1. Optimization problem example

Let a circular cylindrical can has the height h and radius r.

The volume of the can is

$$V = \pi r^2 h$$

Then,

$$h = \frac{V}{\pi r^2}$$

The surface area of the can is

$$S = 2\pi r h + 2\pi r^2$$

We want to minimize the surface area, the objective functions is

$$\min_{h,r\in\mathbb{R}} 2\pi r h + 2\pi r^2$$

As $h = \frac{V}{\pi r^2}$,

$$\begin{aligned} S &= 2\pi r \frac{V}{\pi r^2} + 2\pi r^2 \\ &= \frac{2V}{r} + 2\pi r^2 \end{aligned}$$

Then,

$$\begin{aligned} \frac{\partial S}{\partial r} &= \frac{-2V}{r^2} + 4\pi r \\ 0 &= \frac{-2V}{r^2} + 4\pi r \end{aligned}$$

We will get,

$$\begin{aligned} r^* &= \sqrt[3]{\frac{V}{2\pi}} \\ h^* &= \sqrt[3]{\frac{4V}{\pi}} \end{aligned}$$

Check if second order optimality condition is greater than 0,

$$\frac{\partial^2 S}{\partial^2 r} = \frac{4V}{r^3} + 4\pi > 0 \quad \text{; since r, V, } \pi > 0$$

So that, $r^* = \sqrt[3]{\frac{V}{2\pi}}$ and $h^* = \sqrt[3]{\frac{4V}{\pi}}$

2. Optimally conditions

From the optimality conditions,

$$\begin{aligned} \nabla f(x^*) = 0 \quad (f'(x^*) = 0) \\ \nabla^2 f(x^*) \succ 0 \quad (f''(x^*) > 0) \end{aligned}$$

We have function

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$$f(x_1, x_2) = \frac{3}{2}(x_1^2 + x_2^2) + (1+a)x_1x_2 - (x_1 + x_2) + b, \quad a, b \in \mathbb{R}$$

First order optimality condition,

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = 3x_1 + (1+a)x_2 - 1$$
$$\frac{\partial f(x_1, x_2)}{\partial x_2} = 3x_2 + (1+a)x_1 - 1$$
$$\nabla f(x^*) = \begin{bmatrix} 3x_1 + (1+a)x_2 - 1 \\ 3x_2 + (1+a)x_1 - 1 \end{bmatrix} = 0$$

We can derive,

$$\begin{bmatrix} 3 & 1+a \\ 1+a & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Second order optimality condition,

$$\nabla^2 f(x^*) = \begin{bmatrix} 3 & 1+a \\ 1+a & 3 \end{bmatrix} \succ 0$$

We know that $\nabla^2 f(x^*)$ is positive definite matrix, then the determinant of it is positive.

$$\begin{vmatrix} 3 & 1+a \\ 1+a & 3 \end{vmatrix} > 0$$
$$9 - (1+a)^2 > 0$$
$$(1+a)^2 < 9$$
$$-3 < 1+a < 3$$

So that,

$$-4 < a < 2 \quad \text{and} \quad b \in \mathbb{R}$$

3. Nelder–Mead method

Mishra's Bird function

$$f(x, y) = sin(y)e^{(1-cos(x))^2} + cos(x)e^{(1-sin(y))^2} + (x-y)^2$$

subjected to,

$$(x+5)^2 + (y+5)^2 < 25$$

1. To illustrate the behavior of the method, plot simplex (triangle) for every iterations

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Circle
```

```
# define function for illustration
def fn(point):
    x, y = point
    return np.sin(y)*np.exp((1 - np.cos(x))**2) + np.cos(x)*np.exp((1 - np.sin(y))**2
```

```
# illustrate function
```