MSc Program

Portfolio Sold-Out Problem in Numbers for DeFi Lending Protocols

Student: *Waralak Pariwatphan*
Research Advisor: *Yury Yanovich*

Skoltech

April, 2023

Background: DeFi Meets Classic Finance

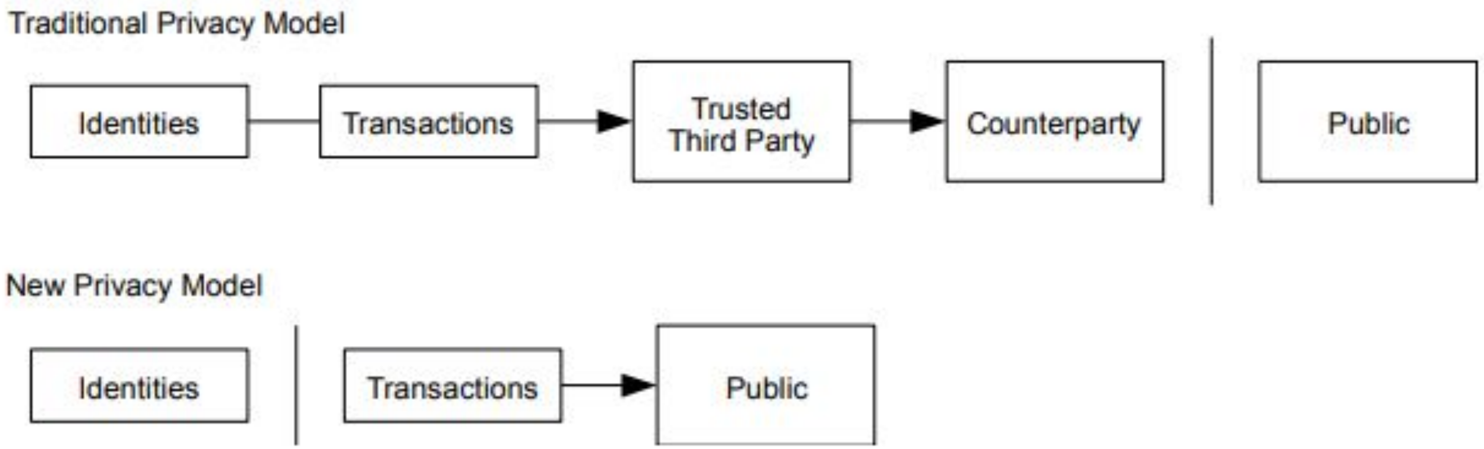
Decentralized Finance: peer-to-peer financial services on public blockchains.

Lending protocols

BANK FOR INTERNATIONAL SETTLEMENTS

Parameters:
- Probability of default (PD)
- Loss Given Default (LGD)
- Liquidity coverage ratio (LCR)

Bank secrecy



[Nakamoto, S. (2008). Bitcoin: A Peer-to-Peer Electronic Cash System. *Www.Bitcoin.Org*, 1–9. https://bitcoin.org/bitcoin.pdf]

The loan mechanism on the Maker protocol

Loan Portfolio Tokenization: Summary

- Bank loans can be tokenized and regrouped into commodity

→

- New competitive tools for small investors
- Free secondary market

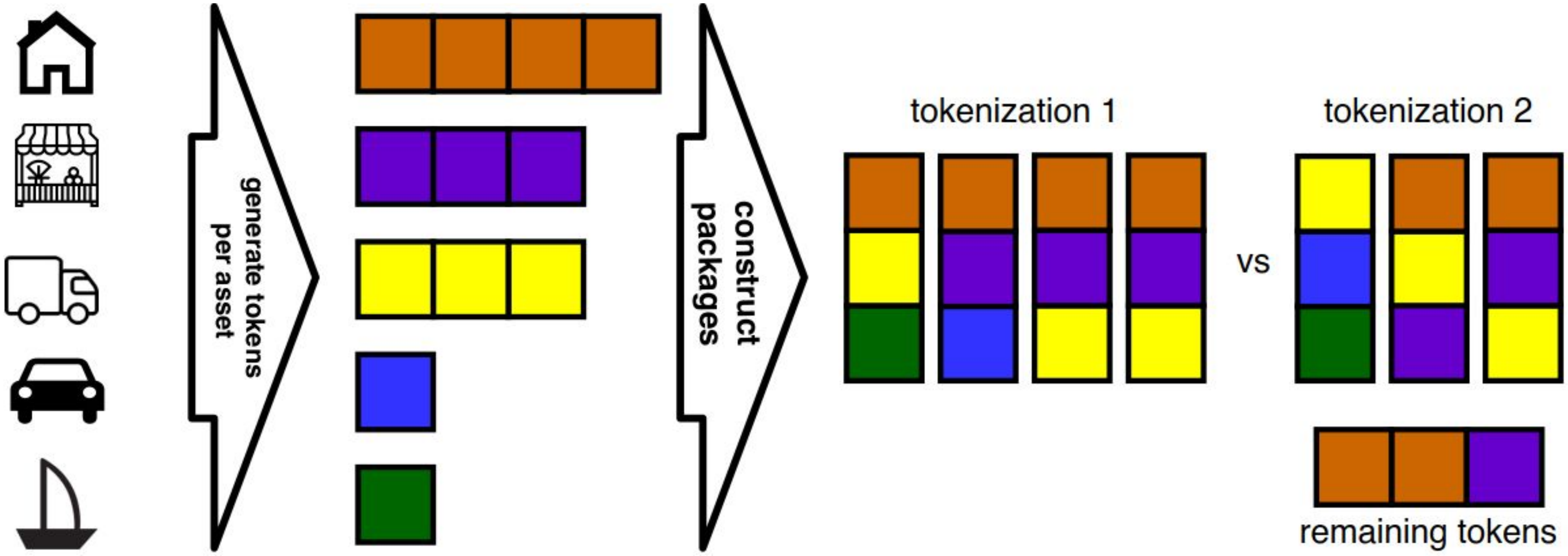
- Lack of bank auditability by government
- Free secondary market

→

- Blockchain needed

Skoltech

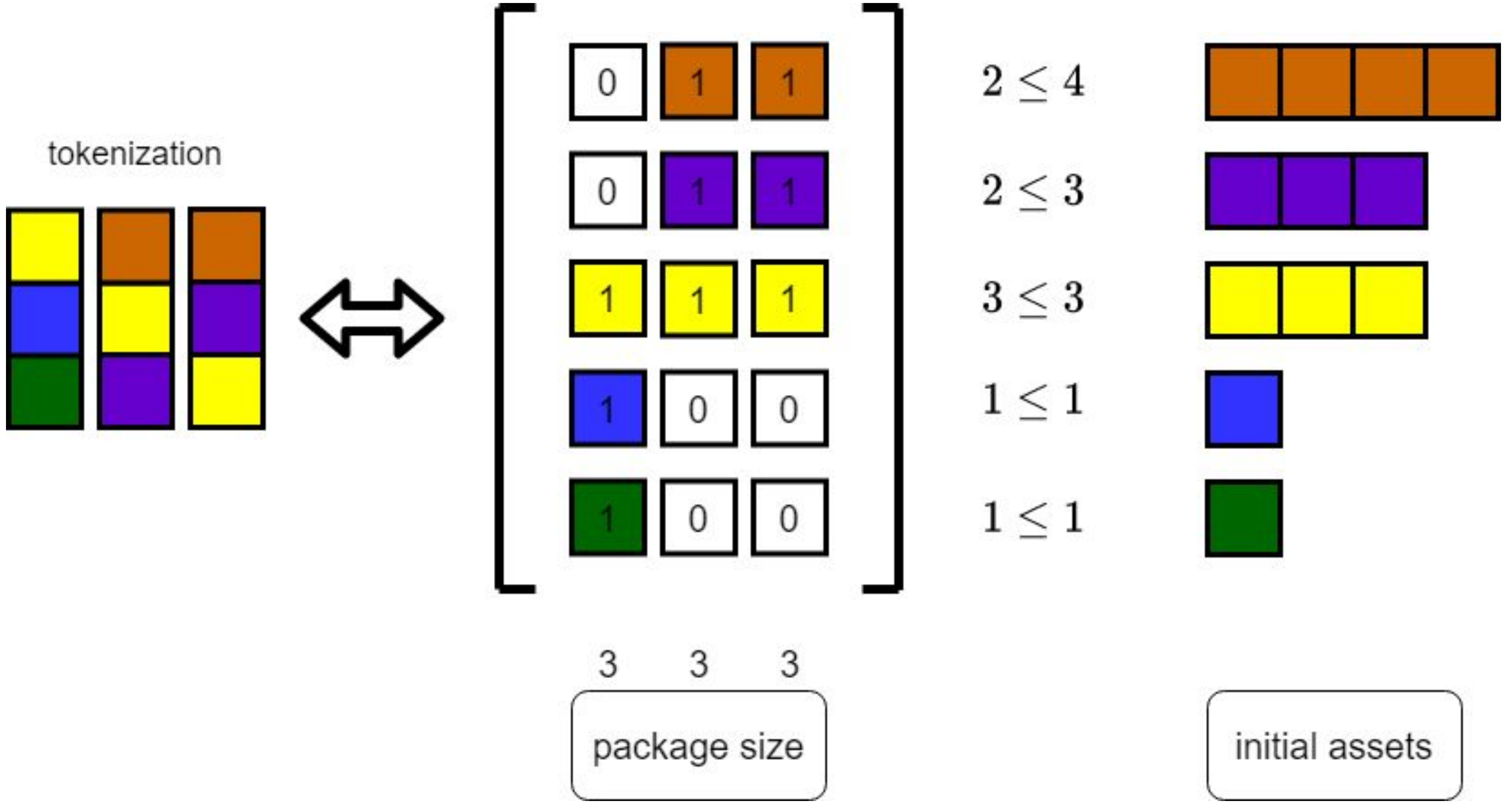
Portfolio Sold-Out: Problem



How to construct as many packages as possible for a given token set?

Skoltech

Portfolio Sold-Out: Objective



Find a matrix with the maximum possible number of columns under given restriction.

Skoltech

Portfolio Sold-Out: Notations

For any positive integer K we denote $\overline{K} = \{1, \dots, K\}$.

The portfolio is characterized by the number of assets $N \geq 1$ and a set of random variables $A_1 \cdot \xi_1, \dots, A_N \cdot \xi_N$, where

- $A_1 \leq \cdots \leq A_N$ are deterministic positive numbers equal to the expected returns of each asset
- random variables $\xi_1, \dots, \xi_N$, describing the uncertainty per unit of return
  - $\mathbf{E}\,\xi_n = 1, \quad n \in \overline{N}$
  - covariance $\mathrm{cov}(\xi_i, \xi_j) = K_{ij}, \quad i, j \in \overline{N}$
  - covariance matrix $\mathbf{K} = (K_{ij})_{i,j \in \overline{N}}$

A **package** composed of the portfolio $(\vec{A}, \vec{\xi})$ is a vector $\vec{c} \in R^N$ such that

- $0 \leq \vec{c} \leq \vec{A}$ and
- $\mathbf{E}\,\vec{c}\vec{\xi}^T = 1.$

Skoltech

Portfolio Sold-Out: Problem

The variance of the package $\vec{c}$ equals

$$V(\vec{c}) = \operatorname{Var} \vec{c}^{T}\vec{\xi} = \vec{c}^{T}\,\mathbf{K}\,\vec{c}.$$

A set of M packages $\mathbf{C}_M = (\vec{c}_1 \mid \ldots \mid \vec{c}_M) \in R^{N\times M}$ is the tokenization of the portfolio $\left(\vec{A}, \vec{\xi}\right)$ if $\sum_{m=1}^{M} \vec{c}_m \leq \vec{A}$.

The variance V of tokenization $\mathbf{C}_M$ is the maximum variance of its packages: $V(\mathbf{C}_M) = \max_{m \in \overline{M}} V(\vec{c}_m)$.

Problem. For a given portfolio $\left(\vec{A}, \vec{\xi}\right)$ and a variance threshold $\sigma^2 > 0$, the portfolio sold-out problem is

$$M \to \max_{M, \mathbf{C}_M:\, V(\mathbf{C}_M) \leq \sigma^2}$$

Skoltech

Portfolio Sold-Out: Special cases

By assets:

- Homogeneous: $\mathbf{K} = \sigma_0^2 \, \mathbf{I}_N$
- Independent : $K_{ij} = 0$ for $i \neq j$
- General : any $\mathbf{K}$ is allowed.

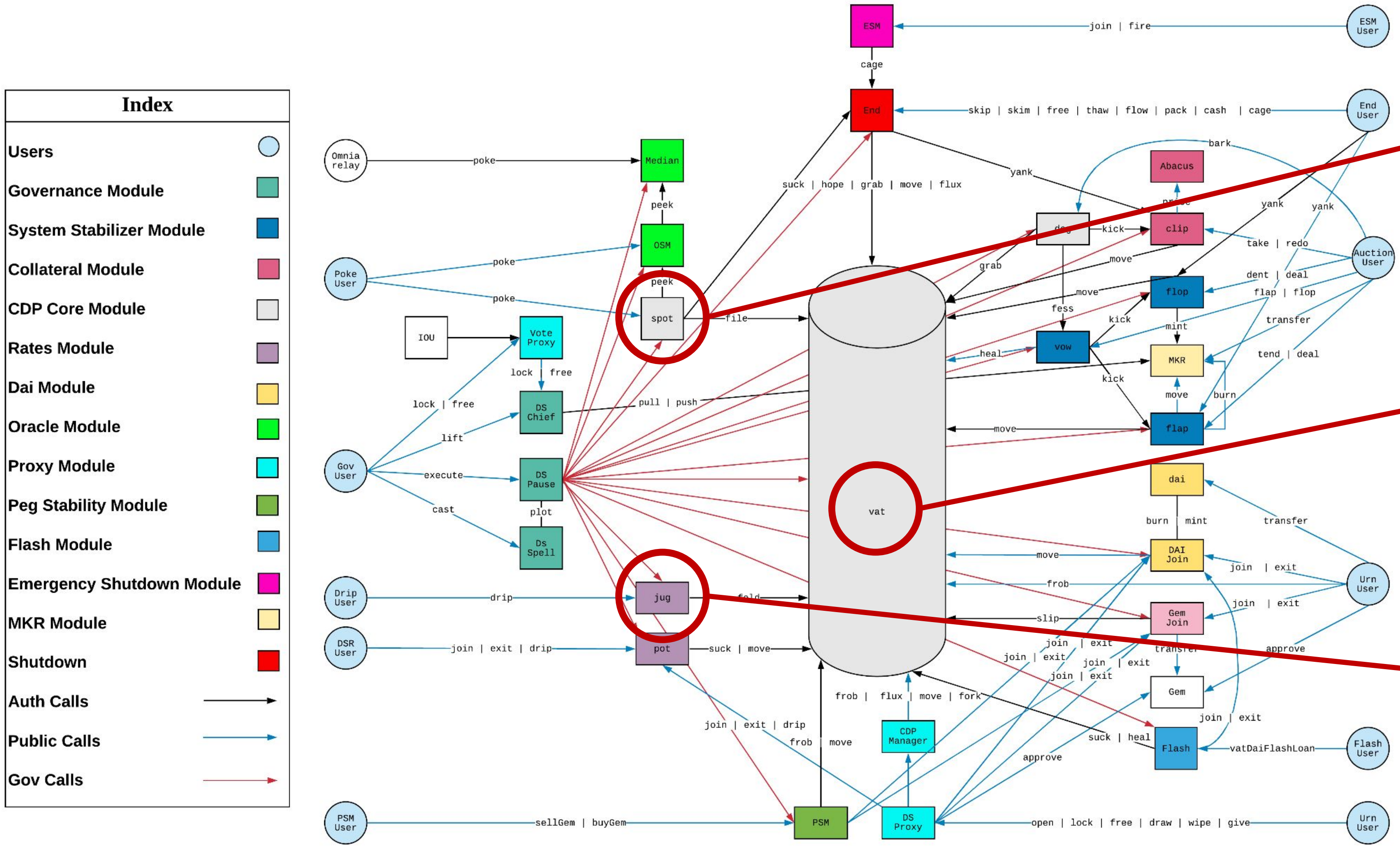
By packets:

- Discrete : $\mathbf{C}_M$ is Boolean matrix
- Continuous: $\mathbf{C}_M$ is real matrix.

	Discrete	Continuous
Homogeneous	optimal solution	optimal solution
Independent	to be solved	to be solved
General	to be solved	numerical solution

Skoltech

MakerDao



MakerDao Structure

ilk.pip
holds the current price of a given ilk (collateral)

Vat
stores Vaults and tracks all the associated Dai and Collateral balances

Jug
accumulate stability fees for a particular collateral type

Skoltech

Objectives

01 Design and develop optimization method to obtain the optimal portfolio sold-out

02 Apply optimization method to the MakerDao dataset

Skoltech

Methods

- Reduce portfolio sold-out to Second-Order Cone Programming (SOCP)
- Prepare MakerDao dataset for tokenization
- Apply SOCP solver to portfolio sold-out problem

Skoltech

Results: General Continuous

Constraints are convex!

Lemma. If the matrix $\mathbf{C}_{\mathrm{M}} \in \mathbf{R}^{\mathrm{N}\times\mathrm{M}}$ is the optimal solution to the general continuous problem, then the matrix $\overline{\mathbf{C}}_{\mathrm{M}} = (\vec{\bar{c}}| \dots |\vec{\bar{c}}$,where $\vec{\bar{c}} = \frac{1}{\mathrm{M}}\sum_{\mathrm{m}\in\overline{\mathrm{M}}} \vec{c}_{\mathrm{m}}$.

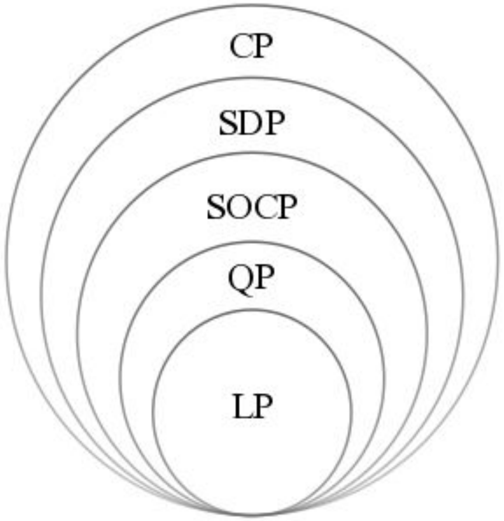
Let $\mathrm{M} \in \mathbf{R}$. Denote $\vec{a} = \mathrm{M} \cdot \vec{c}$.

Theorem. The general continuous problem is equivalent to

$$\|\vec{a}\|_1 \to \max_{\vec{a}:\begin{cases}\vec{a}^{\mathrm{T}}\mathbf{K}\,\vec{a} \le \sigma^2\|\vec{a}\|_1 \\ \vec{0} \le \vec{a} \le \vec{A}\end{cases}}$$

Theorem 2. Portfolio sold-out problem is polynomial reducible to **second order cone programming.**

$\to$ numerical solution.



LP: linear program,
QP: quadratic program,
SOCP second-order cone program,
SDP: semidefinite program,
CP: cone program.

Results: MakerDao Data

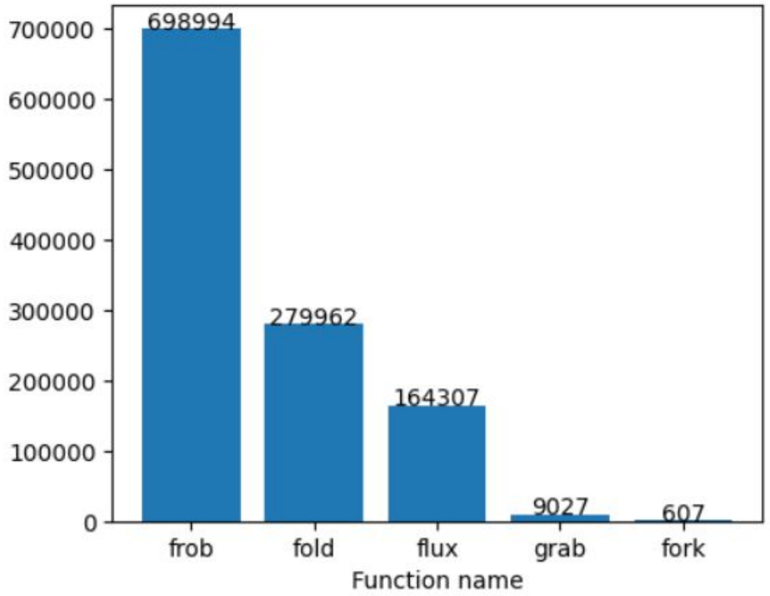


Figure 1: Total number of each function name

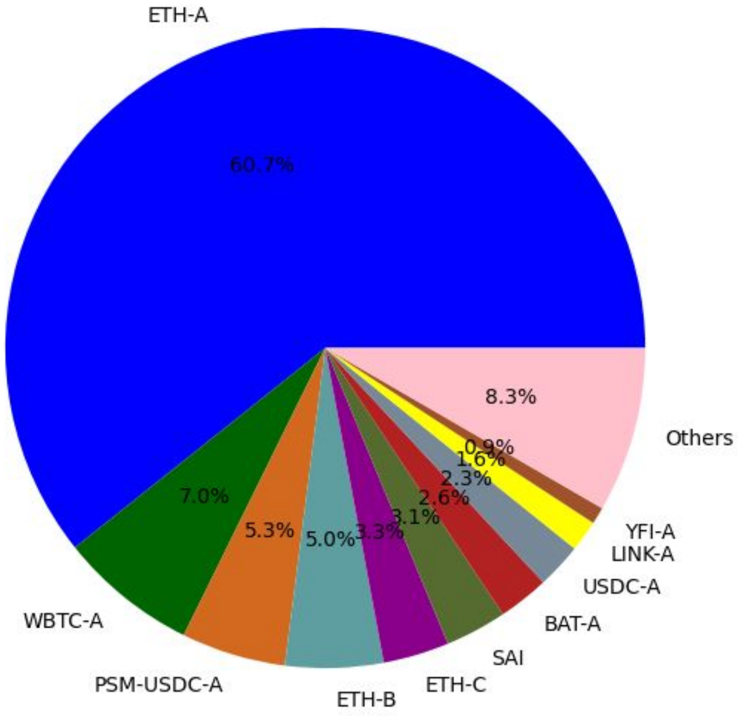


Figure 2: Proportion of each collateral type

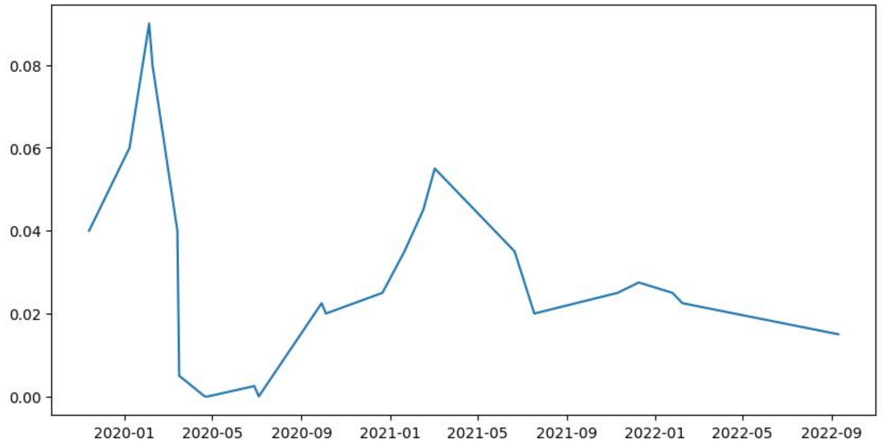


Figure 3: Stability fee rate per year

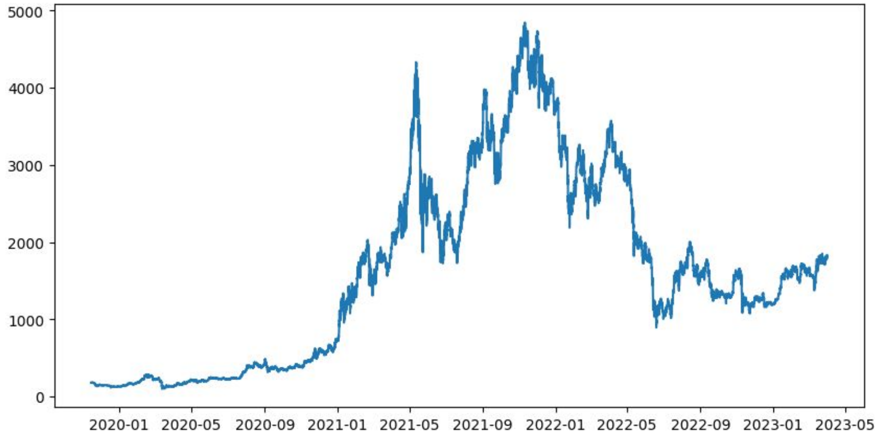


Figure 4: ETH/DAI rate

Skoltech

Results: MakerDao for Portfolio Sold-Out

Daily data

$\vec{A}$ is borrowers with debt amounts.
K is a covariance for their defaults.

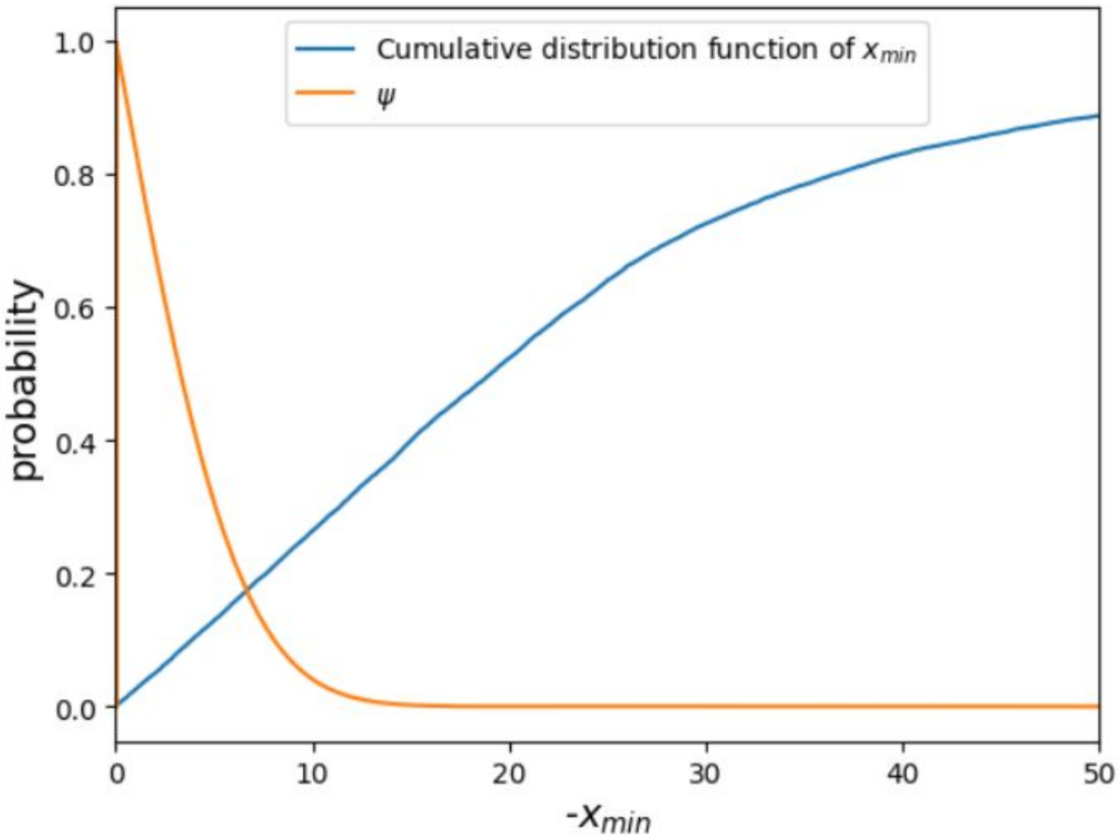


Figure 5: Probability and level of default

Skoltech

Baseline: MakerDAO dataset

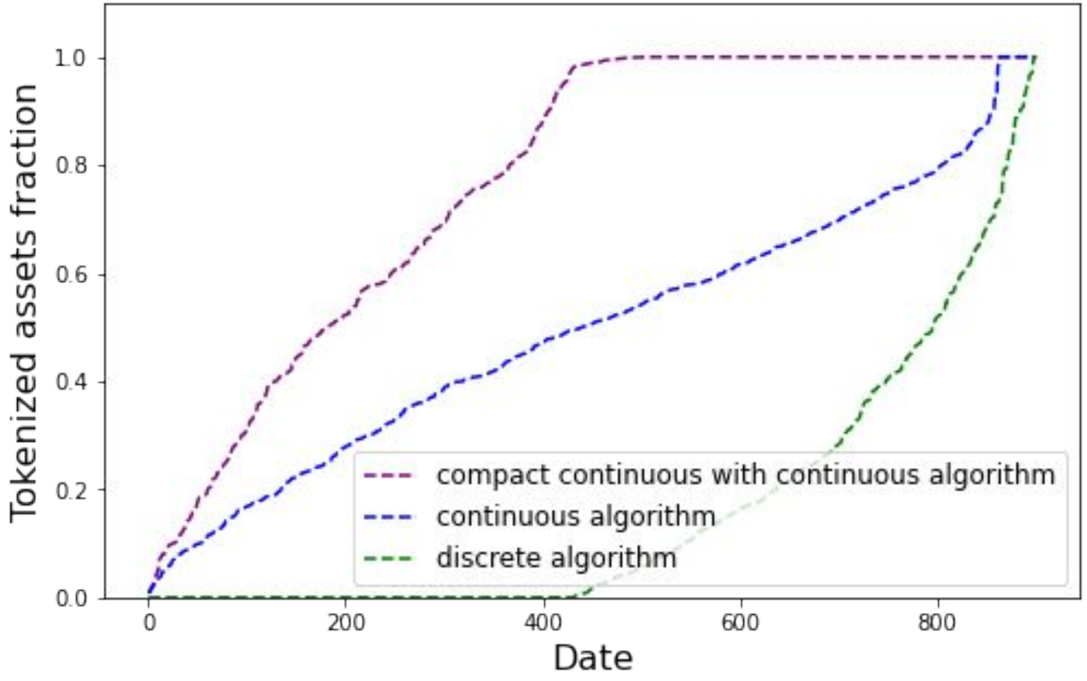


Figure 6: Tokenized into commodity asset fractions

The proportion of tokenized asset into the packages defines as

$$\text{tokenized fraction} = \frac{\text{the total amount of tokenized asset}}{\text{the total amount of initial asset}}$$

Table: The tokenized fractions of different models

Method	Tokenized fraction
Compact continuous with continuous algorithm *	76.58 %
Special case continuous algorithm **	40.01 %
Special case discrete algorithm **	13.65 %

* Solved by the metaheuristics Slime Mould Algorithm (SMA)

** Mis-Model assumption: independent with the same variance.

Skoltech

Conclusions

- Portfolio sold-out problem is reduced to SOCP, which allows optimal numerical solution.
- MakerDao DeFi protocol provides real loan dataset, which is impossible for classic banking system.
- SOCP solver **will** be applied to the MakerDao dataset

Skoltech

Outlook: Planning to Achieve

To complete optimization part with solving SOCP.

Skoltech