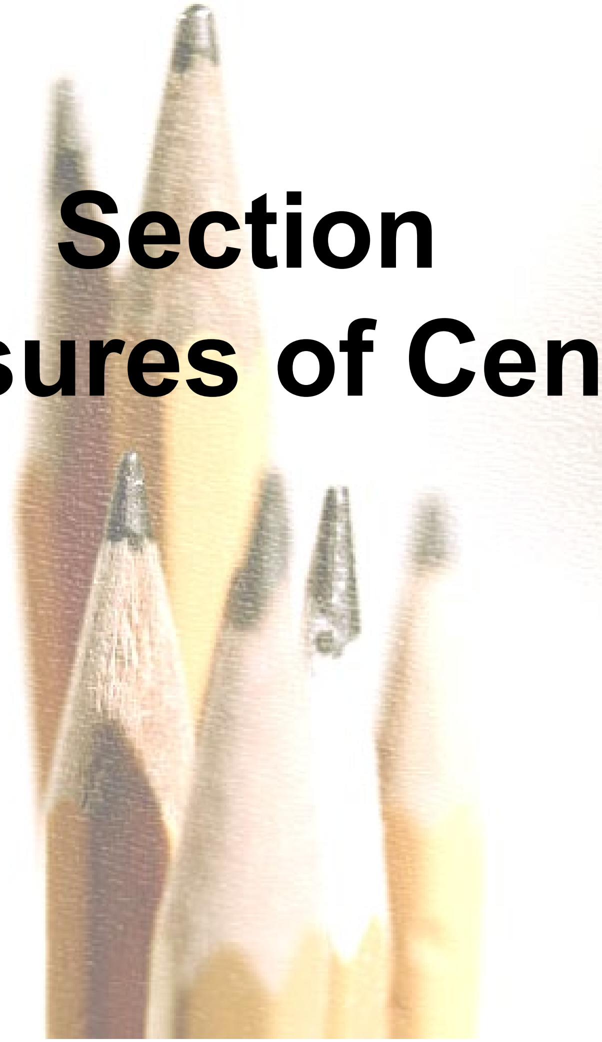
- **Descriptive Statistics**

  In this chapter we'll learn to summarize or describe the important characteristics of a known set of data

- **Inferential Statistics**

  In later chapters we'll learn to use sample data to make inferences or generalizations about a population

Section
Measures of Center



Key Concept

Characteristics of center. Measures of center, including mean and median, as tools for analyzing data. Not only determine the value of each measure of center, but also interpret those values.

Part 1

Basics Concepts of Measures of Center

Measure of Center

- **Measure of Center**

  the value at the center or middle of a data set

Arithmetic Mean

- **Arithmetic Mean (Mean)**

  the measure of center obtained by adding the values and dividing the total by the number of values

What most people call an *average*.

Notation

Σ denotes the **sum** of a set of values.

x is the **variable** usually used to represent the individual data values.

n represents the **number of data values** in a **sample**.

N represents the **number of data values** in a **population**.

Notation

$\bar{x}$ is pronounced 'x-bar' and denotes the mean of a set of **sample** values

$$\bar{x} = \frac{\Sigma x}{n}$$

μ is pronounced 'mu' and denotes the mean of all values in a **population**

$$\mu = \frac{\Sigma x}{N}$$

Mean

- **Advantages**

  Is relatively reliable, means of samples drawn from the same population don't vary as much as other measures of center

  Takes every data value into account

- **Disadvantage**

  Is sensitive to every data value, one extreme value can affect it dramatically; is not a *resistant* measure of center

Median

- **Median**

  the middle value when the original data values are arranged in order of increasing (or decreasing) magnitude

- **often denoted by $\tilde{x}$ (pronounced 'x-tilde')**

- **is not affected by an extreme value - is a resistant measure of the center**

Finding the Median

First *sort* the values (arrange them in order), the follow one of these

1. **If the number of data values is odd, the median is the number located in the exact middle of the list.**

2. **If the number of data values is even, the median is found by computing the mean of the two middle numbers.**

5.40	1.10	0.42	0.73	0.48	1.10
0.42	0.48	0.73	1.10	1.10	5.40

(in order - even number of values – no exact middle shared by two numbers)

$$\frac{0.73 + 1.10}{2}$$

MEDIAN is 0.915

5.40	1.10	0.42	0.73	0.48	1.10	0.66
0.42	0.48	0.66	0.73	1.10	1.10	5.40

(in order - odd number of values)

exact middle **MEDIAN is 0.73**

Mode

- **Mode**
  the value that occurs with the greatest frequency

- Data set can have one, more than one, or no mode

Bimodal	two data values occur with the same greatest frequency
Multimodal	more than two data values occur with the same greatest frequency
No Mode	no data value is repeated

Mode is the only measure of central tendency that can be used with nominal data

Mode - Examples

a. 5.40 1.10 0.42 0.73 0.48 1.10 ⇦ Mode is 1.10

b. 27 27 27 55 55 55 88 88 99 ⇦ Bimodal - 27 & 55

c. 1 2 3 6 7 8 9 10 ⇦ No Mode

Definition

- **Midrange**
  the value midway between the maximum and minimum values in the original data set

$$\text{Midrange} = \frac{\text{maximum value} + \text{minimum value}}{2}$$

Midrange

- **Sensitive to extremes**
  because it uses only the maximum and minimum values, so rarely used

- **Redeeming Features**

  (1) very easy to compute

  (2) reinforces that there are several ways to define the center

  (3) Avoids confusion with median

Round-off Rule for Measures of Center

Carry one more decimal place than is present in the original set of values.

Critical Thinking

Think about whether the results are reasonable.

Think about the method used to collect the sample data.

Part 2

Beyond the Basics of Measures of Center

Mean from a Frequency Distribution

Assume that all sample values in each class are equal to the class midpoint.

Mean from a Frequency Distribution

use class midpoint of classes for variable x

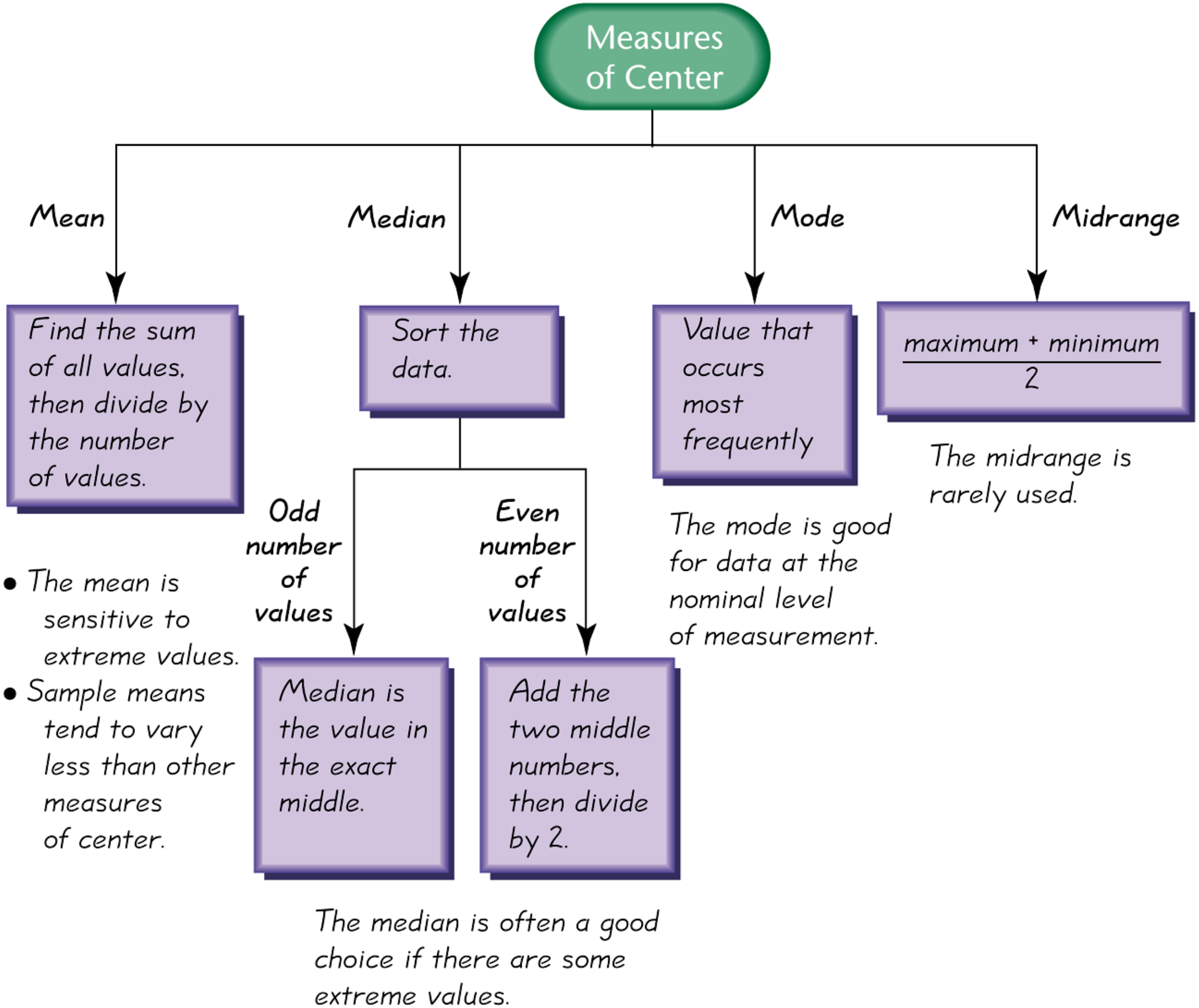
$$\bar{x} = \frac{\Sigma(f \cdot x)}{\Sigma f}$$

Weighted Mean

When data values are assigned different weights, we can compute a weighted mean.

$$\bar{x} = \frac{\Sigma\,(w \cdot x)}{\Sigma\,w}$$

Best Measure of Center



Skewed and Symmetric

- **Symmetric**

  distribution of data is symmetric if the left half of its histogram is roughly a mirror image of its right half

- **Skewed**

  distribution of data is skewed if it is not symmetric and extends more to one side than the other

Skewed Left or Right

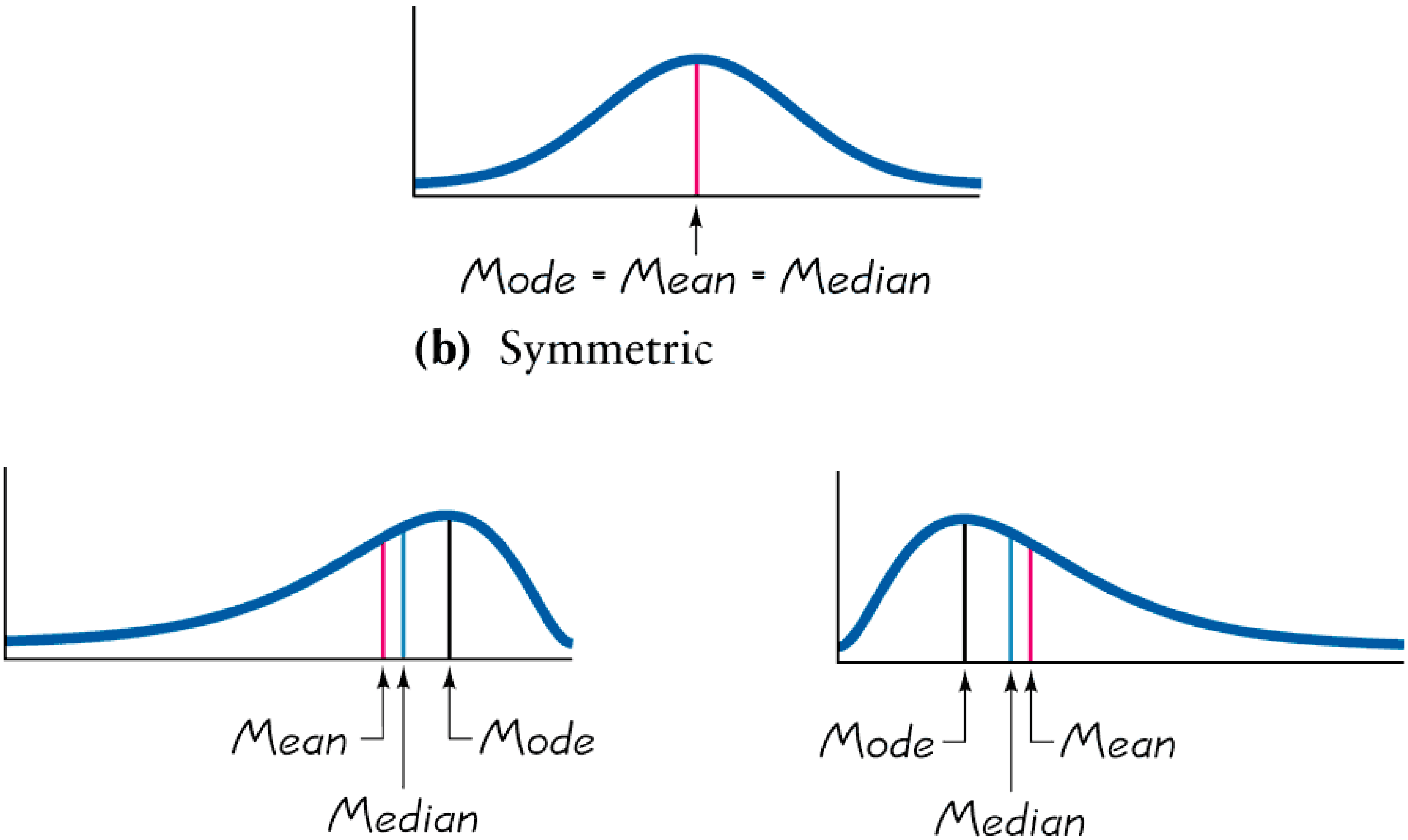
- **Skewed to the left**

  (also called negatively skewed) have a longer left tail, mean and median are to the left of the mode

- **Skewed to the right**

  (also called positively skewed) have a longer right tail, mean and median are to the right of the mode

Skewness



(b) Symmetric

(a) Skewed to the Left (Negatively)

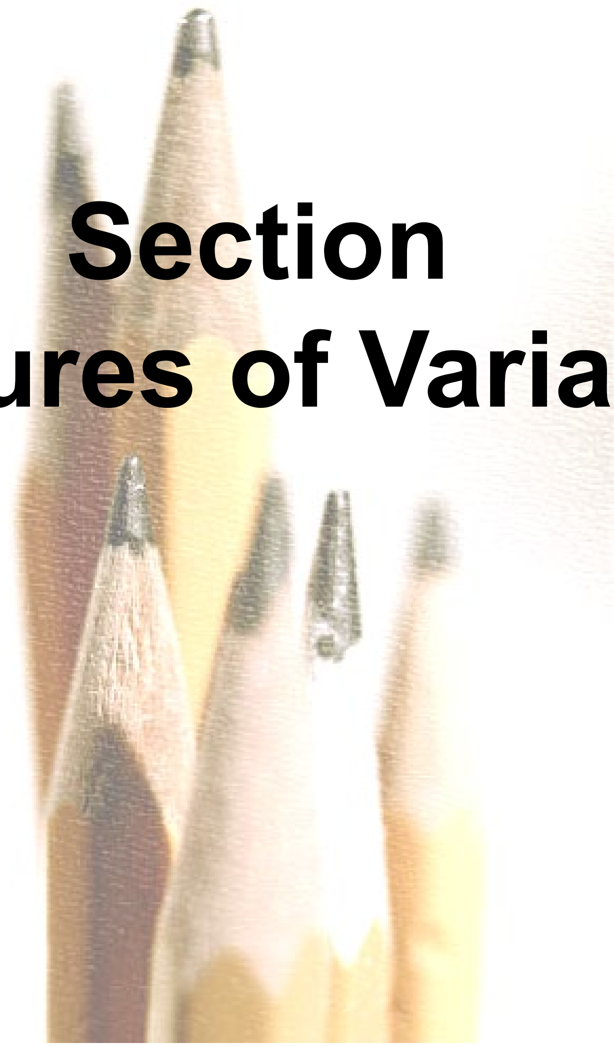
(c) Skewed to the Right (Positively)

Recap

In this section we have discussed:

- Types of measures of center
    - Mean
    - Median
    - Mode
- Mean from a frequency distribution
- Weighted means
- Best measures of center
- Skewness

Section
Measures of Variation



Key Concept

Discuss characteristics of variation, in particular, measures of variation, such as standard deviation, for analyzing data.

Make understanding and interpreting the standard deviation a priority.

Part 1

Basics Concepts of Measures of Variation

Definition

The **range** of a set of data values is the difference between the maximum data value and the minimum data value.

Range = (maximum value) – (minimum value)

It is very sensitive to extreme values; therefore not as useful as other measures of variation.

Round-Off Rule for Measures of Variation

When rounding the value of a measure of variation, carry one more decimal place than is present in the original set of data.

Round only the final answer, not values in the middle of a calculation.

Definition

The standard deviation of a set of sample values, denoted by *s*, is a measure of variation of values about the mean.

Sample Standard Deviation Formula

$$s = \sqrt{\frac{\Sigma\,(x - \bar{x})^2}{n - 1}}$$

Sample Standard Deviation (Shortcut Formula)

$$s = \sqrt{\frac{n\Sigma(x^2) - (\Sigma x)^2}{n\,(n-1)}}$$

Standard Deviation - Important Properties

- The standard deviation is a measure of variation of all values from the **mean**.
- The value of the standard deviation s is usually positive.
- The value of the standard deviation s can increase dramatically with the inclusion of one or more outliers (data values far away from all others).
- The units of the standard deviation s are the same as the units of the original data values.

Comparing Variation in Different Samples

It's a good practice to compare two sample standard deviations only when the sample means are approximately the same.

When comparing variation in samples with very different means, it is better to use the coefficient of variation, which is defined later in this section.

Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}$$

This formula is similar to the previous formula, but instead, the population mean and population size are used.

Variance

- The **variance** of a set of values is a measure of variation equal to the square of the standard deviation.

- Sample variance: s^2 - Square of the sample standard deviation s

- Population variance: σ^2 - Square of the population standard deviation σ

Unbiased Estimator

The sample variance s^2 is an unbiased estimator of the population variance σ^2, which means values of s^2 tend to target the value of σ^2 instead of systematically tending to overestimate or underestimate σ^2.

Variance - Notation

s = *sample* standard deviation

s^2 = *sample* variance

σ = *population* standard deviation

σ^2 = *population* variance

Part 2

Beyond the Basics of Measures of Variation

Range Rule of Thumb

is based on the principle that for many data sets, the vast majority (such as 95%) of sample values lie within two standard deviations of the mean.

Range Rule of Thumb for Interpreting a Known Value of the Standard Deviation

Informally define *usual* values in a data set to be those that are typical and not too extreme. Find rough estimates of the minimum and maximum "usual" sample values as follows:

Minimum "usual" value = (mean) – 2 × (standard deviation)

Maximum "usual" value = (mean) + 2 × (standard deviation)

Range Rule of Thumb for Estimating a Value of the Standard Deviation *s*

To roughly estimate the standard deviation from a collection of known sample data use

$$s \approx \frac{\text{range}}{4}$$

where

range = (maximum value) – (minimum value)

Properties of the Standard Deviation

- Measures the variation among data values
- Values close together have a small standard deviation, but values with much more variation have a larger standard deviation
- Has the same units of measurement as the original data

Properties of the Standard Deviation

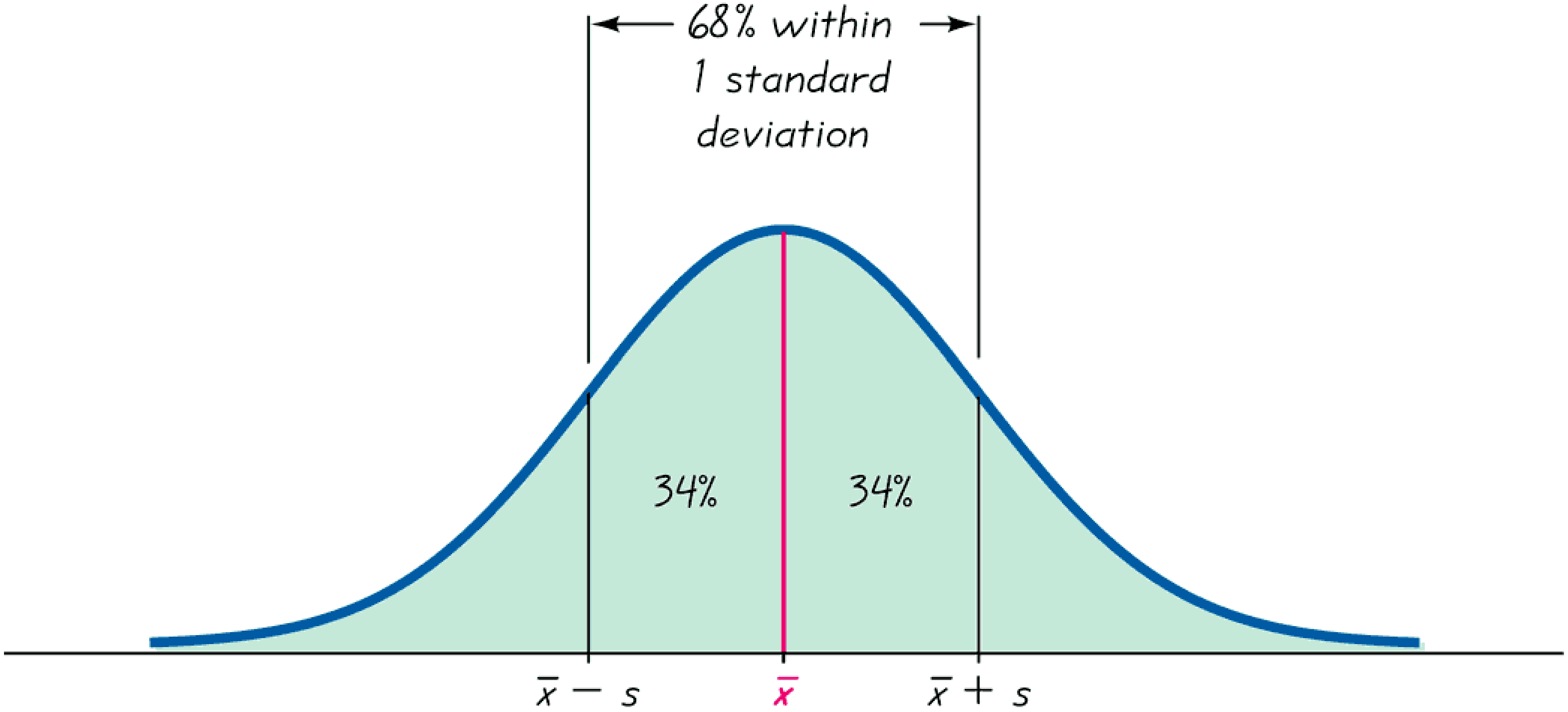
- For many data sets, a value is *unusual* if it differs from the mean by more than two standard deviations
- Compare standard deviations of two different data sets only if the they use the same scale and units, and they have means that are approximately the same

Empirical (or 68-95-99.7) Rule

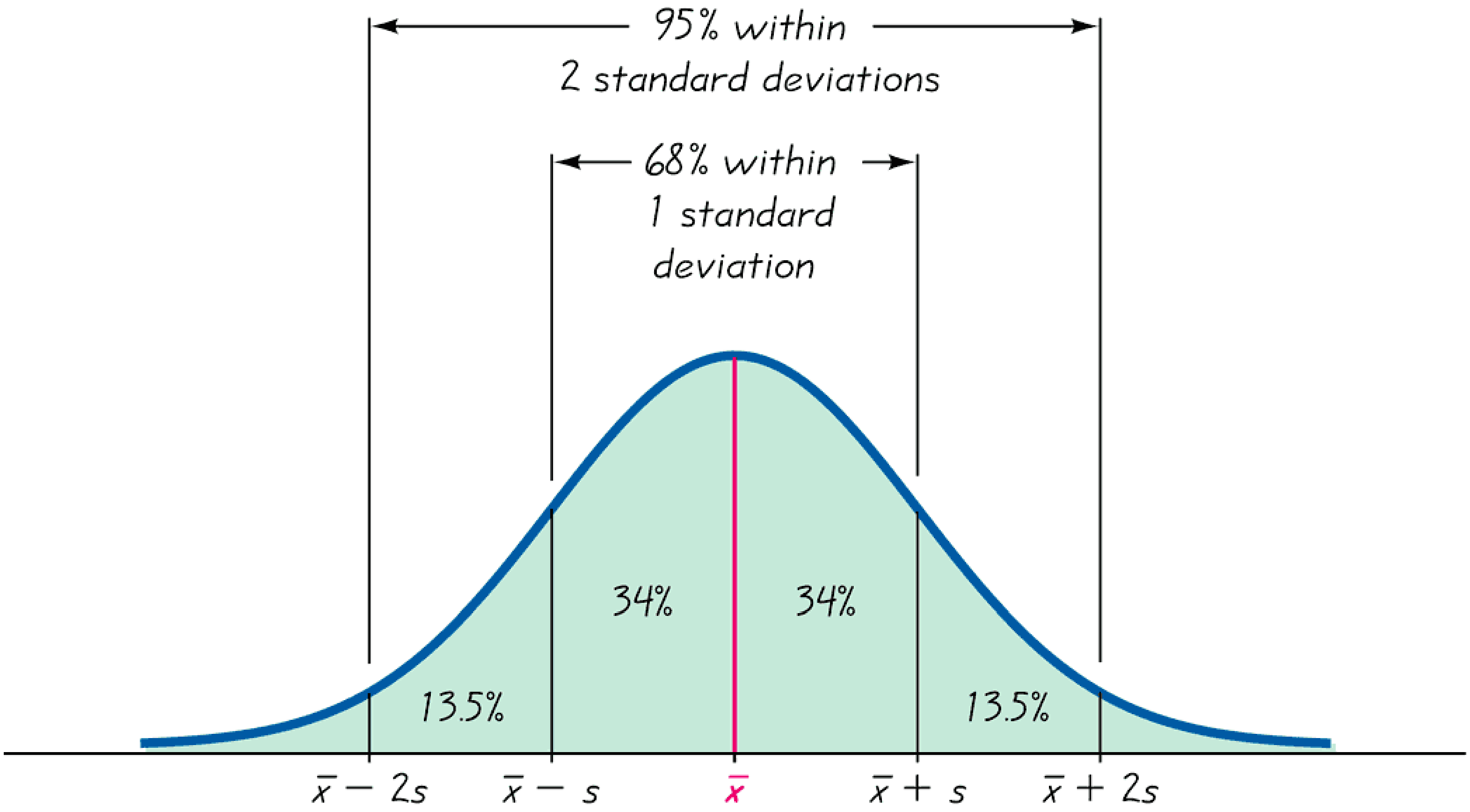
For data sets having a distribution that is approximately bell shaped, the following properties apply:

- About 68% of all values fall within 1 standard deviation of the mean.
- About 95% of all values fall within 2 standard deviations of the mean.
- About 99.7% of all values fall within 3 standard deviations of the mean.

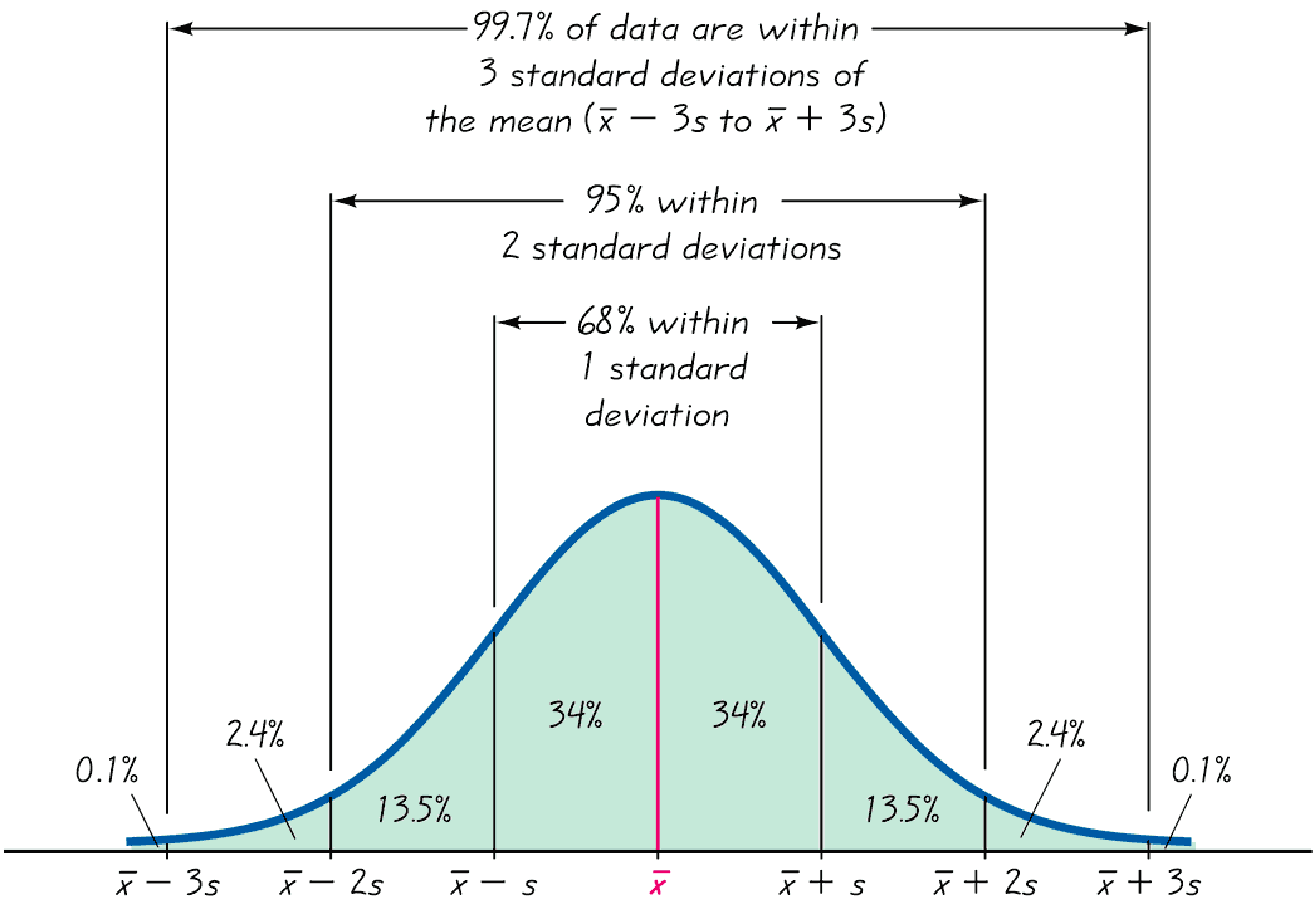
The Empirical Rule



The Empirical Rule



The Empirical Rule



Outliers

- **An outlier is a value that lies very far away from the vast majority of the other values in a data set.**

Important Principles

- An outlier can have a dramatic effect on the mean.
- An outlier can have a dramatic effect on the standard deviation.
- An outlier can have a dramatic effect on the scale of the histogram so that the true nature of the distribution is totally obscured.

Recap

In this section we have looked at:

- Range
- Standard deviation of a sample and population
- Variance of a sample and population
- Range rule of thumb
- Empirical distribution